Firm Size Distortions and the Productivity Distribution: Evidence from France*

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Abstract

We show how size-contingent laws can be used to identify the equilibrium and welfare effects of labor regulation. Our framework incorporates regulation into the Lucas (1978) model and applies this to France where many labor laws start to bind on firms with exactly 50 or more employees. Using data on the population of firms between 2002 and 2007 period, we structurally estimate the key parameters of our model to construct counterfactual size, productivity and welfare distributions. With flexible wages, the deadweight loss of the regulation is below 1% of GDP, but when wages are downwardly rigid welfare losses exceed 5%. We also show, regardless of wage flexibility, that the main losers from the regulation are workers (and to a lesser extent large firms) and the main winners are small firms.

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1 Introduction

A recent literature has documented empirically how distortions that raise the cost of labor or capital affect aggregate productivity through misallocations of resources from more productive to less productive firms. As Restuccia and Rogerson (2008) have argued[1] these distortions may mean that more efficient firms produce too little and employ too few workers. Hsieh and Klenow (2009) show that these misallocations account for a significant proportion of the difference in aggregate productivity between the US, China and India.[2] In this paper, we focus on understanding the impact and the size of one specific distortion on the French firm size distribution: regulations that increase labor costs when firms reach 50 workers.

The idea that misallocations of resources lie behind aggregate productivity gaps is attractive in understanding the differences between the US and Europe.[3] According to the European Commission (1996) the average production unit in the EU employed 23% less workers than in the US. Consistent with this, Figure 1 shows that there appear to be far fewer large French firms compared with the US firms. In particular there is a large bulge in the number of firms with employment just below 50 workers in France, but not in the US. This is illustrated in Figure 2 which shows the exact number of manufacturing firms by number of workers. There is a sharp fall in the number of firms who have exactly 50 employees (160) compared to those who have 49 employees (416).

The burden of French labor legislation substantially increases when firms employ 50 or more workers. As we explain in detail below, firms above this size threshold must create a works council ("comité d'entreprise") with a minimum budget of 0.3% of total payroll, establish a health and safety committee, appoint a union representative and so on. What are the implications for firm size, firm productivity and aggregate productivity from those laws? Intuitively, some more productive firms that would have been larger without the regulation choose to remain below the legal threshold to avoid these costs. In this paper we show how these changes in the firm size distribution can be exploited to infer the level and distribution of the welfare cost of these regulations.

There has been extensive discussion of the importance of labor laws for unemployment and more recently

[1] See also Parente and Prescott (2000), Bloom and Van Reenen (2007) and Petrin and Sivadasan (2010).

[2] In development economics many scholars have pointed to the "missing middle", i.e. a preponderance of very small firms in poorer countries compared to richer countries (see Banerjee and Duflo, 2005, or Jones, 2011). For example, in the late 1980s in India large firms were banned from producing about 800 product groups (Little et al, 1987). Many explanations have been put forward for this such as financial development, taxes, human capital, lack of competition in product markets, and social capital. One possibility, related to our approach, is size related labor regulations. Besley and Burgess (2004), for example, suggest that labor regulation is one of the reasons why the formal manufacturing sector is much smaller in some Indian states compared to others.

[3] Bartelsman, Haltiwanger and Scarpetta (2009) examine misallocation using micro-data across many OECD countries and make a similar point. In particular, they find that the "Olley Pakes" (1996) covariance term between size and productivity is much smaller in France (0.24 in their Table 1) and other European countries compared to the US (0.51 in their Table 1). Bloom, Sadun and Van Reenen (2012) also report a more efficient allocation of employment to better managed firms in the US than in Europe.

productivity (e.g. Layard and Nickell, 1999; European Commission, 2006). The OECD, World Bank and other agencies have developed various indices of the importance of these regulations, based on examination of laws and (sometimes) surveys of managers. It is very hard, however, to see how these can be quantified as "adding up" the regulatory provisions has a large arbitrary component. A contribution of our paper is to offer a methodology for quantifying the tax equivalent of a regulation, albeit in the context of a specific model. Moreover, the calculation is extremely transparent, economically intuitive and can be applied in many contexts.

There are different views on the underlying sources of heterogeneity in firm productivity. We follow Lucas (1978) in taking the stand that managerial talent is the primitive, and that the economy-wide observed resource distribution is, as Manne (1965) felicitously put it, "a solution to the problem: allocate productive factors over managers of different ability so as to maximize output." Managers make discrete decisions or solve problems (Garicano, 2000). Making better decisions, or solving problems that others cannot solve, raises everyone's marginal product. This means that, in equilibrium, better managers must be allocated more resources. In fact, absent decreasing returns to managerial talent, the best manager must be allocated all resources. Given limits to managerial time or attention, the better managers are allocated more workers and more capital to manage. This results in a "scale-of-operations" effect whereby differences in talent are amplified by the resources allocated.[4] Lucas (1978) first explored these effects in an equilibrium setting.[5]

When managers are confronted with legislation that introduces a cost of acquiring a size that is beyond a certain threshold, they may choose to stay below the threshold and stay at an inefficiently small size. By studying the productivity of these marginal managers, we are able to estimate the cost of the legislation, the distortions in them, and thus the welfare cost of the legislation for the entire firm size distribution.[6]

We start by setting up a simple model of the allocation of a single factor, labor, to firms in a world where there are decreasing returns to managerial talent. We use it to study the effect of a step change in labor costs after a particular size and show that there are four main effects:

1. Equilibrium wages fall as a result of the reduction in the demand for workers (i.e. some of the tax incidence falls on workers)

[4] In a model of this kind, the source of decreasing returns are on the production size, and are linked to limits to managerial time. For our purposes here, as Hsieh and Klenow (2009) show, this source of decreasing returns is equivalent to having the decreasing returns come (as is more common in recent literature following Melitz, 2003) from the utility side.

[5] Such a scale of operations effects is at the heart of Rosen's (1982) theory of hierarchies, where efficiency units of labor controlled (and not just number of bodies) matter, and also in Garicano and Rossi-Hansberg (2006) where there is limited quantity-quality substitutability so that matching between workers and managers takes place. Empirically, this technology has been used to explain a wide-range of phenomena, for example the impact of scale of operations effects on CEO wages (Gabaix and Landier, 2008).

[6] Many empirical papers have shown that deregulation (e.g. Olley and Pakes, 1996), higher competition (e.g. Syverson, 2004) and trade liberalization (e.g. Pavcnik, 2002) have tended to improve reallocation by increasing the correlation between firm size and productivity.

2. Firm size increases for all firms below the threshold as a result of the general equilibrium effect on wages
3. Firm size reduces to precisely the regulatory threshold for a set of firms that are not productive enough to justify incurring the regulatory costs
4. Firm size reduces proportionally for all firms that are productive enough to incur the additional cost of regulation.

We use the model to guide our estimation of the impact of these costs. The theory tells us there is a deviation from the "correct" firm size distribution as a result of the regulation. That is, we expect to see a departure from the usual power law firm size distribution[7] as firms bunch up below the threshold of 50 workers. Given factors such as measurement error, however, the observed empirical departure from the power law is not just at 49 workers but also affects firms of slightly smaller sizes. Similarly, there is not precisely zero mass to the right of the threshold, but rather a "valley" were there are significantly fewer firms than we would expect from an unbroken power law. Then, at some point the firm size distribution becomes again a power law, with a lower intercept. The break in the power law from the bulge and valley of firms around the threshold helps empirically identify the magnitude of the regulatory distortion.

Our results are consistent across specifications, and the cost of these regulations is quite precisely estimated. Our key finding is that it is not just the regulation, but the impact of these regulations and downward rigid wages what makes the regulation problematic. When wages are fully flexible, we find that these regulations operate mainly as a variable cost, and are equivalent to a 1.3 percentage point increase in wages across the distribution. If wages are fully flexible, they need to decrease by around 1%. If they do, the regulation has large distributional consequences, but creates small deadweight losses. Profits of large firms drop by as much as workers wages, 1%, and profits of midsize firms grow by 4%. Large deadweight losses only take place when wages are rigid. In this case, the regulation results in a 5% unemployment rate and a 4.3% deadweight loss.

In both cases the regulation has winners and losers. Large firms lose from incurring all of these costs, and workers lose from the lower wages. When wages are flexible, small and medium size firms benefit from the redistribution, as they benefit from paying lower wages and do not have to pay the costs that the regulation involves.

Overall, the labor regulations that we study and that affect firms over 50 workers place a significant burden on the economy if wages are not flexible, by keeping firms below their optimal size and by reducing output. Too many workers work for smaller firms, and too few for large firms, reducing the economy's potential.

[7] See Axtell (2001), Sutton (1997) and Gabaix, (2009). There is a large literature on the size and productivity distribution of firms in macro, trade, finance and IO. Appropriately, the first major study in this area was by Gibrat (1931) who studied French industrial firms, the main focus of the empirical part of our paper.

The most closely related paper to ours is Braguinsky, Branstetter and Regateiro (2011) who seek to explain changes in the support of the Portuguese firm size distribution in the context of the Lucas model with labor regulations[8]. Their calibrations also show substantial effects of the regulations on aggregate productivity. The Portuguese data, however, does not present a clear structural break as ours and thus their approach does not exploit the sharp discontinuity in the data to identify the structural parameters of the model.[9] Our paper is also related to the more general literature using tax "kinks" to identify behavioral parameters (e.g. Saez, 2010; Chetty et al, 2011).

The structure of the paper is as follows. Section 2 describes our theory and some extensions. Section 3 describes the empirical strategy we use to map the theory into the data. Section 4 describes the institutional setting and data. Section 5 contains the main results, which come in three parts. First we show that the main empirical predictions of the model in terms of the size and productivity distribution are consistent with the data. Secondly, we estimate the parameters of the structural model and use this to show that the costs of the regulation are non-trivial. Third, we show that there is considerable heterogeneity across sectors within manufacturing and outside manufacturing. We present various extensions and robustness tests in Section 6, before drawing some conclusions in the final section.

2 Theory

We aim to estimate the distortions in the productivity distribution and the reallocation effect that results from an implicit tax on firm size that starts at a particular threshold. Our strategy relies on analyzing the choices of those firms that prefer to stay at a lower size in order to avoid the tax. Having done that, we will be able to estimate the general equilibrium effects of the tax through the changes in firm size.

We begin by studying regulatory effects on the firm size and productivity distribution in the simplest possible version of the Lucas model. There is only one input in production, labor, and a single sector.[10] The primitive of the model is the pdf $\phi(\alpha)$ of "managerial ability"α, $\phi : [\underline{\alpha}; +\infty[\rightarrow \mathbb{R}$. Ability is defined and measured by how much an agent can raise a team's output: a manager who has ability α and is allocated n workers produces $y = \alpha f(n)$. Larger teams produce more, $f' > 0$, but given limited managerial time, there are decreasing returns to the firm scale that a manager can manage, $f'' < 0$.

The key difference between our setting and the original Lucas model is that we allow for a tax on firm size,

[8] On the quantitative theory side Guner, Ventura amd Yi (2006, 2008) also consider a Lucas model with size-contingent regulation. They calibrate this to uncover sizeable welfare losses. Unlike our paper and Braguinsky et al (2011), however, there is no econometric application.

[9] Braguinsky et al (2011) attribute this to the sheer multitude of size-related regulations in Portugal which makes it hard to identify any sharp cut-off in the size distribution.

[10] For size-contingent regulations in a multi-sector Lucas world see Garcia-Santana and Pijoan-Mas (2010) who apply it to Indian textiles.

which imposes a wedge between the wage the worker receives and the cost to the firm. In our application this "labor tax"involves an extra marginal cost and also a fixed cost component.[11] Moreover, this tax does not grow in a smooth way, but instead it is only borne by firms after they reach a given size N. In what follows and for simplicity, we consider that firm size is continuous and that the regulation binds for firms having size $n > N$.

2.1 Individual Optimization

Let $\pi(\alpha)$ be the profits obtained by a manager with skill α when he manages a firm at the optimal size. These profits are then given by:

$$\pi(\alpha) = \max_{n} \alpha f(n) - w\overline{\tau}n - \overline{k}, \qquad \text{with} \begin{cases} \overline{\tau} = 1, \overline{k} = 0 \text{ if } n \leq N \\ \overline{\tau} = \tau,\ \overline{k} = k \text{ if } n > N \end{cases} \tag{1}$$

where w is the worker's wage, n is the number of workers, k is the fixed cost that must be incurred over threshold N, and τ is the tax, which also applies for firm over a minimum threshold of N (50 workers in our application). Firm size at each side of the threshold is then determined by the first order condition:

$$\alpha f'(n^*_{N,\tau,w}) - \overline{\tau}w = 0, \qquad \text{with} \begin{cases} \overline{\tau} = 1, \overline{k} = 0 \text{ if } n \leq N \\ \overline{\tau} = \tau,\ \overline{k} = k \text{ if } n > N \end{cases} \tag{2}$$

so that $n^*_{N,\tau,w}(\alpha) = f'^{-1}(\frac{\overline{\tau}w}{\alpha})$. Note that $\partial n^*_{N,\tau,w}/\partial\alpha > 0$, $\partial n^*_{N,\tau,w}/\partial\tau < 0$ and $\partial n^*_{N,\tau,w}/\partial w < 0$.

The size constraint is reached at size N and managerial ability α_c (sub-script "c" for "constrained") is given by:

$$\alpha_c^{N,\tau,w} = \frac{w}{f'(N)} \tag{3}$$

Firms can legally avoid being hit by the regulation by choosing to remain small. The cost of this avoidance is increasing in the talent (α) of the individual. If managerial ability is sufficiently high rather than staying at $n = N$ and avoiding the tax managers choose to leverage their talent, become large and pay the tax. The ability level $\alpha_u^{N,\tau,w,k}$ of the "marginal manager" is defined by the indifference condition between remaining small or jumping to be a larger firm and paying the regulatory tax:

$$\alpha_u^{N,\tau,w,k} f(N) - wN = \alpha_u^{N,\tau,w,k} f(n^*_{N,\tau,w}(\alpha_u^{N,\tau,w,k})) - w\tau n^*_{N,\tau,w}(\alpha_u^{N,\tau,w,k}) - k \tag{4}$$

where $n^*_{N,\tau,w}(\alpha_u^{N,\tau,w,k})$ is the optimal firm size for an agent of skill $\alpha_u^{N,\tau,w,k}$ when wages are set at w. Subscript

[11] Previous studies of this problem, such as particularly Kramarz and Michaud (2003) suggest that the fixed cost component are second order relative to the marginal cost component. Empirically, we also find this result.

u refers to firms that are "unconstrained" in the sense that they choose to grow large in spite of the tax.

2.2 Equilibrium

The most skilled individuals choose to be manager-entrepreneurs, since they benefit from their higher ability in two ways. First, for a given firm size n, they earn more profits. Second, the most skilled individuals hire a larger team, $n^*_{N,\tau,w}(\alpha)$. We denote the ability threshold between managers and workers as $\alpha_{\min}$, individuals with ability below $\alpha_{\min}$ will be workers.

A competitive equilibrium is defined as follows:

Definition 1 *Given a distribution of managerial talent $\phi(\alpha)$ over $[\underline{\alpha};+\infty]$, a per worker implicit labor tax τ and a fixed cost k that binds all firms of size $n > N$, and a production function $y = \alpha f(n)$, a competitive equilibrium consists of:*

Definition 2 *(i) a wage level $w^*_{N,\tau,k}$ paid to all workers*

(ii) an allocation $n^*_{N,\tau,k}(\alpha)$ that assigns a firm of size $n^*_{N,\tau,k}$ to a particular manager of skill α

(iii) a triple of cutoffs $\{\alpha^{N,\tau,k}_{\min} \leq \alpha^{N,\tau,k}_{c} \leq \alpha^{N,\tau,k}_{u}\}$, such that $W = [\underline{\alpha}, \alpha^{N,\tau,k}_{\min}[$ is the set of workers, $M_1 = [\alpha^{N,\tau,k}_{\min}, \alpha^{N,\tau,k}_{c}]$ is the set of unconstrained, untaxed managers, $M_2 = [\alpha^{N,\tau,k}_{c}, \alpha^{N,\tau,k}_{u}[$ is the set of size constrained, at $n^*_{N,\tau,k} = N$, but untaxed managers, and $M_3 = [\alpha^{N,\tau,k}_{u}, \infty[$ is the set of taxed managers such that:

Definition 3 *(E1) No agent wishes to change occupation from manager to worker or to change from unconstrained to constrained.*

(E2) The choice of $n^*_{N,\tau,k}(\alpha)$ for each manager α is optimal given their skills, taxes (τ, k) and wages $w^*_{N,\tau,k}$;

(E3) Supply of labor equals demand for labor.

Start with condition (E1): an agent prefers to be a worker if $w^*_{N,\tau,k} > \alpha f\left(n^*_{N,\tau,k}(\alpha)\right) - w^*_{N,\tau,k} n^*_{N,\tau,k}(\alpha)$, or a manager if $w^*_{N,\tau,k} < \alpha f\left(n^*_{N,\tau,k}(\alpha)\right) - w^*_{N,\tau,k} n^*_{N,\tau,k}(\alpha)$, and thus we have[12]:

$$\alpha^{N,\tau,k}_{\min} f\left(n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{\min})\right) - w^*_{N,\tau,k} n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{\min}) = w^*_{N,\tau,k} \tag{5}$$

Equilibrium condition (E2), from the first order condition (2) implies that firm sizes are given by:

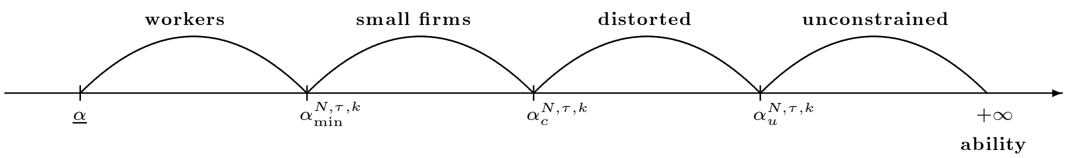
[12] Note that we have $n^*_{N,\tau,k}(\alpha) = n^*_{N,\tau,w^*_{N,\tau,k}}(\alpha)$, where $n^*_{N,\tau,w}$ has been defined in section 2.1.

$$
\begin{aligned}
n^*_{N,\tau,k}(\alpha) &= 0 \quad \text{if } \alpha < \alpha^{N,\tau,k}_{\min} && (6)\\
n^*_{N,\tau,k}(\alpha) &= f'^{-1}\left(\frac{w^*_{N,\tau,k}}{\alpha}\right) \quad \text{if } \alpha^{N,\tau,k}_{\min} \leq \alpha \leq \alpha^{N,\tau,k}_{c} && (7)\\
n^*_{N,\tau,k}(\alpha) &= N \quad \text{if } \alpha^{N,\tau,k}_{c} \leq \alpha < \alpha^{N,\tau,k}_{u} && (8)\\
n^*_{N,\tau,k}(\alpha) &= f'^{-1}\left(\frac{\tau w^*_{N,\tau,k}}{\alpha}\right) \quad \text{if } \alpha^{N,\tau,k}_{u} \leq \alpha < \infty && (9)
\end{aligned}
$$

In $[\alpha^{N,\tau,k}_{\min}, \alpha^{N,\tau,k}_{c}]$ we find firms that are not directly affected by the distortion. The only impact of the regulation comes through the general equilibrium effect of lower wages $w^*_{N,\tau,k}$. Lower wages induces some low-ability individuals to became small firms rather than remain as workers.[13] At $[\alpha^{N,\tau,k}_{c}, \alpha^{N,\tau,k}_{u}[$ we find the constrained firms: those companies that given the choice between (1) paying the regulatory cost ($(\tau - 1)w^*_{N,\tau,k}.n + k$) and (2) choosing their optimal size and paying $w^*_{N,\tau,k}$ but staying at size $n \leq N$, prefer to stay below N. Last, once productivity exceeds a higher threshold α_u, firms are sufficiently productive that they pay the tax in order to produce at a higher level.

Thus we have four categories of agents as the following figure shows:

Equilibrium partition of individuals into workers and firm types
by managerial ability, α



Notes: This figure shows the definitions of different regimes in our model. Individuals with managerial ability below $\alpha^{N,\tau,k}_{\min}$ choose to be workers rather than managers. Individuals with ability between $\alpha^{N,\tau,k}_{\min}$ and $\alpha^{N,\tau,k}_{c}$ are "small firms" who (conditional on the equilibrium wage, which is lower under regulation) do not change their optimal size. Between $\alpha^{N,\tau,k}_{c}$ and $\alpha^{N,\tau,k}_{u}$ are individuals who are affected by the regulatory constraint and choose their firm size to be smaller than they otherwise would have been - we call these individuals/firms who are in a "distorted "regime. Individuals with ability above $\alpha^{N,\tau,k}_{u}$ are choosing to pay the implicit tax rather than keep themselves small.

Finally, from condition (E3) in Definition 1, equilibrium requires that markets clear- that is the supply and demand of workers must be equalized. The supply of workers is $\int_{\underline{\alpha}}^{\alpha^{N,\tau,k}_{\min}} \phi(\alpha)\mathrm{d}\alpha$, and the demand of workers by all available managers, $\int_{\alpha^{N,\tau,k}_{\min}}^{\infty} n^*_{N,\tau,k}(\alpha)\phi(\alpha)\mathrm{d}\alpha$, where $n^*_{N,\tau,k}(\alpha)$ is the continuous and piecewise differentiable function given as above. Thus:

[13] In other words the regulatory distortion creates "too many" entrepreneurial small firms. This seems to be a feature of many Southern European countries which have a large number of small low productivity firms.

$$\int_{\underline{\alpha}}^{\alpha_{\min}^{N,\tau,k}} \phi(\alpha)\mathrm{d}\alpha = \int_{\alpha_{\min}^{N,\tau,k}}^{\infty} n^*_{N,\tau,k}(\alpha)\phi(\alpha)\mathrm{d}\alpha \tag{10}$$

Solving the model involves finding four parameters: the cutoff levels $\alpha_{\min}^{N,\tau,k}$, $\alpha_c^{N,\tau,k}$, $\alpha_u^{N,\tau,k}$, and the equilibrium wage $w^*_{N,\tau,k}$. For this we use the four equations (3), (4), (5) and (10). The equilibrium is unique and we can prove our main proposition over the comparative statics in the equilibrium:

Proposition 1 *The introduction of a tax/variable cost τ of hiring workers starting at firm size N has the following effects:*

1. *Reduces equilibrium wages as a result of the reduction in the demand for workers*
2. *Increases firm size for all firms below the threshold, $[\alpha_{\min}^{N,\tau,0}, \alpha_c^{N,\tau,0}]$, as a result of the general equilibrium effect that reduces wages*
3. *Reduces firm size to the threshold N for all firms that are constrained, that is those in $[\alpha_c^{N,\tau,0}, \alpha_u^{N,\tau,0}[$*
4. *Reduces firm size for all firms that are taxed $[\alpha_u^{N,\tau,0}, +\infty[$*

Proposition 1 looks at the comparative statics for $\tau \geqslant 1$ keeping $k = 0$. We relegate the proofs around $k > 0$ to Appendix 1, as empirically we estimate $k = 0$.

Example. Consider a power law, $\phi(\alpha) = \frac{0.6}{\alpha^{1.6}}$ and returns to scale parameter of $\theta = 0.9$. Figure 3 shows the firm size distribution for a firm size cut-off at 50 employees, and an employment tax of $\tau - 1 = 1\%$ (while $k = 0$). As in the distribution in the data, there is a spike at 49 employees that breaks the power law. Figure 4 reports the productivity α as a function of firm size n. It shows that we should expect a spike in the productivity distribution at the point in which the regulation starts to bind. Essentially the maximum bar of this graph is the most productive firm that is affected by the regulation. We track the firm size simply by moving horizontally to the right in the graph.

