Incentives and the Limits to Deflationary Policy*

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Abstract

I study a version of the Lagos-Wright (2005) model for which the Friedman rule is always a desirable policy, but where implementation may be constrained by the need to respect incentive-feasibility. In the environment I consider, incentives are distorted owing to private information and limited commitment. I demonstrate that a monetary economy can overcome the former friction, but not necessarily the latter. When this is so, there is an incentive-induced lower bound to the rate of deflation away from the Friedman rule. There are also circumstances in which the best incentive-feasible monetary policy may entail a strictly positive rate of inflation. This will be the case, for example, if agents are sufficiently impatient or if there are rapidly diminishing returns to production.

1 Introduction

The Friedman rule asserts that the optimal monetary policy is contractionary; so that a deflationary monetary policy is desirable. This policy prescription is surprisingly robust across a wide class of models, including some that purport to model money seriously. Of course, monetary policy in practice is almost always expansionary. The discrepancy between theory and practice in this regard constitutes somewhat of a puzzle.

One resolution to this puzzle can be found in a recent body of work that studies monetary theory in the context of environments where money is essential. While the Friedman rule can arise as an optimum even here, researchers have identified several cases in which it may not. One such class of models relies on environments where some form of redistribution is desirable; see, for example, Levine (1991), Deviatov and Wallace (2001), Molico (2006), Williamson (2006), and Waller (2006). In another class of models, an expansionary monetary policy can serve to mitigate the distortions induced by search and bargaining frictions;

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see, for example, Shi (1997), Lagos and Rocheteau (2005), and Rocheteau and Wright (2005).

In the literature cited above, the Friedman rule is a feasible policy, but departures from this rule are desirable for one reason or another. In this paper, I suggest that the opposite may true; namely, that the Friedman rule is a desirable policy, but that implementation of this policy may be infeasible.

I consider a model of intertemporal trade based on the analytically tractable quasi-linear environment introduced by Lagos and Wright (2005), but with search frictions absent.[1] When agents lack commitment and idiosyncratic shocks are private information, memory (a public access database containing the full set of individual trading histories and reports) is essential for efficient implementation. When memory is available, the economy functions as a credit system, where purchases and sales of goods are debited and credited with book-entry objects in a centralized system of accounts. If the punishment for default is not overly severe, the constrained-efficient allocation may not correspond to what is achievable in a world with full commitment and public information.

Absent a record-keeping technology, a tangible fiat money instrument is essential for trade. When trade is organized as a system of competitive spot markets and when nominal lump-sum taxes and transfers are feasible, the most desirable monetary policy in this environment is to deflate at the Friedman rule. In doing so, the monetary authority essentially replicates the system of punishments and rewards that would be put in place by an optimal mechanism. I demonstrate, however, that the same frictions that render money (memory) essential in the first place, may at the same time render the Friedman rule infeasible. When this is so, there is an incentive-induced lower bound to the rate of deflation away from the Friedman rule. There are also circumstances in which the best incentive-feasible monetary policy may entail a strictly positive rate of inflation.

2 The Physical Environment

The economy is populated by a mass of *ex ante* identical agents, distributed uniformly on the unit interval. Each period $t = 0, 1, 2, ..., \infty$ is divided into two subperiods which, for convenience, are labeled 'day' and 'night.' All agents have an opportunity to produce or consume during the day; label this good x_t, where $x_t < 0$ denotes production and $x_t > 0$ denotes consumption. At night, agents have an equal probability of realizing either an opportunity to produce or a desire to consume. Let $u(c_t)$ denote the flow utility of consumption and $-g(y_t)$ the flow disutility of production.

[A1] The functions $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are continuously differen-

[1] The Lagos-Wright model itself builds on a long string of developments in monetary theory, beginning with Kiyotaki and Wright (1989).

tiable, with

$$u'' < 0 < u',\ u(0) = 0;\ g', g'' \geq 0,\ g(0) = 0;\ \text{and } \beta u'(0) > g'(0);$$

for some $0 < \beta < 1$.

Preferences over $\{x_t(i), c_t(i), y_t(i)\}_{t=0}^{\infty}$ for agent $i \in [0,1]$ are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \frac{1}{2} \left[u(c_t(i)) - g(y_t(i)) \right] \right\}.$$

As all goods are nonstorable, the economy-wide resource constraints are given by:

$$\begin{aligned} \int x_t(i) di &= 0; \\ \int c_t(i) di &= \int y_t(i) di; \end{aligned}$$

for all $t \geq 0$.

Weighting all agents equally, the planning problem reduces to choosing a y that maximizes *ex ante* utility:

$$W(y) = \left(\frac{1}{1-\beta} \right) \left(\frac{1}{2} \right) \left[u(y) - g(y) \right]. \tag{1}$$

Clearly, the function $W(y)$ is continuously differentiable and strictly concave, with $W(0) = W(\overline{y}) = 0$ for some unique $0 < \overline{y} < \infty$; and achieves a unique maximum $0 < y^* < \overline{y}$ satisfying $u'(y^*) = g'(y^*)$.

That is, the planner assigns $c_t^*(i) = y^*$ and $y_t^*(i) = 0$ if i is a consumer during the night; and $c_t^*(i) = 0$ and $y_t^*(i) = y^*$ if i is a producer during the night. Observe that as $x_t(i)$ enters linearly in preferences, any lottery over $\{x_t(i) : t \geq 0\}$ that satisfies $E_0 x_t^*(i) = 0$ would satisfy the resource constraint and entail no *ex ante* welfare loss. Because it will be relevant in what follows, I am interested in lotteries of the following form. Let $x_0^*(i) = 0$ for all i; with $x_t^*(i) = x^* \geq 0$ if agent i produced at $t-1$ and $x_t^*(i) = -x^*$ if agent i consumed at $t-1$. Of course, in the current context, one may without loss set $x^* = 0$. In what follows, I refer to (x^*, y^*) as the *first-best* allocation.

