Contracting Institutions and Economic Development

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October 2010

[VERY PRELIMINARY]

Abstract

This paper studies the effects of contracting institutions on economic development. A growth model is presented with endogenous incomplete markets, where financial frictions generated by the imperfect enforcement of contracts depend on the future growth of the economy, which determines the costs of being excluded from financial markets after defaulting. As the economy approaches its steady state, frictions and their effect on income become more important because the net benefits of honoring contracts fall. Therefore the model predicts that contracting institutions affect GDP per capita in the last stages of development. This effect is not only due to a slower accumulation of capital, but it is also caused by a misallocation of resources toward labor intensive sectors of production, where self-enforcing incentives are stronger. To validate the model empirically I use cross-country regressions to estimate the effect of contracting institutions on per capita GDP. In line with the main predictions of the model, the econometric evidence shows that this effect is larger in richer economies, and it has taken place mainly over the last 60 years. Unlike contracting institutions, the evidence shows that property right institutions, included in an extension to the model, have had an effect on income per capita throughout the development process.

*I thank Harold L. Cole and Dirk Krueger for helpful comments, discussions and guidance. I also thank Ari Aisen, Ufuk Akcigit, Wei Cui, Flavio Cunha, Jesus Fernandez-Villaverde, Charles Jones, Guido Menzio, and seminar participants at the Midwest Macroeconomic Meetings 2010, the 11th Meeting of LACEA's Political Economy Group, the EconCon at Princeton, the UPenn Macro Lunch, and the Central Bank of Chile for useful comments. All errors are mine.

1 Introduction

The state and associated institutions provide the legal framework that enables private contracts to facilitate economic transactions (North [1981]). Consequently, the quality of these institutions should affect economic efficiency. The extensive empirical literature, described in La Porta et al. [2008], that investigates the link between legal institutions and income per capita has found a strong and significant relationship. Legal traditions, developed centuries ago in Europe, have been used to identified the exogenous component of institutional quality. But these traditions does not only determine the rules governing contracting between private agents, but they are also a determinant of a broader concept of property rights institutions (Levine [2005]). Acemoglu and Johnson [2005] distinguished between two different types of institutions: contracting institutions (CI) -those that enable private contracts between citizens-, and property rights institutions (PRI) -those that protect citizens against expropriation by the government and powerful elites. They document a strong link between CI and legal origin on the one hand, and PRI and initial endowments on the other, which would have influenced the type of institutions established by Europeans in former colonies. The authors then show that, after controlling for the effect of PRI, differences across countries in income per capita today are not related to CI quality indicators.

This paper contributes to the debate about the comparative effects of these two types of institutions on income per capita and development. In order to do that it develops a growth model with endogenous financial frictions and a distinction between CI and PRI. The main prediction of the model is that the effect of the quality of CI on GDP per capita depends on the distance between its current level and its steady-state level. The closer the economy is to its steady-state, the larger are the effects of the quality of these institutions on income per capita. Unlike CI, the effect of PRI does not depend on the stage of economic development of a country, and they affect GDP per capita throughout the development process. The paper revisits the empirical evidence on the growth-institutions nexus in light of the model to validate its main predictions. It finds that the effect of the quality of CI is only significant in the group of rich economies, and that it has taken place mainly during the last half century.

In the model financial frictions arise from the assumption that entrepreneurs, who sign contracts with consumers in order to borrow resources and to invest in physical capital, are able to walk away from the contracts. If they do so it is assumed they can not take full advantage of future production opportunities. The benefit of defaulting is to appropriate the resources borrowed from consumers, which, in equilibrium, are proportional to current output. As the net benefit of honoring the contract is increasing in the expected future growth of the economy, a higher growth rate makes future production opportunities more attractive relative to stealing capital. Financial frictions become less binding, and more efficient contracts are self-enforced. On top of this CI, whose quality is assumed as exogenous, reduce the benefits of defaulting. Thus, even if self-enforcement incentives are weak, optimal contracts can be enforced if the institutional quality is good enough. But if the efficient contract is self-enforced in the absence of these institutions, the quality of the latter does not affect production.

These financial frictions are embedded into the standard neoclassical growth model. Along the transition path towards a steady-state, growth is declining and thus self-enforcement weaken, reaching their lowest level in the steady-state[1]. Therefore financial frictions are more important when the economy is close to its steady-state. There is no default in equilibrium. But the new contracts include debt constraints, lowering capital accumulation and output. The main prediction of the model is that the effect of the quality of CI on income per capita across countries becomes increasingly relevant in the later stages of economic development, when debt constraints generated by financial frictions in the absence of these institutions bind the most. Empirically this implication has at least two dimensions. First, in the cross-sectional dimension we should observe a larger effect of CI in richer economies, because they are probably closer *on average* to their steady-states[2]. Second, in the time series dimension, a larger effect of CI should be observed in recent periods, as most of the countries have industrialized and transitioned toward their steady-states. The paper exploits these two predictions to validate empirically the theoretical model.

An additional implication of the model is that the incentives to default, and therefore the consequences of financial frictions, depend positively on the intensity with which capital is used in production. To explore this feature the model includes two sectors, a capital intensive sector and a labor intensive sector. In this context financial constraints are more binding in the capital intensive sector, generating not only a fall in total savings but also an inefficient allocation of capital (and thus labor) toward the labor intensive sector. Therefore CI affect income per capita not only because capital accumulation slows down but also because of a misallocation of resources that affects economy-wide total factor productivity (TFP) negatively.

As an extension, PRI are included in the model in a very stylized way. In countries with low

[1]The use of an exogenous growth model is for clarity. An endogenous growth model with transitional dynamics would generate the same theoretical prediction. However, when testing the main implications of the model it is assumed there exists conditional convergence, something that it is confirmed in the empirical section.

[2]Under conditional convergence, this assumes there are poor countries with income levels in steady-states similar to those of rich countries.

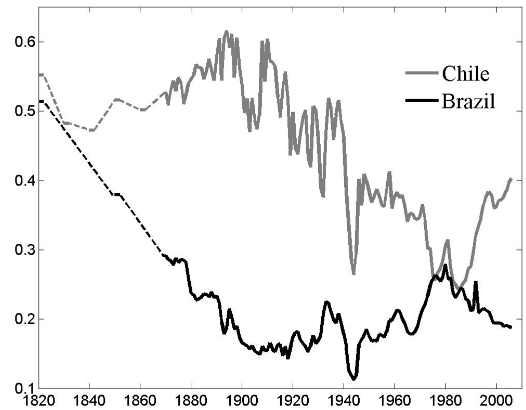

Figure 1: Relative GDP per capita with respect to the US.

quality PRI, governments are able to steal a fixed fraction of capital every period[3]. Thus, the quality of these institutions slows down the transition to the long-run equilibrium, and lower the stock of capital and income per capita in the steady-state.

The paper implements cross-country regressions to compare the timing of the effects of each type of institutions on income per capita. But to illustrate the main empirical predictions we can consider the historical performance of a group of economies. Figure 1 shows GDP per capita relative to the US for two countries, Chile and Brazil, since their independence. The US is used here as the benchmark because of its good institutions, and I consider former colonies from The Americas as they were the first obtaining their independence. Inside Latin America, Chile has shown one of the best scores on indicators capturing the quality of PRI since independence, while Brazil has shown one of the worst. In terms of legal institutions these two countries have a civil law system, which is the system associated with a low efficiency. Consequently they show low scores in CI efficiency indicators[4]. Figure 1 shows an early divergence between Brazil and the others two countries. With respect to the US this gap has persisted for almost two centuries. Chile on the other hand was able to maintain the initial gap with respect to the US until the Second World War. According to the model the difference in PRI has had a deep effect on the differences between these countries since independence, as these institutions affect GDP per capita throughout the development process. But we can also observe that the gap between the US and Chile widened in the post-war period. This path would be the one predicted by the model for two countries with similar quality of

[3]In the paper it is shown that this assumption can be interpreted as a reduced form for a more complex problem.

[4]For PRI this is the case for the preferred measured by Acemoglu and Johnson [2005], constraints on the executive, from the Polity IV database. For CI it is the case for contracting enforcement (Djankov et al. [2008]) and legal formalism (Djankov et al. [2003]). All these indicators are described in the empirical part of the paper.

PRI, but different quality of CI, as they affect GDP per capita only in the last stages of economic development.

The empirical part the paper follows previous IV identification strategies to estimate the effect of institutional quality on income per capita. It also applies a new, although related, identification strategy to increase the size of the sample. Additionally, two modifications to previous specifications are introduced. First the sample is split between rich and poor countries, and second, the level of GDP per capita in 1950 -the first year with enough data- is introduced as a control. Intuitively in this case, if the inclusion of the lagged level of GDP does not change the coefficient on CI, it does not have information about it, and so the quality of CI did not have an effect on that lagged level of GDP. Before introducing these modifications the results confirm the findings by Acemoglu and Johnson [2005]. But in most of the new specifications: (1) the effect of CI is larger and (more) significant for the sample of richer economies, (2) when controlling for the level of GDP per capita in 1950, the effect of CI becomes (more) significant, and (3) the coefficient on PRI shows the opposite pattern. All of these findings are in line with the main predictions of the model[5].

Literature Review

This paper is closely related to the theoretical literature on financial frictions and growth. Most of this literature focuses on informational imperfections as the main source of financial frictions, following Townsend [1979][6]. Since exclusion from future production opportunities as a punishment has not been introduced in these papers, the main implication of the model presented here has not been obtained. Among the papers focusing on pure enforcement problems as the source of financial frictions, the most closely related to this paper is Marcet and Marimon [1992]. Their results are different from the ones presented here, because they analyze the central planner problem, allowing transfers between lenders and borrowers contingent on default decisions. Then, the fact that growth is decreasing over time translates on a path for borrowers' consumption that is increasing over time. Moreover the optimal level of investment is feasible in steady-state as contingent transfers to borrowers are positive. Another paper focusing on imperfect enforcement is the quantitative study by Buera et al. [2010]. There, exclusion from future production opportunities after default-

[5]These results may be useful to explain the conflicting evidence described above. First, Acemoglu and Johnson [2005] focus only in former colonies, while the studies on the effects of legal institutions include richer European countries. Second, because of the temporal pattern for the effect of both types of institutions, and in the presence of measurement problems, to prove the significance of the effect of CI would be easier using, as the dependent variable, the growth in GDP per capita on the last decades -as in the studies on legal institutions-, instead of its unconditional level -as in Acemoglu and Johnson [2005].

[6]See for instance Greenwood and Jovanovic [1990], Castro et al. [2004] Townsend and Ueda [2006], Castro et al. [2009], and Greenwood et al. [2010]

ing is not included, but the ability to overcome financial constraints with internal funds it is. The authors show quantitatively, that even with self-financing the efficient level of investment can not be achieved on sectors with larger financial needs.

Theoretically this paper is also related to the literature on limited enforceability of contracts and imperfect insurance (Kehoe and Levine [1993], Kocherlakota [1996], and Alvarez and Jermann [2000] among others), and on sovereign borrowing (Eaton and Gersovitz [1981], Cole and Kehoe [1995], Kletzer and Wright [2000], and Kehoe and Perri [2002] among others), where enforcement by a third party is totally absent. These papers study theoretically the implications of exclusion from financial markets after defaulting, although not in a growth context. Among the papers on imperfect insurance and incomplete markets, the closest to this paper is the one by Krueger and Perri [2006], who also study the effect of changes in the environment on self-enforcement incentives, although in a different context.

The role of financial frictions on the misallocation of capital has been studied before. In Buera et al. [2010] misallocation is generated across firms with different fixed costs, as they determine external borrowing needs. The aim is to explain the allocation of capital between manufacturing and traditional, small-scale, service industries. In this paper the misallocation is across sectors with different capital intensities, and thus the focus would be on differences between manufacturing and agriculture[7]. The quantitative framework also allows Buera et al. [2010] to study the effect of financial frictions on the misallocation of entrepreneurial abilities. Castro et al. [2009] also study misallocation effects in a model with informational frictions. In their paper misallocation is across industries producing capital goods and those producing consumption goods, and it is due to a larger volatility of idiosyncratic volatility shocks in the former.

Empirically this paper is related to the extensive literature exploring the link between institutions and income per capita. Papers focusing on the role of legal institutions have found a close link between legal origin and their quality. Some of the indicators influenced by the former are investor protection (La Porta et al. [1997] and La Porta et al. [1998]), the formalism of judicial procedures (Djankov et al. [2003]), judicial independence (La Porta et al. [2004]), and the quality of contract enforcement (Djankov et al. [2008]). Using these findings some papers have identified a strong and significant relationship between these institutions and income per capita (Beck et al. [2000], Levine [1998], Levine [1999], and Levine et al. [2000]). As noted above, Acemoglu and Johnson [2005] explore the comparative effects of CI and PRI on income per capita, using the ideas developed

[7]Echevarria [1997] shows that, in terms of GDP composition, the main differences across countries are associated with the size of the agricultural sector relative to manufacturing and services.

by Engerman and Sokoloff [1997], Engerman and Sokoloff [2002], Acemoglu et al. [2001], and Acemoglu et al. [2002] about the effects of initial endowments on institutional quality in former colonies.

There is a related empirical literature showing that legal institutions have affected economic outcomes only in the last century (Rajan and Zingales [2003], La Porta et al. [2008], and Musacchio [2010]). The explanation for these findings is that the quality of legal institutions is not an issue for growth as long as prevalent political interests support them. The model in this paper is able to generate the observed divergence on outcomes without relying on political issues. Moreover, this is consistent with the empirical evidence, as the effects of CI on GDP per capita are independent of the quality of PRI.

With respect to its main implications, this paper is closely related to the work by Acemoglu et al. [2007]. They also show that some policies and institutions, beneficial for growth in the first stages of development, might be harmful in the long-run. This is because countries are better off adopting existing technologies when they are far from the technological frontier, but as they approach this frontier they are better off creating new technologies. Both models would suggest that growth-enhancing reforms an economy will grow fast despite institutional deficiencies. However, the policies identified by the authors -anticompetitive policies and investment subsidies, or others which increase monopolists' gains- are different from the institutions analyzed here.

The predictions related to TFP are in line with the evidence identifying this variable as the main channel trough which legal institutions affect GDP per capita (Beck et al. [2000]), and as the main source of output per worker differences across countries (Hall and Jones [1999]). The model also predicts high rates of return to capital coexisting with low rates in the same economy, consistent with the evidence summarized by Banerjee and Duflo [2005]. Additionally, the relative price of output in the capital-intensive sector falls with the efficiency of CI, as more resources are allocated there. If the production of capital goods is capital-intensive, then investment rates at common international prices co-vary positively with income, as shown by Castro et al. [2009][8].

The next section of the paper presents the model. It first characterizes the perfect enforcement equilibrium. After that the binding pattern of the constraints imposed by low quality CI when allocations are those of the perfect enforcement equilibrium is described. The main results of the paper are related to this issue. Section 2 describes next the imperfect enforcement equilibrium and

[8] In a small open economy the reallocation would raise the value of the labor intensive sector unambiguously. Then a country with efficient CI would specialize in the production of goods for which relationship-specific investments are most important, as found by Nunn [2007].

ends extending the model to the inclusion of PRI. Section 3 presents the econometric evidence, and the last section concludes.

2 The Model

The Economic Environment

The economy is populated by consumers and two types of entrepreneurs, each with measure 1. Each entrepreneur has access to one of two technologies, which differ in factor intensities. There is no entry to, or exit from, entrepreneurship, and entrepreneurs can not switch sectors[9]. Denote by $j = m$ the capital-intensive sector, or manufacturing, and $j = a$ the labor-intensive sector, or agriculture. Denote by $i = c, e_j$ consumers and entrepreneurs with access to technology j respectively. Technologies can be described by the following expression for $j = m, a$,

$$y_j = z_j^{1-\alpha_j} k_j^{\alpha_j} n_j^{\upsilon_j}$$

where z_j captures the level of technology, k_j and n_j are capital and labor allocated to sector j respectively, and α_j and υ_j are positive constants. Assume that both technologies show decreasing returns to scale and that they are both equally intensive in the fixed factor, so $\omega = 1 - \alpha_a - \upsilon_a = 1 - \alpha_m + \upsilon_m > 0$, and $\alpha_m > \alpha_a$. Finally z_m and z_a grow at constant (gross) rates, $\mu_m > 1$ and $\mu_a > 1$ respectively, so this is a deterministic exogenous growth model.

Each consumer is endowed with one unit of labor. Preferences over consumption of both goods are given by the following instantaneous utility function

$$u(c) = c_a^{\eta} c_m^{1-\eta}$$

where $0 < \eta < 1$ is a constant. The representative type i agent maximizes the expected value of his lifetime utility as given by

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{u(c_t)^{1-\sigma^i} - 1}{1 - \sigma^i}\right]$$

where σ^i is the risk aversion coefficient or the inverse of the elasticity of substitution. It is assumed for simplicity that entrepreneurs are risk neutral, so $\sigma^i = 0$ for $i = e_a, e_m$. Consumers are risk

[9]This simplifying assumption, and the fact that there are decreasing returns to scale, imply the existence of profits without entry. In a more general model entrepreneurship may be endogenous. But the fact that lending is constrained at an individual level due to the possibility of default, entry could overcome the effects of financial frictions. However, if entrepreneurs differ in their productivity, new entrants will be less productive and therefore a missalocation of entreprneurial ability will reduce output as well.

averse so $\sigma^c = \sigma > 0$.

The price of the capital-intensive good is normalized to 1, while the price of the labor-intensive good is p. For notational purposes the price of good j is denoted by p_j, so henceforth $p_j = 1$ if $j = m$, and p otherwise. Hence the total value of output in the economy is

$$y = y_m + py_a$$

Only the capital-intensive good can be used as capital, which depreciates at a rate δ each period. This implies the following market clearing conditions for each sector,

$$c_m = \sum_i c_m^i = y_m + (k_m + k_a)(1 - \delta) - k'_m - k'_a \tag{1}$$

$$c_a = \sum_i c_a^i = y_a \tag{2}$$

where c_j^i is the consumption of good j by agent $i = c, e_a, e_m$.

Consumers save a fraction of their income and lend it to entrepreneurs, who do not save. Entrepreneurs finance capital with these resources and, if they find optimal to do so, they pay back to consumers the amount lent plus the market interest rate after production takes place. If they find optimal not to pay back to consumers and if they are not caught doing so, which happens with probability ρ, they appropriate the stock of capital and its return. Then the parameter ρ captures the quality of institutions related to the enforcement of contracts. In case of defaulting, entrepreneurs can not borrow from the consumer anymore, but they can use the amount of capital stolen to produce in the future with the same technology described above. If the entrepreneur is caught, which happens with probability $(1 - \rho)$, he is forced to give back the capital stolen and the return, so he is left without any income source for the future.

Discussion

In order to introduce the need of external borrowing in the model in a simplified way it is assumed entrepreneurs can not save. However, in the presence of financial frictions affecting external financing, agents may be able to invest in their own projects, at the cost of forgoing current consumption, to achieve higher levels of investment. Moreover, self-investment has a positive effect on the maximum amount a borrower can obtain from creditors. In the presence of imperfect enforcement this is because collateral increases the cost of defaulting, as creditors may be able to appropriate some fraction of it[10]. The effect of collateral on external financing depends positively on the credible

[10] In the presence of informational asymmetries this is because collateral reduces the incentives of falsely claiming a bad outcome (Bernanke and Gertler [1989] and Kiyotaki and Moore [1997]).

commitment of the government to reallocate collateral across agents (Kletzer and Wright [2000]), i.e. on the quality of CI. Therefore the worse is the quality of CI, and so the more important are borrowing restrictions, the lower is the efficiency of collateral in aleviating financial frictions.

