

## 1. Jones and Manuelli (1990) explains Piketty (2014)

Consider the following model. There is a representative household with a utility function:

$$U(0) = \int_0^{\infty} e^{-\rho t} \log C(t) dt$$

where  $\rho > 0$ . Labor is  $L = 1$  and there is no population growth.

There is a perfectly competitive final good producer with technology:

$$Y = AK + BK^{\alpha}L^{1-\alpha}$$

where  $A > 0$ ,  $B > 0$ , and  $0 < \alpha < 1$ .

Capital accumulates over time given

$$\dot{K} = AK + BK^{\alpha}L^{1-\alpha} - C - \delta K$$

1. Define an equilibrium for this economy.
2. Characterize the Balanced Growth Path (BGP) of this economy. Among other things, you need to:
  1. Find the optimality conditions of the household and the final good producer.
  2. Find an expression that relates  $A$ ,  $\rho$ , and  $\delta$  with the growth rate of the economy  $g$  along the BGP. Which parametric assumption do we need to impose to ensure that  $g > 0$ ?
  3. Rewrite the differential equations that characterize the dynamics of this economy in terms of two transformed variables:

$$z = \frac{Y}{K}$$
$$\chi = \frac{C}{K}$$

3. Build a phase diagram to characterize the transitional dynamics of this economy using  $z$  and  $\chi$ .
4. Show that the rental rate of capital,  $r$ , declines gradually toward its BGP value, that  $r > g$ , and that capital grows without bound.
5. Derive an expression for capital's share of income:

$$\frac{rK}{Y}$$

and show that, along a BGP with  $g > 0$ , capital's share rises toward one and labor's share falls toward zero

Note that 4. and 5. are some of the main claims in Piketty's book.