Note that the pattern of trades that supports the optimal allocation entails some form of credit arrangement. That is, an agent who enters the night period with a desire for consumption can offer nothing in exchange for the output he acquires, except a promise that he will reciprocate at some future date when the opportunity presents itself. Likewise, a producer must somehow be willing to extend consumers credit in this form. Of course, if agents could perfectly commit to honor their debt obligations, and if agent types (whether consumer or producer) are public information, then the standard welfare theorems apply.

3 Essential Memory

The term *memory* here refers to a public-access record-keeping technology that keeps track of all individual trading histories; see Kocherlakota (1998). To make memory essential, a 'trading friction' in some form must be present. The frictions most commonly employed for this purpose are limited commitment and/or private information.

Assume that there is a complete lack of commitment and that an agent's type is private information. I assume, however, that agents are identifiable in the sense that their trading (and reporting) histories are common knowledge.

Given the quasi-linear nature of preferences, I can appeal to a quasi-linear mechanism (with strong budget balance) and focus on (stationary) feasible allocations of the form (x, y), where:

$$x_t(i) = \begin{cases} +x & \text{if} \quad (c_{t-1}(i), y_{t-1}(i)) = (0, y); \\ -x & \text{if} \quad (c_{t-1}(i), y_{t-1}(i)) = (y, 0); \end{cases}$$

with $x_0(i) = 0$ for all i. Any such allocation generates the welfare function (1); in particular, note that $E_t x_t(i) = 0$.[2]

The lack of commitment implies that an allocation (x, y) must respect *ex post* rationality. If the punishment for noncompliance is autarky, then the allocation (x, y) must satisfy the following set of (sequential) participation constraints (PC):

$$\begin{aligned} -g(y) + \beta\left[x + W(y)\right] &\geq 0; \quad (2) \\ -x + W(y) &\geq 0. \quad (3) \end{aligned}$$

One might note that as each agent is of measure zero, the banishment of any countable number of agents has no measurable impact on aggregates (so that the threat of banishment is legitimately credible).

Private information requires that the allocation (x, y) respects incentive-compatibility (IC); in particular, agents cannot have an incentive to misrepresent their type in the night stage. Note that the way things are set up here, it is impossible for a night-consumer to misrepresent his type, as he does not have the technology to produce. It is possible, however, for a night-producer to report himself as a consumer (he derives zero utility from consumption, but economizes on the utility cost of production). Hence, an incentive-compatible allocation must satisfy:

$$-g(y) + \beta\left[x + W(y)\right] \geq \beta\left[-x + W(y)\right]. \quad (4)$$

[2] Technically, I need to assume that implementation is not hindered by any upper bound on x. This assumption is maintained throughout the paper.

Lemma 1 *If an allocation* (x, y) *satisfies (3) and (4), then (2) is necessarily satisfied.*

Proof. The proof follows immediately from the stated conditions (2), (3), and (4). ■

The implication of lemma 1 is that I can restrict attention to allocations that satisfy PC for the day-producer and IC for the night-producer. These two restrictions can be stated more compactly as follows:

$$\left(\frac{1}{1-\beta}\right)\left(\frac{1}{2}\right)[u(y) - g(y)] \geq x \geq \left(\frac{1}{\beta}\right)\left(\frac{1}{2}\right)g(y). \tag{5}$$

Definition 1 An allocation (x, y) is said to be *incentive-feasible* iff (x, y) satisfies condition (5).

Let $S \subset \mathbb{R}_+^2$ denote the set of incentive-feasible allocations.

Lemma 2 *Given* **[A1]**, *the set* S *is non-empty, convex, and compact. Moreover,* $\exists\ y \in S$ *such that* $y > 0$.

Proof. Lemma 2 is easily established by noting the following facts. First, the upper contour set $x \geq (2\beta)^{-1}g(y)$ is convex owing to the convexity of g. Second, the lower contour set $x \leq W(y)$ is convex owing to the concavity of W. Hence, the intersection of these two sets, S, is convex. As u and g are continuous, S is closed. The strict concavity of W and the convexity of g imply that S is bounded. The set is non-empty as $(x, y) = (0, 0) \in S$. The existence of an incentive-feasible and non-autarkic allocation $y > 0$ is guaranteed by the condition that $u'(0)$ is sufficiently large in the sense that $\beta u'(0) > g'(0)$; see **[A1]**. ■

It follows as a corollary that the problem of choosing (x, y) to maximize $W(y)$ subject to $(x, y) \in S$ is well-defined. Furthermore, as $W(y)$ is strictly concave, there is a unique solution $y > 0$. Associated with this solution is an x (not necessarily unique) satisfying (5). The exact nature of the solution depends on parameters; and in particular, on the discount factor β. The following proposition asserts that the first-best allocation is implementable when agents are sufficiently patient.

Proposition 1 $(x^*, y^*) \in S$ *for any* $\beta \in [\beta_0, 1)$; *with* $0 < \beta_0 < 1$ *defined by* $\beta_0 \equiv g(y^*)/u(y^*)$.