How important is self-investment in attenuating frictions is a quantitative issue not addressed by the model. But the results by Buera et al. [2010] give some insights on this issue. The authors allow entrepreneurs to save in a similar model of imperfect enforcement and show that external financing is increasing on entrepreneurs' wealth[11]. One of the main quantitative findings of the paper is that, while self-financing can alleviate the inefficiencies generated by financial frictions, the efficient level of investment can not be achieved on sectors with larger financial needs. It follows that in the environment proposed by this paper, most likely self-financing would be not enough to alleviate the effects of financial frictions in the long-run.

The assumption behind the main result of the paper; the fact that, after defaulting, entrepreneurs can not take full advantage of future production opportunities, can be implemented in different ways. Albuquerque and Hopenhayn [2004] specifies an outside value function for defaulters that depends positively on the amount borrowed and the technology shock. This function can be interpreted in different ways: the entrepreneur may not be able to re-establish itself as a new firm but can save the amount stolen, or may be excluded from borrowing, saving, and insurance, or only from borrowing, but still may be able to produce. All of these punishments may be temporary or permanent. As it will become clear later, this general approach is valid for this paper as well, as in all the cases the net benefit of defaulting is decreasing on growth. However, in order to clearly illuminate the proposed mechanism it is assumed that defaulting entrepreneurs, although able to produce, are excluded from financial markets permanently. As noted in the introduction this is the most common assumption adopted in the theoretical literature on limited enforceability of contracts and imperfect insurance, and on sovereign borrowing, where enforcement by a third party is totally absent. Moreover it is simple enough to analyze commitment problems only from the side of borrowers and makes the model easy to modify to include equity financing, as it is shown below.

From a theoretical point of view permanent exclusion from financial markets must be a renegotiation proof equilibrium in the absence of any third party enforcement. Kletzer and Wright [2000] show punishment strategies under which a renegotiation-proof, self-enforced contract between a

[11]The authors consider a specific environment where there is only one-side lack of commitment. As the lender can commit, entrepreneurs save their assets with him. Thus all the capital used in the firm is external. Notice that the multiplier effect of wealth on external financing is larger than the case where the entrepreneur invests in his own project, because in case of defaulting the lender apropiates all the entrepreneurs' wealth.

borrower and multiple lenders exist. Although the actions off the equilibrium path are different, the outside value is equal to the value of a reversion to permanent autarky for the borrower[12]. A negotiation-proof equilibrium with permanent reversion to autarky could also be sustained under imperfect information, with a borrower type that values honesty for its own sake, as modeled by Cole and Kehoe [1995]. This can be easily introduced in the model without changing its main implications. Exclusion from trading relationships as a punishment has also been documented empirically in situations where law enforcement is extremely inefficient or non existence (See Greif [1993] for 11th century trade relationships, McMillan and Woodruff [1999] for firms in Vietnam without access to courts, and La Ferrara [2003] for credit transactions inside kin groups in Ghana).

The monitoring technology becomes a critical issue when introducing exclusion as punishment in poor countries, as lenders generally lack the ability to monitor borrowers. It could be argued that under these conditions a borrower can easily renege on their debt with a lender and form a relationship with another one that has no information about his past behavior. However there is an extensive literature showing that the lack of information available for screening borrowers reinforces local credit relationships, as information about borrowers is more easily available at that level. Cull et al. [2006] analyze the emergence of local credit institutions in the US and Europe as early as the 17th century, Kumar and Matsusaka [2009] revise the historical evidence about how local relationships were central in supporting financial transactions in preindustrial societies, and Fafchamps [2004] studies extensively informal credit relationships in Africa. As the evidence shows, in these environments alternative credit relationships are more difficult to establish, validating exclusion as a punishment[13]. Of course this generates other types of inefficiencies, as funds can not be allocated to the best projects. But in that case the role of CI in attenuating them is less clear, as information would constrain their scope anyways.

Competitive Equilibrium

The aggregate state of the world is described by (k, z), where $k = k_m + k_a$. The evolution of k is governed by the function $k' = K(k, z)$, which is exogenously given for all agents. A non arbitrage condition is imposed, inducing $r_j = r$ for $j = a, m$.

[12]The punishment in this case consists on the exclusion from financial markets of the borrower who fails to do a contingent payment until the latter makes a payment to the lender that transfers, in present value, all of the surplus from the relationship to de lender.

[13]Some examples are very informative about the effect of the lack of information networks on credit relationships. In China around 1700 credit supply was dominated by the Shanxi people, so if corruption occurred, they could very easily locate the family of the borrower (Peng [1994]). In India in 1970, Timberg and Aiyar [1984] show that it was common to never took new borrowers for financer brokers, and to keep only children and grandchildren of businessmen with whom they and his father and grandfather had done business as clients.

The dynamic program problem facing the type i representative entrepreneur is

$$V(k,z) = \max_{k_j, n_j, c_m, c_a, c_m^d, c_a^d} \left\{ \max \left(u(c) + \beta V(k', z'),\ \rho \left[u(c^d) + \beta V^d(\bar{k}'_j = (1-\delta)k_j; k', z') \right] + (1-\rho) u(c) \right) \right\}$$

subject to

$$\begin{aligned} c_m + pc_a &= p_j y_j - wn_j - (r+\delta)k_j \\ c_m^d + pc_a^d &= p_j y_j - wn_j \end{aligned}$$

and a given $k' = K(k,z)$. The function $V^d(\bar{k}'_j; k', z')$ is the continuation value of defaulting, and it is equal to

$$V^d(\bar{k}'_j; k', z') = \max_{c_m'^d, c_a'^d} \left\{ u(c'^d) + \beta V^d(\bar{k}''_j; k'', z'') \right\}$$

subject to

$$\begin{aligned} c_m'^d + p' c_a'^d &= p'_j z_j'^{1-\alpha_j} \bar{k}_j'^{\alpha_j} n_j'^{\upsilon_j} - w' n'_j \\ \bar{k}''_j &= (1-\delta)\bar{k}'_j \end{aligned}$$

and a given $k' = K(k,z)$. For simplicity it is assumed that the risk neutral entrepreneur does not replace the fraction of capital that depreciates each period.

The dynamic program problem facing the representative consumer is

$$U(b; k, z) = \max_{c_m, c_a, n, b'_m, b'_a} \left\{ \frac{u(c)^{1-\sigma} - 1}{1-\sigma} + \beta U(b'; k', z') \right\}$$

subject to

$$c_m + pc_a + b'_m + b'_a = wn + (b_m + b_a)(1+r)$$

and an incentive compatibility (IC) constraint for e_j,

$$u(c^{e_j}) + \beta V^{e_j}(k', z') \geq \rho \left[u(c^{de_j}) + \beta V^{de_j}(\bar{k}'_j; k', z') \right] + (1-\rho) u(c^{e_j})$$

or

$$\beta \left[V^{e_j}(k', z') - \rho V^{de_j}(\bar{k}'_j; k', z') \right] \geq \rho \left[u(c^{de_j}) - u(c^{e_j}) \right]$$

where c^{e_j}, V^{e_j}, c^{de_j}, and V^{de_j} are the solutions to the type e_j entrepreneur problem. The LHS of the constraint is the future cost of defaulting, while the RHS is the current benefit of doing it. Consumers also take as given the law of motion for the aggregate economy, $k' = K(k,z)$. The competitive equilibrium can now be defined[14].

[14]A constraint on the demand for capital of entrepreneurs ($k_j \leq b_j$) is included here. This is because, when imperfect enforceability problems are binding, there will be an excess of demand for capital in equilibrium and so entrepreneurs will not be maximizing their constrained utility if this restriction is not included.

Definition 1. *A competitive equilibrium is a set of decision functions* $c^i = C^i(k,z)$, $b'_j = B_j(k,z)$, $n = N(k,z)$, *and* $n_j = N_j(k,z)$, *for* $j = a, m$ *and* $i = c, e_j$, *a set of pricing functions* $w = W(k,z)$, $r = R(k,z)$, *and* $p = P(k,z)$, *and an aggregate law of motion for the capital stock* $k' = K(k,z)$, *such that*

1. *Type* i *entrepreneurs solve their dynamic programming problem, given the aggregate state of the world* (k,z) *and the pricing functions* $W(\cdot)$, $R(\cdot)$, $P(\cdot)$, *and subject to the additional constraint* $k_j \leq b_j$, *with the equilibrium solution satisfying* $k_j = b_j$, $c^{e_j} = C^{e_j}(k,z)$, *and* $n_j = N_j(k,z)$.

2. *Consumers solve their dynamic programming problem, taking as given* (k,z) *and the functions* $W(\cdot)$, $R(\cdot)$, *and* $P(\cdot)$, *with the equilibrium solution satisfying* $c^c = C^c(k,z)$, $b'_j = B_j(k,z)$, *and* $n = N(k,z)$.

3. *The economy wide resource constraints (1) and (2), and the clearing condition for the labor market,* $n = n_a + n_m$, *hold each period.*

Balanced Growth Path and the Stationary Transformation

Along the balanced growth path the interest rate is constant, and c_j grows at a constant rate. Let's call γ_a the constant (gross) rate of c_a, and let's call γ_c the constant growth rate of c_m along the balance growth path. Using the market clearing condition (2), y_a will grow at γ_a as well. On the other hand, given that they all grow at a constant rate, the variables on the RHS of equation (1) must grow at the same constant rate γ_m. Moreover, using the production functions the following must hold, $\gamma_m = \mu_m^{1-\alpha_m}\gamma_m^{\alpha_m}$ and $\gamma_a = \mu_a^{1-\alpha_a}\gamma_m^{\alpha_a}$. Then

$$\gamma_m = \mu_m$$

and

$$\gamma_a = \mu_a^{1-\alpha_a}\mu_m^{\alpha_a}$$

Notice that this is true whether the IC constraint is binding or not.

For prices we can see from the consumer's budget constraint that pc_a and w must grow at the constant rate $\gamma_m = \mu_m$. Then p grows at a constant rate $(\mu_m/\mu_a)^{1-\alpha_a}$. Therefore this economy converges to a state where the relative size of each sector is constant. As income grows at the same rate for all agents, relative consumption across agents is constant as well.

There is no default in equilibrium in this model. However it is necessary to model the path for the variables off the equilibrium path when default occurs. It is possible to show (see Appendix A) that total expenditure on manufactures and agricultural goods, c_m^d and pc_a^d respectively, grow at the same constant rate $(1-\delta)$, for the entrepreneur that defaulted and it is operating in sector j.

Given the conjectured asymptotic growth rate for all variables, one can impose a transformation that will render them stationary in the limit. Define $\hat{x}_t = x_t/g_x^t$, where g_x is the growth rate of some variable x_t when $t \to \infty$. Then the consumer and entrepreneur's problems can be rewritten using this transformation as a model that converges to a stable steady-state which corresponds to an unbounded growth path for the original model.

The transformed dynamic programming problems are presented in Appendix A. The main difference with respect to the original model are the discount factors, that now incorporate all the information related to the non transitional dynamics of the economy. Now $\beta\gamma$ is the discount factor for consumers and the entrepreneur that has not defaulted, where $\gamma = \gamma_a^\eta \gamma_m^{1-\eta}$ is the steady-state growth rate of the consumption basket for those agents. For the entrepreneur that defaulted previously the discount rate is now $\beta\bar{\gamma}$, where $\bar{\gamma} = \gamma((1-\delta)/\mu_m)^{\alpha_j/(1-\upsilon_j)}$ is the asymptotic growth rate of the consumption basket for that entrepreneur. The fact that $\bar{\gamma} < \gamma$ means that the future growth in utility falls after defaulting. An additional modification is included in the market clearing condition for manufactures and the consumers' budget constraint. As both $\hat{k}'$ and $\hat{k}$ appear in the same equation the first one must be adjusted by its steady-state growth rate. The modified equations are shown in Appendix A.

Perfect Enforceability (PE)

In this section I solve the model under PE of contracts. Roe et al. [2010] present similar multisector frictionless growth models. The main difference is that they assume constant returns to scale and consequently they can use zero profits conditions to link prices of goods and factors of production, simplifying the analysis. Another difference is that here we are interested in characterizing not only the growth rates of the endogenous variables but also how these growth rates evolve over time. This, needed to study the case when enforcement is not perfect, demands more analytical work and a restriction on the intertemporal elasticity of substitutions.

The model can be solved in two steps. In particular, given a sequence for the endogenous variables $\hat{k}$ and $\hat{p}$, the consumer problem becomes a static problem, where total expenditures are optimally allocated across both types of goods. The entrepreneurs' problems are also static, so if the sequence for $(\hat{k},\hat{p})$ is known, the rest of the endogenous variables $(\hat{c}^i, \hat{k}_j, n_j, \hat{w}, r)$ can be determined. Therefore first I solve the static equilibrium given the sequence $(\hat{k},\hat{p})$. In order to do this I use a version of the Rybczynski and Stopler-Samuelson theorems to characterize the transition.

In Lemma 1 below I show that these theorems can be applied to the present model under positive rents for entrepreneurs. Once this is done the path for capital and the relative price can be derived from the dynamic problem faced by the consumer, and the market clearing conditions. A well known result for this kind of models that it is proven below but it is worth noticing now is that $\forall \hat{k} < \hat{k}^{\text{SS}}$, $d\hat{k} > 0$, where $\hat{k}^{\text{SS}}$ is the transformed level of capital in steady-state, and $d\hat{k}$ is the change in the stock of capital. Thus the focus in this section is to show the effect of the increase in the stock of capital on the rest of the endogenous variables.

On the supply side the static problem of the entrepreneurs is to choose capital and labor so as to maximize profits. The solution to this problem is characterized by the equalization of marginal productivities and factor prices. These conditions can be used to derive the total cost function and the marginal cost function. After equalizing the latter to the price, the supply function for good j is given by

$$\hat{y}_j{}^{\omega} = \Psi_j \frac{\hat{p}_j^{1-\omega}}{\bar{r}^{\alpha_j} \hat{w}^{\upsilon_j}} \quad j = a, m. \tag{3}$$

where Ψ_j is a positive constant that depends on $\hat{z}_j$, α_j, and υ_j, and $\bar{r} = r + \delta$. This expression becomes a zero-profits condition when $\omega = 0$, i.e. in the constant return to scale case. Using Shephard's Lemma to derive the demands for labor and capital from the cost function, we get the market clearing conditions for both factor markets,

$$n_a + n_m = \Psi'_a \left[\hat{y}_a \left(\frac{\bar{r}}{\hat{w}} \right)^{\alpha_a} \right]^{\frac{1}{1-\omega}} + \Psi'_m \left[\hat{y}_m \left(\frac{\bar{r}}{\hat{w}} \right)^{\alpha_m} \right]^{\frac{1}{1-\omega}} = 1 \tag{4}$$

$$\hat{k}_a + \hat{k}_m = \Psi''_a \left[\hat{y}_a \left(\frac{\hat{w}}{\bar{r}} \right)^{\upsilon_a} \right]^{\frac{1}{1-\omega}} + \Psi''_m \left[\hat{y}_m \left(\frac{\hat{w}}{\bar{r}} \right)^{\upsilon_m} \right]^{\frac{1}{1-\omega}} = \hat{k} \tag{5}$$

Then, given a pair $(\hat{k}, \hat{p})$, these 4 equations totally characterize the supply side of the economy, and they can be solved to get $(\hat{w}, r, \hat{y}_a, \hat{y}_m)$. The following lemma shows how this set of variables are affected by changes in $\hat{k}$, and $\hat{p}$.

Lemma 1. *Rybczynski: fix $\hat{p}$, then,*

$$\frac{\partial \hat{w}}{\partial \hat{k}} > 0, \quad \frac{\partial r}{\partial \hat{k}} < 0, \quad \frac{\partial \hat{y}_m}{\partial \hat{k}} > 0,$$

and $\exists \omega^R > 0$ such that $\forall \omega \in \left[0, \omega^R\right]$,

$$\frac{\partial \hat{y}_a}{\partial \hat{k}} < 0.$$

Stopler-Samuelson: fix $\hat{k}$, then

$$\frac{\partial \hat{w}}{\partial \hat{p}} > 0, \quad \frac{\partial (r/\hat{p})}{\partial \hat{p}} < 0, \quad \frac{\partial \hat{y}_m}{\partial \hat{p}} < 0, \quad \frac{\partial \hat{y}_a}{\partial \hat{p}} > 0$$

and $\exists\omega^S > 0$ *such that* $\forall\omega \in \left[0, \omega^S\right]$,

$$\frac{\partial r}{\partial \hat{p}} < 0, \quad \frac{\partial(\hat{w}/\hat{p})}{\partial \hat{p}} > 0.$$

Proof. See Appendix B. ■

Therefore, as capital increases in this economy Lemma 1 implies that the capital intensive sector expands, while the labor intensive sector shrinks (a Rybczynski's effect). Given diminishing marginal productivities and the complementarity between factors of production, this lowers the interest rate and rises wages. The second part of Lemma 1 implies that an increase in the relative price of the labor intensive good has the opposite effect on production, but the same effect on factor prices because now the change is driven by the demand side (a Stopler-Samuelson's effect). All of this is true on a certain range for rents[15]. Although it comes from the dynamic block of the model, the effect of the change in the stock of capital on the relative price is positive, so the total effect on relative output is ambiguous while the total effect on factor prices is not.

