Framing Competition*

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Abstract

We analyze a model of market competition in which two identical firms choose prices as well as how to present, or "frame", their products. A consumer is randomly assigned to one firm, and whether he makes a price comparison with the other firm is a probabilistic function of the firms' framing strategies. We analyze the Nash equilibria in this model. In particular, we show how the answers to the following questions are linked: (1) Are firms' choices of prices and frames correlated? (2) Can firms earn payoffs in excess of the max-min level? (3) Does greater consumer rationality (in the sense of better ability to make price comparisons) imply lower equilibrium prices? We also argue that our model provides a novel account of the phenomenon of product differentiation.

1 Introduction

Standard models of market competition assume that consumers are perfectly able to form a preference ranking of all the alternatives they are aware of, given search costs and potentially limited information about product characteristics. In reality, consumers do not always carry out all the comparisons that "should" be made. Moreover, whether consumers make preference comparisons often depends on the way the alternatives are presented, or "framed". For instance:

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- The complexity of price schedules often makes it hard to make comparisons. A mobile phone calling plan can condition rates on the timing of the call, the network affiliation of the call's destination, or on total previous usage. Since different calling plans condition on different contingencies, consumers may find it difficult to compute the most attractive one.

- Consumers may fail to regard a market alternative as being relevant to their choice problem, even when they know of its existence. Few conspicuous features of the first alternative they consider may steer them towards making comparisons with some products at the expense of others. For instance, a consumer who is exposed to a hamburger ad or walks by a hamburger stall while considering options for a light meal, may fail to take into account alternatives that do not easily fall into the fast food category to which hamburger is traditionally associated.[1]

This paper studies market competition when consumers have limited ability to compare market alternatives, and when comparability is sensitive to framing. Adapting a formalism first introduced in Eliaz and Spiegler (2007), we construct a model that enriches standard Bertrand competition by incorporating the firms' framing decisions. We are interested in the effects of framing on consumer behavior only in so far as it hinders or facilitates price comparisons, and we ignore framing effects that cause preference reversals. We explore the interaction between firms' pricing and framing decisions, and its implications for industry profits and consumer welfare.

Here are some of the questions that we address: (1) Are pricing and framing equilibrium strategies correlated? (2) Does the consumers' limited, frame-sensitive ability to rank alternatives enable firms to earn collusive profits? (3) How are firms' equilibrium pricing and framing decisions affected when some ways of framing an alternative are more conducive to price comparisons than others? (4) Does greater consumer rationality (in the sense of lower sensitivity to framing) lead to a more competitive equilibrium outcome?

In our model, two profit-maximizing firms produce perfect substitutes at zero cost, and face one consumer who buys one unit if priced below a reservation value. Each firm i choose a price p_i and a format x_i for its products. Given the firms' pricing and framing decisions, the consumer chooses as follows. He is initially assigned to one firm at random, say firm 1. With probability $\pi(x_1, x_2)$, the consumer makes a price comparison and chooses the rival firm's product if strictly cheaper. Otherwise, he buys

[1] This example is based on an experiment by Nedungadi (1989).

from the firm 1. When $\pi(x, y) = \pi(y, x)$ for all formats x, y - a property we dub "*order independence*" - price comparisons are independent of the order in which the consumer considers alternatives.

The framing structure given by π can be viewed as a random graph, where the set of nodes corresponds to the set of formats, and $\pi(x, y)$ is the (independent) probability of a directed link from node x to node y. The graph structure represents the consumer's limited, frame-sensitive ability to make price comparisons. The interpretation of a link from format x to format y is that y is easy to compare to x, or that x triggers associations that make the consumer think of the product framed by y as an equivalent choice whenever he first considers the product framed by x. Because of the graph structure, our framework may be reminiscent of models of spatial competition. However, in the concluding section we show that there are significant differences between the two formalisms, both at the level of individual consumer behavior and at the level of equilibrium analysis.

Formats in our model capture the various ways in which firms can present an intrinsically homogeneous product. We use the term "format" in a broad sense that includes aspects of the products' presentation which may be of no relevance to a consumer's utility and yet affect his propensity to make a price comparison. A format can be a price format, a "language" in which a contract is written, an aspect of the positioning of a product (e.g., the assignment of food products into categories such as snacks or health food), and so on. The utility-irrelevance of framing is a limitation of our model. For instance, a consumer may have preferences over the different contingencies covered by a mobile phone calling plan whereas, in our model, such contingencies are introduced by firms for the sole purpose of facilitating or hindering price comparisons.

The benchmark case of a rational consumer is represented by a complete graph (i.e., every node is linked to every other node with probability one), because the consumer always makes a price comparison and chooses the cheapest alternative. The model collapses to conventional Bertrand competition, with firms charging prices equal to zero in Nash equilibrium.

An illustrative example: A "core-periphery" graph

We use the following example to illustrate the model and some of our main insights. Consider the order-independent graph given by Figure 1:[2]

[2] In this paper, diagrams that represent order-independent graphs are drawn as non-directed graphs. In addition, the diagrams supress self-links. Order-independent graphs and non-directed graphs are payoff equivalent for the firms. The difference is that in the former the link between x and y is realized independently of the link between y and x whereas in the latter they are realized simultaneously.

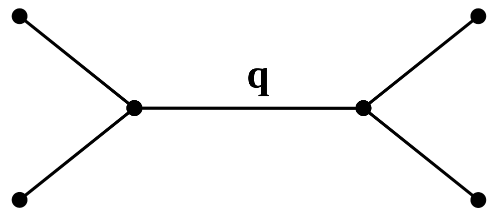

Figure 1

The two "core" nodes in the center can be interpreted as relatively basic price formats that are comparable (and thus linked) with probability q. The four "peripheral" nodes represent more complex formats, each being comparable to one of the basic formats, to which it is linked with probability one. Alternatively, the core nodes may represent broad product categories, while each peripheral node can be interpreted as a refinement of its "parent" broad category.

Let us consider first an extreme case in which the two core formats are incomparable - i.e., $q = 0$. The game played between the two firms has a unique symmetric Nash equilibrium. Firms play a mixed strategy that randomizes independently over formats and prices. The framing strategy assigns probability $\frac{1}{2}$ to each of the two core formats and zero probability to the peripheral formats. Note that this framing strategy has the property that when a firm adopts it, the probability of a price comparison is $\frac{1}{2}$, independently of the rival firm's framing strategy: this framing strategy max-minimizes the probability of a price comparison. The expected equilibrium price is $\frac{1}{2}$, and thus firms earn an equilibrium payoff of $\frac{1}{4}$, which is also the max-min payoff. Profits are positive due to consumers' limited ability to make price comparisons. However, competitive forces are strong enough to rule out additional, collusive gains above max-min payoffs.

Now consider the case in which the two core formats are comparable - i.e., $q = 1$. The framing strategy that mixes uniformly between the two core formats remains the unique strategy that max-minimizes the probability of a price comparison. Consequently, the max-min payoff is still $\frac{1}{4}$. However, in symmetric Nash equilibrium firms do not play this framing strategy. Instead, they mix uniformly over all six formats. Moreover, the framing and pricing strategies are correlated: when the price is in $[\frac{2}{3}, 1]$, firms mix uniformly over the four peripheral formats, and when the price is in $[\frac{2}{5}, \frac{2}{3}]$, firms mix uniformly over the two core formats. Expected equilibrium price is $\frac{2}{3}$, and thus the firms' equilibrium payoff is $\frac{1}{3}$, which exceeds the max-min level.

The graph with $q = 1$ has greater connectivity than the graph with $q = 0$, and thus represents a "more rational" consumer. For any strategy profile of the firms, it leads to fewer decision errors for a consumer. Nevertheless, the expected Nash equilibrium price is higher when $q = 1$. This apparent anomaly is explained by the fact that firms that charge a high (low) price have an incentive to adopt a framing strategy that induces a low (high) probability of a price comparison. When $q = 0$, the framing strategy that mixes uniformly over the two core formats equalizes the probability of a price comparison for all formats. Hence, it is optimal for firms to adopt this framing strategy independently of their price. In contrast, when $q = 1$, mixing uniformly over the two core formats does not suit firms that charge a high price, as core formats are always comparable. When a firm's realized price is high, it is optimal for the firm to choose peripheral formats as are they are less likely to trigger a price comparison. Thus, equilibrium payoffs rise above the max-min level.

Overview of the results

We begin our analysis of Nash equilibria for graphs that satisfy order independence. This analysis, presented in Section 3, highlights a property of graphs, called "*weighted regularity*", which generalizes the familiar regularity property. A graph is weighted-regular if nodes can be assigned weights such that each node has the same total weighted links. (Regularity corresponds to a special case in which the weights are uniform across the entire set of graph nodes.) Under weighted regularity, all formats are equally comparable, once the frequency with which they are played is factored in.

We show that if a graph is weighted-regular, there exists a Nash equilibrium in which the firms' pricing and framing strategies are independent, and their payoffs are equal to the max-min level. The significance of max-min equilibrium payoffs is that competitive forces prevail in that they push industry profits to the lowest level possible given the consumer's limited ability to make price comparisons. Conversely, if firms' pricing and framing strategies are independent in some Nash equilibrium, the graph must be weighted-regular and firms earn max-min payoffs in this equilibrium. Moreover, their pricing strategies must be identical.

We investigate a special class of symmetric Nash equilibria, called "cutoff equilibria", where every format that is played with positive probability is unambiguously associated with prices either above or below a cutoff. We show that a cutoff equilibrium induces max-min payoffs if and only if the graph is weighted-regular. Moreover, the equilibrium framing strategy conditional on prices above (below) the cutoff min-maximizes (max-minimizes) the probability of a price comparison.

We apply the results above to obtain a complete characterization of symmetric Nash equilibria in a class of "bi-symmetric" graphs, that is, graphs in which the connectivity between two formats depends only on which of two categories they belong to. In Section 4, we relax order independence and examine the extent to which these results can be extended.

Related literature

This paper joins recent attempts to formalize in broad terms the role of framing effects in decision making. Rubinstein and Salant (2008) study choice behavior, where the notion of a choice problem is extended to include both the choice set A and a frame f, which is interpreted as observable information which should not affect the rational assessment of alternatives but nonetheless affects choice. A choice function assigns an element in A to every extended choice problem. Rubinstein and Salant conduct a choice-theoretic analysis of such extended choice functions, and relate their framework to the standard model of choice correspondences. In particular, they identify conditions under which extended choice functions are consistent with utility maximization. Bernheim and Rangel (2007) use a similar framework to extend standard welfare analysis to situations in which choices are sensitive to frames.

Our notion of frame dependence differs from the one in the above models. First, we associate frames with individual alternatives, rather than entire choice sets. Second, in our model framing affects the probability that consumers apply a preference ranking, but never leads to preference reversals. Finally, our focus is on market implications rather than choice-theoretic analysis. In this respect, this paper is closest to Eliaz and Spiegler (2007), which first formalized the idea that framing (and marketing devices in general) affects preference incompleteness without reversing preference rankings. The model of consumer behavior in Eliaz and Spiegler is more general in that the consumer's propensity to apply a preference ranking to a pair of market alternatives depends on an arbitrary function of the alternatives' payoff-relevant details as well as their frames. In the market applications analyzed in Eliaz and Spiegler, framing decisions are costly and price setting is assumed away, leading to very different game-theoretic properties.

This paper contributes to a growing theoretical literature on the market interaction between profit-maximizing firms and boundedly rational consumers. Rubinstein (1993) analyzes monopolistic behavior when consumers differ in their ability to understand complex pricing schedules. Piccione and Rubinstein (2003) study intertemporal pricing when consumers have diverse ability to perceive temporal patterns. Spiegler (2006a,b) analyzes markets in which profit-maximizing firms compete over consumers who rely on naive sampling to evaluate each firm. DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006,2008) and Gabaix and Laibson (2006) study interaction with consumers having limited ability to predict their future tastes. See Ellison (2006) for a recent survey.

