

Consider the following model of production in the world economy. Output,  $o$ , is given by

$$o = [(xu)^\sigma + (zs)^\sigma]^{1/\sigma}, \text{ with } \sigma \leq 1,$$

where  $u$  is the amount of unskilled labor used in production and  $s$  denotes the amount of skilled labor. Here  $x$  and  $z$  are shift factors denoting the productivity of unskilled and skilled labor in production. The pair  $(x, z)$  indexes a technology. A firm in a country is free to pick the technology,  $(x, z)$ , that it wants, subject to the following constraint

$$x^\omega + z^\omega = \beta, \text{ with } \omega > \sigma\omega + \sigma \text{ or equivalently } \sigma/(1 - \sigma) < \omega.$$

The questions below will be graded upon both the economic intuition and technical ability that you demonstrate.

1. Let  $w$  be the unskilled wage rate in a country and  $v$  represent the skilled wage rate. How will the firm choose its inputs and technology? Explain the importance of  $\sigma$ .
2. Let  $\bar{U}$  be the supply of unskilled labor in country and  $\bar{S}$  be its supply of skilled labor. How will wages and technology be determined? Explain the importance of  $\sigma$ .
3. What does the above model say about cross-country production in the world economy? What empirical predictions would obtain from such a model? Outline how you might test it? (Assume that  $\sigma > 0$ , which is what empirical researchers have found.)
4. What role does the restriction  $\omega > \sigma\omega + \sigma$  or play? *Hint:* For a given value of  $x^\omega$  the cost function is

$$[w^{\sigma/(\sigma-1)}(x^\omega)^{\sigma/[\omega(1-\sigma)]} + v^{\sigma/(\sigma-1)}(\beta - x^\omega)^{\sigma/[\omega(1-\sigma)]}]^{(\sigma-1)/\sigma} o.$$

Think about corner solutions.