Self-Fulfilling Debt Crises: A Quantitative Analysis*

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Abstract

This paper uses the information contained in the joint dynamics of government's debt maturity choices and interest rate spreads to quantify the importance of belief-driven fluctuations in sovereign bond markets. We consider a benchmark model of sovereign borrowing featuring debt maturity choices, risk averse lenders and rollover crises á la Cole and Kehoe (2000). In this environment, lenders' expectations of a default can be self-fulfilling, and their beliefs contribute to variation in interest rate spreads along with economic fundamentals. Through the lens of the model, these sources of default risk can be identified because they have contrasting implications for the maturity structure of debt chosen by the government. We fit the model to the Italian debt crisis of 2008-2012, finding that rollover risk accounted for 20% of the movements in sovereign bond yields over this episode. Our results have implications for the effects of the liquidity provisions established by the European Central Bank during the summer of 2012.

Keywords: Sovereign Debt Crises, Rollover Risk, Maturity Choices, Risk Premia.

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1 Introduction

The idea that lenders' pessimistic beliefs about the solvency of a government can be self-fulfilling has been often used by economists to explain fluctuations in sovereign bonds yields. For example, it has been a common explanation for the sharp increase in interest rate spreads of southern European economies in 2011, and for their subsequent decline upon the introduction of the OMT bond-purchasing program by the European Central Bank.[1] According to this view, the ECB interventions were desirable because they eliminated non fundamental fluctuations in bonds markets, protecting members of the euro-area from inefficient self-fulfilling crises.

However, assessing empirically whether movements in interest rate spreads are due to self-fulfilling beliefs is challenging, and this makes the evaluation of these "lender of last resort" types of policies problematic. The high interest rate spreads observed in southern Europe, for example, could have been due purely to the bad economic fundamentals of these economies. In this second view, the fall in bond yields after the introduction of OMT would be evidence that the program implicitly raised expectations of future bailouts for peripheral countries, guarantees that were priced by bondholders. This latter interpretation would lead to a less favorable assessment of the ECB intervention because bailouts guarantees induce governments to overborrow and they introduce balance sheet risk for the Central Bank.

The main contribution of this paper is to bring a benchmark model of sovereign borrowing with self-fulfilling debt crises to the data, and to show how it can be used to disentangle fundamental and non-fundamental fluctuations in interest rate spreads. In our model, these two sources of default risk have opposing implications for the maturity structure of debt chosen by the government. Our identification strategy consists in using these model's restrictions, along with observed maturity choices, to infer the likelihood of a self-fulfilling crisis.[2] We fit our model to the Italian debt crisis of 2008-2012, finding that non-fundamental risk accounted for 23% of the observed fluctuations in interest rate spreads. We then use our estimates to evaluate the implications of the OMT program.

We consider the canonical model of sovereign borrowing in the tradition of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008). In our environment,

[1] *Outright Monetary Transactions* (OMT), introduced in September 2012, allowed the ECB to purchases of sovereign bonds in secondary markets without explicit quantity limits. See Section 7.

[2] The idea of using economic choices to learn about the types of risk agents are facing has been exploited in several other contexts. A classic example is the use of consumption data along with the logic of the permanent income hypothesis to separate between between permanent and transitory income shocks. See Cochrane (1994) for an application on U.S. aggregate data, Aguiar and Gopinath (2007) for emerging markets, and Guvenen and Smith (2014) for a recent study using micro data.

a government issues debt of multiple maturities in order to smooth out endowment risk. The government lacks commitment over future policies and, as in Cole and Kehoe (2000), it cannot commit to repay the debt within the period. This last assumption leads to the possibility of self-fulfilling debt crises: if lenders expect a default and do not buy new bonds, the government may find it too costly to service the stock of debt coming due, thus validating lenders' expectations. This can happen despite the fact that a default would not be triggered if lenders were holding more optimistic expectations about repayments. These rollover crises can arise in the model when the stock of debt coming due is sufficiently large and economic fundamentals are sufficiently weak.

In this set up, interest rate spreads vary over time because of "fundamental" and "non-fundamental" uncertainty. Specifically, they may be high because lenders expect the government to default in the near future irrespective of their behavior. Or, they may reflect the expectation of a future rollover crisis. The goal of our analysis is to measure these sources of variation in interest rate spreads. The reason why the debt maturity choices made by the government provide information on these sources of default risk builds on basic properties of the canonical sovereign debt model. When choosing debt maturity, the government weights the contribution of three different forces: its lack of commitment, the incompleteness of the debt contracts, and the risk of roll-over crises.

Consider now a situation where high interest rates reflect mostly the prospect of a future rollover crisis. In this scenario, the government optimally chooses to *lengthen* its debt maturity. By back-loading payments, it reduces the stock of debt that needs to be rolled over in the near future, minimizing in this fashion the possibility of a "run" on its debt. Hence, if high interest rates today are due to the expectation of future self-fulfilling crises, we should observe the government to actively increase the maturity of its debt.

On the contrary, the government *shortens* its debt maturity when high interest rates are purely due to its fundamental inability to commit to future repayments. By doing so, the government is able to improve the terms at which it borrows from lenders, and this is valuable for the former when it faces high borrowing rates. As emphasized in Arellano and Ramanarayanan (2012) and Aguiar and Amador (2014b), short term debt is a better instrument for disciplining the borrowing behavior of the government in the future.[3] Hence, a shortening of debt maturity induces lenders to charge lower default premia on newly issued debt because they expect a lower risk of default in the future. These gains are not counteracted by losses due to a decrease in the insurance provided by the maturity

[3]When the government issues new debt, interest rate spreads increase because of heightened risk of default. This increase in interest rates is particularly costly for the government if it entered the period with mostly short term debt, as a larger fraction of the stock needs to be rolled-over at the increased rates. *Ceteris paribus*, inheriting a large fraction of maturing debt makes the government less willing to borrow.

structure of government debt, as Dovis (2014) shows in a related environment.[4]

This logic implies that changes in the maturity structure of government debt provide useful information to measure fundamental and non-fundamental sources of default risk. In practice, though, such an approach may lead to spurious results if one does not control for shifts in the demand of government securities. Broner, Lorenzoni, and Schmukler (2013) document that risk premia on long term bonds increase systematically during sovereign crises. Neglecting these shifts could undermine our identification strategy: rollover risk could be driving interest rate spreads and yet we could observe a shortening of debt maturity simply because lenders are not particularly willing to hold long term government securities. To address this issue, we allow for time-varying term premia in the model by introducing shocks to the lenders' stochastic discount factor as in Ang and Piazzesi (2003).

We apply our framework to the Italian debt crisis of 2008-2012. We calibrate the lenders' stochastic discount factor by matching the behavior of risk premia on long term German's zero coupon bonds, measured using the predictive regressions of Cochrane and Piazzesi (2005). The parameters of the government's decision problem are calibrated to match the cyclical behavior of Italian public debt, interest rate spreads and real economic activity over our sample. Our calibration delivers empirically plausible debt levels, countercyclical debt issuances, and key moments of the interest rate spreads distribution observed in our sample (1999:Q1-2012:Q2).

Using the calibrated model, we decompose the observed interest rate spreads into a component reflecting the expectation of a future rollover crisis and a component due to fundamental risk. We find that rollover risk explains 23% of the movements in interest rate spreads. Moreover, we show that neglecting the information content of maturity choices results in substantial uncertainty over the split between fundamental and non-fundamental sources of default risk, as the model lacks identifying restrictions to discipline the risk of a rollover crisis.

Finally, we show how our results can be used to evaluate the OMT program. We model OMT as a price floor schedule implemented by a deep pocketed central bank. This policy can be designed to eliminate the possibility of rollover crises without bond purchases being carried out on the equilibrium path. This design, which results in a Pareto improvement, is our normative benchmark. We use our model to test whether

[4]Long term debt provides insurance for the government because capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities, as the market value of debt falls when the marginal utility of the government is high. See Angeletos (2002), Buera and Nicolini (2004) and Debortoli, Nunes, and Yared (2015) for a similar mechanism in an environment without default risk.

the OMT program is indeed implementing this benchmark. Specifically, we construct the "fundamental" interest rate spread that would emerge in a world without rollover crises, and we compare it with the actual Italian spread observed after the policy announcements. We find that this counterfactual spread is 100 basis points above the observed one, this suggesting that the post OMT spread partly reflects the expectation of future bailouts on the equilibrium path.

There is a long and growing literature on multiplicity of equilibria in models of sovereign debt. While the Eaton and Gersovitz (1981) framework tends to generate unique equilibria, the seminal papers of Alesina, Prati, and Tabellini (1989) and Cole and Kehoe (2000) shows that the government's inability to commit to current repayments can lead to self-fulfilling rollover crises. Recent papers assess the importance of rollover risk by introducing this feature in quantitative models of sovereign debt, for example Conesa and Kehoe (2012) and the contemporaneous work of Aguiar, Chatterjee, Cole, and Stangebye (2015) .[5] Our paper contributes to this literature by proposing an identification strategy based on the behavior of debt maturity around default crises.

More generally, our research is related to quantitative analysis of sovereign debt models. Papers that are related to our work include, among others, Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo, Martinez, and Sosa Padilla (2015), Bianchi, Hatchondo, and Martinez (2014), Borri and Verdhelan (2013) and Salomao (2014). Relative to the existing literature, our model features rollover risk, endogenous maturity choices and risk aversion on the side of the lenders. Our analysis shows that the behavior of debt duration is necessary for the identification of rollover risk, while shocks to the stochastic discount factor of the lenders are introduced to control for demand factors that may undermine our identification strategy. Our modeling of the maturity choices differ from previous research and builds on recent work by Sanchez, Sapriza, and Yurdagul (2015) and Bai, Kim, and Mihalache (2014). Specifically, the government in our model issues portfolios of zero coupon bonds with an exponentially decaying duration. The maturity choice is discrete, and it consists on the choice of the decaying factor. This modeling feature simplifies the numerical analysis of the model relative to the formulation of Arellano and Ramanarayanan (2012).

Our analysis on the effects of liquidity provisions is related to Roch and Uhlig (2014) and Corsetti and Dedola (2014). These papers show that these policies can eliminate self-fulfilling debt crisis when appropriately designed. We contribute to this literature by using our calibrated model to test whether the drop in interest rates spreads observed after the

[5] There is also a reduced form literature that addresses related issues, see for example De Grauwe and Ji (2013).

announcement of OMT is consistent with the implementation of such policy or whether it signals a prospective subsidy paid by the ECB.

Finally, our paper is related to the literature on the quantitative analysis of indeterminacy in macroeconomic models, see Jovanovic (1989), Farmer and Guo (1995) and Lubik and Schorfheide (2004). The closest in methodology is Aruoba, Cuba-Borda, and Schorfheide (2014) who use a calibrated New Keynesian model solved numerically with global methods to measure the importance of belief-driven fluctuations for the U.S. and Japanese economy.

Layout. The paper is organized as follows. Section 2 presents the model. Section 3 discusses our key identifying restriction. Section 4 describes the calibration of the model and presents an analysis of its fit and Section 5 discusses some properties of the calibrated model. Section 6 uses the calibrated model to measure the importance of rollover risk during the Italian sovereign debt crisis. Section 7 analyzes the OMT program. Section 8 concludes.

2 Model

2.1 Environment

Preferences and endowments: Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition matrix $\mu\left(\cdot|s_{t-1}\right)$. It is convenient to split the state into two components, $s_t = \left(s_{1,t}, s_{2,t}\right)$ where $s_{1,t}$ is the *fundamental* component and $s_{2,t}$ is the *non-fundamental* component. The fundamental component affects endowments and preferences while the non-fundamental component collects coordination devices orthogonal to the fundamentals.