Our paper is also related to the large literature on product differentiation (for instance, see Anderson, De Palma and Thisse (1992)). Indeed, our model provides a novel interpretation of this phenomenon. In equilibrium, firms offer a homogenous product in a variety of guises, and this variety can be viewed as a kind of product differentiation. Yet, in our model, differentiation does not result from the firms' attempt to cater to diverse taste niches, but from the attempt to make price comparison less likely. The force behind differentiation is the limited comparability between different ways of presenting a homogeneous product, rather than differentiated tastes. >From the point of view of consumer welfare, differentiation in our model has the purely negative effect of raising market prices.

2 The Model

A *graph* is a pair (X, π), where X is a finite set of nodes and $\pi : X \times X \rightarrow [0, 1]$ is a function that determines the probability $\pi(x, y)$ a directed edge links node x to node y. The probability that x is linked to y is independent of other links being realized. Let n denote $|X|$. We refer to nodes as *formats*. Assume that $\pi(x, x) = 1$ for every $x \in X$ - that is, every format is linked to itself. A graph π is *deterministic* if for every distinct $x, y \in X$, $\pi(x, y) \in \{0, 1\}$. A graph π is *order independent* if $\pi(x, y) = \pi(y, x)$ for all $x, y \in X$.

A market consists of two identical, expected-profit-maximizing firms and one consumer. These firms produce at zero cost a homogenous product for which the consumer has a reservation value equal to one. The firms move simultaneously. A pure strategy for firm i is a pair (p_i, x_i), where $p_i \in [0, 1]$ is a price and $x_i \in X$. We allow firm i to employ mixed strategies of the form $\left(\lambda_i, (F_i^x)_{x \in Supp(\lambda_i)}\right)$, where $\lambda_i \in \Delta(X)$ and F_i^x is a *cdf* over $[0, 1]$ for every $x \in Supp(\lambda_i)$. We refer to λ_i as firm i's framing strategy and to F_i^x as firm i's pricing strategy at x. Let $\mu^x \in \Delta(X)$ denote a degenerate probability distribution that assigns probability one to node x. The marginal pricing strategy induced by a mixed strategy $\left(\lambda_i, (F_i^x)_{x \in Supp(\lambda_i)}\right)$ is

$$F_i = \sum_{x \in Supp(\lambda)} \lambda_i(x) F_i^x$$

Given a *cdf* F, let F^- denote its left limit.

Given a realization $(p_i, x_i)_{i=1,2}$ of the firms' strategies, the consumer chooses a firm according to the following rule. He is randomly assigned to a firm - with probability $\frac{1}{2}$ for each firm. Suppose that he is assigned to firm i. If there is a link from x_i to x_j - an event that occurs with probability $\pi(x_i, x_j)$ - the consumer makes a price comparison and chooses firm j if $p_j < p_i$. Otherwise, the consumer chooses the initially assigned firm i.

The consumer's initial assignment to a firm can be interpreted as the first alternative considered in a sequential decision process or as a default option arising from previous decisions. The consumer's choice procedure is biased in favor of the initial firm i: the consumer selects it with probability one when $p_j \geq p_i$ and with probability $1 - \pi(x_i, x_j)$ when $p_j < p_i$. When the graph is order-independent, the sequential aspect of the choice procedure is inessential. In this case, the model is consistent with an additional interpretation in which the consumer is confronted with both alternatives simultaneously, chooses the cheaper one if the formats are linked, and chooses randomly otherwise.

To illustrate the firms' payoff function, consider the following graph:

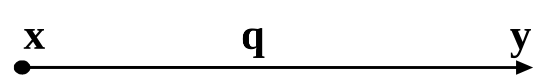

Figure 2

Thus, $\pi(x, y) = q$ and there is no link from y to x. Suppose that firm 1 adopts the format x while firm 2 adopts the format y. If $p_1 < p_2$, firm 1 earns a payoff of $\frac{1}{2}p_1$ while firm 2 earns $\frac{1}{2}p_2$. If $p_1 > p_2$, firm 1 earns $p_1 \cdot (\frac{1}{2} - \frac{1}{2}q)$ while firm 2 earns $p_2 \cdot (\frac{1}{2} + \frac{1}{2}q)$.

When firm i plays the mixed strategy $\left(\lambda_i, (F_i^x)_{x \in Supp(\lambda_i)}\right)$, we can write firm j's expected payoff from the pure strategy (p, x) as follows:

$$\frac{p}{2} \cdot \{1 + \sum_{y \in X} \lambda_i(y) \cdot [(1 - F_i^y(p)) \cdot \pi(y, x) - F_i^{y-}(p) \cdot \pi(x, y)]\}$$

Is consumer choice rational?

Fully rational consumers are represented by a complete graph - i.e. $\pi(x, y) = 1$ for all $x, y \in X$. Rational consumers make a price comparison independently of the firms' framing decisions, and in this case the model is reduced to standard Bertrand competition.

For a typically incomplete graph, the consumer's choice behavior is inconsistent with maximizing a random utility function over price-format pairs. To see why, consider the following deterministic, order-independent graph: $X = \{a, b, c\}$, $\pi(a, b) = \pi(b, c) = 1$ and $\pi(a, c) = 0$. Suppose that $p < p' < p''$. When faced with the strategy profile $((p, a), (p', b))$, the consumer chooses (p, a) with probability one. Similarly, when faced with the strategy profile $((p', b), (p'', c))$, the consumer chooses (p', b) with probability one. However, when faced with the strategy profile $((p, a), (p'', c))$, the consumer chooses each alternative with probability $\frac{1}{2}$. No random utility function over $[0, 1] \times X$ can rationalize such choice behavior. The reason is that the graph represents a binary relation which is intransitive, and this translates into the intransitivity of the implied revealed preference relation over price-format pairs.

Hide and seek

Our analysis will make use of an auxiliary two-player, zero-sum game, which is a generalization of familiar games such as Matching Pennies. The players are referred to as *hider* and *seeker*, denoted h and s. The players share the same action space X. Given the action profile (x_h, x_s), the hider's payoff is $-\pi(x_h, x_s)$ and the seeker's payoff is $\pi(x_h, x_s)$. We will refer to this game as the *hide-and-seek* game associated with a graph. Given a mixed-strategy profile (λ_h, λ_s) in this game, the probability that the seeker finds the hider is

$$v(\lambda_h, \lambda_s) = \sum_{x \in X} \sum_{y \in X} \lambda_h(x) \lambda_s(y) \pi(x, y)$$

To see the relevance of this auxiliary game to our model, suppose that firm 1 plays a mixed strategy with framing strategy λ and an atomless marginal pricing strategy F

over the support $[p^L, p^H]$. When firm 2 considers charging the price p^H, it should select a format that minimizes the probability of a price comparison. Hence, it behaves as a hider in the hide-and-seek game, where the seeker's strategy is λ. Similarly, when firm 2 considers charging the price p^L, it should select a format that maximizes the probability of a price comparison. Hence, it behaves as a seeker in the hide-and-seek game, where the hider's strategy is given by λ. When a firm considers charging an intermediate price, it chooses its framing strategy partly as a hider and partly as a seeker.

The value of the hide-and-seek game is

$$v^* = \max_{\lambda_s} \min_{\lambda_h} v\left(\lambda_h, \lambda_s\right)$$

The max-min payoff of a firm in our model is thus $\frac{1}{2}(1 - v^*)$. The reason is that the worst-case scenario for a firm is that its opponent plays $p = 0$ and adopts the seeker's max-min framing strategy, to which a best-reply is to play $p = 1$ and minimize the probability of a price comparison.

Preliminary analysis of Nash equilibria

We will conduct a detailed analysis of Nash equilibria in the following sections. In this section, we present two basic results. The first characterizes the support of the marginal pricing strategies when both firms make positive profits. The second provides a simple necessary and sufficient condition for the equilibrium outcome to be competitive (that is, both firms charge zero prices).

Proposition 1 *In any Nash equilibrium in which firms make positive profits, there exists a price $p^l \in (0, 1)$ such that for both $i = 1, 2$, F_i is strictly increasing over the interval $[p^l, 1)$.*

Proposition 2 *Let F_i be a Nash-equilibrium marginal pricing strategy for firm i, $i = 1, 2$. Then, $F_1(0) = F_2(0) = 1$ if and only if there exists a format $x^* \in X$ such that $\pi(x, x^*) = 1$ for every $x \in X$.*

Note that a corollary of Proposition 1 is that if firm i earns the max-min payoff $\frac{1}{2}(1 - v^*)$ in Nash equilibrium, then it must be the case that firm j's framing strategy conditional on $p < 1$ is a max-min strategy for the seeker in the associated hide-and-seek game.

The proofs of these results rely on price undercutting arguments that are somewhat more subtle than familiar ones. For instance, suppose that firm 1's marginal pricing strategy has a mass point at some price p^* which belongs to the support of firm 2's marginal pricing strategy. In conventional models of price competition, there is a clear incentive for firm 2 to undercut its price slightly below p^*. In our model, however, price undercutting may have to be accompanied by a change in the framing strategy in order to be effective. Adopting a new framing strategy may be undesirable for firm 2 because it could change the probability of a price comparison when the realization of firm 1's pricing strategy is $p \neq p^*$.

For the rest of the paper, we assume that the necessary and sufficient condition for a competitive equilibrium outcome is violated.

Condition 1 *For every $x \in X$ there exists $y \neq x$ such that $\pi(y, x) < 1$.*

This condition ensures that the firms' max-min payoff is strictly positive - or, equivalently, that the value of the associated hide-and-seek game is strictly below one. Once competitive equilibrium outcomes have been eliminated, any Nash equilibrium must be *mixed*. To see why, assume that each firm i plays a pure strategy (p_i, x_i). If $0 < p_i \leq p_j$, then firm j can deviate to the strategy $(p_i - \varepsilon, x_i)$, where $\varepsilon > 0$ is arbitrarily small, and raise its payoff. If $p_i = 0$, firm i earns zero profits, contradicting the observation that the firms' max-min payoffs are strictly positive. Thus, from now on, we will take it for granted that Nash equilibrium is strictly mixed.

3 Nash Equilibrium under Order Independence

In this section, we analyze mixed strategy equilibria in order-independent graphs. We present the notion of weighted regularity, some general characterization results, and a complete characterization of symmetric equilibria in the class of so-called "bi-symmetric" graphs. We use this characterization to highlight the non-trivial effects that greater consumer rationality has on equilibrium prices in our model. Finally, we discuss the novel account that our model provides for the phenomenon of product differentiation.

3.1 Weighted Regularity

When an order-independent graph is regular - i.e. when $\sum_{y \in X} \pi(x, y) = \bar{v}$ for all nodes $x \in X$ - all formats are equally comparable in the sense that each format has an identical expected number of links. However, this notion of equal comparability ignores the frequency with which different formats are adopted. If, for example, x is an isolated node and both firms choose this format, the consumer will make a price comparison with probability one. Hence, a proper notion of equal comparability should take into account the frequency of adoption of different formats.

Definition 1 *An order-independent graph (X, π) is weighted-regular if there exist $\beta \in \Delta(X)$ and $\bar{v} \in [0, 1]$ such that $\sum_{y \in X} \beta(y) \pi(x, y) = \bar{v}$ for all $x \in X$. We say in this case that β verifies weighted regularity.*

Regularity thus corresponds to a special case in which the uniform distribution over X verifies weighted regularity. Note that the set of distributions that verify weighted regularity is convex. The following are examples of weighted-regular, order-independent graphs.

