Improving GDP Measurement: A Measurement-Error Perspective

Boragan Aruoba, University of Maryland
Francis X. Diebold, University of Pennsylvania
Jeremy Nalewaik, Federal Reserve Board
Frank Schorfheide, University of Pennsylvania
Dongho Song, Boston College

Lawrence R. Klein Legacy Colloquium

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Two U.S. *GDP* Estimates: GDP_E and GDP_I

Both are available for U.S.

– GDP_E used routinely

– GDP_I may also be valuable

We provide a superior estimate.

Literatures

GDP_E vs. GDP_I
(Nalewaik 2010, ...)

Dynamic factor models and optimal signal extraction
(..., Fleischman and Roberts 2011, ...)

Data revision properties
(..., Faust-Rogers-Wright 2005, ...)

Forecast combination
(..., Timmermann 2006, ...)

Warm-up: The Forecast-Error Approach to Combining (Pooling Noisy *GDP* "Forecasts")

$$GDP_C = \lambda GDP_E + (1-\lambda) GDP_I$$

$$\lambda^* = \frac{1-\phi\rho}{1+\phi^2-2\phi\rho}$$

where $\phi = \sigma_E^2/\sigma_I^2$ and $\rho = corr(e_E, e_I)$

Problem: ϕ and ρ are unknown and can't be estimated.

Calibration is one way forward:

Aruoba, Diebold, Nalewaik, Schorfheide and Song (2012), "Improving GDP Measurement: A Forecast Combination Perspective," in Chen and Swanson (eds.), *Causality, Prediction, and Specification Analysis: Essays in Honor of Halbert L. White Jr.*, Springer, 1-26.

Optimal Combining Weights are Far From 0 and 1

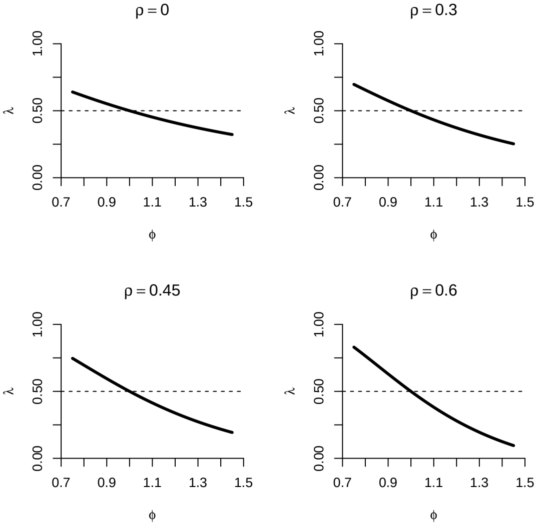

Figure: λ vs. ϕ for Various ρ Values. Reference at $\lambda = 0.50$.

Gains From Combining Are Huge

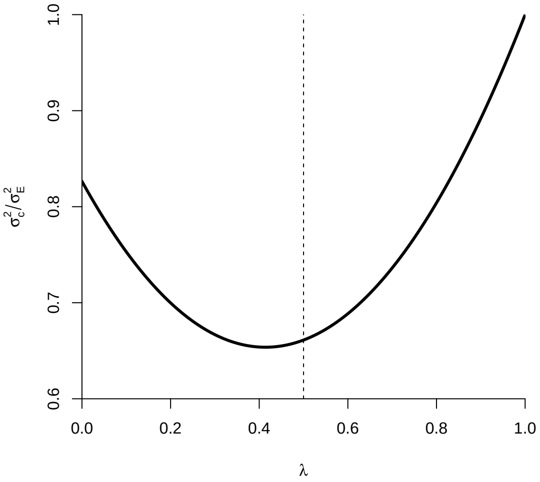

Figure: σ_C^2/σ_E^2 for $\lambda \in [0, 1]$. We set $\phi = 1.10$ and $\rho = 0.45$. Reference at $\lambda = 0.50$.