2.3 Empirical Implications

Our econometric work uses the theory as a guide to estimate the welfare losses that result from this regulation. As is well known, the firm size distribution generally follows a power law.[14] Lucas (1978) shows that Gibrat's law implies that the returns to scale function must be $f(n) = n^\theta$, and that for it to be consistent with a power law, the managerial ability or productivity distribution must also be power, $\phi(\alpha) = c_\alpha \alpha^{-\beta_\alpha}$, $\alpha \in [\underline{\alpha}; +\infty[$ with the constants $c_\alpha > 0$ and $\beta_\alpha > 0$. In this case, from the first order conditions in equations (6) to (9), firm sizes are given, for the equilibrium wage $w^*_{N,\tau,k}$, by:

[14] There is a literature in EconoPhysics that has focued on this. See Axtell (2001) for the US, Ramsden and Kiss-Haypal (2000) for OECD countries and Hernández-Pérez, Angulo-Browna and Tun (2006) developing countries.

$$n^*_{N,\tau,k}(\alpha) = \begin{cases} 0 & \text{if} \quad \alpha < \alpha^{N,\tau,k}_{\min} \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} & \text{if} \quad \alpha^{N,\tau,k}_{\min} \leq \alpha \leq \alpha^{N,\tau,k}_{c} \\ N & \text{if} \quad \alpha^{N,\tau,k}_{c} \leq \alpha < \alpha^{N,\tau,k}_{u} \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)} \alpha^{1/(1-\theta)} & \text{if} \quad \alpha^{N,\tau,k}_{u} \leq \alpha < \infty \end{cases} \tag{11}$$

Given our assumption that the distribution of $\phi(\alpha)$ follows a power law, $\phi(\alpha) = c_\alpha.\alpha^{-\beta_\alpha}$ the distribution of firm sizes $\chi(n)$ is also power (apart from the threshold), since by the change of variable formula, $\chi(n) = \phi(\alpha(n)) * n^{-\theta} w \overline{\tau} \frac{(1-\theta)}{\theta}$ (omitting the threshold). The "broken"power law on n^* is then given by:

$$\chi^*(n) = \begin{cases} c_\alpha (1-\theta) \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}} n^{-\beta} & \text{if} \quad n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{\min}) = n^{N,\tau,k}_{\min} \leq n \leq N = n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{c}) \\ \int_{\alpha_c}^{\alpha_u} \phi(\alpha) \mathrm{d}\alpha = \delta & \text{if} \quad n = N = n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{c}) \\ 0 & \text{if} \quad N < n < n^{N,\tau,k}_{u} = n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{u}) \\ c_\alpha (1-\theta) \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}} \underbrace{\tau^{-\frac{\beta-1}{1-\theta}}}_{\leq 1} n^{-\beta} & \text{if} \quad n^*_{N,\tau,k}(\alpha^{N,\tau,k}_{u}) = n^{N,\tau,k}_{u} \leq n \end{cases} \tag{12}$$

where $\beta = \beta_\alpha.(1-\theta) + \theta$ and δ is the mass of firms whose size is distorted - these are firms that choose to stay below the firm size threshold, rather than growing and paying the additional labor costs, τ and k. Furthermore, $n^{N,\tau,k}_{\min}$ denotes the optimal firm size for the entrepreneur with lowest ability (which is therefore the minimum firm size), and $n^{N,\tau,k}_{u}$ denotes the optimal firm size for the first entrepreneur choosing to pay the tax.

The adding up constraints on δ can be written more conveniently in the size (n) space rather than the ability space. After some straightforward manipulation, relegated to Appendix A, we show that $n^{N,\tau,k}_{\min} = n^*_{N,\tau,k}(\alpha_{\min}) = \theta/(1-\theta)$ as long as $N > \theta/(1-\theta)$ and we can rewrite the pdf of n^* as:

$$\chi^*(n) = \begin{cases} \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1) n^{-\beta} & \text{if} \quad \theta/(1-\theta) \leq n \leq N \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \left(N^{1-\beta} - T n_u^{1-\beta}\right) & \text{if} \quad n = N \\ 0 & \text{if} \quad N < n < n^{N,\tau,k}_{u} \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1) T n^{-\beta} & \text{if} \quad n^{N,\tau,k}_{u} \leq n \end{cases} \tag{13}$$

where $T = \tau^{-\frac{\beta-1}{1-\theta}}$. The upper employment threshold, $n^{N,\tau,k}_{u}$, is unknown and must be estimated alongside β, the power law term, θ and T. Note that the shape parameter β in the power law is unaffected by the regulation; instead, in log-log space, the labor regulations generate a parallel shift in the firm size distribution measured by T (see Figure 5). Thus the key empirical implication is that the tax can be recovered from the jump T in the power law.

In Section 3, we propose an empirical model in which we introduce an error term in the model so that we can take it to the data. Such empirical model must account for two departures in Figure 2 from the predictions in the theory:

1. The departure from the power law does not start at N, but slightly earlier: there is a bump in the distribution starting at around 46 workers.
2. The region immediately to the right of N does not have zero density, but rather there are some firms with positive employment levels just to the right of the regulatory cut-off, N.

The model we propose to account for these departures features a measurement error in employment. This seems reasonable for at least two reasons. First, several different regulations start at size 50, and as explained in greater detail in section 4.1, they rely on slightly different concepts of employment size, defined respectively in the Code du Travail (labor laws), Code du Commerce (commercial law), Code de la Sécurité Social (social security) and in the Code Général des Impôts (fiscal law). The measurement of firm size that we use[15] corresponds to the fiscal definition because it is a mandatory item that is reported in the firm's fiscal accounts - the arithmetic mean of the workforce at the end of the quarter of the fiscal year - and this measure of size is therefore available with accuracy for a larger number of firms. However, it does not exactly corresponds to the concept of size that is relevant for all of the regulations, since no (single) index of size having this property exists. We discuss alternatives to our the measurement error approach at the end of Section 6 where we examine adjustment costs, optimization errors and Leontief production functions.

2.4 Welfare calculations

Total output of a given entrepreneur of skill α and with size $n^*_{N,\tau,k}(\alpha)$, given by replacing the equilibrium wages $w^*_{N,\tau,k}$ in $n^*_{N,\tau,w}(\alpha)$ as given by equation (11), is:

$$y(\alpha, N, \tau, k) = \alpha f(n^*_{N,\tau,k}(\alpha))$$

and thus total output in this economy is given by:

$$Y(N,\tau,k) = \int_{\alpha_{\min}^{N,\tau,k}}^{\alpha_c^{N,\tau,k}} \alpha f(n^*_{N,\tau,k}(\alpha))\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_c^{N,\tau,k}}^{\alpha_u^{N,\tau,k}} \alpha f(N)\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_c^{N,\tau,k}}^{\infty} \alpha f(n^*_{N,\tau,k}(\alpha))\phi(\alpha)\mathrm{d}\alpha$$

And thus the welfare change is then given as follows:

[15] Fiscal definition, Article 208-III-3 du Code Général des Impôts.

$$
\begin{aligned}
\Delta Y &= Y(N,\tau,k) - Y(N,1,0) \\
&= \int_{\alpha_{\min}^{N,\tau,k}}^{\alpha_{\min}^{N,1,0}} \alpha \left[f(n^*_{N,\tau,k}(\alpha)\right] \phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_{\min}^{N,1,0}}^{\alpha_c^{N,\tau,k}} \alpha \left[f(n^*_{N,\tau,k}(\alpha)) - f(n^*_{N,1,0}(\alpha))\right] \phi(\alpha)\mathrm{d}\alpha \\
&\quad + \int_{\alpha_c^{N,\tau,k}}^{\alpha_u^{N,\tau,k}} \alpha \left[f(N) - f(n^*_{N,1,0}(\alpha))\right] \phi(\alpha)\mathrm{d}\alpha \\
&\quad + \int_{\alpha_c^{N,\tau,k}}^{\infty} \alpha \left\{ \left[f(n^*_{N,\tau,k}(\alpha)) - f(n^*_{N,1,0}(\alpha))\right] \right\} \phi(\alpha)\mathrm{d}\alpha \qquad (14)
\end{aligned}
$$

where $n^*_{N,1,0}$ is the first best firm size[16] and $\alpha_{\min}^{N,1,0}$ is the first best cutoff between workers and managers. The deadweight losses are then the result of adding up three effects:

(1) The top row of equation (14) captures two positive effects on total output from the fall in the equilibrium wage arising from the regulation. First, there are some additional firms since marginal workers are drawn into becoming entrepreneurs by cheaper labor. Second, the firms who are below the regulatory threshold (and not paying the tax) will be able to hire more workers as their wages are lower.

(2) There is a first "local" output loss, that is the result of the firms that would have had optimal size but instead are constrained at N workers. This is the second row of equation (14).

(3) Finally, there is the loss from the larger firms in the economy, which incur higher labor costs due to the implicit tax (even after netting off the lower equilibrium wage), and have a size that is too small.

2.5 Wage Rigidity

We also provide a welfare cost-benefit analysis under the assumption that wages are rigid and do not adjust downwards as in the basic model. Frictions in downward wage setting are common, especially in France where the Minimum Wages is high and unions are strong. More generally, there is likely to be a reservation wage below which individuals will not work, particular in nations like France with generous welfare benefits. Incorporating rigid wages requires a small extension of the model. We define the equilibrium with rigid wages in the following way:

Definition 4 *Given a distribution of managerial talent $\phi(\alpha)$ over $[\underline{\alpha}; +\infty[$, a per worker labor tax τ and a fixed cost k that binds all firms of size $n > N$, and a production function $\alpha f(n)$, a competitive equilibrium with rigid wages consists of:*

Definition 5 (*i*) *a wage level $w^{*,RIGID}_{N,\tau,k}$ paid to all employed workers which is computed as the equilibrium wage $w^*_{N,1,0}$ in the undistorted economy (baseline Lucas model and undistorted case of definition 1)*

[16] Note that the parameter N is optional when $\tau = 1$ and $k = 0$.

(ii) *an allocation* $n^{*,RIGID}_{N,\tau,k}(\alpha)$ *that assigns a firm of size* $n^{*,RIGID}_{N,\tau,k}$ *to a particular manager of skill* α

(iii) *a triple of cutoffs*[17] $\{\alpha^{RIGID}_{\min} \leq \alpha^{RIGID}_{c} \leq \alpha^{RIGID}_{u}\}$, *such that* $W = [\underline{\alpha}, \alpha^{RIGID}_{\min}[$ *is the set of* <u>*potential*</u> *workers,* $M_1 = [\alpha^{RIGID}_{\min}, \alpha_c]$ *is the set of unconstrained, untaxed managers,* $M_2 = [\alpha^{RIGID}_{c}, \alpha^{RIGID}_{u}[$ *is the set of size constrained, at* $n^{*}_{N,\tau,k} = N$*, but untaxed managers, and* $M_3 = [\alpha^{RIGID}_{u}, \infty[$ *is the set of taxed managers*

(iv) *an* <u>*unemployment rate*</u> $u^{*,RIGID}_{N,\tau,k}$ *defined as the number of unemployed workers as a share of the total number of potential workers*

such that:

Definition 6 $\left(E1^{RIG}\right)$ *No agent wishes to change occupation from manager to worker or to change from unconstrained to constrained.*

$\left(E2^{RIG}\right)$ *The choice of* $n^{*}_{N,\tau,k}(\alpha)$ *for each manager* α *is optimal given their skills, taxes* τ *and wages* w*;*

$\left(E3^{RIG}\right)$ *Supply of labor is equal to the sum of demand for labor and* <u>*unemployment*</u>*.*

The model with fixed wages is solved in the same way as before; the main differences relate to condition $\left(E1^{RIG}\right)$ and to the labor market equation.

Condition $\left(E1^{RIG}\right)$ now compares the profit a "small" (untaxed) potential entrepreneur with the *expected* wage of a worker, who earns $w^{*,RIGID}_{N,\tau,k}$ when it is employed, but 0 if she is unemployed:

$$\alpha^{RIGID}_{\min} f\left(n^{*,RIGID}_{N,\tau,k}(\alpha^{RIGID}_{\min})\right) - w^{*,RIGID}_{N,\tau,k} n^{*,RIGID}_{N,\tau,k}(\alpha_{\min}) = (1 - u^{*,RIGID}_{N,\tau,k}) w^{*,RIGID}_{N,\tau,k} \tag{15}$$

The labor market equation is also modified, since now the regulation generates unemployment:

$$(1 - u^{*,RIGID}_{N,\tau,k}) \int_{\underline{\alpha}}^{\alpha_{\min}} \phi(\alpha)\mathrm{d}\alpha = \int_{\alpha_{\min}}^{\infty} n^{*RIGID}_{N,\tau,k}(\alpha)\phi(\alpha)\mathrm{d}\alpha \tag{16}$$

The remainder of the welfare analysis is otherwise unaltered.

3 Empirical Strategy

In this section we explain how we apply our theoretical framework to the data. First, we allow for some measurement error which is necessary to fit the employment data. Second, we discuss identification and inference. Third, we show how we can make empirical welfare calculations.

[17] These cutoffs are also parameterized by N, τ and k, but we remove these superscripts to improve readibility.

3.1 Empirical Model

Recall that our starting point is the pdf of n^*, which is, according to the theory, given by equation (13).

Employment is measured with error so we assume that rather than observing $n^*_{N,\tau,k}(\alpha)$ we observe:

$$n_{N,\tau,k}(\alpha, \varepsilon) = n^*_{N,\tau,k}(\alpha)e^{\varepsilon}$$

where the measurement error ε is unobservable. In the data we observe the distribution of n, and thus obtaining the likelihood function requires that we obtain the density function of n. The law of $n|\varepsilon$, has support on $[e^{\varepsilon}; +\infty[$. The conditional cumulative distribution function is given by (see Appendix A):

$$\mathbb{P}(x < n|\varepsilon) = \begin{cases} 0 & \text{if} \quad \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) < \varepsilon \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (ne^{-\varepsilon})^{1-\beta} & \text{if} \quad \ln(n) - \ln(N) \leq \varepsilon \leq \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} T\left(n_u^{N,\tau,k}\right)^{1-\beta} & \text{if} \quad \ln(n) - \ln(n_u^{N,\tau,k}) < \varepsilon \leq \ln(n) - \ln(N) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} T\left(ne^{-\varepsilon}\right)^{1-\beta} & \text{if} \quad \varepsilon \leq \ln(n) - \ln(n_u^{N,\tau,k}) \end{cases}$$

Let ε be normally distributed with mean 0 and variance σ. Integrating over ε we can compute the unconditional CDF simply as:

$$\forall n > 0, \quad \mathbb{P}(x < n) = \int_{\mathbb{R}} \mathbb{P}(x < n|\varepsilon)\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right) \mathrm{d}\varepsilon.$$

In Appendix A we show that no further constraints on the parameters are required for this object to be a CDF:

Lemma 1 *Let ε be normally distributed with mean 0 and variance σ so that the measurement error is log normal. Then the function $\mathbb{P}(x < n)$ is a cumulative distribution function, that is strictly increasing in n, with $\lim_{n \longrightarrow 0} \mathbb{P} = 0$ and $\lim_{n \longrightarrow \infty} \mathbb{P} = 1$ for all feasible values of all parameters, σ, θ, T, β, and $n_u^{N,\tau,k}$.*

Thus taking the derivative of $\mathbb{P}$ formulated in this way we can obtain the density of the observed n. Given such a density, it is straightforward to estimate the parameters of the model by maximum likelihood.[18] Specifically, the maximum likelihood estimation yields estimates of the parameters: $\widehat{\sigma}, \widehat{T}, \widehat{\beta}, \widehat{n}_u$ and $\widehat{k}$.[19]

Figure 5 shows the difference between the pure model where employment was measured without error and the true model where there is measurement error. The solid (blue) line shows the firm size distribution under

[18] In a previous version of the paper we generated OLS estimators of these parameters. However, Bauke (2007) and Howell (2002), both within the physics literature, have shown that least square methods may be unreliable. Gabaix and Ibragimov (2011) make the same point and propose a simple rank-based method with the robust approximations for for standard errors. This methodology is adequate for the analysis of the upper tail of a power law distribution, but not for the medium part as in our case. In Appendix B we show how to obtain OLS estimates of the parameters of interest developing a new methodology borrowed from the time series literature on structural breaks. These results suggest a larger implicit tax of the regulation.

[19] See section 3.2 for the identification of k.

the pure model of Section 2 (same as Figure 3) whereas the hatched line shows the firm size distribution when we allow for measurement error. The smoothness of the hump around 50 will depend on the degree of measurement error - Figure 5 shows that if we increase the measurement error to $\sigma = 0.5$ instead of $\sigma = 0.15$ it is almost impossible to visually identify the effects of the regulation.

3.2 Identification and Inference

ML estimation over the size distribution allows us to obtain most of the parameters of interest. Intuitively, the slope of the line in Figure 5 (which is the same before and after the cut-off) identifies β, the power law parameter. The composite parameter $T = \tau^{-\frac{\beta-1}{1-\theta}}$ which is a function of our key object of interest the implicit tax, τ, is identified from three related features of the data. First, the downward shift of the power law slope around 50 employees. Second, the hump of firms just before the regulatory threshold at 50 employees and third the width of the "valley" in the size distribution between 49 employees and where the power law recovers at n_u. The larger is the implicit tax, the greater will be the downward shift, the hump of firms at the regulatory threshold and the depth of the valley in the firm size distribution. The fixed costs k/w, are identified from the indifference equation (4) of the marginal manager around the regulatory threshold. This will also generate a hump and valley, but will not generate a downward shift in the power law as the marginal cost of labor remains at w. Hence the existence of a large downward shift in the slope of the firm size distribution after the regulatory threshold is powerful evidence of a variable cost component of the regulation. The measurement error, σ, is identified from the size of the random deviations of size from the broken power law.

Given the estimates of T, β, and σ we still need an estimate of returns to scale θ in order to identify the key tax parameter, τ. There are several ways to obtain θ. In principle, it can be recovered from the size distribution itself jointly with the other parameters (see Appendix A2). This method relies on rather strong assumptions over the identity of the smallest firm from the indifference condition between being a worker and a manager in (5). As discussed in Appendix A2, empirically the data is not rich enough to estimate θ from the size distribution alone (although we can reject very large values of the parameter), so we consider several alternatives in order to examine the empirical robustness of our estimates of τ. Our first approach is to calibrate θ from existing estimates. Since this is well recognized to be an important parameter in the macro reallocation literature there are a number of papers to draw on. Basu and Fernald (1997) show a large number of estimates based on US data and suggest a value of 0.8 is reasonable. Most calibrations seem to take a value of around 0.8 (e.g. Guner et al, 2006, use a $\theta = 0.802$ for Japan). Atkeson and Kehoe (2005) using a version of the Lucas model with organizational capital suggest a value of 0.85. We also consider more extreme values of $\theta = 0.5$ (used by Hsieh and Klenow, 2009) and $\theta = 0.9$ in the results section.

A second approach is to use information from the production function. Since we have rich data on firms we can estimate production functions and from the sum of the coefficients on the factor inputs estimate returns to scale. Appendix C.2 details how we do this using a variety of methods such as Levinsohn and Petrin (2003), Olley and Pakes (1996) and the more standard Solow residual approach. A third method is to use the relationship between size and TFP from equation (11) to back out an estimate of the returns to scale.

Given one of these estimates of θ, we have an estimate of the implicit (variable) tax of regulation as:

$$\widehat{\tau} = \widehat{T}^{-\frac{1-\widehat{\theta}}{\widehat{\beta}-1}} \tag{17}$$

We obtain standard errors for the estimates of the tax using block-bootstraping at the industry four-digit level, with 100 replications.

4 Institutional Setting and Data

4.1 Institutions: The French Labor market and Employment Costs

France is renowned for having a highly regulated labor market (see Abowd and Kramarz, 2003; Kramarz and Michaud, 2010). What is less well known is that most of these laws only bind on a firm when it reaches a particular employment size threshold. Although there are some regulations that bind when a firm (or less often, a plant) reaches a lower threshold such as 10 or 25 employees, 50 is generally agreed by labour lawyers and business people to be the critical threshold when costs rise significantly.

In particular, when firms get to the 50 employee threshold they need to undertake the following duties (see Appendix D for a comprehensive overview):

- They must set up a "works council" ("comité d'entreprise") with minimum budget of 0.3% of total payroll.
- They must establish a committee on health, safety and working conditions (CHSCT)
- A union representative (i.e. not simply a local representative of the firm's workers) must be appointed if wanted by workers
- They must establish a profit sharing plan
- They incur higher liability in case of a workplace accident
- They must report monthly and in detail all of the labor contracts to the administration.
- Firing costs increase substantially in the case of collective dismissals of 10 or more workers. This increase is an implicit tax on firm size (e.g. Bentolila and Bertola, 1990) which makes firms reluctant to hire.
- They must undertake to do a formal "Professional assessment" for each worker older than 45.

How important are such provisions for firms? Except in the case of the minimum regulatory budget that is to be allocated to firm councils, which provide an order of magnitude for a lower bound, it is extremely hard to get a handle on this. For example, what is the opportunity cost of managerial time involved in dealing with works councils, union representatives, health and safety committees, etc.?

Our framework is designed to recover the costs of such regulations, specifically to recover both the variable and fixed cost components. While some of them have a larger variable cost component (such as the works council budget) others may involve larger fixed costs.

4.2 Data

Our main dataset is constructed from administrative (fiscal) data covering the universe of French firms[20] between 2002 and 2007. These are based on the mandatory reporting of firms' income statements to tax authorities and hold about 2.2m observations per year. Our main results are on the approximately 200,000 firms active in manufacturing industries (NACE2 classes 15 to 35) as productivity is easier to measure in these industries. We also look at all the other main private non-manufacturing sectors in extensions of the baseline results. The data are the (mandatory) fiscal returns of all French firms ("FICUS") and are the appropriate level for analysis as it is on this administrative unit ("entreprise") that the main laws pertain to.

In addition to accurate information on employment (average number of workers in last quarter of the fiscal year), FICUS contains balance sheet information on capital, investment, wage bills, materials, four digit industry affiliation, etc. that are important in estimating productivity. We also use the DADS (Déclarations Annuelles de Données Sociales) dataset in some of the robustness tests which contains worker-level information on hours, occupation, gender, age, etc. Details of the TFP estimation procedure, which in the baseline specification uses the Levinsohn and Petrin (2003) version of the Olley and Pakes (1996) method[21], are reported in Appendix C.2.

5 Results

5.1 Qualitative analysis of the data

Before moving to the econometrics we first examine some qualitative features of the data to see whether they are consistent with our model. Many commentators have expressed skepticism about the quantitative importance of employment regulations as it is sometimes hard to observe any clear change in the size distribution

[20] See Di Giovanni, Levchenko and Ranciere (2011) and Caliendo, Monte and Rossi-Hansberg (2012) for other work on these data.

[21] Our baseline results use the Levinsohn and Petrin (2003) version of the Olley and Pakes (1996) method of using a control function approach to deal with unobserved productivity shocks and selection when estimating production functions. Because we have a panel of firms we can implement this and estimate the production function coefficients. There are several issues with this approach (see Ackerberg et al, 2007) to estimating production functions so we also estimate TFP using a variety of other methods (see Appendix C.2 for details, results available upon request).

around important legal thresholds[22], so we first focus on this issue. Figure 6 presents the empirical distribution of firm size around the cut-off of 50 employees for two datasets. The dataset we use (FICUS), the fiscal files of the French tax administration, is the population dataset of the universe of French firms that forms the basis of our econometric work. Panel 6.1 in Figure 6 is the same as Figure 2. As previously discussed, there is a sharp discontinuity in size precisely at 50 employees which is strong non-parametric evidence for the importance of the regulation. There are 416 firms with exactly 49 employees and then only 160 with 50 employees. Importantly, the distribution which declines from 31 employees flattens after about 44 employees, just before the stacking up at 49 employees then dropping off a sharp cliff when size hits 50. The top right hand side of Figure 6 shows this in log-log space clearly indicating the evidence of a "broken power law".