Example 3.1: Equivalence relations. Consider a deterministic graph that in which $\pi(x, y) = 1$ if and only if x and y belongs to the same equivalence class of an equivalence relation. Any distribution that assigns equal probability to each equivalence class verifies weighted regularity.

Example 3.2: A cycle with random links. Let $X = \{1, 2, ..., n\}$, where n is even. Assume that for every distinct $x, y \in X$, $\pi(x, y) = \frac{1}{2}$ if $|y - x| = 1$ or $|y - x| = n - 1$, and $\pi(x, y) = 0$ otherwise. A uniform distribution over all odd-numbered nodes verifies weighted regularity.

Example 3.3: Linear similarity. Consider the following deterministic graph. Let $X = \{1, 2, ..., 3L\}$, where $L \geq 2$ is an integer. For every distinct $x, y \in X$, $\pi(x, y) = 1$ if and only if $|x - y| = 1$. A uniform distribution over the subset $\{3k - 1\}_{k=1,...,L}$ verifies weighted regularity.

In addition, note that the graph given by Figure 1 is weighted regular if and only if $q = 0$. The framing strategy that verifies weighted regularity in this case assigns probability $\frac{1}{2}$ to each of the two core nodes.

Lemma 1 *The distribution $\lambda \in \Delta(X)$ verifies weighted regularity in a graph (X, π) if and only if (λ, λ) is a Nash equilibrium in the associated hide-and-seek game.*

Proof. Suppose that λ verifies weighted regularity. If one of the players in the associated hide-and-seek game plays λ, every strategy for the opponent - including λ itself - is a best-reply. Now suppose that (λ, λ) is a Nash equilibrium in the associated hide-and-seek game. Denote $v(\lambda, \lambda) = v^*$. If some format attains a higher probability of a price comparison than v^*, then λ cannot be a best-reply for the seeker. Similarly, if some format attains a lower probability of a price comparison than v^*, then λ cannot be a best-reply for the hider. Therefore, it must be the case that every format generates the same probability of a price comparison - namely v^* - against λ. ■

Thus, a graph is weighted-regular if and only if the associated hide-and-seek game has a symmetric Nash equilibrium.

3.2 Price-Format Independence and Equilibrium Payoffs

A mixed strategy $\left(\lambda, (F^x)_{x \in Supp(\lambda)}\right)$ exhibits *price-format independence* if $F^x = F^y$ for any $x, y \in Supp(\lambda)$. The next proposition shows that if the graph is weighted-regular, there exists a symmetric Nash equilibrium that exhibits price-format independence. Conversely, if the strategies are price-format independent in some Nash equilibrium, then each firm plays a framing strategy that verifies weighted regularity and earns max-min payoffs. In addition, the firms' pricing strategies must be identical. Define the *cdf*

$$G^*(p) = 1 - \frac{1 - v^*}{2v^*} \cdot \frac{1 - p}{p} \tag{1}$$

with support $[\frac{1-v^*}{1+v^*}, 1]$.

Proposition 3 *(i) Suppose that λ^1 and λ^2 verify weighted regularity. Then, there exists a Nash equilibrium in which firm i, $i = 1, 2$, plays the framing strategy λ^i and the pricing strategy $F_i^x(p) = G^*(p)$ for all $x \in X$, and earns max-min payoffs.*
(ii) Let $\left(\lambda_i, (F_i^x)_{x \in Supp(\lambda_i)}\right)_{i=1,2}$ be a Nash equilibrium in which both firms' strategies exhibit price-format independence. Then, λ_1 and λ_2 verify weighted regularity, firms earn max-min payoffs, and their marginal pricing strategy is given by 1.

Proof. *(i)* Suppose that firm i plays the framing strategy λ^i. By the definition of weighted regularity, every format that the rival firm j may adopt attains the same probability of a price comparison v^* against λ^i. We can thus assume that the probability of a price comparison is exogenously fixed at v^*. The pricing strategy F given by (1) has the property that for every p in the support of F, the following equation holds:

$$\frac{1-v^*}{2} = \frac{p}{2} \cdot [1 + v^*(1-F(p)) - v^*F(p)]$$

which is necessary and sufficient for F to be a best-replying pricing strategy to itself, given that the probability of a price comparison is v^*.

(ii) By assumption, $F_i^x = F_i$ for any $x \in Supp(\lambda_i)$, $i = 1, 2$. Therefore, $x \in \arg\min v(\cdot, \lambda_i)$ for every $x \in Supp(\lambda_j)$ - otherwise, it would be profitable to deviate into the pure strategy $(1, y)$ for some $y \in \arg\min v(\cdot, \lambda_i)$. Similarly, $x \in \arg\max v(\cdot, \lambda_i)$ for every $x \in Supp(\lambda_j)$ - otherwise, it would be profitable to deviate into the pure strategy (p^l, y) for some $y \in \arg\max v(\cdot, \lambda_i)$. It follows that (λ_1, λ_2) and (λ_1, λ_2) are Nash equilibria of the associated hide-and-seek game. Hence, as λ_1 and λ_2 max-minimize as well as min-maximize v, (λ_1, λ_1) and (λ_2, λ_2) are also Nash equilibria of the associated hide-and-seek game. Therefore, both λ_1 and λ_2 verify weighted regularity. Relatively standard arguments (see Proposition 1 in Spiegler (2006)) establish that the equilibrium pricing strategy for each firm must be given by (1) if the probability of a price comparison is exogenously fixed at v^*. ∎

For an intuition behind this result, note that when firms play framing strategies that verify weighted regularity, their opponents are indifferent among all formats and can treat the probability of a price comparison v^* as fixed and exogenous. Therefore, we can construct an equilibrium in which firms play framing strategies that verify weighted regularity, and an independent pricing strategy. For the converse, note that the framing strategy that firms play in conjunction with the highest price in the equilibrium distribution minimizes the probability of a price comparison against the equilibrium framing strategy. Similarly, the framing strategy associated with the lowest price in the equilibrium distribution maximizes the probability of a price comparison against the equilibrium framing strategy. If these two framing strategies coincide, then all formats must induce the same probability of a price comparison against the opponent's framing strategy.

To demonstrate this result, let us revisit some of the examples presented in the previous sub-section. In Example 3.2, suppose that firm 1 (2) plays a framing strategy which is a uniform distribution over all odd-numbered (even-numbered) nodes. Both distributions verify weighted regularity. Suppose further that both firms play independently the pricing strategy given by (1), where $v^* = \frac{2}{n}$. This strategy profile constitutes a Nash equilibrium.

In Example 3.3, suppose that both firms play a framing strategy which mixes uniformly over the subset of nodes $\{3k-1\}_{k=1,\ldots,L}$. This distribution verifies weighted regularity. Suppose further that both firms play independently the pricing strategy given by (1), where $v^*=\frac{1}{L}$. This strategy profile constitutes a symmetric Nash equilibrium, in which the consumer makes a price comparison if and only if the firms adopt the same format. In this equilibrium, the formats that are played with positive probability are like "local monopolies": when the consumer faces two different formats, he remains loyal to the one adopted by the firm he is initially assigned to. Price comparisons take place only when both firms use the same format.

Not *all* Nash equilibria in weighted-regular graphs necessarily exhibit price-format independence. This is trivially the case in graphs that contain redundant nodes (i.e., there exist distinct formats x,x' such that $\pi(x,y)=\pi(x',y)$ for every $y\in X$). In this case, we can construct an equilibrium in which the framing strategy verifies weighted regularity, yet the format x is associated with low prices while the format x' is associated with high prices. As we will see in Section 3.3, price-format correlation is possible under weighted-regular graphs even when there are no redundant nodes.

Two questions are still open. Do max-min equilibrium payoffs imply that the graph is weighted-regular? Does weighted regularity imply that equilibrium payoffs cannot exceed the max-min level? We are only able to address these questions under some restrictions on equilibrium strategies.

Proposition 4 *Consider a Nash equilibrium* $\left(\lambda_i,(F_i^x)_{x\in Supp(\lambda_i)}\right)_{i=1,2}$. *If firm 1 earns max-min payoffs and firm 2 play a framing strategy with full support, then* (X,π) *is weighted-regular.*

Proof. The proof is based on the following version of Farkas' lemma. Let Ω be an $l\times m$ matrix and b an l-dimensional vector. Then, exactly one of the following two statements is true: *(i)* there exists $\beta\in\mathbb{R}^m$ such that $\Omega\beta=b$ and $\beta\geq 0$; *(ii)* there exists $\delta\in\mathbb{R}^l$ such that $\Omega^T\delta\geq 0$ and $b^T\delta<0$.

Suppose that (X,π) is not weighted-regular. Let us first show that for every $\mu\in\Delta(X)$ such that $\mu(x)>0$ for all $x\in X$, there exists $\tilde{\mu}\in\Delta(X)$ such that, for all $y\in X$,

$$\sum_{x\in X}\mu(x)\,\pi(x,y)<\sum_{x\in X}\tilde{\mu}(x)\,\pi(x,y)$$

Order the nodes so that $X = \{1, .., n\}$. Any $\beta \in \Delta(X)$ is thus represented by a row vector $(\beta_1, ..., \beta_n)$. Let Π be a $n \times n$ matrix whose ijth entry is $\pi(i, j)$. Note that $\Pi = \Pi^T$. Since (X, π) is not weighted-regular, there exist no $\beta \in \mathbb{R}^n$ and $c > 0$ such that $\Pi\beta^T = (c, c, ..., c)^T$. By Farkas' Lemma, there exists a column vector $\delta \in \mathbb{R}^n$ such that $\Pi\delta \geq 0$ and $(c, c, ..., c)\delta < 0$. Since $\pi(i, i) = 1$ for every $i \in \{1, ..., n\}$ and $\pi(i, j) \geq 0$ for all $i, j \in \{1, ..., n\}$, we can modify δ into a column vector $\tilde{\delta}$ such that $\tilde{\delta}_i > \delta_i$ for every i, $\Pi\tilde{\delta} > 0$ and $\sum_i \tilde{\delta}_i = 0$. Let $\mu \in \Delta(X)$ and $\mu(i) > 0$ for every $i \in \{1, ..., n\}$. By the construction of $\tilde{\delta}$, $\tilde{\mu} = \mu + \alpha\tilde{\delta}$ is also a probability distribution over X, for a sufficiently small $\alpha > 0$. Then

$$\Pi\tilde{\mu}^T = \Pi\mu^T + \alpha\Pi\tilde{\delta} > \Pi\mu^T$$

In particular, every component of the vector $\Pi\tilde{\mu}^T$ is strictly larger than the corresponding component of $\Pi\mu^T$.

By hypothesis, $\lambda_2(x) > 0$ for all $x \in X$. We have shown that there exists another framing strategy $\tilde{\lambda}$ such that every format $y \in X$ induces a strictly higher probability of a price comparison than λ_2. This contradicts that λ_2 is a max-min strategy. ■

The proof of this result relies entirely on the associated hide-and-seek game. It also shows that if, in the hide-and-seek game, there exists a max-min strategy with full support for the seeker, there must exist a symmetric Nash equilibrium.

3.3 Cutoff Equilibria

In this sub-section we study equilibria that exhibit a simple kind of price-format correlation. A symmetric Nash equilibrium in which firms play the strategy $(\lambda, (F^x)_{x \in Supp(\lambda)})$ is a *cutoff equilibrium* if there exist prices $p^l \leq p^m \leq p^h$ such that for all $x \in Supp(\lambda)$, the support of F^x is either $[p^l, p^m]$ or $[p^m, p^h]$. Thus, in a cutoff equilibrium formats are unambiguously associated either with high prices or with low prices. Let λ_H be the framing strategy conditional on the nodes in which the pricing strategy has support $[p^m, p^h]$. Similarly, let λ_L be the framing strategy conditional on the nodes in which the pricing strategy has support $[p^l, p^m]$.