Proof. By condition (5), an incentive-feasible allocation must satisfy:

$$\left(\frac{1}{1-\beta}\right)\left(\frac{1}{2}\right)[u(y) - g(y)] \geq \left(\frac{1}{\beta}\right)\left(\frac{1}{2}\right)g(y);$$

or, with some manipulation,

$$\beta u(y) \geq g(y).$$

As $u(y^*) > g(y^*) > 0$, one can find some $0 < \beta_0 < 1$ for which $\beta_0 u(y^*) = g(y^*)$; so that $\beta u(y^*) > g(y^*)$ for all $\beta \in [\beta_0, 1)$. Associated with y^* is any x^* satisfying (5); i.e.,

$$\left(\frac{1}{1-\beta}\right)\left(\frac{1}{2}\right)[u(y^*) - g(y^*)] \geq x^* \geq \left(\frac{1}{\beta}\right)\left(\frac{1}{2}\right) g(y^*).$$

■

It follows as a corollary that the first-best allocation is not implementable when $\beta \in (0, \beta_0)$. In this latter case, it turns out that both PC and IC bind tightly.[3]

Proposition 2 *If $\beta \in (0, \beta_0)$, then the constrained-efficient allocation (x_0, y_0) is characterized by a $0 < y_0 < y^*$ satisfying $\beta u(y_0) = g(y_0)$ with $x_0 = W(y_0)$.*

Proof. By Proposition 1, $\beta u(y^*) < g(y^*)$ for all $\beta \in (0, \beta_0)$. As $W(y)$ is strictly increasing over the range $[0, y^*)$, one can recast the choice problem here as maximizing y subject to $\beta u(y) \geq g(y)$. Given **[A1]**, the solution clearly entails a $0 < y_0 < y^*$ satisfying $\beta u(y_0) = g(y_0)$. The associated x_0 can be obtained residually from (5); i.e., $x_0 = W(y_0)$. ■

Hence, we arrive at the familiar result that better things are possible for economies beset with trading frictions as patience is increased. Of course, the result that the first-best is potentially implementable for a range of discount factors not arbitrarily close to unity relies heavily on quasi-linearity. But whether the first-best is implementable or not, the key point here is that memory is essential.

4 Essential Money

What I mean here by 'money' is a 'tangible' and non-counterfeitable fiat object. To consider a world where fiat money is valued, we need to think of an environment where 'intangible' memory is essential but prohibitively costly. One way to accomplish this is to assume away any mechanism capable of observing and recording individual trading histories (including reports of individual trading histories). In this case, the only feasible trades will involve *quid-pro-quo* exchanges of goods for some other object that does not correspond to any private obligation. In the economy studied earlier, this other object took the form of intangible memory (a book-entry object). In the absence of memory, a tangible

[3] Diagrammatic representations are provided in Figures 1 and 2.

object like fiat money can serve as a substitute for the missing memory object; again, see Kocherlakota (1998).

Assume that fiat money is perfectly durable and divisible. Following a version of the Lagos-Wright model studied in Rocheteau and Wright (2005), assume that trade is organized as a sequence of competitive spot markets (with zero entry costs). Let (v_d, v_n) denote the values of money prevailing in the day and night markets, respectively (where value is measured in units of output).

4.1 Money Supply

Let M^- denote the total stock of fiat money at the beginning of the day-market (prior to any injection or withdrawal). Assume that this stock expands at the gross rate μ so that:

$$M = \mu M^-, \tag{6}$$

where M denotes the 'next' period's money supply. Assume that the initial money stock is distributed evenly across the population. New money $(\mu - 1)M^-$ is injected (or withdrawn) by way of a lump-sum transfer (or tax):

$$\tau = (\mu - 1)M^-. \tag{7}$$

Assume that this transfer (or tax) is applied at the beginning of each day-market and that $\mu \geq \beta$ (it can be shown the equilibria do not exist for $\mu < \beta$).

Before proceeding, I have to deal with the question of exactly how these transfers or taxes are supposed to occur in world where agents are essentially anonymous. The approach I take here is to assume that there is a mechanism, suitably interpreted here as a government or central bank, that has some limited powers of identification and coercion. In particular, assume that the mechanism can identify which agents have received a contemporaneous transfer or paid a contemporaneous tax.[4] Note that this assumption does not violate the idea that personal trading histories or current money holdings are unobservable by the mechanism. Second, since I want to consider a world where deflation is at least physical possibility, assume that the mechanism can observe money holdings and extract money from those with positive balances in the day-market.[5] Together, these assumptions imply that *nominal* lump-sum taxation is feasible, but that payment is to some extent voluntary; i.e., agents can avoid the tax by refusing

[4] This assumption is necessary, for example, to prevent an agent who has received a transfer to claim otherwise.

[5] Alternatively, if the government cannot observe money holdings, assume that it has the power to exclude agents from the night market (so that the mechanism can play the role of a 'gatekeeper'). One analogy, suggested to me by Chris Waller, is the role played by a bouncer at a nightclub. As far as the bouncer is concerned, each agent arriving at the door is anonymous. But agents can be identified in terms of whether or not the (voluntary) cover charge has been paid. Once this fee has been collected, agents pass through the door and melt away into an anonymous population.

to accumulate money balances.[6]

Because the issue involved may be subtle, let me repeat things in another way. I do not want to simply assume that agents can escape paying a nominal tax with impunity. The consequence of such an assumption would be to render taxes (and hence, deflation) infeasible. While this would obviously serve to place a lower bound on deflation, I want to consider cases where agents may voluntarily pay a tax, so that some deflation may be possible.