The Measurement-Error Approach to Combining (Pooling and Smoothing Noisy *GDP* "Measurements")

Two-Equation Model:

$$\begin{bmatrix} GDP_{Et} \\ GDP_{It} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} GDP_t + \begin{bmatrix} \epsilon_{Et} \\ \epsilon_{It} \end{bmatrix}$$

$$GDP_t = \mu(1-\rho) + \rho GDP_{t-1} + \epsilon_{Gt},$$

$$(\epsilon_{Gt}, \epsilon_{Et}, \epsilon_{It})' \sim iid\ N(\underline{0}, \Sigma)$$

$$0 \leq \rho < 1$$

– Both GDP_E and GDP_I are noisy measures of latent true *GDP*

– Optimal smoothing for *GDP* (over space *and* time)

– Estimation rather than calibration

– Interesting hypotheses regarding the form of Σ

Hypotheses of Interest

Diagonal-Σ: ("standard")

$$\Sigma = \begin{bmatrix} \sigma^2_{GG} & 0 & 0 \\ 0 & \sigma^2_{EE} & 0 \\ 0 & 0 & \sigma^2_{II} \end{bmatrix}$$

Block-Diagonal-Σ: (captures overlap in counts)

$$\Sigma = \begin{bmatrix} \sigma^2_{GG} & 0 & 0 \\ 0 & \sigma^2_{EE} & \sigma^2_{EI} \\ 0 & \sigma^2_{IE} & \sigma^2_{II} \end{bmatrix}$$

Unrestricted-Σ: (motivated by Nalewaik, 2010, *inter alia*)

$$\Sigma = \begin{bmatrix} \sigma^2_{GG} & \sigma^2_{GE} & \sigma^2_{GI} \\ \sigma^2_{EG} & \sigma^2_{EE} & \sigma^2_{EI} \\ \sigma^2_{IG} & \sigma^2_{IE} & \sigma^2_{II} \end{bmatrix}$$

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Identification

Diagonal-Σ model is identified

Block-Diagonal-Σ model is identified

Unrestricted-Σ model is *unidentified*

(We can increase the volatility of true *GDP* innovations and the measurement errors, but decrease the covariance between true *GDP* innovations and the measurement errors, without changing the distribution of observables.)

Identification requires fixing any element of Σ

A Useful Re-Parameterization

Recall:

$$\begin{aligned} GDP_t &= \mu(1-\rho) + \rho GDP_{t-1} + \epsilon_{Gt} \\ \Sigma &= \begin{bmatrix} \sigma^2_{GG} & \sigma^2_{GE} & \sigma^2_{GI} \\ \sigma^2_{EG} & \sigma^2_{EE} & \sigma^2_{EI} \\ \sigma^2_{IG} & \sigma^2_{IE} & \sigma^2_{II} \end{bmatrix} \end{aligned}$$

Reparameterize in terms of the ratio of GDP variance to GDP_E variance:

$$\zeta = \frac{\frac{1}{1-\rho^2}\sigma^2_{GG}}{\frac{1}{1-\rho^2}\sigma^2_{GG} + 2\sigma^2_{GE} + \sigma^2_{EE}}$$

A ζ value less than, but close to, 1 seems most natural

We take $\zeta = 0.80$ as our benchmark

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A Different Approach

Three-Equation Model:

$$\begin{bmatrix} GDP_{Et} \\ GDP_{It} \\ U_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \kappa \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \lambda \end{bmatrix} GDP_t + \begin{bmatrix} \epsilon_{Et} \\ \epsilon_{It} \\ \epsilon_{Ut} \end{bmatrix}$$

$$GDP_t = \mu(1-\rho) + \rho GDP_{t-1} + \epsilon_{Gt},$$

where $(\epsilon_{Gt}, \epsilon_{Et}, \epsilon_{It}, \epsilon_{Ut})' \sim iid\ N(\underline{0}, \Omega)$, with

$$\Omega = \begin{bmatrix} \sigma^2_{GG} & \sigma^2_{GE} & \sigma^2_{GI} & \sigma^2_{GU} \\ \sigma^2_{EG} & \sigma^2_{EE} & \sigma^2_{EI} & 0 \\ \sigma^2_{IG} & \sigma^2_{IE} & \sigma^2_{II} & 0 \\ \sigma^2_{UG} & 0 & 0 & \sigma^2_{UU} \end{bmatrix}$$

What to Use for U?