Lemma 2 *If $(\lambda, (F^x)_{x \in Supp(\lambda)})$ is a cutoff equilibrium strategy with $p^l < p^m < p^h$, then (λ_H, λ_L) is a Nash equilibrium in the associated hide-and-seek game.*

Proof. Let $(\lambda, (F^x)_{x \in Supp(\lambda)})$ be a cutoff equilibrium. Note that a firm charging p^m is indifferent between λ_H and λ_L. Moreover, λ_H minimizes $v(\lambda_H, \lambda)$ and λ_L maximizes $v(\lambda_L, \lambda)$. Denote $\alpha = 1 - F(p^m)$. Then, $\lambda = \alpha\lambda_H + (1-\alpha)\lambda_L$. The payoff from the strategy (p^m, λ_H) can be written as

$$\frac{p^m}{2}\left(1 + v(\lambda_H, \lambda) - 2(1-\alpha)\, v(\lambda_H, \lambda_L)\right)$$

Since λ_H minimizes $v(\cdot, \lambda)$, it must be the case that λ_H minimizes $v(\cdot, \lambda_L)$. The payoff from the strategy (p^m, λ_L) can be written as

$$\frac{p^m}{2}\left(1 + 2\alpha v(\lambda_L, \lambda_H) - v(\lambda_L, \lambda)\right)$$

Since λ_L maximizes $v(\cdot, \lambda)$, it must be the case that λ_L maximizes $v(\cdot, \lambda_H)$. Hence, (λ_L, λ_H) is a Nash equilibrium in the hide-and-seek game. ■

Proposition 5 *Consider a cutoff equilibrium $(\lambda, (F^x)_{x \in Supp(\lambda)})$. (i) If firms earn max-min payoffs, then λ verifies weighted regularity. (ii) If the graph is weighted-regular, then firms earn max-min payoffs.*

Proof. *(i)* Assume that firms earn max-min payoffs in equilibrium. If p^m coincides with p^l or p^h, then $\arg\min_{x \in X} v(\mu^x, \lambda) = \arg\max_{x \in X} v(\mu^x, \lambda)$ hence λ verifies weighted regularity. Let us now suppose that $p^l < p^m < p^h$. Denote $\alpha = 1 - F(p^m)$. Then, $\lambda = \alpha\lambda_H + (1-\alpha)\lambda_L$. Since λ max-minimizes v, λ is a max-min strategy for the seeker in the associated hide-and-seek game. By Lemma 2, (λ_L, λ_H) is a Nash equilibrium in the hide-and-seek game. Therefore, $v(\lambda_L, \lambda_H) = v(\lambda_H, \lambda) = v^*$. Equation **??** implies $v(\lambda_L, \lambda) = v(\lambda_H, \lambda)$. Hence, $\min v(\cdot, \lambda) = \max v(\cdot, \lambda)$ - i.e., λ verifies weighted regularity.

(ii) Suppose that the graphs is weighted-regular, and let $\beta \in \Delta(X)$ be a framing strategy that verifies this property. Therefore, $v(\mu^x, \beta) = v^*$ for every $x \in X$. Now suppose that $(\lambda, (F^x)_{x \in Supp(\lambda)})$ is a cutoff equilibrium in which firms earn payoffs above the max-min level. Then, $v(\lambda_H, \lambda) < v^*$. Since λ_H and λ_L are optimal at p^m:

$$\begin{aligned} 2\alpha v(\lambda_H, \lambda_H) - v(\lambda_H, \lambda) &\geq 2\alpha v^* - v^* \\ 2\alpha v(\lambda_L, \lambda_H) - v(\lambda_L, \lambda) &\geq 2\alpha v^* - v^* \end{aligned}$$

where α denotes the probability that $p > p^m$.

Optimality of λ_L at p^l implies

$$v(\lambda_L, \lambda) \geq v^*$$

Hence, $v(\lambda_L, \lambda_H) \geq v^*$. By equation (**??**), $v(\lambda_H, \lambda_H) > v^*$. Since by definition,

$$v(\lambda_H, \lambda) = \alpha v(\lambda_H, \lambda_H) + (1-\alpha) v(\lambda_H, \lambda_L)$$

we obtain $v(\lambda_H, \lambda) > v^*$, a contradiction. ■

The intuition for this result is as follows. According to Lemma 2, the formats adopted in the low (high) price range of a cutoff equilibrium are "seeking formats" ("hiding formats") that aim to maximize (minimize) the probability of a price comparison. When weighted regularity is violated, there is a real distinction between seeking and hiding formats. When both firms realize a price in the high range, the probability that the consumer chooses correctly is relatively low, because the firms' framing strategy conditional on $p > p^m$ evades a price comparison. In particular, when a firm charges the monopolistic price $p = 1$ it is compared to the rival firm with a probability below v^*, hence its payoff exceeds the max-min level. Thus, the distinction between "seeking" and "hiding" formats gives firms a market power they lack when the graph is weighted-regular (where the distinction between "seeking" and "hiding" formats disappears). The illustrative example in the Introduction demonstrates this effect.

For a non-trivial example of a weighted-regular graph that gives rise to a cutoff equilibrium, consider the deterministic, nine-node graph given by Figure 3. A uniform distribution over the six bold nodes verifies weighted regularity ($v^* = \frac{1}{3}$). One can construct an equilibrium in which this is indeed the framing strategy, and yet framing and pricing decisions are correlated. Specifically, the three peripheral nodes are played with probability $\frac{1}{3}$ each conditional on $p \in [\frac{2}{3}, 1]$, while their internal neighbors are played with probability $\frac{1}{3}$ each conditional on $p \in [\frac{1}{2}, \frac{2}{3})$. The marginal pricing strategy is given by expression (1).

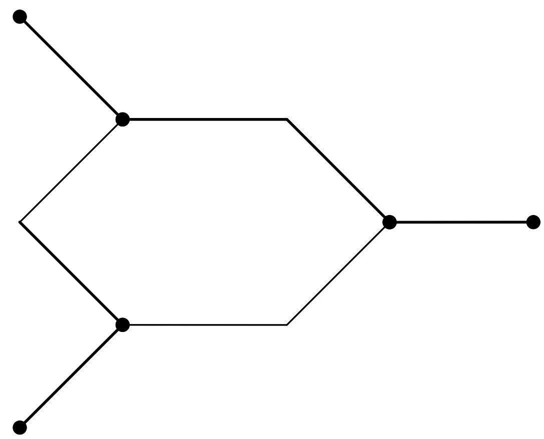

Figure 3

3.4 Bi-Symmetric Graphs

In this sub-section we provide a complete characterization of symmetric Nash equilibrium in a special class of graphs. An order-independent graph (X, π) is *bi-symmetric* if X can be partitioned into two sets, Y and Z, such that for every distinct $x, y \in X$:

$$\pi(x,y) = \{ \begin{array}{lll} q_Y & if & x,y \in Y,\, x \neq y \\ q_Z & if & x,y \in Z,\, x \neq y \\ q & if & x \in Y \text{ and } y \in Z \end{array}$$

where $\min(q_Y, q_Z, q) < 1$.

One natural interpretation is that Y and Z represent two broad ways of spuriously categorizing products. Under this interpretation, it makes sense to assume that two particular brands are more comparable when they are similarly categorized - i.e., $q \leq \min(q_Y, q_Z)$. In contrast, when $q_Y \leq q \leq q_Z$, it is more natural to interpret Y and Z as two broad price formats, where Y represents a more complex format than Z. Define

$$q_I^* = \frac{1 + q_I \cdot (n_I - 1)}{n_I}$$

where $I = Y, Z$ and $n_I = |I|$. Without loss of generality, assume $q_Z^* \geq q_Y^*$.

One can verify that a bi-symmetric graph is weighted-regular if and only if (see Appendix)

$$(q_Y^* - q)(q_Z^* - q) \geq 0$$

When $q_Y^* = q_Z^* = q$, there is a continuum of framing strategies that verify weighted regularity. Otherwise, the framing strategy that verifies weighted regularity assigns probability $\frac{q_Y^* - q}{(q_Y^* - q) + (q_Z^* - q)}$ to the set Z, and mixes uniformly within Y and Z. The value of the hide-and-seek game under weighted regularity is thus

$$v^* = \frac{q_Y^* q_Z^* - q^2}{(q_Y^* - q) + (q_Z^* - q)} \tag{2}$$

except when $q_Y^* = q_Z^* = q$, in which case $v^* = q$.

By Proposition 3, if weighted regularity holds, any distribution that verifies weighted regularity is an equilibrium framing strategy and $G^*(p)$ given in (1) is a price-format independent equilibrium pricing strategy.

When the condition for weighted regularity is not satisfied - i.e., when q is strictly between q_Y^* and q_Z^* - the value of the game is $v^* = q$, since there is a Nash equilibrium in the hide-and-seek game, in which the seeker plays the framing strategy $U(Z)$ (that is, a uniform distribution over Z), while the hider plays $U(Y)$ (that is, a uniform distribution over Y). It can be verified that there exists a cutoff equilibrium in which:

$$\begin{aligned} \lambda_H &\equiv U(Y) \qquad (3) \\ \lambda_L &\equiv U(Z) \\ F(p^m) &= \frac{q - q_Y^*}{q_Z^* - q_Y^*} \end{aligned}$$

Denote $\alpha = 1 - F(p^m)$. Then, firms earn an equilibrium payoff of

$$(1-\alpha) \cdot \frac{1}{2}(1-q) + \alpha \cdot \frac{1}{2}(1 - q_Y^*)$$

which strictly exceeds the max-min level of $\frac{1}{2}(1-q)$. The pricing strategy can be easily derived from (3). We omit it for brevity.

The following proposition states that the above equilibria characterize the set of symmetric equilibria.

Proposition 6 *Let (X, π) be a bi-symmetric graph. In symmetric Nash equilibrium:*
(i) If $(q_Y^* - q)(q_Z^* - q) \geq 0$, firms play a framing strategy that verifies weighted regularity, and independently the pricing strategy given by (1), where v^* is given by (2).
(ii) If $(q_Y^* - q)(q_Z^* - q) < 0$, firms play the cutoff equilibrium given by (3).

Proof. See Appendix. ■

This result provides another demonstration for the non-trivial relation between consumer rationality and equilibrium profits. Let $q_Y^* < q \leq q_Z^*$, and consider the firms' equilibrium payoff as a function of q_Z^*. When $q = q_Z^*$, the graph is weighted-regular and firms earn the max-min payoff $\frac{1}{2}(1-q)$. As q_Z^* goes up, the max-min payoff remains the same, yet equilibrium payoffs rise. This is surprising, because a higher value of q_Z^* corresponds to a "more rational" consumer. To recall the intuition for the example in the Introduction, a higher q_Z^* pushes firms to a framing strategy that places greater weight on "hiding". The firms' market power is strengthened, since the probability of a price comparison is lower.

3.5 A Comment on Asymmetric Equilibria

Nash equilibrium is not necessarily unique and not necessarily symmetric in our model. Recall that in Example 3.2, there exist asymmetric mixed-strategy equilibria, in which firms randomize over disjoint sets of formats. However, in these equilibria, the firms' pricing strategies and profits are the same as in the symmetric equilibrium that this graph generates. Whether this is a general property of equilibria in our model is an open question.