The static demand side problem consists on the allocation of total consumption expenditures among both goods. Define total consumption expenditures for agent $i = c, e_j$ as

$$\hat{\epsilon}^i = \hat{p}\hat{c}_a^i + \hat{c}_m^i \tag{6}$$

Then the Cobb-Douglas instantaneous utility function implies that the fraction of total expenditures spent in each good is constant, so optimal demands for each agent are

$$\hat{p}\hat{c}_a^i = \eta\hat{\epsilon}^i \tag{7}$$

$$\hat{c}_m^i = (1-\eta)\hat{\epsilon}^i \tag{8}$$

The dynamic block of the model consists in finding the law of motion for capital and the relative price. The consumer smooths total expenditures over time. But this must be consistent with the market clearing conditions. In the manufacturing sector any difference between consumption and production is absorbed by changes in investment. In the agricultural sector however the relative price must adjust to achieve market clearing. Therefore to get the path for prices we need to find

[15] The fact that there are decreasing returns to scale reduces factor mobility across sectors due to a scale effect produced by the fixed factor. That lowers the effect of a change in capital on the labor intensive sector. Similarly the effect of a change in the relative price on the interest rate and the real wage is lower with decreasing returns to scale. Additionally, with any level of positive rents, there is also an effect of the stock of capital on factor prices. This effect is zero in the constant returns to scale case. All of these effects are continuous on the level of rents, being constant returns to scale the limit case when $\omega \to 0$.

how the demand, which is a fixed fraction of total expenditures, and the supply, which is given by the static block above, evolve over time in the agricultural sector. Notice first that

$$\hat{\epsilon}^c = \bar{\eta}\hat{p}^{\eta}u(\hat{c}^c) \tag{9}$$

where $\bar{\eta} = (1-\eta)^{\eta-1}\eta^{-\eta}$. Replacing this expression in to the consumer's budget constraint and differentiating the first order condition for savings, we get the optimal path for consumer's instantaneous utility[16],

$$\frac{du(\hat{c}^c)}{u(\hat{c}^c)} = \frac{1}{\sigma}\left(r' - \bar{\beta} - \bar{\mu}_m - \eta\frac{d\hat{p}}{\hat{p}}\right) \tag{10}$$

where $\bar{\beta} = (\beta\gamma^{1-\sigma})^{-1} - 1 > 0$ and $\bar{\mu}_m = \mu_m - 1 > 0$. We can use this expression and differentiate equation (9) to get the optimal path for total consumer's expenditures,

$$d\hat{\epsilon}^c = \hat{\epsilon}^c\left[\frac{1}{\sigma}\left(r' - \bar{\beta} - \bar{\mu}_m\right) + \eta(\frac{\sigma-1}{\sigma})\frac{d\hat{p}}{\hat{p}}\right] \tag{11}$$

In the case of entrepreneurs, total expenditure is equal to total income, $\omega\hat{y}_j$, then totally differentiating output in each sector, we have,

$$d\hat{\epsilon}^{e_j} = \omega\left[d\hat{p}\left(\hat{y}_j + \hat{p}_j\frac{\partial\hat{y}_j}{\partial\hat{p}}\right) + d\hat{k}\ \hat{p}_j\frac{\partial\hat{y}_j}{\partial\hat{k}}\right] \tag{12}$$

The market clearing condition in the agricultural sector and Equation (7) imply that the fraction η of total expenditures must equal the total value of production in that sector,

$$\eta\hat{\epsilon} = \hat{p}\hat{y}_a$$

so

$$\eta d\hat{\epsilon} = d\hat{p}\left(\hat{y}_a + \hat{p}\frac{\partial\hat{y}_a}{\partial\hat{p}}\right) + d\hat{k}\ \hat{p}\frac{\partial\hat{y}_a}{\partial\hat{k}} \tag{13}$$

Then replacing Equations (11) and (12) in (13) we obtain the change in prices that clears the agricultural sector in each period,

$$d\hat{p} = \frac{\frac{\eta}{\sigma}\hat{\epsilon}^c\left(r' - \bar{\beta} - \bar{\mu}_m\right) + d\hat{k}\left[\eta\omega\frac{\partial\hat{y}_m}{\partial\hat{k}} - (1-\eta\omega)\hat{p}\frac{\partial\hat{y}_a}{\partial\hat{k}}\right]}{(1-\eta\omega)\left(\hat{y}_a + \hat{p}\frac{\partial\hat{y}_a}{\partial\hat{p}}\right) - \eta\omega\frac{\partial\hat{y}_m}{\partial\hat{p}} - \frac{\eta^2(\sigma-1)}{\hat{p}\sigma}\hat{\epsilon}^c} \tag{14}$$

The first component that determines the path for prices is the growth rate in total expenditures. If planned total expenditures grow slowly, meaning that the willingness to smooth consumption over time is high, supply would be growing faster than demand. The manufacturing sector clears reallocating more resources to the production of capital, which is consistent with the fast growth

[16] This is an approximation to simplify notation that does not affect the results. In particular I am assuming that $log(1+x) = x$, for $x = r, \beta$, and $\mu_m - 1$.

in savings. On the agricultural sector consumption must equal production, so the growth rate in the relative price has to be lower to close the gap between supply and demand, reallocating more resources to the manufacturing sector and reducing consumption on agricultural goods. The second component is related to the supply side of the economy. As the increase in the stock of capital has a different effect in each sector then an adjustment in the relative price is needed for market clearing, because a fixed amount is spent in each good. In particular, given that Lemma 1 implies that the relative size of the agricultural sector shrinks with the stock of capital, the relative price rises. Given that the first component is always positive, it follows that the RHS of equation (14) is always positive if rents are inside the interval defined in Lemma 1.

Finally, using the consumer's budget constraint, we get an expression for the change in the stock of capital,

$$\mathrm{d}\hat{k} = \frac{\hat{w} + \hat{k}(r - \bar{\mu}_m) - \hat{\epsilon}^c}{\mu_m} \tag{15}$$

Now the model can be solved for any initial state $\hat{k}_0$, knowing $\hat{p}_0$. Given these two variables the supply side is determined using equations (3), (4), and (5). Then equations (14) and (15) can be solved to find the future value of capital and the relative price. Repeating this process it is possible to solve for the entire transition of the economy to its balanced growth path. Therefore the last step is to find $\hat{p}_0$. This level is determined by a transversality condition that rule out Ponzi schemes, so as to identify the point $(\hat{p}_0, \hat{k}_0)$ which is on the stable path. Finally imposing the conditions $d\hat{p} = 0$ and $d\hat{k} = 0$, equations (3), (4), (5), (14), and (15) characterize the steady-state equilibrium. The next proposition describes the transition of the economy from an initial low capital stock to its balanced growth path under PE.

Proposition 1. *Suppose $\rho = 0$ and $\hat{k} < \hat{k}^{ss}$. Then $\forall \omega \leq min(\omega^R, \omega^S)$,*

$$d\hat{k} > 0, \quad d\hat{p} > 0, \quad d\hat{w} > 0, \quad d(\hat{w}/\hat{p}) > 0, \quad dr < 0, \quad d(r/\hat{p}) < 0, \quad d\hat{c}_m > 0, \quad d(\hat{p}\hat{c}_a) = d(\hat{p}\hat{y}_a) > 0.$$

And $\exists \sigma^* > 0$ such that $\forall \sigma \geq \sigma^*$,

$$d\hat{y}_m > 0.$$

Under these conditions,

$$d\left|\frac{dx}{x}\right| = d\left|g_x\right| < 0,$$

for $x = \hat{k}, \hat{p}, \hat{w}, \hat{w}/\hat{p}, r, r/\hat{p}, \hat{c}_m, \hat{p}\hat{y}_a$, and $\hat{p}^{1-\eta}\hat{y}_a$; and $\exists \sigma^{**} > 0$, with $\sigma^{**} > \sigma^*$, such that $\forall \sigma \leq \sigma^{**}$, this is true for $x = \hat{y}_m$ and $\hat{y}_m/\hat{p}^{\eta}$.

Proof. See Appendix B. ■

Figure 2 shows the PE equilibrium. Total capital in the economy grows at a positive rate. As explained above this rises manufacturing output and wages, and lowers agricultural output and the interest rate, if the conditions for Lemma 1 hold. This rises the relative price, offsetting the effects on production but reinforcing the effects on factor prices. The final effect on sectoral production is ambiguous. However total expenditures on both goods grow, so the value of agricultural output grows as well. As manufactures are also used as capital this is not implied for that sector. But as noted above the growth rate of investment depends on the elasticity of substitution, σ. If the latter is high enough consumption smoothing generates an increasing excess of supply in both sectors during the transition. In the agricultural sector this translates into a lower increase in the price, while in the manufacturing sector it generates a higher growth in investment. Therefore, for some range of σ, the growth rate of investment is high enough to ensure that the positive growth in consumption translates into a positive growth in manufacturing output[17]. Finally, an additional feature of the PE equilibrium, which is key to analyze the IE equilibrium later, is that all the variables reduce the rate at which they increase (or decrease) during the transition. As the return on capital falls when the economy approaches its steady-state, capital accumulation slows down, lowering output growth, wages growth, and the rate at which the interest rate decreases.

The IC Constraint Under PE Allocations

Under IE of contracts the IC constraint for entrepreneurs in the consumer's problem is now relevant. The strategy for analyzing the equilibrium with IE is first to characterize when the IC constraint is binding when the outcome is the PE equilibrium, both in steady state and during the transition. Then I show how the path for the endogenous variables is affected when this constraint is binding and I describe the new equilibrium.

Using the fact that entrepreneurs spend a fixed amount on each good, their flow utility can be expressed as the ratio of total income and a geometric average of prices. This implies that the current benefit of defaulting for entrepreneur e_j is $u(\hat{c}^{de_j}) - u(\hat{c}^{e_j}) = \rho(r+\delta)\hat{k}_j/\hat{p}^{\eta}$. Thus, using the new discount rates $\beta\gamma$ and $\beta\bar{\gamma}$, the IC constraint can be rewritten as,

$$IC_j(\hat{k}) = \beta\left[\gamma\bar{V}_j(\hat{k}') - \rho\bar{\gamma}\bar{V}_j^d(\hat{k}_j;\hat{k}')\right] - \frac{\rho(r+\delta)\hat{k}_j}{\hat{p}^{\eta}} \geq 0 \tag{16}$$

[17] Barro and SalaiMartin [2004] also show that the behavior of the investment rate during the transition depends on the elasticity of substitution in an exogenous growth model with one sector.

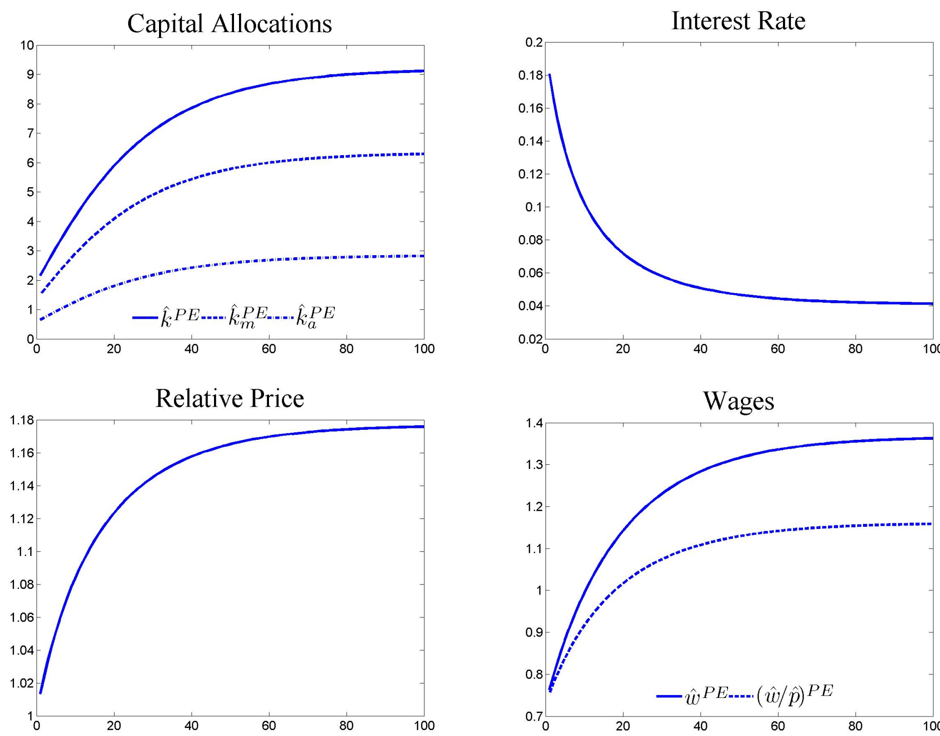

Figure 2: Perfect Enforcement Equilibrium

where

$$\bar{V}_j(\hat{k}') = \frac{(1-\upsilon_j)\hat{p}'_j\hat{y}'_j - (r'+\delta)\hat{k}'_j}{\hat{p}'^{\eta}} + \beta\gamma\bar{V}_j(\hat{k}'')$$

and

$$\bar{V}^d_j(\hat{k}_j;\hat{k}') = \frac{(1-\upsilon_j)\hat{p}'_j\hat{z}^{1-\alpha_j}\hat{k}_j^{\alpha_j}n_j'^{\,\upsilon_j}}{\hat{p}'^{\eta}} + \beta\bar{\gamma}\bar{V}^d_j(\hat{k}_j;\hat{k}'')$$

So the next period flow utility if the entrepreneur honors the contract is the value of output net of factor payments, while the one if the entrepreneur defaults is the value of output, using the current stock of capital, net of labor income. To see if the IC constraint is binding under PE allocations we can use the FOC for capital to substitute $(r+\delta)\hat{k}_j$ by $\alpha_j\hat{y}_j$ above. Rearranging terms we can express the IC constraint under PE allocations (IC^{PE}) as,

$$IC^{\mathrm{PE}}_j(\hat{k}) = \beta\left[\gamma\tilde{V}_j(\hat{k}',\hat{k}) - \rho\bar{\gamma}\tilde{V}^d_j(\hat{k}',\hat{k})\right] - \rho\alpha_j \geq 0$$

where

$$\tilde{V}_j(\hat{k}',\hat{k}) = \frac{\hat{p}^{\eta}}{\hat{p}'^{\eta}}(1-\alpha_j-\upsilon_j)\,\frac{\hat{p}'_j\hat{y}'_j}{\hat{p}_j\hat{y}_j} + \beta\gamma\tilde{V}_j(\hat{k}'',\hat{k})$$

and

$$\tilde{V}_j^d(\hat{k}', \hat{k}) = \frac{\hat{p}^\eta}{\hat{p}'^\eta}(1 - \upsilon_j) \left(\frac{\hat{w}/\hat{p}_j}{\hat{w}'/\hat{p}'_j} \right)^{\frac{\upsilon_j}{1-\upsilon_j}} + \beta\bar{\gamma}\tilde{V}_j^d(\hat{k}'', \hat{k})$$

We can interpret $\tilde{V}$ and $\tilde{V}^d$ as the continuation utility of honoring and not honoring contracts relative to the current gain of doing it. Then, under PE allocations, the relative continuation utility of honoring the contract depends positively on the future growth rate of output in each sector. The higher the former the higher is the growth rate of rents when the entrepreneur maintains the access to consumers' savings. Likewise, the relative continuation utility of defaulting depends negatively on the future growth on wages. If the entrepreneur is excluded from financial markets then the future path for rents is totally determined by the cost of the only variable factor of production, labor. Additionally both continuation utilities depend negatively on the future growth rate of prices, as higher prices in the future reduce the amount of goods that the entrepreneur can consume for a given level of income. The main implications of the model are derived from this expression.

First let us analyze the binding pattern of the IC constraint along the balanced growth path under PE allocations. In this case all the endogenous variables are constant, so the expression above simplifies to

$$IC_j^{\mathrm{PE}}(\hat{k}^{\mathrm{SS}}) = \left(1 + \frac{\omega}{\alpha_j}\right)\phi_\rho - 1 \geq 0$$

where

$$\phi_\rho = \left(\frac{\beta\gamma}{1-\beta\gamma} - \frac{\beta\rho\bar{\gamma}}{1-\beta\bar{\gamma}}\right) / \left(\rho + \frac{\beta\gamma}{1-\beta\gamma}\right) > 0$$

Then the IC constraint will be binding depending on ϕ_ρ, capital intensity, and the degree of decreasing returns to scale. The following proposition formalizes some of the implications.

Proposition 2. *Along the balance growth path,*

$$\frac{\partial IC_j^{PE}(\hat{k}^{SS})}{\partial\omega} > 0, \quad \frac{\partial IC_j^{PE}(\hat{k}^{SS})}{\partial\alpha_j} < 0, \quad \frac{\partial IC_j^{PE}(\hat{k}^{SS})}{\partial\rho} < 0.$$

Moreover,

$$IC_a^{PE}(\hat{k}^{SS}) < 0 \quad \Rightarrow \quad IC_m^{PE}(\hat{k}^{SS}) < 0$$

Proof. It follows from the text and the assumption that $\alpha_m > \alpha_a$. ■

The proposition shows that, along the asymptotic balanced growth path, the IC constraint will be more likely binding in the sector which is more capital intensive -the larger is α_j- and in the sector with lower rents under first best allocations -the lower is ω. By assumption the last term is

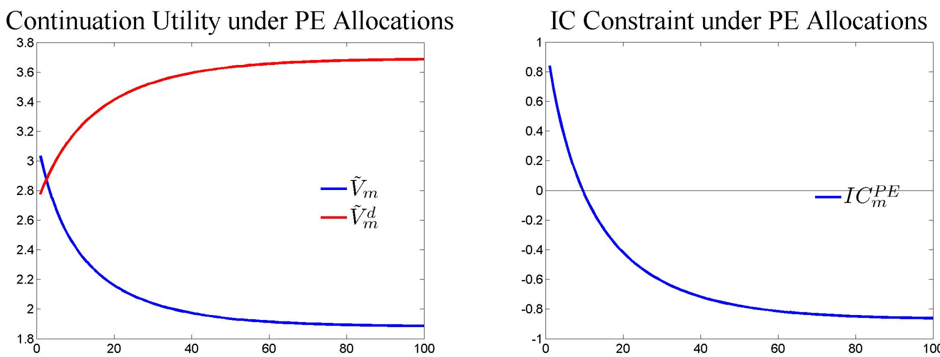

Figure 3: The IC Constraint Under PE Allocations

equal in both sectors, so the IC will be more likely binding in the capital intensive sector. Therefore the IC constraint will be binding in the manufacturing sector if (and not if and only if) it is binding in the agricultural sector. Given that ϕ_ρ is decreasing on ρ, the constraint will be tighter the larger is this parameter. Also notice that if $\rho = 0$ then $\phi_\rho = 1$, and the constraint is not binding in steady state.

Proposition 3. *Suppose $\hat{k} < \hat{k}^{ss}$. Then, under the conditions listed in Proposition 1,*

$$IC_j^{PE}(\hat{k}) < 0 \quad \Rightarrow \quad IC_j^{PE}(\hat{k}') < 0 \quad \forall \hat{k}' > \hat{k} \quad j = a, m.$$

Proof. See Appendix B. ■

As described above, under PE allocations the growth rate of the value of output in both sectors, and hence total rents, slows down as time goes by. Therefore the relative continuation utility of honoring the contract, $\tilde{V}$, decreases over time. On the other hand, as the growth rate of wages slows down, the relative continuation utility of not honoring the contract, $\tilde{V}^d$, increases over time. It follows that the value of the constraint, IC_j^{PE}, is decreasing over time. In Figure (3) we can see this for the numerical exercise shown before. Then, as the economy approaches its steady-state, it becomes more attractive to default as the cost of doing so decreases. As this cost converges to the steady-state cost, this implies that the IC constraint will be binding at some point during the transition only if it is binding along the balanced growth path, and if the IC constraint is binding at any time, then it will be always binding thereafter.

Proposition 4.

$$\forall \rho \in [0, 1]\,, \ \exists \hat{k}^{*PE} > 0 \ \textit{such that} \ \ \forall \hat{k} < \hat{k}^{*PE}, \ IC_j^{PE}(\hat{k}) > 0 \quad j = a, m.$$

Proof. See Appendix B. ■

The benefit of defaulting is proportional to the stock of capital allocated to the entrepreneur, but the benefits of honoring the contract are not. Thus, if aggregate capital is small enough, the benefits of honoring the contract are always bigger that the costs, even for a very high ρ. The proposition means that for any quality of CI, the constraint imposed by the financial frictions that they generate, is only binding after some amount of aggregate capital has been accumulated. This, jointly with Propositions 2 and 3, constitute the main finding of the paper. They imply that the PE allocation either ends to be feasible at some positive level of capital for the first time and stays infeasible for ever, or it is always feasible.

Imperfect Enforceability (IE)

Now the IE equilibrium can be characterized. Suppose all the conditions for Proposition 1 hold, and take the case when $IC_a^{\mathrm{PE}}(\hat{k}^{\mathrm{SS}}) \geq 0$, and $IC_m^{\mathrm{PE}}(\hat{k}^{\mathrm{SS}}) < 0$. Then we know there exists some level of aggregate capital, $\hat{k}^{*\mathrm{PE}}$, at which the PE allocations are not feasible. We also know that the constraint will be binding for the manufacturing sector first. I show a numerical example in Figure (4) to compare the PE and IE equilibriums.

Along the balanced growth path only the manufacturing sector will be constrained. A lower level of capital must be allocated to that sector so the entrepreneur finds optimal to honor the contract and repay to consumers. This can be seen in the following expression derived from Equation (16), which must hold in steady-state,

$$\frac{\alpha_m \hat{y}_m}{\hat{k}_m}\left(1+\frac{\omega}{\alpha_m}\right)\phi_\rho = r$$

The fact that the constraint is binding implies that the term multiplying the marginal productivity of capital, is lower than one. On the other hand the interest rate is not affected by imperfect enforcement along the steady-state. Therefore now the output-capital ratio must be greater than in the PE case. This is achieved trough a reduction on capital. But the consequent fall in manufacturing output lowers the price of the agricultural good. This reduces the marginal product of capital in agriculture, and so a lower level of capital is allocated to that sector as well. Output falls in both sectors lowering the labor demand and wages. The allocation of labor depends on the relative size of the effects. Because technologies have decreasing returns to scale it is not necesarily true that the lower amount of capital reduces employment in the manufacturing sector.