The economy is populated by a large number of lenders and a domestic government. The government receives an endowment (tax revenues) every period, and decides the path of spending G_t. Tax revenues are a constant share τ of the GDP, $Y\left(s_{1,t}\right)$. The government values a stochastic stream of spending $\{G_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(G_t\right), \tag{1}$$

where the period utility function U is strictly increasing, concave, and it satisfies the usual assumptions.

The lenders value flows using the stochastic discount factor $M(s_{1,t}, s_{1,t+1})$. Hence the value of a stochastic stream of payments $\{d_t\}_{t=0}^{\infty}$ from time zero perspective is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \tag{2}$$

where $M_{0,t} = \prod_{j=0}^{t} M_{j-1,j}$.

Market structure: The government can issue a portfolio of non-contingent defaultable bonds to lenders in order to smooth its spending G_t in front of fluctuations in tax revenues. For computational convenience, we restrict the government to issue portfolios of zero-coupon bonds (ZCB) indexed by (B_{t+1}, λ_{t+1}) for $\lambda_{t+1} \in [0, 1]$. A portfolio (B_{t+1}, λ_{t+1}) at the end of period t corresponds to a stock of $(1 - \lambda_{t+1})^{n-1} B_{t+1}$ ZCB of maturity $n \geq 1$. The variable λ_{t+1} captures the duration of the stock of debt: higher λ_{t+1} implies that the repayment profile is concentrated at shorter maturities. For instance, if $\lambda_{t+1} = 1$, then all the debt is due next period. The variable B_{t+1} controls the level of debt issuances, with the face value of issued debt equal to B_{t+1}/λ_{t+1}. We let $q_{t,n}$ be the price of a ZCB of maturity n issued at time t.

The timing of events within the period follows Cole and Kehoe (2000): the government first issues new debt, lenders choose the price of newly issued debt, and finally the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. We assume that if the government defaults, it is excluded from financial markets and it suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value for the government conditional on a default. Lenders that hold inherited debt and the new debt just issued do not receive any repayment.[6] Differently from the timing in Eaton and Gersovitz (1981), the government does not have the ability to commit not to default within the current period. As we will see, this allows for self-fulfilling debt crisis. The budget constraint for the government when it does not default is

$$G_t + B_t \leq \tau Y_t + \Delta_t, \tag{3}$$

where Δ_t is the net amount of resources that the government raises in the period,

$$\Delta_t = \sum_{n=1}^{\infty} q_{t,n} \left[(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t \right]. \tag{4}$$

[6]This is a small departure from Cole and Kehoe (2000), since they assume that the government can use the funds raised in the issuance stage even if it defaults. Our formulation simplifies the problem and it should not change its qualitative features. The same formulation has been adopted in other works, for instance Aguiar and Amador (2014b).

In the above expression we are using the fact that if a government enters the period with a portfolio (B_t, λ_t) and wants to exit the period with a portfolio (B_{t+1}, λ_{t+1}), then it must issue additional $(1-\lambda_{t+1})^{n-1}B_{t+1} - (1-\lambda_t)^n B_t$ ZCB of maturity n.[7] These are the only trades that the government must execute. The government does not have to buy back all its entire portfolio and re-issue the entire stock of debt.

2.2 Recursive Equilibrium

2.2.1 Definition

We consider a recursive formulation of the equilibrium. Let $\mathbf{S} = (B, \lambda, s)$ be the state today at the beginning of the period and $\mathbf{S}'$ the state tomorrow. The problem for the government that has not defaulted yet can be written as

$$V(\mathbf{S}) = \max_{B', \lambda', G, \delta \in \{0,1\}} \delta \left\{ U(G) + \beta \mathbb{E}[V(\mathbf{S}') \, | \mathbf{S}] \right\} + (1-\delta)\underline{V}(s_1) \tag{5}$$

subject to

$$\begin{aligned} G + B &\leq \tau Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= \sum_{n=1}^{\infty} q_n(\mathbf{S}, B', \lambda') \left[(1-\lambda')^{n-1} B' - (1-\lambda)^n B \right], \end{aligned}$$

where $q_n(\mathbf{S}, B', \lambda')$ is the price of a defaultable ZCB of maturity n given the state $\mathbf{S}$ and the government's choices for the new portfolio (B', λ').

The lender's no-arbitrage conditions require that

$$q_n(\mathbf{S}, B', \lambda') = \delta(\mathbf{S}) \, \mathbb{E}\left\{ M(s_1, s_1') \, \delta(\mathbf{S}') \, q_{n-1}(B'', \lambda''; s') \, | \mathbf{S} \right\} \text{ for } n \geq 1, \tag{6}$$

where $B'' = B'(B', \lambda', s')$, $\lambda'' = \lambda'(s', B', \lambda')$, and $q_0(\mathbf{S}, B', \lambda') = 1$. The presence of $\delta(\mathbf{S})$ in equation (6) implies that new lenders receive a payout of zero in the event of a default today. Note that the pricing schedule depends on the initial portfolio of debt, (B, λ), and not only on the exogenous state s and the portfolio at the end of the period, (B', λ'), as in a standard Eaton and Gersovitz (1981) model. This is because the initial portfolio affects the current default decision, $\delta(\mathbf{S})$, which is a key determinant of the price of newly issued

[7] When $(1-\lambda_{t+1})^{n-1}B_{t+1} - (1-\lambda_t)^n B_t$ is negative the government is buying back the ZCB of maturity n. Buy backs of government securities under our formulation are necessary whenever the government shortens the duration of the debt. This is an unrealistic feature of the model as buy backs are hardly observed in the data, but it allows for a greater numerical tractability.

debt under the timing convention in Cole and Kehoe (2000) that we adopt.

A *recursive equilibrium* is value function for the borrower V, associated decision rules $\{\delta, B', \lambda', G\}$ and a pricing function $q = \{q_n\}_{n\geq 1}$ such that $\{V, \delta, B', \lambda', G\}$ are a solution of the government problem (5) and q satisfies the no-arbitrage conditions (6).

2.2.2 Multiplicity of equilibria and Markov selection

This economy features multiple recursive equilibria. Moreover, within an equilibrium, outcomes are not determined entirely by fundamentals. As in Cole and Kehoe (2000), there are states of the world in which lenders' expectations of a default are self-fulfilling: if lenders expects the government to default today, and do not buy new bonds, the government finds it optimal to default while if lenders believe that the government will repay, and they roll-over the maturing debt, the government will indeed repay.

To understand how this situation can arise, it is convenient to define the *fundamental price*,

$$q_n^{\text{fund}}\left(s, B', \lambda'\right) = \mathbb{E}\left\{M\left(s_1, s_1'\right)\delta\left(\mathbf{S}'\right) q_{n-1}\left(B'', \lambda''; s'\right) | \mathbf{S}\right\} \tag{7}$$

that is, the price that lenders will choose if in state (s, B, λ) if the government chooses a new portfolio (B', λ') and it commits to repay in the current period. Also, let

$$\Delta^{\text{fund}}\left(\mathbf{S}, B', \lambda'\right) = \sum_{n=1}^{\infty} q_n^{\text{fund}}\left(s, B', \lambda'\right)\left[(1-\lambda')^{n-1}B' - (1-\lambda)^n B\right]$$

be the amount of resources that the government can raise from lenders at such prices.

Consider a state $\mathbf{S}$ where it is optimal for the government to repay if lenders expect that the government will not default in the current period. Lenders' expectations are validated if

$$\max_{B',\lambda'}\left\{U\left(\tau Y\left(s_1\right) - B + \Delta^{\text{fund}}\left(\mathbf{S}, B', \lambda'\right)\right) + \beta\mathbb{E}\left[V\left(B', \lambda', s'\right)|\mathbf{S}\right]\right\} \geq \underline{V}\left(s_1\right). \tag{8}$$

Consider now this alternative scenario. If the government tries to raise resources from the market, the lenders expect the government to default today and so by equation (6) they set the price of newly issued debt to zero. The lenders' expectations are validated in equilibrium if

$$\underline{V}\left(s_1\right) > U\left(\tau Y\left(s_1\right) - B\right) + \beta\mathbb{E}\left[V\left((1-\lambda)B, \lambda, s'\right)|\mathbf{S}\right]. \tag{9}$$

That is, if it is optimal for the government to default when it cannot issue new debt.[8]

[8]If condition (9) is not satisfied, instead, lenders' expectations cannot trigger a default. This is because

For these beliefs to trigger a default along the equilibrium path, it must also be that the government prefers to default relative to reduce its indebtedness by buying back part of its debt, $\Delta \leq 0$, at the fundamental prices. That is

$$\begin{aligned} \underline{V}(s_1) > \quad & \max_{B',\lambda'} \left\{ U\left(\tau Y(s_1) - B + \Delta^{\text{fund}}(\mathbf{S}, B', \lambda')\right) + \beta \mathbb{E}\left[V(B', \lambda', s') \,|\mathbf{S}\right] \right\} \\ & \text{subject to } \Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \leq 0 \end{aligned} \tag{10}$$

Note that condition (10) implies condition (9).[9]

It is easy to see that for all λ and s there are intermediate values of B such that both (8) and (10) hold.[10] In this region of the state space, the government's default decision depends on lenders' beliefs, debt crisis may be self-fulfilling, and outcomes are indeterminate: lenders may extend credit to the government and there will be no default, or the lenders may not roll-over because they expect no repayment, in which case the government will default, validating lenders' expectations.

We follow most of the literature and use a parametric rule that selects among these possible outcomes. In order to explain our selection mechanism, it is useful to partition the state space in three regions (note that such regions are endogenous and depend on the selection mechanism). Following the terminology in Cole and Kehoe (2000), we say that the government is in the *safe zone*, $\mathcal{S}^{\text{safe}}$, if it does not find optimal to default even if lenders are not willing to roll-over its debt. That is,

$$\mathcal{S}^{\text{safe}} = \Big\{ \mathbf{S} : (10) \text{ does not hold } \Big\}.$$

We say that the government is in the *crisis zone*, $\mathcal{S}^{\text{crisis}}$, if (B, λ, s) are such that it is not optimal for the government to repay debt during a rollover crisis but it is optimal to repay if the lenders roll it over. That is,

$$\mathcal{S}^{\text{crisis}} = \Big\{ \mathbf{S} \ : \ (8) \text{ and } (10) \text{ hold } \Big\}.$$

Finally, the residual region of the state space, the *default zone*, $\mathcal{S}^{\text{default}}$, is the region of the state space in which the government defaults on its debt regardless of lenders' behavior,

$$\mathcal{S}^{\text{default}} = \Big\{ \mathbf{S} \ : (8) \text{ does not hold } \Big\}.$$

it is optimal for the government to repay its debt even if it cannot raise resources by issuing debt. Because of that, an individual lender has an incentive to buy government bonds at a positive price, this ruling out $q = 0$ as an equilibrium price.

[9]In Appendix A we provide a further discussion of conditions (9) and (10).

[10]See Proposition 1 in Aguiar and Amador (2014a) for a formal proof.

Indeterminacy in outcomes arises only when the economy is in the crisis zone.[11]

We consider the following selection mechanism: let the non-fundamental state be $s_2 = (\pi', \xi)$. The variable π' is the probability that there will be a rollover crisis in the next period conditional on the economy being in the crisis zone. We assume that π' follows a first order Markov process, $\pi' \sim \mu_\pi(.|\pi)$. The variable ξ indicates whether a rollover crisis takes place in the current period. Whenever the economy is in the crisis zone, if $\xi = 0$ then lenders roll-over the debt and there is no default. If $\xi = 1$, instead, the lenders do not roll-over the government debt and there is a default. Given our discussion above, $\xi = 1$ with probability π.[12] Conditional on this selection rule, the outcome of the debt auctions are unique in the crisis zone.[13]

The equilibrium outcome is a stochastic process

$$\mathbf{y} = \{\lambda'(B_0, \lambda_0, s^t), B'(B_0, \lambda_0, s^t), \delta(B_0, \lambda_0, s^t), G(B_0, \lambda_0, s^t), q(B_0, \lambda_0, s^t)\}_{t=0}^{\infty}$$

naturally induced by the recursive equilibrium objects. The outcome path depends on properties of the selection, i.e. the process for $\{\pi_t\}$, and on the realization of the non-fundamental state $\{s_{2,t}\}$. Hence default risk is driven by both fundamental and non-fundamental uncertainty, and the goal of our analysis is to measure these sources of default risk.

