Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 13, 2012

Instructions

This exam has 5 questions and a total of 100 points.

You have **two and one-half hours** to complete it.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (20 pts) A competitive firm uses two inputs, x_1 and x_2, to make one output, y. Consider the following possible cost function, where the exponents a and b are positive constants:

$$c(y, \mathbf{w}) = \left(\tfrac{1}{2}w_1^a + \tfrac{1}{2}w_2^b + \sqrt{w_1 w_2}\right) y.$$

   (a) (5 pts) For what values of a and b is c truly a cost function? Prove your answer.

   For the remaining questions, assume a and b satisfy the restrictions you just identified.

   (b) (5 pts) Find the firm's conditional factor demand functions, $\hat{x}_1(y, \mathbf{w})$ and $\hat{x}_2(y, \mathbf{w})$.

   (c) (5 pts) Find the firm's supply function, $y^*(p, \mathbf{w})$.

   (d) (5 pts) Find a production function f for which c is the corresponding cost function.

2. (20 pts) Consider a general equilibrium exchange economy in which all agents have strictly increasing and continuous utility functions.

   (a) (5 pts) Add conditions, if necessary, to the "if" part of the following proposition to make it correct: "If x is a competitive equilibrium allocation, then all the agents have the same marginal rates of substitution at x."

   (b) (5 pts) Add conditions, if necessary, to the "if" part of the following proposition to make it correct: "If all agents' marginal rates of substitution are equal at x, then x is Pareto efficient."

   (c) (5 pts) Provide an example of an economy that does **not** satisfy all the assumptions of your answer to (b), but that nonetheless has a Pareto optimal allocation at which all the agents' marginal rates of substitution are equal. A carefully drawn and labeled figure will suffice.

   (d) (5 pts) Provide an example of an economy that has an allocation at which all the agents' marginal rates of substitution are equal, but which is not Pareto optimal. A carefully drawn and labeled figure will suffice.

3. (20 pts) Consider a two period problem in which there is uncertainty in the second period. There is a single nonstorable consumption good. There are three agents: one entrepreneur and two consumers. The entrepreneur's utility function is c_1, and the two consumers each have the utility function $c_1 + \mathbb{E}[u(c)]$, where c_1 is the agent's first-period consumption and

$$u(c) = 2\ln(c)$$

   is each consumer's vNM utility function for second-period consumption. There are two equally likely states of the world in the second period. The entrepreneur owns a riskless firm (asset), $z = (1, 1)$, that pays 1 in each of the two states of the world in the second period. The entrepreneur will sell shares in the firm in the first period, that is, before the realization of the state.

   Each consumer has endowment 2 in the first period. Consumer 1's second-period (state-dependent) endowment is $e^1 = (1, 0)$ (i.e., 1 unit in the first state and 0 in the second), and consumer 2's is $e^2 = (0, 1)$.

   (a) (9 pts) Suppose the entrepreneur sells shares in the firm, i.e., asset z. Find the competitive equilibrium price, q, of these shares. (Normalize prices so that the price of consumption in the first period is 1.)

(b) (9 pts) Suppose instead that the entrepreneur sells shares in the asset by breaking it into two separate assets that pay $(1, 0)$ and $(0, 1)$, respectively. Find the competitive equilibrium prices, q_1 and q_2, of these assets. (Continue to set the price of consumption in the first period to 1.)

(c) (2 pts) Will the entrepreneur prefer to sell the asset as a single unit or break it into two separate parts?

4. (20 pts) Consider a three-person economy with one public good (x) and one private good (y). An allocation is feasible only if $x \geq 0$ and each $y_i \geq 0$. Each consumer is endowed with $e > 0$ units of private good. The private good cost of producing an amount x of public good is

$$C(x) = \tfrac{1}{16}x^2.$$

The utility function of consumer i is $\theta_i x + y_i$. The type of consumer 3 is commonly known to be $\theta_3 = 0$. The types of consumers 1 and 2 are independently distributed, with

$$\Pr(\theta_i = 0) = \Pr(\theta_i = 1) = \tfrac{1}{2}.$$

Before consumers 1 and 2 learn their types, consumer 3 can sell them a symmetric pivot mechanism for a price P that they can use after they learn their types. Consumers 1 and 2 each pay $\frac{1}{2}P$, and P may be negative or positive. Consumer 3's ultimate income will be $P + e$ plus her profit from operating the mechanism, i.e., the difference between the sum of the taxes that consumers 1 and 2 pay, and the cost of producing the public good.

Refer to a price P as *viable* if (*i*) no consumer's income (private good consumption) is ever negative after P and the pivot taxes are paid and the public good is produced, and (*ii*) each consumer's ex ante utility is not less than e. (Assume the mechanism's dominant strategy equilibrium is the one played.)

Find the interval $[\underline{P}, \bar{P}]$ of viable prices for when (a) $e = 10$ and (b) $e = 8$.

5. (20 pts) Consider a principal-agent model in which the agent has two possible effort levels, e_L and e_H, and two possible outputs, $\underline{\pi} = 0$ and $\bar{\pi} = 90$. If the realized output is π, the agent's effort is e, and the principal pays him a wage w, the principal's utility is $\pi - w$ and the agent's is $u(w, e) = \sqrt{w} - g(e)$. The lowest wage the agent can be paid is 0, and his reservation utility is also 0. The probability and disutility-of-effort functions are given by

$$\Pr(\bar{\pi}|e) = \begin{cases} 1/3 & \text{if } e = e_L \\ 2/3 & \text{if } e = e_H \end{cases} \qquad g(e) = \begin{cases} 0 & \text{if } e = e_L \\ 2 & \text{if } e = e_H \end{cases}.$$

In addition to π, a signal s is generated by the agent's effort. Its set of possible realizations is $\{\varnothing, L, H\}$. For $i, j \in \{L, H\}$, the probability distribution of s conditional on e_i is given by

$$\Pr(s = \varnothing | e = e_i) = 1 - q,$$

$$\Pr(s = j | e = e_i) = \begin{cases} q & \text{for } j = i \\ 0 & \text{for } j \neq i \end{cases},$$

where $q \in (0, 1)$. (Thus, q is the probability that the agent's effort is observed.) In the first-best world, e, π, and s are all contractible. In the second-best world, only π and s are contractible. The principal makes an ultimatum offer of a contract.

(a) (5 pts) Describe the first-best outcome.

(b) (15 pts) Describe, as precisely as possible, the nature of a second-best optimal contract, as a function of the parameter q.