Imagine the problem of an entrepreneur who runs a firm. The entrepreneur has one unit of time, which he must split between working, h, and doing research, r. He produces output in a period according to the linear production function

$$o = zh,$$

where z is a total factor productivity (TFP). By spending time on research in the current period, r, the entrepreneur can improve the likelihood of drawing a good value for tomorrow's technology shock. Next period's TFP is denoted by z'. The research process for improving next period's technology shock can be described as follows: If the entrepreneur spends r units of time on R&D then he can draw a new technological shock, $z' \geq z$, with probability $\pi(r)$. The new value of the technological shock is distributed in line with the conditional *cumulative* distribution function, $H(z'|z)$, where

$$H(z'|z) = 1 - (\frac{z}{z'})^{\psi}, \text{ (Pareto) with } \psi > 1 \text{ and for } z' \geq z.$$

With probability $1 - \pi(r)$ the technological shock will retain its old value, z. Let $\pi(r)$ be an increasing, strictly concave function with $\pi(0) = 0$ and $\pi(1) < 1$. (Your answer will be graded *both* upon the economic intuition you display and the technical ability that you demonstrate.)

1. Formulate the entrepreneur's dynamic programming problem.

2. What form must the value function take? What form will the solution for r have? How fast can the entrepreneur expect his firm to grow at? (*Hint*: The form of the value function and solution for r will be partially explicit and partially implicit.)

3. Suppose the government subsidizes R&D at the rate s. The government raises the funds for the subsidy from elsewhere in the economy. How does this affect expect growth? (*Hint*: How does s affect the value function? How should you figure this out? Use your answer here in conjunction with your results in part 1 and 2.)