The strategy that I adopt in this endeavor is, for the remainder of this section, to assume that agents will voluntarily pay any lump-sum tax $\tau = (\mu - 1)M^{-}$ for any $\mu \geq \beta$. In this case, the optimal monetary policy will be the Friedman rule ($\mu = \beta$). In a sequel, I then use the analysis developed earlier to determine under whether the Friedman rule is incentive-feasible; and if it is not, then determine the lower bound on deflation implied by incentive-feasibility.

4.2 Day Market

Let z denote an agent's nominal money balances at the beginning of the day market (exclusive of any transfer); and let $m \geq 0$ denote the money this person takes into the night market. In the day market, all agents are able to buy or sell output x; this gives rise to a day-market budget constraint:

$$x = v_d(z + \tau - m). \tag{8}$$

Let $D(z)$ denote the utility value of beginning the day with z units of money; and let $N(m)$ denote the utility value of beginning the night with m units of money. Note that $N(m)$ denotes the value before knowing whether one will have a desire to consume or opportunity to produce in the night-market. The value functions D and N must satisfy the recursive relationship:

$$D(z) = \max_{m \geq 0} \left\{ v_d \left(z + \tau - m \right) + N(m) \right\}. \tag{9}$$

Assuming, for the moment, that $N' > 0$ and $N'' < 0$ (these conditions will hold for any $\mu > \beta$), the demand for money in the day market is characterized by:

$$v_d = N'(m). \tag{10}$$

As originally highlighted in Lagos and Wright (2005), money demand at this stage is independent of beginning-of-period money balances z. Furthermore, we see that:

$$D'(z) = v_d; \tag{11}$$

so that D is linear in z.

[6] In particular, note that I do not allow society to impose any real penalty on agents. For example, the government cannot confiscate output or force agents to work.

4.3 Night Market

Consider an agent who brings m units of money into the night market. The agent subsequently realizes whether he is a producer or consumer. Let $P(m)$ and $C(m)$ denote the utility value associated with being a producer and consumer, respectively. Then the *ex ante* value of entering the night market satisfies:

$$N(m) = \frac{1}{2}C(m) + \frac{1}{2}P(m). \tag{12}$$

4.3.1 Consumers

In the night-market, a consumer holding m units of money faces the following budget constraint:

$$z^{+} = m - v_n^{-1}y,$$

where z^{+} denotes money balances carried forward into the next period's day market and y denotes purchases of output at night. Note that the environment prevents the existence of private debt, so that $z^{+} \geq 0$. As demonstrated in Rocheteau and Wright (2005), this constraint will bind tightly for any inflation rate away from the Friedman rule; and will just bind at the Friedman rule. Invoking this latter result, the solution to the consumer's choice problem satisfies:

$$y = v_n m; \tag{13}$$

which yields the value function:

$$C(m) = u\left(v_n m\right) + \beta D(0),$$

from which we derive:

$$C'(m) = v_n u'\left(y\right). \tag{14}$$

4.3.2 Producers

In the night-market, a producer holding m units of money faces the following budget constraint:

$$z^{+} = m + v_n^{-1}y.$$

Note that the constraint $z^{+} \geq 0$ will not bind. Hence, the producer's choice problem is given by:

$$P(m) = \max_{y} \left\{ -g(y) + \beta D(m + v_n^{-1}y) \right\},$$

from which we have:

$$\begin{aligned} P'(m) &= \beta D'\left(z^{+}\right) \\ &= \beta v_d^{+}; \end{aligned} \tag{15}$$

where the latter derivation makes use of (11) updated to the next period. The nighttime supply of output is characterized by:

$$v_n g'(y) = \beta v_d^+. \tag{16}$$

The interesting thing to note here is that producers are willing to produce even in the absence of any explicit future reward promised to them. Instead, the future reward for their current sacrifice is embedded in the belief that the money they accumulate today will have purchasing power in the future day market; i.e., that $v_d^+ > 0$.

Before proceeding, we need information on $N'(m)$. Using (12), together with the results in (14) and (15), we see that:

$$\begin{aligned} N'(m) &= \frac{1}{2}C'(m) + \frac{1}{2}P'(m); \\ &= \frac{1}{2}v_n u'(y) + \frac{1}{2}\beta v_d^+. \end{aligned}$$

Substituting (16) into the equation above yields:

$$N'(m) = \frac{1}{2}v_n \left[u'(y) + g'(y)\right]. \tag{17}$$

4.4 Equilibrium

Combining (10) and (17),

$$2v_d = v_n \left[u'(y) + g'(y)\right]. \tag{18}$$

Multiplying both sides of the expression above by β and updating one period yields:

$$2\beta v_d^+ = \beta v_n^+ \left[u'\left(y^+\right) + g'(y^+)\right].$$

By (16), we see that $\beta v_d^+ = v_n g'(y)$; combining this with the expression above results in:

$$2v_n g'(y) = \beta v_n^+ \left[u'\left(y^+\right) + g'(y^+)\right].$$

In equilibrium, we have $m = M$ and $y = v_n M$; i.e., see (13). Hence, it follows that $(v_n^+/v_n) = 1/\mu$. The expression above may therefore be expressed as:

$$2g'(y) = \left(\frac{\beta}{\mu}\right)\left[u'\left(y^+\right) + g'(y^+)\right].$$

In what follows, I restrict attention to a (non-degenerate) steady-state where $y = y^+ = \hat{y}$. Hence, the expression above may alternatively be expressed as:

$$u'(\hat{y}) = \left[2\left(\frac{\beta}{\mu}\right) - 1\right] g'(\hat{y}). \tag{19}$$

Condition (19) fully characterizes the equilibrium level of nighttime production $\hat{y}$ as a function of parameters; and, in particular, the policy parameter μ. Note that $\hat{y}(\mu) = y^*$ iff $\mu = \beta$ and that $\hat{y}$ is strictly decreasing in μ for any $\mu \geq \beta$. These latter results are well-known in the literature. In particular, the Friedman rule emerges as an optimal policy in this type of environment. (Whether this policy is incentive-feasible, however, remains to be seen).