Imperfect enforceability of contracts has an effect on the equilibrium before the constraint is binding. This is because consumers anticipate the fall in future income due to the constraint, and

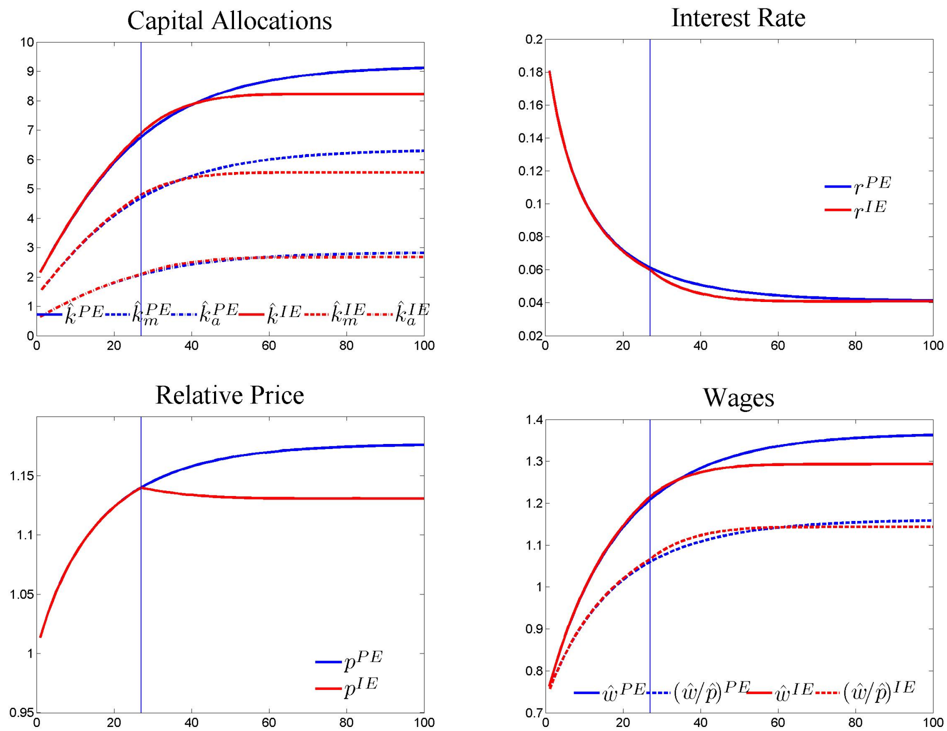

Figure 4: Imperfect Enforceability Equilibrium

their willingness to smooth consumption makes them to increase savings. Then capital grows at a faster rate before the constraint becomes binding. At the moment this happens, which is marked with a vertical line in the graphs, there is a reallocation of resources, as total capital adjusts slowly. In this period less capital is allocated to the manufacturing sector and more capital is allocated to the agricultural sector[18]. This reallocation of capital among sectors generates a fall in the relative price, as demand falls for both goods proportionally. Therefore, despite the reallocation of resources from manufacturing to agriculture, the value of output falls in both sectors. Finally there is an excess of demand for labor, because agriculture is more intensive in this factor, and so the wage rises, specially in terms of the agricultural good.

The interest rate now is equal, in equilibrium, to the marginal productivity of capital in the unconstrained sector, i.e. agriculture. The fall in the relative price and the reallocation of resources to this sector then reduce the interest rate. This has two effects. First the fall in the interest rate

[18]Otherwise the ratio $r/\hat{p}$ should be lower than the PE level, to have $IC_m(\hat{k}^{\mathrm{IE}}) = 0$. As aggregate capital does not change the amount of capital in agriculture would have to be lower than in the PE case. But this is clearly not an equilibrium because in this case, the unconstrained sector would have a marginal productivity of capital larger than the interest rate.

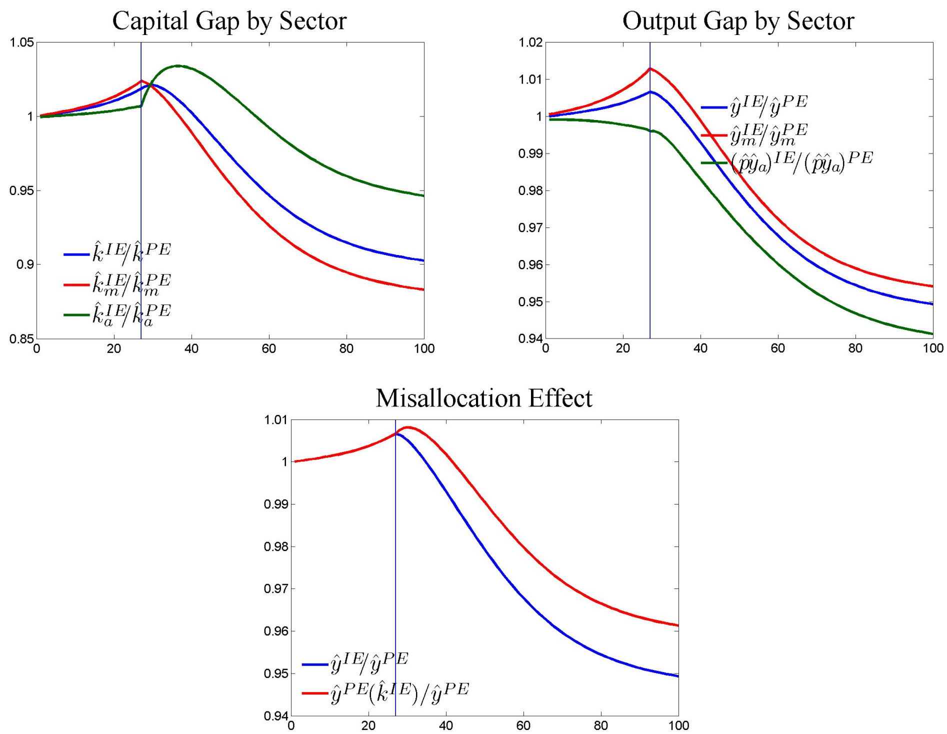

Figure 5: Imperfect Enforceability Equilibrium

relaxes the IC constraint for the manufacturing sector, increasing the stock of capital that can be allocated there. Second, and together with the fall in total income, it reduces the rate at which capital is accumulated in the economy, lowering the growth rate of output in both sectors and of wages. Therefore the initial inefficient reallocation effect becomes a lower capital accumulation effect. The economy converges to its steady-state described above with lower capital and lower output in both sectors.

In Figure 5 the gap between the IE and PE equilibrium allocations can be seen more easily. Initially both sectors attract more capital, specially manufacturing because is more capital intensive. Labor is also reallocated to that sector, so output falls in agriculture under IE. Then, when the constraint becomes binding, the agricultural sector expands and the manufacturing sector shrinks. Total output falls due to this inefficient reallocation of resources. But eventually the slow down in aggregate capital affects both sectors, and both start to shrink relative to the PE case, until they converge to their steady-state levels. In terms of the manufacture good both sector shrink since the beginning because of the change in the relative price. In the last graph in Figure 5 I split the effect on total output per capita into a capital accumulation effect and a misallocation effect. The red line shows the income ratio which can be achieved if the stock of capital resulting from the IE equilibrium could be reallocated efficiently. Therefore this ratio shows the output gap due to the capital accumulation effect. The difference between the red line and the blue line, which shows the gap between IE and PE allocations, is then the misallocation effect. As shown in the graph the misallocation of resources toward the agricultural sector generates the fall in output initially, while the capital accumulation effect becomes more important therefater.

Property Right Institutions

It is interesting to include other kind of distortions in the model to see how they interact with CI. In particular, and given that the empirical evidence on the effects of CI on income per capita has highlighted the effect of PRI, it would be interested to include them in the model. A detailed modeling of the latter type of institutions is not intended here. See for instance Thomas and Worrall [1994] for a formal analysis of these institutions. Instead I assume a very simple mechanism. Every period a government steals a fraction $0 \leq \theta \leq 1$ of total capital. This can be easily justified by the existence of a myopic government which tries to maximize utility out of tax revenues, and that has the technology to expropriate the stock of capital in the economy and to get a fraction θ of it for own consumption. Assume that the government can only tax capital returns. If this is the case then the government will not expropriate if,

$$\tau r k \geq \theta k$$

where τ is the tax rate on capital returns. If the representative consumer can choose the tax rate he will always choose a level such that $\tau r k = \theta k$. This is because even though he does not like taxes, he prefers to be taxed at any rate better than being expropriated by the government. This implies that

$$\tau = \frac{\theta}{r}$$

If we interpret the parameter θ as the efficieny of PRI, an important implication of this extension is that, unlike CI, PRI affect output troughout the development process. The model can be easily modified to include this feature because it is equivalent to an increase on the subjective discount rate $\bar{\beta}$. It is well known that this generates a slow down on the growth process. The reduction in the expected return reduces savings, lowering capital accumulation, and the growth rate of output in both sectors. The relative price falls because of these changes. Lemma 1 implies that factor prices grow at a lower rate as well. Therefore the relative continuation utility of honoring the contract, $\tilde{V}$, falls, while the relative continuation utility of defaulting, $\tilde{V}^d$, rises. It follows that this type of frictions bring forward the time at which the IC constraint becomes binding in the economy. In Figure 6 I show the path for capital and wages for the case that $\theta > 0$. It can be seen the flattening

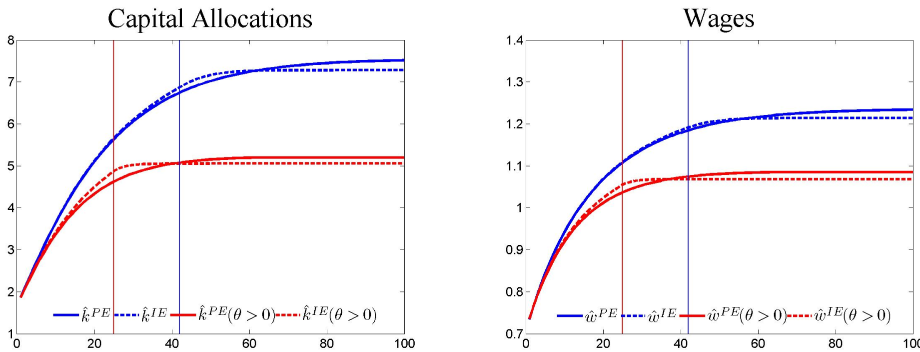

Figure 6: Imperfect Enforceability and Property Right Institutions

on the trajectories of these variables and the effect of this on the binding pattern of the IC constraint.

The fact that the IC constraint becomes binding before in countries with worse PRI imposes a difficulty to the empirical analysis. The main result of the paper is that CI have a negative effect on income only when the country reaches a certain level of capital. The natural strategy to prove this hypothesis empirically is with a large set of countries. If the estimated effect is present only for richer countries, then we would accept it. However, the analysis in this section suggests that if differences on the level of development among countries are due to distortions like the ones captured by the parameter θ in the model, poor countries can also be affected by low quality CI if they are close enough to their steady-state.

3 The Evidence

The previous section shows that the quality of CI affects income per capita. In particular, propositions 2, 3, and 4 imply that this effect would realize only after a certain level of capital has been accumulated. Moreover, if the quality of CI remains the same, the effects on the economy will persist throughout the development process. This is the main implication of the model and the focus of the empirical exercise. Following previous empirical work on the link between the quality of institutions and income per capita, cross-country regressions are implemented in this section. The natural way of testing the model is studying how the effects of CI on GDP per capita vary depending on the distance of this variable with respect to its steady-state level. However the latter is not observable[19]. This section first explains the modifications made to previous specifications

[19]The inclusion in the regressions of a multiplicative term between the two types of institutions seems to be a good way of testing some of the implications of the model. However, given that the indicators for both types of institutions are endogenous, such interaction would not be reliable.

needed to prove the main implications of the model. Then it describes the identification strategy and the data used in the regressions, and finally it presents the results. It is assumed throughout this section that there exist conditional convergence. The estimations below show that this property is present in the sample of countries used here.

Empirical Strategy

An alternative strategy to prove the implications of the model is to compare the effects of CI among rich and poor countries. As the former have transitioned toward their steady-state and expected growth is close to its long-run level, the incentives to honor contracts are weaker, so better CI are needed in order to achieve the first best level of income. Then, in a group of rich economies, the variance of income per capita should be explained by the variance in the quality of CI. For poor economies the implication is not straight forward. As noted above, if these countries are poor because their steady-state level of income per capita is low, CI can also affect them. However we do observe countries with low levels of GDP per capita and high growth rates for long periods of time, meaning there are economies relatively far from their steady-states. It follows that, even though for individual countries the relationship between income and the effects of CI is ambiguous, on average this should not be the case; a larger effect should be observed in the group of richer economies. The first empirical exercise will be then to split the sample of countries according to income per capita and compare the results.

An alternative way of testing the main implication of the model is to investigate the timing of the effects of the quality of CI on GDP per capita. To see this, assume first consumers do not anticipate the fall in output due to inefficient CI. Then the true process for the log of output in a country i will be given by,

$$y_{ti} = \bar{y}_t + (1 - \theta_i)\bar{y}_t - \rho_i I_{ti}$$

with

$$I_{ti} = \begin{cases} \bar{y}_t - \bar{y}_{\tau_i} & \text{if } \ t > \tau_i \\ 0 & \text{otherwise} \end{cases}$$

Here $\bar{y}_t$ is exogenous and increasing, $(\bar{y}_t - \bar{y}_{t-1})$ is decreasing, and τ_i is the moment at which the constraint is binding in i. θ_i and ρ_i capture the quality of PRI and CI respectively as in the model. Assume $\theta_i \in [0, 1]$ and $\rho_i \in [0, 1]$, so if $\theta_i = 0$ and $\rho_i = 0$ country i has perfect institutions, and $y_{ti} = 2\bar{y}_t$. In the country with the worst institutions $\theta_i = 1$ and $\rho_i = 1$, so $y_{ti} = \bar{y}_t$ if $t < \tau_i$, and $y_{ti} = \bar{y}_{\tau_i}$ thereafter.

Suppose we only observe imperfect indicators of institutional quality, $\hat{\theta}_i$ and $\hat{\rho}_i$, for n countries. Assume that the measurement error on these variables is white noise, and that n is large enough so we can express the expected value as the average over the n countries. Then it is easy to see that after running the regression,

$$y_{ti} = \hat{\alpha}_0 + \hat{\alpha}_1(1-\hat{\theta}_i) + \hat{\alpha}_2\hat{\rho}_i + \epsilon_i \tag{17}$$

we get the following results,

$$E(\hat{\alpha}_0) = \alpha_0 = \bar{y}_t$$

$$E(\hat{\alpha}_1) = \alpha_1 = \bar{y}_t$$

$$E(\hat{\alpha_2}) = \alpha_2 = -(1/n)\sum_{t>\tau_i}(\bar{y}_t - \bar{y}_{\tau_i})$$

The coefficient on ρ_i captures only the effects for those countries where the IC constraint is binding at t, i.e. those where $t > \tau_i$. Now suppose that instead of using the unconditional level of output at t we use the change on output between t and t^* as the dependent variable. The new regression is

$$y_{ti} - y_{t^*i} = \hat{\beta}_0 + \hat{\beta}_1(1-\hat{\theta}_i) + \hat{\beta}_2\hat{\rho}_i + \upsilon_i \tag{18}$$

In this case the results will be,

$$E(\hat{\beta}_0) = \alpha_0 - \bar{y}_{t^*}$$

$$E(\hat{\beta}_1) = \alpha_1 - \bar{y}_{t^*}$$

$$E(\hat{\beta}_2) = -(1/n)\left(\sum_{(t>\tau_i)(t^*<\tau_i)}(\bar{y}_t - \bar{y}_{\tau_i}) + \sum_{(t>\tau_i)(t^*>\tau_i)}(\bar{y}_t - \bar{y}_{t^*})\right)$$

The expected value of the coefficient on PRI falls as now only the effect of these institutions between t^* and t is captured by the regression. The expected value of the coefficient on CI on the other hand can fall or remain constant depending on t^*. Take first the case when for some countries the constraint was already binding at t^*, so $t^* > \tau_i$. For those countries the effect captured by the regression falls because of the same reason it falls in the case of PRI. Now instead of $\bar{y}_t - \bar{y}_{\tau_i}$ as in the unconditional case, the effect will be $(\bar{y}_t - \bar{y}_{\tau_i}) - (\bar{y}_{t^*} - \bar{y}_{\tau_i}) = \bar{y}_t - \bar{y}_{t^*}$. For the rest of the countries the effect remains the same because the level of income per capita in t^* does not have any information about ρ_i. But then if the constraint was not binding for any country at time t^*, there will not be any difference between the expected value of the two coefficients. Therefore if $t^* \leq \min_i(\tau_i)$, $E(\hat{\beta}_2) = \alpha_2$. Notice that $\min_i(\tau_i)$ exists by Proposition 4. The empirical strategy then is to include the lag of income per capita into the cross-country regressions and see what happens to the estimated coefficients.

An additional issue to take into account is that the model predicts conditional convergence. The empirical literature has strongly supported this feature as well. This implies that the error term in Equation (18) is correlated with the lagged value of income per capita. If the institutional quality indicators are correlated with the latter, the estimated coefficients will be biased. This is always the case for the coefficient on PRI, but if $t^* \leq \min_i(\tau_i)$ it is not the case for the coefficient on CI. Therefore the lagged value of income per capita is included as an additional regressor, and so the new specification is

$$y_{ti} - y_{t^*i} = \hat{\beta}_0 + \hat{\beta}_1(1 - \hat{\theta}_i) + \hat{\beta}_2\hat{\rho}_i + \hat{\beta}_3 y_{t^*i} + \zeta_i \tag{19}$$

An additional effect of including the lagged value of GDP per capita as an additional regressor is to reduce the variance of the coefficients when shocks to income per capita are persistent enough. In the case of the coefficient on CI, and provided that under the predictions of the model its expected value should not vary when the new regressor is included, its significance may rise. Then, under these conditions, estimating Equation (19) may be easier to identify the significance of this coefficient in the presence of measurement problems, facilitating the identification of the true effect of CI. Although measurement problems are mitigated when IV are used, their effect may still persist (Hausman [2001])[20].

Identification

The literature linking institutions and long-run growth is large, as described above. The main empirical problem facing these studies is that available measures of institutional quality are outcomes and therefore they are affected by actual economic conditions, being causal relationships difficult to identify. To overcome this problem past studies have used instruments to capture the exogenous component of the quality of institutions. These instruments are based on the idea that the nature of the institutional framework is highly persistent and it was mainly shaped by the influence of European countries. In the case of CI it has been widely documented that the main exogenous variation is given by the differences in legal traditions spread by European countries trough conquest, imitation, and colonization (Levine [2005] and La Porta et al. [2008]). On the other hand, Acemoglu et al. [2002] propose a measure of initial endowments as instruments. They show that relatively rich areas in 1500 are now relatively poor countries. Their explanation is that in poorer areas Europeans established institutions of private property that favored long-run growth, while in richer areas they established extractive institutions, which discourage investment and economic

[20] Under measurement problems the inclusion of the lagged value of income per capita can reduce the significance of the coefficients if this variable has better information about the true institutional quality. Under the model predictions this will not be the case for CI if $t^* \leq \min_i(\tau_i)$. The existence of permanent shocks, which in the model are shocks to the quality of institutions, may change the value of the coefficients. However the fact that there exist some t for which the coefficient on CI does not change is still valid. Moreover the literature on the effects of institutions on income per capita shows that the quality of the former is very persistent.

development (see also Engerman and Sokoloff [2002]). Therefore indicators related to initial endowments are good instruments to capture the exogenous component of the quality of PRI. In particular Acemoglu et al. [2002] show that urbanization and population density in 1500 capture very well these determinants. Related to this idea, Acemoglu et al. [2001] propose a measure of settler mortality as an alternative instrument. The idea is that better institutions were established in places where Europeans could settle.

