Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 4, 2014

Instructions

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) There are two possible states of the world and one good, "money". It is commonly known that state s will occur with probability $\pi_s > 0$, for $s = 1, 2$. A state contingent allocation is a pair $(x_1, x_2) \in \mathbb{R}^2_+$. Consider a consumer who has a complete, transitive, and strongly monotonic ordering $\succsim$ over these allocations. Assume $\succsim$ is convex.

   (a) (10 pt) Prove or disprove: This consumer must be weakly risk averse.

   (b) (15 pts) Do the same as in (a), but under the assumption now that the consumer satisfies the expected utility hypothesis. Let u denote the consumer's Bernoulli utility function, and assume it is twice continuously differentiable, with $u' > 0$.

2. (25 pts) Consider a society $N = \{1, \dots, n\}$ and a finite set X of alternatives. Assume $n \geq 2$ and $\#X \geq 3$. Let $\mathfrak{R}$ be the set of complete and transitive binary relations on X. One alternative, $s \in X$, is the *status quo.* For each profile $\vec{R} \in \mathfrak{R}^n$, let G ("good") be the set of alternatives that are weakly Pareto preferred to s :

$$G = \{x \in X : xR_i s \ \forall i \in N\}.$$

   (Note that $s \in G$.) Let B ("bad") be the complementary set, $B = X \backslash G$. For each $\vec{R} \in \mathfrak{R}^n$ define a binary relation $F(\vec{R})$ on X by

$$\begin{aligned} \forall x \in G, y \in B : &\quad xF(\vec{R})y \text{ and not } yF(\vec{R})x \\ \forall x, y \in G : &\quad xF(\vec{R})y \ \Leftrightarrow \ xR_n y \\ \forall x, y \in B : &\quad xF(\vec{R})y \ \Leftrightarrow \ xR_n y \end{aligned}$$

   Answer the following questions, and prove your answers:

   (a) (6 pts) Is F dictatorial?

   (b) (6 pts) Does F satisfy Unanimity?

   (c) (6 pts) Does F satisfy Independence of Irrelevant Alternatives?

   (d) (7 pts) Is F an (Arrow) Social Welfare Function?

3. (25 pts) Consider a pure exchange economy with ℓ goods and n agents.

   (a) (5 pts) Define the core of this economy.

   (b) (5 pts) State the core convergence theorem.

   (c) (5 pts) Assume that each agent has a utility function that is strictly increasing, strictly concave and differentiable. Prove that a competitive equilibrium allocation is in the core.

   (d) (10 pts) Suppose now that there are 2 goods and 4 agents. Agent 1 has utility function u^1 and endowment (w_1^1, w_2^1), and agent 2 has utility function u^2 and endowment (w_1^2, w_2^2). The functions u^1 and u^2 are strictly increasing and strictly concave. Agent 3 has the same utility function and endowment as agent 1, and agent 4 has the same utility function and endowment as agent 2. Prove that in any core allocation, agents 1 and 3 get the same allocation, and agents 2 and 4 get the same allocation.

4. (25 pts) Three hunters will hunt for deer tomorrow in a game park in which there is exactly one deer. Assume the deer will be caught. There are thus three possible states of the world: state s represents the event that hunter s catches the deer, for $s = 1, 2, 3$. The three initial endowment bundles of contingent deer meat are

$$\omega^1 = (1, 0, 0),\ \omega^2 = (0, 1, 0),\ \omega^3 = (0, 0, 1).$$

Today (date $t = 0$) the hunters arrange for how the meat from the deer will be shared tomorrow (date $t = 1$). The utility function of hunter i is

$$U^i(x_i) = \Sigma_{s=1}^{3} \pi_s^i u^i(x_s^i),$$

where x_s^i is his consumption of deer meat in state s, and π_s^i is his belief probability that state s will occur. Assume u^i is continuous, strictly concave, and strictly increasing.

(a) (8 pts) Suppose the hunters agree that the state probabilities are $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$. (Hunter 1 is believed to be twice as likely to catch a deer as is either of the other two.) Show that at any interior Pareto efficient allocation, hunter 1 will consume the same amount of deer meat regardless of who catches the deer. (You can assume for this part that each u_i is differentiable.)

For the remaining parts (b) and (c): assume each hunter is so self-confident that he believes he will surely catch a deer: $\pi_i^i = 1$ for each i (and hence $\pi_s^i = 0$ for $s \neq i$.

(b) (8 pts) Prove that if $x^* = (x^{1*}, x^{2*}, x^{3*})$ is Pareto efficient, then $x_i^{i*} = 1$ for each i.

(c) (9 pts) What is the set of competitive equilibrium prices, letting p_s denote the price at date 0 for contingent deer meat in state s at date 1.