Econ 706 Prelim, August 2013

The three questions are weighted equally. Good luck!

Consider Bayesian analysis of a simple Markovian dynamic model:

$$y_t = \phi y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim iid \; N(0, \sigma^2)$$

$$|\phi| < 1.$$

(1) Derive the natural conjugate prior (NCP) for $\phi|\sigma$. What is the posterior mean of $\phi|\sigma$ and how is it related to the prior mean, the MLE, the prior precision, and the MLE precision? What are the key benefits of using NCPs? What are the key costs? Maintaining use of NCPs, how might you attempt to represent prior ignorance regarding $\phi|\sigma$? What complications arise? If you could use non-NCPs, how might you attempt to represent prior ignorance regarding $\phi|\sigma$? What complications arise? Why might you be interested in posterior inference not on $\phi|\sigma$, but rather on ϕ?

From this point onward, use the NCP for $\phi|\sigma$ (and, when relevant, for $\sigma|\phi$) unless explicitly instructed otherwise.

(2) Design a Markov chain Monte Carlo algorithm (in this case, a Carter-Kohn multi-move Gibbs sampler) to sample from the joint posterior distribution of (ϕ, σ). *Be precise and complete.* How would you assess convergence to steady state of your Gibbs sampler? How would you use the Gibbs draws from the joint posterior to sample from the marginal posterior of ϕ?

(3) Is the sequence of Gibbs draws from the marginal posterior of ϕ likely to be serially uncorrelated? If so, why? If not, what is the likely form of the serial correlation? If you know the spectral density at frequency zero of the Gibbs sequence of ϕ, $f_\phi(0)$, how would you use it to assess the accuracy of your estimate of the posterior mean of ϕ based on the Gibbs sequence? If instead $f_\phi(0)$ is unknown, how would you estimate it consistently using an autoregressive model-based estimator? A lag-window estimator with triangular weighting? In each case, what determines the bandwidth, what conditions must it satisfy to achieve consistency, and how might you select it in practice?