A Likelihood Analysis of Models with Information Frictions[1]
(Job Market Paper)

Preliminary Draft

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Abstract

I develop and estimate a general equilibrium model with imperfectly-informed firms in the sense of Woodford (2001). The model has two aggregate shocks: a monetary shock and a technology shock. In this environment, agents' "forecasting the forecasts of others" can produce realistic dynamics of model variables. These dynamics feature highly-persistent real effects of monetary shocks, and delayed effects of such shocks on inflation. The paper provides a full Bayesian empirical analysis of the model, revealing that it accurately captures the persistent propagation of monetary shocks. Yet in order to deliver this result, the model predicts that firms acquire rather little information about monetary shocks. In order to investigate whether this prediction is plausible, I augment the imperfect-information model with a schedule of information bundles about the two shocks. This type of schedule is extensively used in the rational-inattention literature (e.g., Sims, 2003). I find that the marginal value of information about monetary policy is much higher than technology at the estimated information bundle of the imperfect-information model. Hence, I argue that this model predicts that firms acquire implausibly too little information about monetary policy.

Keywords: Imperfect common knowledge; forecasting the forecasts of others; rational inattention; Bayesian econometrics; persistent real effects of nominal shocks.

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1 Introduction

A number of influential empirical studies of the US economy has documented that money disturbances have highly persistent real effects and delayed impacts on inflation (Christiano *et al.*, 1999, Stock and Watson, 2001). The conventional approach to explaining this evidence relies upon sticky-price models (e.g., Gali and Gertler, 1999, Eichenbaum and Fisher, 2004, Christiano *et al.*, 2005, and Smets and Wouters, 2007). Nonetheless, this type of models can generate sluggish adjustments to monetary disturbances only with a degree of price stickiness that is implausibly large according to recent micro-evidence on price-setting (Bils and Klenow, 2004)[3].

Woodford (2002) proposes an alternative modeling approach: imperfect-common-knowledge models. In his price-setting model, monopolistic producers set their prices under limited information and strategic complementarities. Firms observe idiosyncratic noisy signals regarding the state of monetary policy and solve a signal-extraction problem in order to keep track of the model variables. Since the signal is noisy, firms do not immediately learn the occurrence of monetary disturbances. As a result, the price level fails to adjust enough to entirely neutralize the real effects of nominal shocks. Moreover, because of the idiosyncratic nature of the signals, in the aftermath of a shock firms are also uncertain about what other firms know that other firms know... that other firms know about that shock. Owing to strategic complementarities in price setting, a problem of forecasting the forecast of others of the type envisioned by Townsend (1983b) arises. This feature of the model has been shown to amplify the persistence in economic fluctuations[4] and in the propagation of monetary disturbances to real variables and prices[5]. Moreover, it is worthy emphasizing that in

[3] A modelling solution that preserves sticky prices and is not in conflict with micro-data on price-setting is developed by Altig et al. (2005).

[4] See also Townsend (1983b), Townsend (1983a); Hellwig (2002), Adam (2008), Rondina (2008), Angeletos and La'O (2009), and Lorenzoni (forthcoming A).

[5] See also Phelps (1970), Lucas (1972), Adam (2007), Gorodnichenko (2008), Maćkowiak and Wiederholt (2008), Nimark (2008), Paciello (2008), and Lorenzoni (forthcoming B). See Mankiw and Reis (2002a, 2002b, 2006, 2007), and Reis (2006a, 2006b, 2009) for models where information frictions are modelled as time-dependent updating of agents' information sets.

this model prices adjust frequently, but move only gradually to their complete information levels. Thus the resolution proposed by Woodford (2002) is appealing in that it can potentially explain sluggish adjustments of macro variables without necessarily discording with the micro evidence on price changes.

This paper addresses the following question: can a version of the imperfect-common knowledge model (ICKM) replicate the persistent effects of monetary shocks we observe in the data? The answer is yes but with some caveats. To get this answer, the paper proceeds by constructing a dynamic stochastic general equilibrium (DSGE) model with two shocks: a monetary policy shock and an aggregate technology shock. Firms receive one idiosyncratic noisy signal about each of these two shocks and face strategic complementarities in price setting. The signal-extraction problem and the price-setting problem are similar to Woodford (2002). I estimate the ICKM and a vector autoregressive model (VAR) through Bayesian methods. I consider the impulse response functions (IRFs) implied by this statistical model as an accurate description of the propagation of monetary shocks in the data[6]. From a Bayesian perspective, this conjecture is sensible because the VAR turns out to dominate the ICKM in terms of time series fit (Schorfheide, 2000). I then compare the IRFs of output and inflation to a monetary shock implied by the ICKM to those implied by the VAR. I find that the ICKM successfully captures the sluggish and hump-shaped response of output and inflation to monetary shocks implied by the VAR. Nonetheless, the signal-to-noise ratio of monetary policy is smaller than that of technology by a factor of six. The reason is that the estimated ICKM can generate highly sluggish responses to monetary shocks only if firms acquire little information about monetary policy.

This finding begs the following question: Is it plausible that firms acquire so little information about monetary policy? The answer to this question is no. I reach this conclusion by studying an ICKM augmented with a signal-to-noise schedule. In this model, firms choose

[6] The use of VARs to estimate the impact of money on the economy was pioneered by Sims (1972, 1980). See also Leeper *et al.* (1996) and Christiano *et al.* (1999).

the optimal signal-to-noise ratios along that schedule. Since this type of schedule is widely used in the literature of rational inattention (Sims, 2003), I will call this augmented ICKM rational-inattention model (RIM). A justification for considering this particular type of schedule stems from the maintained assumptions of RIMs: (1) firms cannot attend perfectly to all available information; (2) information needs to be processed by firms before being used for decision-making; (3) firms face limitations on the overall amount of information they can process; (4) an information-theoretic measure, proposed by Sims (2003), is used to quantify the overall amount of processed information. In my RIM, the signal-to-noise ratios turn out to be sufficient statistics for the overall amount of processed information about the two shocks. Hence, the schedule is a set of pairs of signal-to-noise ratios that imply the same overall amount of processed information. I calibrate the schedule to include the signal-to-noise ratios of the estimated ICKM. I then solve for the optimal signal-to-noise ratios in this schedule predicted by the RIM. I find that the marginal value of information about monetary policy is much higher than that about technology at the ICKM point of the schedule. This result suggests that the signal-to-noise ratio relative to monetary policy is implausibly small in the estimated ICKM.

This is the first paper that obtains Bayesian estimates for the parameters of an ICKM à la Woodford (2002). This empirical approach is motivated by the aim of countering the lack of empirical guidance in selecting the size of information frictions, which strongly influences the speed of adjustment of variables to shocks in this type of models.

Furthermore, the paper investigates what the ICK mechanism of generating persistence adds or tacks away from more popular mechanisms, such as Calvo sticky prices. To this end, I consider a sticky-price model à la Calvo (1983) (Calvo model). This model differs from the ICKM in only two respects: (1) firms are perfectly informed, and (2) firms can re-optimize their prices only at random periods, as in Calvo (1983). I estimate the Calvo model through Bayesian techniques. First, I find that, unlike the ICKM and the VAR, the

Calvo model fails to generate hump-shaped responses of output and inflation to monetary shocks. Second, the ICKM fits the data moderately better than the Calvo model.

This paper depart from Woodford (2002) in several respects. First, I study a DSGE model, while Woodford (2002) analyze a price-setting model. Second, Woodford's model has one rather than two shocks. Having an additional shock allows me to get around the problem of stochastic singularity when I evaluate the likelihood function. Specifically, I consider a monetary shock and an aggregate technology shock. Finally, my empirical strategy is likelihood-based, while Woodford (2002) calibrates the parameters of his model.

This paper is also related to the literature of rational inattention (Sims, 2003, 2006; Luo, 2008; Paciello, 2008; Van Nieuwerburgh and Veldkamp, 2008; Woodford, 2008; Maćkowiak *et al.*, 2009; and Maćkowiak and Wiederholt, forthcoming). Maćkowiak and Wiederholt (2009) introduce a model where firms optimally decide how much attention to pay to aggregate and idiosyncratic conditions, subject to a constraint on information flows. When they calibrate their model to match the average absolute size of price changes observed in micro data, they find that nominal shocks have sizeable and persistent real effects.

The remainder of the paper proceeds as follows. Section 2 presents the ICKM and the Calvo model. Section 3 shows the Bayesian analysis of these two models and the VAR. I assess the plausibility of the ICKM's prediction on firms' processed information about monetary policy in section 4. In section 5, I conclude.

2 The models

In this section I describe two DSGE models. The first model is a model with imperfect common knowledge à la Woodford (2002). In this model, information-processing frictions are modelled by assuming that firms have to solve a signal extraction problem in order to estimate the state of the aggregate technology and that of monetary policy. In the second model, all agents have perfect information but they can re-optimize their prices only at random periods, as in Calvo (1983).

2.1 The common structure

The economy is populated by households, firms, financial intermediaries, and a monetary authority (or central bank). Households derive utility from consumption and disutility from supplying labor to firms. Furthermore, households face a cash-in-advance (CIA) constraint. Firms sell differentiated goods to households in a monopolistic competitive environment with a production function that is linear in its unique input, which is labor. There is a continuum $(0, 1)$ of firms and hence of their differentiated goods, indexed by i. Furthermore, there are two shocks: an aggregate technology shock and a monetary policy shock. Financial intermediaries demand deposits from households and supply liquidity facilities to firms in perfectly competitive markets. The monetary authority sets the growth rate of money in the economy.

At the beginning of period t, the households inherit the entire money stock of the economy, M_t. As in Lucas (1982), households decide their money holdings after observing current period innovations to technology and monetary shocks. The households decide how much money D_t to deposit at the financial intermediaries. These deposits yield interest at rate $R-1$. The financial intermediaries receive household deposits and a monetary injection from the monetary authority, which it lends to firms at a fixed fee τ_t. The firms set prices, hire labor service from households, and produce. They use the liquidity facilities provided by the financial intermediaries so as to pay wages W_tH_t, where W_t is the nominal hourly wage, and H_t is hours worked. Households' cash balance increases to $M_t - D_t + W_tH_t$. The CIA constraint requires that households pay for all consumption purchases with the accumulated cash balances. Firms sell their goods to households and then pay back their loans, $L_{i,t}$. Finally, households receive back their deposits inclusive of interest rate and dividends from firms, Π_t.

