Preliminary and Incomplete

Human Capital Risk, Contract Enforcement, and the Macroeconomy

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Abstract

This paper develops a tractable macroeconomic model with human capital risk and limited contract enforcement, and uses a calibrated version of the model to show that limited contract enforcement can explain a substantial part of the observed lack of consumption insurance for young households. The model is consistent with a number of important macro-level and micro-level facts for the US economy: it matches the aggregate capital-to-output ratio and the estimates of earnings risk, and it implies life-cycle profiles of earnings growth and financial wealth that are in line with the empirical evidence.

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1. Introduction

In most countries, human capital represents a large part of aggregate total wealth.[1] Indeed, for many households human capital is the only form of capital they own. Moreover, there is strong evidence that human capital is an asset class with high return and high risk.[2] This begs the question why private insurance for human capital risk is often lacking or incomplete. In this paper, our answer is "limited contract enforcement". More specifically, we first develop a tractable macroeconomic model with human capital risk and limited contract enforcement, and then use a version of the model calibrated to US data to show that limited contract enforcement can explain a significant part of the observed lack of insurance for young households.

There is a simple intuition underlying our result that limited contract enforcement matters a lot for young households. In the calibrated model economy, young households face high and risky human capital returns. They also have almost no financial wealth, which means that they would like to borrow in order to buy insurance and increase human capital investment. However, young households cannot borrow because default is possible and contract enforcement for them is weak. In other words, banks will not lend to young households since their human capital cannot be used as collateral (non-pledgeability of human capital), they have almost no financial capital, and, as shown in this paper, the threat of exclusion from financial markets for 7-10 years is not sufficient to enforce credit contracts. Thus, there is one large group of households, namely the young, for which limited contract enforcement can explain the observed lack of consumption insurance.

The previous macro literature on limited contract enforcement has usually disregarded life-cycle motives, that is, the literature so far has not introduced heterogeneity in *expected*

[1] Labor income is about two thirds of total income, which suggests that human capital is at least as important as physical capital. Estimates of the aggregate stock of physical and human capital usually find that human capital is the larger component.

[2] See section 2 for a discussion of the empirical literature.

human capital returns (expected earnings growth). For the purpose of comparison, we therefore begin our quantitative analysis with a version of the model without life-cycle considerations. In accordance with the previous literature, we find that for realistic assumptions about contract enforcement, the model cannot generate any substantial lack of risk sharing in equilibrium. The analysis also shows that the threat of excluding defaulting households from financial markets for around 7-10 years is not a very effective means of contract enforcement, but that the threat of seizing at least half of the physical capital is a highly effective means of contract enforcement for all households holding close to the average amount of physical capital in the economy.

Though the model developed in this paper is highly tractable, it is still rich enough to allow for a tight link between theory and data. In particular, when human capital shocks are i.i.d., the implied labor income process follows a logarithmic random walk. The random-walk specification has often been used in the empirical literature to model the permanent component of labor income risk, and we choose the parameters of the model so that the implied variance of the innovation term matches the empirical estimates based on data drawn from the Panel Study of Income Dynamics (PSID). In addition to matching the permanent component of US earnings risk, the calibrated model economy also matches the US aggregate capital-to-output ratio. Moreover, we take earnings data from the PSID and financial wealth data from the Survey of Consumer Finance and show that the life-cycle model implies age-profiles of mean earnings growth and financial wealth that are in line with the corresponding age-profiles for US male household heads. In short, the model's implication are in accordance with the empirical evidence along a number of important dimensions.[3]

In addition to the economic contribution, this paper also makes a methodological contribution by developing a tractable framework that might provide a powerful tool for the quantitative analysis of a wide range of interesting macroeconomic issues. More precisely, we show that in our framework recursive equilibria can be computed without knowledge of

[3] See section 6 for details and a discussion of the relevant literature.

the endogenous wealth distribution. Our equilibrium characterization result is based on the property that individual consumption policy functions are linear in total wealth (financial plus human) and individual portfolio choices are independent of wealth. We also show that the maximization problem of individual households is convex so that a simple FOC-approach is applicable. In short, a rather complex, infinite-dimensional fixed-point problem has been transformed into a relatively simple, finite-dimensional fixed-point problem. Indeed, in many applications, including the one we present in this paper, the equilibrium problem is reduced to a low-dimensional fixed-point problem that can be solved numerically at very low computational cost.

At this stage, two general comments might be in order. First, this paper does not show that moral hazard and/or adverse selection play no role in understanding the lack of private insurance. It does, however, suggest that limited contract enforcement is an issue of first-order importance for understanding the working of private insurance markets for the young.[4] Second, following a long tradition in macroeconomics, we use a simple model formulation that treats the production and accumulation of physical capital and human capital symmetrically. We do this not only for tractability reasons, but also because this approach allows us to focus on two properties of human capital that are *per se* unrelated to the production process, namely riskiness and non-pledeability.

The remainder of the paper is structured as follows. Section 2 discusses the literature and section 3 outlines the model. In section 4 we characterize equilibrium policies and show how the computation of equilibria can be simplified. Section 5 provides a first quantitative analysis based on an economy with i.i.d. human capital shocks and no life-cycle considerations. This example allows for a useful comparison with the existing literature. Further, it demonstrates in a transparent manner that the distinct effects of the two dimensions of contract enforcement, namely exclusion from financial market participation and seizure of

[4]For reasons of conceptional clarity, we do not model moral hazard or adverse selection, but we conjecture that binding borrowing constraints for young households will remain important in a nested model with both limited contract enforcement and moral hazard (adverse selection).

capital. In section 6 we analyze the life-cycle economy and report the main findings.

2. Literature

This paper is most closely related to the large literature on limited commitment/enforcement. See, for example, Alvarez and Jermann (2000), Kehoe and Levine (1993,2001), Kocherlakota (1996), and Thomas and Worrall (1988) for seminal theoretical contributions and Krueger and Perri (2006) and Ligon, Thomas, and Worrall (2002) for highly influential quantitative work. We contribute to the theoretical literature by developing a tractable framework and by providing a more general treatment of contract enforcement.[5] We contribute to the quantitative literature by showing that a calibrated macro model with physical capital and limited contract enforcement can generate substantial lack of consumption insurance for a large group of households. In contrast, previous quantitative results in the literature (Krueger and Perri, 2006) suggest that models with limited contract enforcement generate no or only negligible lack of consumption insurance for all households if these models are required to match the aggregate capital-to-output ratio of the US economy.[6] However, the previous literature has not considered household heterogeneity in expected earnings growth rates (life cycle), a feature of the data that we show is important for understanding the macroeconomic effects of limited contract enforcement.

This paper is also related to the macroeconomic literature on human capital. Indeed, our formulation of the production side of the model is motivated by the contributions to

[5] More precisely, the limited enforcement/commitment literature has focused on one dimension of contract enforcement, namely the threat of exclusion from financial markets (risk sharing). The second dimension of contract enforcement, namely the threat of seizing assets, has been emphasized by the large macroeconomic literature on collateral constraints (for example, Kiyotaki and Moore (1997) and Kuebler and Schmedders (2003)). In a certain sense, the current paper provides an integrated approach to these two strands of the literature.

[6] Krueger and Perri (2006) match the cross-sectional distribution of consumption fairly well, but the implied volatility of individual consumption growth is negligible in their model. A similar almost full-insurance result is obtained by Cordoba (2006).

the endogenous growth literature emphasizing human capital accumulation (Lucas (1988), Jones and Manuelli (1990), and Rebelo (1991)). Of course, models in this literature usually postulate a representative household, that is, complete and frictionless financial markets. There has been some work on human capital models with incomplete markets (Krebs (2003) and Huggett, Ventura, and Yaron(2010)), but we are not aware of any macroeconomic study that combines human capital risk with contracting frictions.[7]

There is also a recent, but fast growing, macroeconomic literature on default in incomplete-market models (for example, Chatterjee, S., D. Corbae., M. Nakajima, and V. Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007)). Though these models provide useful insights into a number of important issues, they are necessarily silent about the underlying financial friction that leads to second-best outcomes, which is the main topic of the current paper.

Turning to the empirical literature, there is convincing evidence that individual households face large and highly persistent labor income shocks that have strong effects on individual consumption. For the estimation of individual income risk, see, for example, MaCurdy (1982), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004). For the consumption response, see, for example, Cochrane (1990), Townsend (1994), and Blundell, Pistaferri, and Preston (2008). If we think of labor income as the return to human capital investment, a time-honored tradition in micro- and macro-economics, then these empirical findings show that there is a large amount of uninsurable human capital risk. Finally, a large literature has estimated the returns to education. Although results vary substantially, the estimates cluster around a value of 10 percent (see Krueger and Lindhal (2001) for a survey), which shows that human capital returns of the young are quite high. Estimates of the returns to on-the-job training (Mincer 1962) and investment in children (Caucutt and Lochner, 2005) are often in a similar range.

[7]Wright (2003) considers an "AK" model with i.i.d. shocks and limited enforcement that, in a certain sense, is a simplified version of the current set-up. In an interesting recent study, Lochner and Monge (2010) analyze a model with human capital and enforcement constraints, but they have no risk and their focus and modeling is quite different from ours.

3. Model

The model is a version of the type of human capital model that has been popular in the endogenous growth literature. We consider a production economy with an aggregate constant-returns-to-scale production function using physical and human capital as input factors. There are a large number (a continuum) of individual households with CRRA-preferences who can invest in risk-free physical capital and risky human capital. Human capital investment is risky due to idiosyncratic shocks to the stock of human capital that follow a stationary Markov process with finite support (Markov chain). We confine attention to stationary recursive equilibria. Households have access to a complete set of credit and insurance contracts, but their ability to use the available financial instruments is limited by the possibility of default (endogenous borrowing/short-sale constraints). Defaulting households keep their human capital (non-pledgeability of human capital), lose a fraction of their physical capital (first dimension of contract enforcement), and are excluded from financial markets until a stochastically determined future date (second dimension of contract enforcement).

3.1. Production

There is one all-purpose good that can be consumed, invested in physical capital, or invested in human capital. Production of this one good is undertaken by one firm (a large number of identical firms) that rents capital and labor in competitive markets and uses these input factors to produce output, Y, according to the aggregate production function $Y = F(K, H)$. Here K and H are the (aggregate) levels of physical and human capital employed by the firm. The production function, F, is a standard neoclassical production function, that is, it has constant-returns-to-scale, satisfies a Inada condition, and is continuous, concave, and strictly increasing in each argument. Given these assumptions on F, the derived intensive-form production function, $f(\tilde{K}) = F(\tilde{K}, 1)$, is continuous, strictly increasing, strictly concave, and satisfies a corresponding Inada condition, where we introduced the "capital-to-labor ratio" $\tilde{K} = K/H$. Given the assumption of perfectly competitive labor and capital markets, profit maximization implies

$$\begin{aligned} r_k &= f'(\tilde{K}) \\ r_h &= f(\tilde{K}) + f'(\tilde{K})\tilde{K} \ , \end{aligned} \tag{1}$$

where r_k is the rental rate of physical capital and r_h is the rental rate of human capital. Note that r_h is simply the wage rate per unit of human capital and that we dropped the time index because of our stationarity assumption. Clearly, (1) defines rental rates as functions of the capital to labor ratio: $r_k = r_k(\tilde{K})$ and $r_h = r_h(\tilde{K})$.

