Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 4, 2012

Instructions

This exam has 5 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise. Write clearly so that you might get partial credit.

Good luck!

1. (20 pts) A strictly increasing utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$ gives rise to a demand function $\mathbf{x}(\mathbf{p}, y) = (x_1(\mathbf{p}, y), \ldots, x_n(\mathbf{p}, y))$ defined on $\mathbb{R}^{n+1}_{++}$. Assume it and any other functions you use to answer this question are twice continuously differentiable.

   (a) (8 pts) State all the properties this demand function must necessarily satisfy.

   (b) (12 pts) For each property you listed in (a), sketch a proof of why it must be satisfied.

2. (20 pts)

   (a) (5 pts) Define carefully the core of an exchange economy with a finite number of agents.

   (b) (5 pts) State assumptions under which a Walrasian equilibrium allocation is contained in the core, and prove the result.

   (c) (10 pts) Give an example of an economy in which there is a Walrasian equilibrium allocation that is *not* in the core.

3. (20 pts) Consider a pure exchange economy with two agents, 1 and 2, and two goods, x_1 and x_2. The utility function of agent $i = 1, 2$ is

$$u^i(x_1^i, x_2^i) = \min(x_1^i, x_2^i).$$

   The endowments of the agents are $e^1 = (2, 0)$ and $e^2 = (0, 1)$.

   (a) (5 pts) Describe carefully the set of Pareto efficient allocations.

   (b) (5 pts) Suppose there is a competitive market in which the agents trade to a Walrasian equilibrium. Suppose that through some misfortune, 3/4 of agent 1's endowment is destroyed before trading occurs, leaving him with only $\hat{e}^1 = (1/2, 0)$. Determine how this affects the equilibrium utility of each agent as compared to the equilibrium utility he would have had if the endowment had not been destroyed.

   (c) (10 pts) Suppose now that instead of accidental endowment destruction, each agent can destroy a portion of his endowment before the market opens. When the market then opens, the outcome will be the Walrasian equilibrium for the economy in which the agents have the endowment amounts they did not destroy. Thinking of this as a normal form game in which each agent's strategy is an amount of his endowment to destroy, what can you say about its Nash equilibria?

4. (20 pts) Two bidders for a painting have private values for it that are independently distributed uniformly on $[0,1]$. If bidder i has value v_i, wins with probability q_i, and pays a price p_i, her payoff is $q_i v_i - p_i$. Denote a revelation mechanism as a four-tuple, $\langle q_1, q_2, p_1, p_2 \rangle$, of functions of (v_1, v_2) satisfying $q_i(\cdot) \geq 0$ and $0 \leq q_1(\cdot) + q_2(\cdot) \leq 1$.

   For each of the following symmetric pairs $\langle q_1, q_2 \rangle$ of probability functions, does a pair of price functions $\langle p_1, p_2 \rangle$ exist such that $\langle q_1, q_2, p_1, p_2 \rangle$ is a dominant strategy incentive compatible (DIC) mechanism? A Bayesian incentive compatible (BIC) mechanism? Indicate your reasoning. Lastly, when you believe a pair of price functions exists that yields a DIC or BIC mechanism, find a price pair such that the resulting DIC or BIC mechanism gives zero interim utility to the lowest type of each bidder.

   (a) $q_i^a(v_1, v_2) = \frac{1}{4} v_i$

   (b) $q_i^b(v_1, v_2) = \begin{cases} 1 & \text{if } \frac{3}{4} v_i \leq v_j < v_i \\ \frac{1}{2} & \text{if } v_i = v_j \\ 0 & \text{otherwise} \end{cases}$

   (c) $q_i^c(v_1, v_2) = \begin{cases} 1 & \text{if } 2v_i - 1 \leq v_j < v_i \\ \frac{1}{2} & \text{if } v_i = v_j \\ 0 & \text{otherwise} \end{cases}$

5. (20 pts) Consider a principal-agent model in which the agent has three possible effort levels, $\{e_L, e_M, e_H\}$, and two possible outputs, $\underline{\pi} = 4$ and $\bar{\pi} = 5\frac{1}{9}$. If the realized output is π, the agent's effort is e, and the principal pays him a wage w, the principal's utility is $\pi - w$ and the agent's is

$$u(w, e) = \sqrt{w} - g(e).$$

   The lowest wage the agent can be paid is 0, and his reservation utility is also 0. The probability and disutility-of-effort functions are given by

$$\Pr(\bar{\pi}|e) = \begin{cases} 0 & \text{if } e = e_L \\ \frac{1}{2} & \text{if } e = e_M \\ 1 & \text{if } e = e_H \end{cases} \qquad g(e) = \begin{cases} 0 & \text{if } e = e_L \\ k & \text{if } e = e_M \\ 1 & \text{if } e = e_H \end{cases},$$

   where $k \in (0,1)$. [For future reference, note that $\bar{\pi} - \underline{\pi} = \frac{10}{9}$ and $\frac{1}{2}(\bar{\pi} + \underline{\pi}) = 4\frac{5}{9}$.]

   (a) (5 pts) Find the first-best outcomes, $(\underline{w}^{FB}, \bar{w}^{FB}, e^{FB})$, for each $k \in (0,1)$.

   (b) (5 pts) Find the second-best outcomes, $(\underline{w}^{SB}, \bar{w}^{SB}, e^{SB})$, for each $k \in (\frac{1}{2}, 1)$.

   (c) (10 pts) For $k \in (0, \frac{1}{2})$, take the first step towards finding the second-best outcome by determining the principal's optimal contract for inducing e_M.