2.1.1 The representative household

The representative household derives utility from composite consumption C_t relative to a habit stock. I assume that the habit stock is given by the level of technology A_t . This assumption ensures that the economy evolves along a balanced growth path even if the utility function is additively separable in consumption and leisure. The household derives disutility from hours worked H_t and maximizes

$$\max_{\{C_{i,t},H_t,M_t,D_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\frac{[C_{t+s}/A_{t+s}]^{(1-\phi)} - 1}{1-\phi} - \alpha H_{t+s} \right]$$

such that

$$P_t C_t = M_{t-1} + W_t H_t - D_t \tag{1}$$

$$0 \leq D_t \tag{2}$$

$$C_t = \left(\int_0^1 (C_{i,t})^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} \tag{3}$$

$$M_t = (M_{t-1} + W_t H_t - D_t - P_t C_t) + R_t D_t + \Pi_t \tag{4}$$

where C_t is the amount of the composite consumption at time t, $C_{i,t}$ denoted the amount of the differentiated good produced by the firm i at time t, P_t is the price of the consumption good at time t, β is the discount factor, and $1/\phi$ is the intertemporal elasticity of substitution.

The first constraint is the CIA constraint requiring that the household has to hold money up-front to finance her consumption. The second constraint prevents households from borrowing from the financial intermediaries. The third constraint is the Dixit-Stiglitz aggregator of consumption varieties. The fourth constraint is the law of motion of households' money holdings.

2.1.2 The technology of firms

Every firm has the same technology:

$$Y_{i,t} = A_t N_{i,t} \tag{5}$$

where $Y_{i,t}$ is the output produced by the firm i at time t, and $N_{i,t}$ is the labor input demanded by firm i at time t.

I further assume that the aggregate technology A_t follows a random walk with drift:

$$\ln A_t = A_0 + \ln A_{t-1} + \sigma_a \varepsilon_{a,t} \tag{6}$$

where $\varepsilon_{a,t} \backsim \mathcal{N}(0,1)$. Finally, it turns out to be useful to define:

$$a_t \equiv \ln A_t - A_0 \cdot t \tag{7}$$

2.1.3 Financial intermediaries

Financial intermediaries solve the trivial problem:

$$\max_{\{L_t, D_t\}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s Q_{t+s} \Pi^b_{t+s} \right] \tag{8}$$

st

$$\Pi^b_t = (1 - R_t) D_t + X_t + \tau_t \cdot \mathbb{I}\{L_t > 0\} \tag{9}$$

$$L_t \leq X_t + D_t \tag{10}$$

where Q_t is the time 0 value of a unit of the consumption good in period t to the representative household, L_t is the aggregate amount of liquidity supplied to firms $L_t = \int L_{i,t} di$, and $X_t = M_{t+1} - M_t$ is the monetary injection.

2.1.4 The monetary authority

The monetary authority lets the money stock M_t grow at rate

$$\Delta \ln M_t = (1 - \rho_m) M_0 + \rho_m \Delta \ln M_{t-1} + \sigma_m \varepsilon_{m,t} \tag{11}$$

with $\varepsilon_{m,t} \backsim \mathcal{N}(0,1)$ and where Δ stands for the first-difference operator, the degree of smoothness in conducting monetary policy ρ_m is such that $\rho_m \in [0,1)$. M_0 is a parameter that represents the long-run average growth rate of $\ln M_t$.

Equation (11) can be interpreted as a simple monetary policy rule without feedbacks. The innovations $\varepsilon_{m,t}$ capture unexpected changes of the money growth rate. Changes in M_0, ρ_m, and σ_m correspond to rare regime switch. Moreover, the monetary policy shock $\varepsilon_{m,t}$ is assumed to be orthogonal to the technology shock $\varepsilon_{a,t}$. Finally, it is useful to denote:

$$m_t \equiv \ln M_t - M_0 \cdot t \tag{12}$$

It is simple to show that market clearing for the monetary market requires that:

$$\ln M_t = \ln Y_t + \ln P_t \tag{13}$$

2.2 The ICKM

In the ICKM, firms do not bear any cost when they optimally adjust their prices. Nonetheless, they cannot observe any realizations of the model variables. Firms observe idiosyncratic noisy signals concerning the state of technology a_t and that of monetary policy m_t. Therefore, they will estimate the model variables by using the history of realizations of their signals. For tractability, it is assumed that the other agents (i.e., households, financial intermediaries, and the monetary authority) perfectly observe the past and the current realizations of all the model variables.

In every period, given the technology specified in equations (5)-(6), firms solve:

$$\max_{P_{i,t}} \; \mathbb{E}\left[\beta^t Q_t \left(P_{i,t} Y_{i,t} - W_t N_{i,t} - \tau_t \mathbb{I}\left\{L_{i,t} > 0\right\}\right) | \mathcal{I}_t^i\right] \tag{14}$$

$st.$

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\nu} Y_t, \qquad L_{i,t} = W_t N_{i,t} \tag{15}$$

$$\mathcal{I}_t^i = \left(\left\{\mathbf{z}_{i,\tau}\right\}_{\tau=-\infty}^{t}, \Theta_I\right) \tag{16}$$

where Q_t is the time 0 value of a unit of the consumption good in period t to the representative household, which is treated as exogenous by the firm. $\mathbb{I}\{\cdot\}$ is an indicator function that has value one if the statement within curly brackets is true. $\mathcal{I}_t^i$ is the information set available to firm i at time t. This set contains the history of the idiosyncratic signals $\{\mathbf{z}_{i,\tau}\}_{\tau=-\infty}^{t}$ and the vector of model parameters Θ_I, that is

$$\Theta_I \equiv \left(\nu, \rho_m, \alpha, A_0, M_0, \phi, \beta, \sigma_m, \sigma_a, \tilde{\sigma}_m, \tilde{\sigma}_a\right) \tag{17}$$

It is important to emphasize that I assume that at time 0 firms are endowed with an infinite sequence of signals[7]. Furthermore, the equilibrium laws of motion of all model variables are assumed to be common knowledge among firms.

Firm i's signal model is

$$\mathbf{z}_{i,t} = \begin{bmatrix} m_t \\ a_t \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_m & 0 \\ 0 & \tilde{\sigma}_a \end{bmatrix} \cdot \begin{bmatrix} e_{m,i,t} \\ e_{a,i,t} \end{bmatrix} \tag{18}$$

[7] This assumption simplifies the analysis in that firms will have the same Kalman gain matrix in their signal-extraction problem. Furthermore, this matrix can be shown to be time-invariant. This assumption makes the task of solving the model easier.

where $\mathbf{z}_{i,t} \equiv [z_{m,i,t}, z_{a,i,t}]'$, $\mathbf{e}_{i,t} \equiv [e_{m,i,t}, e_{a,i,t}]'$ and $\mathbf{e}_{i,t} \overset{iid}{\backsim} \mathcal{N}(\mathbf{0}, \mathbb{I}_2)$. Note that a_t and m_t are the exogenous state variables of the model and the signal noises $e_{m,i,t}$ and $e_{a,i,t}$ are assumed to be *iid* across firms and time.

Assuming that the two signals are orthogonal may be considered a strong assumption. After all, firms might learn about a given exogenous state variable by processing signals concerning the other state variable. I find, however, that relaxing this assumption of orthogonality of signals does not substantially affect the main predictions of the estimated model.

2.3 The Calvo model

In the Calvo model all agents perfectly observe the past and current realizations of the model variables. Moreover, the prices charged by each firm are re-optimized only at random periods. The key (simplifying) assumption is that the probability that a given firm will optimally adjust its price within a particular period is independent of the state of the model, the current price charged, and how long ago it was last re-optimized. Firms that do not re-optimize index their prices at the steady-state inflation rate.

I assume that only a fraction $(1-\theta_p)$ of firms re-optimizes their prices, while the remaining θ_p fraction adjusts them to the steady-state inflation π_*. The problem of the firms that are allowed to re-optimize their prices in period t is:

$$\max_{P_{i,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \left[\theta_p^s \beta^{t+s} Q_{t+s} \left(\pi_*^s P_{i,t} - MC_{t+s} \right) \frac{Y_{i,t+s}}{P_{t+s}} - \tau_t \mathbb{I}\left\{ L_{i,t} > 0 \right\} \right] \tag{19}$$

$st.$

$$MC_t = \frac{W_t}{A_t}, \; Y_{i,t+s} = \left(\frac{\pi_*^s P_{i,t}}{P_{t+s}} \right)^{-\nu} Y_{t+s} \tag{20}$$

where Q_{t+s} is the marginal utility of a unit of consumption at time $t+s$ in terms of the utility of the representative household at time t, and MC_{t+s} stands for the nominal marginal costs in period $t+s$. I consider only the symmetric equilibrium at which all firms will choose the same optimal price $P_{i,t} = P_t^*$. On aggregate:

$$P_t^{1-\nu} = \left[(1-\theta_p) P_t^{*(1-\nu)} + \theta_p \ (\pi_* P_{t-1})^{1-\nu} \right] \tag{21}$$

where π_* is the balance-growth-path (gross) inflation rate. I denote Θ_C as the set of parameters of the Calvo model:

$$\Theta_C \equiv (\nu, \rho_m, \alpha, A_0, M_0, \phi, \beta, \theta_p, \sigma_m, \sigma_a) \tag{22}$$

2.4 Detrending, log-linear approximation

In the two models, the exogenous processes (6) and (11) induce both a deterministic and a stochastic trend to all endogenous variables, except labor. I will detrend the non-stationary variables before log-linearizing the models. It is useful to define the stationary variables as follows:

$$y_t \equiv \frac{Y_t}{A_t}, \quad p_t \equiv \frac{A_t P_t}{M_t}, \quad p_{i,t} = \frac{A_t P_{i,t}}{M_t} \tag{23}$$

In order to log-linearize the ICKM,[8] I take the following steps. First, I derive the price-setting equation by solving firms' problem. Second, I transform the variables according to the definitions (23). Third, I log-linearize the resulting price-setting equation around the deterministic steady state. Fourth, I aggregate the log-linearized price-setting equation across firms and obtain the law of motion of price level. Fifth, the law of motion of real output can be easily obtained from combining the law of motion of price level and equation (13).

[8] How to log-linearize and solve the Calvo model is standard and hence omitted. We use the routine *gensys* developed by Sims (2002) to numerically solve this model.