As noted above, Acemoglu and Johnson [2005] take advantage of the strong link between the quality of CI and legal origin on the one hand, and the quality of PRI and initial endowments on the other, to identify the exogenous component of each of these highly correlated variables, and to be able to unbundle their effects on income per capita and financial development. The same strategy is used in this paper in the baseline estimations. Only a group of former colonies are included in the sample. The theory outlined by Acemoglu et al. [2002] for the relationship between initial endowments and institutional quality applies only to these countries. In fact they mention that their empirical results are no longer valid when European countries are included, as predicted by their theory. Additionally, when using this set of instrumental variables the sample is reduced because of the exclusion of former colonies for which these instruments are not available.

The model presented in this paper can be applied to any economy, and so CI could have shaped the economic development of the colonizers during the last century as well. Including more countries not only improve the results but also allows to keep a relatively large number of countries in each group when the sample is split between rich and poor economies. Moreover, and different than this sample-size effect, the fact that former colonies are poorer on average than European countries impede the identification of the effects of CI due to a sample-selection effect. To overcome this problem an alternative identification strategy is conducted. This is based on the idea that, especially in the first stages of development, PRI regulated the relationship between the government or elites, and the rest of the population, while CI regulated the relationship mostly among members within elites.

The case of former colonies is illustrative in this respect. Unlike CI, the existence of native populations in the colonies led Europeans to establish systematically more inefficient PRI there than in their countries. Natives provided a supply of labor that could be forced to work (Acemoglu et al. [2002]). Also the elites had to concentrate a lot of political power in their hands to maintain the social order and to avoid uprisings from other groups (Aguirre [2010]). Indeed, fear to race conflicts was one of the main reasons for the establishment of autocratic regimes throughout The Americas after independence (Williamson [2009], p.233). More generally, the relevance of being a colony has been captured by the use of time since independence as an explanatory variable for ex-

	CI_1	CI_2	PRI_1	PRI_2	CI_1	CI_2	PRI_1	PRI_2	CI_1	CI_2	PRI_1	PRI_2
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**
Constant	3.437^{***}	4.713^{***}	5.271^{***}	6.225^{***}	4.281^{***}	3.976^{***}	6.898^{***}	8.366^{***}	3.124^{***}	4.617^{***}	6.631^{***}	6.491^{***}
	0.120	*0.174*	*0.224*	*0.133*	*0.482*	*0.425*	*0.792*	*0.487*	*0.179*	*0.337*	*0.371*	*0.317*
English Legal Origin	0.672^{***}	-1.909^{***}	-0.046	0.200	0.563^{***}	-2.108^{***}	-0.093	0.071	0.932^{***}	-2.031^{***}	-0.423	0.622
	0.207	*0.245*	*0.391*	*0.209*	*0.215*	*0.230*	*0.456*	*0.245*	*0.228*	*0.361*	*0.545*	*0.399*
Log Population Density in 1500	-0.009	0.086	-0.429^{***}	-0.326^{***}								
	0.043	*0.070*	*0.104*	*0.065*								
Log Settler Mortality					-0.193^{*}	0.174^{*}	-0.340^{**}	-0.468^{***}				
					0.107	*0.093*	*0.151*	*0.100*				
Urbanization in 1500									0.041^{**}	0.027	-0.141^{***}	-0.040
									0.017	*0.032*	*0.044*	*0.033*
R^2	0.307	0.573	0.199	0.317	0.439	0.667	0.070	0.352	0.418	0.562	0.196	0.148
Observations	34	47	63	61	31	40	49	50	31	34	36	37

Table 1: First-Stage Regressions for CI and PRI, Former Colonies

plaining PRI quality (Beck et al. [2003] and Acemoglu et al. [2008]). On the other hand CI did not seem to have to adapt systematically to the new environment in former colonies. Papers focusing solely on the effects of legal institutions on economic outcomes, which include European countries on their estimations, do not distinguish between the efficiency of legal systems between these and former colonies (La Porta et al. [1998], La Porta et al. [1999], La Porta et al. [2008], Djankov et al. [2008]). In fact, one of the main findings by Djankov et al. [2003] is that courts' efficiency and their ability to deliver justice are determined by the characteristics of the legal procedure, rather than to the general underdevelopment of the country. This has been documented by historians as well[21]. Ultimately however, the validity of this strategy to identify the comparative effect of both types of institutions must be checked empirically, as Acemoglu and Johnson [2005] do for the initial endowments and legal origin indicators.

Ethno-linguistic fractionalization captures the idea behind the identification strategy just proposed, and it is available for a large number of countries. In ethnically diverse countries it would be less likely to find sound PRI, as heterogeneity translates into ethnic differences between the elites or the government, and the rest of the population. Fractionalization captures the incentives of ethnic groups in the government to use their power against other groups. Many papers have found a statistically significant relationship between fractionalization and institutional quality (Alesina et al. [2003] and Beck et al. [2003]) and related economic policies (Easterly and Levine [1997]). As CI regulate the relationship between members within the elite, we do not expect a significant effect

[21] For instance, Haring [1947] concludes that "*basically, however, people in the Indies, especially in the domain of private law, lived accordingly to the same judicial criteria as in Spain*" (Haring [1947], p.110), while Rothermund [2007] concludes that in Africa and Asia, "*...the legal systems were taken over by nationalists without any criticism. They had* [not] *protested neutral manifestations* [of foreign rule] *such as laws on the statute books*" (Rothermund [2007], p.252).

	CI_1 (1)	CI_2 (2)	PRI_1 (3)	PRI_2 (4)
Constant	3.271***	4.918***	4.848***	5.718***
	0.189	*0.288*	*0.397*	*0.281*
English Legal Origin	0.588***	−1.490***	0.326	0.321
	0.111	*0.226*	*0.303*	*0.204*
Fractionalization	−0.428	0.051	−1.149**	−0.841**
	0.327	*0.426*	*0.564*	*0.361*
Latitude	1.534***	−2.365***	3.356***	3.902***
	0.345	*0.533*	*0.698*	*0.555*
R^2	0.392	0.465	0.263	0.541
Observations	59	70	88	86

Table 2: First-Stage Regressions for CI and PRI, Full Sample

of this variable on the quality CI. Legal origin is used for capturing the exogenous component of CI, as it is also available for a large sample of countries. It is important to control for others factors in the first-stage, particularly the degree of influence of colonizers in former colonies. Latitude is available for a large sample of countries, and captures, besides other geographical features, factors affecting the incentives for settlement by the colonialists, since tropical endowments represent an inhospitable disease environment for them (Easterly and Levine [2003]). Latitude has been used among others by Hall and Jones [1999], La Porta et al. [1999], Beck et al. [2003], and Easterly and Levine [2003], and we expect a significant effect of it on the quality of the two types of institutions.

As a measure of CI quality the contract enforcement indicator constructed by Djankov et al. [2008] is used. The authors survey insolvency practitioners from 88 countries about how debt enforcement will proceed against an identical hotel about to default on its debt. They use data on time, cost, and the likely disposition of the assets to construct a measure of the efficiency of debt enforcement in each country. This is the best indicator for capturing the parameter ρ in the model since it includes explicitly most of the costs of debt enforcement, and large costs deter legal actions against fraudsters by creditors[22]. Results are also presented with the index of legal formalism constructed by Djankov et al. [2003], which is a measure of the number of legal proceedings for the collection of a bounced check. This index does not measure costs explicitly, although the authors show that it is correlated with the delay in the resolution of disputes.

For PRI indicators Acemoglu and Johnson [2005] use constraints on the executive from the

[22] I drop Angola out of the sample because it is an outlier and biases the results, specially when only former colonies are used. In log terms, as used in the estimations, the value of this variable for Angola is 0.18. That means more than 4.5 standard deviations below the average (3.8) and more than 2 standard deviations below the second worst value, which is 1.9 for Turkey.

	Full Sample		*GDP pc above the median*		*Full Sample*		*Full Sample*		*GDP pc 1950*		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**
Constant	7.971^{***}	0.230	8.636^{***}	4.752^{***}	11.413^{***}	3.967^{***}	8.904^{***}	1.235	7.300^{***}	9.492^{***}	8.154^{***}
	0.141	*0.901*	*0.092*	*0.631*	*0.487*	*1.080*	*0.249*	*1.227*	*0.100*	*0.361*	*0.218*
English Legal Origin	0.123	0.354^{**}	0.309	0.389^{**}	-0.105	0.111	0.408	0.560^{***}	-0.218	-0.275	-0.161
	0.225	*0.138*	*0.206*	*0.163*	*0.231*	*0.152*	*0.324*	*0.203*	*0.165*	*0.190*	*0.253*
Log population density in 1500	-0.367^{***}	-0.050	-0.227^{***}	-0.105^{**}					-0.299^{***}		
	0.067	*0.069*	*0.043*	*0.044*					*0.050*		
Log Settler Mortality					-0.703^{***}	-0.351^{***}				-0.449^{***}	
					0.101	*0.094*				*0.069*	
Urbanization in 1500							-0.070^{***}	0.008			-0.082^{***}
							0.024	*0.020*			*0.022*
Log GDP per capita in 1950		1.060^{***}		0.506^{***}		0.784^{***}		0.940^{***}			
		0.121		*0.080*		*0.097*		*0.140*			
R^2	0.313	0.721	0.476	0.644	0.622	0.783	0.249	0.668	0.353	0.477	0.289
Observations	69	69	34	34	50	50	37	37	69	50	37

Table 3: Direct Effect of Instruments on GDP per capita Today, Former Colonies.

Polity IV database as the preferred measure. This index measures explicitly how constrained the executive is in taking arbitrary decisions. They also use the risk of expropriation ICRG index. As noted by Acemoglu and Johnson [2005] the latter measure is an equilibrium outcome, determined by the actions taken by both the citizens and the elites. Moreover one of its components is Law and Order, which is an assessment of the strength and impartiality of the legal system and the popular observance of the law. These features are closer to the definition of CI than to the one for PRI. For these reasons constraints on the executive is the preferred measure used here, and the ICRG index is included only for comparison and robustness[23].

Finally, for the lagged value of GDP per capita I use 1950. This is because 1950 is the earliest year for which data on GDP per capita for a large number of developing countries is available. The source is Maddison [2008][24]. As the dependent variable I use GDP per capita in 2006, which is the latest in Maddison [2008]. The source for the legal origin variable is Djankov et al. [2008], who focus in particular on the legal origin of a country's bankruptcy laws. Countries classified as socialist legal origin are drop from the sample. Only for a few of them instruments are available, and therefore it is difficult to control for their specific features. Moreover the exclusion of transition economies ensure that the results are not driven by the reclassification of the latter from socialist into the French and German civil law families (La Porta et al. [2008]).

[23] For both constraints on the executive and the ICRG index I use the average since the year 2000.

[24] Maddison [2008] reports data on GDP per capita for earlier periods. The sample is relatively large only for 1913 and 1870, but still the number of countries used in the estimations would not be larger than 30.

	Full Sample		GDP pc above the median		GDP pc 1950
	(1)	(2)	(3)	(4)	(5)
Constant	9.143***	1.035	9.489***	5.490***	7.849***
	0.190	*0.756*	*0.136*	*0.737*	*0.131*
English Legal Origin	0.458**	0.496***	0.419***	0.436***	−0.036
	0.227	*0.147*	*0.161*	*0.156*	*0.191*
Fractionalization	−2.753***	−1.229***	−0.424	−0.083	−1.475***
	0.330	*0.277*	*0.688*	*0.519*	*0.254*
Log GDP per capita in 1950		1.033***		0.494***	
		0.090		*0.089*	
R^2	0.377	0.761	0.090	0.477	0.249
Observations	94	94	43	43	94

Table 4: Direct Effect of Instruments on GDP per capita Today, Full Sample.

Empirical Results

Table 1 shows the first-stage regressions for the sample of former colonies. The dependent variables are the log of contract enforcement (CI_1), legal formalism (CI_2), constraints on the executive (PRI_1), and the ICRG index (PRI_2). As expected legal origin has a strong and significant effect on the CI measures and a non significant effect on the PRI measures, in all the specifications. In the case of initial endowments, only when population density is used we observe a significant effect on PRI but not on CI indicators. When the other variables are used they are significant explaining the preferred PRI measure, but they are sometimes significant explaining the quality of CI. Because of this and the fact that the sample is larger when population density is used, the preferred specification for the second-stage estimations for former colonies uses the latter as an instrument.
In Table 2 the first-stage regression for the full sample of countries is shown. As expected we can see that legal origin (fractionalization) has a significant effect on the quality of CI (PRI) indicators, and a non significant effect on PRI (CI) indicators, confirming the identification strategy proposed above. Also as expected, latitude is highly significant for all the institutional quality measures.
Before estimating the second-stage regressions it might be interesting to explore the direct effect of the instruments on GDP per capita. Given that, in general, the evidence shows that legal origin closely determines CI and, on the other hand, initial endowments closely affect PRI, this exercise can be useful to infer the effects of each type of institutions without a specific measure of their quality. Acemoglu et al. [2002] perform a similar exercise for former colonies. The authors find a negative and statistically significant effect of initial endowments variables -population density and urbanization in 1500-, on today's GDP per capita. This is what they call the Reversal of Fortune. Moreover, and more relevant for this paper, they show that after controlling for these features other possible explanatory variables, including legal origin, do not have a statistically significant effect on today's GDP per capita.

	Contract Enforcement				Legal Formalism			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Constant	2.661*	−0.382	1.465	−1.358*	3.945***	1.328	0.611	0.419
	1.510	*1.246*	*1.646*	*0.723*	*0.950*	*1.081*	*1.516*	*0.867*
Log Enforcement Efficiency	0.240	0.435**	−0.154	0.347				
	0.335	*0.190*	*0.339*	*0.219*				
Legal Formalism					−0.084	−0.164**	0.055	−0.141
					0.118	*0.072*	*0.122*	*0.089*
Constraints on Executive	0.851***	0.367			0.839***	0.234		
	0.156	*0.380*			*0.156*	*0.390*		
Expropriation Protection			1.130***	0.348			1.140***	0.239
			0.208	*0.360*			*0.212*	*0.399*
Log GDP per capita in 1950		0.682*		0.823***		0.850**		0.924***
		0.405		*0.263*		*0.415*		*0.295*
R^2	0.313	0.721	0.313	0.721	0.313	0.721	0.313	0.721
Observations	69	69	69	69	69	69	69	69

Table 5: Second-Stage Results, Former Colonies.

Table 3 uses legal origin, population density in 1500, settler mortality, and urbanization in 1500 as explanatory variables. The effect of legal origin on the unconditional level of GDP per capita today is non significant in all the specifications where the lagged value of GDP per capita is not included as a regressor. This confirms the results by Acemoglu et al. [2002]. The fact that former colonies are poorer on average may be explaining this result in light of the predictions of the model. In column (3) we can see that when only countries with GDP per capita above the median are included, the coefficient on legal origin rises when controlling for population density, although it is still not significant, probably because of the small sample size. The second test proposed above can also be implemented here. When controlling for GDP per capita in 1950, the coefficient rises in the three cases, and it becomes significant when population density and urbanization are included (columns (2) and (8)). The coefficients on initial endowments on the other hand become smaller and even non significant in some cases. Finally in the last three columns GDP per capita in 1950 is the dependent variable. Legal origin is not significant, while the measures of initial endowments are so, confirming the previous results. Table 4 shows the same exercise but using fractionalization and legal origin as explanatory variables. In column (1) we can see that legal origin and fractionalization are statistically significant. This differs from Acemoglu et al. [2002] result, probably because the sample is different. More interesting for this paper are the results of the two tests described above. In column (3) the sample is restricted to countries with GDP per capita above the median. The significance of the effect of legal origin rises, while fractionalization becomes non significant. The second test also shows the expected results. After controlling for GDP per capita in 1950 (column (2)) the significance of the effect of legal origin rises, and the coefficient on fractionalization remains

	Contract Enforcement				Legal Formalism			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Constant	2.739*	−0.658	1.178	−0.935	3.246***	1.394*	1.893	1.133
	1.503	*0.965*	*1.620*	*0.907*	*1.253*	*0.777*	*1.554*	*0.870*
Log Enforcement Efficiency	0.098	0.522**	0.137	0.493*				
	0.407	*0.262*	*0.403*	*0.270*				
Legal Formalism					−0.030	−0.169**	−0.042	−0.163*
					0.125	*0.085*	*0.124*	*0.089*
Constraints on Executive	0.969***	0.393			0.967***	0.258		
	0.225	*0.311*			*0.228*	*0.339*		
Expropriation Protection			0.938***	0.474			0.936***	0.317
			0.218	*0.375*			*0.221*	*0.417*
Log GDP per capita in 1950		0.648***		0.537*		0.824***		0.743**
		0.232		*0.311*		*0.254*		*0.354*
Independent in 1950	−0.266	0.075	0.853***	0.586***	−0.286	0.023	0.821***	0.367*
	0.508	*0.404*	*0.300*	*0.218*	*0.461*	*0.391*	*0.247*	*0.216*
R^2	0.481	0.725	0.481	0.725	0.481	0.725	0.481	0.725
Observations	69	69	69	69	69	69	69	69

Table 6: Second-Stage Results, controlling for Independence. Former Colonies.

significant but it is reduced by a half. These results are confirmed when GDP per capita in 1950 is used as the dependent variable in column (5).

The second-stage results for former colonies are presented in Table 5. Legal origin and population density in 1500 are used as instruments. As the indicator for CI the first four columns use contract enforcement, and the rest use legal formalism. Results are similar so I describe only the first four columns. Contracting enforcement is never significant when not controlling for the lagged value of GDP per capita (columns (1) and (3)). This confirms the results by Acemoglu and Johnson [2005]. However, when the latter variable is included as a control, we can see that the CI coefficient rises, and it even becomes significant when using the preferred indicator for PRI (column (2)). In the case of PRI we observe the opposite pattern, the coefficients are significant when not controlling for the lagged value of GDP per capita, and fall and become non significant, when this is done. It is not shown but similar results are obtained when settler mortality or urbanization in 1500 are used as instruments.

Including the lagged value of GDP per capita in 1950 could be an issue in the sense that some of the countries in the sample were still colonies at that time. Therefore the change on income per capita since then could have been influenced by the process of decolonization and independent from institution quality. But if indeed PRI exported to former colonies were systematically more ineficient than in Europe, and CI did not suffer such a systematic change, then the coefficient on PRI may be including some of these developments. As explained above this can also help to iden-

	Full Sample				*GDP pc above the median*				*GDP pc below the median*			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**
Constant	1.97^{***}	3.41^{*}	1.16^{*}	-0.96	5.05^{***}	3.93^{***}	5.06^{***}	3.71^{***}	5.48^{***}	-0.56	4.44^{***}	-0.25
	0.67	*2.08*	*0.67*	*0.92*	*0.49*	*1.42*	*0.48*	*0.87*	*1.14*	*2.84*	*1.22*	*0.95*
Log Enforcement Efficiency	-0.03	-0.10	-0.05	0.44	0.89^{***}	0.85^{***}	0.80^{**}	0.87^{***}	-0.33	0.38	-0.48	0.20
	0.36	*0.43*	*0.43*	*0.42*	*0.32*	*0.31*	*0.33*	*0.33*	*0.41*	*0.58*	*0.41*	*0.39*
Constraints on Executive	1.20^{***}	1.44^{***}			0.16	0.11			0.62^{***}	-0.04		
	0.17	*0.54*			*0.22*	*0.38*			*0.22*	*0.73*		
Expropriation Protection			1.13^{***}	0.52			0.18	0.05			0.77^{***}	0.17
			0.20	*0.34*			*0.18*	*0.24*			*0.23*	*0.41*
Log GDP per capita in 1950		-0.34		0.59^{**}		0.19		0.25		1.00		0.87^{***}
		0.49		*0.23*		*0.33*		*0.16*		*0.63*		*0.25*
R^2	0.62	0.78	0.59	0.76	0.51	0.66	0.52	0.66	0.16	0.62	0.17	0.62
Observations	86	86	86	86	43	43	43	43	43	43	43	43

Table 7: Second-Stage Results by Income Groups. Full Sample

tify the true coefficient on CI if the implications of the model are true and there are measurement problems in the institutional indicators. To see if this is true Table 6 includes a dummy variable as a regressor, which takes the value of one when the country got its independence before 1950. The dummy is only significant when the ICRG index is used for capturing PRI. As expected countries that were not independent before 1950 grew slower than the rest of the countries, a difference that is not captured by the initial level of GDP per capita in this case. Qualitatively all the results in Table 5 hold, in particular the non significance of the coefficients on CI when the unconditional level of GDP per capita is used as a dependent variable. Now however, although their size do not change, they become significant when controlling for the lagged value of GDP per capita even when the ICRG index is used as a PRI indicator.