3 Measuring Rollover Risk: The Role of Maturity

The goal of our analysis is to measure which part of interest rate spreads can be attributed to rollover risk and which part to fundamental default risk. Consider the following decomposition of interest rate spread between a one period defaultable bond and the risk

[11] It is in principle possible that outcomes are not fully determined by fundamentals in the safe zone if condition (9) holds while condition (10) does not. In this case, the government may be prevented from borrowing at fundamental prices but there is no default along the equilibrium path since the government prefers to buy back some of the debt at fundamental prices rather than defaulting. We abstract from this potential scenario in our analysis.

[12] The variable π does not enter the state space because the realization of ξ informs the players on whether there is a rollover crisis today, while π' is sufficient, along with the other state variables, to form expectations on rollover crises going forward.

[13] However, we cannot ensure that the equilibrium value function, decision rules and pricing functions are unique as the operator that implicitly defines a recursive equilibrium may have multiple fixed points. In order to overcome this issue, we restrict our attention to the limit of the finite horizon version of the model. Under our selection rule, the finite horizon model features a unique equilibrium and so does its limit.

free interest rate, r_t^*:

$$\frac{r_{1,t} - r_t^*}{r_{1,t}} = \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\pi_t + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} - \text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right) \quad (11)$$

The first term on the right side represents the pure rollover risk component as it measures the probability of a rollover crisis next period. This can happen with probability π_t if the government falls in the crisis zone next period. The second term measures the pure fundamental component of default risk. The government defaults with probability one, irrespective of lenders' beliefs, if $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}$. The third term, $\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right)$, reflects a premium that lenders demand for holding the defaultable bonds.

Our main insight is that government's choices regarding debt maturity provides valuable information to distinguish between the fundamental and the rollover risk component. If the rollover risk component is large then the government has an incentive to lengthen the maturity of its debt since long term debt is less susceptible to rollover risk as first shown in Cole and Kehoe (2000). On the contrary, absent rollover risk, previous research - for instance Arellano and Ramanarayanan (2012), Aguiar and Amador (2014b) and Dovis (2014) - has shown that a shortening of maturity is typically the optimal response of the government when facing high interest rates driven by fundamental risk. We will then use this differential behavior to identify the rollover risk component. Next, we describe in more details the relation between maturity choices and sources of default risk.

3.1 Maturity choices and rollover risk

Consider rollover risk first. The key observation here is that the probability of rollover risk in (11), $\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\pi_t$, has two components: an exogenous one, π, and an endogenous one, $\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$. In fact, the current government can affect the probability of being in the crisis zone next period by choosing its new debt portfolio, (B', λ'). Because a rollover crisis is inefficient, when the exogenous probability of a rollover crisis next period, π, is high, the current government wants to reduce the endogenous component. As emphasized in Cole and Kehoe (2000), the government can do so by lengthening the debt maturity structure and/or reducing the value of debt.

To understand why lengthening debt maturity reduces the crisis zone, consider the condition defining the safe zone, [14]

$$U(\tau Y(s_1') - B') + \beta \mathbb{E}[V((1-\lambda')B', \lambda', s'')] \geq \underline{V}(s') \,. \quad (12)$$

[14] For simplicity, we focus on condition (9) instead of (10).

Suppose that the government lengthens the maturity of its debt while keeping constant the amount of resources it raises. This is achieved by decreasing λ' and reducing B' by the appropriate amount. By doing so, the government reduces the payments coming due in the next period and it increases $U(\tau Y(s_1') - B')$ at the cost of higher future payments that reduce the continuation value $\mathbb{E}[V((1-\lambda')B', \lambda', s'')]$ in (12). In the crisis zone, the government is credit constrained, in that it would like to borrow more at higher debt prices, and so the marginal utility of consumption next period is higher than the marginal reduction in expected utility from period two onward.[15] Hence the proposed variation increases the left side of (12). Therefore, lengthening debt maturity reduces the likelihood of falling into the crisis zone next period.[16]

3.2 Maturity choices in absence of rollover risk

We turn now to discuss the dynamics of the maturity composition of debt in relation to fundamental default risk. Previous works on incomplete market models without commitment have shown that the government has an incentive to shorten its maturity when the probability of future fundamental default increases because of an adverse shock hitting the government. For concreteness, in our environment, think about a negative shock to tax revenues when $\pi = 0$. This literature has emphasized two channels as the main determinants of the maturity composition of debt: the *incentives* and the *insurance* channel.

The incentives channel makes short term debt relatively more desirable than long term debt. This is because the value of long term debt does not only depend on the default probability next period, but also on the issuance decisions of future governments as those affect future default probabilities. This creates a time inconsistency problem because future governments do not internalize the negative effect that new issuances have on the price of long term debt today and issue more debt than what optimal from the current government's perspective. Lenders today anticipate this behavior and demand higher interest rates for holding long term debt. Hence, if the current government issues long term debt, the equilibrium outcome will feature high interest rates and a higher probability of future defaults relative to what it would obtain if he had commitment over future issuances.[17] This time inconsistency problem between current and future governments is

[15] See the proof of Proposition 2 in Appendix B to see this point formally in a three-period version of the model.

[16] We show in the Appendix for a simple three period example, that in the extreme case where there are no fundamental shocks, $\pi > 0$, and it is optimal to repay the debt absent a rollover crisis (no fundamental default risk), the government will issue only long term debt in this economy.

[17] Interest rates are higher precisely because lenders correctly anticipate a higher probability of future defaults.

not present for short term debt because its value - conditional on repayment - does not depend on future issuances. This makes short term debt more attractive from the perspective of the current government as it keeps interest rates and the probability of future defaults low.[18]

The insurance channel makes long term debt is desirable because it is a better asset than short term debt to provide insurance absent outright default. To illustrate this point, consider a situation in which the government is hit by a negative shock to its tax revenues that increases its future prospects of default. If all inherited debt is short term, the government must refinance all of its stock of debt at the new high interest rates and so either its current consumption or its continuation value must decline. If instead part of the inherited debt is long term, only a fraction of its stock of debt must be refinanced at the high interest rates and so the government can keep its current consumption relatively high without reducing its continuation value. The opposite happens in response to a positive shock to tax revenues when the prospects of future default decrease and the interest rates are low. From an ex-ante perspective, this creates an incentive to issue long term debt because this increases government's consumption when the market value of debt falls (higher future probability of default) and the marginal utility of the government is high.

The relative strength of the incentive and insurance channel shapes the optimal portfolio decisions in the absence of rollover risk. Previous work and basic economic logic suggest that the government finds it optimal to reduce the maturity of its debt when facing higher fundamental default risk in response to a negative shock to tax revenues when $\pi = 0$. This is because of two reasons.

First, the time inconsistency problem associated with long term debt is more severe when default risk is high. This is because in these states, output is low and/or inherited debt is high and so the government would like to issue more debt in order to smooth out consumption. If such debt is long term though, it will face high interest rates and higher future default probabilities. To avoid this, the government tilts its maturity more toward short term debt. See Aguiar and Amador (2014b) for a similar argument.

Second, the need to hold long term debt for insurance reasons falls when default risk increases. As discussed in Dovis (2014), this happens because pricing functions are more sensitive to shocks when the economy approaches the default region. Hence the larger ex-post variance of the price of long-term debt allows for more insurance because the market value of long term debt falls more in future bad states. *Ceteris paribus*, this makes consumption in the next period less sensitive to shocks.

[18] In Appendix B, we isolate this channel in the context of a three-period model in which there are no shocks at $t = 1$. In this case, the government does not issue long term debt and all debt is short term.

Thus both forces call for the government to tilt its maturity toward shorter maturity when fundamental default risk is high. This is consistent with the findings in the quantitative model of Arellano and Ramanarayanan (2012). We will show that this prediction is confirmed in our calibrated model.

3.3 Confounding factors

The preceding discussion suggests that we can use the dynamics of government's debt maturity to measure the importance of rollover risk. It is important to stress some potential pitfalls of our approach. As in other structural work, our analysis is not robust to misspecifications of the trade-offs that Treasury departments face in practice when managing public debt during crises. One concern could be that debt managers follow simple rule of thumbs that abstract from the state of the economy when choosing the maturity of new issuances. This would make maturity choices uninformative about the underlying sources of default risk. However, several historical episodes are consistent with the idea that rollover risk is indeed a major concern for public debt management. Appendix A discusses two of these episodes, the Italian experience in the early 1980s and that of Finland during the European debt crisis. While far from a systematic analysis, the narrative of these events indicates that rollover risk can be a key consideration for the management of debt maturity in practice.

A second concern is that observed maturity choices can be affected by shifts in the demand of government bonds. This could be problematic for our approach. A government that is facing a rollover crisis may not be willing to lengthen debt maturity if at the same time lenders demand higher compensation for holding these assets. Hence, rather that reflecting little rollover risk, a shortening of debt maturity may be the optimal response of a government who finds increasingly expensive to issue long term debt. This view finds some support in the data, as previous research by Broner, Lorenzoni, and Schmukler (2013) has documented that risk premia on long term securities systematically increase during debt crises. In our quantitative analysis we are going to control for these confounding factors by considering a stochastic discount factor for lenders that generate time variation in the risk premium on long term assets.

4 Quantitative Analysis

We now fit the model to Italian data during the 1999:Q1-2012:Q2 period. This section proceeds in three steps. Section 4.1 describes the parametrization and the calibration strategy. Section 4.2 reports the results of the calibration. Finally, we study the fit of the model in Section 4.3.

4.1 Parametrization and Calibration Strategy

We model the lenders' stochastic discount factor, $M_{t,t+1} = \exp\{m_{t,t+1}\}$, following Ang and Piazzesi (2003),

$$\begin{aligned}
m_{t,t+1} &= -(\phi_0 + \phi_1 \chi_t) - \frac{1}{2}\kappa_t^2 \sigma_\chi^2 - \kappa_t \varepsilon_{\chi,t+1}, \\
\chi_{t+1} &= \mu_\chi (1-\rho_\chi) + \rho_\chi \chi_t + \varepsilon_{\chi,t+1} \qquad \varepsilon_{\chi,t+1} \sim \mathcal{N}(0, \sigma_\chi^2), \\
\kappa_t &= \kappa_0 + \kappa_1 \chi_t .
\end{aligned} \tag{13}$$

In this formulation, expected excess returns on long term bonds are proportional to χ_t (see Appendix E). Hence, shocks to χ_t induce movements in risk premia over long term assets, allowing us to control for time variation in the relative demand of long and short term bonds. For future reference, we index the parameters of the stochastic discount factor with $\theta_1 = [\phi_0, \phi_1, \kappa_0, \kappa_1, \mu_\chi, \rho_\chi, \sigma_\chi]$.

The government discounts future flow utility at the rate β. The utility function is

$$U(G_t) = \frac{(G_t - \underline{G})^{1-\sigma} - 1}{1-\sigma},$$

where $\underline{G}$ is the non-discretionary level of public spending. We interpret $\underline{G}$ as capturing the components of public spending that are hardly modifiable by the government in the short run, such us wages of public employees and pensions. As we will discuss in Section 4.3, this specification helps our model matching the level and cyclicality of public debt.