2.5 Source of persistence in the ICKM

Let me introduce some notation. Firm i's expectations of order zero about the state of monetary policy are the state itself, that is $m_t^{(0)}(i) \equiv m_t$. Firm i's first-order expectations about the state of monetary policy are denoted by $m_t^{(1)}(i) \equiv \mathbb{E}\left[m_t|\mathcal{I}_t^i\right]$. Average first-order expectations about the state of monetary policy can be computed as follows $m_t^{(1)} \equiv \int m_t^{(1)}(i)\, di$. Firm i's second-order expectations are the firm i's first-order expectations of the average first order expectations, or more concisely $m_t^{(2)}(i) \equiv \mathbb{E}\left[m_t^{(1)}|\mathcal{I}_t^i\right]$. By rolling this argument forward I obtain the average j-th order expectation, for any $j \geq 0$,

$$m_t^{(j)} \equiv \int m_t^{(j)}(i)\, di \tag{24}$$

Moreover, firm i's $(j+1)$-th order expectations about the state of monetary policy, for any $j \geq 0$, are:

$$m_t^{(j+1)}(i) \equiv \mathbb{E}\left[m_t^{(j)}|I_t^i\right] \tag{25}$$

In the Calvo model, the speed of adjustment of variables to shocks is determined by the size of the Calvo parameter θ_p. In the ICKM, the speed of adjustment of variables to a shock is affected by the signal-to-noise ratio associated with that shock and the strategic complementarity in price setting. The strategic complementarity in price setting measures the extent to which firms want to react to the expected average price P_t. It is simple to show that in the ICKM the degree of strategic complementarity is determined by $1-\phi$.

The law of motion of price level in the ICKM is:

$$\ln P_t = \left[\sum_{j=0}^{\infty} (1-\phi)^j \phi \left(m_{t|t}^{(j+1)} - a_{t|t}^{(j+1)}\right)\right] - \ln \bar{y} + M_0 - A_0 \tag{26}$$

where $m_{t|t}^{(j)}$ and $a_{t|t}^{(j)}$ are the average j-th order expectations about the state of monetary policy and technology at time t. From equation (13) and equation (26) and after straightforward manipulations it is easy to derive the law of motion of real output:

$$\ln Y_t = \left[m_t - \sum_{j=0}^{\infty} (1-\phi)^j \, \phi m_{t|t}^{(j+1)} \right] + \sum_{j=0}^{\infty} (1-\phi)^j \, \phi a_t^{(j+1)} - \ln \bar{y} + A_0 \tag{27}$$

Note that both price level and output are affected by weighted averages of the infinite hierarchy of higher-order expectations about the exogenous states.

Equation (27) shows that monetary shocks have real effects in the ICKM as long as they are not fully anticipated by the average higher-order expectations of firms. More specifically, if the realization of m_t is common knowledge among firms, then $m_t^{(j)} = m_t$ for all j and the terms inside the square brackets cancel out. This shows that if monetary policy is common knowledge among firms, money is neutral in the in the ICKM.

Equations (26)-(27) make it clear that there are two sources of persistence in the ICKM. First, the sluggish adjustment of higher-order expectations and, second, the strategic complementarity in price setting. The former effect is determined by the signal-to-noise ratios that influence the precision of signals[9]. The more imprecise the signals are, the more sluggishly the average expectations of every order will respond to shocks.

The strategic complementarity (i.e. $1 - \phi$) influences the persistence of output and inflation by affecting the relative weights in the weighted averages of higher-order expectations in equations (26)-(27). More precisely, the lower the strategic complementarity is, the smaller the weights of the average expectations of higher order are. The economic intuition is that the degree of strategic complementarity affects how strongly firms want to react to prices set by other firms. The stronger firms' reaction to other firms' pricing behavior is, the more they care about what other firms think that other firms think .... about the exogenous state of the economy. In other words, strategic complementarity is the factor triggering the mechanism of forecasting the forecast of others.

[9] Since, in the ICKM, firms observe two orthogonal signals, the speed of propagation may differ between the two shocks. Evidence that macroeconomic variables react at different speed to monetary and to technology shocks has been found by Paciello (2009). Hence, this property of the ICKM is appealing.

It is important to emphasize that the signal structure (18) in the ICKM implies that signals ($\mathbf{z}_{i,t}$) provide less and less information about expectations of higher and higher order. Therefore, the higher the order of average expectations, the more sluggishly they will adjust to shocks. Since larger strategic complementarity raises the weights associated with the average expectations of higher order in equation (26)-(27), it boosts the persistence of output and inflation responses to shocks. For any given size of incomplete information, strategic complementarity in price setting plays a crucial role in amplifying the persistence in the propagation mechanism of shocks.

2.6 Model solution

When one characterizes rational expectation equilibria (REE) in models with incomplete information, a typical challenge is dealing with an infinite-dimensional state vector (*infinite regress*)[10] (Townsend, 1983b). The reason is that the laws of motion of infinitely many higher-order expectations have to be characterized in order to solve the model. This task is clearly unmanageable. In my ICKM, this problem arises when there is strategic complementarity in price-setting (i.e., $\phi > 0$). Yet, here, this issue can be elegantly resolved as in Woodford (2002), since it is possible to re-define the state of the model as a weighted average of infinitely many higher-order expectations. This leads to a state space of finite dimension[11]. A detailed description of the method that numerically solves the ICKM is in a technical appendix, which is available upon request.

[10] See Nimark (2009) for a thorough explanation of this problem.

[11] Different methods have been developed to solve dynamic models with incomplete information. Following Townsend (1983b), the customary approach of solving this class of models is to assume that the realizations of states at some arbitrary distant point in the past are perfectly revealed. Rondina and Walker (2009) have challenged this approach by showing that such a truncation leads to reveal the entire history of the realizations of states to agents, regardless of the point of truncation. See Nimark (2008) for a truncation-based method that preserves the recursive structure.

3 Empirical analysis

I fit the ICKM and the Calvo model to observations on output and price level. I place a prior distribution on parameters and conduct Bayesian inference. First, I present the data set, the measurement equations, the prior distributions and the posterior distributions for model parameters. I then address the question of whether the ICKM provides an accurate description of the propagation mechanism of monetary shocks to output and inflation. To do that, I introduce a largely parameterized VAR model. I conjecture that if the responses of output and inflation to monetary shocks implied by the ICKM closely replicate the ones implied by the VAR, the answer to the question above is positive. From a Bayesian perspective, this conjecture is sensible as long as the VAR model attains a higher posterior probability than the ICKM, as pointed out in Schorfheide (2000). I verify that this is indeed true by comparing the marginal data densities of the ICKM and the VAR. Finally, I discuss the IRFs of the three models.

3.1 The data

The data are quarterly and range from the third quarter of 1954 to the fourth quarter of 2005. I use the U.S. per capita real GDP and the U.S. GDP deflator from Haver Analytics (Haver mnemonics are in italics). Per capita real GDP is obtained by dividing the nominal GDP (*GDP*) by the population 16 years and older (*LN16N*) and deflating using the chained-price GDP deflator (*JGDP*). The GDP deflator is given by the appropriate series (*JGDP*).

3.2 Measurement equations

Denote the U.S. per capita real GDP, and the U.S. GDP deflator as $\{GDP_t,\ t = 1, 2, ...T\}$, and $\{DEFL_t,\ t = 1, 2, ...T\}$, respectively. The measurement equations are:

$$\ln GDP_t = \widehat{y}_t + a_t + A_0 \cdot t + \ln \overline{y} \tag{28}$$

$$\ln DEFL_t = \widehat{p}_t + m_t - a_t + (M_0 - A_0) \cdot t + \ln \overline{p} \tag{29}$$

where the subscript $\widehat{}$ means log-deviations of a variable from its perfect-information symmetric steady-state value, $\ln \overline{y}$ is the logarithm of the steady-state value of y_t, and $\ln \overline{p}$ is the logarithm of the steady-state value of p_t.

The Kalman filter can be used to evaluate the likelihood function of the models. Yet, the filter must be initialized and a distribution for the state vector in period $t = 0$ has to be specified. As far as the vector of stationary state variables is concerned, I use their unconditional distributions. I cannot initialize the vector of non-stationary state variables (i.e. m_t, a_t) in the same manner, since their unconditional variance is not defined. I follow the approach introduced by Chang *et al.* (2007), who propose to factorize the initial distribution as $p(\mathbf{s}_{1,t})\, p(\mathbf{s}_{2,t})$, where $\mathbf{s}_{1,t}$ and $\mathbf{s}_{2,t}$ are the vector of stationary and non-stationary variables, respectively. They suggest setting the first component $p(\mathbf{s}_{1,t})$ equal to the unconditional distribution of $\mathbf{s}_{1,t}$, whereas the second component $p(\mathbf{s}_{2,t})$ is absorbed into the specification of the prior.

3.3 Prior distributions

I use the same prior distributions for those parameters that are common in the two models. I fix the value of ν equal to 10. This implies a mark-up of about 11%, which is in line with what is suggested by Woodford (2003). Moreover, the discount factor, β, is fixed to 0.99, which is a plausible discount factor when the model periods are interpreted as quarters. Table 1a shows the prior distributions for the parameters used in both the ICKM and the Calvo model.

The parameter ϕ affects the elasticity of substitution. I center the prior for this parameter at one. This value implies a logarithmic utility function in consumption as it is widely used in the DSGE literature. As shown in section , this parameter turns out to influence the strategic complementarity in price setting and hence has a crucial role in generating persistence in the model Woodford (2002).]

The models imply that the stock of M_t is equal to the nominal output. See equation (13). Hence, the autoregressive parameter of monetary policy, ρ_m, and the standard deviation of the monetary policy shock, σ_m, can be estimated by using presample observations of the (detrended) U.S. per capita real GDP and the (detrended) U.S. GDP deflator. This presample dataset is obtained from Haver Analytics and ranges from the first quarter of 1949 to the second quarter of 1954.

The prior of the standard deviation of the technology shock, σ_a, is centered at 0.007. This value is regarded as plausible by the real business cycle literature (Prescott, 1986).

In absolute terms, I set the priors for standard deviations of signal noise, $\tilde{\sigma}_m$, and $\tilde{\sigma}_a$, so as to ensure that signals are quite informative about the business-cycle-frequency variations of model variables[12]. In relative terms, these prior specifications are chosen so as to make the two signals about equally informative about the corresponding state variables[13].

The prior for the Calvo parameter θ_p is centered at 0.67, implying an average duration of price contracts of three quarters. This value is regarded as consistent with the survey evidence discussed in Blinder *et al.* (1998). The parameter α is not identifiable, since I do not have hours worked among my observables.