The next panel of Figure 6 compares FICUS with another dataset, DADS, that is also frequently typically used by labor economists. In Panel 6.2 we aggregate employment up to the appropriate level for each FICUS firm. This enables us to investigate different measures of employment such as employment dated on 31st December or full-time equivalents. The discrete jump at 50 shows up here almost as clearly as the FICUS data. The bottom panels of Figure 6 uses Full-Time Equivalents (over one calendar year) which shows less of a jump than the straight count of employees in the previous panels; we rationalize this fact in section 6.3. Figure 6 illustrates the importance of good data - one of the reasons that other studies have not identified such a clear discontinuity around the regulatory threshold is that they may have been using data with greater measurement error than our own. Recall that Figure 5 illustrates the problem of how measurement error can disguise the effect of the regulation.

Figure 7 shows the firm size distribution over a larger range between 1 and 1,000 employees. Overall, firm size seems to approximate a power law in the employment size distribution prior to the bulge around 50. After 50, there is a sharp fall in the number of firms and the line more flat than expected before resuming what looks like another power law. Broadly, outside a "distorted" region around 50 employees, one could describe this pattern a "broken power law" with the break at 50.[23] The finding of the power-law for firm size in France is similar to that for many other countries and has been noted by other authors (e.g. Di Giovanni and Levchenko, 2010), but the finding of the break in the law precisely around the main labor market regulation is new to the academic literature (the only exception is Ceci-Renaud and Chevalier, 2011). As is well known the power law fits rather less well for the very small firms. Additionally, there does appear to be some break in the power law at firm size 10. This corresponds to the size thresholds from other pieces of labor and accounting regulations (see Appendix D). In order to avoid conflating these issues we focus our analysis on firms with 10 or more employees, and therefore on the additional costs generated by regulations at threshold

[22] For example, Schivardi and Torrini (2008) and Boeri and Jimeno (2005) on Italian data, Braguinsky et al (2011) on Portuguese data or Abidoye et al (2010) on Sri Lankan data. The authors find that there is slower growth just under the threshold consistent with the regulation slowing growth, but they find relatively little effect on the cross-sectional distribution. This may be because of the multitude of regulations, variable enforcement or measurement error in the employment data (see sub-section 2.3).

[23] See Howell (2002) for examples of how to estimate these types of distributions. More generally see Bauke (2007) for ways of consistently estimating power laws.

50 relative to average labor cost for firms having 10 to 49 employees. In principle however, the methods used here could be generalized to other breaks in the power law.

Our basic model, following Lucas, has the implication that more talented managers leverage their ability over a greater number of workers (Figure 4). Figure 8 tests this property and plots the mean TFP levels by firm size. Panel A does this for firms between 5 and 100 employees whereas Panel B extends the threshold out to firms with up to 1,000 employees. In all panels productivity appears to rise monotonically with size, although there is more heteroskedacity for the larger firms as we would expect because there are fewer firms in each bin. The relationship between TFP and size is broadly log-linear. What is particularly interesting for our purposes, however, is the "bulge" in productivity just before the 50 employee threshold. We mark these points in red. This looks consistent with our model where some of the more productive firms who would have been just over 50 employees in the counterfactual world, choose to be below 50 employees to avoid the cost of the regulation. Firms just below the cut-off are a mixture of firms who would have had a similar employment level without the implicit tax and those firms whose size is distorted by the size-related regulation.

We exploit the relationship between size and TFP to identify the θ, returns to scale parameter in some of the empirical estimates.

5.2 Econometric Implementation

The key parameters are estimated from the size distribution of firms using the ML procedure described above. We begin in Table 1 with a set of baseline results using calibrated values of θ for the entire sample of French manufacturing firms 2002-2007.[24] We begin by using a calibrated value of $\theta = 0.8$ from Basu and Fernald (1997) and Guner et al (2006, 2008) in column (1). The slope of the power law, β, is about 1.8 and highly significant. The upper employment threshold, n_u, is estimated to be about an employment level of 58 and we obtain a standard deviation of the measurement error of just over 0.10, which suggests significant, but not major amounts of mismeasured employment. Turning to the estimates of the tax equivalent costs of the regulation, we obtain $T = 0.948$ which is determined in part by the implicit variable labor tax which we estimate to be $\tau = 1.013$ and highly significant. This implies that the regulation increases variable costs by 1.3 percentage points which is a moderately large and important effect. By contrast the fixed cost component of the regulation is insignificant at the 5% level, incorrectly signed and small in magnitude.[25] We will therefore focus on variable costs of regulation in the rest of the paper. Figure 9 shows the data and the fit of the model using the estimated parameters. Although not perfect, we seem to do a reasonable job at mimicking the size distribution even around the regulatory threshold.

[24] We use a sample of firms with between 10 and 1,000 employees correcting our estimates for censoring at the lower and upper known thresholds. We do this because there are other regulations that bite at 10 employees.

[25] The estimate obtained for k/w is of -0.496, which suggests that the fixed cost component of the regulation is 17,649 Euros (since average labor cost is 35,583 Euros) which is small. For example, for a 250 employee firm this will be less than 0.2% of total labor costs.

Column (2) of Table 1 considers an alternative calibration of $\theta = 0.85$ from Atkeson and Kehoe (2005). The results are very stable, although as expected the estimate of the marginal tax falls from 1.3% to 1%. This is because the importance of the distortions of the tax depend on returns to scale. When returns to scale are close to unity the most efficient firms have a very large share of output, so it only takes a small distortionary tax to have a large effect on the size distribution. As decreasing returns set in it takes a much larger estimate of τ to rationalize any given distorted distribution (the "downward shift", "hump" and "hole" in firm size). Column (3) considers $\theta = 0.50$ and column (4) a $\theta = 0.90$. The first case is formally equivalent to a minimum firm size of one employee which is empirically consistent with the data and implies a large tax of 3.3 percentage points. The second case reduces the implicit tax to 0.4 percentage points, but this specification appears to be rejected by the data since the log-likelihood drops substantially.

Table 2 uses the returns to scale parameter directly estimated from a production function (see Table A1 and Appendix C). The first column just reproduces our baseline results from column (1) of Table 1. Column (2) contains the results using the estimations from the production function giving a value of $\theta = 0.855$ that is highly significant. Other parameter estimates remain stable and we obtain an estimate of $\tau = 1.010$ only slightly lower than the baseline case of $\tau = 1.013$. Column (3) uses the TFP estimates from the production function to estimate the TFP-size relationship as in Figure 10. We recover an estimate of θ from the slope of this relationship which is $\theta = 0.799$ and re-estimate all the parameters. This generates a generally stable values with an estimate of $\tau = 1.013$, again very close to the baseline results.

5.3 Changes in the level and distribution of welfare

Our model allows us to fully calculate the impact of the regulation on the firm size distribution, output and welfare. As shown in Section 2, the slope of the power law does not change as a result of the implicit tax. The impact of the tax is a parallel move upwards of the firm size distribution at sizes $n < 49$, a spike at $n = 49$, and a parallel move down for $n > 58$. The counterfactual firm size is a power law with the exponent β we calculated in our analysis, and $\tau = 1$. The position of the intercept is pinned down by the labor market condition, which requires that the total number of agents in the economy is constant, and by the minimum firm size which in our specification is pinned down by the returns to scale parameter and also stays constant.[26]

We need to choose an upper bound for firm to make these calculations as formally the power law will give positive mass to firms of near infinite size which is obviously not a feature of real world data. Like the returns to scale parameter θ (see Section 3.2), this upper bound is in principle obtained from the firm size distribution, but in practice, since we only the use data over the range 10 to 1,000 employees and use a conditional specification of the likelihood, we do not have enough information on this upper bound to actually estimate it. Our baseline calculations use a "calibrated" firm of size 10,000 as the maximum although we

[26] Note that this is not the case when the regulation binds for firms of all sizes (i.e. $N = 0$ and $\tau > 1$), or when wages are rigid. See Section 2 and Appendix A for the details of the derivation.

also provide estimates (see Appendix Table A2) where we vary the upper bound with very little qualitative effect on the results, for reasons we shall explain.

Figure 10 presents the change in the firm size distribution in the world with regulation (bold line) and without regulation (dashed line) using the estimated parameters from our model. As the theory led us to expect, in the counterfactual unregulated economy there are fewer firms under 49 employees. This is because in the regulated economy (i) there is a spike at 49 employees for those firms who are optimally avoiding the regulation and (ii) since equilibrium wages have fallen there is an expansion in the number of small firms. Compared to the unregulated economy, the regulated world has fewer large firms since there is an additional implicit tax to pay. This downward shift in the size distribution is less dramatic because these large firms also benefit from the lower equilibrium wages caused by the regulation which offsets some (but not all) of the regulatory burden.

Figure 11 examines the distribution of output across entrepreneurs of different ability in the regulated and unregulated economies again using our estimated parameters. Without loss of generality we normalize the maximum ability to unity. Empirically, individuals with managerial ability of 0.341 employ exactly 49 employees in the unregulated economy. Entrepreneurs of this ability or below produce more in the regulated economy because they benefit from lower labor costs. We estimate that individuals with ability levels between 0.341 and 0.369 will choose to employ exactly 49 workers to avoid the regulation. For entrepreneurs in this interval, although their output continues to rise with ability as in the unregulated economy, it rises at a slower rate because their firms are not growing in size as their ability increases. The entrepreneurs with high ability who choose to pay the implicit tax and will grow at the same rate (with respect to ability) as in the unregulated world. However, since labor costs are higher for these firms (lower equilibrium wages only partially offset the higher implicit tax), their output is a bit below that of the unregulated world.

Figure 12 examines the income changes for individuals of different ability in the regulated and unregulated economies. The difference between the bold and dashed lines indicates the distributional effects of the regulation. It can be immediately seen that there are two groups of losers and one group of winners from the regulation. Low ability individuals (below 0.21) are the biggest losers: these are workers who suffer from approximately a 1% fall in their wages. At the other end of the spectrum are large firms who also lose profits, although slightly less than 1%. The winners are the "middle classes" comprised of the small firms who enjoy lower labor costs and a group of workers who are induced to become entrepreneurs by the lower equilibrium wages due to the regulation (those with ability in the range 0.21 to 0.34). As in the Figure 11, individuals with ability between 0.34 and 0.37 are those firms who choose to avoid the regulation. Most of them are better off under regulation because of lower wages, but a few of the more able are actually worse off as they could be larger firms without the implicit tax.

The exact numbers underlying the welfare calculations in Figures 11 and 12 are in Table 4. Column (1) labeled “flexible wages” has a full employment equilibrium (row 1). 3.6% of firms are on the spike at 49 employees and 9% of firms choose to pay the implicit tax. Equilibrium wages falls by over $\ln\left(w^*_{N,1,0}\right) - \ln\left(w^*_{N,\tau,0}\right) \approx 1\%$ (rows 4 and 10.a). Labor costs rise by 0.2% for large firms and they are 1.2% smaller. Small firms' costs fall by 1% and these firms are 5.4% larger.

The pure deadweight output loss is quite small with flexible wages (0.02% of GDP in row 9b), but the overall welfare loss depends on how one regards the implicit tax revenues which are 0.8% of GDP in row 9a. The pure administrative (e.g. reporting) cost element of this can be regarded as deadweight loss, but much of the implicit tax are wage payments to the unionists, lawyers and HR staff and could be regarded as transfers. Of course, these groups may also absorb managerial time and cause disruption and therefore output reductions which would need to be netted out. The loss may also be smaller or larger to the extent that the workers value the amenities provided by the labor regulation[27].

How much do workers value these amenities? We examine this empirically by looking at wages around the threshold (see Figure 13). If workers prefer to work at firms that have there are such insurance benefits we would expect wages to be lower after the threshold of 50 employees. As expected the wage is upward sloping, but there does not appear to be a significant fall in wages after the regulatory threshold. However one regards the implicit tax, however, the total welfare loss from the regulation is not large - under 1% of GDP (row 9c).[28]

We focus next on the distributional consequences. Row 10 details the winners and losers. As suggested by Figure 12 workers and large firms lose about 1% (rows 9a and 9e) whereas the smaller firms all gain (rows 9b-9d).

The second column of Table 4 considers the case of downwardly rigid wages due to say the strong minimum wage or union strength (about 90% of all workers in France are covered by a collective bargain[29]). In this case unemployment rate of 5.2% emerges in the regulated economy as wages do not fall for those who are employed (note that the unemployment rate in France was between 8% and 9% in our sample period). The welfare loss rises to 5.1% of GDP if implicit taxes are included (row 9a) or 4.3% if they are excluded (row 9b). The large welfare loss arises from the 5.36% aggregate income loss of those in the labor force (many of whom now do not have jobs) and a 6.5% fall in the profits of large firms as they now have to accept the full burden of the regulation and cannot offset this against lower equilibrium wages. Similarly, small firms gain nothing as labor costs are no lower. Interestingly, the distributional consequences have a similar flavor as before. The “working class” loses out from the regulation because they have less jobs rather than lower

[27] In the case where the amount of tax collected is distributed to workers on top of their wage, their income would decrease by 0.06% instead of 1.07% (as compared to the case without tax).

[28] The modest magnitude of the welfare cost is perhaps unsurprising as the regulation does not cause the rank order of firm size to change with the ability distribution. Hopenhayn (2012) shows in a general context that first order welfare losses from misallocation require some rank reversals between ability and size.

[29] http://www.eurofound.europa.eu/eiro/country/france.pdf

wages. All workers are homogenous so they make an *ex ante* decision to enter based on their expected wage (relate to expected profits from being an entrepreneur) and there is a random draw *ex post* to determine who will be unemployed. In our risk neutral set up low ability individuals lose out to a similar degree regardless of the degree of wage inflexibility. Highly able individuals lose out substantially in terms of much lower profits if wages are rigid, but less if wages are flexible. In terms of political economy, this may be why large firms lobby hard against increases in the minimum wage.

Although the empirical maximum of firm size is 86,587 in the data, we choose a more realistic upper bound[30] of 10,000 since there are on average only 5 firms per year having a size greater than 10,000 (out of an average of 170,000 firms with positive employment in manufacturing industries). Table A2 shows that our quantitative estimates of welfare are not much changed when we vary our assumption about the upper bound of firm size (using alternative values of 500, 1,000 and 5,000 employees). As we drop more of the larger firms unsurprisingly, welfare costs are slightly lower as the largest firms lose more from paying higher labor costs. But these differences are not dramatic. For example, the output loss when we take the maximum firm size to be 500 is 3.9% of GDP in the case of rigid wages compared to 4.3% in our baseline case. Consequently, although we may be understating the welfare losses by using an upper bound of 10,000, the effect is likely to be small.

In summary, we have two main quantitative results. First, aggregate welfare losses from the regulation are less than 1% of GDP when wages are flexible, but are around 5% of GDP if wages are downwardly rigid. Second, the regulation redistributes income away from workers and larger firms and towards those of mediocre managerial ability. The first result is well known in the literature (the incidence of a labor tax will partly fall on workers), quantifying its magnitude in a specific equilibrium setting is original. Furthermore, the second result on distributional consequences of regulation is, we believe novel and unexpected, especially when quantified from a structural econometric model.

6 Extensions and Robustness

In this section we consider several extensions to our framework and robustness tests of the results.

6.1 Industry Heterogeneity

Holding the parameters constant across industries is an attempt to focus on the macro-economic consequences of the regulation. But there is nothing in our approach that requires we do this. Consequently we have investigated various ways of allowing the coefficients to vary across industries. We begin with simply splitting the industries into "high tech" and "low tech" following OECD definitions (these are based on R&D intensity). The estimates of parameters are given in columns (4) and (5) of Table 2 and the analogous production function

[30] As explained in what follows, choosing a higher value for the upper bound would increase welfare costs.

estimates are in the last two columns of Table A1. There does appear to be significant heterogeneity with the estimated implicit tax insignificantly different from zero in the high-tech sectors and bearing more heavily in low-tech sectors (1.3%).

Next, in Table 4, we examine the other main sectors outside manufacturing. We use calibrated values of $\theta = 0.80$ and $\theta = 0.85$. The first two columns repeat the baseline estimates using these values. We estimate the models for the other four large sectors of the French economy outside manufacturing in the rest of the table - Transport (columns (3) and (4)), Construction (columns (5) and (6)), Wholesale and Distribution Trade (columns (7) and (8)) and Business Services (columns (9) and (10)). The implicit tax seems to be more important in both Transport (2.5%) and Construction (2.0%) than it was in manufacturing (1.3%). In business services, by contrast, the regulation seems to be estimated to be insignificantly different from zero.

Finally, we estimated the production functions separately by three digit sectors and used the full ML technique with estimated production functions as in column (2) of Table 2. This allows the scale (θ) and all other parameters to be freely estimated. Some of the industries have insufficient number of firms to perform this estimation but we are still able to do this for a large number. The results are in Figure A1 which again demonstrates a substantial degree of heterogeneity with some sectors with estimates of the implied tax from near zero to over 50%. The heterogeneity of the implicit taxes is related to industry characteristics in an intuitive way. For example, when labor costs are a smaller share of total value added, the estimated $\tau's$ tend to be bigger and when the capital-labor ratio of the sector is high they tend to be smaller. The distortion associated with the regulation is less damaging in sectors when labor is a less important factor (Marshall's "importance of being unimportant").

6.2 Changing the organizational structure of corporations

An obvious way in which a business group could respond to the regulation is simply by misreporting employment size. The authorities and unions are well aware of this incentive and threaten hefty fines and prison sentences for employers who lie to the fiscal or social security authorities. A more subtle way of dealing with the regulation is by splitting a company into smaller subsidiaries. For example, a firm which wished to grow to 50 employees could split itself into two 25 employee firms controlled by the group CEO. There are costs to such a strategy - the firm will have to file separate fiscal and legal accounts, demonstrate that the affiliates are operating autonomously and suffer from greater problems of loss of control.[31]

One way to check for this issue is to split the sample into those firms that are standalone businesses and those that are subsidiaries/affiliates of larger groups. We split our firms into those that are standalone companies without subsidiaries and those which are part of larger groups. Panel A of Figure 13 compares the power law for these two types of firms. For both standalone firms and affiliates we can observe the broken

[31] A more extreme reaction of the firm would be to engage in franchising. This has some further costs as the CEO no longer has claims over the residual profits of the franchisee and loses much control. In any case, franchising is rare in manufacturing.

power law at 50. The fact that it exists for standalone firms (which cover the majority of workers for firms in Figure 13) implies that our results are not being driven solely by corporate restructuring. Panel B repeats the distribution for the standalones, but now considers affiliates aggregated up to the *group* level. Although the power law is still broken at 50 it is less pronounced than at the subsidiary level in Panel A which could indicate some degree of corporate restructuring in response to the regulation.

For the latter we aggregate employment to the group level. In Figure 14 we can see a clear discontinuity around 50 employees for the group size as well as the standalone firms. This suggests that firms are not able to avoid the regulation simply by restructuring their form. This suggests that corporate restructuring does not full undue the regulation.

6.3 Other margins of adjustment to the regulation

The simplest version of the model focuses on the decision over firm size based on employment. However, there are many other possible margins of adjustments that firms could use to avoid the regulation. This can be allowed for in the model by re-writing output as $y = \alpha[h(n, x)]^{\theta}$ instead of $y = \alpha n^{\theta}$ where x are the other factors of production such as hours per worker or human capital. If there was perfect substitutability between labor and these other factors then the firm could avoid the size-distortion we have discussed. More realistically when there is imperfect substitution the firm can mitigate some of the costs of the regulation through substitution. Of course, having to sub-optimally substitute into other factors of production generates some welfare loss by itself.

The most obvious way the firm could adjust is by increasing the amount of hours per worker rather than expand the number of employees. We find clear evidence that firms respond in this way in Figure 15 as the number of annual hours increases just before the threshold of 50 employees. This is a combination of firms making workers do more overtime hours and substituting towards full time workers and away from part-timers. This is reassuring as it suggests that firm size is not just being misreported to avoid the regulation - firms are genuinely changing their activities in a theoretically expected direction.

There are many other possible margins of adjustment such as using more skilled workers, increased capital intensity and a greater use of outsourced workers. Appendix C3 shows some evidence that firms are using all these margins of adjustment in order to mitigate the costs of the regulation. Note that since we observe all these margins we are able to take account of them in our estimation of the production function[32]. They should therefore not in principle bias our estimates of TFP.

Finally, there is the issue of whether firms could just lie about their employment size to the authorities in order to evade the regulation. There is certainly evidence of this from tax returns (e.g. Almunia and Lopez-Rodriguez, 2012), although generally researchers are surprised at how low these rates are given the

[32] The estimated equation incorporates capital as a factor of productions, and labour is measured in terms of hours.

incentives (e.g. Kleven, Kreiner and Saez, 2009). There is, of course, a cost to these evasion strategies in the form of legal fines and possible criminal charges and we doubt this can be a major factor accounting for the results. First, we have shown that there is adjustment on a number of margins that is consistent with a real effect of the regulation such as productivity and hours of work. There is no incentive for the firm to report these other items in a systematically misleading way. Second, hiding taxable revenues is much easier than hiding workers who have a very physical presence with a legal contract, health and pension rights. Third, in 2009 alone the French state employed 2,190 "agents de controle" to monitor firms compliance.[33] That is about one agent per 100 firms with ten or more employees, a rather high degree of observation. Third, hidden workers could be considered an unmodelled additional factor of production (x) as above. As we show in the next sub-section our model performs reasonably well in predicting output even abstracting away from such considerations.

6.4 Overall Performance of the Model

Firms do appear to be adjusting to the regulation around the threshold, in particular by attempting to increase hours rather than raw labor when they get close to 50 employees. However, if firms are able to perfectly substitute into these other types of activity the regulation has a smaller welfare effect than what we compute. To address the magnitude of this problem we analyze the implications of our model for predicting the distribution of output.

Recall that we are using the distribution of firm size and employment to estimate the parameters of our economic model. We do not use information on output (except very indirectly in the experiments that look at TFP, such as column (2) of Table 2). If alternative margins of adjustment were important, then the observed distribution of output would be very different (and total output larger) than what is predicted by our model. In particular, we expect to under-estimate the share of output produced by large firms, and to over-estimate the share of output produced by small firms. As explained in the previous sub-section, large firms can reduce their regulatory cost by using existing workers more intensively (e.g. hours), increasing workforce quality through upgrading skill composition, using greater capital, etc. All these will cause them to produce more output than we would predict in our basic model.

Table 5 contains the results of this exercise. Panels A and B compare the actual and predicted distributions of firms and employment. Unsurprisingly we match these pretty closely as this is the data that we are using to fit our parameters. Panel C is the greater challenge as we do an "out of sample" prediction– the output estimation. The proportion around the hump of 49-57 workers is nearly spot on at 3%. As expected, we under-estimate the output for larger firms, but not by too much. Our parameters suggest that 69% of output should be in firms with over 58 employees whereas the number is 72.8% in the data. As shown by the standard

[33] http://www.insee.fr/fr/ffc/ipweb/ip1399/ip1399.pdf

errors we are well within the 95% confidence intervals for the data which is a pretty good job for a simple model.

6.5 A Comparison with the Hsieh-Klenow (2009) approach

In a pioneering and influential paper, Hsieh and Klenow (2009) take a related approach to examining regulatory distortions by examining the distributions of size and productivity in the US, India and China. Like us, they found that such distortions could be substantial at the macro-economic level. Their approach, however, focuses on the variation in marginal revenue productivity (MRP) as an indicator for distortions because, when factor prices are the same MRPs should be equalized across firms even when underlying managerial ability is heterogeneous. In our context, we follow Hsieh and Klenow (2009) and estimate the distortion (τ) from the change in the MRP. The relation can be seen in Appendix Table A2. Naively using data around the threshold to look at the change in MRP would be incorrect, however, as this local distortion in the MRP is due to the decision of firms in the 50-60 range to optimally choose to be at 49. The treatment effect in the RDD would reflect the local distortion and not the global distortion. The Regression Discontinuity Design (RDD) approach would lead to misleading estimates of the implicit tax. Instead, our approach would suggest comparing the MRP for firms away from the threshold. When implement this idea using the Hsieh Klenow method of value added per worker (relative to the industry average) as an index of MRP we obtain an estimate of $\tau = 0.046$(Appendix Table A4).[34] Using our own estimates of MRP from the model ($\alpha\theta\,(n^*)^{\theta-1}$) we obtain a value of $\tau = 0.032$ (Appendix Table A4). Although we prefer our more structural approach, we note that these estimates are similar to those in Table 1 column (3) where we use the Hsieh and Klenow's (2009) preferred measures of the returns to scale, θ to obtain a $\tau = 0.033$. Our preferred estimates use a higher θ and generate a lower estimate of the implicit tax.