With $\hat{y}$ determined by condition (19), the remaining equilibrium variables are easy to compute. The value of money in the night-market is $\hat{v}_n = \hat{y}/M$. Hence, the value of money in the night-market grows at rate $(1/\mu)$; and it is easy to establish that the same must be true for the value of money in the day-market; which, by condition (16) must satisfy:

$$\hat{v}_d = (M^- \beta)^{-1} \hat{y} g'(\hat{y}). \tag{20}$$

The distribution of money holdings at the beginning of the day-market can be derived as follows. First, we know from (10) that all agents enter the night-market with identical money balances M^-. Those that become producers acquire M^- dollars in the night-market from those that become consumers. Hence, producers exit the night-market with $2M^-$ dollars and consumers exit with zero dollars.

Agents face a day-market budget constraint:

$$x = v_d(z + \tau - m);$$

see (9). For ex-producers, $z = 2M^-$ and $\tau = (\mu - 1)M^-$. In equilibrium, $m = M = \mu M^-$. Therefore, in equilibrium, we have:

$$\hat{x} = \hat{v}_d M^-;$$

or, substituting in (20),

$$\hat{x}(\mu) = \beta^{-1} \hat{y} g'(\hat{y}) \equiv \chi(\hat{y}). \tag{21}$$

That is, $\chi(\hat{y})$ denotes the daytime 'reward' for agents who produced $\hat{y}$ in the last period night-market; and $-\chi(\hat{y})$ denotes the daytime 'punishment' for agents who consumed $\hat{y}$ in the last period night-market. This object is a function of the money-growth (inflation) rate μ through the effect that this parameter has on $\hat{y}$. In particular, observe that $\hat{x}$ is strictly decreasing in μ.

I summarize this section by noting how the monetary equilibrium appears to replicate aspects of the quasi-linear mechanism described earlier. Recall that to induce truthful revelation, an optimal mechanism must offer nighttime producers a sufficiently large future reward in the form of x. Here, we see that the monetary mechanism associates the award $\chi(\hat{y}) > 0$ for the past sacrifice $\hat{y} > 0$. This future reward at the same time constitutes a future punishment for those who consumed at night. To induce participation on the part of those saddled with this implicit debt obligation, the punishment cannot be made too large.

When this is the case, ex-consumers willingly sacrifice $\chi(\hat{y})$ units of day-output to rebalance their money holdings. These latter agents are willing to accumulate money at this stage, as only money can be used to purchase output in upcoming night-market. That is, only by accumulating money can ex-consumers insure themselves against any future desire to consume. It is in this sense in which a monetary equilibrium can potentially replicate the punishment/reward system that might alternatively be designed by an optimal mechanism. Whether agents do in fact have an incentive for truthful revelation and participation in the monetary equilibrium is a matter that I turn to next.

5 Incentive-Feasible Deflation

The first question I ask is whether producers in the night-market will indeed have an incentive to reveal themselves as such.

Proposition 3 *Any monetary equilibrium allocation* $(\hat{x}(\mu), \hat{y}(\mu))$ *is incentive-compatible.*

Proof. Recall that an allocation (x, y) is IC if $x \geq (2\beta)^{-1} g(y)$. Hence, using (21), a monetary equilibrium is IC if:

$$\beta^{-1} \hat{y} g'(\hat{y}) \geq (2\beta)^{-1} g(\hat{y});$$

or,

$$g'(\hat{y}) \geq \frac{1}{2} \frac{g(\hat{y})}{\hat{y}}.$$

As convexity implies that $g' \geq g/y$, the proposition follows. ■

Proposition 3 implies that the monetary equilibrium associated with the Friedman rule is incentive-compatible. With incentive-compatibility ensured, it remains to check that the *ex post* rationality condition (PC) is satisfied for this allocation.

Before proceeding, I invoke an assumption that simplifies the exposition without detracting from the main qualitative results that are about to follow.

[A2] $y g'(y) = \theta g(y)$ for all y for some $\theta \geq 1$.

The following proposition asserts that if agents are sufficiently patient, then the monetary allocation associated with the Friedman rule is PC.

