Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

June 2, 2014

Instructions

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) A strictly increasing utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ gives rise to a demand function $\mathbf{x}(\mathbf{p}, y) = (x_1(\mathbf{p}, y), x_2(\mathbf{p}, y))$. It is continuously differentiable in a neigborhood N of some $(\mathbf{p}^0, y^0) \gg \mathbf{0}$. Theory tells us much about the nature of such demand functions: use what it tells us to answer the following questions.

   (a) (3 pts) Write the definition of $\eta_i(\mathbf{p}, y)$, the income elasticity for good i.

   (b) (11 pts) Suppose the demand for good 1 takes the form

   $$x_1(\mathbf{p}, y) = \alpha_1(\mathbf{p}) g_1(y)$$

   for all $(\mathbf{p}, y) \in N$. What is the most that this implies about $\eta_i(\mathbf{p}, y)$ on N?

   (c) (11 pts) Now suppose in addition that in N, demand for good 2 takes the same form,

   $$x_2(\mathbf{p}, y) = \alpha_2(\mathbf{p}) g_2(y),$$

   and $\mathbf{x}(\mathbf{p}, y)$ satisfies the law of reciprocity:

   $$\frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1}.$$

   What can you now say about the two income elasticity functions on N?

2. (25 pts) A competitive firm uses hops to make beer via a production function $f : \mathbb{R} \to \mathbb{R}$. The price of beer is $p > 0$ and the cost of new hops is $w > 0$. The firm has $x_0 > 0$ hops left over from last year, but an unknown amount of these old hops will either grow or spoil before production begins this year; the amount of them that will be usable is $x_0 + \theta\tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is a nondegenerate random variable with mean zero, $\theta > 0$ is a parameter to allow easy comparative statics, and $\theta\tilde{\varepsilon} \in (-x_0, x_0)$. The firm will purchase more hops, $x \geq 0$, to maximize its expected profit,

   $$\mathbb{E}\left\{pf(x_0 + \theta\tilde{\varepsilon} + x) - wx\right\}.$$

   Assume f is smooth with derivatives $f' > 0$ and $f'' < 0$, and that the solution, $x^*(x_0, w, p, \theta)$, is positive. Make, if necessary, additional reasonable assumptions under which the signs of the partial derivatives,

   $$x^*_{x_0},\ x^*_w,\ x^*_p,\ x^*_\theta,$$

   can be determined, and find their signs under those assumptions.

3. (25 pts) Consider an exchange economy with two goods and two households with endowments

$$e^1 = (1, 0) \text{ and } e^2 = (0, 1)$$

In each of the following specifications of utility functions, state whether a competitive equilibrium exists. If any do exist, compute them all. If none exist, explain why not and which part of the standard existence proof fails (be specific about which assumption fails).

(a) $u^1(x_1, x_2) = \min(x_1, 2x_2),\ u^2(x_1, x_2) = \min(2x_1, x_2)$

(b) $u^1(x_1, x_2) = \max(x_1, x_2),\ u^2(x_1, x_2) = \max(x_1, x_2)$

(c) $u^1(x_1, x_2) = (x_1)^2 + (x_2)^2,\ u^2(x_1, x_2) = x_1 x_2$

4. (25 pts) Consider an economy with a linear production set. Suppose there are 2 agents and 4 commodities. Commodities 1 and 2 are consumption goods while commodities 3 and 4 are skilled and unskilled labor. Suppose there are 4 possible activities transforming skilled and unskilled labor into commodities 1 and 2:

$$a_1 = (1, 0, -3, 0) \quad a_2 = (0, 1, -1, 0)$$
$$a_3 = (1, 0, 0, -4) \quad a_4 = (0, 1, 0, -2)$$

Suppose there is a single firm whose production set Y is the closed cone generated by these activity vectors:

$$Y = \{y \in R^L : y = \Sigma_{m=1}^4 \alpha_m a_m \text{ for some } \alpha \in R_+^4\}$$

The agents' utility functions are $u^1(c_1, c_2, c_3, c_4)$ and $u^2(c_1, c_2, c_3, c_4)$, and their endowments are $e^1 = (0, 0, 1, 0)$ and $e^2 = (0, 0, 0, 2)$.

(a) Define a competitive equilibrium.

(b) Show that if activities $a_2,\ a_3$, and a_4 are used in equilibrium, a_1 cannot be used.

(c) Show that it cannot be the case that $a_1,\ a_3$, and a_4 are used in equilibrium.

(d) Suppose now that agents' utility functions are

$$u^1(c_1, c_2, c_3, c_4) = \log(c_1) + \log(c_2)$$
$$u^2(c_1, c_2, c_3, c_4) = \log(c_1) + \log(c_2)$$

Compute a competitive equilibrium in which activities $a_2,\ a_3$ and a_4 have 0 profit, and hence might be used in equilibrium.