For a special class of graphs, we are able to establish the uniqueness of Nash equilibrium. We say that a graph (X, π) is *symmetric* if $\pi(x, y) = q$ for all distinct x and y.

Proposition 7 *Suppose that (X, π) is symmetric with $q < 1$. The Nash equilibrium is unique. Both firms play the framing strategy $U(X)$. Moreover, F_i^x is given by (1) for every $x \in X$, $i = 1, 2$, where*

$$v^* = \frac{1 + q(n-1)}{n}$$

Proof. See Appendix. ■

Thus, framing asymmetries across firms and price levels are impossible in equilibrium. Note that symmetric graphs are a special case of bi-symmetric graphs in which $q_Y = q_Z = q$.

4 Relaxing Order Independence

In this section we explore some properties of Nash equilibria when the graph violates order independence. We begin by extending the notion of weighted regularity.

Definition 2 *A graph (X, π) is weighted-regular if there exist $\beta \in \Delta(X)$ and $\bar{v} \in [0,1]$ such that $\sum_{y \in X} \beta(y)\, \pi(x,y) = \sum_{y \in X} \beta(y)\, \pi(y,x) = \bar{v}$ for all $x \in X$. We say β verifies weighted regularity.*

The equivalence between weighted regularity and the existence of symmetric equilibrium in the associated hide-and-seek game, established for order-independent graphs, needs to be qualified when order independence is relaxed.

Lemma 3 *(i) If λ verifies weighted regularity, then (λ, λ) is a Nash equilibrium in the hide-and-seek game; (ii) If (λ, λ) is a Nash equilibrium in the hide-and-seek game and $\lambda(x) > 0$ for every $x \in X$, then λ verifies weighted regularity.*

Proof. The proof of part (i) is identical to the order-independent case. Let us turn to part (ii). Suppose that (λ, λ) is a Nash equilibrium in the hide-and-seek game. Since λ is a best-reply for the hider against λ, $v(\mu^x, \lambda) \geq v(\lambda, \lambda)$ for every $x \in X$. By the full-support assumption, if there is a frame $x \in X$ for which $v(\mu^x, \lambda) > v(\lambda, \lambda)$, then $\sum_{x \in X} \lambda(x) v(\mu^x, \lambda) > v(\lambda, \lambda)$. The L.H.S. of this inequality is by definition $v(\lambda, \lambda)$, a contradiction. Similarly, since λ is a best-reply for the seeker against λ, $v(\lambda, \mu^x) \leq v(\lambda, \lambda)$ for every $x \in X$. By the full-support assumption, if there is a frame $x \in X$ for which $v(\lambda, \mu^x) < v(\lambda, \lambda)$, then $\sum_{x \in X} \lambda(x) v(\lambda, \mu^x) < v(\lambda, \lambda)$. The L.H.S. of this inequality is by definition $v(\lambda, \lambda)$, a contradiction. It follows that for every $x \in X$, $v(\mu^x, \lambda) = v(\lambda, \mu^x) = v(\lambda, \lambda)$, hence the graph is weighted regular. ■

To see how the full support assumption is necessary for the second part of this lemma, consider the deterministic graph given by Figure 4. The hide-and-seek game induced by this graph has a symmetric Nash equilibrium in which both the hider and the seeker play y and z with probability $\frac{1}{2}$ each. However, the graph is not weighted-regular.

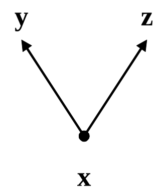

Figure 4

The full-support qualification carries over to the next result, which is a close variation on Proposition 3.

Proposition 8 *(i) Suppose that λ^1 and λ^2 verify weighted regularity. Then, there exists a Nash equilibrium in which each firm $i = 1, 2$ plays the framing strategy λ^i and the pricing strategy $F_i^x(p) = G^*(p)$ for all $x \in X$, and earns max-min payoffs.*
(ii) Let $\left(\lambda_i, (F_i^x)_{x \in Supp(\lambda_i)}\right)_{i=1,2}$ be a Nash equilibrium in which the pricing strategies exhibit price-format independence and the framing strategies have full support for both firms. Then, λ_1 and λ_2 verify weighted regularity, firms earn max-min payoffs, and their marginal pricing strategy is given by (1).

Proof. Analogous to the proof of Proposition 3. ■

One can extend the notion of bi-symmetric graphs by allowing asymmetric connectivity between the sets Y and Z - that is, $\pi(y, z) = q_{YZ}$ and $\pi(z, y) = q_{ZY}$ for every $y \in Y$, $z \in Z$, where $q_{YZ} \neq q_{ZY}$ (while maintaining the assumption that connectivity is symmetric and constant *within* each of the two sets). The reader can easily verify that such graphs are never weighted regular. The following example demonstrates that they admit a type of price-format correlation different from the one captured by cutoff equilibria. In particular, the supports of the pricing strategies can be nested in one another.

Let $X = \{x, y\}$, and assume $\pi(y, x) = q$ and $\pi(y, x) = 0$. There is a symmetric Nash equilibrium in which the firms play a framing strategy that satisfies $\lambda(x) = \frac{1}{2-q}$, and a pricing strategy given by:

$$
\begin{aligned}
F^x(p) &= \frac{1}{2p+pq}(3p+q-1) \\
F^y(p) &= \frac{1}{2p+pq}(3p+pq-1)
\end{aligned}
$$

over the interval $[\frac{1}{3+q}, 1]$, and

$$
\begin{aligned}
F^x(p) &= \frac{1}{2p}\left(3p+q-pq^2-1\right) \\
F^y(p) &= 0
\end{aligned}
$$

over the interval $[\frac{1-q}{3-q^2}, \frac{1}{3+q}]$. Note that firms earn max-min payoffs in this equilibrium.

An incumbent-entrant model

Equilibrium analysis when order independence is relaxed is greatly simplified if the assumption that the consumer's initial firm assignment is random is dropped. Suppose that the consumer is initially assigned to firm 1, referred to as the Incumbent. Firm 2 is referred to as the Entrant. In this case, firm 1's max-min payoff is $1-v^*$, while firm 2's max-min payoff is zero.

Proposition 9 *Any Nash equilibrium $\left(\lambda_i, (F_i^x)_{x\in Supp(\lambda_i)}\right)_{i=1,2}$ of the Incumbent-Entrant model has the following properties:*
(i) (λ_1, λ_2) constitutes a Nash equilibrium in the associated hide-and-seek game in which firm 1 (2) is the hider (seeker).
(ii) Firm 1's equilibrium payoff is $1-v^*$ while firm 2's equilibrium payoff is $v^*(1-v^*)$.
(iii) The firms' marginal pricing strategies over $[1-v^*, 1)$ are given by:

$$
\begin{aligned}
F_1(p) &= 1-\frac{1-v^*}{p} \\
F_2(p) &= \frac{1}{v^*}\cdot[1-\frac{1-v^*}{p}]
\end{aligned}
$$

and F_1 has an atom of size $1-v^*$ on $p=1$.

Proof. See Appendix. ■

The simplicity of the equilibrium characterization in this case results from the firms' unambiguous incentives when choosing their framing strategies. The Incumbent has an unequivocal incentive to avoid a price comparison (because then it is chosen with probability one), while the Entrant has an unequivocal incentive to enforce a price comparison (because otherwise it is chosen with probability zero). Note that firm 2's equilibrium profit does not behave monotonically in the price comparison probability v^*. The reason is that when comparison is very unlikely, industry profits are high but the Incumbent has significant market power, whereas when price comparison is very likely, the Incumbent's market power is greatly diminished but industry profits are eroded because of the stronger competitive pressure.

5 Conclusion: Remarks on Product Differentiation

We conclude with a discussion of how our model relates to the phenomenon of product differentiation. The mixing over formats that we observe in Nash equilibrium can be viewed as a type of product differentiation. Variety is conventionally viewed as the market's response to consumers' differentiated tastes. In contrast, in our model the firms' product is inherently homogenous; differentiation is a pure reflection of the firms' attempt to avoid price comparisons.

Our model can be interpreted as an unconventional model of spatial competition. Think of firms as stores and of nodes as possible physical locations of stores. A link from one location x to another location y indicates that it is costless to travel from x to y. The absence of a link from x to y means that it is impossible to travel in this direction. According to this interpretation, the consumer follows a myopic search process in which he first goes randomly to one of the two stores (independently of their locations). Then, he travels to the second store if and only if this "trip" is costless. Finally, the consumer chooses the cheapest firm that his search process has elicited (with a tie-breaking rule that favors the initial firm.)

Although this re-interpretation is reminiscent of the literature on spatial competition, there is a crucial difference. In conventional models of spatial competition, consumers are attached to specific locations and select the nearest firm, as long as travelling costs are not prohibitively high (in which case they choose neither firm). Thus, a consumer who is attached to a location x does not care at all about the cost of transportation between two stores if neither of them is located at x. In contrast, in our model, consumer choice is always sensitive to the probability of a link between the firms' locations. Recall that in our model consumer choice may be impossible to rationalize with a random utility function over pairs (p, x). In contrast, conventional models of spatial competition (and product differentiation in general) are based on the assumption that consumer choice is consistent with a random utility function over price-location pairs.

The different consumer behavior induced by the two classes of models implies differences in equilibrium outcomes. First, recall our observation in Section 2 that pure-strategy Nash equilibria that support non-zero prices fail to exist. Second, some important effects in our model are impossible in conventional spatial competition models. For example, consider a spatial competition model that fits the graph of Figure 1. In particular, assume that the consumer is attached to each core node with probability α and to each peripheral node with probability $(1-2\alpha)/4$. It can be shown that in symmetric equilibrium of this model, firms assign zero probability to the peripheral nodes for every value of α and q.

It may be interesting to explore - especially for empirical purposes - a model that synthesizes the two approaches to product differentiation. Suppose that instead of a single consumer, there is a population of consumers, where each consumer type is characterized by two primitives: a graph π_θ and a willingness-to-pay function $u_\theta : X \rightarrow \{0,1\}$. The function u_θ essentially describes the set of product formats (or brands) that type θ likes. Aggregate consumer behavior will thus reflect the distribution of this extended notion of consumer types. In particular, observed behavior that may be impossible to reconcile with conventional models of differentiated tastes may be accounted for by such an extended model that combines taste heterogeneity and limited comparability.

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6 Appendix: Proofs

6.1 Proposition 1

Consider a Nash equilibrium in which firms earn strictly positive payoffs. For each firm $i = 1, 2$, let p_i^l denote the infimum of the support of F_i. Denote $p^l = \min(p_1^l, p_2^l)$. Suppose that there is an interval (p, p'), $p^l \leq p < p' \leq 1$, such that $F_2(p) = F_2^-(p')$. Then, F_1 cannot assign any weight to the interval (p, p'). Otherwise, firm 1 can make higher profits by deviating from any strategy (p'', x) in the support of its equilibrium strategy, $p'' \in (p, p')$, to the strategy $(p'' + \varepsilon, x)$, where $\varepsilon > 0$ is sufficiently small. Thus,

$F_1(p) = F_1^-(p')$. Let us now show that there exists no $x \in X$ such that (p, x) is a best-reply for any firm i against firm j's strategy. If neither F_1 nor F_2 have a mass point at p, then firm i can profitably deviate to $(p + \varepsilon, x)$, where $\varepsilon > 0$ is sufficiently small. Now suppose that F_2^x, say, has a mass point at p for some $x \in X$. Such a mass point is a best-reply for firm 2 only if firm 1 also has a mass point at (p, y) for some y for which $\pi(x, y) > 0$ - otherwise, deviating to $(p+\varepsilon, x)$ would be profitable for firm 2, for a sufficiently small $\varepsilon > 0$. But this means that firm 1 can profitably deviate from (p, y) to $(p - \varepsilon, y)$ for a sufficiently small $\varepsilon > 0$. This contradicts the hypothesis that both F_1 and F_2 are flat in the interval (p, p').