Table 1b presents the implied prior distributions for the strategic complementarity in price settings and the signal-to-noise ratios. As discussed in section 2.5, these parameter values crucially influence the persistence in the ICKM. Note that the prior median for $1-\phi$

[12] We achieve that by setting the prior medians of the coherences between the process of the state variables, in first difference, and their corresponding signals such that these are not smaller than 0.50 at business-cycle frequencies (3-5 years). The coherence ranges from 0 to 1 and measures the degree to which two stationary stochastic processes are jointly influenced by cycles of a given frequency (Hamilton, 1994).

[13] I quantify the amount of information that signals convey about states as in Sims (2003). The formal definition of this measure is provided in section 4.1.

implies no strategic complementarity in price settings. However, note that this prior is quite broad. This implies that this crucial parameter value will be mainly learned from the likelihood. Finally, note that since the processes driving the two exogenous states are different, the prior medians for the signal-to-noise ratios do not have to be the same to make the firms equally informed about the two states.

3.4 Posterior distributions

Given the priors and the likelihood functions implied by the models, a closed-form solution for the posterior distributions for parameters cannot be derived. However, I are able to evaluate the posteriors numerically through the random-walk Metropolis-Hastings algorithm. How these procedures apply to macro DSGE models is exhaustively documented by An and Schorfheide (2007). I generate $1,000,000$ draws from the posteriors.

The posterior medians and 90% credible sets are shown in table 2. The posterior median of the Calvo parameter θ_p implies that firms reset their prices about every two years. This frequency of price adjustments is implausible, according to the existing microeconometric analyses on price changes. Nonetheless, this result is not surprising. In fact, it is well-known that small-scale DSGE models with sticky prices à la Calvo can match the persistence of the macro data only with price contracts of very long duration (Bils and Klenow, 2004). I might fix this problem by setting a tighter prior for the Calvo parameter, but I find that this would seriously undermine the fit of the Calvo model.

The coefficient $(1 - \phi)$ controls the degree of strategic complementarity in price settings. As shown in section 2.5, this coefficient is very important, since it affects the persistence of the impulse response functions (IRFs) of output and price level to shocks. The prior median of strategic complementarity $(1 - \phi)$ was set at 0 (i.e., the utility function is logarithmic in consumption). Hence, Bayesian updating points toward more strategic complementarity in price setting. This raises the persistence in the mechanism of shock propagation for any finite values of signal-to-noise ratios. Figure 1 compares the prior and the posterior distributions for the strategic complementarity $(1-\phi)$. It is apparent that the Bayesian updating clearly pushes this parameter value to a larger strategic complementarity than what is conjectured in the prior.

Moreover, the posterior median of the signal-to-noise ratio regarding the state of monetary policy, $\tilde{\sigma}_m/\sigma_m$, is large relative to that associated with the state of technology, $\tilde{\sigma}_a/\sigma_a$. These estimates imply that the signal regarding the state of technology is more precise than the signal concerning the state of monetary policy. The signal to noise ratio concerning the state of monetary policy is smaller by a factor of six.

3.5 MDD-based comparisons

From a Bayesian perspective, the issue of whether the ICKM fits the data better than the Calvo model can be addressed by comparing the marginal data densities (MDDs) of these two models. This measure is widely used for Bayesian comparisons of non-nested models (Kass and Raftery, 1995, Schorfheide 2000, and An and Schorfheide, 2007). The fact that Calvo model has one parameter less than the ICKM is not problematic, since MDD-based comparisons penalize models for their number of parameters.

Let me denote the ICKM and the Calvo model with $\mathcal{M}_I$ and $\mathcal{M}_C$, respectively. The data used for estimation are denoted by $\tilde{Y} = \{\ln GDP_t, \ln DEFL_t\,,\ t = 1, \ldots T\}$. The MDDs for the ICKM, $P\left(\tilde{Y}|\mathcal{M}_I\right)$, and the Calvo model, $P\left(\tilde{Y}|\mathcal{M}_C\right)$, are defined as:

$$P\left(\tilde{Y}|\mathcal{M}_I\right) = \int \mathcal{L}\left(\Theta_I|\tilde{Y}, \mathcal{M}_I\right) p\left(\Theta_I|\mathcal{M}_I\right) d\Theta_I \tag{30}$$

$$P\left(\tilde{Y}|\mathcal{M}_C\right) = \int \mathcal{L}\left(\Theta_C|\tilde{Y}, \mathcal{M}_C\right) p\left(\Theta_C|\mathcal{M}_C\right) d\Theta_C \tag{31}$$

where $\mathcal{L}(\cdot)$ stands for the likelihood function, and $p(\cdot|\cdot)$ denotes the posterior distribution. The model with the largest marginal data density is the one that fits the data better. I use Geweke's harmonic mean estimator (Geweke, 1999) to approximate the MDDs of these two

DSGE models.

Moreover, I consider a vector autoregressive model VAR(4):

$$\tilde{\mathbf{Y}}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \tilde{\mathbf{Y}}_{t-1} + \mathbf{\Phi}_2 \tilde{\mathbf{Y}}_{t-2} + \mathbf{\Phi}_3 \tilde{\mathbf{Y}}_{t-3} + \mathbf{\Phi}_4 \tilde{\mathbf{Y}}_{t-4} + \boldsymbol{\epsilon}_t \tag{32}$$

where $\tilde{\mathbf{Y}}_t = [\ln GDP_t, \ln DEFL_t]'$ and $\mathbf{\Sigma}_\epsilon \equiv \mathbb{E}\left(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t'\right)$. I fit this VAR(4) to the same data set as that presented in section 3.1. The Minnesota random walk prior (Doan *et al.*, 1984) is implemented in order to obtain a prior distribution for the VAR parameters. Moreover, I obtain $100,000$ posterior draws through Gibbs sampling. In order to compute the MDD of the VAR model I apply the method introduced by Chib (1995).

The log of the MDDs of the VAR and those of the two DSGE models are reported in table 3. The ICKM has a larger MDD than Calvo model. This implies that the model of imperfect common knowledge fits the data better than the Calvo model. From this result, it follows that the ICKM is better than Calvo model in approximating the true probability distribution of the data generating process under the Kullback-Leibler distance (Fernández-Villaverde and Rubio-Ramírez, 2004).

Moreover, the VAR strongly outperforms both DSGE models in fitting the data. This result is not surprisingly, since the two DSGE models are very stylized compared to this statistical model. Since the VAR attains the largest marginal likelihood, it can be regarded as a reliable benchmark in a Bayesian evaluation of the relative performance of the two DSGE models in approximating the responses of output and inflation to monetary disturbances (Schorfheide, 2000). This is the object of the next section.

3.6 IRF-based comparisons

In order to identify the monetary shock in the VAR, I use the restriction that monetary policy has no long-run real effects (e.g., Blanchard and Quah, 1989). Note that this identification scheme is consistent with both the ICKM and the Calvo model.

The IRFs of real output and inflation to a monetary shock implied by the VAR and the two DSGE models are plotted in figures 2 and 3, respectively. The size of the shock is normalized so as the reaction of variables upon impact is the same in all models. As also found by other studies (e.g., Christiano *et al.*, 2005), the VAR-based IRFs document highly persistent and hump-shaped effects of monetary disturbances upon output and inflation.

The Calvo model does not seem to be well-suited to accounting for the hump-shaped pattern of the VAR response, whereas the ICKM appears to be successful in this respect. Moreover, it is worthwhile noticing that the IRF of real output implied by the ICKM peaks three quarters after the occurrence of the shock, exactly as suggested by the benchmark VAR. On the contrary, the Calvo model predicts that the largest response of real output arises two quarters after the occurrence of the shock.

Furthermore, the VAR IRF emphasizes the presence of delayed effects of monetary shocks on inflation, which do not seem to be quite captured by the two DSGE models. The IRF of inflation implied by the VAR reaches its peak after four quarters, while, according to the ICKM, this happens after three quarters.

The ICKM - albeit very stylized - successfully capture the persistent and hump-shaped response of output and inflation to monetary shocks implied the broadly parameterized VAR. This leads me to conclude that the estimated ICKM provides an accurate description of the propagation mechanism of monetary shocks.

4 A deeper look at the source of persistence in the ICKM

As discussed in section 2.5, in the ICKM, the persistence of the propagation of monetary shocks depends on the degree of strategic complementarity, $(1-\phi)$, and the signal-to-noise ratio of monetary policy $\sigma_m/\tilde{\sigma}_m$. The larger the signal-to-noise-ratio is, the faster firms learn about the occurrence of a monetary shock, and then, *ceteris paribus,* the larger the speed of adjustment of variables to monetary shocks is. In section 3.4, I show that the posterior median for the signal-to-noise ratio of monetary policy is smaller than that of technology by a factor of six (see table 2).

I then ask the following question: is it plausible that firms acquire so little information about monetary policy? To answer this question, I augment the ICKM with a signal-to-noise schedule. This type of schedule is extensively used in the literature of rational inattention (Sims, 2003). Henceforth, I will call this augmented ICKM rational-inattention model (RIM). I calibrate the schedule to include the estimated signal-to-noise ratios of the ICKM. In the RIM, firms are allowed to choose the optimal signal-to-noise ratios concerning monetary policy and technology along that schedule. The ultimate objective is to compare the marginal value of information on monetary policy relative to technology at those points of the schedule that are predicted by the RIM and by the estimated ICKM. The more similar these marginal values are, the more plausible the estimated signal-to-noise ratios in the ICKM have to be regarded under the lens of the rational-inattention theory.

4.1 Signal-to-noise schedule

Rational-inattention models rely upon four maintained assumptions. First, information about all model variables is freely available to decision makers. Second, information needs to be processed before being used for decision-making. Third, firms face limitations on the amount of information they can process per unit of time. Fourth, an information-theoretic measure is used to quantify the amount of processed information, as proposed by Sims (2003).

The measure proposed by Sims (2003) quantifies the reduction of uncertainty about states that occurs after having observed the last realization of signals. More formally,

$$\kappa \equiv H\left(m_t, a_t | z_{m,i}^{t-1}, z_{a,i}^{t-1}\right) - H\left(m_t, a_t | z_{m,i}^{t}, z_{a,i}^{t}\right) \tag{33}$$

where $H(\cdot)$ denotes the conditional entropy, which measures the uncertainty about a random variable. The conditional entropy is defined as

$$H\left(m_t, a_t | z_{m,i}^{\tau}, z_{a,i}^{\tau}\right) = \int\int \log_2\left[p\left(m_t a_t | z_{m,i}^{\tau}, z_{a,i}^{\tau}\right)\right] p\left(m_t a_t | z_{m,i}^{\tau}, z_{a,i}^{\tau}\right) dm_t da_t$$

where $p\left(m_t | z_{1,i}^{t}\right)$ is the conditional probability density function of m_t.

