Microeconomic Theory II
Preliminary Examination
August 4, 2014

The exam is worth 120 points in total.

There are 4 questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **(30 points)** Consider the following simultaneous move game played by Bruce and Sheila (with Bruce being the row player, and Sheila the column player):

	L	R
T	x, x	$0, 0$
B	$0, 0$	x, x

Only Bruce knows the value of x, which is either 4 or 8; Sheila assigns equal probability to the two values.

(a) Describe the incomplete information game (i.e., describe the strategies and payoffs). [5 points]

(b) Describe all pure and mixed equilibria of this game. [10 points]

Suppose now that Bruce, after observing the value of x, can either choose to play the simultaneous game with Sheila, or by paying a cost of 3, play the game sequentially, taking the first move publicly, before Sheila makes her move.

(c) Describe a Nash equilibrium of this dynamic game in which Bruce pays the cost of 3 to play sequentially with Sheila for some value of x. Is the equilibrium sequential? Why or why not? [15 points]

Question 2 is on the next page.

2. **(30 points)** A seller can either exert effort (H) or not (L) in the production of a good. A buyer must decide, not knowing the effort choice of the seller, whether to buy (B) the good or not (D). The resulting game is (with the seller being the row player and the buyer the column player):

	B	D
H	$2,3$	$-1,0$
L	$3,2$	$0,0$

(a) Describe the set of feasible payoffs and the subset of individually rational payoffs. [5 points]

(b) Suppose the game is infinitely repeated, and both buyer and seller have discount factor $\delta \in (0,1)$. For some values of δ, there is a simple pure strategy equilibrium of the infinitely repeated game with outcome path $(HB)^\infty$. Describe it, and the bounds on δ for which the profile is a subgame perfect equilibrium. [15 points]

(c) How would your answer to Question 2(b) change if the buyer makes his purchase decision after observing the seller's effort choice? [10 points]

Question 3 is on the next page.

3. **(30 points)** Sheela is considering hiring Bhuvi. If Bhuvi undertakes this employment, he chooses effort $e \in \Re_+$, and suffers a disutility of effort $g(e) = e^2$. Given his effort, Sheela realizes a profit π that is a draw from a uniform distribution on $U[0, 6e]$.

Sheela is risk-neutral, but Bhuvi is risk-averse and has mean variance preferences. In other words, given a random wage w, and taking effort level e, Bruce gets an expected utility of

$$\mathbb{E}[w] - \text{Var}(w) - g(e),$$

while Sheela makes a net profit of

$$\mathbb{E}[\pi - w].$$

(a) Suppose Bhuvi's outside option is 0, and that the effort he takes is observable and contractible. What is the optimal contract for Sheela to offer? What is the first best level of effort she will implement? [5 points].

(b) Suppose effort is not observable. Suppose further that Sheela offers a linear wage contract:

$$w(\pi) = c + \alpha\pi,$$

with $\pi \in [0, 1]$. In other words, Bhuvi gets a lumpsum of c plus a fraction α of the realized profits π. What is the optimal effort level for Bhuvi as a function of c and α? [10 points]

(c) Write down Sheela's optimization problem to find her optimal linear contract. What is the optimal value of c and α? What effort level does Bhuvi take? [15 points]

Question 4 is on the next page.

4. **(30 points)** A public utility commission is charged with regulating a monopoly provider of electricity. The cost function of the monopoly is given by:

$$C(q, \theta) = \begin{cases} 0 & \text{if } q = 0, \\ K + \theta q & \text{if } q > 0, \end{cases}$$

where q is the quantity produced, $K > 0$ is the publicly known fixed cost and $\theta \in [0, 1]$ is the marginal cost. The inverse demand curve for electricity is

$$p(q) \equiv \max\{1 - q, 0\}.$$

Supposing there are no income effects for the good, the consumer surplus is given by,

$$V(q) = \int_0^q p(\tilde{q})d\tilde{q} - p(q)q.$$

The regulator determines whether or not the firm should operate, the quantity q the firm should produce, as well as a subsidy that will be offered to the firm. The firm cannot be forced to operate—it will shut down if it makes negative profits, i.e. if $\Pi(q) \equiv p(q)q - C(q, \theta) + s < 0$.

The regulator wishes to maximize the total of consumer surplus and firm profits less the subsidy, i.e. $V(q) + \Pi(q) - s$.

(a) Suppose the marginal cost $\theta > 0$ is publicly known. Solve the regulator's problem and interpret. Give conditions on K and θ under which the regulator shuts down the firm.
[10 points]

(b) Suppose now that marginal cost is known only to the monopoly. The regulator believes that it may be either of $\theta_H > \theta_L$, with both equally likely. The regulator's problem is now the determine the quantities q_H, q_L and subsidies s_H, s_L as a function of a cost report from the firm.
[20 points]

   i. Explicitly write down the regulator's problem, labelling the incentive compatibility and individual rationality constraints.

   ii. Assuming that the regulator optimally allows both types to operate, simplify the regulator's problem. Which constraints are redundant?

   iii. Argue that $q_L \geq q_H$ in the optimal solution.

   iv. Which constraints hold with equality in the optimal solution?