6.6 Alternative reasons for the existence of firms in the "valley"

The fact that there are any firms to the right of the regulatory threshold at 50 employees is a theoretical puzzle that we account for by allowing for measurement error in employment. Empirically, our measurement error hypothesis does quite well (Figure 9) if anything, overpredicting the number of firms. However, there are a number of alternative hypothesis worth considering involving dynamics, shocks and optimization errors of different kinds.

Adjustment Costs. A first alternative hypothesis that could instead explain firms choosing the "dominated" firm sizes on the range immediately above 50 employees would be the existence of adjustment costs. Suppose for concreteness that firms receive shocks to their target employment numbers (e.g. via α shocks). In

[34] This is using the average for firms between 20 to 42 workers compared to firms with 57 to 200 workers. Reasonable changes of the exact thresholds make little difference.

a world with quadratic adjustment costs, firms would want to get to new target employment, but would converge slowly. This planned employment dynamic might take a firm through the “valley” (50,58) as jumping over it to a new optimal employment might simply be too costly in terms of adjustment costs. This implies that firms are “passing through” the valley and will be disproportionately likely to move out of this area.

To investigate this we examine adjustment dynamics. Indeed, Figure 16 plots the proportion of firms making significant adjustments in employment at different points of the firm size distribution. The left hand panels examine the proportion of firms increasing employment by more than 12%, 10%, 8%, 6% and 4% respectively. There is a general tendency for firms to make smaller proportionate changes in employment the larger they are, but the most striking feature is how the pattern alters around the regulatory threshold. Firms to the right of the threshold are much more likely to either grow or shrink than those who are either larger or smaller. This indicates that the “valley” is an uncomfortable place to be: firms are swiftly moving either in or our of it. Similarly, firms to the left of the threshold are much less likely to grow as this would mean they would have to start paying the extra implicit tax. We obtain similar results if we use value added as our measure of firm growth instead of employment.

Although these patterns are all consistent with an adjustment cost explanation, our measurement error hypothesis can also explain both facts. We would expect mean reversion – firms’ measured employment in the valley is actually smaller or larger than their true data. So long as the measurement error is i.i.d. we would expect greater absolute measured changed in employment in subsequent periods. Thus the dynamics in Figure 16 are consistent with both the measurement error and adjustment costs hypothesis.

In terms of the static data however, the adjustment costs hypothesis does not account as well for the presence of a bulge that spreads on several firm sizes just below 49 employees. Absent measurement error, firms that do not want to grow beyond 50 would stop growing at precisely 49, not at 46. Thus we believe that the adjustment cost hypothesis cannot satisfactorily for the precise deviation from the power law we observe in the data. This is not to say adjustment costs are unimportant, but rather they are not the dominant reason for firms in the valley.

Employment shocks. Consider the possibility there are *unexpected shocks* to employment. Suppose firms face an exogenous quit rate, and that they set a hiring rule to achieve a target employment level. In this case, negative shocks to quits could mean the firm ends up with too many workers. For example, suppose a firm wants 49 workers, can only make hiring decisions at the beginning of the year (like the junior job market in economics) and expects 5 people to leave every year. Hence the firm hires 5 people every year. If there is a negative shock to quits one year when no one leaves, the firm will end up with 54 workers and have to bear the costs of regulation. Being aware of this risk, however, implies that firms will not target a steady state of 49, but something below to avoid getting into the dominated area. This hypothesis therefore predicts that the “hump” would be at a firm size *below* 49 workers. Since the main hump is exactly at 49 workers we can rule out this hypothesis since it implies a counterfactual prediction.

Leontief Production Functions. Similarly hard to square with the patterns we observe in the data is the hypothesis that production is in fixed proportions (*Leontief),* and firms with sizes in the {50,58} range are at their optimal employment level given their capital needs, location needs etc. This would imply, again counterfactually, that there would be no particularly strong tendency by these firms to grow or shrink in that firm size range relative to any other range Again, we can rule out this hypothesis, given what we have already observed about the dynamic patterns in the data.

Bounded rationality. We can imagine a wide range of bounded rationality models, and although some are clearly inconsistent with the evidence, some could probably account for the patterns we observe in the data. A first variant would have firms making optimization errors because they do not know how to set up the employment level targets, so that they choose $\not{n}$ but end up with n^*. But if firms know they might make such optimization errors they will try to stay further away from the 50 workers threshold, so that the bulge, again counterfactually like in the previous case, would be at a lower than 49 employee firm size.

A second type of optimization error has a more behavioral economics flavor where firms simply ignore the regulatory cost they incur from being more than 50. Firms here head unaware into the dominated territory, choose a dominated size, and end up producing at a profit level that is "too low." Since most of the firm regulations that start at size 50 have been in place for many decades in France and are well known in the media, this model seems less likely than for a newly introduced policies. Nevertheless, if firms do use such behavioral rules and do not learn from their mistakes firms in the 50 to 58 size band will have particularly low profits. Kleven and Waseem (2012) show that looking at the profits of such firms would actually allow us to learn the cost of the regulation, assuming they are indeed "too low".[35] Following this idea we examine the profitability of firms around the threshold in Figure 17. We do not find a sharp discontinuity in profitability around the regulatory threshold which casts doubt on the second variety of the bounded rationality model.

Summary. Although there could be other reasons for the density of firms in the valley to the right of the regulatory boundary at 50 employees, none of the obvious ones seems obviously better than our simple measurement error story.

7 Conclusions

How costly is labor market regulation? This is a long-debated subject in policy circles and economics. We have tried to shed light on this issue by introducing a structural methodology that combines a simple theoretical general equilibrium approach based on the Lucas (1978) model of the firm size and productivity distribution. We introduce size-specific regulations into this model, exploiting the fact that in most countries

[35] Their paper, like ours, has a strictly dominated region like ours where the average tax rate is higher for some inviduals. Nevertheless, there is positive mass in the valley to the right of this tax knotch

labor regulation only bites when firms cross specific size thresholds. We show how such a model generates predictions over the equilibrium size and productivity distribution and moreover, can be used to generate an estimate of the implicit tax of the regulation. Intuitively, firms will optimally choose to remain small to avoid the regulation, so the size distribution becomes distorted with "too many" firms just below the size threshold and "too few" firms just above it. Furthermore, the distribution of productivity is also distorted: some of those firms just below the cut-off are "too productive" as they have been prevented from growing to their optimal size by the regulation. We show how the regulation creates welfare losses by (i) allocating too little employment to more productive firms who choose to be just below the regulatory threshold, (ii) allocating too little employment to more productive firms who bear the implicit labor tax (whereas small firms do not) and (iii) through reducing equilibrium wages (due to some tax incidence falling on workers) encourages too many individuals to become small entrepreneurs rather than working as employees for more productive entrepreneurs.

We implement this model on the universe of firms in the French private economy. France has onerous labor laws which bite when a firm has 50 employees, so is ideally suited to our framework. We find that the qualitative predictions of the model fit very well. First, there is a sharp fall off in the firm size distribution precisely at 50 employees resembling a "broken power law" and second, there is a bulge in productivity just to the left of the size threshold.

We then estimate the key parameters of the theoretical model from the firm size distribution. Our approach delivers quite a stable and robust cost of the employment regulation which seems to place an additional cost on labor of around 1% of the wage. We show that we expect this cost to translate into relatively small output losses when wages are flexible (under 1% of GDP) but large losses (over 5% of GDP) when wages are downwardly rigid. Furthermore, there are large distributional effects regardless of wage flexibility with workers losing substantively and small firms benefiting from the regulation. This is unlikely to be an intended consequence of the laws.

This is just the start of our research program. Size-contingent regulations are ubiquitous and our methodology can be used for other regulations, other parts of the size distribution, other industries[36] and other countries. One drawback of our approach is that it is static. We have abstracted away, for example, how firm TFP may evolve over time as firms invest to improve their technology or managerial ability (e.g. Bloom and Van Reenen, 2007). Such investments enable small firms to grow and since size-contingent regulations "tax" this growth over the threshold, they may well discourage investment and therefore inhibit the dynamics of growth in the economy. We have also not delved deeply into dynamics of employment or wage setting (e.g. Robin, 2011)

[36] For example, the retail sector has a large number of size-contingent regulations with "big boxes" being actively discouraged in many countries and US cities (e.g. Bertrand and Kramarz, 2002, or Baily and Solow, 2001).

Despite these caveats, we believe that our approach is a simple, powerful and potentially fruitful way to tackle the vexed problem of the impact of regulation on modern economies.

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A Omitted Proofs

A.1 Comparative Statics in the variable cost and fix costs of regulation

In the main text, Proposition 1 focused on comparative statics when we have no fixed cost of the regulation ($k = 0$) as this is what we obtain empirically. We can also examine what happens when we hold the implicit tax fixed at unity ($\tau = 1$) , but consider only the fixed cost component. In the unique equilibrium:

Proposition 2 *The introduction of a tax/fixed cost k of hiring workers starting at firm size N has the following effects:*

1. *Reduces equilibrium wages as a result of the reduction in the demand for workers*
2. *Increases firm size for all firms below the threshold, $[\alpha_{\min}^{N,1,k}, \alpha_c^{N,1,k}]$, as a result of the general equilibrium effect that reduces wages*
3. *Reduces firm size to the threshold N for all firms that are constrained, that is those in $[\alpha_c^{N,1,k}, \alpha_u^{N,1,k}[$*
4. *Increases firm size for all firms that are taxed $[\alpha_u^{N,1,k}, +\infty[$ as a result of the general equilibrium effect that reduces wages*

Most of the comparative statics are the same as Proposition 1, but there is an important difference in terms of the firms who pay the regulatory tax (point 4). These firms are larger rather than smaller because there is no increase in the variable cost of labor which remains at w (recall it is $w\tau$ and we have assumed that $\tau = 1$). In terms of Figure 5 there is a bulge and valley around the threshold, but no downward shift in the intercept of the firm size distribution.

A.2 Adding up constraint on δ

In this sub-section of the Appendix, we remove subscripts (or superscripts) N, τ, k to improve readibility.

We first show that $n_{\min} = \theta/(1-\theta)$. This first follows from equation 5. Using the power law assumption and the TFP/size relation in equation (11), this relation can be re-written as:

$$n_{\min} = \alpha_{\min}^{\frac{1}{1-\theta}} \left(\frac{\theta}{w^*_{N,\tau,k}} \right)^{\frac{1}{1-\theta}} = \frac{\theta}{1-\theta}$$

It is also useful to define functions n_1 and n_2 respectively as:

$$\begin{aligned} n_1(\alpha) &= \left(\frac{\theta}{w^*_{N,\tau,k}} \right)^{\frac{1}{1-\theta}} \alpha^{\frac{1}{1-\theta}} \\ n_2(\alpha) &= \left(\frac{\theta}{w^*_{N,\tau,k}} \right)^{\frac{1}{1-\theta}} \tau^{-\frac{1}{1-\theta}} \alpha^{\frac{1}{1-\theta}} \end{aligned}$$

Note that equation (11) implies that $n^*_{N,\tau,k}(\alpha) = n_1(\alpha)$ if $\alpha_{\min} \leq \alpha \leq \alpha_c$ and $n^*_{N,\tau,k}(\alpha) = n_2(\alpha)$ if $\alpha_u \leq \alpha$.

Armed with this tools, how do we derive equation (13) from equation (12)? The firm size distribution is given by the broken power law in equation (12):

$$\chi^*(n) = \begin{cases} c_\alpha (1-\theta) \left(\frac{\theta}{w^*_{N,\tau,k}} \right)^{\frac{\beta-1}{1-\theta}} n^{-\beta} & \text{if} \quad \theta/(1-\theta) = n_{\min} \leq n \leq N = n^*_{N,\tau,k}(\alpha_c) \\ \int_{\alpha_c}^{\alpha_u} \phi(\alpha) \mathrm{d}\alpha = \delta & \text{if} \quad n = N = n^*_{N,\tau,k}(\alpha_c) \\ 0 & \text{if} \quad N < n < n_u = n^*_{N,\tau,k}(\alpha_u) \\ c_\alpha (1-\theta) \left(\frac{\theta}{w^*_{N,\tau,k}} \right)^{\frac{\beta-1}{1-\theta}} \underbrace{\tau^{-\frac{\beta-1}{1-\theta}}}_{\leq 1} n^{-\beta} & \text{if} \quad n^*_{N,\tau,k}(\alpha_u) = n_u \leq n \end{cases}$$

We work on *two* restrictions on this pdf:

- The constraint on δ can be re-written more conveniently in terms of firm size rather than productivity. We can express δ equivalently in terms of "regimes" n_1 or n_2 (the two are equivalent up to a variable change):

$$
\begin{aligned}
\delta &= \int_{\alpha_c}^{\alpha_u} \phi(\alpha)\mathrm{d}\alpha \\
&= \int_{n_1(\alpha_c)}^{n_1(\alpha_u)} c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.n^{-\beta}\mathrm{d}n = \int_{n_1(\alpha_c)=N}^{\tau^{\frac{1}{1-\theta}}.n_2(\alpha_u)=\tau^{\frac{1}{1-\theta}}.n_u} c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.n^{-\beta}\mathrm{d}n \\
&= \int_{n_2(\alpha_c)}^{n_2(\alpha_u)} c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.\tau^{-\frac{\beta-1}{1-\theta}}.n^{-\beta}\mathrm{d}n = \int_{\tau^{-\frac{1}{1-\theta}}.n_1(\alpha_c)=N.\tau^{-\frac{1}{1-\theta}}}^{n_2(\alpha_u)=n_u} c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.\tau^{-\frac{\beta-1}{1-\theta}}.n^{-\beta}\mathrm{d}n \\
&= \underbrace{c_\alpha.\frac{1-\theta}{\beta-1}.\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}}_{=C}.\left(N^{1-\beta} - \underbrace{\tau^{-\frac{\beta-1}{1-\theta}}}_{=T}.n_u^{1-\beta}\right)
\end{aligned}
$$

- Equation (12) is a pdf, so this adds up to 1 (with support on $[\theta/(1-\theta);+\infty[$):

$$
\begin{aligned}
\delta &= 1 - c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.\frac{N^{1-\beta}-\left(\frac{\theta}{1-\theta}\right)^{1-\beta}}{1-\beta} - c_\alpha.(1-\theta).\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}\tau^{-\frac{\beta-1}{1-\theta}}.\frac{-n_u^{1-\beta}}{1-\beta} \\
&= 1 - c_\alpha.\frac{1-\theta}{\beta-1}.\left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{\frac{\beta-1}{1-\theta}}.\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - N^{1-\beta} + \tau^{-\frac{\beta-1}{1-\theta}}.n_u^{1-\beta}\right)
\end{aligned}
$$

Taken together, these relations imply:

$$\delta = C.\left(N^{1-\beta} - T.n_u^{1-\beta}\right) = 1 - C.\left[\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - \left(N^{1-\beta} - T.n_u^{1-\beta}\right)\right]$$

Therefore:

$$C = \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \geqslant 1 \tag{18}$$

and:

$$
\chi^*(n) = \begin{cases}
\left(\frac{1-\theta}{\theta}\right)^{1-\beta}(\beta-1)n^{-\beta} & \text{if} \quad \theta/(1-\theta) \leq n \leq N \\
\left(\frac{1-\theta}{\theta}\right)^{1-\beta}\left(N^{1-\beta} - Tn_u^{1-\beta}\right) & \text{if} \quad n = N \\
0 & \text{if} \quad N < n < n_u \\
\left(\frac{1-\theta}{\theta}\right)^{1-\beta}(\beta-1)Tn^{-\beta} & \text{if} \quad n_u \leq< n
\end{cases}
$$

A.3 Proof of Lemma 1

When employment is measured with error, we can only observe the following quantity:

$$n(\alpha,\varepsilon) = n^*(\alpha).e^{\varepsilon}$$

We can then write the conditional CDF of this variable denoted by x below:

$$\mathbb{P}(x < n|\varepsilon) = \begin{cases} 0 & \text{if} \quad n.e^{-\epsilon} \leq \frac{\theta}{1-\theta} \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).\int_{\frac{\theta}{1-\theta}}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if} \quad \frac{\theta}{1-\theta} \leq n.e^{-\epsilon} \leq N \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\underbrace{(\beta-1).\int_{\frac{\theta}{1-\theta}}^{N} x^{-\beta}\mathrm{d}x}_{\left(\frac{\theta}{1-\theta}\right)^{1-\beta}-N^{1-\beta}} + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(N^{1-\beta} - T.n_u^{1-\beta}\right) & \text{if} \quad N \leq n.e^{-\epsilon} \leq n_u \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - T.n_u^{1-\beta}\right) + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).T.\int_{n_u}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if} \quad n_u \leq n.e^{-\epsilon} \end{cases}$$

$$= \begin{cases} 0 & \text{if} \quad \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \leq \varepsilon \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if} \quad \ln(n) - \ln(N) \leq \varepsilon \leq \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.n_u^{1-\beta} & \text{if} \quad \ln(n) - \ln(n_u) \leq \varepsilon \leq \ln(n) - \ln(N) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if} \quad \varepsilon \leq \ln(n) - \ln(n_u) \end{cases}$$

Assuming that ε is a Gaussian noise with mean 0 and variance σ, and denoting by φ the Gaussian pdf and by Φ the Gaussian cdf, we can compute the unconditional probability as:

$$\begin{aligned} \forall n > 0, \quad \mathbb{P}(x < n) &= \int_{\mathbb{R}} \mathbb{P}(x < n|\varepsilon)\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &= \int_{\ln(n)-\ln(N)}^{\ln(n)-\ln\left(\frac{\theta}{1-\theta}\right)} \left[1 - C.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon + \int_{\ln(n)-\ln(n_u)}^{\ln(n)-\ln(N)} \left[1 - C.T.n_u^{1-\beta}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &\quad + \int_{-\infty}^{\ln(n)-\ln(n_u)} \left[1 - C.T.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\ &= \underbrace{\Phi\left(\frac{\ln(n) - \frac{\ln(C)}{\beta-1}}{\sigma}\right)}_{=A(n)} - \underbrace{C.T.n_u^{1-\beta}.\left[\Phi\left(\frac{\ln(n)-\ln(N)}{\sigma}\right) - \Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}\right)\right]}_{=B(n)} \\ &\quad \underbrace{- C.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[\Phi\left(\frac{\ln(n) - \frac{\ln(C)}{\beta-1}}{\sigma} - \sigma.(\beta-1)\right) - \Phi\left(\frac{\ln(n)-\ln(N)}{\sigma} - \sigma.(\beta-1)\right)\right]}_{=C(n)} \\ &\quad \underbrace{- C.T.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma} - \sigma.(\beta-1)\right)}_{=D(n)} \end{aligned}$$

In fact there is no additional constraint in the parameters, because we can show that this function is strictly increasing (straightforward from the way we constructed it), with limits 0 in 0 and 1 in $+\infty$:

$$\begin{aligned} A(n) &\underset{n\to+\infty}{\longrightarrow} 1 & A(n) &\underset{n\to 0}{\longrightarrow} 0 \\ B(n) &\underset{n\to+\infty}{\longrightarrow} Cst \times (1-1) = 0 & B(n) &\underset{n\to 0}{\longrightarrow} Cst \times (0-0) = 0 \\ C(n) &\underset{n\to+\infty}{\longrightarrow} 0 \times (1-1) = 0 & C(n) &\underset{x\to 0}{\longrightarrow} +\infty \times (0-0) = 0 \text{ (*)} \\ D(n) &\underset{n\to+\infty}{\longrightarrow} 0 \times 1 = 0 & D(n) &\underset{n\to 0}{\longrightarrow} +\infty \times 0 = 0 \text{ (*)} \end{aligned}$$

To solve the two problematic cases, marked with $(*)$, let us consider $F(n)$ defined for $F \in \mathbb{R}$ as:

$$
\begin{aligned}
F(n) &= n^{1-\beta}.\Phi\left(\frac{\ln(n)}{\sigma}+F\right) = \frac{\Phi\left(\frac{\ln(n)}{\sigma}+F\right)}{n^{\beta-1}} \\
\text{(L'Hôpital's rule)} \quad &\underset{n\to 0}{\sim} \frac{\frac{1}{n.\sigma}\varphi\left(\frac{\ln(n)}{\sigma}+F\right)}{(\beta-1).n^{\beta-2}} \\
&\underset{n\to 0}{\sim} \frac{1}{\sigma.\sqrt{2\pi}.(\beta-1)}.e^{-\frac{F^2}{2}}.\underbrace{n^{1-\beta}.e^{-\frac{1}{2}.\frac{(\ln n)^2}{\sigma^2}}.e^{-\frac{1}{2}.F.\frac{\ln(n)}{\sigma}}}_{e^{\ln(n).\left(1-\beta-\frac{1}{2}.\frac{F}{\sigma}-\frac{\ln(n)}{2.\sigma^2}\right)}} \\
&\underset{n\to 0}{\longrightarrow} 0
\end{aligned}
$$

Last, the density that corresponds to the CDF is given by:

$$
\begin{aligned}
\chi(n) = {} & \frac{1}{\sigma.n}.\varphi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}\right) - \frac{1}{\sigma.n}.C.T.n_u^{1-\beta}\left[\varphi\left(\frac{\ln(n)-\ln(49)}{\sigma}\right)-\varphi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}\right)\right] \\
& -C.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.(1-\beta).\left(\Phi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}-\sigma.(\beta-1)\right)-\Phi\left(\frac{\ln(n)-\ln(49)}{\sigma}-\sigma.(\beta-1)\right)\right) \\
& -C.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\frac{1}{\sigma}.\left(\varphi\left(\frac{\ln(n)-\frac{\ln(C)}{\beta-1}}{\sigma}-\sigma.(\beta-1)\right)-\varphi\left(\frac{\ln(n)-\ln(49)}{\sigma}-\sigma.(\beta-1)\right)\right) \\
& -C.n^{-\beta}.T.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[(1-\beta).\Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}-\sigma.(\beta-1)\right)+\frac{1}{\sigma}.\varphi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}-\sigma.(\beta-1)\right)\right] \quad (19)
\end{aligned}
$$

We use standard ML techniques to estimate the parameters in equation (19).

Note that we obtain an estimate of C from this procedure from which we can, in principle recover an estimate of the coefficient θ from equation (18). This is unlikely to be a powerful way of identifying the scale parameter, however. We found empirically that the likelihood was very flat when trying to estimate θ in this way, suggesting it was not well identified: this is in particular due to the fact that we only estimate the conditional size distribution for firms having 10 to 1,000 employees (while we expect θ to be identified form the curvature of the distribution "on the left", for the smallest firms). Note that column 4 in table 1 shows however that very large values of θ are rejected by the data, because the obtained ln-likelihood drops. Instead, as discussed in the main text we generate estimates of θ from three alternative routes (i) calibration, (ii) estimates from the production function and (iii) using the TFP-size relationship. We use this when estimating equation (19) together with equation (18). For example, a calibrated $\theta = 0.5$ implies that C = 1. When we use methods (ii) and (iii) and estimate θ using the productivity distribution, we take into account the variance around the estimation of θ in calculating the correct variance-covariance matrix.

B Least squares estimation of broken Power Law

We discuss here an alternative to our ML approach. Taking as our starting point the power law for firm sizes, we can proceed as follows:

$$
\ln \chi(n) = \ln k - \beta \ln n + \delta(D_{n>n_u}) + \sum_{i=n_c}^{n_u} d_i \quad (20)
$$

where $D_{n>n_u}$ is a dummy variable that turns on to 1 for firms above the threshold n_u and is zero otherwise), but we have added d_i dummies that pick up the average number of firms in the distorted size categories, i.e. between the upper (n_u) and lower (n_c) employment thresholds. Equation (20) is estimated subject to the constraint $\sum_{i=n_c}^{n_u} d_i = 0$.