Since signals and exogenous states are orthogonal, it is easy to see that $\kappa = \kappa_m + \kappa_a$, where κ_m and κ_a stand for the information flows regarding monetary policy and technology, respectively. These information flows are defined as:

$$\kappa_m \equiv H\left(m_t | z_{m,i}^{t-1}\right) - H\left(m_t | z_{m,i}^{t}\right) \tag{34}$$

$$\kappa_a \equiv H\left(a_t | z_{a,i}^{t-1}\right) - H\left(a_t | z_{a,i}^{t}\right) \tag{35}$$

I can define the mappings g_m and g_a that link signal-to-noise ratios and information flows[14] as follows:

$$\kappa_m = g_m\left(\sigma_m, \tilde{\sigma}_m, \rho_m\right), \quad \kappa_a = g_a\left(\sigma_a, \tilde{\sigma}_a\right) \tag{36}$$

Hence, for any given autocorrelation parameter of the growth rate of money, ρ_m, the signal-to-noise ratios turn out to be sufficient statistics for the overall amount of information processed about the two exogenous states. Therefore, the signal-to-noise schedule can be defined as a set of pairs of signal-to-noise ratios that imply the same overall amount of processed information.

Given the mappings in (36) and the prior (posterior) draws for the parameter of the ICKM, I can approximate the moments of the prior (posterior) distribution for the information flows κ_m and κ_a through standard Monte Carlo methods. Table 4 shows the prior and posterior medians for those parameters and their 90% credible intervals.

Figure 4 compares the prior and the posterior distributions of the allocation of attention to technology, that is $\kappa_a / \left(\kappa_m + \kappa_a\right)$. This graphical comparison emphasizes that, starting

[14] The mapping g_a can be analytically derived, while the mapping g_m can be computationally approximated. A technical appendix on this issue is available upon request.

from a very broad and agnostic prior for the allocation of attention, the posterior distribution attributes a large portion of firms' attention towards technology (about 82%). Hence, according to the data, the adjustment of output and inflation to monetary shocks is rather slow in the estimated ICKM, as confirmed by figures 2 and 3. That firms process rather little information about monetary policy in the estimated ICKM lines up with the relative size of the estimated signal-to-noise ratios shown in table 2. Furthermore, in figure 4 the posterior appears to be far tighter than the prior, suggesting that the data are informative about this parameter.

4.2 The rational inattention model

In period zero[15], firms choose the precision of their signals[16] by solving:

$$\max_{\kappa_m,\kappa_a} \mathbb{E}\left[\sum_{t=1}^{\infty} \beta^t Q_t \hat{\pi}_t \left(P^*_{i,t}, P_t, m_t, a_t, \tau_t\right) |\mathbf{z}_i^0\right], \tag{37}$$

st

$$P^*_{i,t} \equiv \mathbb{E}\left[(1-\phi)\ln P_t + \phi m_t - \phi a_t |\mathbf{z}_i^t\right] \tag{38}$$

$$\mathbf{z}_{i,t} = \begin{bmatrix} m_t \\ a_t \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_{m,i} & 0 \\ 0 & \tilde{\sigma}_{a,i} \end{bmatrix} \mathbf{e}_{i,t} \tag{39}$$

$$\tilde{\sigma}_m = g_m^{-1}\left(\kappa_m, \sigma_m, \rho_m\right), \quad \tilde{\sigma}_a = g_a^{-1}\left(\kappa_a, \sigma_a\right) \tag{40}$$

$$\kappa_m + \kappa_a \leq \kappa, \text{ any t} \tag{41}$$

[15] Firms are not allowed to reconsider the allocation of attention in any period after $t = 0$. Since firms' period profit function is quadratic and all shocks are Gaussian, it can be shown that this assumption does not give rise to a problem of time inconsistency of firms' policies. See Maćkowiak and Wiederholt (forthcoming).

[16] Since (1) the period profit function is quadratic, (2) all shocks are Gaussian and (3) firms are assumed to have received an infinite sequence of signals at time $t = 0$, the objective function of the allocation-of-attention problem can be shown to be the same across firms. See Maćkowiak and Wiederholt (forthcoming). Thus, every firm will find it optimal to choose the same allocation of attention. These three assumptions are also sufficient to obtain that the information flows, κ_m and κ_m, do not vary over time in the information-processing constraint (41).

where $\hat{\pi}_t(\cdot)$ is the log-quadratic approximation of the period profit function (14) and $\mathbf{e}_{i,t} \overset{iid}{\backsim} \mathcal{N}(\mathbf{0}, \mathbb{I}_2)$. The model economy is assumed to be at its deterministic steady-state in period 0. Moreover, I assume that firms have received an infinite sequence of signals at time 0. Note also that the mapping $g_m^{-1}(\cdot)$ and $g_a^{-1}(\cdot)$ are the inverse of the functions (36). The constraint (41) is the information-processing constraint and sets an upper-bound to the overall amount of information firms can process in every period t.

In this problem, firms decide how to allocate their overall available attention, which is quantified by the parameter κ, between observing monetary policy and technology. It is worthwhile emphasizing that one only needs to set the value of the free parameter κ in order to pin down the schedule of information bundles (κ_m, κ_a) implied by the RIM. I use the estimated value of this parameter κ in the ICK (see table 4) so as to calibrate the schedule. Then I solve for the optimal information flows (κ_m^*, κ_a^*) along this schedule by solving the allocation-of-attention problem (37)-(41).

Note that when firms decide how to allocate their attention, they are aware that this choice will affect their period profit function (14) and in turn the optimal price-setting policy (38) in any subsequent period.

The marginal rate of profit is defined as:

$$\text{MRP} \equiv \frac{\partial \Pi / \partial \kappa_m}{\partial \Pi / \partial \kappa_a}$$

where Π is the sum of discounted profits:

$$\Pi \equiv \sum_{t=1}^{\infty} \beta^t Q_t \hat{\pi}_t \left(P_{i,t}^*, P_t, m_t, a_t \right)$$

It is very simple to see that the MRP in the RIM is equal to unity. In the estimated ICKM, however, MRP may be different from one. The reason is that the estimated information flows (κ_m, κ_a) may differ from the optimal ones that are predicted by the RIM (κ_m^*, κ_a^*). In fact, at the posterior medians, the MRP in the ICKM is 48.20, hugely bigger than in the RIM. Moreover, I compute the value of the profit Π at the optimal information flows (κ_m^*, κ_a^*) and at the estimated (κ_m, κ_a). I find that the profit Π in the ICKM is about 2.5 times smaller than in the RIM. These results lead me to conclude that the estimated ICKM implies that firms acquire implausibly too little information about monetary policy.

4.3 A robustness check

In the RIM the allocation of attention to technology, that is $\kappa_a^* / (\kappa_m^* + \kappa_a^*)$, is larger than that estimated in the ICKM. This implies that variables adjust to monetary shocks faster in the RIM than in the ICKM. Now the crucial question is: does the ICKM model really requires that firms acquire implausibly little information about monetary policy to match the persistent adjustment of variables to monetary shocks?

In order to answer this question, I set the RIM's parameters to be equal to the posterior medians of the ICKM parameters (see table2). I then solve[17] the RIM and obtain the optimal information flows. They are $\kappa_m^* = 0.33$ and $\kappa_a^* = 0.22$. These information flows significantly differ from the posterior medians of κ_m and κ_a in the ICKM (see table 4). Then, given the estimated parameters in table 2 and the optimal information flows, κ_m^* and κ_a^*, I compute the IRFs implied by the RIM.

Figures 5-8 show the IRFs of output and inflation to a monetary shocks and to a technology shock implied by the ICKM, the RIM, and the benchmark VAR. In Figures 5-6 and 8 the size of the shock is normalized so as to produce the same impact upon variables in every model. In figure 7 the size of the shock is set so as to induce the same long-run response of output in the three models.

[17] The rational inattention model is solved in four steps. First, I guess the values of the information flows κ_m and κ_a. Second, given this guess, I numerically characterize the law of motion of the price level exactly as I do when solving the ICKM (see section 2.6). Third, I obtain the optimal information flows, k_m^* and κ_a^*, by solving the allocation-of-attention problem in (37)-(41). Fourth, I check whether $\|\overrightarrow{\kappa} - \overrightarrow{\kappa}^*\| < \varepsilon$, for vectors $\overrightarrow{\kappa} \equiv (\kappa_m, \kappa_a)'$ and $\overrightarrow{\kappa}^* \equiv (\kappa_m^*, \kappa_a^*)'$, with $\varepsilon > 0$ and small. If this criterion is not satisfied, I do another loop by setting the guess $\overrightarrow{\kappa} = \overrightarrow{\kappa}^*$. Otherwise, STOP.

Speeds of adjustment in the two structural models are very different. Specifically, output and inflation adjust very fast to monetary policy shocks in the RIM. This is not consistent with what is documented by the VAR. Hence, I conclude that the ICKM requires that firms acquire implausibly little information about monetary policy in order to generate the persistent propagation of monetary disturbances that is found in the data.

Furthermore, both the ICKM and the RIM do not quite capture the response of output and inflation to the technology shocks that is implied by the benchmark VAR. The ICKM seems to do worse than the RIM in capturing the speed of adjustment to technology shocks that is implied by the VAR. Furthermore, a notable fact of the RIM IRFs is that output and inflation respond in an hump-shaped fashion as found in the IRFs of the benchmark VAR and in many other empirical studies (e.g., Christiano *et al.*, 1999).

5 Concluding remarks

I develop a DSGE model with imperfect common knowledge in the sense of Woodford (2002). The model features two aggregate shocks: a monetary shock and a technology shock. I obtain Bayesian estimates for the model parameters. I find that even though the model is very stylized, its impulse response functions of real output and inflation to a monetary shock closely match those implied by a largely parameterized VAR. Quite remarkably, output and inflation reacts in an hump-shaped and persistent fashion to monetary shocks, as it is widely documented by other influential empirical studies (e.g., Christiano *et al.*, 1999).