Proposition 4 $(\hat{x}(\mu), \hat{y}(\mu)) \in S$ *for* $\mu = \beta$ *and any* $\beta \in [\beta_1, 1)$; *with* $0 < \beta_1 < 1$ *defined by*

$$\beta_1 = \left[\frac{2\theta g(y^*)}{u(y^*) - g(y^*) + 2\theta g(y^*)} \right].$$

Proof. Recall that an allocation (x, y) is PC if $W(y) \geq x$. Hence, using (21), a monetary equilibrium is PC if:

$$u(\hat{y}) - g(\hat{y}) \geq \left(\frac{1-\beta}{\beta}\right) 2\hat{y}g'(\hat{y}).$$

By **[A2]**, this condition may alternatively be expressed as:

$$\beta\left[u(\hat{y}) + (2\theta - 1)g(\hat{y})\right] \geq 2\theta g(\hat{y}). \tag{22}$$

As the term in square brackets is positive and greater than the RHS of the inequality, we can find a $0 < \beta_1 < 1$ such that:

$$\beta_1\left[u(y^*) - g(y^*) + 2\theta g(y^*)\right] = 2\theta g(y^*); \tag{23}$$

so that (22) holds for any $\beta \geq \beta_1$, with $\hat{y}(\beta) = y^*$ and $\hat{x}(\beta) = \chi(y^*)$. ■

Together, propositions 3 and 4 imply that if agents are sufficiently patient, then the Friedman rule is incentive-feasible (the lump-sum taxes required to support the allocation are paid voluntarily) and the monetary equilibrium achieves the first-best allocation.[7]

If, on the other hand, $\beta < \beta_1$, then $\hat{y}(\beta) = y^*$ is no longer incentive-feasible; in particular, PC binds tightly.

Proposition 5 *When $\beta < \beta_1$, there is an incentive-induced lower bound on deflation $\mu_1 > \beta$ determined by $\hat{y}(\mu_1) = y_1$; with $0 < y_1 < y^*$ satisfying $W(y_1) = \chi(y_1)$.*

Proof. If $\beta < \beta_1$, then by proposition 4, $W(y^*) < \chi(y^*)$. As $W(y)$ is strictly increasing over the range $[0, y^*)$, one can recast the choice problem here as maximizing $\hat{y}$ subject to $W(\hat{y}) \geq \chi(\hat{y})$. The solution clearly entails a $0 < y_1 < y^*$ satisfying $W(y_1) = \chi(y_1$; with an associated money growth rate defined implicitly by $y_1 = \hat{y}(\mu_1)$; i.e., see equation (19). To see that $\mu_1 > \beta$, observe that for a given y, equation (19) can be restated as:

$$\mu = \beta\Omega(y);$$

where

$$\Omega(y) \equiv \left[\frac{u'(y) + g'(y)}{2g'(y)}\right].$$

Observe that $\Omega(y^*) = 1$ and that $\Omega'(y) < 0$. Since $y_1 < y^*$, $\Omega(y_1) > 1$, so that $\mu_1 > \beta$. ■

Proposition 5 establishes the conditions that prevent a monetary authority from deflating at the Friedman rule. When agents are impatient, the Friedman rule implies a very rapid deflation (and correspondingly high levels of nominal

[7] See Figure 3.

taxation). If agents are sufficiently impatient, then ex-consumers in the day-market will not find it worthwhile to make the sacrifice necessary to rebalance their money holdings (and pay the requisite taxes). When this is so, the monetary authority is constrained to choose some $\mu > \beta$ to induce participation.

One interesting question to ask is whether there are circumstances in which the monetary authority is prevented from deflating at all ($\mu_1 > 1$). In other words, when might a monetary authority be constrained to inflate at a positive rate, even when deflation is desirable?

To investigate this question, consider condition (23), which determines β_1 as a function of parameters. One parameter of interest is θ, which determines the convexity of the function $g(y)$. In particular, note that as θ increases, the disutility associated with nighttime production increases more rapidly as output expands. Define $\beta_1(\theta)$ via condition (23).

Lemma 3 *$\beta_1(\theta) \to 1$ as $\theta \to \infty$.*

Proof. The result follows immediately from (23). ■

Hence, as the marginal disutility of production is increased, the range of discount factors for which the Friedman rule is incentive-feasible shrinks.

Now, fix a pair (θ, β) such that $\beta < \beta_1(\theta)$. In this case, the best monetary equilibrium allocation is determined by $y_1(\theta)$ satisfying condition $W(y_1) = \chi(y_1)$ as described in proposition 5.

Lemma 4 *Consider a pair (β, θ) such that $\beta < \beta_1(\theta)$. Then $y_1(\theta') \to 0$ as $\theta' \to \infty$ for all $\theta' \geq \theta$.*

Proof. Note that lemma 3 guarantees that $y_1(\theta')$ continues to characterize the best monetary allocation as θ' is increased beyond θ, for the given β under consideration. Next, using (21) and **[A2]**, rewrite $W(y_1) = \chi(y_1)$ as:

$$W(y_1) = \beta^{-1}\theta g(y_1).$$

As $W(y)$ is strictly increasing over the range $[0, y^*)$ and $y_1 \in (0, y^*)$, it follows that y_1 is monotonically decreasing in θ. ■

Finally, with $y_1(\theta)$ determined by lemma 4, the best incentive-feasible inflation rate $\mu_1(\theta)$ is determined, as described in proposition 5, by the condition

$$\mu_1(\theta) = \beta\Omega(y_1(\theta)).$$

Proposition 6 *$\mu_1(\theta) \to \infty$ as $\theta \to \infty$.*

Proof. By lemma 4, $y_1(\theta) \to 0$ as $\theta \to \infty$. As $\Omega'(y) < 0$, it follows that $\mu_1(\theta) \to \infty$ as $\theta \to \infty$. ■

Hence, if production is sufficiently painful at high levels of output (very rapid diminishing returns to scale), a constrained-efficient monetary policy will entail a strictly positive inflation, despite the fact that deflation is desirable from a policy perspective.