3.2. Time and Uncertainty

Time is discrete and indexed by $t = 0, 1, \ldots$. There is a continuum of individual households of unit mass. Household solve a decision problem with an infinite planning horizon, but they may face a possibly state-dependent probability of death. A household who dies is replaced by a newborn. Idiosyncratic risk, including the death event, is represented by a Markov shock process, $\{s_t\}$, where each s_t takes on a finite number of possible values. In the life-cycle economy of section 6, one dimension of s indexes the age of the household. We denote by $s^t = (s_1, \ldots, s_t)$ the history of idiosyncratic shocks up to period t and let $\pi(s^t) = \pi(s_t|s_{t-1}) \ldots \pi(s_2|s_1)$ stand for the probability that s^t occurs. In period $t = 0$, the type of an individual household is characterized by his initial state, $x_0 = (k_0, h_0, s_0)$, where s_0 denotes the initial shock, k_0 the initial stock of physical capital, and h_0 the initial stock of human capital (note that s_0 is not included in s^t). We take as given an initial measure, μ, over initial types. Since we confine attention to stationary equilibria, this initial distribution is chosen to be the implied stationary distribution. The initial state of newborns is also drawn from this stationary distribution (conditional on age, if applicable).

3.3. Preferences

Households have identical preferences over consumption plans, $\{c_t\}$, where $\{c_t\}$ denotes a sequence of functions (random variables), c_t, mapping shock histories, s^t, into consumption levels, $c_t(s^t)$. Similar notation will be used for investment plans (see below). Households are risk-averse and their preferences allow for a time-additive expected utility representation. That is, for a household of initial type (k_0, h_0, s_0), expected life-time utility is given by:

$$U(\{c_t\}|k_0, h_0, s_0) \doteq E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)|k_0, h_0, s_0\right], \tag{2}$$

where the expectations, $E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)|k_0, h_0, s_0\right]$, stands for $\sum_{s^t}\sum_{t=0}^{\infty} \beta^t u(c_t(s^t, k_0, h_0, s_0))\pi(s^t|s_0)$ and β is the discount factor. We assume that preferences are homothetic. Given assumption (2), this means that the one-period utility function exhibits constant relative risk aversion:

$$u(c) = \begin{cases} c^{1-\gamma}/(1-\gamma) & if\ \gamma \neq 1 \\ ln\, c & otherwise \end{cases} \tag{3}$$

3.4. Budget Constraint

Each household can invest in physical capital, k, or human capital, h. In addition, he can buy and sell a complete set of financial contracts (assets) with state-contingent payoffs. More specifically, there is one contract (Arrow security) for each state, and we denote by $a_{t+1}(s_{t+1})$ the quantity bought in period t (sold if negative) of the contract that pays off one unit of the good in period $t+1$ if s_{t+1} occurs. Given his initial type, (k_0, h_0, s_0), a household chooses a plan, $\{c_t, k_t, h_t, \vec{a}_t\}$, where the notation $\vec{a}$ indicates that in each period the household chooses a vector of contract holdings. A budget-feasible plan has to satisfy the sequential budget constraint

$$\begin{aligned} (1+\tau_c)c_t + x_{kt} + (1-\tau_h)x_{ht} + \sum_{s_{t+1}} a_{t+1}(s_{t+1})q(s_{t+1}) &= r_k k_t + r_h h_t + a_t(s_t) \\ k_{t+1} &= (1-\delta_k)k_t + x_{kt} \\ h_{t+1} &= (1-\delta_h(s_t))h_t + x_{ht} \\ c_t \geq 0\ ,\ \ k_{t+1} \geq 0\ ,\ \ h_{t+1} &\geq 0\ , \end{aligned} \tag{4}$$

where $q(s_{t+1})$ is the price of a financial contract that pays off if s_{t+1} occurs, x_{kt} and x_{ht} are investment in physical and human capital, respectively, and δ_k and $\delta_h(s_t)$ are the corresponding depreciation rates. In (4) we have also introduced a (linear) consumption tax, τ_c, and a human capital investment subsidy, τ_h. Note that (4) has to hold in realizations, that is, it has to hold for all histories, s^t.

The budget constraint (4) follows Jones and Manuelli (1990) and Rebelo (1991) by focusing on the direct monetary costs of human capital investment. In contrast, Lucas (1988) and Ben-Porath (1967) consider the indirect costs that arise when households have to allocate a fixed amount of time between work and human capital investment. We have chosen the specification (4) for two reasons. First, it keeps the model highly tractable. Second, it treats the production of physical and human capital fully symmetricly, which seems a useful abstraction given that our focus is on two properties of human capital that are *per se* unrelated to the production process, namely the riskiness and non-pledgeability of human capital.

The specification of the budget constraint (4) makes three implicit assumptions. First, it lumps together general human capital (education, health) and specific human capital (on-the-job training). Second, our current formulation of the household problem neglects the labor-leisure decision, but extending the model to allow for a labor-leisure choice is straightforward. Third, (4) does not impose a non-negativity constraint on human capital investment ($x_{hit} \geq 0$). In our numerical applications this inequality is always satisfied so that adding this constraint would not change the equilibrium, but this is not a general property of the model.

The random variable δ_{ht} represents uninsurable idiosyncratic labor income risk. A negative human capital shock could be due to the loss of firm- or sector-specific human capital subsequent to job termination (worker displacement). The budget constraint (4) assumes that the wage payment is received in each period, but it is straightforward extension of the current model to allow the wage rate, r_h, to depend on the idiosyncratic shock (unemployment). A decline in health (disability) provides a second example for a negative human capital shock. In this case, both general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shock.

So far, we have not imposed any restrictions on trading of financial assets. In this paper, we augment the sequential budget constraint by a short-sale constraint that is loose enough so that it will never bind in equilibrium.

3.5. Financial Intermediation

In this paper, we confine attention to equilibria in which financial contracts are priced in a risk-neutral manner,

$$q_t(s_{t+1}) \quad = \quad \frac{\pi(s_{t+1}|s_t)}{1+r_f} \ , \tag{5}$$

where r_f is the interest rate on financial transactions. Note that this interest rate is in general different from the rate of return on physical capital investment, $r_k - \delta_k$. The pricing equation (5) can be interpreted as a zero-profit condition for financial intermediaries. More precisely, consider financial intermediaries that sell insurance contracts to individual households and invest the proceeds in the risk-free asset that can be created from the complete set of financial contracts and yields a certain return r_f. Given that financial intermediaries face linear investment opportunities and assuming no quantity restrictions on the trading of financial contracts for financial intermediaries, equilibrium requires that financial intermediaries make zero profit, namely condition (5).

3.6. Participation/Enforcement Constraint

In addition to the standard budget constraint, the household has to satisfy a sequential enforcement (participation) constraint, which ensures that at no point in time individual households have an incentive to default on their financial obligations. More precisely, individual consumption plans have to satisfy

$$E\left[\sum_{n=0}^{\infty} \beta^n u(c_{t+n})|k_0, h_0, s_0, s^t\right] \geq V_d(k_t, h_t, s_t) \ , \tag{6}$$

where V_d is the value function of a defaulting household (autarky value) defined as follows.

For a household who defaults in period t, all short and long positions in financial assets are canceled, $a_t(s_t) = 0$, which is the reason why the function V_d does not depend on financial asset holdings. Further, there are two types of punishment for default. First, upon default households lose a fraction ϕ of their physical capital, but they keep all their human capital, which reflects the fact that human capital is non-pledgeable.[8] Second, defaulting households will be excluded from participation in financial markets, $a_{t+n}(s_{t+n}) = 0$, until a stochastically determined future date that occurs with probability $(1 - p)$ in each period, that is, the probability of remaining in (financial) autarky is p.[9] After regaining access to financial markets, the household's expected continuation value is $V^e(k, h, s)$, where (k, h, s) is the individual state at the time of regaining access. For the individual household the function V^e is taken as given, but we will close the model and determine this function endogenously by requiring that $V^e = V$, where V is the equilibrium value function associated with the maximization problem of a household who participates in financial markets. In other words, we assume rational expectations.[10] Note that the two parameters ϕ and p correspond to two different dimensions of contract enforcement, and that an increase in either parameter amounts to an improvement in contract enforcement.

Finally, with respect to the use of physical and human capital after default, we assume that defaulting households still participate in the market for physical and human capital, that is, they rent out their physical and human capital at the going market rate. In short, a defaulting household chooses a continuation plan, $\{c_{t+n}, k_{t+n}, h_{t+n}\}$, facing the sequential budget constraint (4) with $a_{t+n} = 0$.

3.7. Equilibrium

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[8] One might object that future wage payments can be garnished. However, even though in the current model this is formally equivalent to seizing human capital, in reality these two enforcement strategies have very distinct effects: the seizure of human capital only affects human capital invest investment decisions, whereas the seizure of wage payments also affects work and search effort. Moreover, at least in the US this is not an empirically relevant case, since almost all households file for personal bankruptcy according to Chapter 7.

[9] The previous literature has usually assumed $p = 1$ (permanent autarky). There is also a literature that only rules out short positions after default (see Hellwig and Lorenzoni (2009) for a recent example). *Mutatis mutandis*, our theoretical results will also go through for this case.

[10] See also Krueger and Uhlig (2006) for a similar approach. Note that the credit (default) history of an individual household is not a state variable affecting the expected value function, V^e. Thus, we assume that the credit (default) history of households is information that cannot be used for contracting purposes.

In equilibrium, a household of type (k_0, h_0, s_0) chooses a plan, $\{c_t, k_t, h_t, \vec{a}_t\}$. The collection of plans, one for each initial type, defines a global plan or allocation. Given a global plan and assuming a law of large numbers holds, we find the aggregate capital stock held by all households is $E[k_{t+1}] = \int_{k_0,h_0,s_0} \sum_{s^t} k_{t+1}(s^t, k_0, h_0, s_0)\pi(s^t|s_0)\mu(k_0, h_0, s_0)$. In equilibrium, the level of physical and human capital demanded by the firm must be equal to the corresponding aggregate levels supplied by households. Because of the constant-returns-to-scale assumption, only the ratio of physical to human capital is pinned down by this market clearing condition. That is, in equilibrium we must have

$$\tilde{K} = \frac{E[k_{t+1}]}{E[h_{t+1}]} , \tag{7}$$

where $\tilde{K}$ is the capital-to-labor ratio chosen by the firm.