6.2 Proposition 2

Define $X^A = \{x \in X : \pi(y, x) = 1 \text{ for all } y \in X\}$ and $X^B = X - X^A$. Suppose that $F_1(0) = 1$. Then, firm 1 makes zero profits. It follows that $F_2(0) = 1$ and hence firm 2 also makes zero profits. Obviously, $Supp(\lambda_i) \subseteq X^A$, $i = 1, 2$, as if $\lambda_i(x) > 0$ and $\pi(y, x) < 1$ for some y, firm j can make positive profits charging $p = 1$ and choosing y.

If $F_1(0) < 1$, then firm 2 makes positive profits. Thus, $F_2(0) < 1$ and firm 1 also makes positive profits. We first show that there must exist $y \in Supp(\lambda_i)$ such that $\pi(x, y) < 1$ for some $x \in Supp(\lambda_j)$, $i \neq j$, $i, j = 1, 2$. Suppose instead that $\pi(x, y) = 1$ for all $x \in Supp(\lambda_2)$, $y \in Supp(\lambda_1)$. By Proposition 1, the upper bound of the support of F_i is equal to 1 for $i = 1, 2$. Take a node z in the support of λ_2 such that the upper bound of the support of F_i^z is equal to one. The profits of firm 2 are equal to

$$\sum_{x \in X} \left(\frac{1}{2} - \frac{1}{2} F_1^{x-}(1) \right) \lambda_1(x)$$

Choosing a price equal to $1 - \varepsilon$ and a node x^* in X^A, firm 2 obtains

$$(1 - \varepsilon) \sum_{x \in X} \left(\frac{1}{2} - \frac{1}{2} \pi(x^*, x) F_1^x(1 - \varepsilon) + \frac{1}{2}(1 - F_1^x(1 - \varepsilon)) \right) \lambda_1(x)$$

Since firm 2's payoff is positive, $F_1^{x-}(1) < 1$ for some $x \in Supp(F_1)$. But then, for ε sufficiently small, the second expression is larger than the first expression, a contradiction.

Now let p^* be the lowest price p in $Supp(F_1) \cup Supp(F_2)$ for which there exist $x \in Supp(\lambda_j)$ and $y \in Supp(\lambda_i)$, where $i \neq j$, such that $p \in Supp(F_i^y)$ and $\pi(x, y) < 1$. Without loss of generality, suppose that $p^* \in Supp(F_2^y)$. Firm 2's payoff from the pure strategy (p^*, y) is

$$p^* \sum_{x \in X} \left(\frac{1}{2} - \frac{1}{2} \pi(y, x) F_1^{x-}(p^*) + \frac{1}{2} \pi(x, y)(1 - F_1^x(p^*)) \right) \lambda_1(x)$$

If firm 2 deviates to the pure strategy $(p^* - \varepsilon, x^*)$, $x^* \in X^A$, it will earn

$$(p^* - \varepsilon) \sum_{x \in X} \left(\frac{1}{2} - \frac{1}{2} \pi(x^*, x) F_1^x(p^* - \varepsilon) + \frac{1}{2}(1 - F_1^x(p^* - \varepsilon)) \right) \lambda_1(x)$$

By the definition of p^*, if $F_1^{x-}(p^*) > 0$, then $\pi(y, x) = 1$. Since $\pi(x, y) < 1$ for some $x \in Supp(\lambda_1)$, for ε sufficiently small, the second expression is larger than the first expression, a contradiction.

6.3 Proposition 6

Define

$$\begin{aligned} a &= 1 + q_Y(n_Y - 1) - qn_Y \\ b &= 1 + q_Z(n_Z - 1) - qn_Z \end{aligned}$$

One can verify that weighted regularity holds if and only if the system

$$\begin{bmatrix} a & -b \\ n_Y & n_Z \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

has a non-negative solution - that is, if and only if $ab \geq 0$ (or, equivalently, if and only if $(q_Y^* - q)(q_Z^* - q) \geq 0$).

Let $\left(\lambda, (F^x)_{x \in Supp(\lambda)}\right)$ be a symmetric Nash equilibrium strategy, and let F denote the equilibrium marginal pricing strategy. By Proposition 1, and due to the symmetry of equilibrium, F is continuously and strictly increasing over the support $[p^l, 1]$, $p^l < 1$. Let S_x denote the support of F^x, and let p^{xl} and p^{xu} denote the infimum and supremum of S_x. Let $v^x(\lambda)$be the probability that the consumer makes a price comparison conditional on the event that one firm adopts the format x. That is:

$$v^x(\lambda) = \sum_{y \in X} \lambda(y) \pi(x, y) \tag{4}$$

The following claims establish Proposition 6.

Lemma 4 *In a symmetric Nash equilibrium of a bi-symmetric graph, $F(p)$ is continuous on $[p^l, 1]$.*

Proof. It follows from standard arguments, due to the symmetry of equilibrium. ■

Lemma 5 *Suppose that (X, π) is bi-symmetric. In a symmetric Nash equilibrium, $\lambda(x) = \lambda(x')$ for any $x, x' \in Y$ or $x, x' \in Z$, $i = 1, 2$.*

Proof. Let $X^{\max} = \arg\max_{x \in X} \lambda(x)$. Suppose that $X^{\max} \cap Y \neq \varnothing$ and that $\max_{x \in X} \lambda(x) > \lambda(y)$ for some $y \in Y$. Let $\bar{p}$ be the highest price in $\cup_{x \in X^{\max}} S_x$ and suppose that $\bar{p} \in S_{\hat{x}}$, for $\hat{x} \in X_i^{\max}$. Firm i's payoff from the pure strategy $(\bar{p}, \hat{x})$ is

$$\left(\begin{array}{c} q_Y \lambda(y)(1 - F^y(\bar{p})) + \\ \sum_{x \in Y - (\hat{x}, y)} (1 - F^x(\bar{p})) q_Y \lambda(x) + \sum_{x \in Z} (1 - F^x(\bar{p})) q \lambda(x) + \dfrac{1}{2} \left(1 - v^{\hat{x}}(\lambda)\right) \end{array} \right),$$

If the firm deviates to the strategy $(\bar{p}, y)$, it earns

$$\bar{p} \left(\begin{array}{c} \lambda(y)(1 - F^y(\bar{p})) \\ \sum_{x \in Y - (\hat{x}, y)} (1 - F^x(\bar{p})) q_Y \lambda(x) + \sum_{x \in Z} (1 - F^x(\bar{p})) q \lambda(x) + \dfrac{1}{2} (1 - v^y(\lambda)) \end{array} \right).$$

Since $v^{\hat{x}}(\lambda) > v^y(\lambda)$, this deviation is profitable.

Let $X^{\min} = \arg\min_{x \in X} \lambda(x)$. Suppose that $X^{\min} \cap Y \neq \varnothing$ and that $\min \lambda(x) < \lambda(y)$ for some $y \in Y$. Let $\tilde{p}$ be the highest price in S_y. The profit of firm i from the pure strategy $(\tilde{p}, \hat{x})$ where $\hat{x} \in X^{\min}$ is

$$\tilde{p} \left(\begin{array}{c} \lambda(\hat{x}) \left(1 - F^{\hat{x}}(\tilde{p})\right) + \\ \sum_{x \in Y - (\hat{x}, y)} (1 - F^x(\bar{p})) q_Y \lambda(x) + \sum_{x \in Z} (1 - F^x(\bar{p})) q \lambda(x) + \dfrac{1}{2} \left(1 - v^{\hat{x}}(\lambda)\right) \end{array} \right),$$

The profit of firm i from the pure strategy $(\tilde{p}, y)$ is

$$\tilde{p} \left(\begin{array}{c} q_Y \lambda(\hat{x}) \left(1 - F^{\hat{x}}(\tilde{p})\right) \\ \sum_{x \in Y - (\hat{x}, y)} (1 - F^x(\bar{p})) q_Y \lambda(x) + \sum_{x \in Z} (1 - F^x(\bar{p})) q \lambda(x) + \dfrac{1}{2} (1 - v^y(\lambda)) \end{array} \right).$$

Since $v^{\hat{x}}(\lambda) < v^y(\lambda)$, the profit at $\hat{x}$ is larger than the profit at y, a contradiction. ■

Lemma 6 *Suppose that (X, π) is bi-symmetric. In a symmetric Nash equilibrium, for any $p \in [p^l, 1]$, $F^x(p) = F^{x'}(p)$ whenever $x, x' \in Y$ or $x, x' \in Z$.*

Proof. Suppose that $F^y(p) > F^{y'}(p)$ for $y, y' \in Y$. Firm i's payoff from the pure strategy (p, y) is

$$p\left(\begin{array}{c}(1-F^y(p))\lambda(y)+q_Y\left(1-F^{y'}(p)\right)\lambda(y)+\\ \sum_{x\in Y-(y,y')}(1-F^x(p))q_Y\lambda(x)+\sum_{x\in Z}(1-F^x(\bar{p}))q\lambda(x)+\frac{1}{2}(1-v^y(\lambda))\end{array}\right)$$

If the firm deviates to the pure strategy (p, y'), it earns

$$p\left(\begin{array}{c}\left(1-F^{y'}(p)\right)\lambda(y)+q_Y(1-F^y(p))\lambda(y)+\\ \sum_{x\in Y-(y,y')}(1-F^x(p))q_Y\lambda(x)+\sum_{x\in Z}(1-F^x(\bar{p}))q\lambda(x)+\frac{1}{2}\left(1-v^{y'}(\lambda)\right)\end{array}\right).$$

Since $v^y(\lambda) = v^{y'}(\lambda)$ by Lemma 5, this deviation is profitable. ■

Lemma 7 *Suppose that $\lambda(x) = 0$ for some $x \in X$ in some symmetric Nash equilibrium of a bi-symmetric graph (X, π). Then, λ verifies weighted regularity.*

Proof. Without loss of generality, assume that $\lambda(x) = 0$ for some $x \in Y$. By the above lemmas, λ is a uniform distribution over Z. Thus, in particular, $\lambda(x') = 0$ for all $x' \in Y$. Therefore, If $v^z(\lambda) = v^{z'}(\lambda)$ for any $z, z' \in Z$, and $v^y(\lambda) \neq v^{y'}(\lambda)$ for any $y, y' \in Y$. If $v^z(\lambda) \neq v^y(\lambda)$ for some $y \in Y$ and $z \in Z$, then it must be profitable to deviate either to the pure strategy $(1, y)$ or to the pure strategy (p^l, y). It follows that λ verifies weighted regularity. ■

Lemma 8 *Consider a symmetric Nash equilibrium of a bi-symmetric graph (X, π) such that $\lambda(x) > 0$ for all $x \in X$. Then:*
(i) If the graph is not weighted-regular, either $p^{yu} = p^{zl}$ or $p^{zu} = p^{yl}$ for any $y \in Y$ and $z \in Z$.
(ii) If $p^{yu} = p^{zl}$ or $p^{zu} = p^{yl}$ for any $y \in Y$ and $z \in Z$, the graph is not weighted-regular.