Nonetheless, I argue that the estimated model predicts that firms acquire little information about monetary shocks to an extent that is not plausible. I draw this conclusion from evaluating a simplified rational-inattention model à la Sims (2003). This model is an imperfect-common knowledge model where firms are allowed to choose the optimal information flows about the two shocks along a schedule that is commonly used in the literature of rational inattention. I show that the marginal value of information about monetary policy is much higher than that about technology at the point on the schedule predicted by the estimated imperfect-common-knowledge model. Furthermore, I find that the imperfect-common-knowledge model requires that firms acquire implausibly little information about monetary policy to generate the persistent propagation of monetary disturbances that is observed in the data.

Yet I believe that it would be wrong to conclude that imperfect-common-knowledge models à la Woodford (2002) have no chance to generate persistent adjustments of model variables under plausible parameterizations. There is margin to modify the standard imperfect-common-knowledge model so as to make its predictions on firms' allocation of attention among shocks plausible and, at the same time, to retain the persistent propagation of monetary shocks. A highly promising approach for future research is to fit an imperfect-common-knowledge model that features aggregate and idiosyncratic shocks to a data set that includes micro data on price changes. Idiosyncratic shocks are expected to capture the large average absolute size of price changes at the micro level. Rational inattentive firms will find it optimal to pay a lot of their attention to these high-volatile disturbances and less to low-volatile aggregate shocks. For instance, Maćkowiak and Wiederholt (forthcoming), Maćkowiak *et al.*, 2009, and Maćkowiak and Wiederholt (2008) have incorporated high-volatile idiosyncratic shocks into rational-inattention models. When these papers calibrate their models to micro data, they find that rational-inattention models can indeed generate sluggish propagations of nominal shocks.

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Tables and Figures (intended for publication)

Table 1a: Prior distributions

Name	Range	Density	Median	90% Interval
ρ_m	$[0,1)$	Beta	0.50	$[0.18, 0.83]$
A_0	$\mathbb{R}$	Normal	0.00	$[-0.41, 0.41]$
M_0	$\mathbb{R}$	Normal	0.00	$[-0.41, 0.41]$
ϕ	$\mathbb{R}+$	Gamma	1.00	$[0.24, 1.74]$
$100\sigma_m$	$\mathbb{R}+$	InvGamma	1.6	$[0.44, 12.82]$
$100\sigma_a$	$\mathbb{R}+$	InvGamma	0.7	$[0.51, 0.87]$
$100\tilde{\sigma}_m$	$\mathbb{R}+$	InvGamma	5.02	$[2.11, 7.92]$
$100\tilde{\sigma}_a$	$\mathbb{R}+$	InvGamma	1.07	$[0.24, 1.87]$
θ_p	$[0,1)$	Beta	0.67	$[0.37, 0.99]$

Table 1b: Implied prior distributions (ICKM)

Name		ICKM	
		Median	90% Interval
$1-\phi$	strategic complementarity	0.00	$[-0.50, 0.63]$
$\sigma_m/\tilde{\sigma}_m$	signal-to-noise ratio MP	0.56	$[0.06, 3.64]$
$\sigma_a/\tilde{\sigma}_a$	signal-to-noise ratio tech.	0.95	$[0.10, 1.80]$

Table 2: Posterior distributions

	ICKM		Calvo Model	
Name	Median	90% Interval	Median	90% Interval
ρ_m	0.34	$[0.24, 0.45]$	0.24	$[0.15, 0.33]$
$100A_0$	0.45	$[0.36, 0.55]$	0.43	$[0.11, 0.74]$
$100M_0$	1.34	$[1.18, 1.49]$	1.34	$[1.20, 1.48]$
ϕ	0.41	$[0.06, 0.77]$	0.80	$[0.13, 1.58]$
$100\sigma_m$	0.88	$[0.81, 0.91]$	0.89	$[0.82, 0.97]$
$100\sigma_a$	0.86	$[0.70, 1.02]$	2.66	$[2.04, 3.36]$
$100\tilde{\sigma}_m$	9.75	$[4.40, 15.01]$	–	–
$100\tilde{\sigma}_a$	1.45	$[0.60, 2.31]$	–	–
θ_p	–	–	0.88	$[0.82, 0.94]$
$1-\phi$	0.64	$[0.07, 0.95]$	–	–
$\sigma_m/\tilde{\sigma}_m$	0.10	$[0.04, 0.15]$	–	–
$\sigma_a/\tilde{\sigma}_a$	0.63	$[0.33, 0.96]$	–	–

Table 3: Logarithms of Marginal Data Densities (MDDs)

	Models		
	ICKM	Calvo	VAR(4)
log MDD	1547.01	1529.38	1727.04

Table 4 Implied prior and posterior distributions

	Prior		
Variables	Descriptions	Median	90% Interval
κ_m	information flow MP	0.52	$[0.07, 2.51]$
κ_a	information flow tech.	0.67	$[0.12, 1.23]$
$\kappa_m + \kappa_a$	overall level of attention	1.28	$[0.29, 3.20]$
$\frac{\kappa_a}{\kappa_m+\kappa_a}$	allocation of attention to tech.	0.53	$[0.11, 0.82]$
	Posterior		
Variables	Descriptions	Median	90% Interval
κ_m	information flow MP	0.10	$[0.05, 0.18]$
κ_a	information flow tech.	0.45	$[0.22, 0.79]$
$\kappa_m + \kappa_a$	overall level of attention	0.55	$[0.27, 0.94]$
$\frac{\kappa_a}{\kappa_m+\kappa_a}$	allocation of attention to tech.	0.82	$[0.76, 0.87]$

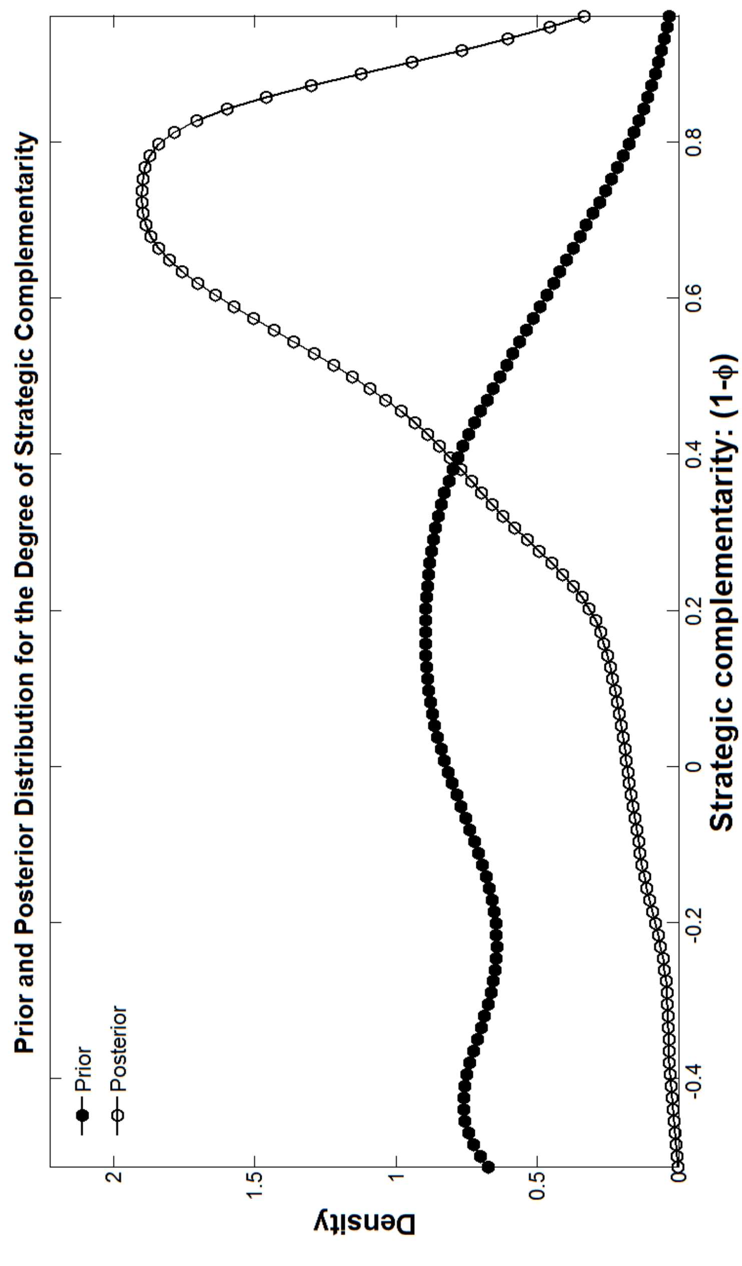
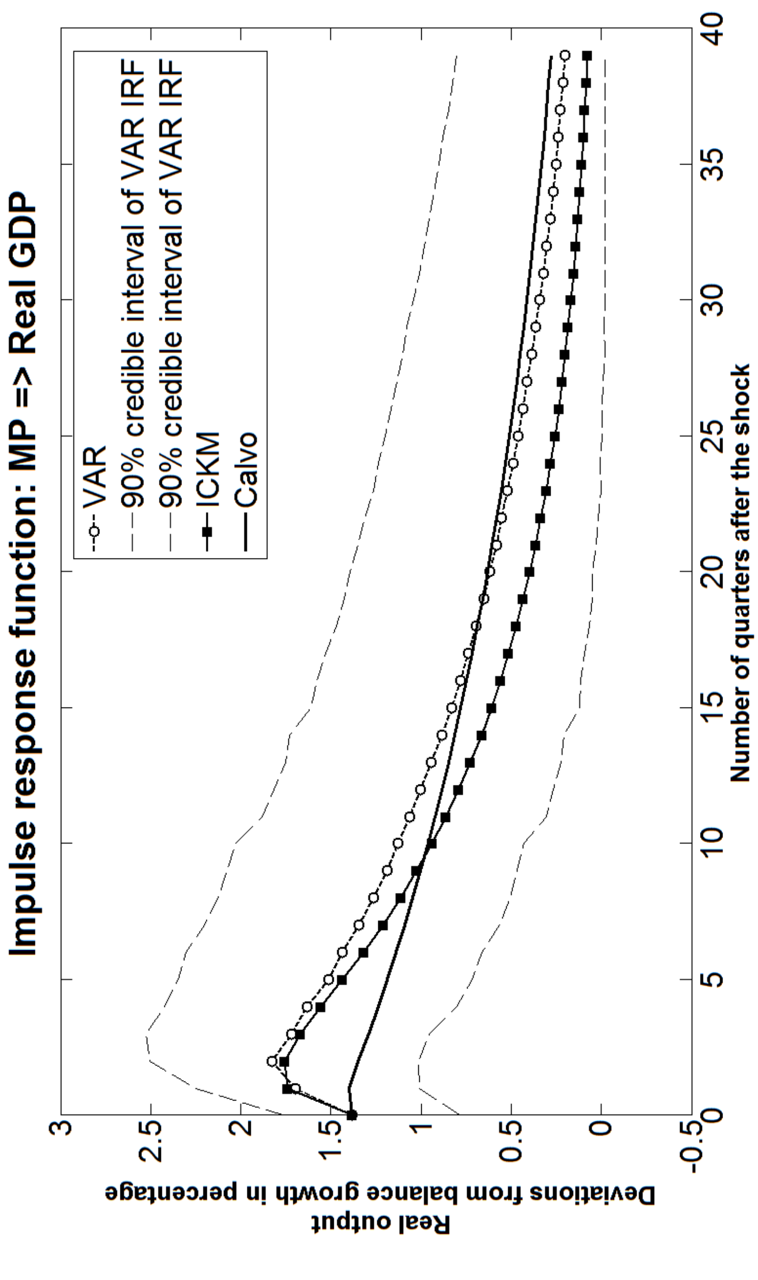
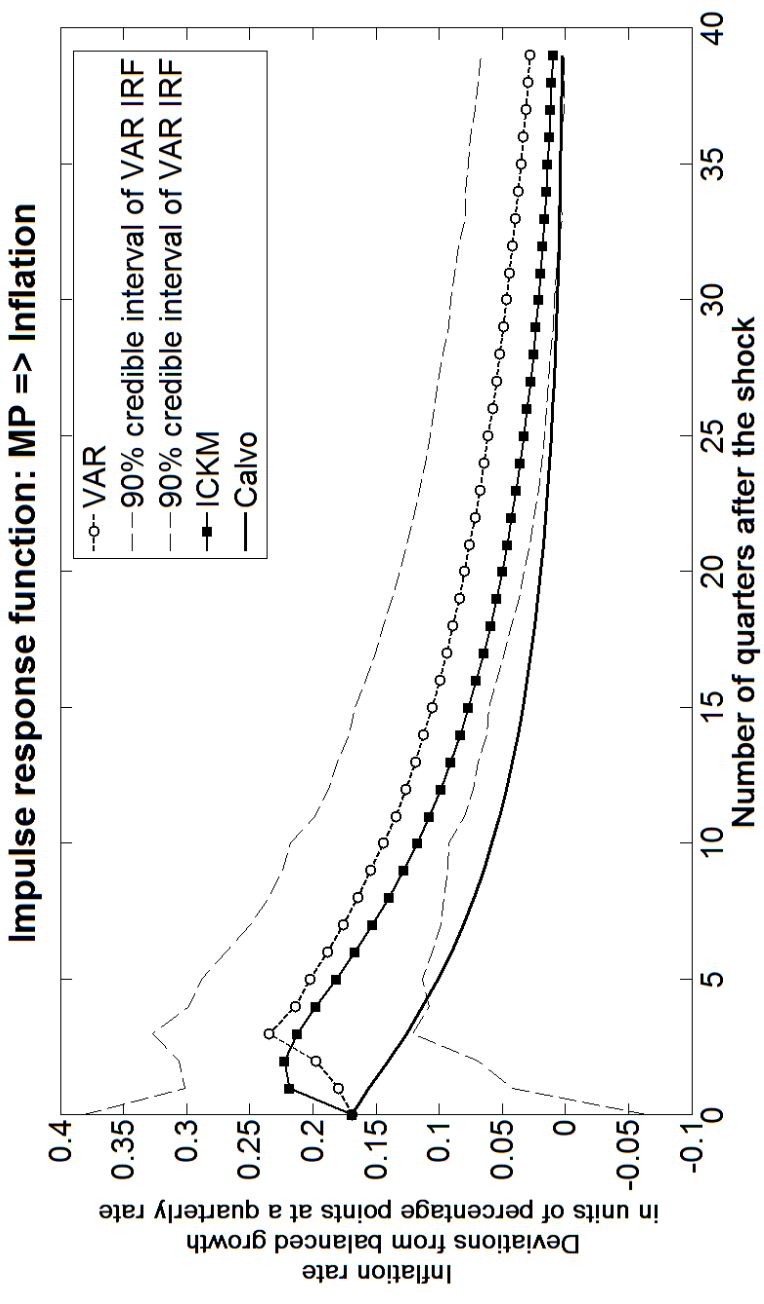