6 Comparative Institutions

Proposition 1 asserts that when $\beta \geq \beta_0$, an institution that can make use of memory and enforce exclusion can implement the first-best allocation. Absent memory, a fiat money object is essential. If money holdings are observable, a plausible conjecture is that proposition 1 will continue to hold, with fiat money serving as a perfect substitute for the missing memory.[8]

Most discussions of optimal monetary policy, however, are couched in terms of a specific institutional setup; namely, markets that operate according to some version of conventional price-theory. The specific institutional setup I adopted above was a system of competitive spot markets, where economic activity is coordinated by market-clearing prices, and where memory (hence credit) is unavailable so that trade must take place via a *quid-pro-quo* exchange of money for goods. To allow for manipulation of the money supply, I assumed that lump-sum nominal transfers/taxes are feasible. Proposition 4 asserts that when $\beta \geq \beta_1$, this trading structure (together with a well-designed monetary policy) can also implement the first-best allocation. But as the imposition of this particular trading system is *ad hoc*, one might reasonably ask whether it is also restrictive.

Proposition 7 $0 < \beta_0 < \beta_1 < 1$.

Proof. By proposition 1, $\beta_0 u(y^*) = g(y^*)$; or

$$[u(y^*) - g(y^*)] = \left(\frac{1-\beta_0}{\beta_0}\right) g(y^*).$$

By proposition 4, $\beta_1 [u(y^*) - g(y^*) + 2\theta g(y^*)] = 2\theta g(y^*)$, or

$$[u(y^*) - g(y^*)] = \left(\frac{1-\beta_1}{\beta_1}\right) 2\theta g(y^*).$$

Combining these two expressions:

$$\left(\frac{1-\beta_0}{\beta_0}\right) = \left(\frac{1-\beta_1}{\beta_1}\right) 2\theta.$$

Since $\theta \geq 1$, it follows that $\beta_0 < \beta_1$. ■

[8] One might rely, for example, on the arguments developed in Kocherlakota (2002).

Hence, there is a region $[\beta_0, \beta_1) \subset (0,1)$ over which an optimal mechanism can implement the first-best allocation, but the competitive market structure cannot.[9] Similar results can be derived in the context of other parameters (e.g., θ).

By proposition 2, when $\beta \in (0, \beta_0)$, the first-best allocation is not implementable even with an optimal mechanism. In this case, the constrained-efficient allocation is characterized by $\beta u(y_0) = g(y_0)$ with $x_0 = W(y_0)$. In contrast, the best allocation achievable in a monetary equilibrium is characterized by $W(y_1) = \chi(y_1) = x_1$,with μ_1 implicitly defined by $\hat{y}(\mu_1) = y_1$.

Proposition 8 $0 < y_1 < y_0 < y^*$ *for all* $\beta \in (0, \beta_0)$.

Proof. Rewrite the condition $\beta u(y_0) = g(y_0)$ as

$$W(y_0) = \beta^{-1}\left(\frac{1}{2}\right)g(y_0);$$

and rewrite condition $W(y_1) = \chi(y_1)$ as

$$W(y_1) = \beta^{-1}\theta g(y_1).$$

Again, note that W is strictly increasing over the range $[0, y^*)$ and that both y_0 and y_1 fall within this range. Since $\theta > 1/2$, it follows that $y_1 < y_0$. ■

As welfare $W(y)$ is strictly increasing over the range $[0, y^*)$, the proposition implies $W(y_1) < W(y_0)$. Hence, welfare under the mechanism is strictly higher relative to what can be achieved with competitive markets for discount factors over the range $(0, \beta_1)$; and welfare under the mechanism and market-system coincides for discount factors over the range $[\beta_1, 1)$.

7 Related Literature

A paper closely related to my own has recently been circulated by Hu, Kennan and Wallace (2007). Their model is closer in spirit to the original Lagos-Wright (2005) framework in that they model the 'night' stage as a decentralized market where agents are matched pairwise and probabilistically. The authors restrict attention to core allocations immune to various notions of defection. In their model, money is essential because agents lack commitment and because monitoring (record-keeping) is absent.[10] The quasi-linear environment renders their analysis (like my own) highly tractable.

Their first key result is that the first-best allocation is implementable even without intervention (i.e., a constant money supply) when agents are sufficiently

[9] See Figure 4.

[10] The authors ignore incentive-compatibility, but Proposition 5 suggests that this is not a relevant abstraction.

patient. This can never be the case in my model (unless β is arbitrarily close to unity). But their result reveals that this must be because monetary trade in my model is restricted to occur in competitive markets; so that marginal, rather than non-marginal conditions are relevant in my model. Put differently, a competitive monetary equilibrium allocation is always in the core, but the set of core allocations is in general much larger; and hence less restrictive.

Their second key result is that, if consumers are restricted to make take-it-or-leave it offers, and if agents are free to skip the day-market, a lump-sum tax financed deflation in no way expands the set of implementable allocations. This result would follow in my analysis too if I, like the authors, assume that agents can escape a nominal tax obligation with impunity (the constrained-efficient monetary policy in this case would be a constant money supply). The authors suggest that the ability to avoid taxes with impunity is a logical by-product of no-commitment. But as my analysis shows, this is not necessarily the case. In particular, agents in my model also lack commitment and have an ability to skip the day market. However, if a mechanism has some limited (but reasonable) powers of identification and coercion, agents may voluntarily pay a nominal tax, so that some deflation is possible.

8 Conclusion

The paper considers a model of intertemporal trade based on the analytically tractable quasi-linear environment introduced by Lagos and Wright (2005). When agents lack commitment and idiosyncratic shocks are private information, memory is essential for efficient implementation.