Finally the second-stage results for the full sample are presented in Table 7. Only contract enforcement is used but the result with legal formalism are unchanged. The first four columns show the results for the full sample. We can see that now the coefficient on CI is never significant, even when controlling for the lagged value of GDP per capita in 1950. This is a more heterogeneous sample and therefore it is more difficult to find a significant effect of CI[25]. The rest of the table show the same estimations but only for the group of richer (than the median) economies (columns (5) to (8)), and for the rest (columns (9) to (12)). The CI indicators are highly significant and positive, even when not controlling for the lagged value of GDP per capita, when only considering rich economies, and they are non significant when considering only poor economies. On the other

[25] Controlling for independence helps in doing this. Results are similar that those in Table 6. I do not show them as I focus here on the results when splitting the sample.

hand the effect of PRI seems to be just the opposite. It is stronger in poorer countries and weaker in richer countries.

Taking all the results together we can conclude that the effect of CI on GDP per capita seem to be more important in richer economies. Additionally, the value of GDP per capita in 1950 does not seem to have relevant information about the quality of CI. This can be evidence that before 1950, when the average level of GDP per capita was lower, the constraint of having low quality CI was not as binding as today. These results do not translate into PRI, confirming the idea that, unlike CI, the former affect income per capita throughout the development process.

Appendix A: The Transformed Dynamic Programming Problems

Given that $\hat{z}$ is constant, the aggregate state of the world is now described by $\hat{k}$, where $\hat{k} = \hat{k}_m + \hat{k}_a$. First it is necessary to compute the steady-state growth of consumption for an entrepreneur that has defaulted in a previous period t^*. Income for this entrepreneur in period $t > t^*$ will be

$$p_{jt} z_{jt}^{1-\alpha_j} \bar{k}_{jt}^{\alpha_j} n_{jt}^{\upsilon_j} - w_t n_{jt}$$

which, using the demand for labor, is

$$(1-\upsilon_j)\left[\frac{p_{jt}\upsilon_j^{\upsilon_j} z_{jt}^{1-\alpha_j} \bar{k}_{jt}^{\alpha_j}}{w_t^{\upsilon_j}}\right]^{\frac{1}{1-\upsilon_j}}$$

using the asymptotic growth rates of p and w, and the fact that $\bar{k}_{jt} = (1-\delta)\bar{k}_{jt-1}$, the asymptotic growth rate of income will be

$$\gamma_m^d = \mu_m^{\frac{1-\alpha_j-\upsilon_j}{1-\upsilon_j}} (1-\delta)^{\frac{\alpha_j}{1-\upsilon_j}}$$

Given the specification used for the instantaneous utility function, both c_m^d ans pc_a^d grow at that rate, meaning that c_a^d will grow at

$$\gamma_a^d = \mu_m^{\frac{1-\alpha_j-\upsilon_j}{1-\upsilon_j}} (1-\delta)^{\frac{\alpha_j}{1-\upsilon_j}} \left(\frac{\mu_a}{\mu_m}\right)^{1-\alpha_a}$$

Now $\hat{c}_{jt}^d = c_{jt}^d/(\gamma_j)^{t^*}(\gamma_j^d)^{t-t^*}$ can be defined, a variable that will be constant in steady-state. It follows that, for $t > t^*$,

$$u(c^d) = \bar{\gamma} u(\hat{c}^d) = \bar{\gamma} \hat{c}_a^{d\,\eta} \hat{c}_m^{d\ 1-\eta}$$

and

$$V_{jt+1}^d(\bar{k_j}; k') = \bar{\gamma} V_{jt+1}^d(\hat{k}_j; \hat{k}')$$

where

$$\bar{\gamma} = (\gamma_m^d)^{1-\eta}(\gamma_a^d)^{\eta} = \gamma \frac{(1-\delta)^{\frac{\alpha_j}{1-\upsilon_j}}}{\mu_m}$$

For non-defaulting entrepreneurs and consumers, consumption of good j grows at the constant rate γ_j as noted in the text. Then we define $\hat{c}_{jt}^i = c_{jt}^i/\gamma_j^t$ for $i = c, e_j$ and $j = a, m$, which will be constant in steady-state, and

$$u(c^i) = \gamma u(\hat{c}^i) = \gamma \hat{c}_a^{i\,\eta} \hat{c}_m^{i\ 1-\eta}$$

The latter definitions also apply to defaulting entrepreneurs during the period in which they default. The last step is to transform the budget constraints and the market clearing conditions. Notice that in every case the LHS and RHS grow at the same rate in steady state so transforming them is simple. When savings are included however, as is the case of the consumer budget constraint, we have

$$\frac{b_{t+1}}{\gamma_m^t} = \gamma_m \frac{b_{t+1}}{\gamma_m^{t+1}} = \gamma_m \hat{b}_{t+1}$$

so the corresponding adjustment must to be made. This is also the case in the market clearing condition for the manufacturing good.

Now the transformed dynamic program problem facing the type i representative entrepreneur can be written as

$$V(\hat{k}) = \max_{\hat{k}_j, n_j, \hat{c}_m, \hat{c}_a, \hat{c}_m^d, \hat{c}_a^d} \left\{\max\left(u(\hat{c}) + \beta\gamma V(\hat{k}),\ \rho\left[u(\hat{c}^d) + \beta\bar{\gamma} V^d(\hat{k}_j; \hat{k}')\right] + (1-\rho)u(\hat{c})\right)\right\}$$

subject to

$$\hat{c}_m + \hat{p}\hat{c}_a = \hat{p}_j\hat{y}_j - \hat{w}n_j - (r+\delta)\hat{k}_j$$
$$\hat{c}_m^d + \hat{p}\hat{c}_a^d = \hat{p}_j\hat{y}_j - \hat{w}n_j$$

and $\hat{k}' = \hat{K}(\hat{k})$. Where

$$V^d(\hat{k}_j; \hat{k}') = \max_{\hat{c}_m'^d, \hat{c}_a'^d}\left\{u(\hat{c}'^d) + \beta\bar{\gamma} V^d(\hat{k}_j; \hat{k}'')\right\}$$

subject to

$$\hat{c}_m'^d + \hat{p}'\hat{c}_a'^d = (r'+\delta)\hat{k}_j$$

And the transformed consumers' problem will be

$$U(\hat{b};\hat{k}) = \max_{\hat{c}_m,\hat{c}_a,n,\hat{b}'_m,\hat{b}'_a} \left\{ \frac{u(\hat{c})^{1-\sigma}-1}{1-\sigma} + \beta\gamma^{1-\sigma}U(\hat{b}';\hat{k}') \right\}$$

subject to

$$\hat{c}_m + \hat{p}\hat{c}_a + \mu_m(\hat{b}'_m + \hat{b}'_a) = \hat{w}n + (\hat{b}_m + \hat{b}_a)(1+r)$$

and the IC constraint,

$$\beta\left[\gamma V^{e_j}(\hat{k}') - \rho\bar{\gamma}V^{de_j}(\hat{k}_j;\hat{k}')\right] \geq \rho\left[u(\hat{c}^{de_j}) - u(\hat{c}^{e_j})\right]$$

and to the law of motion for the aggregate state, $\hat{k}' = \hat{K}(\hat{k})$.

Finally, the transformed market clearing conditions are,

$$\hat{c}_m = \sum_i \hat{c}^i_m = \hat{y}_m + (\hat{k}_m + \hat{k}_a)(1-\delta) - \mu_m(\hat{k}'_m - \hat{k}'_a)$$

$$\hat{c}_a = \sum_i \hat{c}^i_a = \hat{y}_a$$

Appendix B: Proofs

Proof of Lemma 1

Differentiate equations (4) and (5) using equation (3) to get

$$(1-\alpha_a n_a - \alpha_m n_m)\, dlog\hat{w} + (\alpha_a n_a + \alpha_m n_m)\, dlog\bar{r} - n_a dlog\hat{p} = 0$$

$$(\upsilon_a k_a + \upsilon_m k_m)\, dlog\hat{w} + \left(\hat{k} - \alpha_a k_a - \alpha_m k_m\right) dlog\bar{r} - k_a dlog\hat{p} = \hat{k}(1-\alpha_a-\upsilon_a) dlog\hat{k}$$

It is useful to write down the following expression,

$$D = \omega n_a k_a + (1-\alpha_a-\upsilon_m) n_a k_m + (1-\alpha_m-\upsilon_a) n_m k_a + \omega n_m k_m > 0$$

The fact that $D>0$ follows from the fact that $(1-\alpha_j-\upsilon_j)$ is greater than zero for $j=a,m$, and

$$(1-\alpha_a-\upsilon_m) n_a k_m + (1-\alpha_m-\upsilon_a) n_m k_a = \omega n_a k_m + \omega n_m k_a + (\upsilon_a - \upsilon_m)(n_a k_m - n_m k_a) > 0$$

which follows from $(n_a k_m - n_m k_a) > 0$ because,

$$\frac{n_a k_m}{k_a n_m} - 1 = \frac{\upsilon_a \alpha_m}{\upsilon_m \alpha_a} - 1 > 0.$$

Rybczynski: Fix $\hat{p}$. Then use the two equations above to express $dlog\hat{w}$ and $dlog\bar{r}$ in terms of $dlog\hat{k}$,

$$\frac{dlog\bar{r}}{dlog\hat{k}} = -\frac{\hat{k}}{D}\left[\omega(1-\alpha_a n_a - \alpha_m n_m)\right] \tag{20}$$

$$\frac{dlog\hat{w}}{dlog\hat{k}} = \frac{\hat{k}}{D}\left[\omega(\alpha_a n_a + \alpha_m n_m)\right] \tag{21}$$

The first expression is negative and the second one is positive. Notice also that both expressions are zero when $\omega = 0$, i.e. with CRS. To see the effects on output use Equation (3) to get, for $j=a,m$,

$$\frac{dlog\hat{y}_j}{dlog\hat{k}} = \frac{-\alpha_j}{\omega}\frac{dlog\bar{r}}{dlog\hat{k}} + \frac{-\upsilon_j}{\omega}\frac{dlog\hat{w}}{dlog\hat{k}}$$

Replacing the expressions for the change in factor prices we get

$$\frac{dlog\hat{y}_a}{dlog\hat{k}} = \frac{\hat{k}}{D}\left(\alpha_a - (\alpha_a+\upsilon_a)(\alpha_a n_a + \alpha_m n_m)\right) \tag{22}$$

$$\frac{dlog\hat{y}_m}{dlog\hat{k}} = \frac{\hat{k}}{D}\left(\alpha_m - (\alpha_m+\upsilon_m)(\alpha_a n_a + \alpha_m n_m)\right) \tag{23}$$

Then, for sector $j=a,m$ the effect will be positive if

$$(\alpha_a n_a + \alpha_m n_m)(\alpha_j + \upsilon_j) < \alpha_j$$

Take first $j=m$. Given that $(\alpha_a n_a + \alpha_m n_m) < \alpha_m$, a sufficient condition is,

$$\alpha_m + \upsilon_m \le 1$$

which is obviously true. In the case of $j=a$ the expression can be positive or negative. Suppose first $\omega=0$, and so $\alpha_j+\upsilon_j=1$. In this case the LHS will be greater and then the effect will be negative. Then with CRS we have the Rybczynski result (the increase in capital has a positive (negative) effect in the sector intensive in capital(labor)). However, when ω increases the LHS falls, and eventually the effect becomes positive. Then by continuity and monotonicity, $\exists \omega^R > 0$ such that $\forall \omega \in \left[0, \omega^R\right]$ the effect is negative.

Stopler-Samuelson: Fix $\hat{k}$. Then use the system of equations above to express $dlog\hat{w}$ and $dlog\bar{r}$ in terms of $dlog\hat{p}$,

$$\frac{dlog\hat{w}}{dlog\hat{p}} = \frac{1}{D}\left[\omega n_a k_a + \omega n_a k_m + \alpha_m(n_a k_m - n_m k_a)\right] \tag{24}$$

$$\frac{dlog\bar{r}}{dlog\hat{p}} = \frac{1}{D}\left[\omega n_a k_a + \omega n_m k_a + \upsilon_m(n_m k_a - n_a k_m)\right] \tag{25}$$

The first expression is positive for any ω because $(n_a k_m - n_m k_a) > 0$ as shown above. The second expression can be either positive or negative. If $\omega = 0$ then we have the Stopler-Samuelson result with CRS because the expression is negative. As ω increases however the term rises, eventually becoming positive. Then by monotonicity and continuity, $\exists\omega^{S1} > 0$ such that $\forall\omega \in \left[0, \omega^{S1}\right]$ the expression is negative. For the ratio $r/\hat{p}$ is enough to show $dlog\bar{r}/dlog\hat{p} < 1$, or

$$\omega n_a k_a + \omega n_m k_a + \upsilon_m(n_m k_a - n_a k_m) < D$$

which can be simplified to

$$\upsilon_a n_m k_a < (1 - \alpha_a) n_a k_m + \omega n_m k_m$$

Notice that $n_a k_m > n_m k_a$ and therefore it is enough to show

$$0 \leq \omega n_m k_a + \omega n_m k_m = \omega n_a \hat{k}$$

which is always true. For the real wage it is enough to show that $dlog\hat{w}/dlog\hat{p} > 1$, or

$$\omega n_a k_a + \omega n_a k_m + \alpha_m(n_a k_m - n_m k_a) > D$$

Doing some algebra this is equivalent to,

$$\alpha_a n_a k_m - (1 - \upsilon_a) n_m k_a > \omega n_m k_m$$

Notice that if $\omega_j = 0$ then the expression above simplifies to

$$\alpha_a(n_a k_m - n_m k_a) > 0$$

which is true. So we have the Stopler Samuelson result for CRS. If ω increases then the LHS of the expression becomes bigger and eventually becomes larger than the LHS. Then by continuity, $\exists\omega^{S2} > 0$ such that $\forall\omega \in \left[0, \omega^{S2}\right]$ the expression is positive. Therefore define $\omega^S = min(\omega^{S1}, \omega^{S2})$. Now we can use these expressions and Equation (3) to see the effects on output. We get

$$\frac{dlog\hat{y}_m}{dlog\hat{p}} = -\frac{1}{\omega D}\left[(\alpha_m + \upsilon_m)\omega n_a k_a + \omega(\alpha_m n_m k_a + \upsilon_m n_a k_m)\right] \tag{26}$$

$$\frac{dlog\hat{y}_a}{dlog\hat{p}} = \frac{1}{\omega D}\left[(\alpha_a + \upsilon_a)\omega n_m k_m + \omega(\upsilon_a n_m k_a + \alpha_a n_a k_m)\right] \tag{27}$$

It is easy to see that the first expression is negative and the second positive, for any ω.

QED.

Proof of Proposition 1

Monotonicity of $\hat{k}$

In order to show that capital is increasing when its level is below its steady-state level I first show that the equilibrium under perfect enforceability is efficient, in the sense that it coincides to the solution to the central planner problem consisting on maximizing consumers utility subject to the constraint that entrepreneurs utility can not be lower than a certain level. Then, in the central planner problem it is possible to show that the consumer value function is strictly increasing and concave on capital, and that the policy function for capital is strictly increasing on capital. These properties are used to finally show that capital is increasing (decreasing) if below (above) its steady-state level.

To show that the equilibrium is efficient take first the static block of the central planner problem. This problem consist in allocating factors of production among sectors, and allocating consumption across agents and goods, given the current and future stock of capital. The problem is as follows,

$$\Phi(\hat{k}, \hat{k}') = \max_{\hat{c}^c_j, \hat{k}_j, \hat{n}_j} u(\hat{c}^c)$$

subject to

$$\sum_i \hat{c}^i_a \leq \hat{z}_a^{1-\alpha_a} \hat{k}_a^{\alpha_a} n_a^{\upsilon_a}$$

$$\sum_i \hat{c}^i_m \le \hat{z}_m^{1-\alpha_m}\hat{k}_m^{\alpha_m} n_m^{\upsilon_m} + (1-\delta)\hat{k} - \mu_m \hat{k}'$$

$$\hat{k}_a + \hat{k}_m \le \hat{k}$$

$$\hat{n}_a + \hat{n}_m \le 1$$

$$\kappa \le u(\hat{c}^{e_j}) \qquad e_j = e_a, e_m$$

Where $\kappa > 0$ can change over time but it is not a choice variable for the central planner. Denote by $\phi_a, \phi_m, \lambda_k, \lambda_n, \theta_{e_j}$ the multipliers for the constraints. The FOC associated to the allocation of consumption across goods for the consumer are,

$$u_j(\hat{c}^c) = \phi_j \qquad j = a, m$$

In the case of the allocation of consumption goods for entrepreneurs we have

$$u_j(\hat{c}^{e_j})\theta_{e_j} = \phi_j \qquad j = a, m \text{ and } e_j = e_a, e_m$$

Using these two expressions and after some algebra we get,

$$\frac{\phi_a}{\phi_m}\hat{c}^i_a = \eta\left(\frac{\phi_a}{\phi_m}\hat{c}^i_a + \hat{c}^i_m\right) \qquad i = c, e_a, e_m \tag{28}$$

and

$$\hat{c}^i_m = (1-\eta)\left(\frac{\phi_a}{\phi_m}\hat{c}^i_a + \hat{c}^i_m\right) \qquad i = c, e_a, e_m \tag{29}$$

On the supply side the FOC are,

$$\alpha_j \hat{z}_j^{1-\alpha_j}\hat{k}_j^{\alpha_j - 1} n_j^{\upsilon_j} = \frac{\lambda_k}{\phi_j} \qquad j = a, m \tag{30}$$

$$\upsilon_j \hat{z}_j^{1-\alpha_j}\hat{k}_j^{\alpha_j} n_j^{\upsilon_j - 1} = \frac{\lambda n}{\phi_j} \qquad j = a, m \tag{31}$$

Then if we define $\phi_a/\phi_m = \hat{p}$, $\lambda_k/\phi_a = \bar{r}/\hat{p}$, and $\lambda_n/\phi_a = \hat{w}/\hat{p}$, equations (28) and (29) are identical to equations (7) and (8), while equations (30) and (31) are identical to the FOC of the entrepreneurs problem used to derived equations (3), (4), and (5). It follows that the solution to the static central planner problem is identical to the solution to the static descentralized problem.