In the quantitative analysis, we also introduce a utility cost for adjusting debt maturity,

$$\alpha \left(\frac{4}{\lambda'} - \bar{d} \right)^2 .$$

This adjustment cost serves two purposes. First, it leads to well defined maturity choices in regions of the state space where the governments would have been otherwise indifferent over λ', ameliorating the convergence properties of the numerical algorithm used to solve the model.[19] Second, it gives the model enough flexibility to match the level and volatility of debt maturity.

If the government enters a default state, it is excluded from international capital markets and it suffers an output loss d_t. These default costs are a function of the country's income, and they are parametrized following Chatterjee and Eyigungor (2012),

$$d_t = \max\{0, d_0 Y_t + d_1 Y_t^2\}.$$

If $d_1 > 0$, then the output losses are larger when income realizations are above average. We also assume that, while in autarky, the government has a probability ψ of reentering capital markets. If the government reenters capital markets, it starts the decision problem with zero debt.

The output process, $Y_t = \exp\{y_t\}$, depends on the factor χ_t and on its innovations as follow,

$$y_{t+1} = \rho_y y_t + \rho_{y\chi}(\chi_t - \mu_\chi) + \sigma_y \varepsilon_{y,t+1} + \sigma_{y\chi}\varepsilon_{\chi,t+1}, \qquad \varepsilon_{y,t+1} \sim \mathcal{N}(0,1). \tag{14}$$

We allow for correlation between χ_t and y_t in order to capture the cyclicality of risk premia in sample.

The probability of lenders not rolling over the debt in the crisis zone next period follows the stochastic process $\pi_t = \frac{\exp\{\tilde{\pi}_t\}}{1+\exp\{\tilde{\pi}_t\}}$, with $\tilde{\pi}_t$ given by

$$\tilde{\pi}_{t+1} = \pi^* + \sigma_\pi \varepsilon_{\pi,t+1}, \qquad \varepsilon_{\pi,t+1} \sim \mathcal{N}(0,1). \tag{15}$$

We let $\theta_2 = [\sigma, \tau, \underline{G}, \psi, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}, \beta, d_0, d_1, \pi^*, \sigma_\pi, \overline{d}, \alpha]$ denote the parameters associated to the government decision problem.

Our strategy consists in calibrating $\theta = [\theta_1, \theta_2]$ in two steps. In the first step, we choose θ_1 to match the behavior of risk premia over *non-defaultable* long term bonds, measured using the term structure of German's ZCBs. Implicit in this approach is the assumption that the lenders in the model are the marginal investors for these assets as well: thus, we can measure their preferences for short versus long term bonds by studying the behavior of the term structure of German interest rates. We focus on bonds that are arguably not subject to default risk over the sample because of two reasons. First, the absence of a default during the event under analysis makes the measurement of risk premia on Italian

[19] Maturity choices in the model are not determined absent default risk and with risk neutral lenders.

bonds more challenging because of a "peso problem". Second, this approach allows us to calibrate θ_1 without solving the government decision problem, which is numerically complex. In the second step, and conditional on θ_1, we calibrate θ_2 by matching key facts about Italian public finances over our sample.

4.2 Calibration

We start by setting the parameters of the lenders' stochastic discount factor to fit the behavior of expected excess returns on long term German ZCBs. These latter are measured following the procedure developed by Cochrane and Piazzesi (2005). Let $q_t^{*,n}$ be the log price on a non-defaultable ZCB maturing in n quarters, $rx_{t+1}^n = q_{t+1}^{*,n-1} - q_t^{*,n} + q_t^{*,1}$ the associated *realized* excess log returns, $f_t^n = q_t^{*,n-1} - q_t^{*,n}$ the time t log forward rate for loans between $t+n-1$ and $t+n$, and $y_t^1 = -q_t^{*,1}$ the log yield on a ZCB maturing next quarter. We denote by $\mathbf{rx}_{t+1}$ and $\mathbf{f}_t$ vectors collecting, respectively, excess log returns and log forward rates for different maturities. Quarterly data (1973-2013) on the term structure of ZCBs for German federal government securities is obtained from the Bundesbank online database, see Appendix C.

We proceed in two stages. In the first stage, we estimate by OLS a regression of the realized log excess returns averaged across maturities on all the forward rates in $\mathbf{f}_t$,

$$\overline{rx}_{t+1} = \gamma_0 + \gamma'\mathbf{f}_t + \overline{\eta}_t. \tag{16}$$

In the second stage, we estimate the regressions

$$rx_{t+1}^n = a_n + b_n(\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t) + \eta_t^n, \tag{17}$$

where $[\hat{\gamma}_0, \hat{\gamma}]$ is the OLS estimator derived in the first stage. Expected excess returns on a ZCB maturing in n period can then be measured using the fitted values of this second stage regression.[20]

We choose θ_1 so that the pricing model defined by the equations in (2) and (13) fits key properties of short term real interest rates and expected excess returns on a bond with residual maturity of five years ($n = 20$). Specifically, we select ϕ_0 and ϕ_1 to match the mean and standard deviation of the yields on the German short term real rate over the sample. The remaining parameters are chosen to match, in model simulated data, the coefficients of an AR(1) estimated on $\{\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t\}_t$ as well as the OLS point estimates of the

[20]From equation (17) we can see that *expected* excess returns on a bond maturing in n period equal $\mathbb{E}_t[rx_{t+1}^n] = \alpha_n + \beta_n(\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f_t})$.

parameters in equation (17), $[\hat{a}_{20}, \hat{b}_{20}, \hat{\sigma}_{\eta^{20}}]$. Appendix E reports the results of the Cochrane and Piazzesi (2005) regressions and describes in more details the calibration of θ_1. Panel A of Table 1 reports the numerical values of the calibrated parameters.

We can also use the model's restrictions to construct an empirical counterpart to the χ_t shock. Expected excess returns on long term bonds are affine in χ_t, implying that

$$\chi_t = \frac{\mathbb{E}_t[rx^n_{t+1}] - \tilde{A}_n(\theta_1)}{\tilde{B}_n(\theta_1)}, \tag{18}$$

with $\tilde{A}_n(.)$ and $\tilde{B}_n(.)$ defined in Appendix E. We can therefore construct the time path of χ_t by substituting in the right hand side of equation (18) the expected excess returns on the five years German bonds estimated with the Cochrane and Piazzesi (2005) methodology.

We next turn to the calibration of θ_2. We fix σ to 2, and we set $\psi = 0.0492$, a value that implies an average exclusion from capital markets of 5.1 years following a default, in line with the evidence in Cruces and Trebesch (2013). The tax rate is normalized in order to have steady state revenues of 1. The spending requirement $\underline{G}$ is set to 0.70, in order to replicate the average ratio of wages of public employees and transfers to government revenues during the 1999-2012 period.

We map y_t to the deviations of Italian log real GDP from a deterministic trend. The real GDP series is obtained from OECD *Quarterly National Accounts*. We estimate the process in equation (14) for the 1999:Q1-2012:Q2 period using this series and the series for $\hat{\chi}_t$ obtained earlier. As $\rho_{y\chi}$ is not significantly different from zero, we impose the restriction $\rho_{y\chi} = 0$. The point estimates of this restricted model are $\rho_y = 0.939$, $\sigma_{y\chi} = -0.002$ and $\sigma_y = 0.008$.

The remaining parameters, $[\beta, d_0, d_1, \pi^*, \sigma_\pi, \bar{d}, \alpha]$, are chosen to match key features of the behavior of Italian public finances. As commonly done in the literature, we include in the set of empirical targets statistics that summarize the behavior of the debt to output ratio and interest rate spreads. Specifically, we consider the sample mean of the debt to output ratio, the correlation between debt issuances and detrended real GDP, and the mean, standard deviation and skewness of interest rate spreads on Italian ZCB with a residual maturity of five years.[21] We also incorporate in the targets the sample mean and standard deviation of an indicator of debt maturity for Italian government debt.[22] These

[21]The debt to output ratio is total gross debt of the Italian central government expressed in percentages of annualized GDP. Interest rate spreads are yields differential between German and Italian government securities with a residual maturity of five years. See Appendix C for definitions and data sources.

[22]Specifically, we use data from the Italian Treasury and construct the weighted average of the times of principal and coupon payments for outstanding bonds issued by the Italian central government. This

moments provide information on the parameters of the adjustment cost function. There is, instead, little guidance in the literature on the choice of variables that provide information on π^* and σ_π. Our approach consists in incorporating in the set of empirical targets the R^2 of the following regression,

$$\begin{aligned} \text{spr}_t &= a_0 + \sum_{j=1}^{2} a_{j,\text{gdp}} \text{gdp}_t^j + \sum_{j=1}^{2} a_{j,\text{debt}} \text{debt}_t^j + \sum_{j=1}^{2} a_{j,\chi} \hat{\chi}_t^j \\ & \quad + b_1(\text{gdp}_t \times \text{debt}_t) + b_2(\text{gdp}_t \times \hat{\chi}_t) + b_3(\text{debt}_t \times \hat{\chi}_t) + e_t. \end{aligned} \tag{19}$$

The residual in equation (19) measures variation in interest rate spreads that is orthogonal to the fundamental state variables in the model, and it should therefore provide information about the volatility of π_t. We estimate equation (19) by OLS for the 2000:Q1-2012:Q2 period, obtaining an R^2 of 75%.

Because the numerical solution of the model is computationally costly, we first experiment with these five parameters to obtain a range of values that is empirically relevant. At this stage, we fix π^* to -6, a value that implies a 1% annualized probability of a rollover crisis conditional on the economy being in the crisis zone next period. We next solve the model on a grid of points for $[\beta, d_0, d_1, \sigma_\pi, \alpha, \overline{d}]$, and select the parametrization that minimizes a weighted distance between sample moments and their model implied counterparts.[23] Appendix F presents the algorithm used for the numerical solution of the model, while Panel B of Table 1 reports the calibrated values for the model's parameters.

The value of β is 0.9875, substantially higher than that of existing work that focus on emerging markets. In line with previous research, we find that convex output costs are necessary to fit the behavior of interest rate spreads. The numerical values of $[d_0, d_1]$ imply output losses upon defaults of 6.25% when output is at its average level, and 10.75% when output is 9% below trend. These numbers are not inconceivable, given that a government default in Italy would lead to a major disruption of financial intermediation for the private sector (Bocola, 2014), and most likely damage trade relations with other euro-area partners.[24]

indicator maps to λ'^{-1} in our model. See Appendix C for detailed definitions and data sources.

[23] Model implied moments are computed on a long simulation ($T = 5000$) of the model. When computing the statistics, we exclude the first 40 quarters after a default. We weight the distance between a sample moment and its model counterpart by the inverse of the sample moment absolute value.

[24] To best of our knowledge, Hebert and Schreger (2015) represents the only attempt in the literature to directly measure the output costs of a sovereign default. By using variation in legal rulings in the case of Republic of Argentina v. NML Capital, the authors estimate an output costs of sovereign default between 2.4% and 6% of GDP for the Argentinian economy.