Proof. (i) Suppose that the graph is not weighted-regular and $v^z(\lambda) < v^y(\lambda)$. By the above Lemmas, at nodes in Y have the same F^y and all nodes in Z have the same F^z. Therefore, $S_y \cap S_z \neq \varnothing$, for any $y \in Y$ and $z \in Z$. The following equations must hold in equilibrium.

$$\begin{array}{l}\lambda(z)\,qn_Z(1-F^z(p^{yu}))+\frac{1}{2}(1-v^y(\lambda))=\\ \lambda(z)(1+q_Z(n_Z-1))(1-F^z(p^{yu}))+\frac{1}{2}(1-v^z(\lambda))\end{array}$$

$$\lambda(z)\,qn_Z + (1+q_X(n-1))\,\lambda(y)\left(\left(1-F^y\left(p^{zl}\right)\right)\right) + \frac{1}{2}\left(1-v^y(\lambda)\right) =$$
$$\lambda(z)\left(1+q_Z(n_Z-1)\right) + qn\lambda(y)\left(\left(1-F^y\left(p^{zl}\right)\right)\right) + \frac{1}{2}\left(1-v^z(\lambda)\right)$$

which simplify to

$$b\lambda(z)\left(1-F^z\left(p^{yu}\right)\right) = \frac{v^z(\lambda)-v^y(\lambda)}{2}$$
$$b\lambda(z) - a\lambda(y)\left(1-F^y\left(p^{zl}\right)\right) = \frac{v^z(\lambda)-v^y(\lambda)}{2}$$

Hence, $b<0$. Since the graph is weighted regular, $a>0$. It can be easily verified that the above equations can hold only if $F^z\left(p^{yu}\right)=0$ and $F^y\left(p^{zl}\right)=1$. If $v^z(\lambda)>v^y(\lambda)$, a symmetric argument establishes the claim.

(ii) Suppose that $p^{yu}=p^{zl}$. Note that

$$v^z(\lambda)-v^y(\lambda) = b\lambda(z) - a\lambda(y)$$

In equilibrium

$$b\lambda(z) = \frac{b\lambda(z)-a\lambda(y)}{2}$$

Since $\lambda(y), \lambda(z) > 0$, we have $ab<0$. A symmetric argument establishes the claim for the case $p^{zu}=p^{yl}$. ■

Lemma 9 *Consider a symmetric Nash equilibrium of a bi-symmetric graph* (X,π) *such that* $\lambda(x)>0$ *for any* $x\in X$*. If* $p^{yu}\neq p^{zl}$ *and* $p^{zu}\neq p^{yl}$ *for any* $y\in Y$ *and* $z\in Z$*, then the graph is weighted-regular. Moreover, max-min payoffs are obtained, and* $F^z(p)=F^y(p)$ *for any* $p\in\left[p^l,1\right]$*.*

Proof. Lemma 8 implies that if $p^{yu}\neq p^{zl}$ and $p^{zu}\neq p^{yl}$ for any $y\in Y$ and $z\in Z$ then the graph is weighted-regular. As in the proof of Lemma 8, the following equilibrium conditions must hold

$$b\lambda(z)\left(1-F^z\left(p^{yu}\right)\right) = \frac{b\lambda(z)-a\lambda(y)}{2}$$

$$b\lambda(z) - a\lambda(y)\left(1-F^y\left(p^{zl}\right)\right) = \frac{b\lambda(z)-a\lambda(y)}{2}$$

First note that, if either $b=0$ or $a=0$, then either $\lambda(y)=0$ or $\lambda(z)=0$, and the claim follows by Lemma 7. Hence suppose that $ab>0$. Setting $\left(1-F^z\left(p^{yu}\right)\right)=\sigma$ and

$\left(1 - F^y\left(p^{zl}\right)\right) = \delta$, rewrite the system in matrix notation as

$$\begin{bmatrix} b\sigma - \frac{b}{2} & \frac{a}{2} \\ \frac{b}{2} & -a\delta + \frac{a}{2} \end{bmatrix} \begin{bmatrix} \lambda(z) \\ \lambda(y) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This system has a non-null solution if and only if

$$-\sigma - \delta + 2\sigma\delta + 1 = 0$$

which is only possible, for $0 \leq \delta, \sigma \leq 1$, when $\delta = 1, \sigma = 0$ or $\delta = 0, \sigma = 1$. In the former case, $v_i^z(\lambda) = v_i^y(\lambda)$. In the latter case,

$$b\lambda(z) = \frac{b\lambda(z) - a\lambda(y)}{2}$$

and hence positive solutions for $\lambda(z), \lambda(y)$ do not exist when $ab > 0$. Thus in equilibrium, $F^z(p^{yu}) = 1$, $F^y\left(p^{zl}\right) = 0$, and $v^z(\lambda) = v^y(\lambda)$. Consequently, for any $p \in [p^l, 1]$

$$b\lambda(z)(1 - F^z(p)) = a\lambda(y)(1 - F^y(p))$$

Since $v^z(\lambda) - v^y(\lambda) = b\lambda(z) - a\lambda(y) = 0$, we have $F^z(p) = F^y(p)$. ■

It follows from the above lemmas that if $q_Y^* < q < q_Z^*$, then a symmetric Nash equilibrium must be a cutoff equilibrium. Moreover, there are two cases to consider: either λ_H is a uniform distribution over Y and λ_L is a uniform distribution over Z, or λ_H is a uniform distribution over Z and λ_L is a uniform distribution over Y. To determine which is the actual case, and to pin down the framing strategy λ, we rely on the condition that firms are indifferent between playing $y \in Y$ and $z \in Z$ at the cutoff price p^m (where $p^m = p^{zu} = p^{yl}$ in the former case, and $p^m = p^{zl} = p^{yu}$ in the latter case).

In the former case, the condition is given by the equation

$$\lambda(y) n_Y q - \lambda(z) n_Z q_Z^* = \lambda(y) n_Y q_Y^* - \lambda(z) n_Z q$$

for arbitrary $y \in Y$ and $z \in Z$. In the latter case, the condition is given by the equation

$$\lambda(z) n_Z q - \lambda(y) n_Y q_Y^* = \lambda(z) n_Z q_Z^* - \lambda(y) n_Y q$$

for arbitrary $y \in Y$ and $z \in Z$. Since $q_Y^* < q < q_Z^*$, the latter case is ruled out, and only the former case remains, yielding the precise expression for λ.

6.4 Proposition 7

Given a framing strategy λ and a node $x \in X$, define $v^x(\lambda)$ as in (4). Note that in a symmetric graph, $v^x(\lambda) = v^y(\lambda)$ if and only if $\lambda(x) = \lambda(y)$. The proof follows from the following claims.

Lemma 10 *$F_i(p)$ is continuous on $[p^l, 1)$, $i = 1, 2$.*

Proof. Consider first the case $q = 0$. Suppose that F_j^x has a mass point at $p \in [p^l, 1)$. Firm i's payoff from the pure strategy (p, x) is

$$p \cdot \left[\left(1 - F_j^x(p)\right)\lambda_j(x) + \frac{1}{2}\left(F_j^x(p) - F_j^{x-}(p)\right)\lambda_j(x) + \frac{1}{2}\left(1 - \lambda_j(x)\right)\right]$$

The firm's payoff from $(p + \varepsilon, x)$ is bounded from above by

$$(p+\varepsilon) \cdot \left[\left(1 - F_j^x(p)\right)\lambda_j(x) + \frac{1}{2}\left(1 - \lambda_j(x)\right)\right]$$

and the firm's payoff from $(p - \varepsilon, x)$ is bounded from below by

$$(p-\varepsilon) \cdot \left[(1 - F_j^{x-}(p))\lambda_j(x) + \frac{1}{2}\left(1 - \lambda_j(x)\right)\right]$$

If $\varepsilon > 0$ is sufficiently small, the third expression is strictly larger than the first two. Since the second expression is increasing in ε, it follows that for small ε, F_i^x assigns zero probability to the interval $(p, p + \varepsilon)$. But this means that firm j can profitably deviate from (p, x) to $(p + \frac{\varepsilon}{2}, x)$.

The proof for the case $q > 0$ is more conventional. If F_j^x has a mass point at some $p \in [p^l, 1)$, then in order for F_i to be a best-reply, it must be flat on an interval $[p, p+\varepsilon)$, contradicting Proposition 1. ■

Lemma 11 *$\lambda_i(x) = \dfrac{1}{n}$ for all $x \in X$, $i = 1, 2$.*

Proof. Let $X_i^{\max} = \arg\max_{x \in X} \lambda_i(x)$. Let S_{ix} be the support of F_i^x, let p_i^{xl} and p_i^{xu} be the infimum and supremum of S_{ix}, and let X_i^l be the set of nodes such that $p_i^{xl} = p^l$ any $x \in X_i^l$. By Lemma **??**, X_i^l is non-empty and is a subset of $X_j^{\max}$, $i \neq j$. Suppose that $\max_{x \in X} \lambda_1(x) > \dfrac{1}{n}$. We consider two cases.

Case 1: $X_i^l = X_j^l$. We first show that $X_i^l = X_j^{\max}$, $i \neq j$. Suppose not and let $x' \in X_i^l$ and $x'' \in X_j^{\max} - X_i^l$. By hypothesis, $x'' \notin X_j^l$. For any sufficiently small $\varepsilon > 0$, firm i's payoff from the pure strategy $(p^l + \varepsilon, x')$ is

$$\left(p^l + \varepsilon\right) \left(\begin{array}{c} \left(1 - F_j^{x'}\left(p^l + \varepsilon\right)\right) \lambda_j\left(x'\right) + q\lambda_j\left(x''\right) + \\ \sum_{x \in X - (x', x'')} \left(1 - F_j^x\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x\right) + \frac{1}{2}\left(1 - v_i^{x'}\left(\lambda_j\right)\right) \end{array} \right)$$

If the firm deviates to $(p^l + \varepsilon, x'')$, it earns

$$\left(p^l + \varepsilon\right) \left(\begin{array}{c} \lambda_j\left(x''\right) + \left(1 - F_j^{x'}\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x'\right) + \\ \sum_{x \in X - (x', x'')} \left(1 - F_j^x\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x\right) + \frac{1}{2}\left(1 - v_i^{x''}\left(\lambda_j\right)\right) \end{array} \right)$$

Since $x', x'' \in X_j^{\max}$, $\lambda_j\left(x'\right) = \lambda_j\left(x''\right)$ and hence $v_i^{x'}\left(\lambda_j\right) = v_i^{x''}\left(\lambda_j\right)$. Since by hypothesis $F_j^{x'}\left(\left(p^l + \varepsilon\right)\right) > 0$, firm i's deviation is profitable.

By definition, $\max_{x \in X} \lambda_2\left(x\right) \geq \frac{1}{n}$. Suppose that this inequality holds with equality. Then, $X_2^{\max} = X$. But then, since $X_1^{\max} = X_2^{\max}$, $\max_{x \in X} \lambda_1\left(x\right) = \frac{1}{n}$, a contradiction. It follows that $\max_{x \in X} \lambda_2\left(x\right) > \frac{1}{n}$. Hence, $p_i^{xu} < 1$ for any $x \in X_i^l$, $i = 1, 2$. Suppose that $p_1^{x'u}$ is the highest p_i^{xu} for any $x \in X_i^l$, $i = 1, 2$. Let x'' be such that $x'' \notin X_2^{\max}$. Firm 1's payoff from the pure strategy $(p_1^{x'u}, x')$ is

$$p_1^{x'u} \left(\begin{array}{c} q\lambda_j\left(x''\right)\left(1 - F_j^{x''}\left(p_1^{x'u}\right)\right) + \\ \sum_{x \in X - (x', x'')} \left(1 - F_j^x\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x\right) + \frac{1}{2}\left(1 - v_i^{x'}\left(\lambda_j\right)\right) \end{array} \right)$$

If the firm deviates to $(p_1^{x'u}, x'')$, it earns

$$\left(p^l + \varepsilon\right) \left(\begin{array}{c} \lambda_j\left(x''\right)\left(1 - F_j^{x''}\left(p_1^{x'u}\right)\right) + \\ \sum_{x \in X - (x', x'')} \left(1 - F_j^x\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x\right) + \frac{1}{2}\left(1 - v_i^{x''}\left(\lambda_j\right)\right) \end{array} \right)$$

Since $v_i^{x'}\left(\lambda_j\right) > v_i^{x''}\left(\lambda_j\right)$, the deviation is profitable.