Absent a record-keeping technology, a tangible fiat money instrument is essential for trade. When trade is organized as a system of competitive spot markets and when nominal lump-sum taxes and transfers are feasible, the most desirable monetary policy in this environment is to deflate at the Friedman rule. In doing so, the monetary authority essentially replicates the system of punishments and rewards that would be put in place by an optimal quasi-linear mechanism. However, a point overlooked in the literature is the fact that the same frictions that render money (memory) essential may at the same time render the Friedman rule infeasible. When this is so, there is an incentive-induced lower bound to the rate of deflation away from the Friedman rule. In some circumstances, the best incentive-feasible monetary policy may entail a strictly positive rate of inflation.

Figure 1
First-Best Implementation ($\beta > \beta_0$)

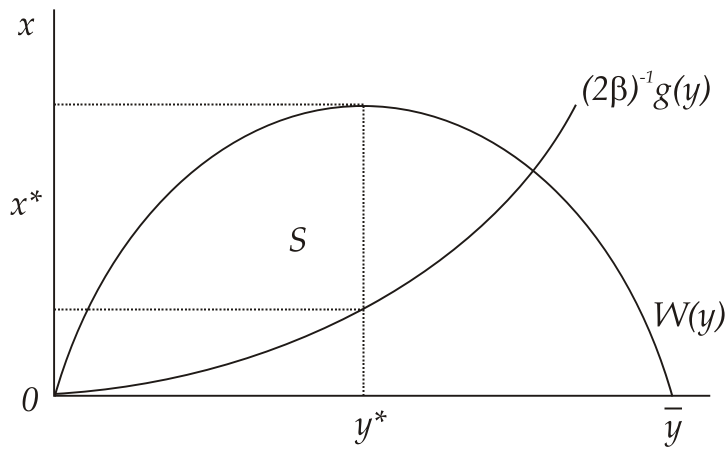


Figure 2
Constrained-Efficient Implementation ($\beta < \beta_0$)

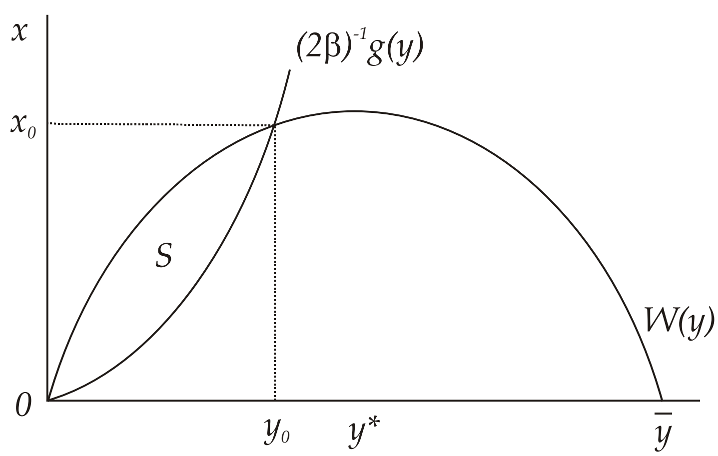


Figure 3
First-Best Monetary Equilibrium ($\beta_0 < \beta_1 < \beta$)

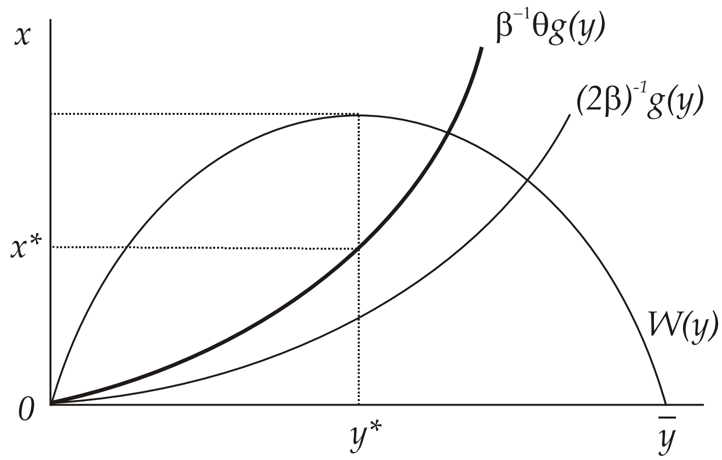
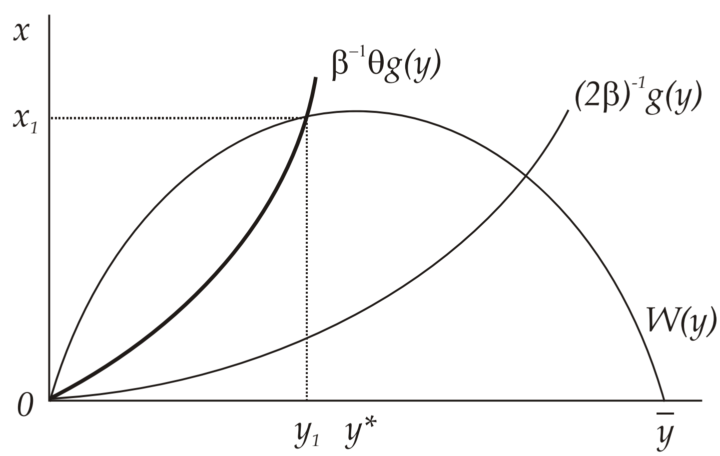


Figure 4
Constrained-Efficient Monetary Equilibrium
($\beta_0 < \beta < \beta_1$)



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