We take U to be the change in the unemployment rate

– Clearly unemployment rate changes load on GDP growth

– Unemployment data are constructed from household surveys, and very little household survey data are used to construct GDP_E and GDP_I

– Hence unemployment measurement errors are reasonably assumed to be orthogonal to those of GDP_E and GDP_I

Empirics, 1960Q1-2011Q4

Estimation

Posterior Means and Ninety Percent Coverage Regions

For the 2-equation model with $\zeta = 0.80$, we have

$$GDP_t = \underset{[2.79,3.35]}{3.08}(1 - 0.57) + \underset{[0.51,0.62]}{0.57} GDP_{t-1} + \epsilon_{Gt}$$

$$\Sigma = \begin{bmatrix} \underset{[6.54,7.70]}{7.09} & \underset{[-1.15,-0.29]}{-0.69} & \underset{[-0.74,-0.04]}{-0.38} \\ \underset{[-1.15,-0.29]}{-0.69} & \underset{[3.14,4.77]}{3.90} & \underset{[0.80,1.85]}{1.29} \\ \underset{[-0.74,-0.04]}{-0.38} & \underset{[0.80,1.85]}{1.29} & \underset{[1.98,2.82]}{2.36} \end{bmatrix}$$

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Posterior Means and Ninety Percent Coverage Regions

For the 3-equation model, we have

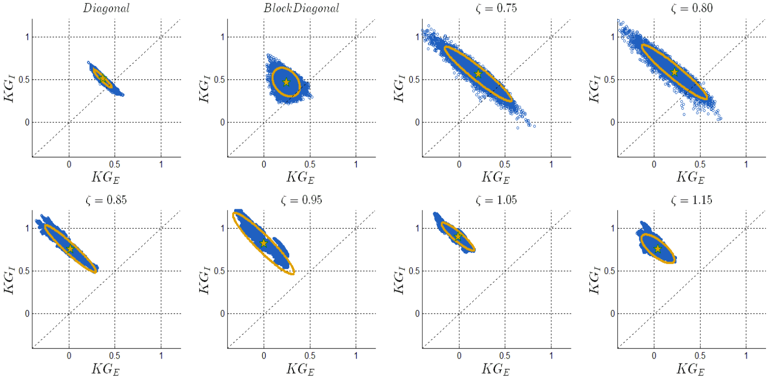
$$\begin{bmatrix} GDP_{Et} \\ GDP_{It} \\ U_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \underset{[1.53,1.71]}{1.62} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \underset{[-0.55,-0.50]}{-0.52} \end{bmatrix} GDP_t + \begin{bmatrix} \epsilon_{Et} \\ \epsilon_{It} \\ \epsilon_{Ut} \end{bmatrix}$$

$$GDP_t = \underset{[2.60,2.95]}{2.78}(1-0.58) + \underset{[0.54,0.63]}{0.58} GDP_{t-1} + \epsilon_{Gt}$$

$$\Omega = \begin{bmatrix} \underset{[6.73,7.35]}{6.96} & \underset{[-1.27,-0.84]}{-1.10} & \underset{[-1.03,-0.59]}{-0.82} & \underset{[1.27,1.66]}{1.46} \\ \underset{[-1.27,-0.84]}{-1.10} & \underset{[4.17,4.79]}{4.57} & \underset{[1.70,2.12]}{1.95} & 0 \\ \underset{[-1.03,-0.59]}{-0.82} & \underset{[1.70,2.12]}{1.95} & \underset{[2.54,3.27]}{3.07} & 0 \\ \underset{[1.27,1.66]}{1.46} & 0 & 0 & \underset{[0.50,0.71]}{0.59} \end{bmatrix}$$

The Importance of GDP_I

Kalman Gains



Blue clouds are 25,000 posterior draws. Gold ellipsoids are ninety percent posterior coverage regions. Gold stars are posterior medians.

Empirics III

$(\hat{\rho},\ \hat{\sigma}^2_{GG})$ for *GDPplus* vs. GDP_E and GDP_I

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Empirics III

$(\hat{\rho},\ \hat{\sigma}^2_{GG})$ for *GDPplus* vs. GDP_E and GDP_I

–*GDP* dynamics much more persistent than previously thought.

– High measurement error in GDP_E and GDP_I, injects downward bias into persistence estimates based on either alone.

– As expected, bias is worse for GDP_E than for GDP_I.