Case 2: $X_i^l \neq X_j^l$. We first show that $X_i^l \cap X_j^l = \varnothing$. Suppose not and let $x' \in X_i^l \cap X_j^l$ and $x'' \in X_i^l - X_j^l$. For any sufficiently small $\varepsilon > 0$, firm i's payoff from the pure strategy $(p^l + \varepsilon, x')$ is

$$\left(p^l + \varepsilon\right) \left(\begin{array}{c} \left(1 - F_j^{x'}\left(p^l + \varepsilon\right)\right) \lambda_j\left(x'\right) + q\lambda_j\left(x''\right) + \\ \sum_{x \in X - (x', x'')} \left(1 - F_j^x\left(p^l + \varepsilon\right)\right) q\lambda_j\left(x\right) + \frac{1}{2}\left(1 - v_i^{x'}\left(\lambda_j\right)\right) \end{array} \right)$$

If the firm deviates to $(p^l+\varepsilon, x'')$, it earns

$$(p^l+\varepsilon)\left(\begin{array}{c}\lambda_j(x'')+\left(1-F_j^{x'}\left(p^l+\varepsilon\right)\right)q\lambda_j(x')+\\ \sum_{x\in X-(x',x'')}\left(1-F_j^{x}\left(p^l+\varepsilon\right)\right)q\lambda_j(x)+\dfrac{1}{2}\left(1-v_i^{x''}(\lambda_j)\right)\end{array}\right)$$

Since $x', x''\in X_j^{\max}$, $\lambda_j(x')=\lambda_j(x'')$ and hence $v_i^{x'}(\lambda_j)=v_i^{x''}(\lambda_j)$. As by hypothesis $F_j^{x'}\left(\left(p^l+\varepsilon\right)\right)>0$, firm i's deviation is profitable.

Let $\bar{X}_i=\{x\in X\mid x\notin X_j^{\max}\text{ or }p_i^{xl}\geq p_j^{xl},\ i\neq j\}$. Let $\bar{p}_i=\min_{x\in\bar{X}_i}p_i^{xl}$. Since $X_i^l\cap X_j^l=\varnothing$, $\bar{p}_i>p^l$. Also, as $\max_{x\in X}\lambda_1(x)>\min_{x\in X}\lambda_1(x)$, $\bar{p}_i<1$. Suppose that $\bar{p}_i\leq\bar{p}_j$. We first show that it cannot be the case that $\bar{p}_i=p_i^{x'l}$ for $x'\notin X_j^{\max}$. Let x'' be a node in $X_j^{\max}\cap X_i^l$. Firm i's payoff from the pure strategy $(\bar{p}_i,x')$ is

$$\bar{p}_i\left(\begin{array}{c}\lambda_j(x')\left(1-F_j^{x'}(\bar{p}_i)\right)+q\lambda_j(x'')+\\ \sum_{x\in X-(x',x'')}\left(1-F_j^{x}(\bar{p}_i)\right)q\lambda_j(x)+\dfrac{1}{2}\left(1-v_i^{x'}(\lambda_j)\right)\end{array}\right)$$

If the firm deviates to $(\bar{p}_i,x'')$, it earns

$$\bar{p}_i\left(\begin{array}{c}\lambda_j(x'')+q\lambda_j(x')\left(1-F_j^{x'}(\bar{p}_i)\right)+\\ \sum_{x\in X-(x',x'')}\left(1-F_j^{x}(\bar{p}_i)\right)q\lambda_j(x)+\dfrac{1}{2}\left(1-v_i^{x''}(\lambda_j)\right)\end{array}\right)$$

Since $v_i^{x''}(\lambda_j)-v_i^{x'}(\lambda_j)=(1-q)\left(\lambda_j(x'')-\lambda_j(x')\right)$, it can be verified that the deviation is profitable.

Suppose that $\bar{p}_i=p_i^{x'l}$ for $x'\in X_j^{\max}$. If $p_j^{x'l}<\bar{p}_i$, a contradiction is easily obtained as for any $x\in X_i^l$, the strategy $(\bar{p}_i,x)$ is a profitable deviation (recall that $\bar{p}_i\leq\bar{p}_j$ by hypothesis). If $p_j^{x'l}=\bar{p}_i$, then $\bar{p}_i=\bar{p}_j$ and, by the same argument, $F_j^x(p)=0$ for any $x\in X_i^l$ in an interval $[p^l,\bar{p}_i+\varepsilon)$ for some $\varepsilon>0$ (if not, $p_i^{xl}<\bar{p}_j=p_j^{xl}$ for some node x in X_i^l). Hence, $x'\notin X_i^l$. Let $x''\in X_i^l$. Then, for any $\gamma\in(0,\varepsilon)$, firm i's payoff from the pure strategy $(p+\gamma,x')$ is

$$(p+\gamma)\left(\begin{array}{c}\lambda_j(x')\left(1-F_j^{x'}(p+\gamma)\right)+q\lambda_j(x'')+\\ \sum_{x\in X-(x',x'')}\left(1-F_j^{x}(p+\gamma)\right)q\lambda_j(x)+\dfrac{1}{2}\left(1-v_i^{x'}(\lambda_j)\right)\end{array}\right)$$

If the firm deviates to $(p+\gamma,x'')$, it earns

$$(p+\gamma)\left(\begin{array}{c}\lambda_j(x'')+q\lambda_j(x')\left(1-F_j^{x'}(p+\gamma)\right)+\\ \sum_{x\in X-(x',x'')}\left(1-F_j^{x}(p+\gamma)\right)q\lambda_j(x)+\dfrac{1}{2}\left(1-v_i^{x''}(\lambda_j)\right)\end{array}\right)$$

Since $\lambda_j(x') = \lambda_j(x'')$, $v_i^{x'}(\lambda_j) = v_i^{x''}(\lambda_j)$ and $F_j^{x'}(p+\gamma) > 0$, one can verify that the deviation is profitable. ■

Lemma 12 *F_1 and F_2 have no mass points.*

Proof. We already saw that F_1 and F_2 have no mass points below $p = 1$. Suppose that F_1 has a mass point at $p = 1$. By standard arguments, F_2 has no mass point at $p = 1$. By Lemma 11, firm 1's payoff is then

$$\frac{1}{2} \cdot \frac{(1-q)(n-1)}{n}$$

Then, if firm 2 plays the pure strategy $(1-\varepsilon, x)$, it earns at least

$$(1-\varepsilon)\frac{1}{n}(1 - F_1^x(1-\varepsilon)) + \frac{(1-\varepsilon)}{2}\frac{(1-q)(n-1)}{n}$$

Since, $\lim_{\varepsilon \longrightarrow 0} F_1^x(1-\varepsilon) < 1$, firm 2's payoff is strictly above firm 1's payoff. But this means that the supports of F_1 and F_2 have a different infimum, contradicting Proposition 1 ■

Lemma 13 *$F_i^x(p) = F_j^{x'}(p)$ for any $p \in [p^l, 1]$, $x, x' \in X$, $i, j \in \{1, 2\}$.*

Proof. We first show that $F_i^x(p)$ must be strictly increasing for any $x \in X$ and $i = 1, 2$. Let $[p', p'')$ be such that $F_1^y(p) > F_1^{y'}(p)$ for $p \in [p', p'')$ and $F_i^x(p) = F_j^{x'}(p)$ for any $p \in [p'', 1]$, $x, x' \in X$, $i, j \in \{1, 2\}$. Then, firm 2's payoff from the pure strategy (p, y), $p \in [p', p'')$, is

$$p \cdot \frac{1}{n} \cdot \begin{pmatrix} (1 - F_1^y(p)) + q\left(1 - F_1^{y'}(p)\right) + \\ \sum_{x \in X-(y,y')} q(1 - F_1^x(p)) + \dfrac{(1-q)(n-1)}{2n} \end{pmatrix}$$

If the firm deviates to (p, y'), it earns

$$p \cdot \frac{1}{n} \cdot \begin{pmatrix} \left(1 - F_1^{y'}(p)\right) + q(1 - F_1^y(p)) + \\ \sum_{x \in X-(y,y')} q(1 - F_1^x(p)) + \dfrac{(1-q)(n-1)}{2n} \end{pmatrix}$$

Since $F_1^y(p) > F_1^{y'}(p)$, the deviation is profitable. Hence, $F_2^y(p)$ is constant on $[p', p'')$. Since F_2 is strictly increasing, there exists a node y'' such that $F_2^y(p) > F_2^{y''}(p)$ for

$p \in [p', p'')$. It follows that F_1^y is also constant on $[p', p'')$. Now let $\hat{p}$ be lowest price p' for which F_i^y is constant on $[p', p'')$ for $i = 1, 2$. Since $F_2^y(\hat{p}) > F_2^{y''}(\hat{p})$ and $F_1^y(\hat{p}) > F_1^{y'}(\hat{p})$, a contradiction is obtained showing, by analogous arguments, the strategy $(\hat{p}, y)$ is not a best-reply.

Since every $F_i^x(p)$ is strictly increasing for any $x \in X$ and $i = 1, 2$, it can easily be verified that its value is determined by a system of linear equations which has a unique, symmetric solution. ■

6.5 Proposition 9

(i) Whenever $p_1 \leq p_2$, the consumer chooses firm 1 with probability one. Whenever $p_1 > p_2$, the consumer chooses firm 2 if and only if he makes a price comparison. Therefore, for every price p that firm 1 considers which lies strictly above the lower bound of F_2, the firm has an incentive to choose a format x that minimizes $v(\cdot, \lambda_2^L(p))$, where $\lambda_2^L(p)$ denotes firm 2's framing strategy conditional on $p' < p$. Similarly, for every price p that firm 2 considers which lies strictly below the upper bound of F_1, the firm has an incentive to choose a format x that maximizes $v(\lambda_1^H(p), \cdot)$, where $\lambda_1^H(p)$ denotes firm 1's framing strategy conditional on $p' > p$. It can be verified that Proposition 1 extends to the Incumbent-Entrant model. Therefore, F_1 and F_2 have the same lower bound $p^l < 1$ and the same upper bound $p^h = 1$. Therefore, in Nash equilibrium, firm 1's framing strategy conditional on $p > p^l$ and firm 2's framing strategy conditional on $p < 1$ constitute a Nash equilibrium in the associated hide-and-seek game. These framing strategies are equal to the firms' marginal equilibrium framing strategies, because as we will verify below, F_1 does not have an atom on p^l and that F_2 does not have an atom on $p = 1$.

(ii) Since $p = 1$ is in the support of F_1 and firm 2's framing strategy conditional on $p < 1$ max-minimizes v, firm 1's equilibrium payoff is $1 - v^*$. Since firm 1 is chosen with probability one when it charges p^l, and since p^l is in the support of F_1, it follows that $p^l = 1 - v^*$. But since p^l is also the infimum of the support of F_2, and since firm 1's framing strategy conditional on $p > p^l$ min-maximizes v, it follows that firm 2's payoff is $v^* \cdot (1 - v^*)$.

(iii) The formulas of F_1 and F_2 follow directly from the condition that every $p \in (1 - v^*, 1)$ maximizes each firm's profit given the opponent's strategy, and the characterization of firm 1's framing strategy conditional on $p > p^l$ and firm 2's framing strategy conditional on $p < p^h$.