

Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 6, 2011

Instructions

This exam has 5 questions and a total of 100 points.

You have two hours to complete it.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise. Write clearly so that you might get partial credit.

Good luck!

1. (20 pts) Suppose that $u(\cdot)$ is a continuous utility function representing a locally nonsatiated preference relation on the consumption set $X = \mathbb{R}_+^L$. Write $v(p, w)$ for the indirect utility function, where $p \in \mathbb{R}_{++}^L$ is the price vector and $w \in \mathbb{R}_+$ is the income.

- (a) (5 pts) Show that v is nonincreasing in p and strictly increasing in w . Point out where the property of locally nonsatiated preference is used in your argument.
- (b) (5 pts) Give an example of a utility function u that satisfies the previous properties, and the corresponding v does not strictly decrease in some p_ℓ .
- (c) (5 pts) Show that v is quasi-convex in (p, w) .
- (d) (5 pts) Let $p^n \rightarrow p$ and $w^n \rightarrow w$ be sequences of prices and incomes, where $p \in \mathbb{R}_{++}^L$ and $w \in \mathbb{R}_+$. Show that

$$\liminf_n v(p^n, w^n) \geq v(p, w).$$

Point out where continuity and local nonsatiation are used in your argument. (Merely citing the maximum theorem is not a valid argument.)

2. (20 pts)

- (a) State carefully Brouwer's fixed point theorem.
- (b) State carefully a general equilibrium theorem that uses Brouwer's theorem and explain how Brouwer's theorem is used in the proof.
- (c) The conclusion of the theorem you identified in (b) may fail to hold if the economy is such that the assumptions of Brouwer's theorem are not satisfied. Give an example of an economy for which this is the case. (A graphical example is sufficient.)
- (d) Now give an example of an economy for which the conclusion of the general equilibrium theorem in (b) does hold, even though the assumptions of Brouwer's theorem do not. (A graphical example is sufficient.)

3. (20 pts) Consider an economy with two states and a single good that can be consumed in each state. There are two agents, $i = 1, 2$, each of whom has a von Neumann-Morgenstern utility function with constant absolute risk aversion coefficient $\alpha > 0$ (they are equally risk averse). Their initial endowments are $\omega^1 = (2, 0)$ and $\omega^2 = (0, 1)$.

- (a) Compute the Walrasian equilibrium for this economy, assuming the states are equally likely.
- (b) Without solving for a new equilibrium, what is the intuition about how the equilibrium might change from (a) if α increases?
- (c) Without solving for a new equilibrium, what is the intuition about how the equilibrium might change from (a) if the probability of state 1 increases?
- (d) Without solving for a new equilibrium, what is the intuition about how the equilibrium might change from (a) if agent 1's endowment in state 1 increases?
- (e) Given a Walrasian equilibrium, let g_i be the number such that the utility agent i would obtain by consuming $g_i \omega^i$ is equal to her equilibrium utility. (Note that $g_i \geq 1$.) Call g_i the *gains from trade* of agent i in this equilibrium. Without solving for a new equilibrium, what is the intuition about how the agents' gains from trade change from (a) if α increases?
- (f) Can you say anything about how the gains from trade change from (a) when the probability of state 1 goes to 1?

4. (15 pts) Consider a society $N = \{1, \dots, n\}$ and a set X of social alternatives. Assume $n \geq 2$ and X is a finite set such that $\#X \geq 3$. Let \mathfrak{R} be the set of all preference orders on X , and let \mathfrak{B} be the set of all nonempty subsets of X .

Let S be a nonempty subset of N satisfying $|S| > 1$. Consider the Social Choice Correspondence defined on $\mathfrak{B} \times \mathfrak{R}^n$ by

$$C(B, \vec{R}) := \{x \in B : \forall y \in B, \exists i \in S \text{ such that } x R_i y\}.$$

State and prove whether C satisfies the following Arrow axioms: IIA (Independence of Irrelevant Alternatives), UN (Unanimity), and RA (Rationality).

5. (25 pts) Two traders share the ownership of an asset. Trader 1 initially owns $q^0 \in [0, 1]$ shares, and trader 2 owns $1 - q^0$ shares. Trader i has value v_i for the asset; if there is no trade and no transfers are made, the utilities of traders 1 and 2 are, respectively, $q^0 v_1$ and $(1 - q^0)v_2$. If there is trade to new shares q , and an amount of money t_i is transferred to trader i , their payoffs are $q v_1 + t_1$ and $(1 - q)v_2 + t_2$. Each v_i is privately known to trader i , and (v_1, v_2) is commonly known to have been drawn from a uniform distribution on the unit square. There is no third party who can pay either trader; money can be destroyed; shares of the asset cannot be destroyed (so $q \in [0, 1]$ is necessary); and any $t_i \in \mathbb{R}$ is consumable by trader i .
- (3 pts) Define a feasible, interim individually rational revelation mechanism for this environment.
 - (3 pts) Describe a first-best efficient allocation for this environment.
 - (4 pts) What does the Myerson-Satterthwaite theorem tell us about this environment?
 - (15 pts) Prove whether or not there are values of q^0 for which there exists a revelation mechanism that is dominant strategy incentive compatible, achieves a first best efficient allocation, and is interim individually rational.