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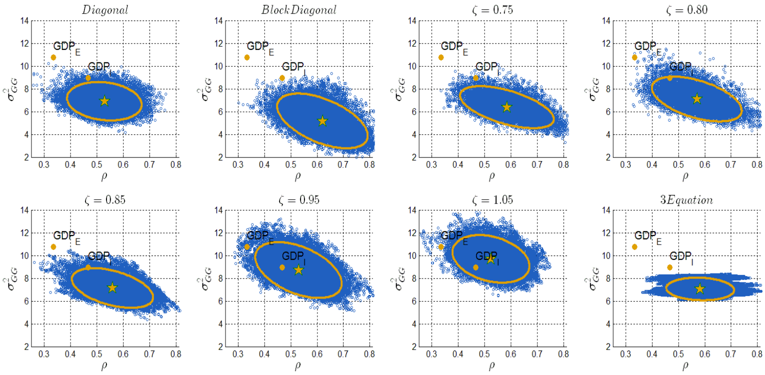
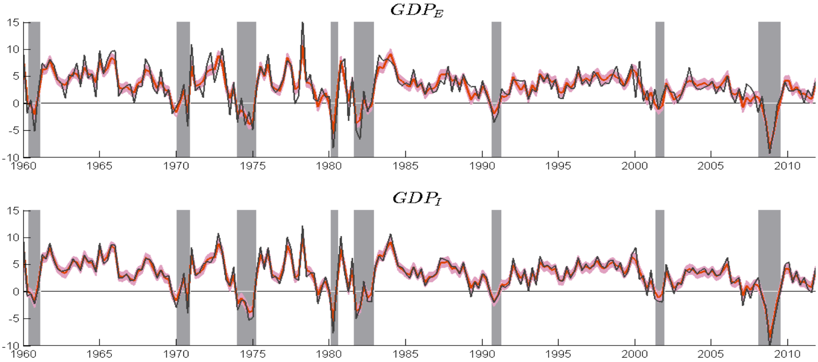
$(\hat{\rho}, \hat{\sigma}^2_{GG})$ Pairs Across Posterior Draws



Blue clouds are 25,000 posterior draws. Gold ellipsoids are ninety percent posterior coverage regions. Gold stars are posterior medians. Gold points are $(\hat{\rho}, \hat{\sigma}^2)$ values from $AR(1)$ regressions fit to GDP_E alone or GDP_I alone.

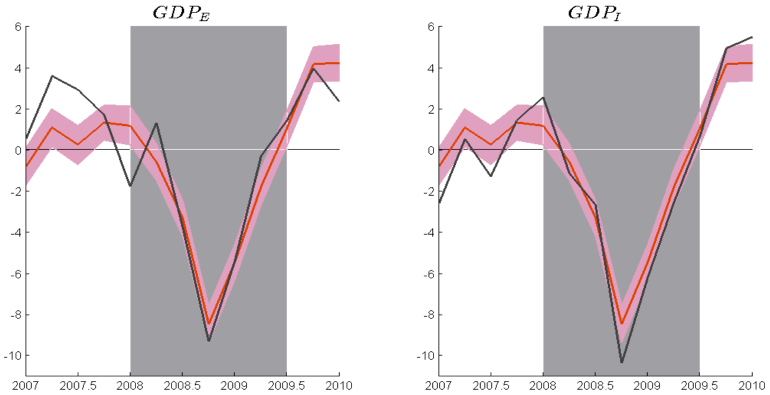
Sample Path Properties of *GDPplus*

$GDPplus$ vs. GDP_E and GDP_I Sample Paths



In each panel we show the sample path of *GDPplus* in red together with a light-red posterior interquartile range, and we show one of the competitor series in black. We obtain *GDPplus* from the 2-equation model with $\zeta = 0.80$.

$GDPplus$ vs. GDP_E and GDP_I Sample Paths, 2007Q1-2009Q4



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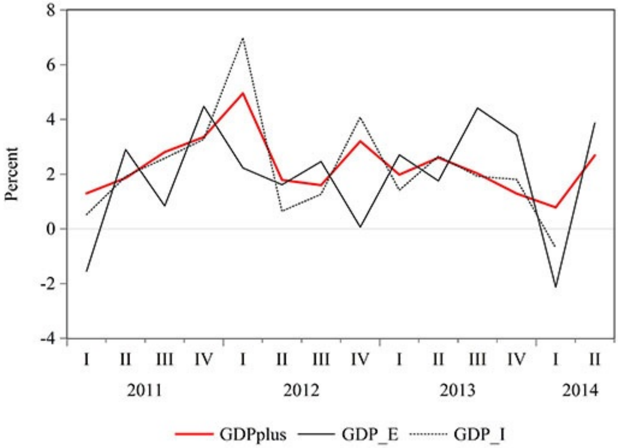
GDPplus vs. GDP_E and GDP_I Sample Paths, 2011Q1-2014Q2 (i.e., Latest Available)



Moving Forward

GDPplus is the natural benchmark U.S. *GDP* estimate

Now produced by Federal Reserve Bank of Philadelphia

– Updated in real time and written to the web (revisions and new releases)

`http://www.phil.frb.org/research-and-data/real-time-center/gdpplus/`