Prior and Posterior Distribution for the Degree of Strategic Complementarity
Prior
Posterior
Density
0
0.5
1
1.5
2
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Strategic complementarity: (1-ϕ)

Figure 1


Impulse response function: MP => Real GDP
VAR
90% credible interval of VAR IRF
90% credible interval of VAR IRF
ICKM
Calvo
Real output
Deviations from balance growth in percentage
Number of quarters after the shock


Impulse response function: MP => Inflation
VAR
90% credible interval of VAR IRF
90% credible interval of VAR IRF
ICKM
Calvo
Inflation rate
Deviations from balanced growth
in units of percentage points at a quarterly rate
Number of quarters after the shock
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
5
10
15
20
25
30
35
40

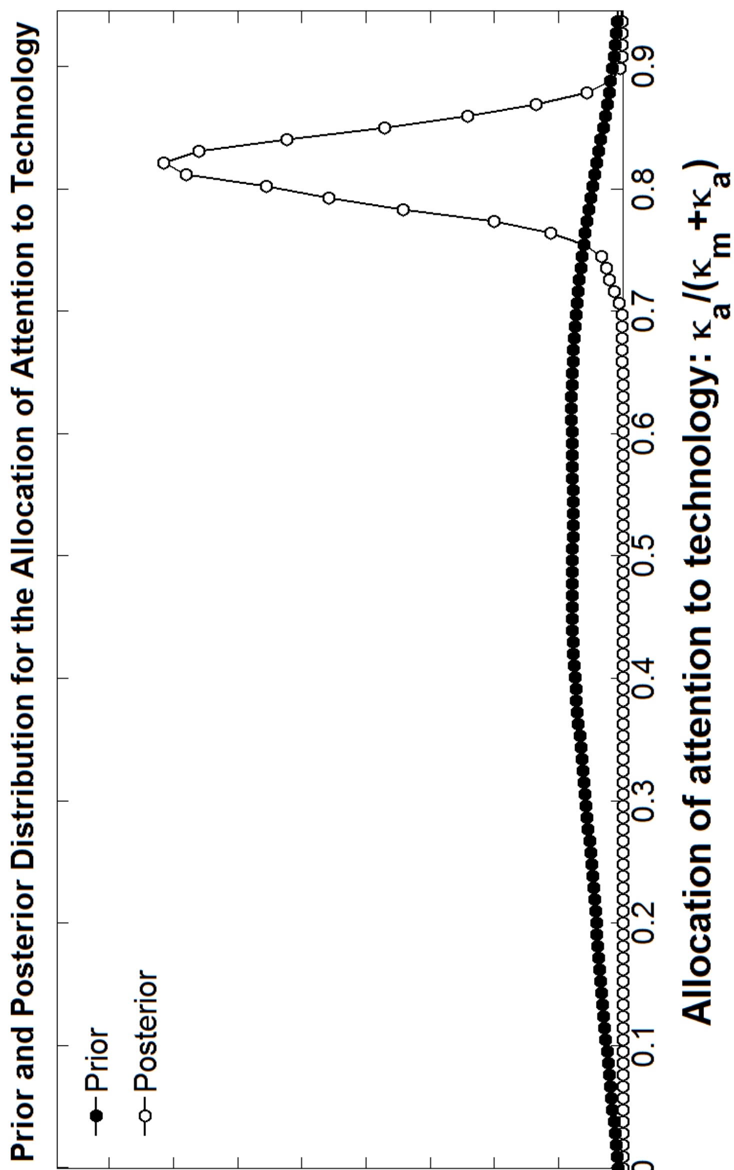

Prior and Posterior Distribution for the Allocation of Attention to Technology
Prior
Posterior
Density
0
2
4
6
8
10
12
14
16
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Allocation of attention to technology: $\kappa_a/(\kappa_m+\kappa_a)$

Figure 4

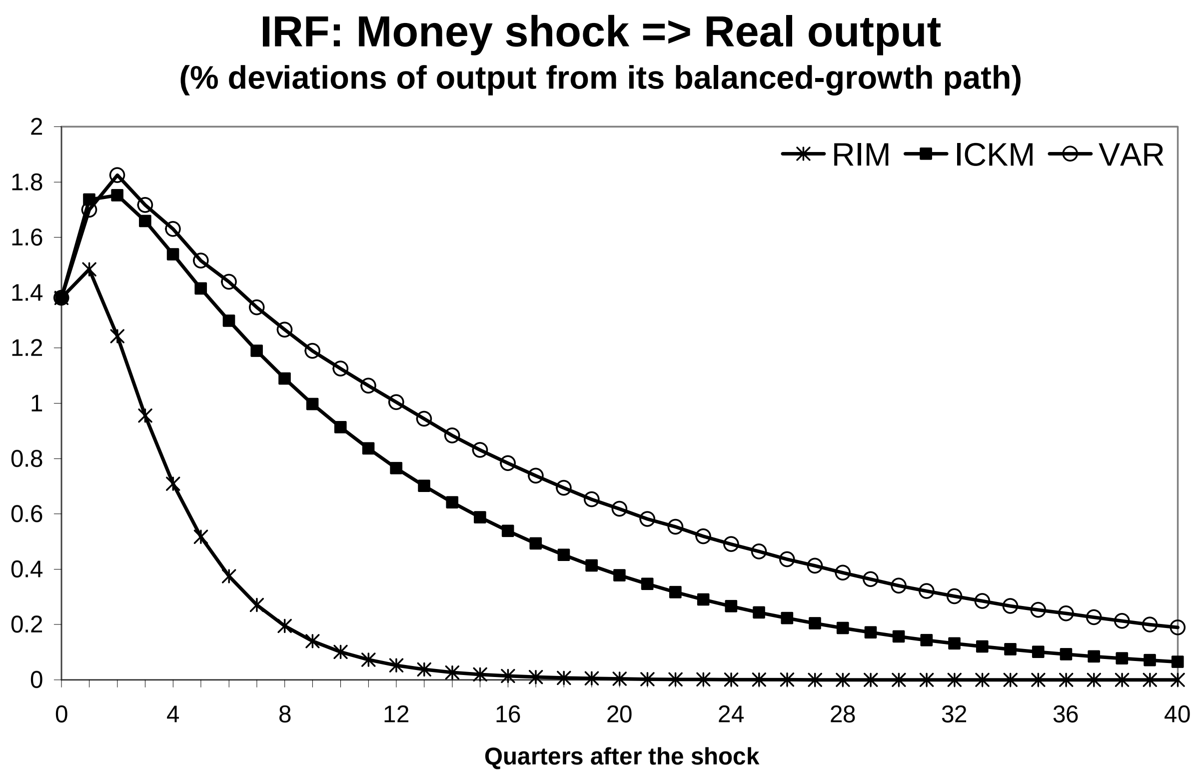

IRF: Money shock => Real output
(% deviations of output from its balanced-growth path)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
4
8
12
16
20
24
28
32
36
40
Quarters after the shock
RIM
ICKM
VAR