Table 1: **Model calibration**

Parameter	Value	Targets
Panel A: Pricing model		
ϕ_0	0.003	Mean of risk free rate
ϕ_1	0.242	Standard deviation of risk free rate
κ_0	-0.228	Method of Simulated Moments
κ_1	-2322.111	Method of Simulated Moments
μ_χ	0.008	Method of Simulated Moments
ρ_χ	0.906	Method of Simulated Moments
σ_χ	0.008	Method of Simulated Moments
Panel B: Government's decision problem		
σ	2.000	Conventional value
ψ	0.049	Cruces and Trebesch (2013)
τ	1.000	Normalization
$\underline{G}$	0.700	Non discretionary public spending
ρ_y	0.939	Estimates from eq. (14)
σ_y	0.008	Estimates from eq. (14)
$\sigma_{y\chi}$	-0.002	Estimates from eq. (14)
$\frac{\exp\{\pi^*\}}{1+\exp\{\pi^*\}} \times 400$	1.000	
σ_π	1.150	Method of Simulated Moments
β	0.988	Method of Simulated Moments
d_0	-0.415	Method of Simulated Moments
d_1	0.522	Method of Simulated Moments
α	0.522	Method of Simulated Moments
$\bar{d}$	0.522	Method of Simulated Moments

4.3 Model Fit

We start by documenting the model's ability to match the empirical targets. The first and second columns of Table 2 show that the model has good in sample fit. The face value of debt is 113.20% of annual GDP on average, close to the 102% observed on average in our sample. Debt issuances are negatively correlated with output, as they are in the data. The distribution of interest rate spreads is remarkably similar to that observed in our data. Spreads are on average small in model simulated data, but they can experience sudden spikes when the government gets closer to the default region: because of that, the implied distribution of interest rate spreads is right skewed, and it has large excess kurtosis. Hence, the model is consistent with the observation that debt crises are rare and extreme events for advanced economies. Finally, the model generates an empirically plausible relation between interest rate spreads and economic fundamentals, as captured by the R^2 of equation (19).

Table 2: **Calibration targets and additional statistics**

Panel A: Empirical Targets				
Statistic	Data	Baseline	CRRA Utility	$\pi_t = 0$
Average debt-to-output ratio	102.20	113.20	30.00	47.96
Correlation $\Delta b'$ and y	-0.20	-0.10	0.20	0.29
Average spread	0.67	0.51	1.10	0.00
SD of spread	1.05	0.95	0.56	0.00
Skewness of spread	2.06	3.25	2.29	0.00
Kurtosis of spread	6.49	16.00	16.49	0.00
R^2 of regression (19)				
Panel B: Additional Statistics				
Default rate	0.00%	0.06%	0.10%	0.00%
SD of debt-output ratio	5.30	3.21	0.98	2.37
Average prob. of crisis zone		87.26%	0.00%	0.00%

Notes: The debt-to-output ratio is the quarterly outstanding debt of the central government scaled by annual GDP (Eurostat). Sample moments are computed over the 2001:Q1-2012:Q2 window. Moments in the model are computed as described in footnote 23.

We can now discuss some of these results in more details.

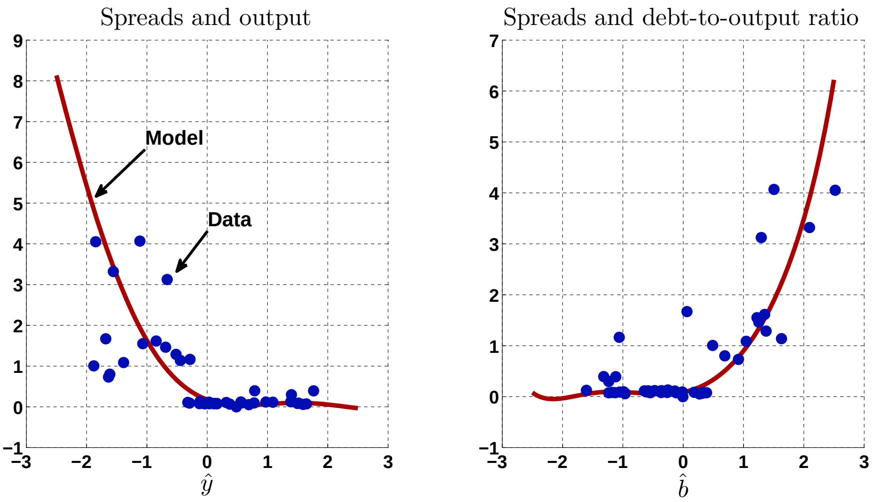
Cyclicality of debt issuances: Calibrations of sovereign debt models presented in the literature typically produce pro-cyclical debt issuances, with the government borrowing more when hit by a positive y_t shocks. This would be at odd with the behavior of Italian public debt during the 2008-2012 crisis, which increased by roughly 20% of annual GDP throughout the episode. In order to explain how we correct this issue, note that the cyclicality of debt issuances in sovereign default models is driven by the interplay of two forces. On the one hand, the government wishes to borrow more in bad times because of consumption smoothing motives. On the other hand, it may not be able to do so because investors tighten lending in bad times due to the heightened risk of a default by the government. In standard calibrations, this second effect dominates, implying that the government borrows more in good times. In our calibration, instead, the first effect dominates.

These differences are mostly due to the interplay of two parameters, β and $\underline{G}$. Calibrations considered in the literature often set a very low β and use CRRA utility ($\underline{G} = 0$). In those environments, the government is impatient relative to the risk free rate and it has fairly small precautionary motives: thus, it behaves myopically, borrowing as much as possible in every period. Because of this feature, the cyclicality of debt issuances mirrors that of the pricing schedule, and the government borrows more in good times because *it can*. In our calibration, instead, the government does not borrow to its capacity in good times because it is less impatient and because the non-homotheticity in the utility function leads to stronger precautionary motives. Once hit by a bad income shock, however, the government increases borrowing because deleveraging is very costly when income is close to the consumption commitments of the government. This behavior leads to countercyclical debt issuances, and it allows the model to fit Italian public debt in our sample.

[to be completed] We can verify this interpretation by comparing our baseline specification to a specification where $\beta = 0.9$ and $\underline{G} = 0$. ....

Interest rate spreads and economic fundamentals: As shown earlier, our model replicates the R^2 of regression (19). One might wonder whether this implies that the relation between interest rate spreads, debt and output is close to what observed in the data. To explore this issue, the solid line in the left panel of Figure 1 plots the average relation between interest rate spreads and output in the model, while the dots report combinations of these two variables in our sample.

Figure 1: **Interest rate spreads sensitivity to output and debt**



Notes: The left panel is constructed as follows. We simulate a $T = 5000$ realization from the model. After standardizing the output series, we fit the following regression, $s_t = a_0 + \sum_{j=1}^{5} a_j y_t^j + e_t$ on the model simulated data. The red solid line represents the fitting curve of this regression. The blue dots are combinations of interest rate spreads and linearly detrended GDP (standardized). The right panel reports the same experiment, this time for the debt-to-output ratio.

We can observe that the model implied elasticities of interest rate spreads to y are highly nonlinear: a decline in output when the economy is doing well has essentially no effects on interest rate spreads on average. However, this elasticity achieves a value of -3.75 when output is one standard deviation below its mean. These elasticities are empirically plausible, in terms of shape and magnitude. The right panel of the figure plots this same information for debt. Again, the implied elasticities of interest rate spreads to the debt-to-output ratio are highly nonlinear, and they well capture the relation between these two variables in the data. This state dependence in interest rate elasticities is what generates the right skewness and excess kurtosis documented in Table 2.

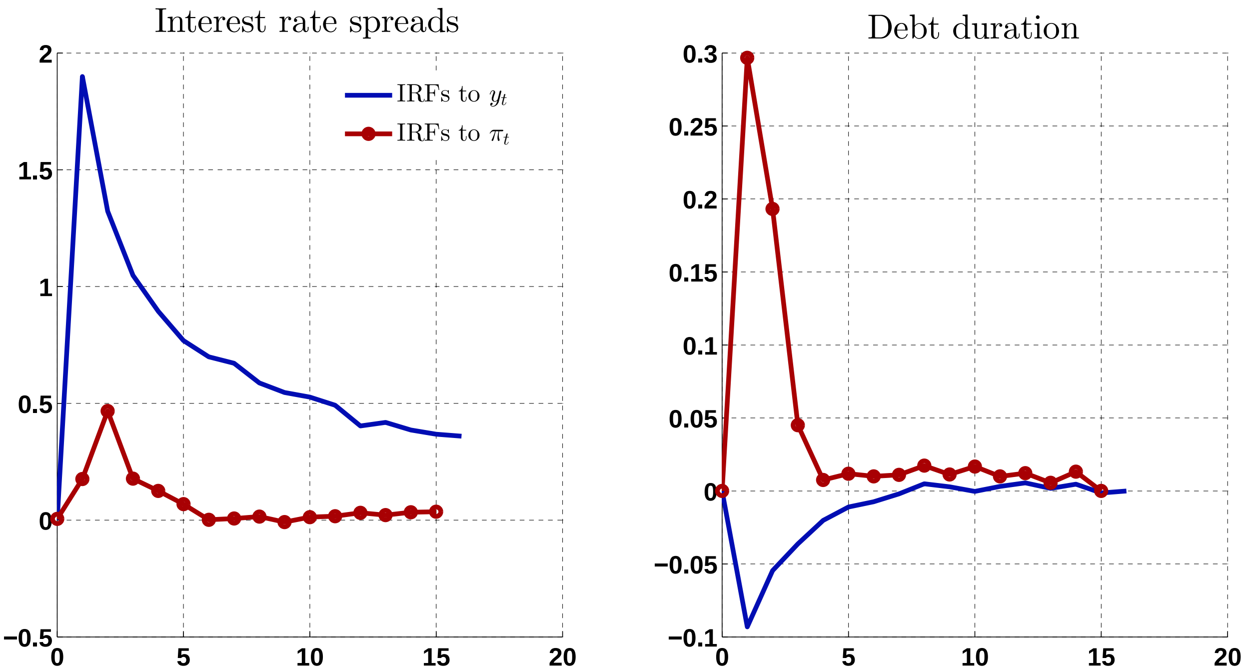
The choice of β and $\underline{G}$ is again important for generating empirically plausible elasticities for interest rate spreads. In the $\beta = 0.90$, $\underline{G} = 0.00$ specification, for example, interest rate spreads are sensitive to income shocks also in good times. The dotted line in Figure 1 depicts this case. Again, this is due to the fact that the government always borrow at its maximal capacity when impatience dwarf precautionary motives, implying that he is at risk of a default even in good times.

Interest rate spreads and non-fundamental risk: [to be completed] The high sensitivity of interest rate spreads to changes in output and debt, summarized by the R^2 of equation (19), does not necessarily imply that the bulk of their variation is due to fundamental risk. In the model, movements in output and debt affects the likelihood of being in the crisis zone in the future: if $\pi_t > 0$, these movements affects interest rate spreads also through their impact on non-fundamental uncertainty. To better gauge the role of this latter, we can compare the volatility of interest rate spreads in our baseline simulations to those of a simulation in which π_t is set to zero in every period. This latter statistic in reported in panel B of Table 2. We can verify that non-fundamental risk accounts on average for x% of the volatility in interest rate spreads, more than what the R^2 of the regression in (19) naively suggests.

5 Sources of Default Risk and Maturity Choices

[To be completed] We now turn to show that our calibrated model is consistent with the logic outlined in Section 3. Our identification argument to measure the importance of rollover and fundamental risk is based upon the different movement of the maturity structure of debt associated to an increase in each component. In Figure 2 we verify this. We plot the average response of interest rate spreads and debt duration - measured as $1/\lambda'$ - to a negative income shock when $\pi = 0$ and to a positive shock to π.

Figure 2: **The dynamics of interest rate spreads and debt duration**



Notes: The blue solid line reports impulse response functions (IRFs) of interest rate spreads and debt duration to a 3 standard deviations income shock in the model without rollover risk. The red circled line reports IRFs to a 3 standard deviation increase in π_t. IRFs are calculated by simulation, and they are expressed as deviation from the ergodic mean. Interest rate spreads are expressed in annualized percentages while debt duration in years.

The solid blue line plots the average response of interest rate spreads and debt duration to a negative shock to tax revenues, τY, conditional on the realization of $\pi_t = 0$ for all t. The fact that $\pi_t = 0$ implies that the rollover risk component is zero and all default risk arises because of fundamentals. We can see that when interest rate spread goes up because of an increase in fundamental default risk, the government shortens the maturity of its debt. This conforms with our discussion in section 3 and previous works that find that high fundamental default risk is associated with a shorter maturity of debt. See in particular Arellano and Ramanarayanan (2012).