The second market clearing condition requires that no resources are created or destroyed by trading in financial contracts. In other words, the total value of all financial asset holdings has to sum to zero:

$$\sum_{s_{t+1}} E[q_t(s_{t+1})a_{t+1}(s_{t+1})] \;=\; 0 \, . \tag{8}$$

Straightforward calculation shows that the two market clearing conditions in conjunction with the budget constraint (4) imply the standard aggregate resource constraint (goods market clearing). Moreover, using the pricing condition (6), the asset market clearing condition (8) reads $E[a_{t+1}] = 0$.

We assume that the government runs a balanced budget period by period. Thus, the government budget constraint is:

$$\tau_c E[c_t] = \tau_h E[x_{ht}] \, . \tag{9}$$

In a recursive equilibrium, optimal plans of individual households, $\{c_t, k_t, h_t, \vec{a}_t\}$, are generated by a policy function, g, that maps $(k_t, h_t, a_t(s_t), s_t)$ into $(k_{t+1}, h_{t+1}, \vec{a}_{t+1})$. To sum up, we have the following equilibrium definition:

Definition A stationary recursive equilibrium is a collection of rental rates (r_k, r_h), a financial interest rate, r_f, an aggregate capital-to-labor ratio, $\tilde{K}$, an expected value function, V^e, and a household policy function, g, so that

i) Utility maximization of households: for each household type, (k_0, h_0, s_0), the household policy function, g, generates a plan, $\{c_t, k_t, h_t, \vec{a}_t\}$, that maximizes expected lifetime utility (2) subject to the sequential budget constraint (4) and the sequential participation constraint (6).

ii) Profit maximization of firms: aggregate capital-to-labor ratio and rental rates satisfy the first-order conditions (1).

iii) Financial intermediation: financial contracts are priced according to (5)

iv) Market clearing: equations (7) and (8) hold.

v) Rational expectations: $V^e = V$.

vi) Stationarity: the initial distribution, μ, is the stationary distribution of the Markov transition function induced by g and the transition matrix π.

vii) The government budget constraint (9) holds.

4. Equilibrium Characterization

In this section, we present the main theoretical results that provide the foundation for the computation of equilibria. We begin with the principle of optimality and a monotone operator theorem for the household problem with enforcement constraints (proposition 1). We then state the solution for the household problem in financial autarky (problem 2). These two results taken together allow us to derive the solution to the household problem with enforcement constraint and to show that the optimal policy function is linear in total wealth (proposition 3). This linearity property gives rise to the equilibrium characterization stated in proposition 4.

4.1. Household Problem

It is convenient to introduce new variables that emphasize that individual households solve a standard inter-temporal portfolio choice problem (with additional participation constraints). To this end, introduce the following variables:

$$
\begin{aligned}
w_t &= k_t + (1-\tau_h)h_t + \sum_{s_t} q_{t-1}(s_t)a_t(s_t) \\
\theta_{kt} &= \frac{k_t}{w_t} \ , \ \ \theta_{ht} = \frac{h_t}{w_t} \ , \ \ \theta_{at}(s_t) = \frac{a_t(s_t)}{w_t} \\
1 + r_t &= (1 + r_k - \delta_k)\theta_{kt} + (1 + r_h/(1-\tau_h) - \delta_h(s_t))\theta_{ht} + \theta_{at}(s_t)
\end{aligned} \tag{10}
$$

In (10) the variable w_t stands for beginning-of-period wealth consisting of real wealth, $k_t + h_t$, and financial wealth, $\sum_{s_t,S_t} q_{t-1}(s_t)a_t(s_t)$. The variable $\theta_t = (\theta_{kt}, \theta_{ht}, \vec{\theta}_{at})$ denotes the vector of portfolio shares and $(1+r)$ is the gross return to investment. Note that wealth including current asset payoffs and depreciation is $(1 + r_t)w_t$, and this variable can equally well be used as an individual state variable. Using the new notation (10) and substituting out the investment variables, x_{kt} and x_{ht}, the budget constraint (4) becomes

$$
\begin{aligned}
w_{t+1} &= (1 + r_t)\, w_t - (1 + \tau_c)c_t \\
1 &= \theta_{kt} + \theta_{ht} + \sum_{s_t} q_{t-1}(s_t)\theta_{at}(s_t) \\
c_t &\geq 0 \ , \ \ w_t \geq 0 \ , \ \ \theta_{kt} \geq 0 \ , \ \ \theta_{ht} \geq 0 \ .
\end{aligned} \tag{11}
$$

Clearly, (11) is the budget constraint corresponding to an inter-temporal portfolio choice problem with linear investment opportunities and no exogenous source of income.

For the maximization problem of a household facing the budget constraint (11) and the sequential enforcement constraint (6), we conjecture that the solution is recursive in the state variable (w, θ, s) and satisfies the Bellman equation

$$
\begin{aligned}
V(w,\theta,s) \quad &= \quad \max_{w',\theta'} \left\{ u\left((1 + r(\theta,s))w - w'/(1+\tau_c)\right) + \beta \sum_{s''} V\left(w',\theta',s'\right)\pi(s'|s) \right\} \\
s.t. \quad & 1 \ = \ \theta'_k + \theta'_h + \sum_{s'} \frac{\pi(s'|s)\theta'_a(s')}{1+r_f} \\
& 0 \ \leq \ w' \ \leq \ (1 + r(\theta,s))w \ , \ \ \theta'_k \geq 0 \ , \ \ \theta'_h \geq 0 \ , \ \ \theta'_a(s') \geq -\bar{\theta}_a(s') \\
& V(w',\theta',s') \ \geq \ V_d(w',\theta',s')
\end{aligned} \tag{12}
$$

Let T be the operator associated with the Bellman equation (12) defined in the canonical way. In contrast to the standard case without a participation constraint, this operator is in general not a contraction. However, it is still a monotone operator. Monotone operators might have multiple fixed points, but under certain conditions we can construct a sequence that converges to the maximal element of the set of fixed points. This maximal solution is also the value function (principle of optimality). More precisely, if the condition that for all s

$$\begin{aligned} \forall\theta' &: \beta \sum_{s'} (1 + r(\theta', s'))^{1-\gamma} \pi(s'|s) < 1 \quad if \ 0 < \gamma < 1 \\ \exists\theta' &: \beta \sum_{s'} (1 + r(\theta', s'))^{1-\gamma} \pi(s'|s) < 1 \quad if \ \gamma > 1 \end{aligned} \tag{13}$$

holds,[11] then we have the following results:

Proposition 1. Suppose that condition (13) is satisfied and that the value function of a household in financial autarky, V_d, is continuous. Let T stand for the operator associated with the Bellman equation (12). Then

i) There is a unique continuous solution, V_0, to the Bellman equation (11) without participation constraint.

ii) $\lim_{n\to\infty} T^n V_0 = V_\infty$ exists and is the maximal solution to the Bellman equation (12)

iii) V_∞ is the value function, V, of the sequential household maximization problem.

Proof. See Krebs and Wright (2010).

We now turn to the maximization problem of a household in default. Along the lines of (11), we can define wealth and portfolio variables so that the sequential budget constraint of a defaulting household becomes:

$$w_{t+n+1} = [1 + r_{d,t+n}]\, w_{t+n} - c_{t+n} \tag{14}$$

[11] Note that for the log-utility case, no condition of the type (13) is required.

$$1 = \theta_{k,t+n} + \theta_{h,t+n}$$
$$c_{t+n} \geq 0 \ , \ w_{t+n} \geq 0 \ , \ \theta_{k,t+n} \geq 0 \ , \ \theta_{h,t+n} \geq 0 \ ,$$

where the investment return is now $1 + r_d = [1 + \theta_k(r_k - \delta_k) + \theta_h(r_h - \delta_h(s))]/(1 + \tau_c)$. The Bellman equation corresponding to the maximization problem of a household in default reads:

$$\begin{aligned} V_d(w, \theta_k, \theta_h, s) \quad = \quad & \max_{w', \theta'_k, \theta'_h} \Big\{ u((1 + r_d(\theta_k, \theta_h, s))w - w'/(1 + \tau_c)) + \beta p \sum_{s'} V_d(w', \theta'_k, \theta'_h, s') \\ & + \beta(1-p) \sum_{s'} V^e\left(w', \theta'_k, \theta'_h, s'\right) \pi(s'|s) \Big\} \qquad (15) \\ s.t. \quad & 1 = \theta'_k + \theta'_h \ , \ \theta'_k \geq 0 \ , \ \theta'_h \geq 0 \ . \end{aligned}$$

Using a standard contraction mapping argument, one can show that (15) has a unique continuous solution if V^e is continuous and if the condition

$$\begin{aligned} \forall \theta' \ : \ \beta \sum_{s''} \left(1 + r_d(\theta'_k, \theta'_h, s')\right)^{1-\gamma} \pi(s'|s) \ &< \ 1 \quad if \ \ 0 \ < \ \gamma \ < \ 1 \qquad (16) \\ \exists \theta' \ : \ \beta \sum_{s'} \left(1 + r_d(\theta'_k, \theta'_h, s')\right)^{1-\gamma} \pi(s'|s) \ &< \ 1 \quad if \ \ \gamma \ > \ 1 \end{aligned}$$

is satisfied for all s. Furthermore, this solution is also the value function associated with the sequential maximization problem of a defaulting household (principle of optimality). Hence, we can confine attention to the Bellman equation (15) when dealing with the individual household problem after default.

A simple guess-and-verify approach shows that if the expected value function has the functional form

$$V^e(w, \theta_k, \theta_h, s) \ = \ \begin{cases} \tilde{V}^e(s)\left(1 + r_d(\theta_k, \theta_h, s)\right)^{1-\gamma} w^{1-\gamma} & if \ \gamma \ \neq \ 1 \\ \tilde{V}^e(s) \ + \ \frac{1}{1-\beta} log(1 + r_d(\theta_k, \theta_h, s)) \ + \ \frac{1}{1-\beta} log w & otherwise \end{cases} \ , \qquad (17)$$

then the autarky value function is given by

$$V_d(w, \theta_k, \theta_h, s) \ = \ \begin{cases} \tilde{V}_d(s)\left(1 + r_d(\theta_k, \theta_h, s) - \phi\theta_k\right)^{1-\gamma} w^{1-\gamma} & if \ \gamma \ \neq \ 1 \\ \tilde{V}_d(s) \ + \ \frac{1}{1-\beta} log(1 + r_d(\theta_k, \theta_h, s) - \phi\theta_k) \ + \ \frac{1}{1-\beta} log w & otherwise \end{cases} \ , \qquad (18)$$

where the intensive-form value function, $\tilde{V}_d$, is the unique solution to the following intensive-form Bellman equation:

$$\begin{aligned}\tilde{V}_d(s) \quad = \quad & \max_{\tilde{c},\theta'_k,\theta'_h} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta p(1-\tilde{c})^{1-\gamma} \sum_{s'} \left(1 + r_d(\theta'_k, \theta'_h, s')\right)^{1-\gamma} \tilde{V}_d(s')\pi(s'|s) \right. \\ & \left. + \beta(1-p)(1-\tilde{c})^{1-\gamma} \sum_{s'} \left(1 + r_d(\theta'_k, \theta'_h, s')\right)^{1-\gamma} \tilde{V}^e(s')\pi(s'|s) \right\} \end{aligned} \tag{19}$$

$$s.t. \quad 1 \ = \ \theta'_k + \theta'_h \quad , \quad 0 \leq \ \tilde{c} \ \leq \ 1 \quad , \quad \theta'_k \geq 0 \quad , \quad \theta'_h \geq 0$$

for $\gamma \neq 1$ and

$$\begin{aligned}\tilde{V}_d(s) \quad = \quad & \max_{\theta'_k,\theta'_h} \left\{ log(1-\beta) \ + \ \frac{\beta}{1-\beta} log\beta \ + \ \frac{\beta}{1-\beta} \sum_{s'} log(1 + r_d(\theta'_k, \theta'_h, s'))\pi(s'|s) \right. \\ & \left. + \beta \sum_{s'} \left(p\tilde{V}_d(s') + (1-p)\tilde{V}^e(s') \right) \pi(s'|s) \right\} \end{aligned}$$

$$s.t. \quad 1 \ = \ \theta'_k + \theta'_h \quad , \quad \theta'_k \geq 0 \quad , \quad \theta'_h \geq 0$$

for the log-utility case.