Figure 5

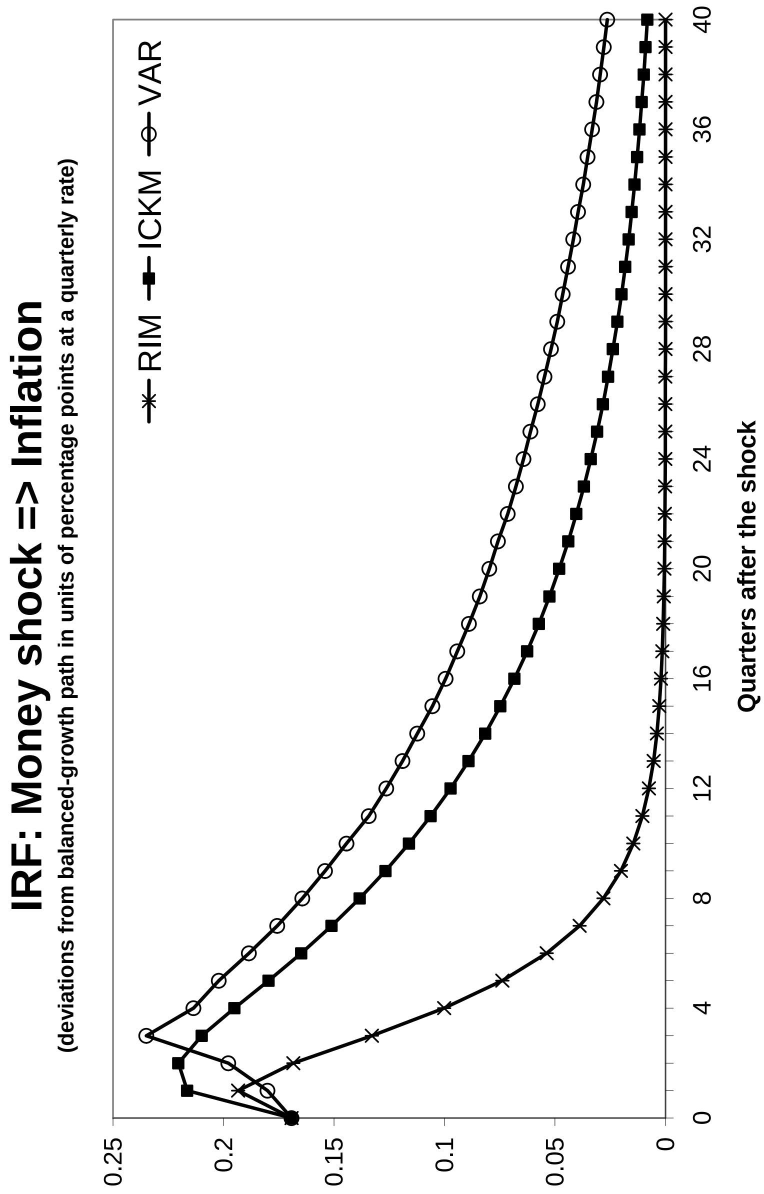

IRF: Money shock => Inflation
(deviations from balanced-growth path in units of percentage points at a quarterly rate)
RIM
ICKM
VAR
0.25
0.2
0.15
0.1
0.05
0
0
4
8
12
16
20
24
28
32
36
40
Quarters after the shock

Figure 6

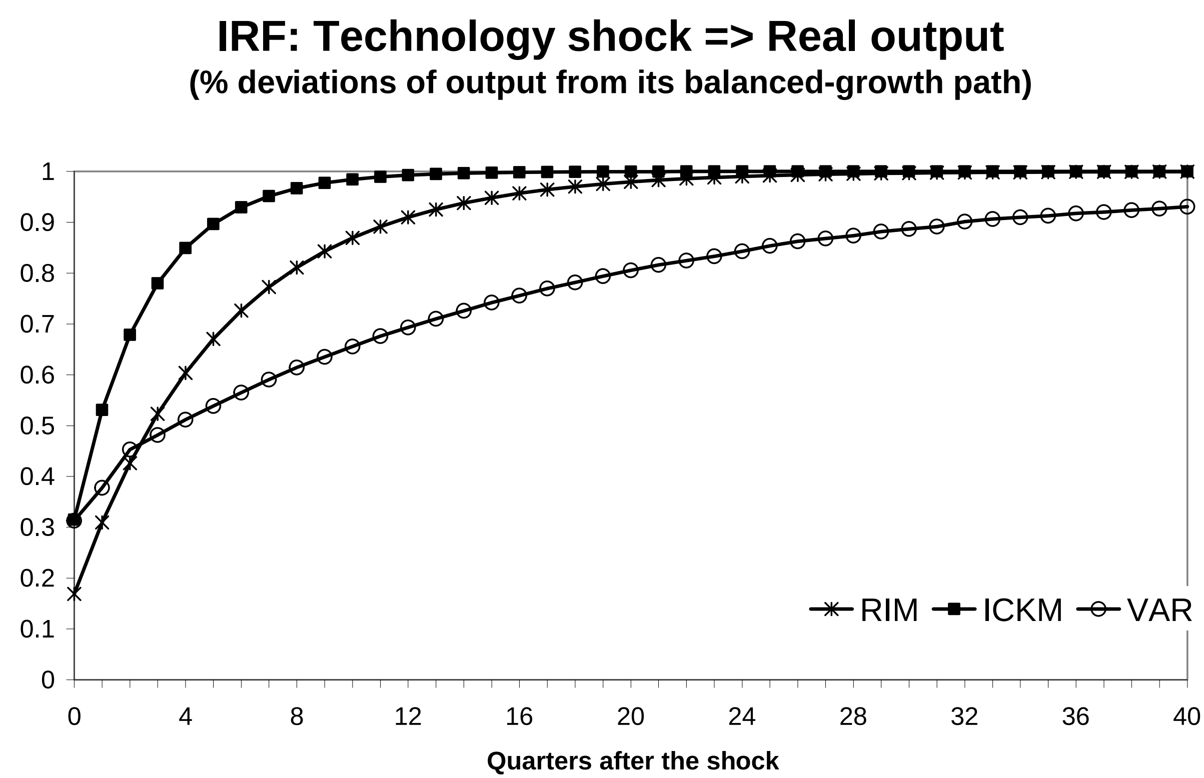

IRF: Technology shock => Real output
(% deviations of output from its balanced-growth path)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
16
20
24
28
32
36
40
Quarters after the shock
RIM
ICKM
VAR

Figure 7

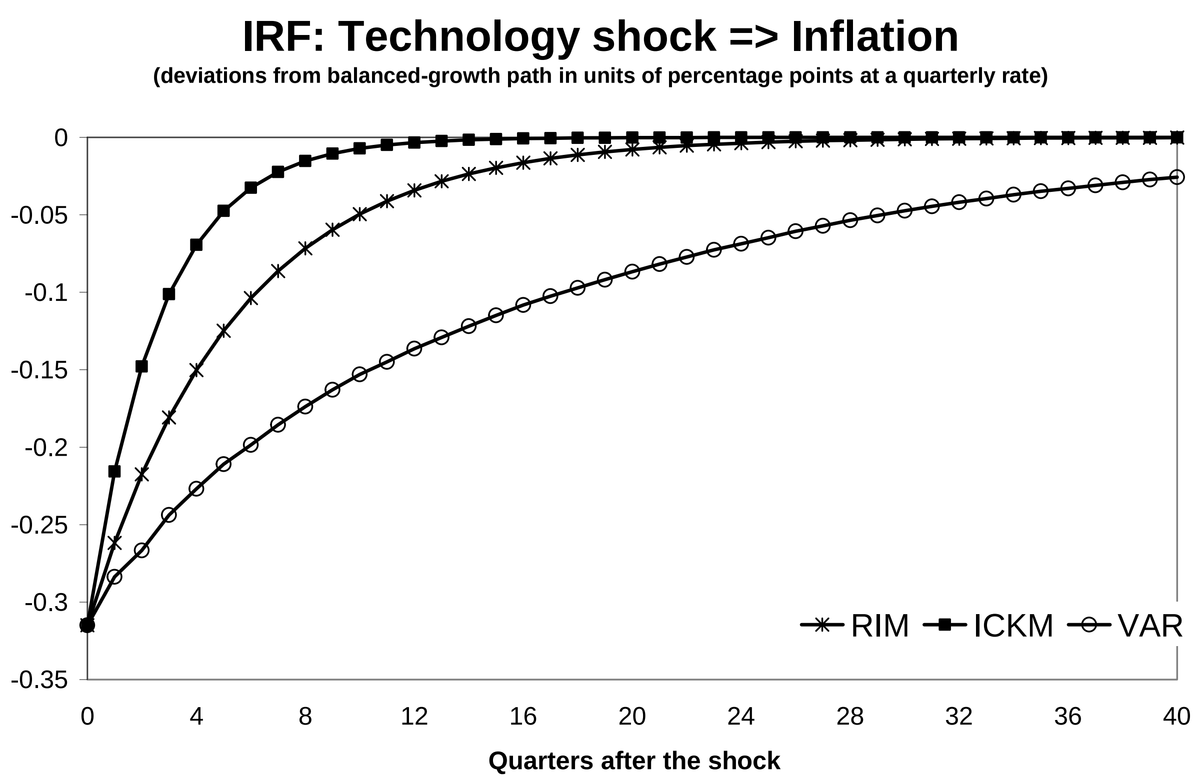

IRF: Technology shock => Inflation
(deviations from balanced-growth path in units of percentage points at a quarterly rate)
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
0
4
8
12
16
20
24
28
32
36
40
Quarters after the shock
RIM
ICKM
VAR

Figure 8