Microeconomic Theory II
Preliminary Examination
June 2, 2014

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(45 points)** Consider the following normal form game played by Bruce and Sheila:

Bruce \ Sheila	L	R
T	$-1, 0$	$3, 3$
M	$1, x$	$0, 0$
B	$0, 0$	$4, 1$

(a) Suppose $x = -1$. What are all the pure and mixed strategy Nash equilibria of this game when $x = -1$? When $x = 1$? [5 points]

Consider now the **once** repeated game, with players observing first period play before making second period choices. Payoffs in the repeated game are the sum of payoffs in the two periods.

(b) Describe the information sets and pure strategies of the extensive form of the repeated game. [5 points]

(c) Suppose $x = -1$. Describe a pure strategy Nash equilibrium of the once repeated game in which Bruce plays T in the first period. Is it subgame perfect? Explain why or why not. [5 points]

(d) Suppose $x = +1$. Describe a pure strategy subgame perfect equilibria of the once repeated game in which Bruce plays T in the first period. [5 points]

Consider now the **infinitely** repeated game, with players observing past play before making choices (so that the game has perfect monitoring). Payoffs in the infinitely repeated game are the average discounted sum of payoffs, with discount factor $\delta \in (0, 1)$.

(e) Suppose $x = +1$. For some values of δ, there is a simple pure strategy equilibrium of the infinitely repeated game with outcome path $(TR)^\infty$. Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [10 points]

(f) Suppose $x = -1$. For some values of δ, there is a pure strategy equilibrium of the infinitely repeated game with outcome path $(TR)^\infty$. This profile is necessarily more complicated than the profile from part 1(e). Why? Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [15 points]

Question 2 is on the next page.

2. **(15 points)** Bruce owns a car he would like to sell to Sheila. Only Bruce knows the quality of the car: it is either good or bad (a lemon). Sheila wants to buy the car only if it is good. Bruce can try to convince Sheila about the car's quality before Sheila makes her purchase decision. To keep things simple, suppose that Bruce receives a payoff of 1 if he sells the car, and 0 otherwise, and that Sheila receives a payoff of $+1$ from buying a good car, a payoff of -1 from buying a bad car, and 0 otherwise.

   (a) Describe the associated extensive form game, where we interpret "Bruce can try to convince Sheila about the car's quality" as Bruce can announce "good" or "bad" before Sheila makes her decision, and this announcement has not direct payoff implications. Be precise in any additional assumptions you must make to obtain a fully specified extensive form game. [3 points]

   (b) Is there an equilibrium of this game in which Sheila buys the car if and only if it is good? Why or why not? [5 points]

   (c) Suppose that in addition to wishing to sell his car, Bruce does not like lying. How would you alter Bruce's payoffs to reflect this? How does this change your answer to part 2(b) (you may restrict attention to pure strategies)? [7 points]

Question 3 is on the next page.

3. **(35 points)** Suppose that the payoff to a firm from hiring a worker of type θ with education e at wage w is

$$f(e,\theta) - w = 4e\theta^2 - w.$$

The utility of a worker of type θ with education e receiving a wage w is

$$w - c(e,\theta) = w - \frac{e^4}{\theta}.$$

The worker's ability is privately known by the worker. There are at least two firms. The worker (knowing his ability) first chooses an education level $e \in \mathbb{R}_+$; firms then compete for the worker by simultaneously announcing a wage; finally the worker chooses a firm. Treat the wage determination as in class, a function $w : \mathbb{R}_+ \to \mathbb{R}_+$ determining wage as a function of eduction.

Suppose the support of the firms' prior beliefs ρ on θ is $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L = 1$ and $\theta_H = 4$.

(a) What is the full information education level for each type of worker? [5 points]

(b) Is there a perfect Bayesian equilibrium in which both types of worker choose their full information eduction level? Be sure to verify that all the incentive constraints are satisfied. [5 points]

(c) Suppose e_L is the education level undertaken by θ_L, while e_H is the education level taken by θ_H in a separating Perfect Bayesian Equilibrium. What can you conclude about e_H and e_L? Be as precise as possible. [10 points]

(d) Suppose the firms' prior beliefs ρ are that the worker has type θ_H with probability $\frac{2}{3}$, and type θ_L with probability $\frac{1}{3}$. Is there a pooling Perfect Bayesian equilibrium in this setting? If yes, describe a pooling PBE and argue that it is one. If not, why not? [10 points]

Now suppose the support of the firms' prior beliefs on θ is $\Theta = \{1, 3, 4\}$.

(e) Is there a perfect bayesian equilibrium in which each type of worker chooses her full information education level? [5 points]

Question 4 is on the next page.

4. **(25 points)** A seller sells to a buyer with type $\theta \in \Theta = [-1, 1]$. If a buyer of type θ receives the good with probability q in return for a payment of p, she has a net utility of:

$$u(q, p|\theta) = e^{(\theta^2)} q - p.$$

The seller uses a direct revelation mechanism. The buyer announces his type θ and receives the good with probability $q(\theta)$ in return for a payment of $p(\theta)$.

(a) From first principles, state and prove necessary and sufficient conditions on $q(\cdot)$ for there to exist a price $p(\cdot)$ such that $(q(\cdot), p(\cdot)$ satisfies IC. Further, derive $p(\cdot)$ as a function of $q(\cdot)$ in this setting. In other words, derive and prove the counterpart of the Fundamental IC Lemma in this setting. [20 points]

(b) The seller's type is $\omega \in \Omega = [1, 2]$, and if she sells the the good to the buyer with probability q in return for a payment of p, she has a net utility of:

$$u(q, p|\theta) = p - e^{(\frac{\omega}{2})} q.$$

Suppose the buyer's and seller's types are private to them. Suppose further the buyer's type is drawn uniformly from Θ, while the seller's type is drawn uniformly from Ω, and the two draws are independent. Does there exist an incentive compatible trading mechanism that is individually rational and has no expected subsidy? Why or why not? [5 points]