Following Axtell (2001), we can estimate equation (20) through OLS[37], conditional on the 'structural

[37] See Gabaix and Ibragimov (2008) for improvements in the OLS procedure using ranks, which is preferred for small samples and for the upper part of the distribution (not the middle, our focus).

breaks' at n_c and n_u.To find these structural break points, we follow Bai (1997) and Bai and Perron (1998) in their study of structural breaks in time series models. In our context, their result implies that for each partition $\{\{1,...n_c\},\{n_c...n_u\},\{n_u,...\}$, one obtains the OLS estimators of $\{k,\beta,\delta_1,\delta_2\}$ subject to constraint $\sum_{i=n_c}^{n_u} d_i = 0$.[38] Letting the sum of squared errors generated by each of these partitions be $SSE(n_u,n_c)$, our estimates of the 'break points', n_u and n_c are:

$$(\widehat{n}_u,\widehat{n}_c) = \arg\min_{n_u,n_c} SSE(n_u,n_c) \tag{21}$$

Bai and Perron (1998) show that, for a wide range of error specifications (including heteroskedastic like in our case) the break points are consistently estimated, and converge at rate $\acute{N}$,where $\acute{N}$ is the maximum firm size as long as $n_u - n_c > \varepsilon\acute{N}$, and $n_c < n_u$, (the break points are asymptotically distinct) which is true in our framework since we know $n_c < N < n_u$.

Armed with these parameter estimates we can the proceed to estimate τ using the results above. One intuitive way of seeing the procedure is as follows. Fix the lower employment threshold (say 43) and estimate the power law (conservatively) only on the part of the employment distribution below this and on the upper part of the size distribution that is undistorted (say under 43 and over 100).[39] This procedure generates a mass of firms (entrepreneurs) displaced to the "bulge" in the distribution between n_c and N (i.e. 43 and 50) as shown in Figure 9. These firms are drawn from between N and n_u, and since we know the counterfactual slope of the power paw over this region, we can reallocate these firms so as to minimize the deviation from this counterfactual power law. n_u is estimated as the maximum employment bin which is attained in this procedure.

Rather than fixing n_c, the Bai and Perron (1998) procedure estimates this efficiently by minimizing a sum of squares criterion along with the other parameters in the model as in equation (21).

This procedure gives us all the parameters necessary to estimate the implicit cost of the regulation which we calculate is equivalent to a labor tax of around 26% ($\tau = 1.26$).

C Using information from the productivity distribution

In this appendix, we remove subscripts (or superscripts) N,τ,k to improve readibility.

C.1 Incorporating TFP into the estimation method

We can do much better if we have direct information on the TFP Distribution. Estimation is a challenge here (see next sub-section), but let us initially assume we have reliable on TFP. First, recall from equation (11) the relationship between firm size and TFP:

$$n^*_{N,\tau,k}(\alpha) = \begin{cases} 0 & \text{if} \quad \alpha < \alpha_{\min} \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} & \text{if} \quad \alpha_{\min} \le \alpha \le \alpha_c \\ N & \text{if} \quad \alpha_c \le \alpha < \alpha_u \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)}\alpha^{1/(1-\theta)} & \text{if} \quad \alpha_u \le \alpha < \infty \end{cases}$$

The empirical model adds a stochastic error term to this to obtain:

[38] Perron and Qu (2006) show that the framework can accomodate linear restrictions on the parameter; and that the consistency and rate of convergence results hold and the limiting distribution is unaffected. However, our constraint is non-linear and no results exist on whether the results hold.

[39] We could in principle use all firms as small as one employee and up the largest firm in the economy. In practice the Power Law tends to be violated at these extremes of the distribution in all countries (e.g. Axtell (2001), so we follow that standard approach of trimming the upper and lower tails. We show that nothing is sensitive to these exact maximum and minimum employment thresholds as can be seen from the various figures.

$$n^*_{N,\tau,k}(\alpha) = \begin{cases} 0 & \text{if} \quad \alpha < \alpha_{\min} \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} e^{\varepsilon} & \text{if} \quad \alpha_{\min} \leq \alpha \leq \alpha_c \\ N e^{\varepsilon} & \text{if} \quad \alpha_c \leq \alpha < \alpha_u \\ \left(\frac{\theta}{w^*_{N,\tau,k}}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)} \alpha^{1/(1-\theta)} e^{\varepsilon} & \text{if} \quad \alpha_u \leq \alpha < \infty \end{cases}$$

Or

$$\begin{aligned} \ln n_1 &= \frac{1}{1-\theta} \ln \alpha + \frac{1}{1-\theta} \ln \left(\frac{\theta}{w^*_{N,\tau,k}} \right) + \varepsilon \\ \ln n_2 &= \ln(N) + \varepsilon \\ \ln n_3 &= \frac{1}{1-\theta} \ln \alpha + \frac{1}{1-\theta} \ln \tau + \frac{1}{1-\theta} \ln \left(\frac{\theta}{w^*_{N,\tau,k}} \right) + \varepsilon \end{aligned}$$

Combining these together:

$$\ln n = \ln n_1 I_{\{\alpha_{\min} \leq \alpha \leq \alpha_c\}} + \ln n_2 I_{\{\alpha_c \leq \alpha < \alpha_u\}} + \ln n_3 I_{\{\alpha_u \leq \alpha\}} \tag{22}$$

where I is an indicator function for a particular regime. If we have a measure of firm-specific α, TFP, then we can estimate equation (22). This is one way to obtain an estimate of θ that is needed to calculate the implicit tax of regulation. Alternatively, we can estimate θ directly as the returns to scale parameter directly from a production function. We show the results from both methods in Table 2.

C.2 Estimation of TFP

There is no one settled way of best estimating TFP on firm level data and there are many approaches suggested in the literature. Fortunately, at least at the micro-level, different methods tend to produce results where the correlation of TFP estimated by different methods is usually high (see Syverson, 2010).

In the baseline result we follow the method of Levinsohn and Petrin (2003) who propose extending the Olley and Pakes (1996) control function method to allow for endogeneity and selection. Olley and Pakes proposed inverting the investment rule to control for the unobserved productivity shock (observed to firm but unobserved to econometrician) that affects the firm's decision over hiring (and whether to stay in business). Because of the problem of zero investment regimes (common especially among smaller firms that we use in our dataset) Levinsohn and Petrin (2003) recommended using materials as an alternative proxy variable that (almost) always takes an observed positive value.

We use this estimator to estimate firm-level production functions on French panel data 2002-2007 (using the unbalanced panel) by each of the four-digit manufacturing industries in our dataset. We also did the same for the retail sector and the business services sector. The production functions take the form (in each industry):

$$\ln y_{it} = \beta_n \ln n_{it} + \beta_k \ln k_{it} + \omega_{it} + \tau_t + \eta_{it} \tag{23}$$

where y = output (value added), n = labour, k= capital, ω is the unobserved productivity shock, τ_t is a set of time dummies and η is the idiosyncratic error of firm i in year t. From estimating the parameters of the production function we can then recover our estimate of the persistent component of TFP.

There are of course many problems with these estimation techniques. For example, Ackerberg et al (2006) focus on the problem of exact multicollinearity of the variable factors conditional on the quasi-fixed factors given the assumption that input prices are assumed to be common across firms. Ackerberg et al (2007) suggest various solutions to this issue.

We consider alternative ways to estimate TFP including the more standard Solow approach. Here we assume that we can estimate the factor coefficients in equation (23) by using the observed factor shares in revenues. We do this assuming constant returns to scale, so $\beta_n = \frac{wn}{py}$ and $\beta_k = 1 - \frac{wn}{py}$. We used the four digit industry factor shares averaged over our sample period for the baseline but also experimented with some firm-specific (time invariant) factor shares. As usual these alternative measures led to similar results.

A problem with both of these methods is that we do not observe firm-specific prices so the estimates of TFP as we only control for four digit industry prices. Consequently, the results we obtain could be regarded as only revenue-based TFPR instead of quantity-based TFPQ (see Hsieh and Klenow, 2009). TFPQ is closer to what we want to theoretically obtain as our estimate of α.In practice, there is a high correlation between these two measures as shown by Foster et al (2008) who have actual data on plant level input and output prices. So it is unclear whether this would make too much of a practical difference to our results.

An alternative approach would be to follow de Loecker (2010) and put more structure on the product market. For example, assuming that the product market is monopolistically competitive enables the econometrician in principle to estimate the elasticity of demand and correct for the mark-up implicit in TFPR to obtain TFPQ. We will pursue this in future work.

D More Details of some Size-Related Regulations in France

The main bite of labor (and some accounting) regulations comes when the firm reaches 50 employees. But there are also some other size-related thresholds at other levels. The main other ones comes at 10-11 employees. For this reason we generally trim the analysis below 10 employees to mitigate any bias induced in estimation from these other thresholds. For more details on French regulation see inter alia Abowd and Kramarz (2003) and Kramarz and Michaud (2010), or, more administratively and exhaustively, Lamy (2010).

D.1 Main Labor Regulations

The unified and official way of counting employees is defined since 2004 in the Code du Travail[40], articles L.1111-2 and 3; it provides a concept of firm size defined at a precise date (i.e. it is not an average). Before that date, the concept of firm size was different across labor regulations.

From 200 employees:

- Obligation to appoint nurses (Code du Travail, article R.4623-51)
- Provision of a local for union representatives (Code du Travail, article R.2142-8)

From 50 employees:

- Monthly reporting of the detail of all labor contracts to the administration (Code du Travail, article D.1221-28)
- Obligation to establish a staff committee ("comité d'entreprise") with business meeting at least every two months and with minimum budget = 0.3% of total payroll (Code du Travail, article L.2322-1-28, threshold exceeded for 12 months during the last three years)
- Obligation to establish a committee on health, safety and working conditions (CHSC) (Code du Travail, article L.4611-1, threshold exceeded for 12 months during the last three years)
- Appointing a shop steward if demanded by workers (Code du Travail, article L.2143-3, threshold exceeded for 12 consecutive months during the last three years)
- Obligation to establish a profit sharing (Code du Travail, article L.3322-2, threshold exceeded for six months during the accounting year within one year after the year end to reach an agreement)
- Obligation to do a formal "Professional assessment" for each worker older than 45 (Code du Travail, article L.6321-1)
- Higher duties in case of an accident occurring in the workplace (Code de la sécurité sociale and Code du Travail, article L.1226-10)
- Obligation to use a complex redundancy plan with oversight, approval and monitoring from Ministry of Labor in case of a collective redundancy for 9 or more employees (Code du Travail, articles L.1235-10 to L.1235-12; threshold based on total employment at the date of the redundancy)

From 25 employees:

- Duty to supply a refectory if requested by at least 25 employees (Code du Travail, article L.4228-22)

[40] The text is available at:
http://www.legifrance.gouv.fr/affichCode.do?cidTexte=LEGITEXT000006072050&dateTexte=20120822

- Electoral colleges for electing representatives. Increased number of delegates from 25 employees (Code du Travail, article L.2314-9, L.2324-11)

From 20 employees:

- Formal house rules (Code du Travail, articles L.1311-2)
- Contribution to the National Fund for Housing Assistance;
- Increase in the contribution rate for continuing vocational training of 1.05% to 1.60% (Code du Travail, articles L.6331-2 and L.6331-9)
- Compensatory rest of 50% for mandatory overtime beyond 41 hours per week

From 11 employees:

- Obligation to conduct the election of staff representatives(threshold exceeded for 12 consecutive months over the last three years) (Code du Travail, articles L.2312-1)

From 10 employees:

- Monthly payment of social security contributions, instead of a quarterly payment (according to the actual last day of previous quarter);
- Obligation for payment of transport subsidies (Article R.2531-7 and 8 of the General Code local authorities, Code général des collectivités territoriales);
- Increase the contribution rate for continuing vocational training of 0.55% to 1.05% (threshold exceeded on average 12 months).

Note that, in additions to these regulations, some of the payroll taxes are related to the number of employees in the firm.

D.2 Accounting rules

The additional requirements depending on the number of employees of entreprises, but also limits on turnover and total assets are as follows (commercial laws, Code du Commerce, articles L.223-35 and fiscal regulations, Code général des Impôts, article 208-III-3):

From 50 employees:

- Loss of the possibility of a simplified presentation of Schedule 2 to the accounts (also if the balance sheet total exceeds 2 million or if the CA exceeds 4 million);
- Requirement for LLCs, the CNS, limited partnerships and legal persons of private law to designate an auditor (also if the balance sheet total exceeds 1.55 million euros or if the CA is more than 3.1 million euros, applicable rules of the current year).

From 10 employees:

- Loss of the possibility of a simplified balance sheet and income statement (also if the CA exceeds 534 000 euro or if the balance sheet total exceeds 267 000 euro, applicable rule in case of exceeding the threshold for two consecutive years).

Table 1: Parameter estimates (calibrating returns to scale, θ)

	(1)	(2)	(3)	(4)
Method	Unconstrained (θ calibrated from Basu and Fernald, 1997)	Unconstrained (θ calibrated from Atkeson and Kehoe, 2005)	Unconstrained (θ calibrated at 0.5, Hsieh-Klenow, 2009)	Unconstrained (θ calibrated at 0.9)
θ, scale parameter	0.8	0.85	0.5	0.9
β, power law	1.822	1.822	1.822	1.829
	(0.059)	(0.059)	(0.059)	(0.057)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.948	0.965
	(0.018)	(0.018)	(0.018)	(0.015)
n_u, upper employment threshold	57.898	57.898	57.898	52.562
	(0.024)	(0.024)	(0.024)	(0.002)
σ, variance of measurement error	0.104	0.104	0.104	0.036
	(0.025)	(0.025)	(0.025)	(0.008)
τ, implicit tax, variable cost	1.013	1.010	1.033	1.004
	(0.005)	(0.004)	(0.013)	(0.002)
k/w, implicit tax, fixed cost	-0.496	-0.372	-1.243	-0.201
	(0.257)	(0.192)	(0.653)	(0.099)
Mean (Median) # of employees	55.7 (23)	55.7 (23)	55.7 (23)	55.7 (23)
Observations	238,701	238,701	238,701	238,701
Firms	57,008	57,008	57,008	57,008
Ln Likelihood	-1,065,936	-1,065,936	-1,065,936	-1,066,165

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different estimates of θ; the methods are indicated in the different columns.

Table 2: Parameter estimates (exploiting information from the Production Function to estimate returns to scale, θ)

Method	(1) Baseline (column (1) of Table 1)	(2) Using Production Function estimates	(3) TFP/Size relationship	(4) High-Tech Sectors (Using Production Function estimates)	(5) Low-Tech Sectors (Using Production Function estimates)
θ , scale parameter	0.8	0.855	0.799	0.882	0.848
		(0.013)	(0.025)	(0.012)	(0.016)
β , power law	1.822	1.822	1.822	1.625	1.864
	(0.059)	(0.052)	(0.055)	(0.063)	(0.063)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.948	0.997	0.929
	(0.018)	(0.020)	(0.023)	(0.012)	(0.032)
n_u, upper employment threshold	57.898	57.899	57.898	50.000	58.328
	(0.024)	(1.133)	(1.342)	(2.474)	(1.603)
σ, variance of measurement error	0.104	0.104	0.104	0.000	0.114
	(0.025)	(0.020)	(0.024)	(0.029)	(0.034)
τ , implicit tax, variable cost	1.013	1.010	1.013	1.001	1.013
	(0.005)	(0.004)	(0.005)	(0.003)	(0.007)
k/w, implicit tax, fixed cost	-0.496	-0.359	-0.498	-0.026	-0.514
	(0.257)	(0.177)	(0.219)	(0.129)	(0.304)
Mean (Median) # of employees	55.7 (23)	55.7 (23)	55.7 (23)	78.3 (29)	51.3 (23)
Observations	238,701	238,701	238,701	38,713	199,988
Firms	57,008	57,008	57,008	9,099	48,139
Ln Likelihood	-1,065,936	-	-	-	-

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different estimates of θ ; the methods are indicated in the different columns. Standard errors are calculated using bootstrap in columns (2) to (5). "Using TFP-Size relationship" calculates $\theta = 1 - \frac{\partial \ln \alpha}{\partial \ln n}$ where $\frac{\partial \ln \alpha}{\partial \ln n}$ is calculated from the coefficient of a regression of ln(TFP) on ln(employment) on firms with 10 to 45 workers. "Using the production function" calculates θ as the sum of the coefficients on the factor inputs obtained from TFP estimation (see Table A1). "High tech" sectors are based on R&D intensity as defined by the OECD.

Table 3: Welfare and Distributional Analysis

(Regulated Economy - Unregulated Economy)	(1)	(2)
Variable	**FLEXIBLE WAGES**	**RIGID WAGES**
1. Unemployment rate	0%	5.217%
2. Percentage of firms avoiding the regulation, δ	3.593%	3.438%
3. Percentage of firms paying tax (compliers)	9.036%	8.646%
4. Change in labor costs (wage reduction) for small firms (below 49)	-1.074%	0
5. Change in labor costs (wage reduction but tax increase), Large firms (above 49)	0.232%	1.306%
6. Excess entry by small firms (percent increase in number of firms)	4.419%	4.409%
7. Increase in size of small firms	5.370%	0
8. Increase in size of large firms	-1.160%	-6.530%
9. Annual welfare loss (as a percentage of GDP):		
a. Implicit Tax	0.804%	0.801%
b. Output loss	0.016%	4.302%
c. Total (Implicit Tax + Output loss)	0.820%	5.103%
10. Winners and losers:		
a. Change in (expected) wages for workers who remain in labor force	*-1.074%*	*-5.358%*
b. Average gain by entering entrepreneurs of small firms	1.603%	*-2.687%*
c. Average profit gain by small unconstrained firms	4.296%	0
d. Average profit gain by firms constrained at 49	2.447%	*-1.849%*
e. Change in profit for large firms	*-0.928%*	*-5.224%*

Notes: This is based on the baseline of Table 1 column (1), under the additional assumption that maximum firm size is 10,000. We set (insignificant) fixed cost of the regulation to zero, i.e. k =0 and τ -1 = 1.3%. In column (1), model solved assuming wages fully adjust (section 2.4). In column (2), model solved assuming that wages are rigid (section 2.5). “Percentage of firms avoiding the regulation” (Row 2) corresponds to δ in main text. “Percentage of firms paying tax” (Row 3) corresponds to mass of agents with productivity greater than in α_u relative to agents with productivity greater than α_{min}. “Change in labor costs for small firms” (Row 4) corresponds to the general equilibrium wage effect. “Change in labor costs for large firms” (Row 5) corresponds to Row 4 + the estimated implicit tax (τ -1 = 1.3%). “Excess entry” (Row 6) corresponds to the difference in the ln(mass of agents having productivity greater than $\alpha_{\min}^{N,\tau,k}$) minus ln(mass of agents having productivity greater than $\alpha_{\min}^{N,1,0}$. "Increase in size of small firms" (Row 7) corresponds to $\ln\left(n^*_{N,\tau,0}\right) - \ln\left(n^*_{N,1,0}\right)$ for firms having productivity smaller than $\alpha_u^{N,\tau,k}$; “increase in size of large firms” (Row 8) corresponds to $n^*_{N,\tau,0}$-$n^*_{N,1,0}$ for firms having productivity greater than $\alpha_c^{N,\tau,k}$. “Implicit Tax” (Row 9a) corresponds to the total amount of implicit tax ($\int_{\alpha \geq \alpha_c}(\tau - 1). w^*_{N,\tau,0}. n^*_{N,\tau,0}(\alpha)\phi(\alpha)\mathrm{d}\alpha$ as a share of total output $Y_{N,\tau,0}$. “Output loss” (Row 9b) corresponds to $\ln\left(Y_{N,1,0}\right) - \ln\left(Y_{N,\tau,0}\right)$. For “winners and losers” (Row 10), we compute the average (percentage point) changes in expected wages or profits for agents in each of the following bins: 9a. labor force $\left[\underline{\alpha}; \alpha_{\min}^{N,\tau,0}\right[$; 9b. new entrepreneurs $\left[\alpha_{\min}^{N,\tau,0}; \alpha_{\min}^{N,1,0}\right[$; 9c. small firms $\left[\alpha_{\min}^{N,1,0}; \alpha_c^{N,\tau,0}\right]$; 9c. constrained firms $\left[\alpha_c^{N,\tau,0}; \alpha_u^{N,\tau,0}\right[$;9d. large firms $\left[\alpha_u^{N,\tau,0}; 1\right]$. In the “rigid wages” case in column (2), expected wages are computed as $(1 - u^{*,RIG}_{N,\tau,k}). w^*_{N,1,0}$ where u is the unemployment rate.

Table 4: Variation in estimates across different sectors

Industry	(1) Manufacturing industries	(2)	(3) Transport	(4)	(5) Construction	(6)	(7) Trade	(8)	(9) Business services	(10)
θ	0.8	0.85	0.8	0.85	0.8	0.85	0.8	0.85	0.8	0.85
β, power law	1.822	1.822	1.878	1.878	2.372	2.372	2.128	2.128	2.001	2.001
	(0.059)	(0.059)	(0.098)	(0.099)	(0.147)	(0.147)	(0.100)	(0.100)	(0.087)	(0.087)
$T = \tau^{\frac{1-\beta}{1-\theta}}$	0.948	0.948	0.898	0.989	0.871	0.872	0.885	0.885	0.984	0.984
	(0.018)	(0.018)	(0.029)	(0.029)	(0.016)	(0.016)	(0.022)	(0.021)	(0.020)	(0.020)
n_u, upper employment threshold	57.898	57.898	55.312	55.318	57.874	57.873	57.151	57.151	58.254	58.256
	(0.024)	(0.024)	(0.016)	(0.016)	(0.013)	(0.013)	(0.015)	(0.015)	(0.018)	(0.018)
σ,measurement error	0.104	0.104	0.060	0.060	0.089	0.089	0.084	0.084	0.106	0.106
	(0.025)	(0.025)	(0.023)	(0.023)	(0.037)	(0.037)	(0.050)	(0.050)	(0.024)	(0.024)
τ, implicit variable tax	1.013	1.010	1.025	1.019	1.020	1.015	1.022	1.016	1.003	1.003
	(0.005)	(0.004)	(0.010)	(0.007)	(0.004)	(0.003)	(0.006)	(0.004)	(0.004)	(0.024)
k/w, implicit tax, fixed cost	-0.496	-0.372	-1.137	-0.850	-0.842	-0.631	-0.951	-0.711	0.003	0.002
	(0.257)	(0.192)	(0.492)	(0.367)	(0.179)	(0.133)	(0.288)	(0.215)	(0.188)	(0.150)
Mean	55.7	55.7	48.2	48.2	29.3	29.3	36.8	36.8	45.3	45.3
(Median) # of employees	(23)	(23)	(23)	(23)	(17)	(17)	(19)	(19)	(20)	(20)
Observations	238,701	238,701	70,479	70,479	159,440	159,440	255,812	255,812	205,835	205,835
Firms	57,008	57,008	14,487	14,487	41,768	41,768	66,848	66,848	61,906	61,906

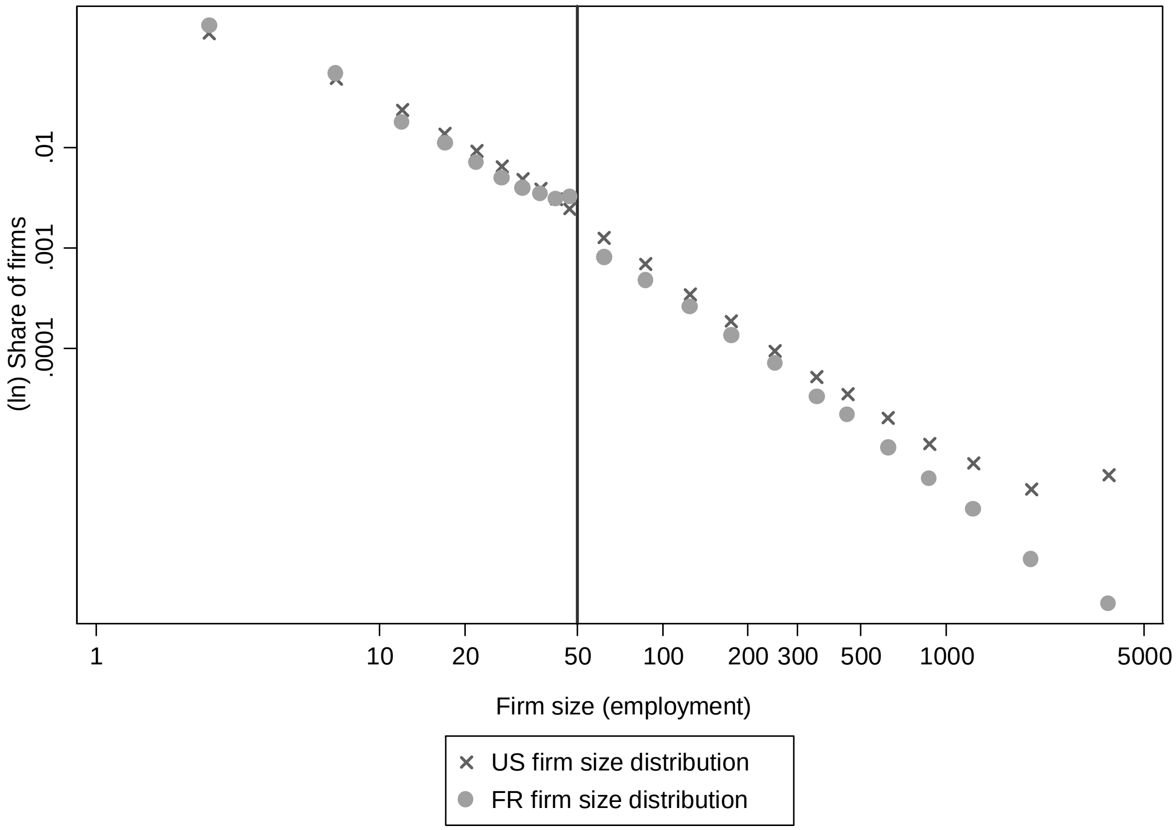
Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 2002-2007 of firms with 10 to 1,000 employees. These estimates of the implicit tax are based on different (calibrated) values of θ that are indicated in the different columns.

