Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 2013

Instructions

This exam has 5 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (20 pts) Let $Y \subseteq \mathbb{R}^N$ denote a production set and $\pi : \mathbb{R}^N_+ \to \mathbb{R}$ a profit function.

   (a) (5 pts) Suppose you are given the set Y. Show how the associated profit function π can be derived.

   (b) (15 pts) Suppose instead that you are given the function π, and told that it is the profit function associated with a closed and convex Y that satisfies free disposal. Show how Y can be derived, and prove that your method works.

2. (20 pts) Consider a two-good exchange economy, where the goods are right shoes and left shoes. Each agent is endowed with a single shoe, either right or left. Shoes are indivisible and must be consumed in whole units, so the consumption sets are $\{(x_L, x_R) | \, x_l$ and x_R are nonnegative integers$\}$. Each agent has Leontief preferences represented by the utility function $u^i(x^i_L, x^i_R) = \min\{x^i_L, x^i_R\}$. In other words, they value only pairs of shoes and extra unmatched shoes are of no use to an agent; more pairs of shoes are strictly preferred to fewer pairs.

   (a) If there are two agents in the economy with $e^i = (1, 0)$ and $e^2 = (0, 1)$,what are the Walrasian equilibria? (Normalize prices $p = (p_L, p_R)$ so that $p_L + p_R = 1$.) (For this entire problem allow agents to demand parts of a shoe, but require an equilibrium allocation to be in whole units.)

   (b) For the economy in (a), what are the core allocations under a definition of the core that requires a blocking coalition to make all its members strictly better off?

   (c) For the economy in (a), what are the core allocations under a definition of the core that requires a blocking coalition to make some of its members strictly better off and the rest weakly better off?

   (d) Consider an economy with $2m+1$ agents with preferences as above, where m is a positive integer. Suppose $m+1$ agents have endowment $e = (1, 0)$, and m agents have endowment $e = (0, 1)$. What are the Walrasian equilibria?

   (e) Consider an economy with three agents with preferences as above. Suppose two agents have endowment $e = (1, 0)$, and the third agent has endowment $e = (0, 3)$. What are the Walrasian equilibria?

   (f) What is the core of the economy in (e), under the definition of the core that requires a blocking coalition to make all its members strictly better off?

   (g) What is the core of the economy in (e), under the definition of the core that requires a blocking coalition to make some of its members strictly better off and the rest weakly better off?

3. (20 pts) Consider a standard two-period economy, dated $t = 0$ and $t = 1$. Agents consume in both periods. There are three states of nature in the second period. There is a single consumption good, and it is used as a numeraire; hence, the spot price of a unit of consumption at either date is 1. At date 0 agents can trade in two primary securities. Security 1 has the second-period payoff vector $r_1 = (1, 0, 0)$, and security 2 has the second-period payoff vector $r_2 = (1, 2, 3)$. The prices of these securities at date 0 are $q_1 = 0.4$ and $q_2 = 1.5$.

   In addition, there are two *derivative* securities, denoted 3 and 4. Security 3 is a call option on security 2 with a strike price of 1, and a price $q_3 = 0.7$. Security 4 is a call option on security 2 with a strike price of 2, and a price $q_4 = 0.3$.

   All quantities are defined in units of the consumption good.

   (a) Show that this system is arbitrage free.

   (b) Assume these prices arise in an incomplete markets equilibrium with the specified four securities.

      i. What is the date 0 price of a contingent claim to deliver one unit of consumption at date 1 in state 3?

      ii. What would be the market price of a put option on asset 4 with a strike price of 1?

      iii. What would be the risk-free interest rate on a loan taken at date $t = 0$?

4. (20 pts) A seller has 1 unit of an indivisible good. There are 2 buyers. Each buyer's value is drawn uniformly and independently from the unit interval, $V = [0, 1]$.

   (a) (6 pts) The seller is considering a direct revelation mechanism with an allocation rule such that the buyer with the highest value wins the good, as long as this value is larger than $\frac{1}{2}$. Suppose the lowest possible type of the buyer pays 0. Give an analytical formula for the interim allocation rule and the interim payment rule. What is the expected revenue of the seller?

   (b) (6 pts) Use the Fundamental IC lemma to describe a Bayes-Nash equilibrium in a first-price sealed-bid auction with a reserve price of $\frac{1}{2}$. (Remember that in a first-price auction, all buyers simultaneously submit bids. The highest bid above the reserve price (if any) wins; the winner pays his bid; the losers pay nothing. If no bid is larger than or equal to the reserve price, then no bidder makes a payment and the seller retains the good.)

   (c) (8 pts) Let us return to studying the direct revelation mechanism. Recall that we implicitly assumed that the outside option of all buyers was 0 in our analysis. Suppose instead that the outside option of each buyer is 0.2, which means that a buyer will only participate in the auction if his interim expected surplus is 0.2 or larger.

      i. (3 pts) Taking the interim surplus from part (b) as given, what would be the range of values of buyers who choose to participate in the auction?

      ii. (5 pts) In equilibrium, what is the lowest value of a buyer who participates in this mechanism? (Note that in choosing whether to participate, a buyer correctly anticipates the participation decision of other buyers.)

5. (20 pts) Consider the following hidden action model with three possible actions, e_1, e_2, and e_3. There are two possible profit outcomes, a high profit $H = 10$ and a low profit $L = 0$. The probabilities of high profit given each action are

$$P(H|e_1) = \frac{2}{3}, \quad P(H|e_2) = \frac{1}{2}, \quad P(H|e_3) = \frac{1}{3}.$$

The agent's costs for taking each action are

$$g(e_1) = \frac{5}{3}, \quad g(e_2) = \frac{8}{5}, \quad g(e_3) = \frac{4}{3}.$$

Finally, the agent has a utility function $v(w) = \sqrt{w}$ for money, and a reservation utility of 0. The principal is risk neutral.

(a) (5 pts) What is the optimal contract when the agent's action is observable?

(b) (5 pts) Now suppose the action is unobservable. Is e_2 implementable by some contract? If yes, provide such a contract. If not, identify the range of values of $g(e_2)$ such that e_2 is implementable by some contract.

(c) (5 pts) What is the optimal contract when the agent's action is unobservable?

(d) (5 pts) What is the optimal contract when the agent's action is unobservable, but the agent is risk neutral?