The circled line in Figure 2 plots the average response of interest rate spreads and debt duration to an increase in π_t. Everything else equal, this increases the probability of a rollover crisis conditional on the economy being in the crisis zone the next period. As expected from our discussion in section 3, this leads to an increase in debt maturity. When the exogenous component of rollover risk increases, the government lengthen the maturity of its debt to reduce the endogenous component of the crisis zone.

6 Decomposing Italian Spreads

We now turn to the main experiment of the paper, and measure the importance of non-fundamental risk during the debt crisis of 2008-2012. In Section 6.1 we combine the calibrated model with Italian data in order to retrieve the path for the exogenous shocks $\{y_t, \chi_t, \pi_t\}$. We use this path to measure the non-fundamental component of interest rate spreads. In Section 6.2 we discuss the information content of maturity choices by repeating this exercise in a version of the model where the government can not change the maturity of its outstanding debt. Finally, section 6.3 performs a sensitivity analysis.

6.1 Measuring non-fundamental risk

The model defines the nonlinear state space system

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t), \end{aligned} \tag{20}$$

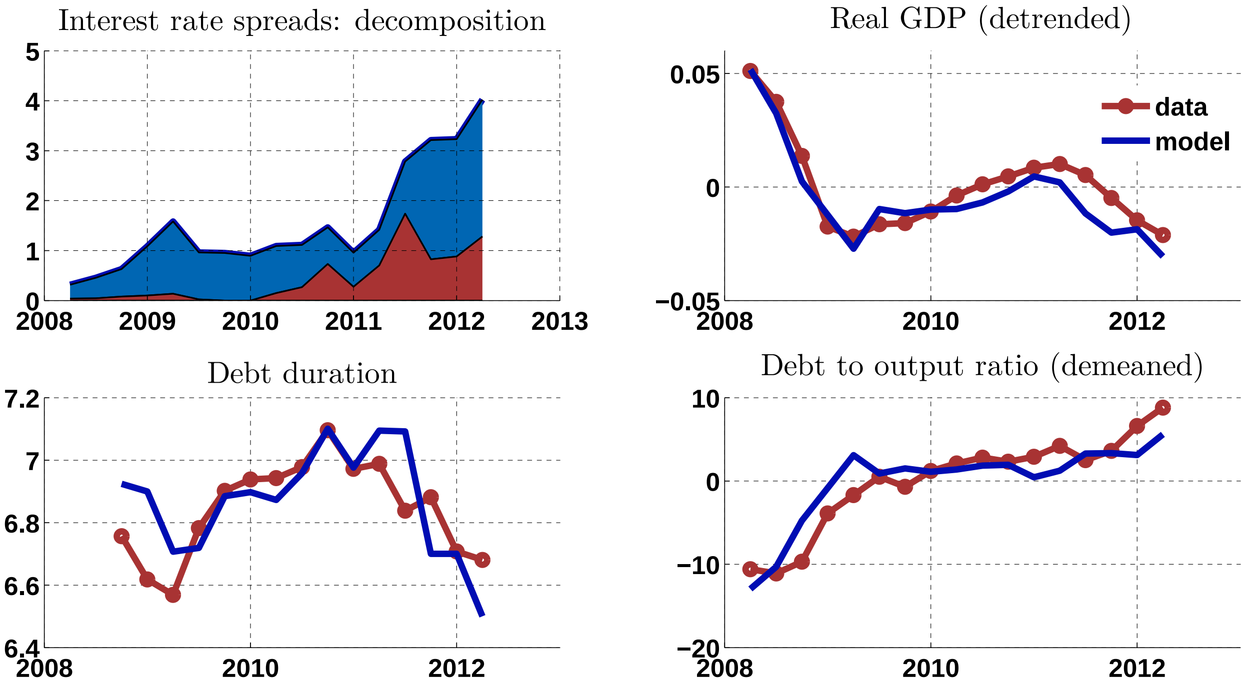
with $\mathbf{Y}_t$ being a vector of observable variables, $\mathbf{S}_t = [B_t, \lambda_t, y_t, \chi_t, \pi_t]$ the state vector, and ε_t the vector collecting the structural shocks. The vector η_t contains classical measurement errors. The functions $g(.)$ and $f(.)$ are obtained using the model's numerical solution.

The vector of observables includes detrended real GDP, the data counterpart to χ_t constructed using equation (18), the interest rate spread series, and the data counterpart to λ'. Given the time path of these variables over the 2008:Q1-2012:Q2 period, we retrieve the realization of the state vector using the relation between states and observables implied by the model. Technically, we carry out this step by applying the particle filter (Fernández-Villaverde and Rubio-Ramírez, 2007) to the system in (20), see Appendix G for a detailed description. It is important to stress that the inference on $[y_t, \chi_t]$ in our approach is disciplined by actual observations because the measurement equation incorporates empirical counterparts to these shocks. The truly unobservable process is the realization of π_t.

Equipped with the path for the exogenous shocks, we next use the structural model to measure the contribution of non-fundamental risk to interest rate spreads. We feed the model with a realization for the structural shocks that is equivalent to the point estimates obtained earlier, with the exception that π_t is set to 0 throughout the sample. The difference between this counterfactual time series and the one filtered from the data isolates this component of interest rate spreads. We will refer to this as the contribution of rollover risk to interest rate spreads. In terms of the decomposition discussed in Section 3, it measures the direct effects that non-fundamental risk has on the likelihood of a future rollover crisis, and its indirect effects on default incentives and risk premia.

Figure 3 reports the results of this experiment. In the first panel we report the filtered interest rate spreads and the component accounted for by rollover risk. The other three panels of the figure report the data series (solid lines) and the associated time series filtered by the model (circled lines) for output, debt duration and debt to output ratio. (The two differ because of the presence of measurement errors.) The model replicates the time path of observables fairly accurately, despite having fewer shocks. This confirms the good in sample fit documented earlier. We can also see that the model generates a trajectory for the debt-to-output ratio that is close to the one observed in the data, even though we did not explicitly include this variable in the set of observables.

Figure 3: **Decomposition of interest rate spreads**



Notes: The top left panel reports the decomposition of the yields differentials between an Italian and a German ZCB with a residual maturity of five year. The red area is the rollover risk component, the blue area is the fundamental risk component. The top right panel and the two bottom panels report Italian detrended GDP, debt duration and debt to output ratio along with the point estimates for such variables implied by the model.

The fundamental component accounts for most of the increase in interest rate spreads over our sample. In 2008:Q1, this component was essentially equal to 30 basis points, while at the end of the episode it was roughly 75% of observed interest rate spreads. This increase is the result of two major developments that occurred during this period. First, the Italian economy experienced a major recession: output went from being 4% above trend in 2008:Q1 to being 3% below trend at the end of the sample. In the model, these shocks push the government closer to the default region, raising the required premia on its debt. However, a visual inspection of the behavior of output reveals that income shocks, by themselves, would not be sufficient to explain the sharp increase in interest rate spreads observed after 2011. The innovations to y in 2008:Q3 and 2011:Q3 are comparable in size, but their impact on the fundamental component of interest rate spreads is substantially larger toward the end of the sample. This non-linearity is explained by the second major development taking place during the event, the increase in public debt. Negative innovations to y at the end of the sample have more sizable implications for the likelihood of a default because the Italian government was more levered at that point in time.

It is important to notice the relation between the rollover risk component (red area) and the behavior of debt duration. In 2011, this component initially increases as interest spreads increase while the duration of debt was high. In the last part of the sample, when interest rate spreads increase, the component that we can attribute to rollover risk decreases because the debt duration of the stock of Italian debt decreases. Hence the model interprets such reduction in debt duration as a reduction in exogenous probability of rollover risk, π.

[To be completed]

6.2 The information content of maturity choices

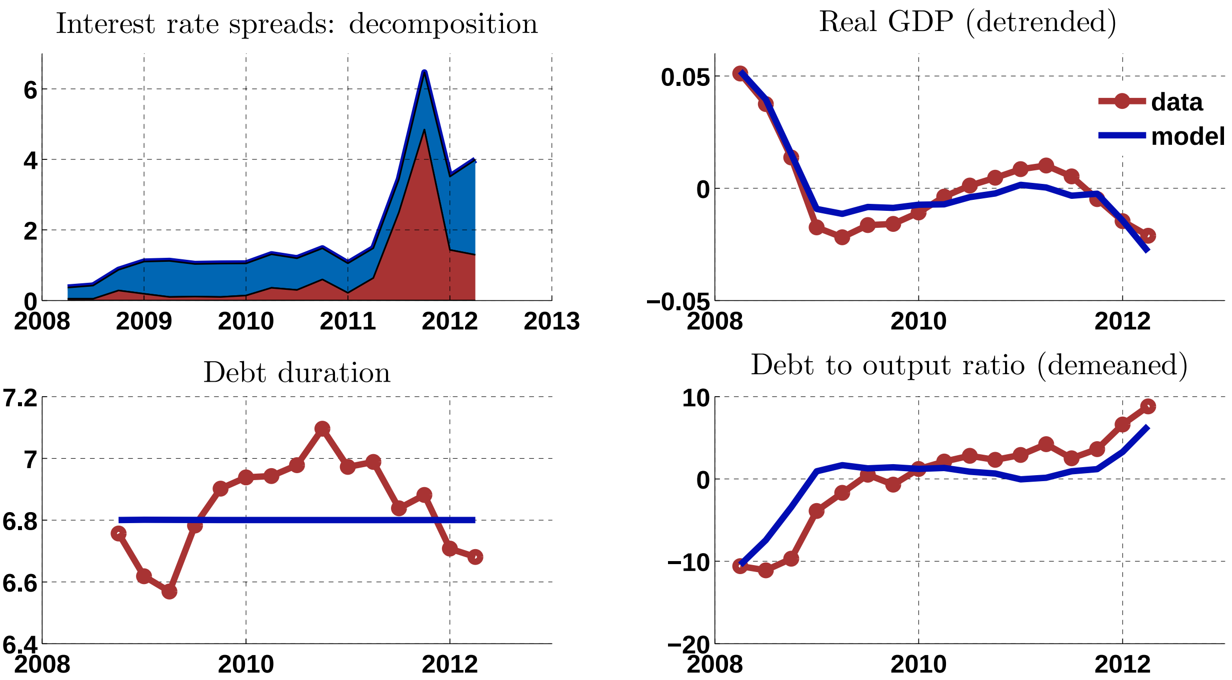
We now repeat the filtering experiment, this time excluding the debt duration series from the set of observables and fixing the λ to its ergodic mean. Figure 4 below reports the outcomes of this exercise.

Absent data on debt duration, the model does not have clear identifying restrictions that can be used to discipline π_t, and it attributes to this term variation in interest rate spreads that can not be accounted by fundamentals $[y_t, \chi_t]$. As one can see from the first panel in the figure, the model is able to account for the peak in interest rate spreads observed in the data. The bulk of the increase is attributed to the rollover risk component (red area). [To be completed]

6.3 Sensitivity analysis

[To be added]

Figure 4: **Decomposition of interest rate spreads without maturity choices**



Notes: The top left panel reports the decomposition of the yields differentials between an Italian and a German ZCB with a residual maturity of five year where the debt duration is fixed exogenously at its ergodic mean and it is not used as observable. The red area is the rollover risk component, the blue area is the fundamental risk component. The top right panel and the two bottom panels report Italian detrended GDP, debt duration and debt to output ratio along with the point estimates for such variables implied by the model.

7 Evaluating OMT Announcements

[This section is based on a previous calibration of the model. we are in the process of updating it.]