Proposition 2. Suppose condition (16) is satisfied and the expected value function, V^e, is given by (17). Then the value function, V_d, associated with the maximization problem of an individual household in financial autarky has the functional form (18), where the intensive-form value function, $\tilde{V}_d$, is the unique solution to the intensive-form Bellman equation (19).

Suppose now that the expected value function, V^e, has the function form (17). Using propositions 1 and 2 and an induction argument, we show in the appendix that under assumption (13) the value function, V, has the functional form

$$V(w, \theta, s) = \begin{cases} \tilde{V}(s)(1 + r(\theta, s))^{1-\gamma} w^{1-\gamma} & if \ \gamma \ \neq \ 1 \\ \tilde{V}(s) \ + \ \frac{1}{1-\beta} log\left(1 + r(\theta, s)\right) \ + \ \frac{1}{1-\beta} log w & otherwise \end{cases} \tag{20}$$

and that the corresponding optimal policy function, g, is

$$c(w, \theta, s) \quad = \quad \begin{cases} \tilde{c}(s)(1 + r(\theta, s))w & if \ \gamma \ \neq \ 1 \\ (1-\beta)(1 + r(\theta, s))w & otherwise \end{cases} \tag{21}$$

$$w'(w,\theta,s) = \begin{cases} (1-\tilde{c}(s)(1+r(\theta,s))w & if\ \gamma \neq 1 \\ \beta(1+r(\theta,s))w & otherwise \end{cases}$$

$$\theta'(w,\theta,s) = \theta'(s)\ .$$

In other words, the value function has the functional form of the underlying utility function, consumption and next-period wealth are linear functions of this-period wealth, and next-period portfolio choices only depend on the current shock. Moreover, proposition 3 also shows that the intensive-form value function, $\tilde{V}$, together with the optimal consumption and portfolio choices, $\tilde{c}$ and θ, can be found by solving an intensive-form Bellman equation that reads

$$\begin{aligned} \tilde{V}(s) &= \max_{\tilde{c},\theta'} \left\{ \frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta(1-\tilde{c})^{1-\gamma} \sum_{s'} (1+r(\theta',s'))^{1-\gamma}\, \tilde{V}(s')\pi(s'|s) \right\} \\ s.t.\quad 1 &= \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1+r_f} \\ & 0 \le \tilde{c} \le 1\ ,\ \theta'_k \ge 0\ ,\ \theta'_h \ge 0\ ,\ \theta'_a(s') \ge \bar{\theta}_a(s') \\ & \left(\frac{\tilde{V}(s')}{\tilde{V}_d(s')}\right)^{\frac{1}{1-\gamma}} (1+r(\theta',s')) \ \ge\ 1 + r_d(\theta'_k,\theta_h,s') - \phi\theta'_k \end{aligned} \tag{22}$$

for $\gamma \neq 1$ and

$$\begin{aligned} \tilde{V}(s) &= \max_{\theta'} \left\{ log(1-\beta) + \frac{\beta}{1-\beta} log\beta + \frac{\beta}{1-\beta} \sum_{s'} log(1+r(\theta',s')\pi(s'|s) \right. \\ &\quad \left. + \beta \sum_{s'} \tilde{V}(s')\pi(s'|s) \right\} \\ s.t.\quad 1 &= \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s'|s)}{1+r_f} \\ & \theta'_k \ge 0\ ,\ \theta'_h \ge 0\ ,\ \theta'_a(s') \ge \bar{\theta}_a(s') \\ & e^{(1-\beta)\left(\tilde{V}(s')-\tilde{V}_d(s')\right)} (1+r(\theta',s')) \ \ge\ 1 + r_d(\theta'_k,\theta'_h,s') - \phi\theta'_k \end{aligned}$$

for the log-utility case.

Proposition 3. Suppose that condition (13) is satisfied and that the autarky value function has the functional form (18) (the expected value function, V^e, has the functional form (17)). Then the value function, V, has the functional form (20) and the optimal policy function is given by (21). Moreover, the intensive-form value function, $\tilde{V}$, and the corresponding optimal consumption and portfolio choices, $\tilde{c}$ and θ', are the maximal solution to the intensive-form Bellman equation (22). This maximal solution is obtained by iterating on the solution, $\tilde{V}_0$, of the intensive-form Bellman equation (22) without participation constraint:

$$\tilde{V} \;=\; \lim_{n\to\infty} \tilde{T}^n \tilde{V}_0 \ ,$$

where $\tilde{T}$ is the operator associated with the intensive-form Bellman equation (21)

Proof Appendix A.

Note that proposition 3 cannot simply be proved by the guess-and-verify method we have used to prove proposition 2 for the case of financial autarky. The reason is that there may be multiple solutions to the Bellman equation (12). In other words, the operator associated with the Bellman equation is monotone, but not a contraction. However, proposition 3 ensures that we have indeed found the value function associated with the original utility maximization problem, and also provides us with a iterative method to compute this solution. Note further that the constraint set in (12) is linear since the return functions are linear in θ. Thus, the constraint set is convex and we have transformed the original utility maximization problem into a convex problem. In other words, the non-convexity problem alluded to in the introduction has been resolved.

4.2. Intensive-Form Equilibrium

Proposition 3 shows how to rewrite the maximization problem of individual households into a recursive problem that is wealth-independent. One implication of the intensive-form representation of the individual maximization problem is that optimal portfolio choices are wealth independent. This result, in turn, implies that the market clearing conditions (7) and (8) can be re-written as intensive-form market clearing conditions that are independent of the wealth distribution. There is, however, still a distribution that matters for the determination of equilibrium, and this is the distribution of relative wealth shares of a household with current shock s_t,

$$\Omega_t(s_t) \doteq \frac{E\left[(1+r_t)w_t|s_t\right]}{E\left[(1+r_t)w_t\right]} .$$

Note that $E[\Omega_t] = \sum_{s_t} \Omega_t(s_t)\pi(s_t) = 1$ and that $(1+r_t)w_t$ is individual wealth including current asset payoffs and asset depreciation. Note also that Ω is finite-dimensional, whereas the set of distributions over (w, θ) is infinite-dimensional.

Using this definition of Ω, in the appendix we show that the market clearing condition (7) is equivalent to the intensive-form market clearing condition

$$\tilde{K}' = \frac{\sum_s \theta'_k(s)(1-\tilde{c}(s))\Omega(s)\pi(s)}{\sum_s \theta'_h(s)(1-\tilde{c}(s))\Omega(s)\pi(s)} \tag{23}$$

and that (8) is equivalent to

$$0 = \sum_{s,s'} \theta'_a(s';s)(1-\tilde{c}(s))\pi(s'|s)\pi(s)\Omega(s) . \tag{24}$$

Moreover, using the definition of Ω and the optimal policy function, g, defined by (21), we also show that the equilibrium law of motion for the Ω-distribution induced by g together with the stationarity condition read:

$$\Omega(s') = \frac{\sum_s (1-\tilde{c}(s))(1+r(\theta'(s),s'))\Omega(s)\pi(s|s')}{\sum_s \sum_{s'} (1-\tilde{c}(s))(1+r(\theta'(s),s'))\Omega(s)\pi(s|s')\pi(s')} . \tag{25}$$

Finally, the government budget constraint (9) becomes:

$$\tau_c \sum_s \tilde{c}(s)(1+r(\theta,s))\pi(s)] = r_h \frac{\tau_h}{1-\tau_h} \sum_s \theta_h(s)\pi(s) . \tag{26}$$

In sum, a recursive equilibrium can be found by solving (22)-(26) if the solution satisfies for all s the condition

$$\beta \sum_{s'} \left(1+r(\theta',s')\right)^{1-\gamma} \pi(s'|s) < 1 . \tag{27}$$

Note that inequality (27) simply ensures that condition (13) is satisfied so that proposition 2 is applicable.

Proposition 4. Suppose that $(\theta, \tilde{c}, \tilde{V}, r_f, \Omega)$ is an intensive-form equilibrium, that is, the consumption-portfolio choice $(\tilde{c}, \theta)$ together with the intensive-form value function $\tilde{V}$ are the maximal solution to the intensive-form Bellman equation (22) satisfying condition (27), the market clearing conditions (23) and (24) are satisfied, and Ω satisfies (25). Then $(g, \tilde{V}, r_f, \mu)$ is a stationary recursive equilibrium, where g is the individual policy function associated with $(\theta, \tilde{c})$ and μ is the stationary distribution associated with the Markov transition function induced by g and π.

Proof. See appendix A.

5. Quantitative Analysis: One-Type Economy

In this section, we present the quantitative analysis of a model without life-cycle considerations. We assume a logarithmic utility function, $u(c) = logc$, and Cobb-Douglas production function, $f(\tilde{k}) = A\tilde{K}^{\alpha}$. We know from proposition 3 that with log-utility the consumption-to-wealth ratio is shock-independent: $\tilde{c}(s) = 1 - \beta$. In addition, we assume that human capital shocks are i.i.d. As shown in the appendix, in this case portfolio choices are independent of the current shock, though individual consumption growth rates are in general shock-dependent. Thus, all households make the same consumption-saving choice and the same portfolio choice regardless of the history of idiosyncratic shocks, and this motivates our terminology "one-type economy". An additional feature of an economy with i.i.d. human capital shocks is that the implied earnings process follows a log random walk, a property that will prove useful when calibrating the model economy.