Table 5: Comparison estimated results with of actual data

(Actual data)	**(1) Firms having 10 to 48 workers**	**(2) Firms having 49 to 57 workers**	**(3) Firms having 58 to 1,000 Workers**
Panel A: Firm Distribution			
Distribution of firms (actual)	0.762	0.035	0.204
Distribution of firms (predicted)	0.760	0.039	0.201
	(0.016)	(0.007)	(0.016)
Panel B: Employment distribution			
Distribution of employment (actual)	0.295	0.032	0.672
Distribution of employment (predicted)	0.277	0.035	0.688
$n=n^*[\alpha].e^{\varepsilon}$; $\sigma_{\varepsilon} = 0.104$	(0.024)	(0.007)	(0.027)
Panel C: Output Distribution			
Distribution of output (actual)	0.242	0.029	0.728
Distribution of output (predicted)	0.275	0.034	0.691
$y = \alpha.n^{\theta} = \alpha.(n^*[\alpha])^{\theta}.e^{\theta\varepsilon}$, $\sigma_{\varepsilon} = 0.104$	(0.028)	(0.007)	(0.028)
	[0.219;0.331]	[0.020;0.048]	[0.635;0.747]

Notes: The "actual" distribution is computed over our data used for estimation between 2002-2007: the population of French manufacturing firms with 10 to 1,000 employees (see tables 1, 2 and 4). The "predicted" distribution is computed using our empirical model described in Section 3 (incorporating a measurement error term ε) and our baseline estimate reported in Table 1 column (1).

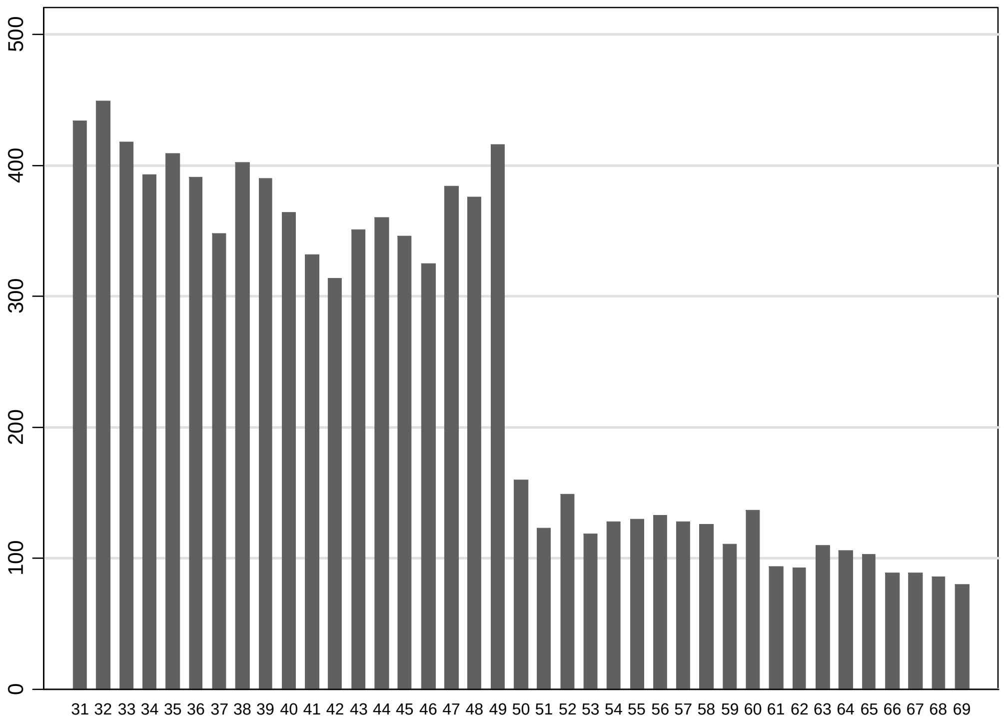
Figure 1: The Firm size distribution in the US and France



Source: FICUS 2002 for France and Census, LBD 2003 for the US. Population databases of all firms.

Notes: This is the distribution of firms (not plants). Authors' calculations

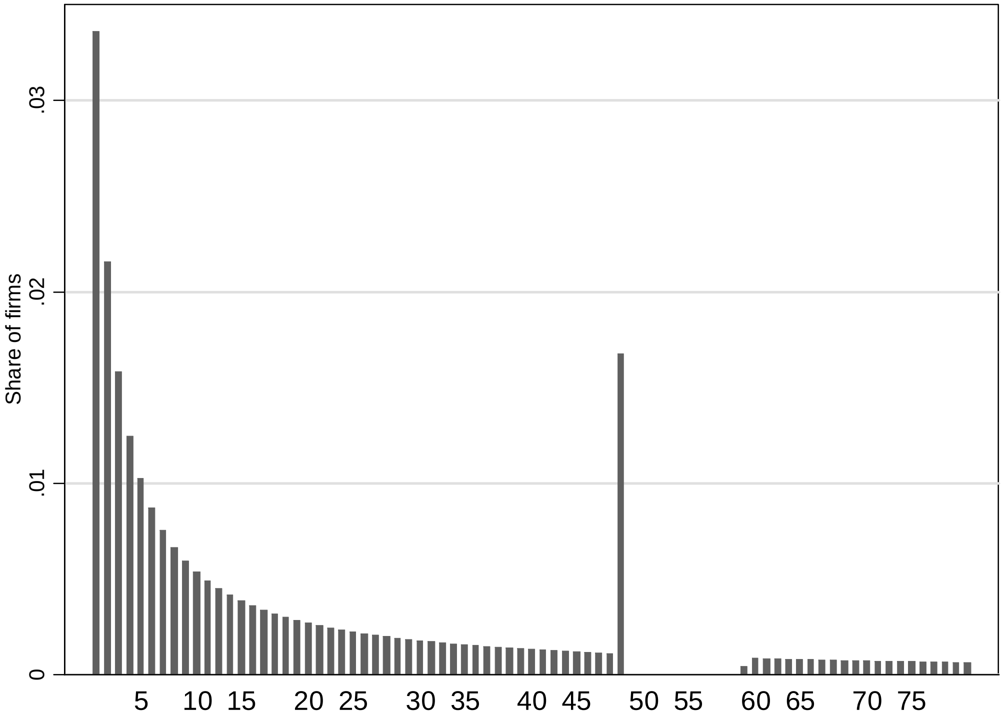
Figure 2: Number of Firms by employment size in France



Source: FICUS, 2002

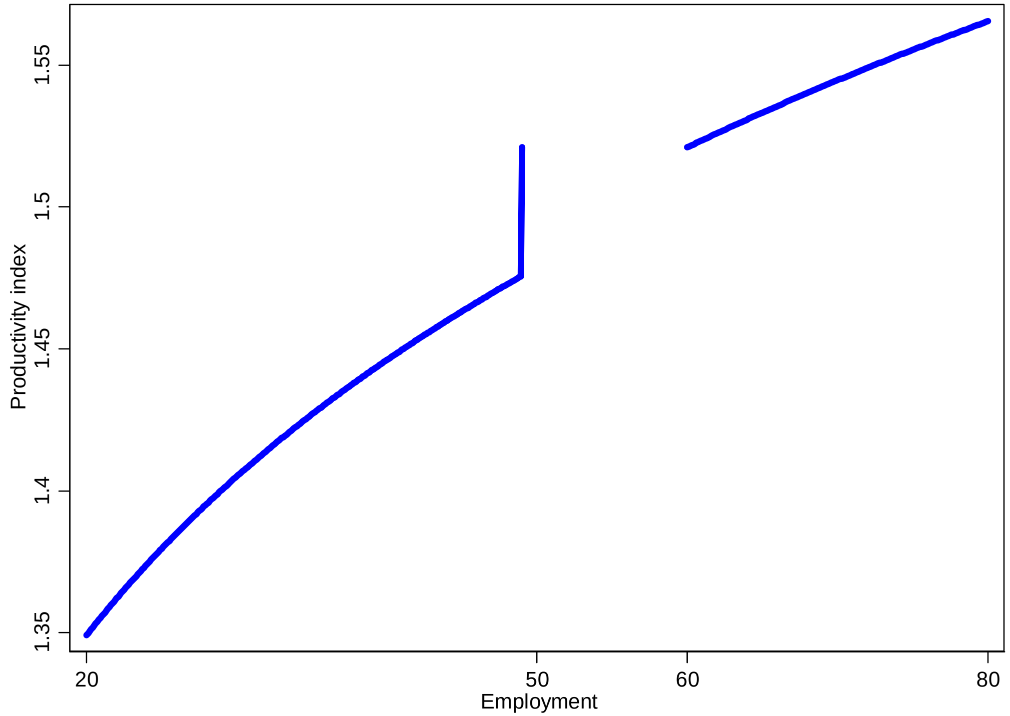
Notes: This is the population of manufacturing firms in France with between 31 and 69 employees. This plots the number of firms in each exact size category (i.e. raw data, no binning). There is a clear drop when regulations begin for firms with 50 or more employees.

Figure 3: Theoretical Firm size distribution with regulatory constraint



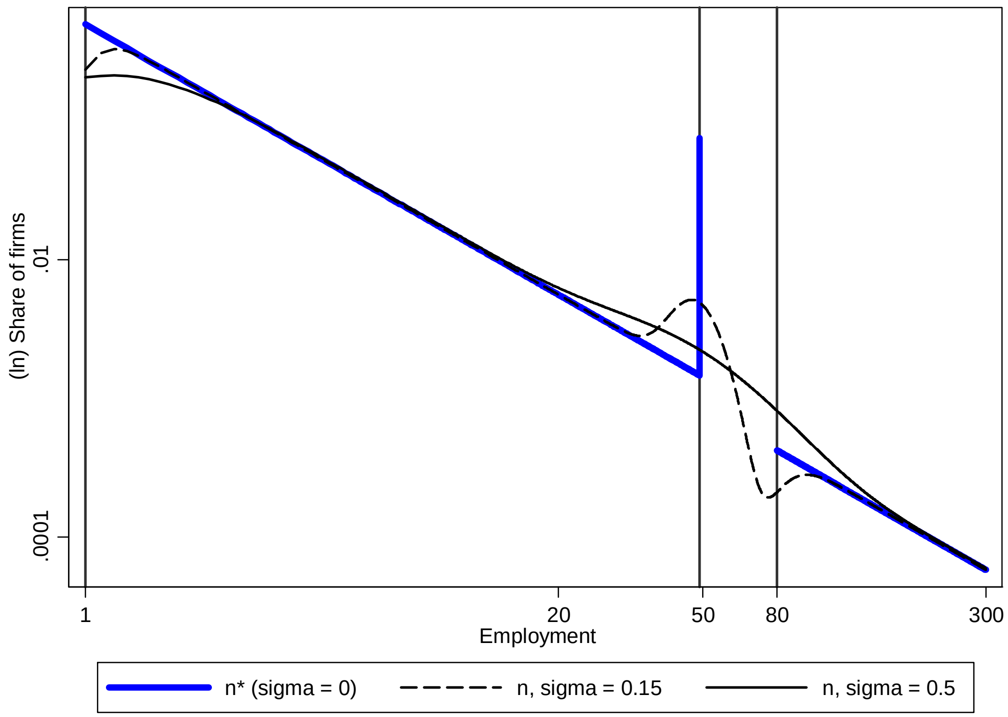
Notes: This figure shows the theoretical firm size distribution with exponentially increasing bins. The tallest bar represents the point at which the size constraint bins. Parameters: $\beta_\alpha = 1.6$, $\tau = 1.01$, $n_u = 60$, $\theta = 0.9$, $\beta = 1.06$.

Figure 4: Theoretical Relationship between TFP (managerial talent) and firm size



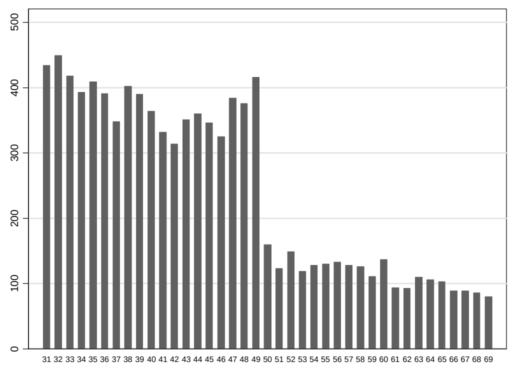
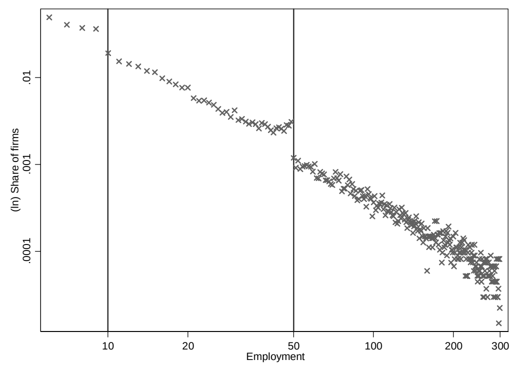
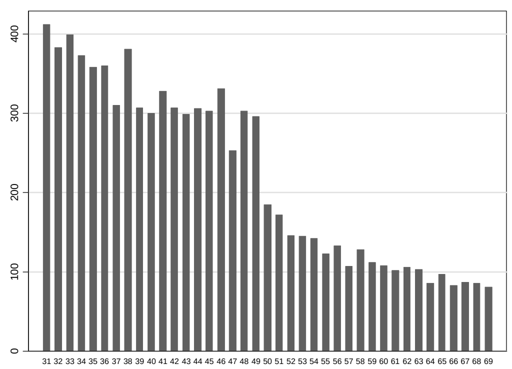
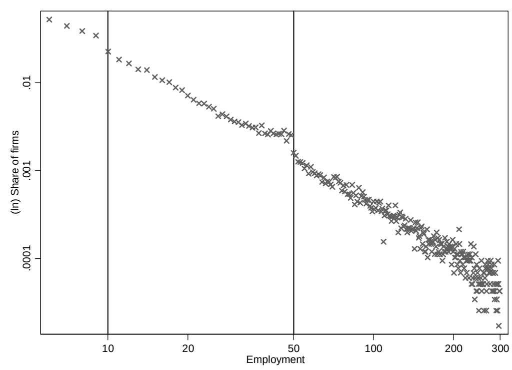
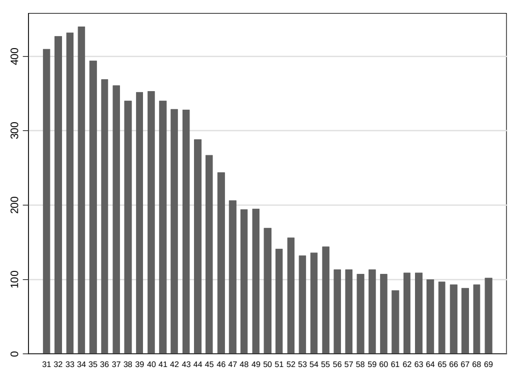
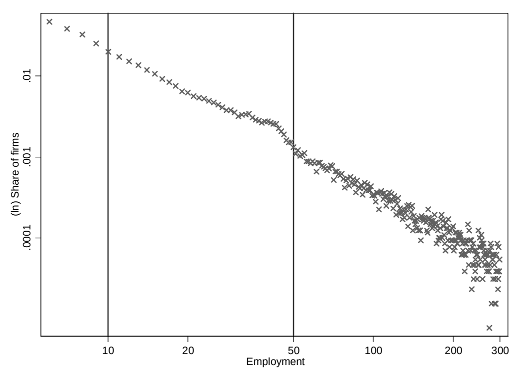
Notes: This figure shows the theoretical relationship between TFP and firm size. There is a mass of firms at employment size=50 where the regulatory constraint binds. Parameters: $\beta_\alpha = 1.6$, $\tau = 1.01$, $n_u = 60$, $\theta = 0.9$, $\beta = 1.06$.

<u>**Figure 5: The Theoretical Firm Size Distribution when employment is measured with error**</u>



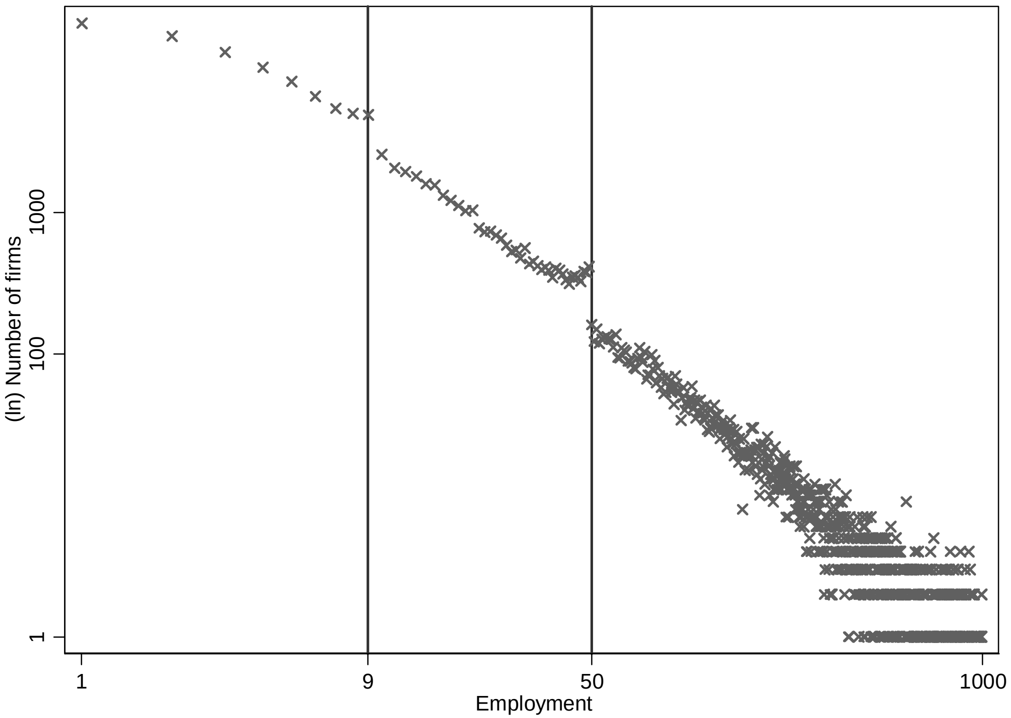
Note: The solid (blue) line shows the theoretical firm size distribution (broken power law), n*. The dashed line shows the new firm size distribution when we extend the model, to allow employment size to be measured with error with σ = 0.15. The solid dark line increases the measurement error to σ = 0.5

Figure 6: The effect on the measured firm size distribution using Alternative Datasets and definitions of employment

<table>
<tr><td>Bar plot</td><td>Log-log plot</td></tr>
<tr><td colspan="2">Panel A: FICUS 2002: Fiscal source (corporate tax collection to fiscal administration)
Arithmetic average of quarterly head counts</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Panel B: DADS 2002: Payroll tax reporting to social administration
"Declared" workers on Dec. 31st: cross-sectional count, taking part of part-timers</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Panel C: DADS 2002: Payroll tax reporting to social administration
"Full-time equivalent" (FTE), computed by the French statistical institute</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Note: Data sources are indicated in the headers of the table. All datasets relate to the year 2002.

Figure 7: Share of Firms by employment size, 2002

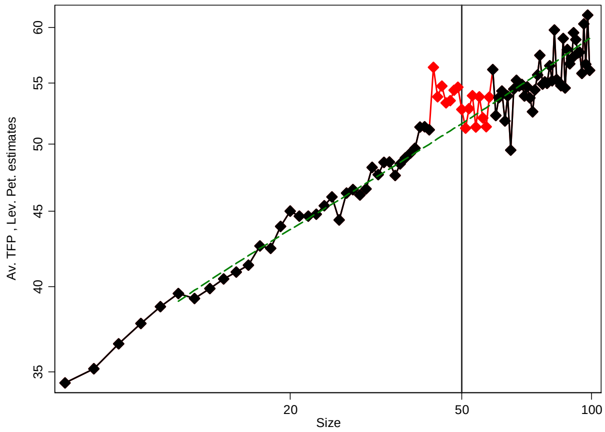


Source: FICUS, 2002

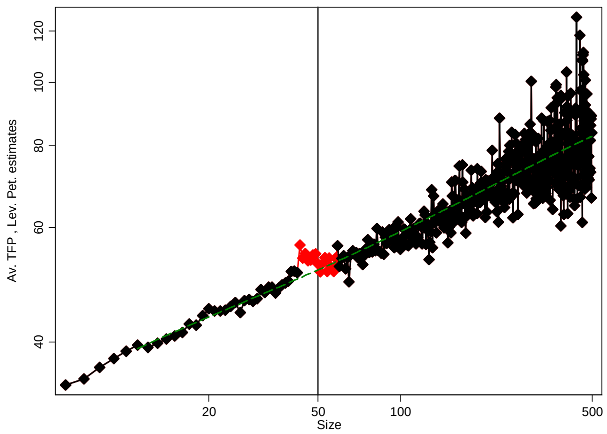
Notes: This is the population of manufacturing firms in France with between 1 and 1000 employees. This plots the number of firms in each exact size category (i.e. raw data, no binning). As in figure 2, there is a clear drop at 50, but also at 10 employees. These thresholds correspond to various size based regulations.