As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright bond purchases in secondary sovereign bond markets. The technical framework of these operations was formulated on September 6 of the same year. The Outright Monetary Transaction (OMT) program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.[25]

Even though the ECB never purchased government bonds within the OMT framework, the mere announcement of the program had significant effects on interest rate spreads of

[25]OMTs consist in direct purchases of sovereign bonds of members of the euro-area in secondary markets. These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions. There are two main characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, OMTs are *conditional* on the country being in a European Financial Stability Facility/European Stability Mechanism macroeconomic adjustment or precautionary program.

peripheral countries. Altavilla, Giannone, and Lenza (2014) estimate that OMT announcements decreased the Italian and Spanish 2 years government bonds by 200 basis points. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing belief-driven inefficient fluctuations in sovereign bond markets of euro-area peripheral countries. Accordingly, OMT has been regarded thus far as a very successful program. In this Section we use our calibrated model to evaluate this interpretation.

7.1 Modeling OMT

We model OMT as follows. At the beginning of each period, after all uncertainty is realized, the government can ask for assistance. In such case the Central Bank (CB) commits to buy government bonds in secondary markets at a price $q_{n,CB}\left(\mathbf{S}, B', \lambda'\right)$ that may depend on the state of the economy, $\mathbf{S}$, on the quantity of debt issued, B', and on the maturity of the portfolio, λ'. We assume that assistance is conditional on the fact that the debt issued is below a cap $\bar{B}_{n,CB}\left(\mathbf{S}, \lambda'\right) < \infty$, also set by the CB. The limit can depend on the state of the economy and on the duration of the stock of the debt portfolio, and it captures the conditionality of the assistance in the secondary markets. Moreover, it rules out Ponzi-schemes. OMT is fully characterized by a policy rule $\left(q_{n,CB}\left(\mathbf{S}, B', \lambda'\right), \bar{B}_{n,CB}(\mathbf{S},\lambda')\right)$. We assume that the CB finances bond purchases with a lump sum tax levied on the lenders. We further assume that such transfers are small enough that they do not affect the stochastic discount factor $M_{t,t+1}$.

The problem for the government described in (5) changes as follows. We let $a \in \{0, 1\}$ be the decision to request CB assistance, with $a = 1$ for the case in which assistance is requested. Then we have:

$$V\left(\mathbf{S}\right) = \max_{\delta \in \{0,1\}, B', \lambda', G, a \in \{0,1\}} \delta \left\{U(G) + \beta \mathbb{E}[V\left(\mathbf{S}'\right)|\mathbf{S}]\right\} + (1-\delta)\underline{V}\left(s_1\right) \tag{21}$$

subject to

$$\begin{aligned} G + B &\leq \tau Y(s_1) + \Delta\left(\mathbf{S}, a, B', \lambda'\right), \\ \Delta\left(\mathbf{S}, a, B', \lambda'\right) &= \sum_{n=1}^{\infty} q_n\left(s, a, B', \lambda'\right)\left[(1-\lambda')^{n-1}B' - (1-\lambda)^n B\right] \\ B'_n &\leq \bar{B}_{n,CB}\left(\mathbf{S},\lambda'\right) \quad \text{if } a = 1, \end{aligned}$$

while the lenders no-arbitrage conditiona requires that

$$q_n\left(\mathbf{S},a,B',\lambda'\right)=\max\{aq_{n,CB}\left(\mathbf{S},B',\lambda'\right);\delta\left(\mathbf{S}\right)\mathbb{E}\left\{M\left(s_1,s_1'\right)\delta\left(\mathbf{S}'\right)q_{n-1}'|\mathbf{S}\right\}\text{ for }n\geq 1 \tag{22}$$

where $q_{n-1}'=q_{n-1}\left(s',B'',\lambda''\right)$ with $B''=B'\left(s',B',\lambda'\right)$, $\lambda''=\lambda'\left(s',B',\lambda'\right)$, $a'=a\left(s',B',\lambda'\right)$, and the initial condition is $q_0'=1$. The max operator on the right hand side of equation (22) captures the fact that lenders have now the option to sell the bond to CB at the price $q_{n,CB}$ in case the government asks for assistance $(a=1)$. Because of that, pricing schedules now depend on current and future decisions of the government to activate OMTs.

Given a policy rule $(q_{CB},\bar{B}_{CB})$, a *recursive competitive equilibrium with OMT* is value function for the borrower V, associated decision rules δ,B',λ',G,a and a pricing function q such that V,δ,B',G are a solution of the government problem (21) and the pricing functions satisfy the no-arbitrage condition (22). For exposition, it is convenient to define also the *fundamental equilibrium outcome* $\mathbf{y}^*=\left\{\delta_t^*,B_{t+1}^*,\lambda_{t+1}^*,G_t^*,q_{n,t}^*\right\}$ as the equilibrium outcome that maximizes the utility for the government given an initial portfolio of debt. We denote the objects of a recursive competitive equilibrium associated with the fundamental outcome with a superscript "$*$".

We now turn to show that an appropriately designed policy rule can uniquely implement the fundamental equilibrium outcome, our normative benchmark.[26]

Proposition 1. *The OMT rule can be chosen such that the fundamental equilibrium outcome is uniquely implemented and assistance is never activated along the path. In such case, OMT is a weak Pareto improvement relative to the equilibrium without OMT (strict if the equilibrium outcome without OMT does not coincide with the fundamental equilibrium).*

Proof. One way to uniquely implement the fundamental equilibrium outcome is to set $q_{n,CB}(\mathbf{S},B',\lambda')=q_n^*\left(s,B',\lambda'\right)$ and $\bar{B}_{n,CB}\left(\mathbf{S},\lambda'\right)\leq(1-\lambda)^{n-1}B^{*\prime}\left(\mathbf{S}\right)$ if $\lambda=\lambda^{*\prime}\left(\mathbf{S}\right)$ and zero otherwise. Such construction is not necessary. A less extreme alternative is to design policies such that for all $\mathbf{S}$ for which there is no default in the fundamental equilibrium, $\delta^*\left(\mathbf{S}\right)=1$, there exists at least one (B',λ') with $(1-\lambda')^{n-1}B'\leq\bar{B}_{n,CB}\left(\mathbf{S},\lambda'\right)$ such that

$$U\left(Y-B+\Delta\left(\mathbf{S},1,B',\lambda'\right)\right)+\beta\mathbb{E}[V^*\left(B',\lambda',s'\right)|\mathbf{S}]\geq\underline{V}(s_1), \tag{23}$$

and for all (B',λ') such that $(1-\lambda')^{n-1}B'\leq\bar{B}_{n,CB}\left(\mathbf{S},\lambda'\right)$ the fundamental equilibrium is

[26] Clearly, the model has incomplete markets and all sorts of inefficiencies (especially when considering an environment with long-term debt). We are going to abstract from policy interventions that aims to ameliorate such inefficiencies. OMT is only targeted at eliminating "bad" equilibria. Such features will also survive in models with complete markets or in environment where some notion of constrained efficiency can be achieved as in Dovis (2014).

always preferable, in that

$$U\left(Y - B + \Delta\left(\mathbf{S}, 1, B', \lambda'\right)\right) + \beta\mathbb{E}[V^*\left(B', \lambda', s'\right)|\mathbf{S}] \leq V^*\left(\mathbf{S}\right). \tag{24}$$

Under (23) and (24), no self-fulfilling run is possible and the choices of B' and λ' are the same that would arise in the fundamental equilibrium outcome.□

By setting a floor on asset prices, the CB allows the government to access financial markets even when lenders are not rolling over the debt. This access to credit market allows the government to repay the maturing debt when facing a run, and it eliminates the self-fulfilling aspect of rollover crises. Quantity limits on debt issuances guarantee that the government does not choose a B' that is larger than the one arising in the fundamental equilibrium.[27] In this fashion, the CB can achieve a Pareto improvement without actually carrying out bond purchases on the equilibrium path.

While the drop in interest rate spreads of southern European economies observed after the introduction of OMT is consistent with this interpretation, it does not provide by itself evidence that the policy operated through this channel. In fact, a decline in interest rate spreads following OMT announcements is also consistent with the interpretation that the policy raised bondholders' expectations of future bailouts for euro-area peripheral countries. To understand this point, suppose that in a given state the fundamental price for the portfolio of debt is $q^{*'}$. Assume now that the CB sets an assistance price $q'_{CB} > q'^{*}$. From equation (22), the announcement of this policy leads to an increase in the price today (equivalently, a reduction in interest rate spreads). This would entail a redistribution of resources from the lenders to the government, and it would have different welfare implications relative to the case described in Proposition 1.

We now propose a procedure to evaluate whether the reduction in interest rate spreads observed after the OMT announcements was due to the elimination of self-fulfilling crises.

7.2 A Simple Test

To explain our approach, suppose that the CB credibly commits to our normative benchmark. The announcement of this intervention eliminates rollover risk in every state of the world, and interest rate spreads jumps to their fundamental value. This fundamental level of interest rate spreads represents a lower bound on the post-OMT spread under the hypothesis that the program was directed exclusively to prevent rollover crises. Our

[27]Under OMT the government acts as a price taker and it has an incentive to overborrow relative to the fundamental equilibrium outcome.

approach consists in comparing the spreads observed after the OMT announcements to their fundamental value: if the latter is higher than the observed one, it would be evidence against the hypothesis that the policy operated exclusively through a reduction in the prospect of future runs.

We perform this test using our calibrated model. Our procedure consists in three steps:

1. Obtain decision rules from the fundamental equilibrium.
2. Feed these decision rules with our series for the fundamental shocks $\{\chi_t, y_t\}$. Obtain counterfactual post-OMT fundamental spreads.[28]
3. Compare post-OMT spreads with the counterfactual ones.

Table 3 reports the results. In the first column we have the Italian spreads observed after the OMT announcements, while the second column presents the counterfactual spreads constructed with the help of our model. We can verify that the observed spreads lie below the one justified by economic fundamentals under the most optimistic interpretation of OMT. In 2012:Q4, the observed spread on our spread series was 222 basis points, while our model suggests that the spread should have been 354 basis points if the program was exclusively eliminating rollover risk. Therefore, our model suggests that the decline in the spreads observed after the OMT announcements partly reflects the anticipation of a future intervention of the ECB in secondary sovereign debt markets. This is not surprising given our result in Section 6: since rollover risk was almost negligible in 2012:Q2, the observed drastic reduction in the spreads should partly reflect the value of an implicit put option for holders of Italian debt guaranteed by the ECB.

Table 3: **Actual and fundamental sovereign interest rate spreads in Italy**

	Actual spreads	**Spreads justified by fundamentals**
2012:Q3	348.24	379.80
2012:Q4	222.25	354.60

[28] The estimates of the state vector obtained with the particle filter end in 2012:Q2. For the 2012:Q3-2012:Q4 period, we set y_t equal to detrended Italian real GDP and χ_t equal to the one estimated from the German yield curve.

8 Conclusion

This paper has proposed a strategy to bring to the data the sovereign debt model of Eaton and Gersovitz (1981) modified to allow for self-fulfilling debt crises as in Cole and Kehoe (2000). In this class of models, the observed maturity choices of the government allow to distinguish between fundamental and non-fundamental sources of variation in interest rate spreads. We apply this identification strategy to Italian data during the debt crisis of 2008-2011. Our preliminary results indicate that fluctuations in non-fundamental risk accounted for a modest fraction of the increase in sovereign borrowing costs. This finding suggests that the sharp reduction in spreads observed upon the establishment of the OMT program reflects the expectation of future bailouts of peripheral euro-area countries.