5.1 Earnings Process

We assume that the human capital depreciation rate is $\delta_h(s_t) = \bar{\delta}_h + \eta(s_t)$, where $\{\eta_t\}$ is i.i.d. with $E[\eta_t] = 0$. As mentioned before, in this case portfolio choices are independent of the current shock realizations. Moreover, in the model economy labor income of an individual household in period t is given by $y_{ht} = r_h h_t$, so that the growth rate of labor income is equal to the growth rate of human capital: $y_{h,t+1}/y_{ht} = h_{h,t+1}/h_t$. Using the policy function (21) in conjunction with the state-independence of portfolio choices, which also implies $\theta_k + \theta_h = 1$, we find:

$$\begin{aligned} log y_{h,t+1} - log y_{ht} &= log h_{h,t+1} - log h_{ht} \\ &= log\beta + log\left(1 + \theta_k(r_k - \delta_k) + (1-\theta_k)(r_h - \bar{\delta}_h - \eta_t)\right) \\ &\approx log\beta + \theta_k(r_k - \delta_k) + (1-\theta_k)(r_h - \bar{\delta}_h) - (1-\theta_k)\eta_t \ , \end{aligned} \tag{28}$$

where the last line uses the approximation $log(1+x) \approx x$. Thus, the log of earnings follows a random walk with drift and innovation term $\epsilon_t = (1-\theta_k)\eta_t$.[12] The random walk specification is often used by the empirical literature to model the permanent component of labor income risk (MacCurdy (1982), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten et al. (2004)), and the estimates of the standard deviation of the innovation term obtained by this literature can in principle be used to calibrate our process of human capital shocks.

One reason why calibration based on either (28) might be problematic is possible misspecification of the earnings process. More specifically, the empirical literature on labor income risk can broadly divided into two strands: one that, after controlling for observable characteristics, assumes that income profiles are homogeneous (MacCurdy (1982)) and one that assumes heterogeneity of income profiles (Lillard and Weiss (1979)). The first strand usually finds a large random walk component or at least a highly persistent component close to a random walk, whereas the second strand often finds that the estimated persistence parameter significantly differs from the random walk specification. For example, Guvenen (2007) finds that the estimated auto-correlation coefficient drops from 0.988 to 0.821 after

[12]We have ϵ_t instead of ϵ_{t+1} in equation (28), and the latter is the common specification for a random walk. However, this is not a problem if the econometrician observes the idiosyncratic depreciation shocks with a one-period lag. In this case, (28) is the correct equation from the household's point of view, but a modified version of (28) with ϵ_{t+1} replacing ϵ_t is the specification estimated by the econometrician.

income heterogeneity has been taken into account.[13] However, based on Monte Carlo simulations, Hryshko (2009) finds that the random walk hypothesis cannot be rejected and that there is little evidence in favor of heterogeneous income growth rates. Moreover, Meghir and Pistaferri (2010) suggest that these two theories might not be mutually exclusive: when Baker and Solon (2003) estimate the parameters of a generalized earnings process that allows for profile heterogeneity, a random walk component, and a transitory component modeled as an AR(1), they find that the variance of the random walk component is precisely estimated and large (though smaller than the estimate when no profile heterogeneity is allowed).

Another reason why calibration based on (28) could be problematic is the following. In the model, changes in earnings are due to changes in human capital, which in turn are driven by two forces: the shock s_t and human capital investment. The model assumes that human capital is immediately adjustable at no cost, so that (28) tends to overstate the ability of households to smooth consumption. To address this issue, we neglect this margin of adjustment when calibrating the model, which can be done by replacing actual human capital investment by its mean value. This yields the following expression for equilibrium earnings growth,

$$log y_{h,t+1} - log y_{ht} \;\approx\; log\beta + log(1 + \bar{r} - \eta_t) \;, \tag{29}$$

where the approximation sign indicates that we have replaced x_{ht} by $E[x_{ht}]$ and $\bar{r} = \theta_k(r_k - \delta_k) + (1 - \theta_k)(r_h - \bar{\delta}_h)$. Thus, the log of earnings still follows a random walk with drift, but the innovation term is now $\epsilon_t = \eta_t$ (using again $log(1 + x) \approx x$) instead of $\epsilon_t = (1 - \theta_k)\eta_t$ as suggested by equation (28). In other words, for a given estimate of σ^2_ϵ, calibration based on (29) leads to a lower value of σ_η than calibration based on (28) as long as $0 \leq (1 - \theta_k) \leq 1$. When calibrating the model economy we use (29) and set $\sigma^2_\epsilon = \sigma^2_\eta$, that is, we use an approach that leads to a relatively low estimate of human capital risk.

There are reasons why the model might understate or overstate the true amount of human capital risk. First, the assumption that earnings innovations are (log)-normally distributed

[13] A similar point has been made by Browning, Ejrnaes, and Alvarez (2006).

is likely to understate the true amount of human capital risk that households face if, as the evidence indicates (Geweke and Keane, 2000), the actual distribution of earnings innovations has a fat lower tail. In a similar vein, the literature on the long-term consequences of job displacement (Jacobson, LaLonde, and D. Sullivan (1993)) has found wage losses of displaced workers that are somewhat larger than suggested by our mean-variance framework. Second, a constant σ represents less uncertainty than a σ that fluctuates with business cycle conditions and has the same mean. Third, there is is also a reason why the model might overstate human capital risk if not all earnings risk is human capital risk. More specifically, if some component of labor income is independent of human capital investment (for example genetic skills) and if this component is random, then some part of the variance of labor income estimated by the empirical literature is not human capital risk.

5.2. Calibration

We follow Krueger and Perri (2006) by choosing a capital income share of $\alpha = 0.3$ and require the model to match a physical capital return, $r_k - \delta_k$, of 4 percent per annum[14] and an aggregate capital-to-output ratio of 2.6. Since $r_k = \alpha Y/K$, this yields $\delta_k = 0.3 : 2.6 - .04 = 0.07538$. There are several empirical studies that suggest that human capital depreciation rates are significantly lower than physical capital depreciation rates. In our baseline calibration, we choose $\delta_h = .04$. The value of β is chosen so that the model matches a given value of earnings growth. The average growth rate of mean male earnings over the period 1968-2001 calculated from the PSID data is equal to 0.19 percent per annum (see section 6).

Finally, the two parameters measuring the two dimensions of contract enforcement, ϕ and p, will be allowed to vary between zero and one. As we change the values of these two parameters, we also vary the other parameter values so that we continue to match our targets for saving and growth, that is, we re-calibrate the model. However, the results reported

[14]Using real financial returns as a proxy for physical capital returns, anything between 1% (T-bills) and 7 % (stocks) seems defensible and has been used in the literature.

below do not change substantially if we keep the other parameter values fixed while varying ϕ and p (comparative statics).

5.3. Consumption and Welfare

We compute equilibria based on the equilibrium characterization result from section 4. Our computational algorithm is discussed in more detail in Appendix B1.

We first consider the amount of risk sharing as a function of the two enforcement parameters p and ϕ. We measure the degree of risk sharing as the ratio of the standard deviation of consumption growth in two different economies: one without financial markets (no risk sharing) and one with financial markets and enforcement constraints. This measure varies between 0 and 1 and an increase means that more risk sharing is possible in equilibrium. We report all results for two different scenarios with respect to the consequences of default. In the first scenario, we assume that defaulting households lose access to financial markets, but continue to supply physical and human capital in the corresponding factor markets (financial autarky). In the second scenario, defaulting households lose access to financial markets and use their physical and human capital for home production (total autarky).

Figure 1 shows the amount of risk sharing for different values of p holding the other enforcement parameter constant at $\phi = 0$. From this figure, we infer that contract enforcement through the exclusion from financial market participation is not very effective. More specifically, if a defaulting household spends on average 7 years in (financial) autarky, $1 - p = 1/7$, a choice that is motivated by the features of the US bankruptcy code (see Livshits et al. (2007)), then there is very little risk sharing in equilibrium: 15 % if defaulting households are forced into full autarky and 30 % if the threat is financial autarky. Moreover, even if defaulting households are excluded from financial markets forever, $p = 1$, consumption volatility is only reduced by around one half.

Figure 2 shows that the welfare cost associated with the lack of consumption insurance is substantial. In the spirit of Lucas (2003), we compute this welfare cost as the percent of lifetime consumption we have to give a household to make him as well off as a comparable household who bears no consumption risk (full insurance). If $1 - p = 1/7$, then the welfare cost is 6 percent, respectively 4 percent, of lifetime consumption, and even if $p = 1$ the welfare cost is still substantial. In comparison, Lucas (2003) finds welfare costs of business cycles that are, for the same degree of risk aversion, around two orders of magnitude smaller. In other words, idiosyncratic labor income risk is substantial and welfare gains from insurance are large, but in order to buy the insurance households would have to borrow a large amount, which they cannot do because the enforcement of credit contracts is weak.[15]

Figure 3 shows our results for different values of ϕ keeping the other enforcement parameter constant at $1 - p = 1/7$. From figure 3 we can see that enforcement through the seizure of capital is highly effective.[16] More specifically, for $\phi = 0$ there is very little risk sharing in equilibrium, but if defaulting households lose only fifty percent of their capital, $\phi = .5$, risk sharing is very close to complete and the corresponding volume of insurance is large. In other words, the equilibrium allocation is almost indistinguishable from the equilibrium allocation without enforcement frictions. Finally, if all the capital is seized, $\phi = 1$, then we obtain full risk sharing in equilibrium. Note that $\phi = 1$ is the parameter value chosen in Krueger and Perri (2006). Finally, figure 4 shows the corresponding welfare costs.

6. Quantitative Analysis: Life-Cycle Economy

In this section, we present the second part of our quantitative analysis. As in the previous part, we consider a version of the model with logarithmic utility function, $u(c) = logc$, and Cobb-Douglas production function, $f(\tilde{k}) = A\tilde{k}^{\alpha}$. However, we now assume that the shock process, $\{s_t\}$, has two components, $s_t = (s_{1t}, s_{2t})$, that are stochastically independent.

[15] One might argue that households could scale down on their investment in physical and/or human capital and use the proceeds to buy sufficient insurance, but any such change in the consumption-portfolio choice would entail efficiency losses that, in our model, are quite large.

[16] This result is true for any value of the enforcement parameter p. In particular, it is also true if results barely change if we set $p = 1$ (no return to financial markets) or $p = 1$ (immediate return to financial markets).

The first component, $\{s_{1t}\}$, is an i.i.d. process representing, as in the previous analysis, household-level shocks to human capital. The second component, $\{s_{2t}\}$, is a variable that keeps track of the age of the individual, s_{2t}. Mean human capital depreciation rates increase with age so that human capital holdings decrease with age. We assume that human capital of retired households depreciates fast enough so that they will choose to hold no human capital. Agents face a constant probability of death and an agent who dies is reborn as a young person. We assume that agents do not care about their offspring, that is, the one-period discount factor takes on the value β if no death occurs and 0 if death occurs. This last assumption renders the model formally equivalent to an OLG model with no inter-generational link.