Figure 8: TFP Distribution around the regulatory threshold of 50 employees

Panel A: Short Employment span

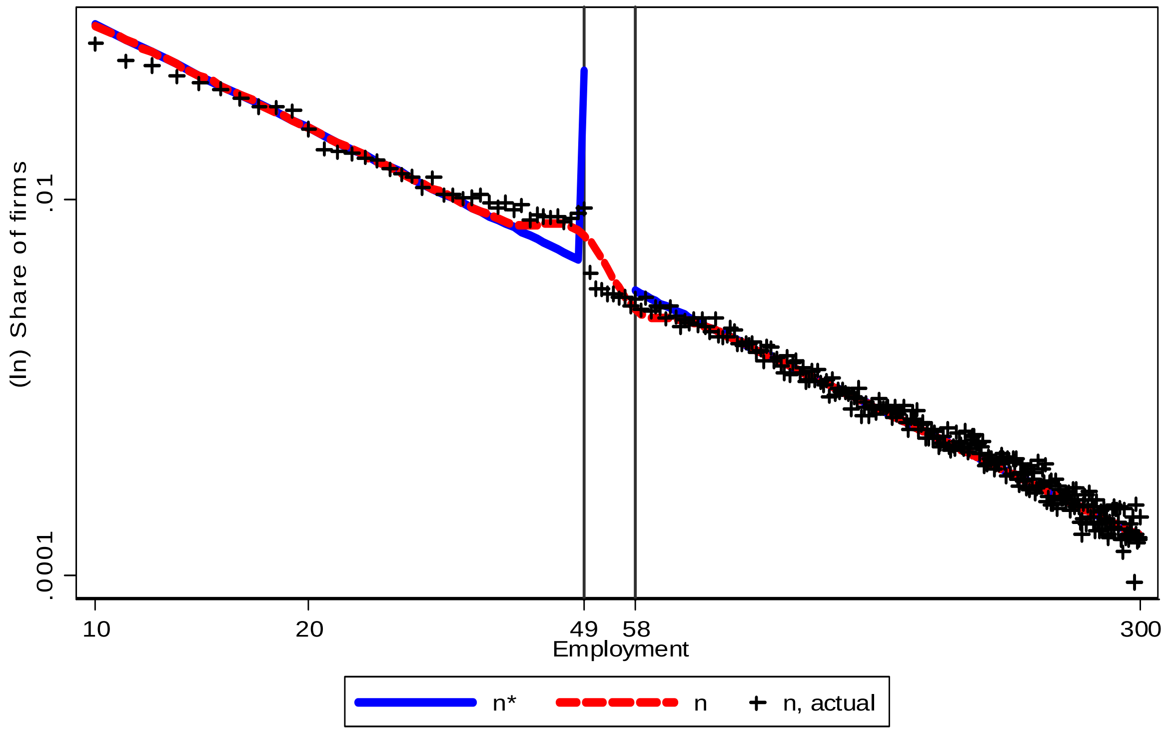


Panel B: Longer Employment span



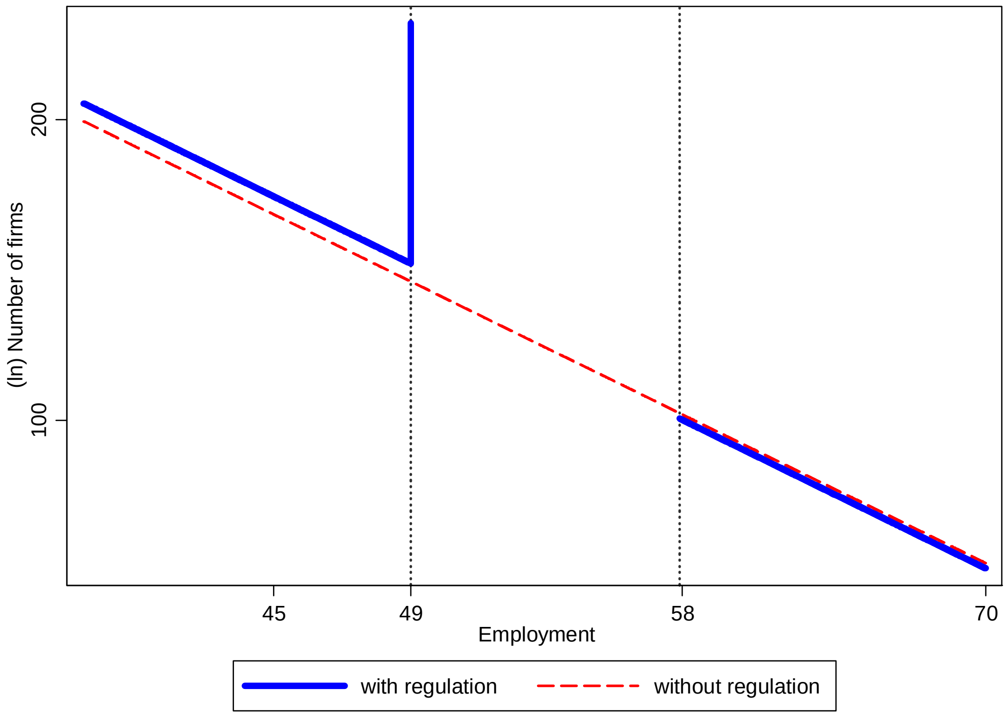
Notes: This figures plots the mean level of TFP by firm employment size using an upper support of 100 (Panel A) or 500 (Panel B). A fourth order polynomial is displayed in both panels using only data from the "undistorted" points (potentially "distorted" points are shown in red).

Figure 9: Firm Size Distribution and Broken Power Law: Data and Fit of Model



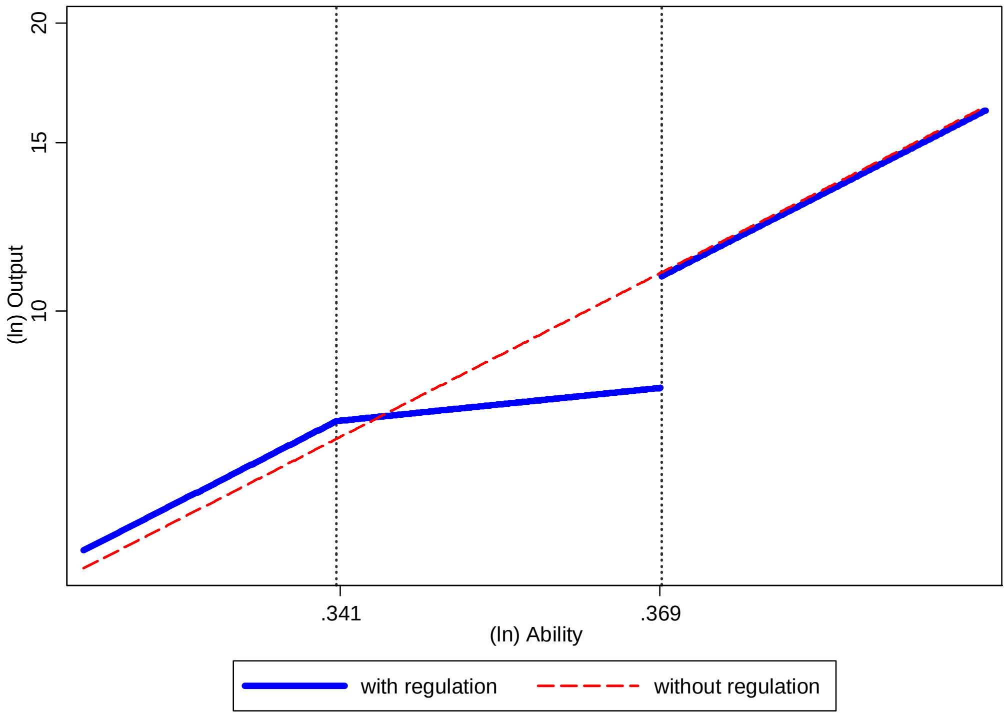
Notes: This shows the difference between the fit of the model (dashed red line) which allows for measurement error with the actual data. Estimates correspond to the baseline specification reported in tables 1 to 3, column 1. We also include the "pure" theoretical predictions (in dark blue solid line).

Figure 10: Firm Size Distribution with and without regulation



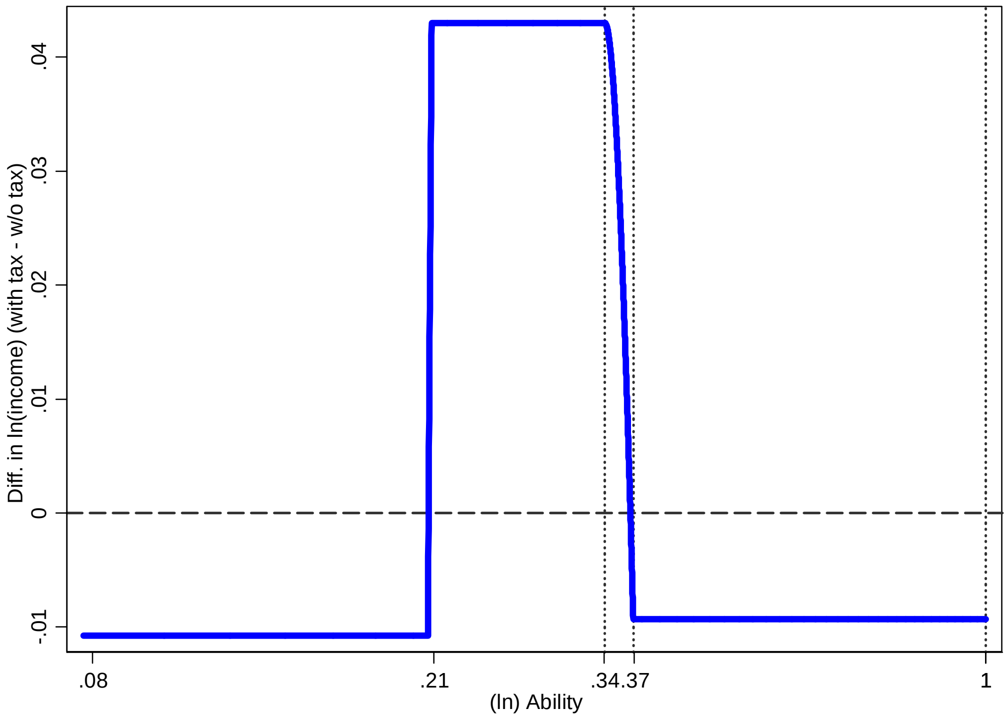
Notes: This figure compares the firm size distribution in the regulated economy (bold line) from a world without regulation (dashed line) based on the estimated parameters from our model (baseline specification reported in Table 1, column (1)).

Figure 11: Output Across Entrepreneurs



Notes: This graph compares the relation between ability (productivity) and output for individuals of different managerial ability in the regulated economy (bold line) and the unregulation economy (dashed line) based on the estimated parameters from our model. Maximum ability has been normalized to 1. An ability level of 0.341 corresponds to a firm size of 49 and an ability level of 0.369 corresponds to a firm size of 58. The underlying estimates correspond to the baseline specification reported in Table 1 column (1).

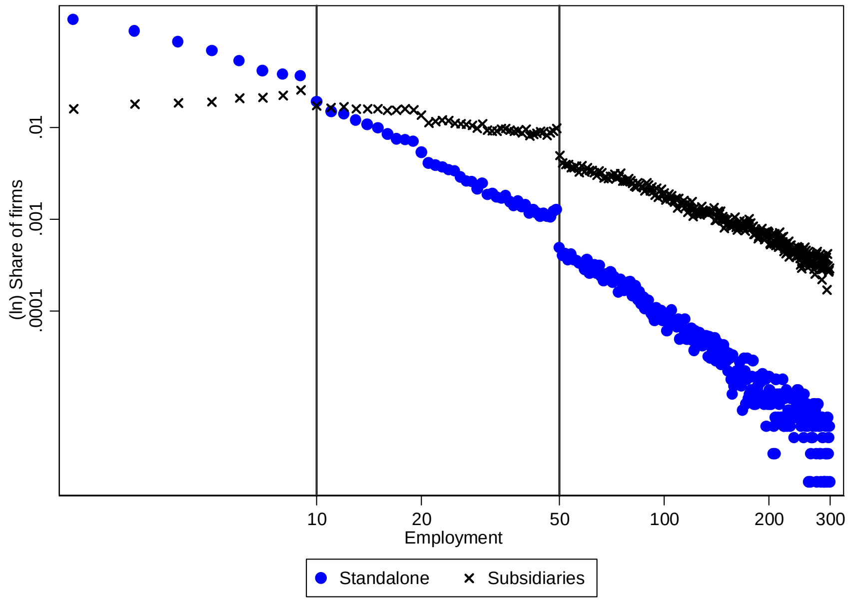
Figure 12: Distributional Effects of the Regulation



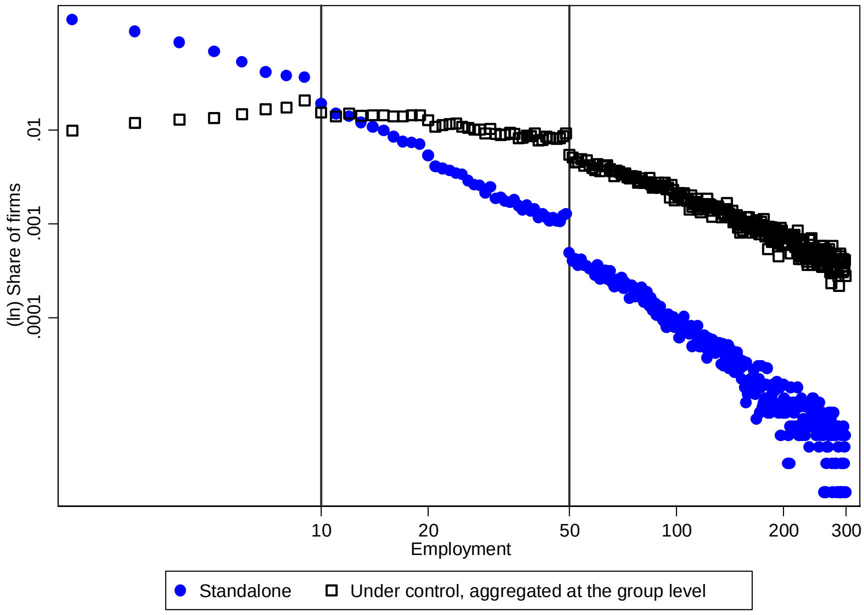
Notes: This graph compares the change in ln(income) for individuals of different managerial ability in the regulated economy relative to the unregulated economy based on the estimated parameters from our model. The dark blue line is our baseline base. Maximum ability has been normalized to 1. Individuals with an ability level below 0.21 are workers in the regulated economy. An ability level of 0.34 corresponds to a firm size of 49 and an ability level of 0.37 corresponds to a firm size of 58. The underlying estimates correspond to the baseline specification reported in Table 1 column (1).

Figure 13: Corporate Restructuring in Response to the Regulation? Independent Firms vs. Corporate groups

Panel A: Standalone firms vs. affiliates of larger groups



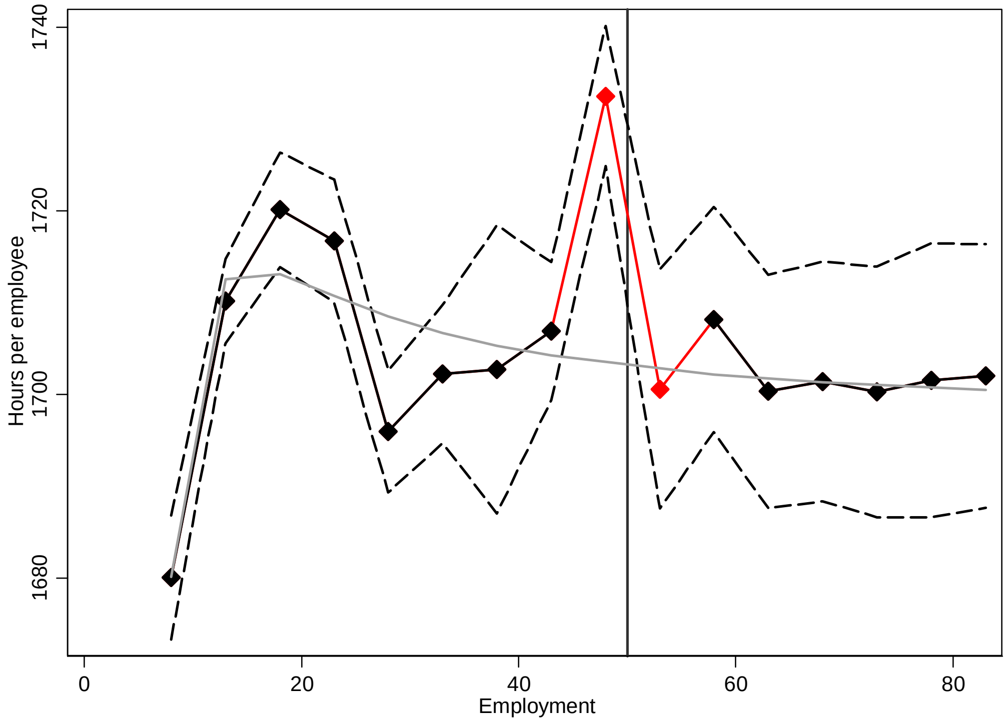
Panel B: Standalone firms vs. groups (i.e. all affiliates aggregated at the group level)



Notes: "Standalone" are independent firms that are not subsidiaries or affiliates of larger groups (blue dots). In Panel A we compare these to affiliates of larger groups with size measured at the affiliate level. A broken power law is visible in both distributions. In Panel B we repeat the standalone distribution but now compare this to affiliates aggregated to the group level (in France, we do not count overseas employees). Although there is a break in the power law for both type of firms it is stronger for the standalone firms as we would expect. The subsidiaries are not driving the results.

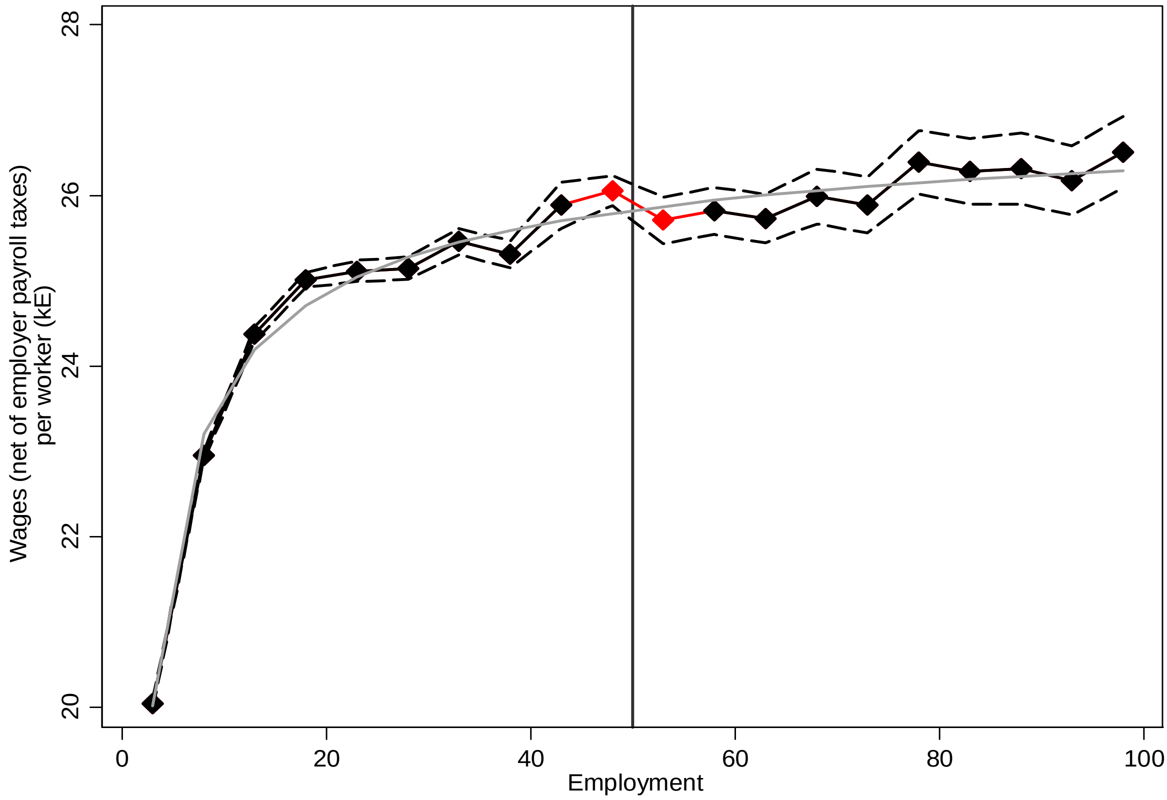
Figure 14: Adjustment in the hours margin around the threshold

(annual hours per worker)



Notes: Annual average hours per worker - combined FICUS and DADs data for 2002. 95% confidence intervals shown.

Figure 15: Workers do not appear to be accepting significantly lower wages in return for "insurance" of employment protection



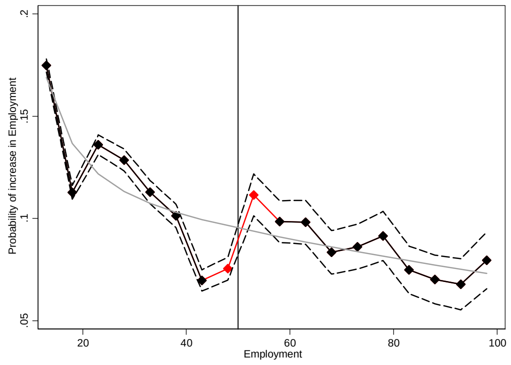
Source: FICUS, 2002.

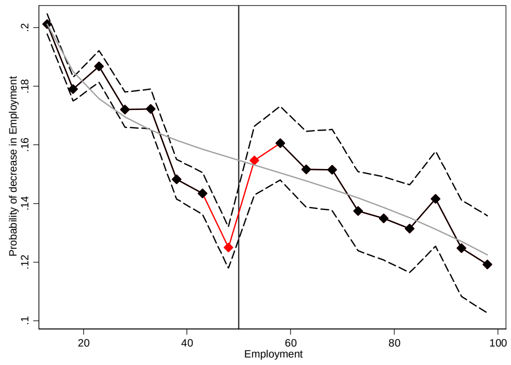
Notes: Wages is the nominal wage (net of payroll tax) by employer size. 95% confidence intervals shown.

