Microeconomic Theory II
Preliminary Examination
University of Pennsylvania

June 4, 2012

The exam is worth 120 points in total.

There are 4 questions. Do all questions.

Start each question in a new book, clearly labeled.

Fully justify all answers and show all work (in particular, describing an equilibrium means providing a full description and proving that it has the desired properties).

Label all diagrams clearly.

Write legibly.

If you need to make additional assumptions, state them clearly.

Good luck!

1. **(20 points)** A consumer lives for two periods. In period 2 she will purchase a commodity bundle $x = (x_1, x_2)$ to maximize her utility $u(x) = x_1^\alpha x_2^\alpha$ subject to her budget constraint $p_1x_1 + p_2x_2 \le y$. These prices are fixed positive constants, known even in period 1.

   In period 1 the consumer invests in a risky asset that returns $(1 + \tilde{r})z$ in period 2 if she invests an amount z. Her wealth is $w > 0$, and she is restricted to choosing $z \in [0, w]$. Her income in period 2 when she invests z will thus be the random variable $\tilde{y} = w + \tilde{r}z$. The asset has a positive expected return: $\mathbb{E}\tilde{r} > 0$. She chooses z to maximize the expected utility she will ultimately obtain in period 2.

   For each $\alpha > 0$, determine whether the consumer's optimal investment in the asset, $z^*(p, w)$, is decreasing, constant, or increasing in w.

2. **(30 points)** Suppose Bruce and Sheila play the following game:

		Sheila		
		L	C	R
	T	$4, 2$	$0, 0$	$-10, -1$
Bruce	M	$0, 0$	$2, 4$	$-10, -1$
	B	$-1, -10$	$-1, -10$	$-20, -20$

   (a) What are all the pure and mixed strategy Nash equilibria of this game? [5 points]

   Suppose Bruce has the option of publicly eliminating one of his choices before the game is played, at a utility cost of 1. (In other words, if Bruce eliminates a choice, his payoffs in the resulting smaller 2×3 game are all reduced by 1.) Denote the elimination of a_1 by $\neg a_1$, and the option of no elimination by X. Since the elimination is public, Bruce's choice from $\{X, \neg T, \neg M, \neg B\}$ is publicly observed by Sheila before they simultaneously choose their actions.

   (b) Describe a pure strategy Nash equilibrium in which Bruce does not eliminate any choices and MC is played. Is it subgame perfect? Why or why not? [10 points]

   (c) Describe *all* the pure strategy subgame perfect equilibria. [10 points]

   (d) Are all these subgame perfect equilibria equally plausible? [5 points]

Question 3 is on the next page.

3. **(40 points)** This question concerns the one-sided bargaining model of an uninformed seller with zero cost facing a buyer whose valuation v can only take on two values, 3 or 5. The seller's beliefs assign a prior probability α to the value 3. The seller makes offers to the buyer. The seller's cost (value) is zero, and the buyer and seller have a common discount factor $\delta \in (0, 1)$.

   (a) What are the perfect Bayesian equilibria of the one period model (that is, the model in which the seller makes a take-it-or-leave-it offer to the buyer). Your answer will be a function of α. [5 points]

   Consider now the two period model, that is, if the buyer rejects the offer in the first period, then the seller make a final offer in period 2, after which the game ends.

   (b) Define a pure strategy profile in this two period game. Define a pure strategy perfect Bayesian equilibrium (PBE) (if you prefer, interpret PBE as almost perfect Bayesian equilibrium). Be precise in the restrictions imposed on behavior. [**Hint:** Among other things, PBE restricts period 2 pricing after a rejected out-of-equilibrium period 1 price. How? Note that you are not (yet) being asked to characterize equilibrium play.] [5 points]

   (c) Prove that, in any pure strategy PBE, both types of buyer must accept any first period price strictly smaller than 3. [5 points]

   (d) Prove that, in any pure strategy PBE, if a first period price $p_1 > 3$ is rejected, then the seller's posterior in the beginning of the second period must assign probability at least α to the low value buyer. [5 points]

   (e) Suppose $\alpha = \frac{2}{3}$. Describe the unique pure strategy PBE. [10 points]

   (f) Suppose $\alpha = \frac{1}{3}$. Prove that there is no pure strategy PBE. [**Hint:** Suppose p_1^* is the first period price in a candidate pure strategy PBE. How should the seller respond to a rejection of a deviation to a price $p_1 \neq p_1^*$? The restriction to pure strategies is important.] [10 points]

Question 4 is on the next page.

4. **(30 points)** Consider the stage game where player 1 is the row player and 2, the column player:

	L	R
T	$2,2$	$0,4$
B	$6,0$	$1,1$

(a) Describe the set of feasible and individually rational payoffs, and illustrate diagrammatically. [5 points]

(b) Suppose the game is infinitely repeated, with perfect monitoring. Players 1 and 2 are both long-lived, and have the same discount factor, $\delta \in (0,1)$. Construct a three state automaton that for large δ is a pure strategy subgame perfect equilibrium, and yields an average discounted payoff to player 1 that converges to 4 as δ converges to 1. Prove that the automaton has the desired properties. [It may help to recall that $1 - x^2 = (1-x)(1+x)$ and $1 - x^3 = (1-x)(1+x+x^2)$ for all x.] [20 points]

(c) Prove that using Nash reversion as a punishment does not lower the range of δ for which the equilibrium outcome path described in part 4(b) is consistent with equilibrium. [5 points]