6.1. Earnings Process

We assume that $\delta_h(s_{1t}, s_{2t}) = \bar{\delta}_h(s_{2t}) + \eta(s_{1t})$, where as before $\{\eta_t\}$ is i.i.d. with $E[\eta_t] = 0$. However, the mean of the human capital depreciation rate, $\bar{\delta}_h$, is now state-dependent (age-dependent): $\bar{\delta}_h = \bar{\delta}_h(s_2)$. As shown in the appendix, the households' portfolio choices are still independent of the i.i.d. shocks s_{1t}, but they now dependent on age, s_{2t}. Using this property and again the policy function (21), we find analogous to (22):

$$log y_{h,t+1} - log y_{ht} \approx log\beta + \theta_k(r_k - \delta_k) + (1-\theta_k)(r_h - \bar{\delta}_h(s_{2t})) - (1-\theta_k)\eta(s_{1t}) \ . \quad (30)$$

Thus, the log of earnings follows a random walk with age-dependent drift. *Mutatis Mutandis*, the discussion of section 5.1 applies, which means that we will use the same value for σ_η^2 when calibrating the model economy. Note that the empirical literature cited above usually removes the effect of age on earnings before estimating the earnings process, so that the age-dependent heterogeneity we have introduced here should not alter the calibration of σ_η^2.

Taking the expectations over (29) with respect to the i.i.d. shocks, $\eta(s_{1t})$, we find mean earnings growth, $E[log y_{h,t+1} - log y_{ht} | s_{2t}]$, as a function of age, s_{2t}. Below we choose the age-dependent mean depreciation rates, $\bar{\delta}_h(s_2)$, so that the implied earnings profile matches the empirical estimates of the earnings profile over the life cycle for male US household heads.

6.2. Calibration

For simplicity, we assume that human capital risk is age-independent and choose $\sigma_\eta = .15$ as in one-type calibration. We follow Huggett et al. (2010) and assume that agents enter the labor market at age 23 and retire at age 61. For agents participating in the labor market, $s_2 \in \{23, \ldots, 60\}$, we choose the age-dependent depreciation rates, $\bar{\delta}_h(s_2)$, so that we match the mean-earnings age profile of male household heads as estimated in Huggett et al. (2010) (see in particular figure 1a – time effect).[17] For retired households, we assume that the human capital depreciation rate is high enough so that they choose not to invest in human capital. Note that retired households face no risk and that their consumption growth rate, g_{re}, is given by $1 + g_{re} = \beta(1 + r_f)$.

We set the first enforcement parameter to $1 - p = 1/7$ and the second enforcement parameter to $\phi = 1$. We choose the remaining parameters/targets of the model as before.

6.3. Results

To be written

[17]We thank Mark Huggett for providing us with the estimates.

Appendix A

A.1. Proof of Proposition 3

Let V_0 be the solution of the Bellman equation (12) without the participation constraint. Simple guess-and-verify shows that V_0 has the following functional form:

$$V_0(w,\theta,s) \;=\; \begin{cases} \tilde{V}_0(s)\,(1+r(\theta,s))^{1-\gamma}\,w^{1-\gamma} & if\ \gamma\ \neq\ 1 \\ \tilde{V}_0(s)\,+\,\frac{1}{1-\beta}log(1+r(\theta,s))\,+\,\frac{1}{1-\beta}w & otherwise \end{cases} \qquad (A1)$$

where $\tilde{V}_0$ is the solution to the intensive-form Bellman equation (22) without participation constraint. Let the operator T be the operator associated with the Bellman equation (12). We show by induction that if $V_n = T^n V_0$ has the functional form, then $V_{n+1} = T^{n+1} V_)$ has the functional form. For $n = 0$ the claim is true because V_0 has the functional form. Suppose now V_n has the functional form. We then have

$$\begin{aligned} V_{n+1}(w,\theta,s) \;&=\; TV_n(w,\theta,s) \qquad (A2)\\ &=\; \max_{w',\theta'}\left\{\frac{((1+r(\theta,s))w-w')^{1-\gamma}}{1-\gamma}\;+\;\sum_{s'}\tilde{V}_n(s')(1+r(\theta',s'))^{1-\gamma}(w')^{1-\gamma}\pi(s'|s)\right\}\\ s.t.\quad 1 \;&=\; \theta'_k+\theta'_h+\sum_{s'}\frac{\theta'_a(s')\pi(s'|s)}{1+r_f}\\ &0\;\le\; w'\;\le\;(1+r(\theta,s))w\;\;,\;\;\theta'_k\ge 0\;\;,\;\;\theta'_h\ge 0\;\;,\;\;\theta'_a(s')\ge\bar{\theta}_a(s')\\ &\tilde{V}_n(s')\,(1+r(\theta'_k,\theta'_h,\theta_a(s')))^{1-\gamma}\,(w')^{1-\gamma}\\ &\ge\;\tilde{V}_d(s')(1+r(\theta'_k,\theta'_h,0,s')-\phi\theta'_k(s))^{1-\gamma}(w')^{1-\gamma} \end{aligned}$$

for $\gamma \neq 1$ and

$$\begin{aligned} V_{n+1}(w,\theta,s) \;&=\; TV_n(w,\theta,s)\\ &=\; \max_{w',\theta'}\Bigg\{log\,(1+r(\theta,s))w-w')\;+\;\beta\sum_{s'}\tilde{V}_n(s')\pi(s'|s)\\ &\qquad +\frac{\beta}{1-\beta}\sum_{s'}log(1+r(\theta',s'))\pi(s'|s)\;+\;\frac{\beta}{1-\beta}log\,w'\Bigg\}\\ s.t.\quad 1\;&=\;\theta'_k+\theta'_h+\sum_{s'}\frac{\theta'_a(s')\pi(s'|s)}{1+r_f}\\ &0\;\le\;w'\;\le\;(1+r(\theta,s))w\;\;,\;\;\theta'_k\ge 0\;\;,\;\;\theta'_h\ge 0\;\;,\;\;\theta'_a(s')\ge\bar{\theta}_a(s') \end{aligned}$$

$$\tilde{V}_n(s') + \frac{1}{1-\beta} log(1 + r(\theta'_k, \theta'_h, \theta'_a(s'), s')) + \frac{1}{1-\beta} log\, w'$$
$$\geq \tilde{V}_d(s') + \frac{1}{1-\beta} log(1 + r(\theta'_k, \theta'_h, 0, s')) + \frac{1}{1-\beta} log\, w'$$

for the log-utility case. Clearly, the solution to the maximization problem defined by the right-hand-side of (A2) has the form

$$\begin{aligned} w'_{n+1} &= (1 - \tilde{c}_{n+1}(s))(1 + r(\theta_{n+1}, s))w \\ \theta'_{n+1} &= \theta'_{n+1}(s) \ , \end{aligned} \qquad (A3)$$

where the subscript $n+1$ indicates step $n+1$ in the iteration. Thus, we have

$$V_{n+1}(w, \theta, s) = \begin{cases} \tilde{V}_{n+1}(s) \, (1 + r(\theta, s))^{1-\gamma} \, w^{1-\gamma} & if \ \gamma \neq 1 \\ \tilde{V}_{n+1}(s) + \frac{1}{1-\beta} log(1 + r(\theta, s)) + \frac{1}{1-\beta} w & otherwise \end{cases} , \qquad (A4)$$

where $\tilde{V}_{n+1}$ is defined accordingly.

From proposition 1 we know that $V_\infty = \lim_{n\to\infty} T^n V_0$ exists and that it is the maximal solution to the Bellman equation (12) as well as the value function of the corresponding sequential maximization problem.. Since the set of functions with this functional form is a closed subset of the set of semi-continuous functions, we know that V_∞ has the functional form. This prove proposition 3.

A.2. Proof of Proposition 4

From proposition 3 we know that individual households maximize utility subject to the budget constraint and participation constraint if condition (13) is satisfied. It is easy to see that condition (13) is satisfied if the proposed portfolio choice, θ', satisfies condition (27). Thus, it remains to show that the intensive-from market clearing conditions (7) and (8) are equivalent to the market clearing conditions (23) and (24) and that the law of motion (25) describes the equilibrium evolution of the relative wealth distribution.

The aggregate stock of physical capital is

$$K_{t+1} = E\left[\theta_{k,t+1} w_{t+1}\right] \qquad (A5)$$

$$
\begin{aligned}
&= E\left[\theta_{k,t+1}(1-\tilde{c}_t)(1+r_t)w_t\right] \\
&= E\left[E\left[\theta_{k,t+1}(1-\tilde{c}_t)(1+r_t)w_t|s_t\right]\right] \\
&= E\left[\theta_{k,t+1}(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right] \\
&= E\left[(1+r_t)w_t\right]\ \frac{E\left[\theta_{k,t+1}(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right]}{E\left[(1+r_t)w_t\right]} \\
&= E\left[(1+r_t)w_t\right]\ E\left[\theta_{k,t+1}(1-\tilde{c}_t)\Omega(s_t)\right]\ ,
\end{aligned}
$$

where the second line follows from the budget constraint, the third line from the law of iterated expectations, the fourth line from the fact that $\theta_{k,t+1}$ and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. A similar expression holds for the aggregate stock of human capital, H_{t+1}, held by households. This proves the equivalence of the intensive-form market clearing condition (23) with the market clearing condition (7).

For the aggregate value of all financial asset holdings we find:

$$
\begin{aligned}
E[\theta_{a,t+1}w_{t+1}] &= E[\theta_{a,t+1}(1-\tilde{c}_t)(1+r_t)w_t] \qquad (A6)\\
&= E[E[\theta_{a,t+1}(1-\tilde{c}_t)(1+r_t)w_t|s_t]] \\
&= E[\theta_{a,t+1}(1-\tilde{c}_t)E[(1+r_t)w_t|s_t]] \\
&= E\left[(1+r_t)w_t\right]\ \frac{E[\theta_{a,t+1}(1-\tilde{c}_t)E[(1+r_t)w_t|s_t]]}{E\left[(1+r_t)w_t\right]} \\
&= E\left[(1+r_t)w_t\right]\ E\left[\theta_{a,t+1}(1-\tilde{c}_t)\Omega(s_t)\right]\ .
\end{aligned}
$$

where the first line follows from the budget constraint, the second line from the law of iterated expectations, the third line from the fact that $\theta_{a,t+1}$ and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. This proves the equivalence of the intensive-form market clearing condition (24) with the market clearing condition (8).

Finally, the law of motion for Ω can be found as:

$$
\begin{aligned}
\Omega_{t+1}(s_{t+1}) &= \frac{E\left[(1+r_{t+1})w_{t+1}|s_{t+1}\right]}{E\left[(1+r_{t+1})w_{t+1}\right]} \qquad (A7)\\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t\right]} \\
&= \frac{E\left[E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_t\right]|s_{t+1}\right]}{E\left[E\left[(1+r_{t+1})(1-\tilde{c}_t)(1+r_t)w_t|s_t\right]\right]}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right]} \\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)E\left[(1+r_t)w_t|s_t\right]\right]} \times \\
&\quad \frac{E\left[(1+r_t)w_t\right]}{E\left[(1+r_t)w_t\right]} \\
&= \frac{E\left[(1+r_{t+1})(1-\tilde{c}_t)\Omega_t(s_t)|s_{t+1}\right]}{E\left[(1+r_{t+1})(1-\tilde{c}_t)\Omega_t(s_t)\right]},
\end{aligned}
$$

where the second line follows from the budget constraint, the third line from the law of iterated expectations, the fourth line from the fact that θ_{t+1} and $\tilde{c}_t$ are independent of wealth and s^{t-1}, and the last line from the definition of Ω. This completes the proof of proposition 4.