Figure 16: Firms just above the regulatory threshold are much more likely to either contract or grow than we would expect. Firms just below the threshold are much *less* likely to contract or grow

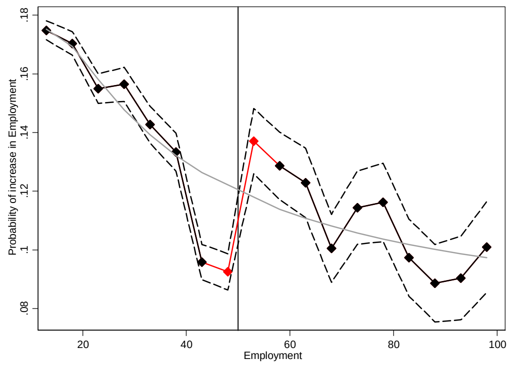
Probability of INCREASE in EMPLOYMENT	Probability of DECREASE in EMPLOYMENT

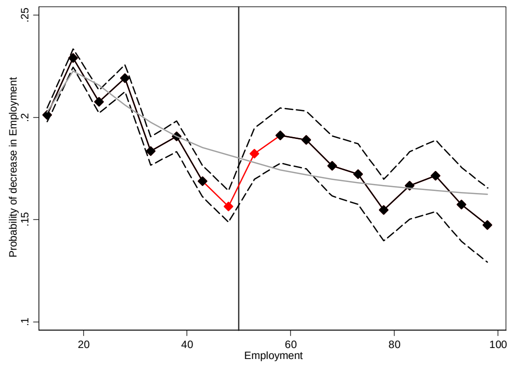
By more than 12%



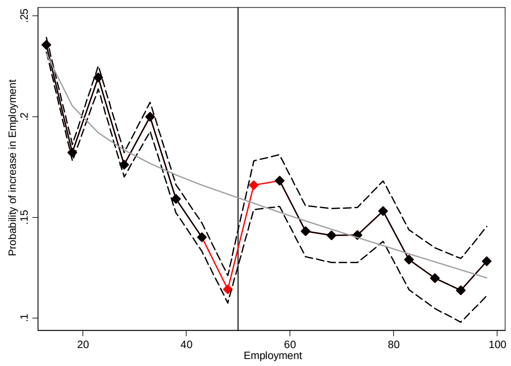


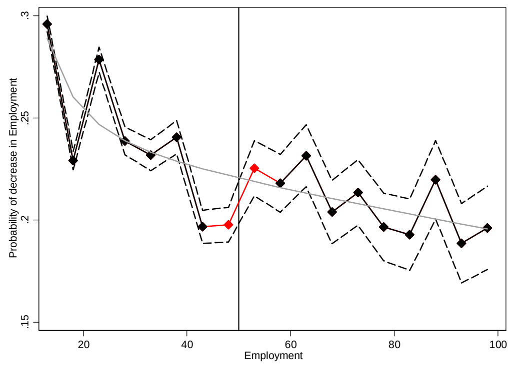
By more than 10%



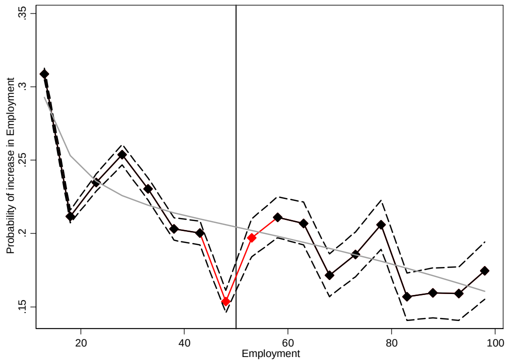


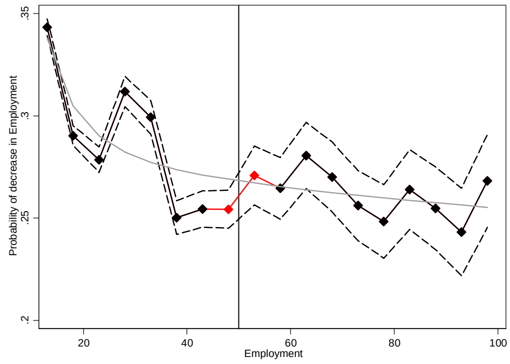
By more than 8%



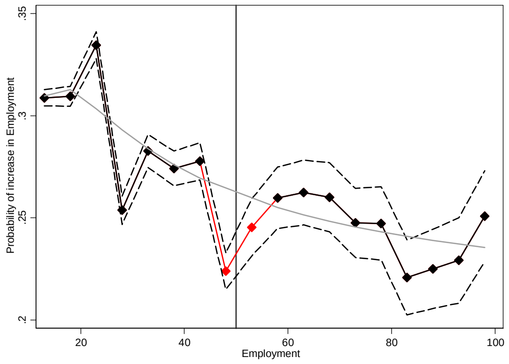


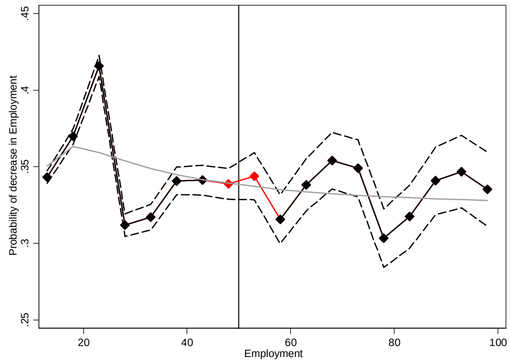
By more than 6%





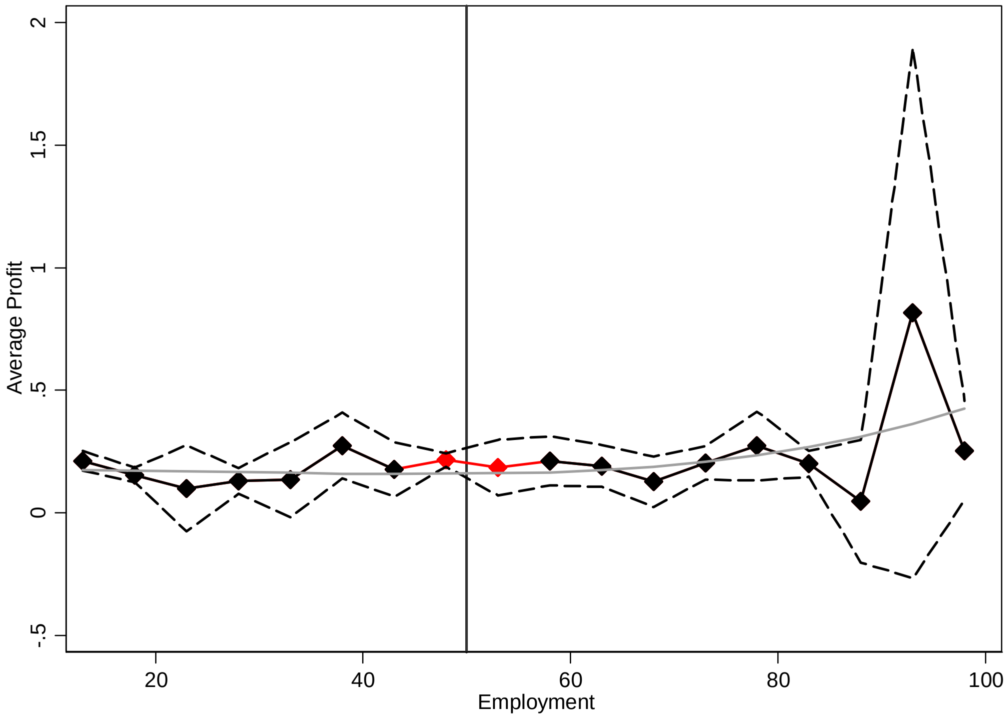
By more than 4% (average probability 2002-2007)





Notes: These graphs examine the proportion of firms whose employment grew (left hand panel) or shrank (tight hand panel) by more than 12%, 10%, 8%, 6% and 4%. We take firms whose size falls into the relevant band at t and then examine subsequent growth between t and t+1. We do this for all firms and for all years separately between 2002 and 2007. 95% confidence intervals shown. Firms who are just above the regulatory threshold are more likely to grow or shrink than we would expect from a polynomial trend. Similarly firms just below the threshold are much less likely to grow or shrink.

Figure 17: Profitability of firms around the regulatory threshold



Notes: Profitability is measured by gross profits divided by value added.

Appendices

Table A1: Production Function Estimation

	(1) Baseline	(2) High Tech Sectors	(3) Low Tech Sectors
Labor	0.739	0.756	0.735
	(0.004)	(0.011)	(0.004)
Capital	0.116	0.126	0.113
	(0.004)	(0.012)	(0.003)
Observations	219,938	35,233	184,705
Firms	53,127	8,410	44,931

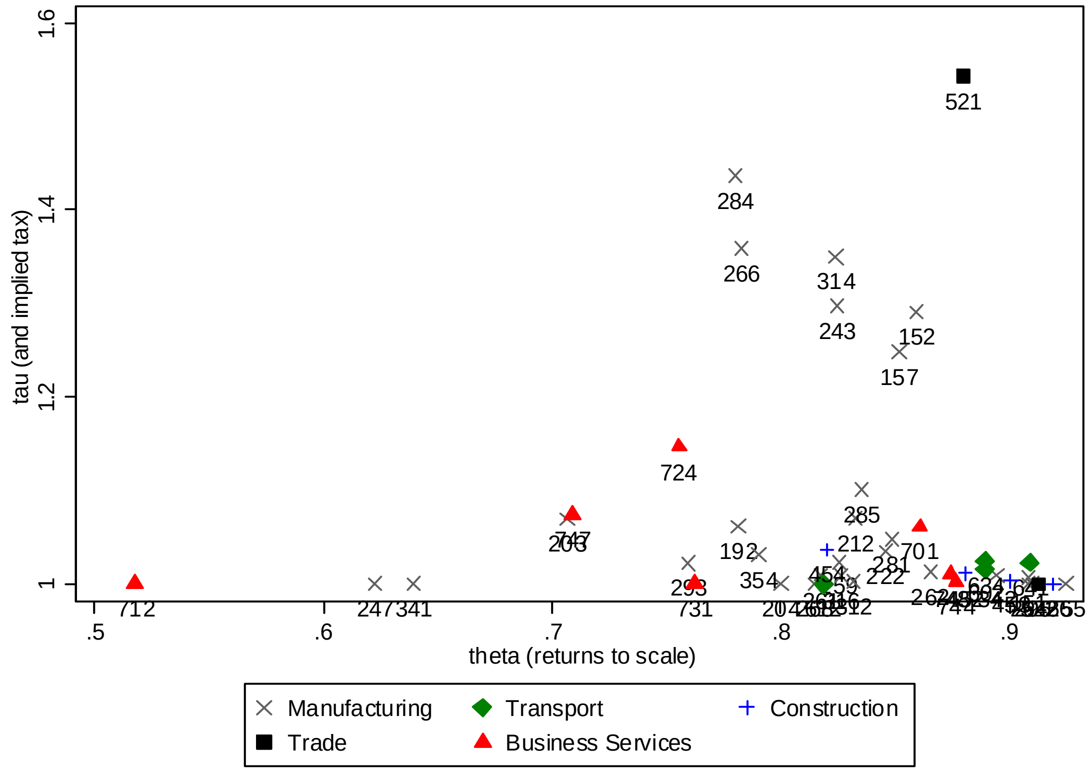
Notes: Parameters estimated by Levinsohn-Petrin (2003) method. Estimation on unbalanced panel 2002-2007 of population of French manufacturing firms with 10 to 1,000 employees (with retrospective information for 2001) for FICUS. "High tech" sectors are based on R&D intensity as defined by the OECD.

Table A2: Welfare Analysis under alternative assumptions for the upper bound of firm size

(Regulated Economy - Unregulated Economy)	Upper bound = 500		Upper bound = 1,000		Upper bound = 5,000	
	Flexible Wages	Rigid Wages	Flexible Wages	Rigid Wages	Flexible Wages	Rigid Wages
1. Unemployment rate	0%	4.066%	0%	4.433%	0%	5.032%
2. Percentage of firms avoiding the regulation	3.652%	3.528%	3.624%	3.490%	3.597%	3.447%
3. Percentage of firms paying tax (compliers)	7.516%	7.259%	8.244%	7.939%	8.928%	8.557%
4. Change in labor costs (wage reduction),Small firms (below 49)	-0.833%	0	-0.910%	0	-1.035%	0
5. Change in labor costs (wage reduction + implicit tax), Large firms (above 49)	0.473%	1.306%	0.396%	1.306%	0.271%	1.306%
6.Excess entry by small firms (percent increase in number of firms)	3.487%	3.472%	3.778%	3.763%	4.265%	4.254%
7. Increase in size of small firms	4.167%	0	4.549%	0	5.176%	0
8. Increase in size of large firms	-2.364%	-6.530%	-1.981%	-6.530%	-1.354%	-6.530%
9. Annual welfare loss:						
a. Implicit Tax (as a share of GDP)	0.525%	0.522%	0.618%	0.615%	0.762%	0.759%
b. Output loss	0.033%	3.353%	0.028%	3.655%	0.019%	4.149%
c. Implicit Tax + Output loss	0.558%	3.875%	0.646%	4.270%	0.781%	4.908%
10. Winners and losers:						
a. Change in wages for workers who remain workers	*-0.833%*	*-4.151%*	*-0.910%*	*-4.534%*	*-1.035%*	*-5.163%*
b. Average gain by entering entrepreneurs	1.245%	*-2.080%*	1.359%	*-2.273%*	1.546%	*-2.589%*
c. Average profit gain by small unconstrained firms	3.333%	0	3.639%	0	4.141%	0
d. Average profit gain by firms constrained at 49	1.484%	*-1.849%*	1.790%	*-1.849%*	2.292%	*-1.849%*
e. Change in profit for large firms	*-1.891%*	*-5.224%*	*-1.585%*	*-5.224%*	*-1.083%*	*-5.224%*

Notes: This is the same analysis as in Table 3 except we allow the assumption over the largest firm size to vary between 500, 1000 and 5000 (compared to 10,000 in the baseline case). We set (insignificant) fixed cost of the regulation to zero, i.e. k =0 and τ -1 = 1.3%. In column (1), model solved assuming wages fully adjust (section 2.4). In column (2), model solved assuming that wages are rigid (section 2.5). "Percentage of firms avoiding the regulation" (Row 2) corresponds to δ in main text. "Percentage of firms paying tax" (Row 3) corresponds to mass of agents with productivity greater than in α_u relative to agents with productivity greater than α_{min}. "Change in labor costs for small firms" (Row 4) corresponds to the general equilibrium wage effect. "Change in labor costs for large firms" (Row 5) corresponds to Row 4 + the estimated implicit tax (τ -1 = 1.3%). "Excess entry" (Row 6) corresponds to the difference in the ln(mass of agents having productivity greater than $\alpha_{\min}^{N,\tau,k}$) minus ln(mass of agents having productivity greater than $\alpha_{\min}^{N,1,0}$. "Increase in size of small firms" (Row 7) corresponds to $\ln(n^*_{N,\tau,0}) - \ln(n^*_{N,1,0})$ for firms having productivity smaller than $\alpha_u^{N,\tau,k}$; "increase in size of large firms" (Row 8) corresponds to $n^*_{N,\tau,0}$-$n^*_{N,1,0}$ for firms having productivity greater than $\alpha_c^{N,\tau,k}$. "Implicit Tax" (Row 9a) corresponds to the total amount of implicit tax ($\int_{\alpha\geq\alpha_c}(\tau-1).\, w^*_{N,\tau,0}.\, n^*_{N,\tau,0}(\alpha)\phi(\alpha)\mathrm{d}\alpha$ as a share of total output $Y_{N,\tau,0}$. "Output loss" (Row 9b) corresponds to $\ln(Y_{N,1,0}) - \ln(Y_{N,\tau,0})$. For "winners and losers" (Row 10), we compute the average (percentage point) changes in expected wages or profits for agents in each of the following bins: 9a. labor force $[\underline{\alpha}; \alpha_{\min}^{N,\tau,0}[$; 9b. new entrepreneurs $[\alpha_{\min}^{N,\tau,0}; \alpha_{\min}^{N,1,0}[$; 9c. small firms $[\alpha_{\min}^{N,1,0}; \alpha_c^{N,\tau,0}]$; 9c. constrained firms $[\alpha_c^{N,\tau,0}; \alpha_u^{N,\tau,0}[$;9d. large firms $[\alpha_u^{N,\tau,0}; 1]$. In the "rigid wages" case in column (2), expected wages are computed as $(1-u^{*,RIG}_{N,\tau,k}).\, w^*_{N,1,0}$ where u is the unemployment rate.

Figure A1: Heterogeneity of Results by three digit sector



Notes: These are the results from industry-specific estimation on the same lines as column (2) of Table 2

Figure A2:

Implications for MRPL, marginal revenue productivity of labor, in our model

Relation (correspondence) between productivity (α, left axis) or marginal product of labor ($\alpha.\theta.(n^*(\alpha))^{\theta-1}$, right axis) and size:

$$\alpha.\theta.(n^*(\alpha))^{\theta-1} = w \text{ for } n < N$$
$$\alpha.\theta.(n^*(\alpha))^{\theta-1} = \tau.w \text{ for } n > n_u$$

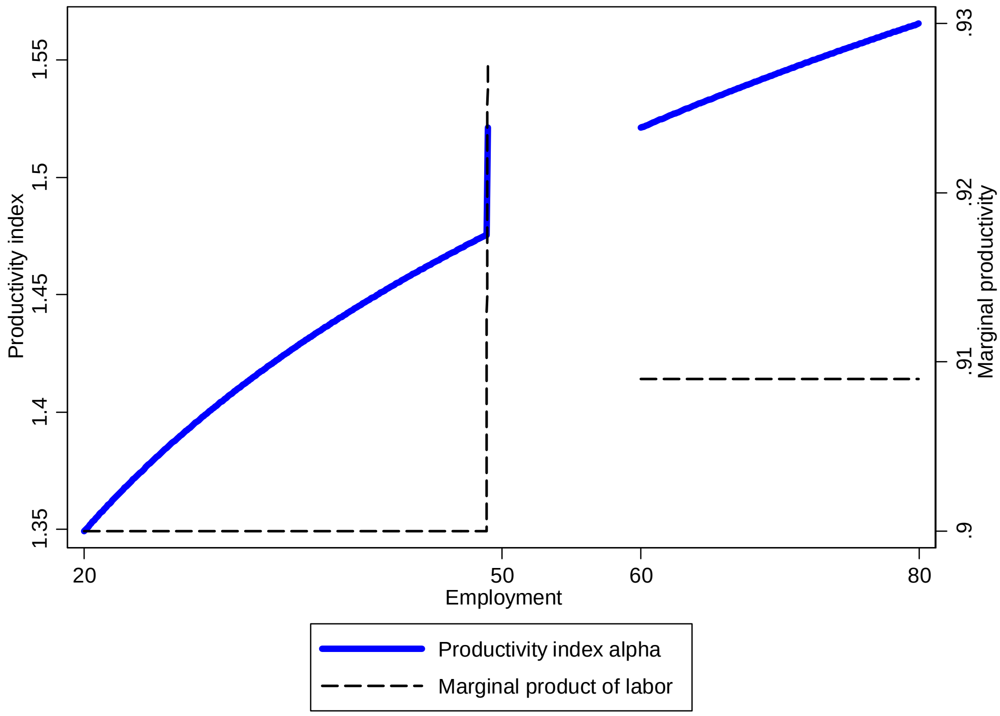
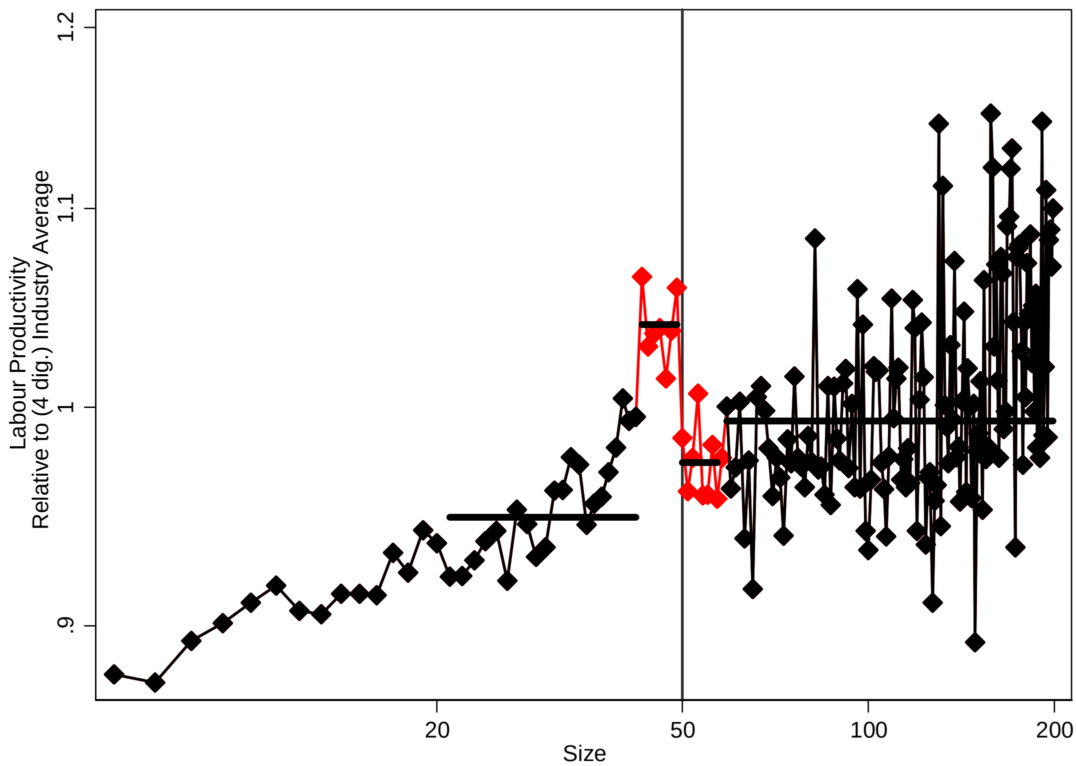
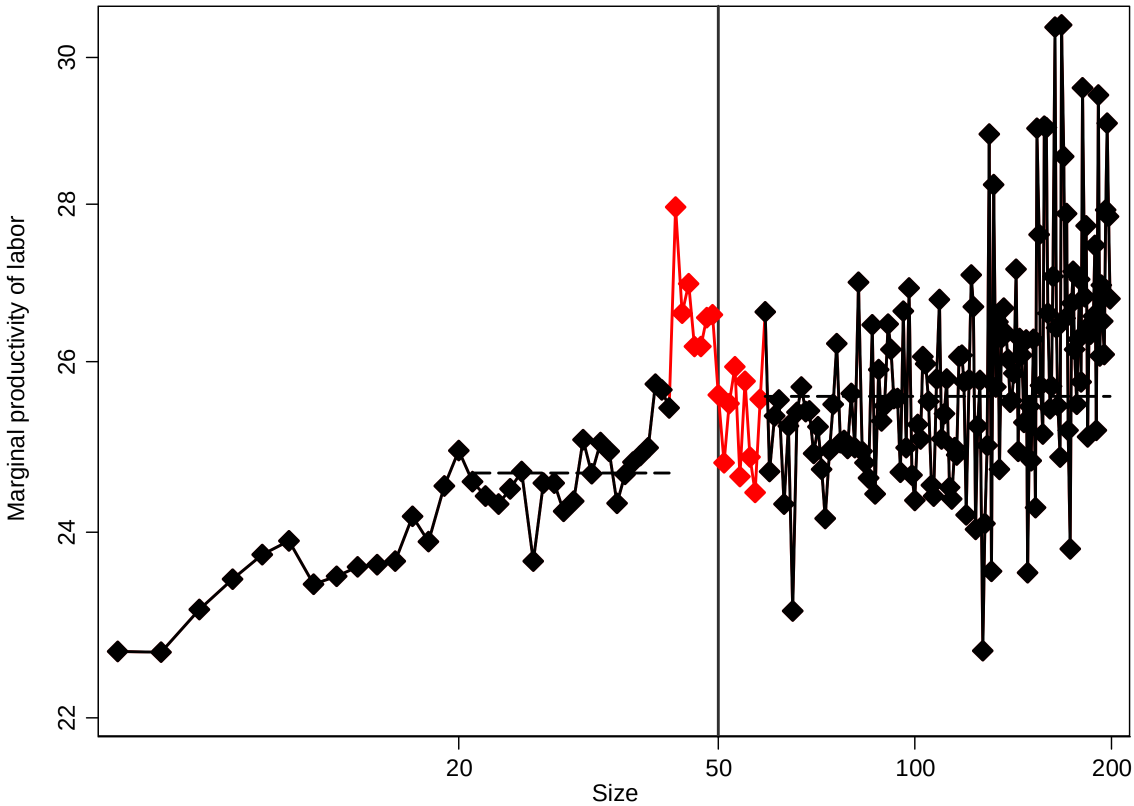


Figure A3: Marginal Revenue Productivity and Firm Size (value added per worker relative to industry average)



Notes: This plots a measure of the MRPL, marginal revenue productivity of labor, as measured by value added per worker (relative to the four digit industry average) by firm size. The "Hsieh-Klenow" estimate of the implicit tax distortion is the difference between average productivity for firms between 20 and 42 employees (0.948) and average productivity for firms between 57 and 200 employees (0.993). This implies a log difference of 0.0464 or 4.64%.

Figure A4: Marginal Revenue Productivity and Firm Size (value added per worker relative to industry average)



Notes: This plots a measure of the MRPL, marginal revenue productivity of labor, as measured by $\alpha.\theta.(n^{*}(\alpha))^{\theta-1}$ by firm size. The "Hsieh-Klenow" estimate of the implicit tax distortion is the difference between average productivity for firms between 20 and 42 employees (24.675) and average productivity for firms between 57 and 200 employees (25.584). This implies a log difference of 0.0315 or 3.15%.