Now using the envelope theorem we get

$$\Phi_{\hat{k}}(\hat{k}, \hat{k}') = \phi_m(1-\delta) + \lambda_k = \phi_m(1 + \alpha_j \hat{z}_j^{1-\alpha_j}\hat{k}_j^{\alpha_j - 1} n_j^{\upsilon_j} - \delta)\phi_m(1 + mp\hat{k}_m - \delta) > 0 \tag{32}$$

Notice also that $\Phi_{\hat{k}\hat{k}}(\hat{k}, \hat{k}') < 0$. Also

$$\Phi_{\hat{k}'}(\hat{k}, \hat{k}') = -\phi_m \mu_m < 0 \tag{33}$$

The dynamic central planner problem only involves choosing the sequence of capital that maximizes the utility of consumers, subject to the sequence of the solution to the static problem. Given that entrepreneurs do not save they are included in this problem only as a restriction in the sense that the central planner has to maximize consumer's utility but subject to keeping the level of utility of the entreprneurs fixed at the optimal level derived in the static problem. Then the central planner maximizes,

$$\Omega = \max_{\{\hat{k}_t\}_0^\infty} \sum_t (\beta\gamma^{1-\sigma})^t \frac{\Phi(\hat{k}_t, \hat{k}_{t+1})^{1-\sigma} - 1}{1-\sigma}$$

subject to the static problem defined for each t. Using equations (32) and (33), the FOC is

$$\frac{u(\hat{c}^c_t)}{u(\hat{c}^c_{t+1})}^{-\sigma} = \frac{\beta\gamma^{1-\sigma}(\phi_{m,t+1}(1-\delta) + \lambda_{k,t+1})}{\phi_{m,t}\mu_m}$$

which, using some algebra, the definitions for the multipliers above, and the solution to the statice problem, is

$$\frac{u(\hat{c}^c_t)}{u(\hat{c}^c_{t+1})}^{-\sigma} = \beta\gamma^{1-\sigma}\left(\frac{\hat{p}_{t+1}}{\hat{p}_t}\right)^{\eta}\frac{1 + r_{t+1}}{\mu_m}$$

This expression is identical to the first order condition used to derive Equation (10) in the text. It follows that the descentralized equilibrium is equivalent to the central planner problem defined here.

We can express the dynamic central planner problem recursively as

$$W(\hat{k}) = \max_{\hat{k}'} \Phi(\hat{k},\hat{k}')^{1-\sigma} + \beta\gamma^{1-\sigma} W(\hat{k}')$$

subject to all the restrictions stated above. The solution to this problem is described by the policy function

$$\hat{k}' = g(\hat{k})$$

The FOC and the enveloped condition for this problem are the following,

$$\Phi(\hat{k},\hat{k}')^{-\sigma}\Phi_{\hat{k}'} = \beta\gamma^{1-\sigma} W_{\hat{k}'}(\hat{k}') \tag{34}$$

$$W_{\hat{k}}(\hat{k}) = \Phi(\hat{k},\hat{k}')^{-\sigma}\Phi_{\hat{k}} \tag{35}$$

From the discussion above we know that the solution to this problem is equivalent to the descentralized equilibrium and that $\Phi(\hat{k},\hat{k}')$ is increasing and concave on $\hat{k}$. This implies that $W(\hat{k})$ is also increasing and concave on $\hat{k}$.

Another property of this problem is that $g(\hat{k})$ is increasing on $\hat{k}$. To see this suppose that it is not true and $g(\hat{k})$ is decreasing on $\hat{k}$. Then $\exists\ \hat{k}_t,\ \hat{k}_\tau$ such that $\hat{k}_\tau > \hat{k}_t$ and $g(\hat{k}_\tau) < g(\hat{k}_t)$. Because $W_{\hat{k}}(\hat{k})$ is decreasing on $\hat{k}$ (by concavity and monotonicity of W), we have $W_{g(\hat{k}_\tau)}(g(\hat{k}_\tau)) > W_{g(\hat{k}_t)}(g(\hat{k}_t))$. This and the FOC from Equation (34) $\Phi(\hat{k}_\tau, g(\hat{k}_\tau))^{-\sigma}\Phi_{g(\hat{k}_\tau)} > \Phi(\hat{k}_t, g(\hat{k}_t))^{-\sigma}\Phi_{g(\hat{k}_t)}$ and so, by concavity of Φ, $g(\hat{k}_t) > g(\hat{k}_\tau)$. But this contradicts the first equality. It follows that $g(\hat{k})$ is increasing on $\hat{k}$.

Now it is possible to show that $d\hat{k} > 0$, $\forall \hat{k} < \hat{k}^{\text{SS}}$. Suppose that $\hat{k}^*$ amd $\hat{k}'^*$ are part of the solution sequence, with $\hat{k}'^* = g(\hat{k}^*)$. By concavity of W we know that

$$(\hat{k}^* - \hat{k}'^*)\,(W_{\hat{k}} - W_{\hat{k}'}) \leq 0$$

Using the FOC in Equation (34), and the envelope condition in Equation (35), this expression becomes,

$$(\hat{k}^* - \hat{k}'^*)\left(\Phi(\hat{k},\hat{k}')^{-\sigma}\Phi_{\hat{k}} - \frac{\Phi(\hat{k},\hat{k}')^{-\sigma}\Phi_{\hat{k}'}}{\beta\gamma^{1-\sigma}}\right) \leq 0$$

or, using the results for the static problem,

$$(\hat{k}^* - \hat{k}'^*)\left(1 + mp\hat{k}_m - \delta - \frac{\mu_m}{\beta\gamma^{1-\sigma}}\right) \leq 0$$

Notice that in steady-state

$$1 = \frac{\beta\gamma^{1-\sigma}(\phi(1-\delta)+\lambda_k)}{\phi_m\mu_m} \quad \rightarrow \quad \frac{\mu_m}{\beta\gamma^{1-\sigma}} = 1 + mp\hat{k}_m^{\text{SS}} - \delta$$

so we have

$$(\hat{k}^* - \hat{k}'^*)\left(mp\hat{k}_m - mp\hat{k}_m^{\text{SS}}\right) \leq 0$$

or, replacing the multipliers to get the decentralized expresion,

$$(\hat{k}^* - \hat{k}'^*)(r - r^{\text{SS}}) \leq 0$$

From Lemma 1 we know that if $\hat{k} < \hat{k}^{\text{SS}}$, then $r > r^{\text{SS}}$. But then in this case we have that $\hat{k} < \hat{k}'$.

Monotonicity of $\hat{p}, \hat{w}, \hat{w}/\hat{p}, r, r/\hat{p}$

Suppose $\omega^* \leq min(\omega^R, \omega^S)$. Obiously $\hat{\epsilon}^c > 0$. Also $r' - \bar{\beta} - \bar{\mu} = r' - r^{\text{SS}} > 0$ from the discussion above when $\hat{k} < \hat{k}^{\text{SS}}$. Then using Lemma 1 and noticing that $\omega \in [0, \omega^R]$, every term in Equation (14) is positive but the last term in the denominator when $\sigma > 1$. However it is possible to show that

$$(1 - \eta\omega_j)\hat{y}_a - \frac{\eta^2(\sigma-1)}{\hat{p}\sigma}\hat{\epsilon}^c > 0$$

which is sufficient to show $d\hat{p} > 0$. Notice that $\hat{\epsilon}^c < \hat{\epsilon} - \hat{\epsilon}^{e_a} = (1 - \eta\omega)\hat{p}\hat{y}_a/\eta$, so it is enough to show

$$1 - \eta\frac{\sigma - 1}{\sigma} > 0$$

which is always true. It follows that $d\hat{p} > 0$. Given $d\hat{p} > 0$ and $d\hat{k} > 0$, the result for factor prices follows directly from Lemma 1.

Monotonicity of $\hat{c}_m, \hat{p}\hat{c}_a, \hat{p}\hat{y}_a, \hat{y}_m$

Using equations (7), (8) and (9) we can express optimal demands in the following way,

$$\hat{p}\hat{c}_a^c = \left(\frac{\eta}{1-\eta}\right)^{1-\eta}\hat{p}^\eta u(\hat{c}^c)$$
$$\hat{c}_m^c = \left(\frac{1-\eta}{\eta}\right)^{\eta}\hat{p}^\eta u(\hat{c}^c)$$

From the central planner problem above we know $u(\hat{c}^c)$ is increasing during the transition, because it is increasing on $\hat{k}$ and $\hat{k}$ is increasing during the transition. From above we know also that $\hat{p}$ is increasing during the transition. It follows that both $\hat{c}_m$ and $\hat{p}\hat{c}_a = \hat{p}\hat{y}_a$ are increasing during the transition.

To prove that $\hat{y}_m$ is monotone we need to show that investment is. First define $z = 1 - s = \hat{c}_m/\hat{y}_m$, where s is the saving or investment rate. Then we have that

$$\gamma_z = \frac{dz}{z} = \frac{d\hat{c}_m}{\hat{c}_m} - \frac{d\hat{y}_m}{\hat{y}_m}$$

Using Equation (8) and totally differentiating $\hat{y}_m$ we have

$$\gamma_z = \frac{d\hat{\epsilon}^c}{\hat{\epsilon}^c} - \frac{d\hat{k}}{\hat{y}_m}\frac{\partial\hat{y}_m}{\partial\hat{k}} - \frac{d\hat{p}}{\hat{y}_m}\frac{\partial\hat{y}_m}{\partial\hat{p}}$$

which is equivalent to,

$$\gamma_z = \frac{1}{\sigma}\left[r - \bar{\beta} - \bar{\mu}_m + \eta(\sigma-1)\frac{d\hat{p}}{\hat{p}}\right] - \left[\frac{\hat{y}_m - \hat{k}(\bar{\mu}_m + \delta) - \hat{c}_m}{\mu_m\hat{y}_m}\right]\frac{\partial\hat{y}_m}{\partial\hat{k}} - \frac{d\hat{p}}{\hat{p}}\frac{\partial log\hat{y}_m}{\partial log\hat{p}}$$

Using Lemma 1, we know that,

$$\frac{\partial\hat{y}_m}{\partial\hat{k}} = \frac{\hat{y}_m n_a}{n_a\hat{k}_m - n_m\hat{k}_a} = \frac{\upsilon_a}{\alpha_m - \alpha_a}(r-\delta) = \psi(r-\delta) \tag{36}$$

$$\frac{\partial log\hat{y}_m}{\partial log\hat{k}} = \frac{\alpha_m\psi\hat{k}}{\hat{k}_m}$$

$$\frac{\partial log\hat{y}_m}{\partial log\hat{p}} = \frac{1}{\alpha_m - \alpha_a}\left[\alpha_m - \frac{\partial log\hat{y}_m}{\partial log\hat{k}}\right] \tag{37}$$

Notice that $1 - z^{\text{SS}} = s^{\text{SS}} = (\bar{\mu}_m + \delta)\hat{k}^{\text{SS}}/\hat{y}_m^{\text{SS}}$. Replacing these expressions above we get,

$$\gamma_z = \frac{\psi r}{\mu_m}\left[z - \frac{\psi\sigma - \mu_m}{\psi\sigma}\right] - \frac{1}{\sigma}\left[\delta + \bar{\beta} + \bar{\mu}_m - \eta(\sigma-1)\frac{d\hat{p}}{\hat{p}}\right] + s^{\text{SS}}\frac{\hat{y}_m^{\text{SS}}\alpha_m\psi\hat{k}}{\hat{k}^{\text{SS}}\mu_m\hat{k}_m} - \frac{d\hat{p}}{\hat{p}}\left(\frac{\alpha_m}{\alpha_m - \alpha_a}\right)\left(1 - \frac{\psi\hat{k}}{\hat{k}_m}\right)$$

or, using the fact that $r^{\text{SS}} = \delta + \bar{\beta} + \bar{\mu}_m$,

$$\gamma_z = \frac{\psi r}{\mu_m}\left[z - \frac{\psi\sigma - \mu_m}{\psi\sigma}\right] + (\delta + \bar{\beta} + \bar{\mu}_m)\left[\frac{s^{\text{SS}}\psi\hat{k}_m^{\text{SS}}\hat{k}}{\mu_m\hat{k}_m\hat{k}^{\text{SS}}} - \frac{1}{\sigma}\right] + \frac{d\hat{p}}{\hat{p}}\left[\frac{\eta(\sigma-1)}{\sigma} - \frac{\alpha_m}{\alpha_m - \alpha_a}\left(1 - \frac{\psi\hat{k}}{\hat{k}_m}\right)\right]$$

Suppose first $\omega = 0$. Then with some algebra and using Equation (14) we get,

$$\frac{d\hat{p}}{\hat{p}} = \frac{\frac{\eta}{1-\eta}\frac{z}{\sigma}(r - \bar{\beta} - \bar{\mu}_m) - \frac{1-z}{\mu_m}\hat{p}\frac{\partial\hat{y}_a}{\partial\hat{k}} + \frac{\hat{k}}{\mu_m\hat{y}_m}(\bar{\mu}_m + \delta)\hat{p}\frac{\partial\hat{y}_a}{\partial\hat{k}}}{\frac{\eta}{1-\eta}z\left(1 - \frac{\eta(\sigma-1)}{\sigma}\right) + \frac{\partial\hat{y}_a}{\partial\hat{p}}\frac{\hat{p}^2}{\hat{y}_m}}$$

Again from Lemma 1 we know that,

$$\frac{\partial \hat{y}_a}{\partial \hat{k}} = -\frac{1}{\hat{p}} \frac{\upsilon_m}{\alpha_m - \alpha_a} r = \psi' r$$

with $\psi + \psi' = 1$. Using this and some algebra we get the following expression for the change in prices,

$$\frac{d\hat{p}}{\hat{p}} = \frac{\frac{\eta}{1-\eta}\frac{z}{\sigma}(r - \bar{\beta} - \bar{\mu}_m) - \frac{1-z}{\mu_m}\psi' r - \frac{\hat{k}\psi'\alpha_m}{\mu_m \hat{k}_m}(\bar{\mu}_m + \delta)}{\frac{\eta}{1-\eta} z \left(1 - \frac{\eta(\sigma-1)}{\sigma}\right) - \frac{\alpha_m}{\alpha_m - \alpha_a}\left(1 - \frac{\psi \hat{k}}{\hat{k}_m}\right)}$$

Replacing this term in the expresion above, and after some algebra, we get,

$$\gamma_z = \left(r + \delta - \eta \frac{d\hat{p}}{\hat{p}}\right)\left[z - \left(1 - \frac{1}{\sigma}\left(\frac{\alpha_a - \alpha_m}{\alpha_a} + \frac{\alpha_m \hat{k}}{\alpha_a \hat{k}_m}\right)\right)\right] + \left(\delta + \bar{\beta} + \bar{\mu}_m\right)\frac{\hat{k}\hat{k}_m^{\mathrm{SS}}}{\hat{k}_m \hat{k}^{\mathrm{SS}}}\left[s^{\mathrm{SS}} - \frac{1}{\sigma}\left(\frac{\alpha_a - \alpha_m}{\alpha_a} + \frac{\alpha_m \hat{k}}{\alpha_a \hat{k}_m}\right)\right] \tag{38}$$

Now suppose that σ is such that

$$s^{\mathrm{SS}} > \frac{1}{\sigma(1-\eta)} \frac{\hat{k}^{\mathrm{SS}}}{\hat{k}_m^{\mathrm{SS}}} \tag{39}$$

and so the second term in Equation (38) is positive, because $\hat{k}/\hat{k}_m = (\alpha_a/\alpha_m)z(\eta/(1-\eta)) + 1 \leq (\alpha_a/\alpha_m)(\eta/(1-\eta)) + 1$. Then if at some point in time,

$$z > \left(1 - \frac{1}{\sigma}\left(\frac{\alpha_a - \alpha_m}{\alpha_a} + \frac{\alpha_m \hat{k}}{\alpha_a \hat{k}_m}\right)\right)$$

we have that $\gamma_z > 0$ for ever. But this contradicts the existance of a steady state and so

$$z < \left(1 - \frac{1}{\sigma}\left(\frac{\alpha_a - \alpha_m}{\alpha_a} + \frac{\alpha_m \hat{k}}{\alpha_a \hat{k}_m}\right)\right) \quad \forall t$$

Differentiating Equation (38) we get

$$\dot{\gamma_z} = d\left(r + \delta - \eta \frac{d\hat{p}}{\hat{p}}\right)\left[z - \left(1 - \frac{1}{\sigma}\left(\frac{\alpha_a - \alpha_m}{\alpha_a} + \frac{\alpha_m \hat{k}}{\alpha_a \hat{k}_m}\right)\right)\right] + \left(r - \bar{\beta} - \bar{\mu}_m - \eta \frac{d\hat{p}}{\hat{p}}\right)\gamma_z z \left(1 + \frac{\eta}{\sigma(1-\eta)}\right) + \left(r + \delta - \eta \frac{d\hat{p}}{\hat{p}}\right)\gamma_z z + (\lambda + \hat{b} +$$

(40)

If $\gamma_z > 0$ at some point, and under the conditions above, $\dot{\gamma_z} > 0$ because the differntial is negative from Equation (10). Then in this case $\gamma_z > 0$ for ever. But this contradicts the existance of a steady state and therefore $\gamma_z < 0$. Then we have that if σ is such that expression (39) holds, $\gamma_z = \gamma_{\hat{c}_m} - \gamma_{\hat{y}_m} < 0$. But we showed that $\gamma_{\hat{c}_m} > 0$, so $\gamma_{\hat{y}_m} > 0$.

Concavity of $\hat{k}$

The growth rate of $\hat{k}$ is $\gamma_{\hat{k}} = \bar{\gamma}_{\hat{k}}/\mu_m$, where

$$\bar{\gamma}_{\hat{k}} = \frac{\hat{y}}{\hat{k}} - \frac{\hat{\epsilon}}{\hat{k}} - (\bar{\mu}_m + \delta)$$

As entrpreneurs do not save this is equivalent to,

$$\bar{\gamma}_{\hat{k}} = \frac{(1-\omega)\hat{y}}{\hat{k}} - \frac{\hat{\epsilon}^c}{\hat{k}} - (\bar{\mu}_m + \delta)$$

To prove that the growth rate of $\hat{k}$ decreases during the transition it is enough to show that $d(\bar{\gamma}_{\hat{k}}) < 0$. Differentiating the expression above we get

$$d(\bar{\gamma}_{\hat{k}}) = (1-\omega)\frac{\partial \hat{y}/\partial \hat{k}}{\hat{k}} d\hat{k} + (1-\omega)\frac{\partial \hat{y}/\partial \hat{p}}{\hat{k}} d\hat{p} - \frac{(1-\omega)\hat{y}}{\hat{k}^2} d\hat{k} - d\left(\frac{\hat{\epsilon}^c}{\hat{k}}\right)$$

Rearenging terms and using Equation (11) this is equivalent to

$$d(\bar{\gamma}_{\hat{k}}) = -\frac{\hat{w}n}{\hat{k}}\gamma_{\hat{k}} + (1-\omega)\frac{\partial\hat{y}/\partial\hat{p}}{\hat{k}}d\hat{p} - \frac{\hat{\epsilon}^c}{\hat{k}}\left[\frac{1}{\sigma}\left(r - \bar{\beta} - \bar{\mu}_m + \eta(\sigma-1)\frac{d\hat{p}}{\hat{p}}\right) - \frac{1}{\mu_m\hat{k}}\left((1-\omega)\hat{y} - \epsilon^c - \hat{k}(\bar{\mu}_m + \delta)\right)\right]$$

or,

$$d(\bar{\gamma}_{\hat{k}}) = -\frac{\hat{w}n}{\hat{k}}\gamma_{\hat{k}} + (1-\omega)\frac{\partial\hat{y}/\partial\hat{p}}{\hat{k}}d\hat{p} + \frac{\hat{\epsilon}^c}{\hat{k}}\left[(r-\bar{\mu}_m)\left(\frac{1}{\mu_m} - \frac{1}{\sigma}\right) + \frac{\bar{\beta}}{\sigma} - \frac{\eta(\sigma-1)}{\sigma}\frac{d\hat{p}}{\hat{p}} + \frac{\hat{w}n - \hat{\epsilon}^c}{\mu_m\hat{k}}\right]$$

Take the direct effect of prices trough the supply side and the demand side,

$$H = \frac{d\hat{p}}{\hat{k}}\left[(1-\omega)\partial\hat{y}/\partial\hat{p} - \frac{\hat{\epsilon}^c}{\hat{p}}\frac{\eta(\sigma-1)}{\sigma}\right]$$

The last term in the square brakets is $\partial\hat{\epsilon}^c/\partial\hat{p}$, the change in total demand due to a change in prices. Given that the agricultural good is only used for consumption, and using the market clearing condition for that sector and the fact that entrepreneurs do not save, we get

$$H = \frac{d\hat{p}}{\hat{k}}\left[(1-\omega)\partial\hat{y}_m/\partial\hat{p} - \partial\hat{c}_m/\partial\hat{p}\right]$$

The first term in the square brakets is negative from Lemma 1. The second term is equal to $(1-\eta)\partial\hat{\epsilon}^c/\partial\hat{p}$, which is negative if $\sigma > 1$. Then, assuming the latter inequality we have that $H < 0$, and hence

$$d(\bar{\gamma}_{\hat{k}}) < -\frac{\hat{w}n}{\hat{k}}\gamma_{\hat{k}} + \frac{\hat{\epsilon}^c}{\hat{k}}\left[(r-\bar{\mu}_m)\left(\frac{1}{\mu_m} - \frac{1}{\sigma}\right) + \frac{\bar{\beta}}{\sigma} + \frac{\hat{w}n - \hat{\epsilon}^c}{\mu_m\hat{k}}\right] \equiv -\frac{\hat{w}n}{\hat{k}}\gamma_{\hat{k}} + \frac{\hat{\epsilon}^c}{\hat{k}}\left[A + \frac{\hat{w}n - \hat{\epsilon}^c}{\mu_m\hat{k}}\right]$$

The only case when $d(\bar{\gamma}_{\hat{k}})$ can be positive is when the term inside the square brakets its positive. Then the rest of the proof asumes that this is the case, and so

$$\frac{\hat{\epsilon}^c}{\mu_m\hat{k}} < \frac{\hat{w}n}{\mu_m\hat{k}} + A$$

But given this inequality we get

$$d(\bar{\gamma}_{\hat{k}}) < -\frac{\hat{w}n}{\mu_m\hat{k}}\left[(r-\bar{\mu}_m) - A\right] + A^2 + \frac{A\left(\hat{w}n - \hat{\epsilon}^c\right)}{\mu_m\hat{k}}$$

Notice that the term inside the first square brakets is negative, so, given the inequality above, only if $A > 0$ the RHS can be positive. Then assume $A > 0$ for the rest of the proof.