Appendix B

B.1. Computation with I.I.D. Shocks

Assuming log-utility and an i.i.d. shock process, the intensive-form Bellman equation (22) determining the intensive-form value function becomes

$$
\tilde{V} = \max_{\theta'} \left\{ log(1-\beta) + \frac{\beta}{1-\beta} log\beta + \frac{\beta}{1-\beta} \sum_{s'} log(1+r(\theta',s'))\pi(s') + \beta\tilde{V} \right\} \tag{B1}
$$

$$
\begin{aligned}
s.t. \quad 1 &= \theta'_k + \theta'_h + \sum_{s'} \frac{\theta'_a(s')\pi(s')}{1+r_f} \\
& \theta'_k \geq 0 \ , \ \theta'_h \geq 0 \ , \ \theta'_a(s') \geq \bar{\theta}_a(s') \\
& e^{(1-\beta)\left(\tilde{V}-\tilde{V}_d\right)} (1+r(\theta',s')) \ \geq \ 1 + r_d(\theta'_k,\theta'_h,s') - \phi\theta'_k
\end{aligned}
$$

From (B1) it immediately follows that the intensive-form value function, $\tilde{V}$, and the optimal portfolio choice, θ, are independent of s, though they still depend on the aggregate capital-to-labor ratio, $\tilde{K}$, and the interest rate, r_f. Given that portfolio choice are state-independent, the market clearing conditions become

$$
\begin{aligned}
\tilde{K} &= \frac{\theta_k}{\theta_h} \\
0 &= \sum_{s'} \theta_a(s')\pi(s')
\end{aligned} \tag{B2}
$$

Note that we have $\Omega(s) = \Omega = 1$ so that equation (25) is automatically satisfied. Note also that the second market clearing condition in conjunction with the budget constraint imply $\theta_k + \theta_h = 1$.

Equations (B1) and (B2) determine an intensive-form equilibrium. We solve (B1) and (B2) iteratively as follows:

1. Pick an aggregate capital-to-labor ratio, $\tilde{K}$, which determines the rental rates r_k and r_h and therefore the autarky value function $\tilde{v}_d$.

2. Given these values for r_k and r_h and $\tilde{v}_d$, search for a vector $(\tilde{v}, \theta_k, \theta_h, \theta_a, r_f)$ that solves (B1) and the second part of (B2). This gives us a new value of $\tilde{K}$ defined by θ_k/θ_h.

3. Iterate on $\tilde{K}$ until the first market clearing condition in (B2) is satisfied.

Note that for step 2, we make heavy use of the first-order conditions associated with the maximization problem (B1), which are sufficient given the concavity of the objective function and the convexity of the choice set.

B.2. Computation with I.I.D. Shocks and Age-Dependent Mean

Assuming again log-utility and a shock process with two components, $s_t = (s_{1t}, s_{2t})$, as described in section 6. Since the first component is i.i.d., the Bellman equation (21) determining the intensive-form value function becomes

$$
\begin{aligned}
\tilde{V}(s_2) \quad = \quad & \max_{\theta'} \left\{ log(1-\beta) + \frac{\beta}{1-\beta} log\beta + \frac{\beta}{1-\beta} \sum_{s'_1, s'_2} log(1 + r(\theta', s'_1, s'_2))\pi(s'_1, s'_2|s_2) + \beta\tilde{V}(s_2) \right\} \\
s.t. \quad 1 \ = \ & \theta'_k + \theta'_h + \sum_{s'_1, s'_2} \frac{\theta'_a(s'_1, s'_2)\pi(s'_1, s'_2|s_2)}{1 + r_f} \\
& \theta'_k \geq 0 \ , \ \theta'_h \geq 0 \ , \ \theta'_a(s'_1, s'_2) \geq \bar{\theta}_a(s'_1, s'_2) \qquad (B3) \\
& e^{(1-\beta)\left(\tilde{V}(s_2) - \tilde{V}_d(s_2)\right)} (1 + r(\theta', s'_1, s'_2)) \ \geq \ 1 + r_d(\theta'_k, \theta'_h, s'_1, s'_2) - \phi\theta'_k
\end{aligned}
$$

From (B3) it immediately follows that the optimal portfolio choice, θ, is independent of s_1, but in general depends on s_2. Thus, the market clearing conditions become

$$\begin{aligned}
\tilde{K} &= \frac{\sum_{s_2} \theta_k(s_2)\pi(s_2)\Omega(s_2)}{\sum_{s_2} \theta_h(s_2)\pi(s_2)\Omega(s_2)} \qquad (B4) \\
0 &= \sum_{s_2, s_1', s_2'} \theta_a(s_2; s_1', s_2')\pi(s_1', s_2'|s_2)\pi(s_2)\Omega(s_2)
\end{aligned}$$

Equations (B3) and (B4) determine an intensive-form equilibrium. We solve (B3) and (B4) iteratively as follows:

1. Pick an aggregate capital-to-labor ratio, $\tilde{K}$, which determines the rental rates r_k and r_h and therefore the autarky value function $\tilde{v}_d$.

2. Given these values for r_k and r_h and $\tilde{v}_d$, search for a vector $(\tilde{v}, \theta_k, \theta_h, \theta_a, r_f)$ that solves (B3) and the second part of (B4). Note that $\tilde{v}$ and the portfolio vector, $(\theta_k, \theta_h, \theta_a)$, now has age-dependent components. Define a new value of $\tilde{K}$ through the portfolio choices of households in the canonical way.