Suppose for now that the following is true,

$$\frac{\hat{\epsilon}^c}{\mu_m\hat{k}} > \left[1 + \frac{\hat{w}n}{\hat{k}(r-\bar{\mu}_m)}\right]A \tag{41}$$

then, knowing that $\mu_m \geq 1$ and $A > 0$,

$$d(\bar{\gamma}_{\hat{k}}) < -\frac{\hat{w}n}{\mu_m\hat{k}}\left[(r-\bar{\mu}_m) - A\right] + A^2 + \frac{A\hat{w}n}{\mu_m\hat{k}} - \left[1 + \frac{\hat{w}n}{\mu_m\hat{k}(r-\bar{\mu}_m)}\right]A^2$$

and it takes just algebra to show that

$$d(\bar{\gamma}_{\hat{k}}) < -\frac{\hat{w}n}{\mu_m\hat{k}(r-\bar{\mu}_m)}\left[(r-\bar{\mu}_m) - A\right] < 0$$

so capital grows at a decreasing rate during the transition.

Then the last step is to show that expression (41) holds. First, adding the consumers' budget constraints and using the growth rate of consumption expenditures (Equation 10) we get, for any period τ,

$$\hat{\epsilon}^c_\tau = \frac{(1+r_\tau)\sum_{t=\tau}^{\infty} \frac{r_t \hat{k}_\tau + \mu_m^{(t-\tau)} n\hat{w}_t}{\prod_{s=\tau}^{t}(1+r_s)}}{\sum_{t=\tau}^{\infty} \beta^{(t-\tau)/\sigma} \left(\frac{\mu_m}{\mu_a}\right)^{(t-\tau)(1-\alpha_a)\eta(1-1/\sigma)} \left(\frac{\hat{p}_t}{\hat{p}_\tau}\right)^{\eta(1-1/\sigma)} \frac{\prod_{s=\tau}^{t}(1+r_s)^{(1/\sigma-1)}}{(1+r_\tau)^{(1/\sigma-1)}}}$$

It is easy to see that $\hat{\epsilon}^c_\tau$ is decreasing on r_{τ^*}, for any $\tau^* > \tau$, if $\sigma \leq 1$. If $\sigma > 1$ the denominator decreases with r_{τ^*} so it is not clear if in that case $\hat{\epsilon}^c_\tau$ is increasing or decreasing on r_{τ^*}. To see that it is decreasing take the case when $\sigma \to \infty$, which is the case when the effect of r_{τ^*} on the denominator is the largest. In that case we have

$$\hat{\epsilon}^c_{\tau\sigma\to\infty} = \frac{(1+r_t)\sum_{t=\tau}^{\infty} \hat{p}_t^{\eta} \frac{\left(\frac{r_\tau}{\hat{p}_t^{\eta}}\hat{k}_\tau + \mu_m^{(t-\tau)}\frac{n\hat{w}_t}{\hat{p}_t^{\eta}}\right)}{\prod_{s=\tau}^{t}(1+r_s)}}{\sum_{t=\tau}^{\infty} \left(\frac{\mu_a}{\mu_m}\right)^{(t-\tau)(1-\alpha_a)\eta} \left(\frac{\hat{p}_t}{\hat{p}_\tau}\right)^{\eta(1-1/\sigma)} \frac{\prod_{s=\tau}^{t}(1+r_s)}{(1+r_\tau)}}$$

First notice that $d(\frac{r_t}{\hat{p}_t^{\eta}}\hat{k}_\tau + \mu_m^{(t-\tau)}\frac{n\hat{w}_t}{\hat{p}_t^{\eta}}) = d(\frac{r_t}{\hat{p}_t^{\eta}})(\hat{k}_\tau - \hat{k}_t) > 0$. This follows from Lemma RSS and the monotonicity of capital. Then, given the transversality condition, $\beta\gamma^{1-\sigma} < 1$, and the monotonicity of the relative price, the numerator grows faster than the denominator. It follows that $\hat{\epsilon}^c_\tau$ is decreasing on r_{τ^*}. We can replace the former and the term inside the parenthesis by their initial levels and get,

$$\hat{\epsilon}^c_\tau > \frac{\sum_{t=\tau}^{\infty} \left(\frac{\hat{p}_t}{\hat{p}_\tau}\right)^{\eta} \frac{\left(r_\tau \hat{k}_\tau + \mu_m^{(t-\tau)} n\hat{w}_\tau\right)}{(1+r_\tau)^{(t-\tau)}}}{\sum_{t=\tau}^{\infty}(\beta\gamma^{1-\sigma})^{(t-\tau)/\sigma}\mu_m^{(t-\tau)(1-1/\sigma)} \left(\frac{\hat{p}_t}{\hat{p}_\tau}\right)^{\eta(1-1/\sigma)} (1+r_\tau)^{(t-\tau)(1/\sigma-1)}}$$

Using the same argument above to show that the RHS was decreasing on r_{τ^*}, it is easy to show that the RHS now is increasing on $\hat{p}_{\tau^*}$, for some $\tau^* > \tau$. Then we get,

$$\begin{aligned}
\hat{\epsilon}^c_\tau \quad &> \frac{\sum_{t=\tau}^{\infty} \frac{\left(r_\tau \hat{k}_\tau + \mu_m^{(t-\tau)} n\hat{w}_\tau\right)}{(1+r_\tau)^{(t-\tau)}}}{\sum_{t=\tau}^{\infty}(\beta\gamma^{1-\sigma})^{(t-\tau)/\sigma}\mu_m^{(t-\tau)(1-1/\sigma)}(1+r_\tau)^{(t-\tau)(1/\sigma-1)}} \\
&= (1+r_\tau)\left[\hat{k}_\tau + \frac{n\hat{w}_\tau}{(r_\tau - \bar{\mu}_m)}\right]\left(1 - (\beta\gamma^{1-\sigma})^{1/\sigma}(1+r_\tau)^{1/\sigma-1}\mu_m^{1-1/\sigma}\right)
\end{aligned}$$

Then, provided that $1 + r > \mu_m$,

$$(\beta\gamma^{1-\sigma})^{1/\sigma}(1+r_\tau)^{1/\sigma-1}\mu_m^{1-1/\sigma} > 1 - \frac{\frac{\hat{\epsilon}^c_\tau}{\mu_m \hat{k}_\tau}}{\left[1 + \frac{n\hat{w}_\tau}{\hat{k}_\tau(r_\tau - \bar{\mu}_m)}\right]}$$

Inequality (41) follows after taking logs and noticing that $\mu_m \geq 1$.

Concavity of $\hat{p}, \hat{w}, \hat{w}/\hat{p}, r, r/\hat{p}$

For the relative price take first the case when rents are zero ($\omega = 0$). Equation (13) can be written as

$$\gamma_{\hat{\epsilon}} = \gamma_{\hat{p}}\left(1 + \frac{\partial log\hat{y}_a}{\partial log\hat{p}}\right) + \gamma_{\hat{k}}\left(\frac{\partial log\hat{y}_a}{\partial log\hat{k}}\right) \tag{42}$$

From Lemma 1 we know that,

$$\frac{\partial log\hat{y}_a}{\partial log\hat{k}} = -\frac{(1-\alpha_m)\alpha_a\hat{k}}{(\alpha_m - \alpha_a)\hat{k}_a} < 0$$

$$\frac{\partial log\hat{y}_a}{\partial log\hat{p}} = -\frac{1}{(\alpha_m - \alpha_a)(1-\alpha_m)}\frac{\partial log\hat{y}_a}{\partial log\hat{k}} - \frac{\alpha_a(\alpha_a\hat{k}_a + \alpha_m\hat{k}_m)}{\hat{k}_a(\alpha_m - \alpha_a)^2} > 0$$

replacing these two expressions above we get

$$d(\gamma_{\hat{p}})\left(1+\frac{\partial log\hat{y}_a}{\partial log\hat{p}}\right)=d(\gamma_{\hat{\epsilon}})-d(\gamma_{\hat{k}})\left(\frac{\partial log\hat{y}_a}{\partial log\hat{k}}\right)+\underbrace{\left(\frac{\gamma_{\hat{p}}}{(\alpha_m-\alpha_a)(1-\alpha_m)}-\gamma_{\hat{k}}\right)d\left(\frac{\partial log\hat{y}_a}{\partial log\hat{k}}\right)+\gamma_{\hat{p}}d\left(\frac{\alpha_a(\alpha_a\hat{k}_a+\alpha_m\hat{k}_m)}{\hat{k}_a(\alpha_m-\alpha_a)^2}\right)}_{H}$$

The term inside the parenthesis on the LHS is positive, while the first two terms on the RHS are negative. Hence, to get the desired result, it is suficient to prove that the rest of the expression on the RHS is negative ($H<0$). After some algebra we get

$$H=\frac{(1-\alpha_m)\alpha_m\hat{y}_m}{(\alpha_m-\alpha_a)\hat{p}\hat{y}_a}\gamma_z\left(\frac{\gamma_{\hat{p}}}{(\alpha_m-\alpha_a)}-\gamma_{\hat{k}}\right)$$

where $\gamma_z<0$ was defined above. Using equation (42) to substitute for $\gamma_{\hat{p}}$ and more algebra, we get

$$H=\frac{(1-\alpha_m)\alpha_m\hat{y}_m}{(\alpha_m-\alpha_a)^2\hat{p}\hat{y}_a}\gamma_z\left(\gamma_{\hat{\epsilon}}-\alpha_m\gamma_{\hat{k}}\right)$$

Therefore it is enough to show that $\bar{H}=\gamma_{\hat{k}}-\gamma_{\hat{\epsilon}}<0$ to get the result. Notice that

$$\bar{H}=\frac{1}{\mu_m}\left[\frac{\hat{y}}{\hat{k}}-\frac{\hat{\epsilon}}{\hat{k}}-(\bar{\mu}_m+\delta)\right]-\frac{1}{\sigma}\left(r-\bar{\mu}_m-\bar{\beta}+\eta(\sigma-1)\gamma_{\hat{p}}\right)$$

rearrenging terms, and using A defined in the proof of the concavity of $\hat{k}$,

$$\bar{H}=A-\frac{\eta(\sigma-1)}{\sigma}\gamma_{\hat{p}}+\frac{wn-\hat{\epsilon}}{\mu_m\hat{k}}$$

But inequality (41) implies

$$\frac{\hat{\epsilon}}{\mu_m\hat{k}}>A+\frac{\hat{w}n}{\hat{k}(r-\bar{\mu}_m)}A>A+\frac{\hat{w}n}{\mu_m\hat{k}}\left(1-\frac{\mu_m}{\sigma}\right)$$

Therefore $\bar{H}<0$, and so $d(\gamma_{\hat{p}})<0$, if $\sigma>\mu_m$, because $\gamma_{\hat{p}}>0$.

For wages notice first that the elasticity with respect to capital is zero with CRS. Then,

$$d(\gamma_{\frac{\hat{w}}{\hat{p}}})=d(\gamma_{\hat{p}})\left(\frac{\partial log\hat{w}/\hat{p}}{\partial log\hat{k}}\right)+\gamma_{\hat{p}}d\left(\frac{\partial log\hat{w}/\hat{p}}{\partial log\hat{k}}\right)$$

The first term on the RHS is negative because the elasticity is positive from Lemma 1. But the last term is zero because the elasticity is constant:

$$d\left(\frac{\partial log\hat{w}/\hat{p}}{\partial log\hat{k}}\right)=d\left(\frac{\alpha_m(n_a\hat{k}_m-n_m\hat{k}_a)}{(\alpha_m-\alpha_a)^2n_m\hat{k}_a/(1-\alpha_m)\alpha_a}-1\right)=\frac{(1-\alpha_m)\alpha_a\alpha_m}{(\alpha_m-\alpha_a)^2}d\left(\frac{n_a\hat{k}_m}{n_m\hat{k}_a}\right)=0$$

where the last result follows from the fact that $n_a\hat{k}_m/n_m\hat{k}_a=(1-\alpha_a)\alpha_m/(1-\alpha_m)\alpha_a$. Hence $d(\gamma_{\frac{\hat{w}}{\hat{p}}})<0$ and, because $d(\gamma_{\hat{p}})<0$, $\gamma_{\hat{w}}<0$.

For the interest rate the proof is very similar.

$$d(\gamma_r)=d(\gamma_{\hat{p}})\left(\frac{\partial logr}{\partial log\hat{k}}\right)+\gamma_{\hat{p}}d\left(\frac{\partial logr}{\partial log\hat{k}}\right)$$

Now the first term is positive and, again, the last term is zero. Therefore $d(\gamma_r)>0$, and so $d(\gamma_{\frac{r}{\hat{p}}})>0$.

Concavity of $\hat{c}_m, \hat{p}\hat{c}_a, \hat{p}\hat{y}_a, \hat{y}_m$

Consumption of both goods and agricultural output, in terms of manufacturing goods, grow at the same rate than total expenditures (ϵ). Then the result follows from the fact that $d(\gamma_\epsilon) < 0$. Given that $d(\gamma_{\epsilon/\hat{p}^\eta}) < 0$ as well, the result for manufacturing output in terms of flow utility, $\hat{p}^{1-\eta}\hat{y}_a$ follows as well.

For manufacturing output, first notice that there is a value of the elasticity of substitution, $\bar{\sigma}$ such that s is constant during the transition. In that case it is clear that $\gamma_{\hat{y}_m} = \gamma_{\hat{c}_m} = \gamma_{\hat{p}\hat{c}_a} > 0$, so $\sigma^* < \bar{\sigma}$, and $d(\gamma_{\hat{y}_m}) < 0$ and $d(\gamma_{\hat{y}_m/\hat{p}^\eta}) < 0$. Finally, by continuity of Equation (40) on σ, $\exists \sigma^{**}$, where $\sigma^* < \bar{\sigma} < \bar{\sigma}^{**}$ such that $\forall \sigma < \sigma^{**}$, $d(\gamma_{\hat{y}_m}) < 0$ and $d(\gamma_{\hat{y}_m/\hat{p}^\eta}) < 0$.

QED.

Proof of Proposition 3

It is enough to show that $IC_j^{\mathrm{PE}}(\hat{k})$ is monotonically decreasing on $\hat{k}$, and therefore it is sufficient to show that $\tilde{V}_j(\hat{k}', \hat{k})$ is monotonically decreasing, and $\tilde{V}_j^d(\hat{k}', \hat{k})$ is monotonically increasing, on $\hat{k}$.

First define the sequences $\{g_t\}_{t=0}^\infty$ and $\{V_t\}_{t=0}^\infty$, where $\forall t$,

$$V_t = \sum_{s=t}^{\infty} \lambda^{s-t} \left[\prod_{h=t}^{s} g_h \right]$$

Suppose the first sequence is monotone. We are interested to know under what properties for the first sequence the second sequence is monotonically increasing or decreasing. Notice that

$$V_{t+1} - V_t = V_{t+1}(1 - \lambda g_t) - g_t$$

Now if $\{g_t\}_{t=0}^\infty$ is strictly increasing then

$$V_{t+1} > \frac{g_t}{1 - \lambda g_t}$$

It follows that

$$V_{t+1} - V_t = V_{t+1}(1 - \lambda g_t) - g_t > 0$$

so the second sequence is also strictly increasing.

Alternatively, if $\{g_t\}_{t=0}^\infty$ is strictly decreasing then

$$V_{t+1} < \frac{g_t}{1 - \lambda g_t}$$

It follows that

$$V_{t+1} - V_t = V_{t+1}(1 - \lambda g_t) - g_t < 0$$

so the second sequence is also strictly decreasing.

Now define

$$g_t = g(\hat{k})_t^{\hat{y}} = \frac{\hat{p}_t^\eta (\hat{p}_{jt+1}\hat{y}_{jt+1})}{\hat{p}_{t+1}^\eta (\hat{p}_{jt}\hat{y}_{jt})}$$

and $\lambda = \beta\gamma$, then $V_t = \frac{\tilde{V}_j(\hat{k}', \hat{k})}{\omega\beta\gamma}$.
Also define

$$g_t = g(\hat{k})_t^{\hat{w}/\hat{p}_j} = \frac{\hat{p}^\eta \hat{w}'/\hat{p}_j'}{\hat{p}'^\eta \hat{w}/\hat{p}_j}$$

and $\lambda = \beta\bar{\gamma}$, then $V_t = \frac{\tilde{V}_j(\hat{k}', \hat{k})}{\rho\beta\bar{\gamma}}$.

Hence it is enough to show that $g(\hat{k})_t^{\hat{y}}$ is strictly decreasing on $\hat{k}$, and that $g(\hat{k})_t^{\hat{w}/\hat{p}_j}$ is strictly increasing on $\hat{k}$. But this follows directly from Proposition 1.

QED.

Proof of Proposition 4

Take the limit of $IC_j^{\mathrm{PE}}(\hat{k})$ when $\hat{k}$ goes to zero. The only term that does not converge to zero, independently of ρ, is $\tilde{V}_j(\hat{k}', \hat{k})$, which, given that $\hat{k}^{\mathrm{SS}} > 0$ and therefore $d\hat{k} > 0$ when $\hat{k} \to 0$, is positive, again, independently of ρ. It follows that

$$\forall \rho \in [0,1]\,, \quad \lim_{\hat{k} \to 0} IC_j^{\mathrm{PE}}(\hat{k}) > 0$$

and that, by continuity, $\forall \rho \in [0,1]$, $\exists \hat{k}^{*\mathrm{PE}} > 0$ such that $\forall \hat{k} < \hat{k}^{*\mathrm{PE}},\ IC_j^{\mathrm{PE}}(\hat{k}) > 0$, for $j = a, m$..

QED.

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