3. Iterate on $\tilde{K}$ until the first market clearing condition in (B4) is satisfied.

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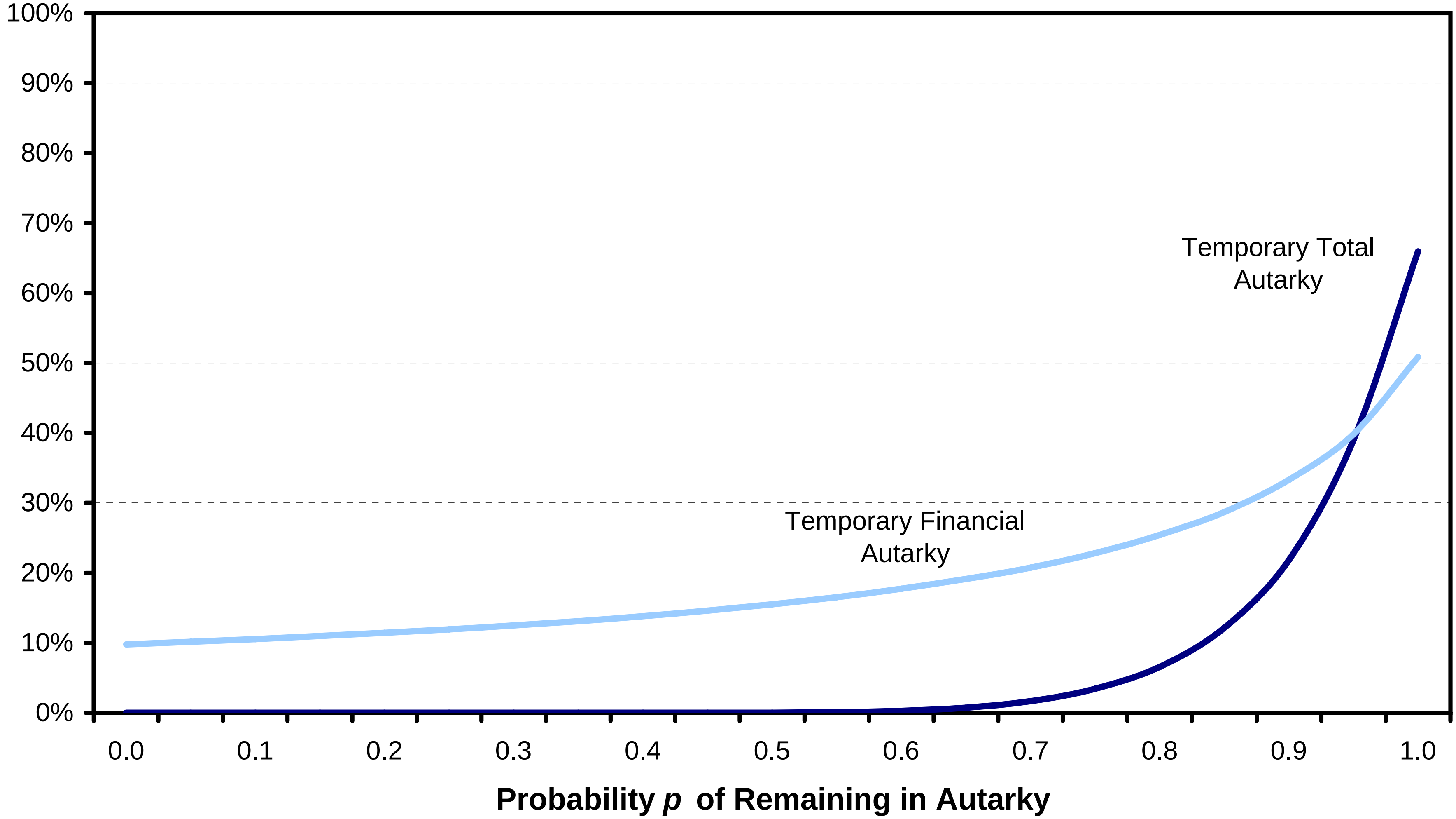
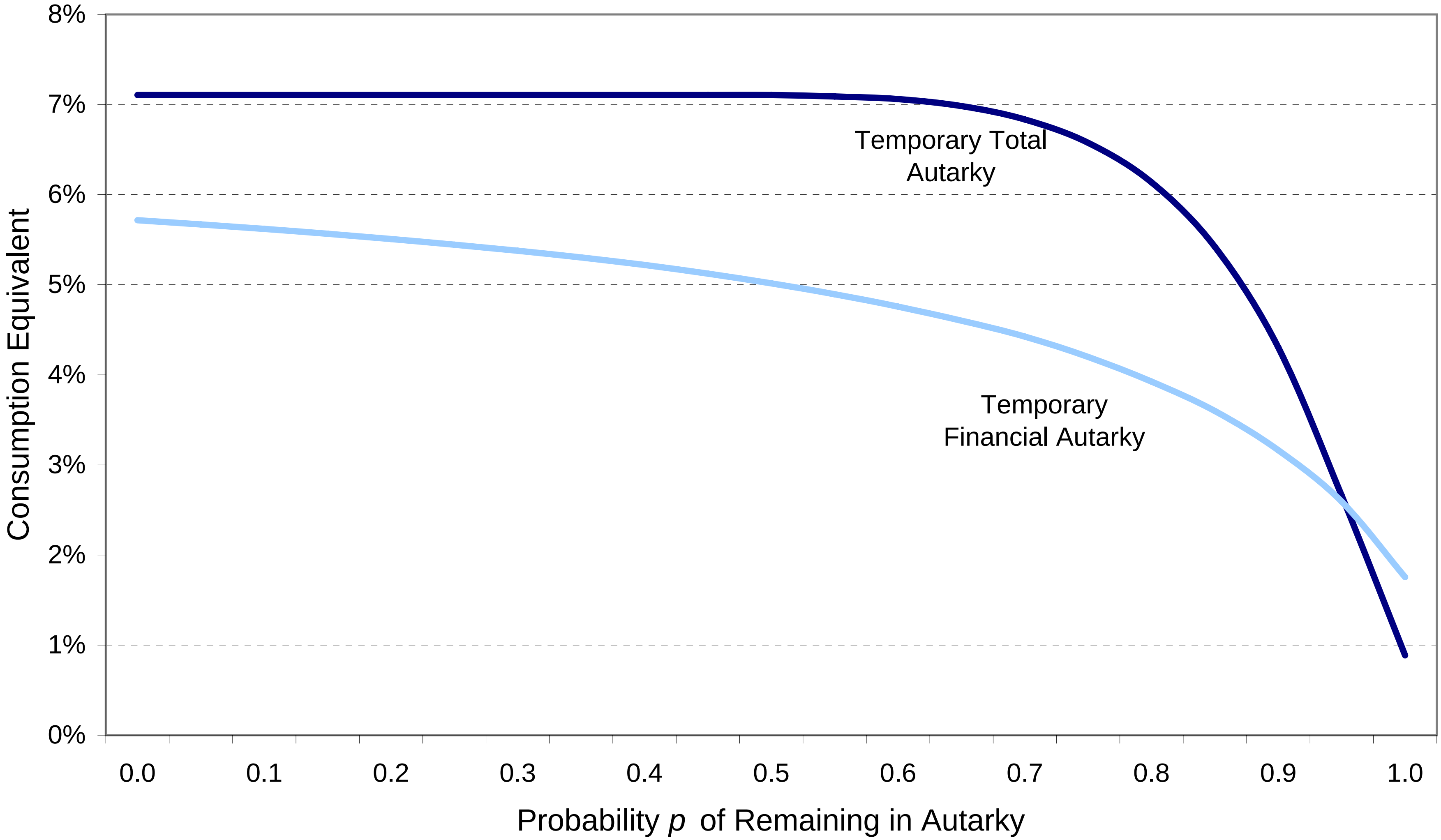
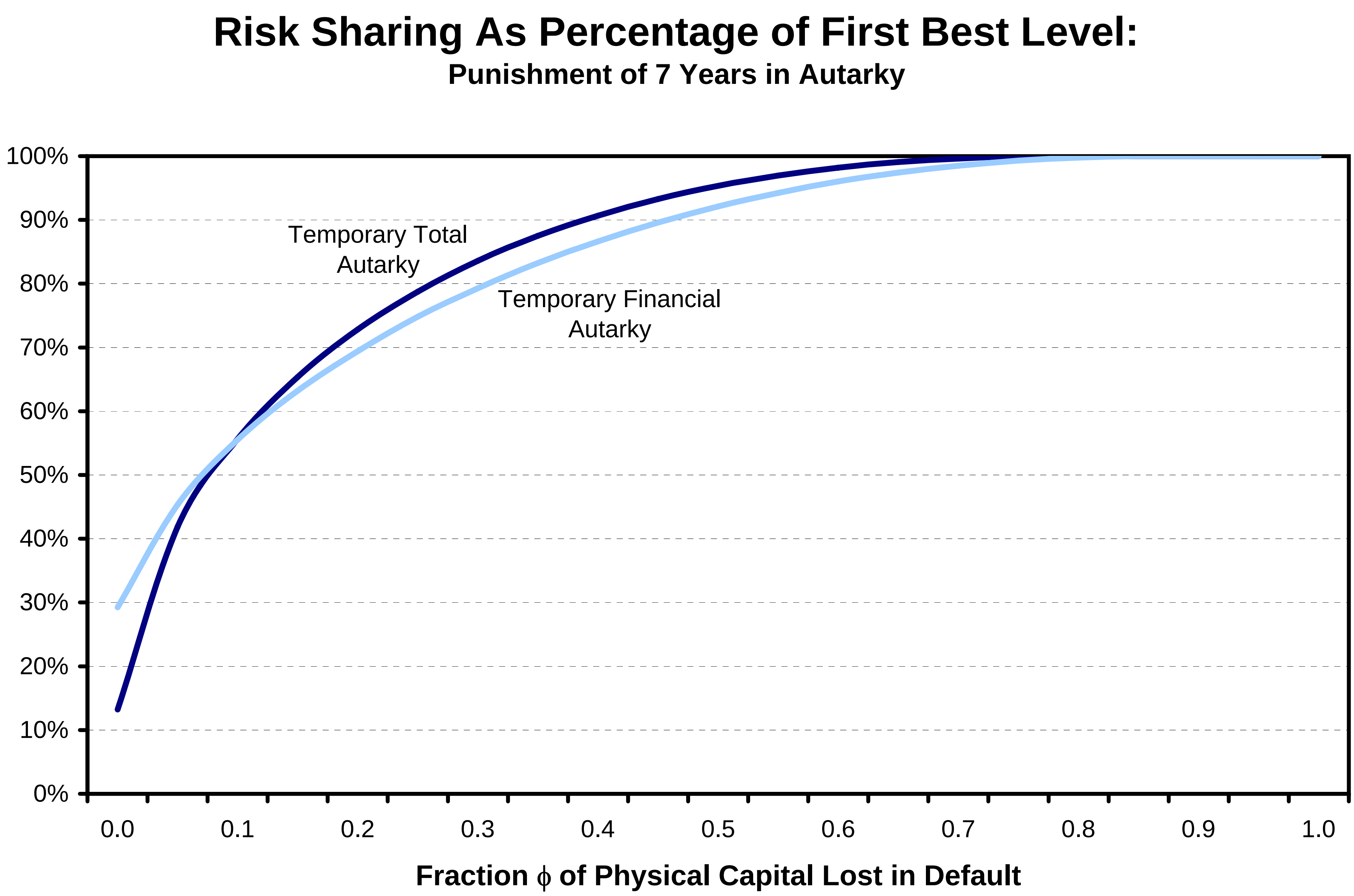
Risk Sharing As Percentage of First Best Level:
Fraction of Capital Seized ϕ = 0

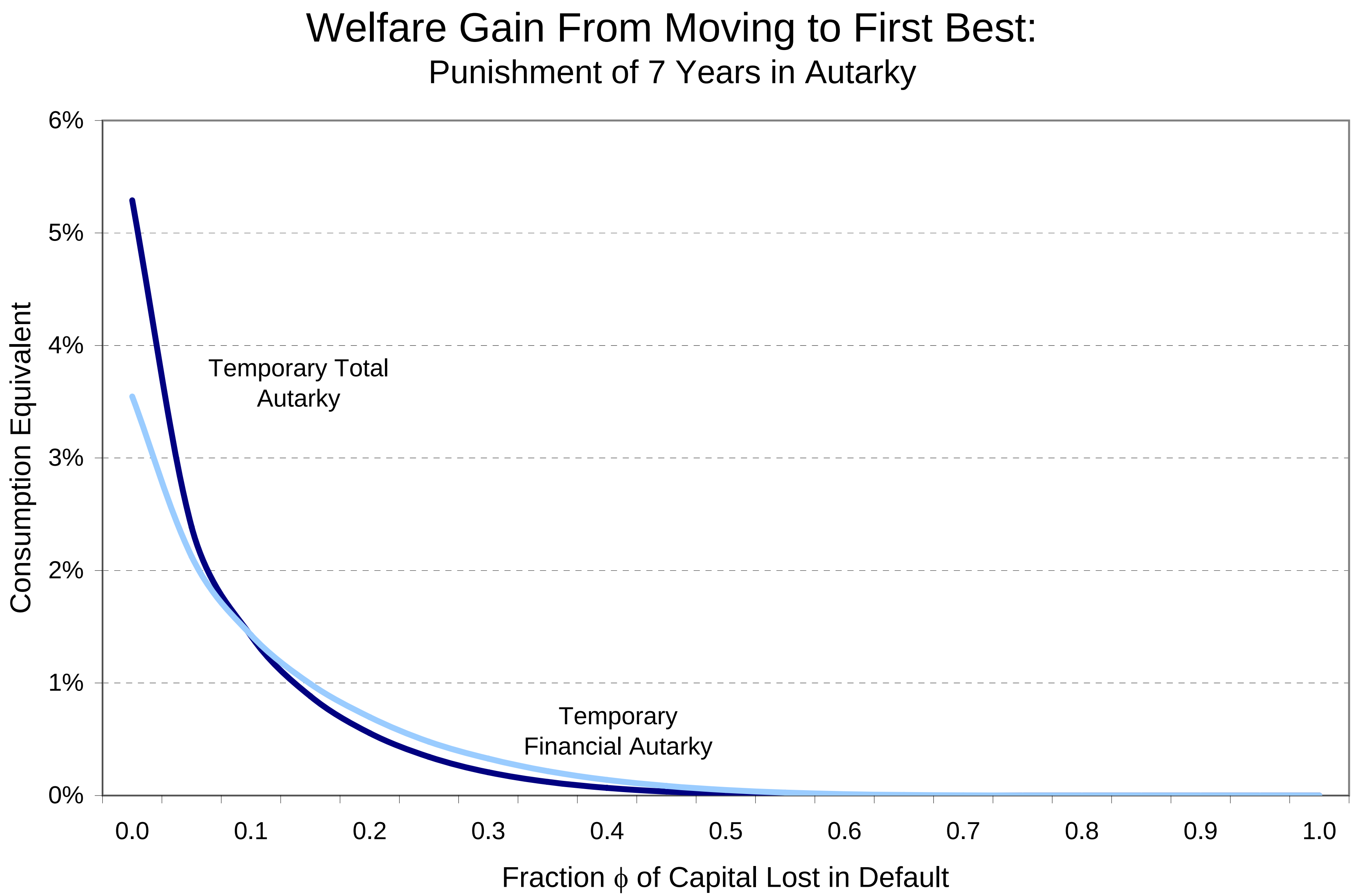


Welfare Gain From Moving to First Best:
Punishment of 7 Years in Autarky





Risk Sharing As Percentage of First Best Level:
Punishment of 7 Years in Autarky
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Temporary Total Autarky
Temporary Financial Autarky
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fraction ϕ of Physical Capital Lost in Default


Welfare Gain From Moving to First Best:
Punishment of 7 Years in Autarky
Consumption Equivalent
6%
5%
4%
3%
2%
1%
0%
Temporary Total
Autarky
Temporary
Financial Autarky
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Fraction ϕ of Capital Lost in Default