The analysis considered belief-driven fluctuations that arise from rollover risk as introduced in Cole and Kehoe (2000). We did not consider other types of multiplicity considered in the literature, for example the one emphasized in Calvo (1988) and recently studied by Lorenzoni and Werning (2013) and Navarro, Nicolini, and Teles (2015), or the one in Broner, Erce, Martin, and Ventura (2014). Future research should investigate which feature of the data can be used to discipline empirically these other sources of multiplicity.

Our approach is not limited to sovereign bond markets, and it could be applied in other environments where self-fulfilling expectations may be important drivers of default risk. For example, one could use changes in the liability structure of financial intermediaries in periods like the Great Depression to assess whether bankruptcies of these institutions were driven by insolvency, or whether they were due to "bank runs" á la Diamond and Dybvig (1983).

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Appendix

A Timing, Rollover Risk, and Crisis Zone

In this section, we carefully define the crisis zone in the model. Recall that the timing within the period is as follows:

- Enter with state $\mathbf{S} = (B, \lambda, s)$;
- Taking as given the pricing schedule, $q(\mathbf{S}, B', \lambda') = \{q_n(\mathbf{S}, B', \lambda')\}_{n=1}^{\infty}$, the government chooses its new portfolio of debt, $(B', \lambda')(\mathbf{S})$ to solve

$$V(\mathbf{S}) = \max_{B', \lambda'} \left\{ U(G(\mathbf{S})) + \beta \mathbb{E} V\left(B', \lambda', s'\right), \underline{V}(s_1) \right\}$$

  where

$$G(\mathbf{S}) = \tau Y(s_1) - B + \sum_{n=1}^{\infty} q_n\left(\mathbf{S}, B', \lambda'\right)\left[(1-\lambda')^{n-1} B' - (1-\lambda)^n B\right];$$

- Lenders choose the price for the government bonds, $q(\mathbf{S}, B', \lambda')$, according to the no-arbitrage conditions (6);
- Finally, the government decides whether to default on its debt or not. The default decision is given by $\delta(\mathbf{S}, B', \lambda', \{q_n\}) \in \{0, 1\}$,[29] with $\delta = 1$ if

$$U\left(\tau Y(s_1) - B + \sum_{n=1}^{\infty} q_n\left[(1-\lambda')^{n-1} B' - (1-\lambda)^n B\right]\right) + \beta \mathbb{E} V\left(B', \lambda', s'\right) \geq \underline{V}(s_1)$$

  and $\delta = 0$ otherwise.

For notational convenience, it is useful to define the price of one unit of an arbitrary portfolio of maturity λ given that the government's portfolio is (B', λ') as

$$Q\left(\mathbf{S}, B', \lambda' | \lambda\right) = \sum_{n=1}^{\infty} (1-\lambda)^{n-1} q_n\left(\mathbf{S}, B', \lambda'\right).$$

We denote by $\mathcal{S}^{max}$ the *largest* region of the state space for which a default is possible.[30]

[29] We allow δ to depend on arbitrary $\{q_n\}$ to have notation to think about off-path situation. It is clear the the problem in (5) is enough to determine default decision along the equilibrium path.

[30] We can think of $\mathcal{S}^{max}$ as the collection of states in which the government defaults if lenders choose the worst possible price from the government's perspective conditional on satisfying the lenders' no-arbitrage condition.

The next lemma characterizes the set $\mathcal{S}^{max}$. To this end, define

$$\Omega(\mathbf{S}) \equiv \max_{B',\lambda'} U\left(\tau Y(s_1) - B + \Delta^{\text{fund}}(\mathbf{S}, B', \lambda')\right) + \beta \mathbb{E} V(B', \lambda', s') \tag{25}$$

subject to $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \leq 0$.

Lemma 1. *Given $V(B, \lambda, s)$ and $Q(\mathbf{S}, B', \lambda')$, $\mathbf{S} \in \mathcal{S}^{max}$ if and only if*

$$\underline{V}(s) > \Omega(\mathbf{S}) \tag{26}$$

Proof of Lemma 1. The necessity part is obvious. If (26) does not hold then the government will never default when the inherited state is $\mathbf{S}$ because it has the option to buy back part of the debt. Note that imposing the fundamental pricing function - the highest possible prices - in (25) is without loss of generality. In fact, since the government is buying-back debt, a decrease in price will only increase the value of Ω.

Consider now the sufficiency part. First note that $\mathbf{S} \in \mathcal{S}^{max}$ if for all (B', λ') such that $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \geq 0$ we have

$$U(\tau Y(s_1) - B) + \beta \mathbb{E} V(B', \lambda', s') < \underline{V}(s_1) \tag{27}$$

and for all (B', λ') such that $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') < 0$ we have

$$U\left(\tau Y(s_1) - B + \Delta^{\text{fund}}(\mathbf{S}, B', \lambda')\right) + \beta \mathbb{E} V(B', \lambda', s') < \underline{V}(s_1) \tag{28}$$

where in (27) we use the fact that when net issuances are positive, $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \geq 0$, the worst price for the government's perspective is zero and in (28) we use the fact that when net issuances are negative, $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') < 0$, the worst price for the government's perspective is the fundamental price. If (27) and (28) are satisfied, it is then rational for lenders to expect a default and it is optimal for the government to default. We can further simplify (27) by noticing that it is sufficient to check such condition only for (B', λ) such that $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') = 0$ because the continuation value $\mathbb{E} V(B', \lambda', s')$ is decreasing in B'. Combining this simplified condition (27) with condition (28) implies that $\mathbf{S} \in \mathcal{S}^{max}$ if (26) holds, proving the claim. *Q.E.D.*

We can then define the crisis zone as $\mathcal{S}^{crisis} = \mathcal{S}^{max} \setminus \mathcal{S}^{fund}$.

We can simplify further the characterization in Lemma 1, by assuming that the programming problem in (25) is concave. If this is the case, whenever the crisis zone is not empty ($\mathcal{S}^{max} \neq \mathcal{S}^{fund}$) it must be that the constraint in (25) binds. Otherwise the government could achieve the fundamental equilibrium. Then $\mathbf{S}$ is in $\mathcal{S}^{max}$ if

$$U\left(\tau Y\left(s_1\right) - B\right) + \beta W\left(\mathbf{S}\right) < \underline{V}(s_1)$$

where

$$W\left(\mathbf{S}\right) \equiv \max_{B',\lambda'} \mathbb{E} V\left(B', \lambda', s'\right) \text{ subject to } \Delta^{\text{fund}}\left(\mathbf{S}, B', \lambda'\right)=0.$$

B Three-Period Model

To illustrate in the most transparent way the key trade-offs that govern the optimal maturity composition of debt we consider a three-period version of the economy. At $t = 0$ the government can issue two types of securities: a zero coupon bond maturing in period 1, b_{01}, and a zero coupon bond maturing in period 2, b_{02}. In period 1, the government issues only a zero coupon bond maturing in period 2, b_{12}. It is convenient to present the model starting from the last period. At $t = 2$, the government does not issue new debt and its only choice is whether to default on the previously issued debt ($\delta_2 = 0$),

$$V_2\left(b_{02} + b_{12}, s_2\right) = \max_{\delta_2} \delta_2 U\left(\tau Y_2 - b_{02} - b_{12}\right) + (1 - \delta_2)\underline{V}_2.$$

At $t = 1$, the government issues b_{12} and it decides whether to default ($\delta_1 = 0$). The decision problem at $t = 1$ is

$$V_1\left(b_{01}, b_{02}, s_1\right) = \max_{\delta_1, G_1, b_{12}} \delta_1 \left\{U\left(G_1\right) + \beta \mathbb{E}_1[V_2\left(b_{02} + b_{12}, Y_2\right)]\right\} + (1 - \delta_1)\underline{V}_1$$

subject to

$$G_1 + b_{01} \leq \tau Y_1 + q_{12}\left(s_1, b_{02} + b_{12}\right) b_{12}$$

Finally at $t = 0$ the government issues both short and long term debt to solve

$$V_0\left(s_0\right) = \max_{G_0, b_{01}, b_{02}} U\left(G_0\right) + \beta \mathbb{E}_0[V_1\left(b_{01}, b_{02}, s_1\right)]$$

subject to

$$G_0 + D_0 \leq \tau Y_0 + q_{01}\left(s_0, b_{01}, b_{02}\right) b_{01} + q_{02}\left(s_0, b_{01}, b_{02}\right) b_{02},$$

with D_0 being the debt inherited from the past. To avoid issues associated with dilution of legacy debt, we assume that the government does not inherit long-term debt. We further assume that D_0 is sufficiently small that the government does not default at $t = 0$. Price schedules q_{01}, q_{02}, and q_{12} must be consistent with lenders no-arbitrage condition

$$\begin{aligned} q_{12}\left(s_1, b_{02}+b_{12}\right) &= \mathbb{E}_1\left[M\left(s_1, s_2\right)\delta_2\left(s_2, b_{02}+b_{01}\right)\right] \\ q_{01}\left(s_0, b_{01}, b_{02}\right) &= \mathbb{E}_0\left[M\left(s_0, s_1\right)\delta_1\left(s_1, b_{01}, b_{02}\right)\right] \\ q_{02}\left(s_0, b_{01}, b_{02}\right) &= \mathbb{E}_0\left[M\left(s_0, s_1\right)\delta_1\left(s_1, b_{01}, b_{02}\right)M\left(s_1, s_2\right)\delta_2\left(s_2, b_{02}+b_{01}\right)\right] \end{aligned}$$

For simplicity we assume that lenders are risk neutral so $M(s_0, s_1) = M(s_1, s_2) = m$.

B.1 All default risk is rollover risk

Proposition 2. *In the three period economy, if there is only rollover risk and fundamental default never happen at $t = 1, 2$ then $b_{01} = 0$ and all debt is long term.*

Proof of Proposition 2. The proof is by contradiction. Suppose $b_{01} > 0$. Consider then the following variation: increase b_{02} by $\varepsilon/q_{02} > 0$, and decrease b_{01} by $\varepsilon/q_{01} > 0$ so that G_0 in unchanged. Moreover, notice that - under the assumption that there is no fundamental default risk - the optimal allocation that can be achieved at $t = 1$ starting from (b_{01}, b_{02}, s) can be achieved with $(b_{01} - \varepsilon/q_{01}, b_{02} + \varepsilon/q_{02})$. In fact, since there are no shocks at $t = 1, 2$ we have that $q_{12} = m$ and at the original allocation the following Euler equation is satisfied:

$$mU'(G_1) = \beta U'(G_2) \tag{29}$$

and so it is clear that achieving same G_1, G_2 is budget feasible and optimal.

We next turn to show that the proposed variation reduces the crisis zone. In fact, at $t = 1$ there can be a rollover crisis only if

$$U\left(\tau Y_1 - b_{01}\right) + \beta EV_2\left(b_{02}, Y_2\right) \leq \underline{V}_1 \tag{30}$$

$$U\left(\tau Y_1 - b_{01} + \varepsilon/q_{01}\right) + \beta EV_2\left(b_{02} + \varepsilon/q_{02}, Y_2\right) \leq \underline{V}_1 \tag{31}$$

The fact that (29) holds at the original allocation implies that if $b_{12} > 0$ then

$$q_{12}U'\left(\tau Y_1 - b_{01}\right) > \beta EV_2'\left(b_{02} + b_{12}\right) \iff \frac{1}{q_{01}}U'\left(\tau Y_1 - b_{01}\right) > \frac{1}{q_{02}}\beta EV_2'\left(b_{02} + b_{12}\right)$$

So we have that

$$
\begin{aligned}
U\left(\tau Y_1 - b_{01} + \varepsilon / q_{01}\right) + \beta E V_2\left(b_{02} + \varepsilon / q_{02}, Y_2\right) &\approx \left[U\left(\tau Y_1 - b_{01}\right) + \beta E V_2\left(b_{02}, Y_2\right)\right] \\
&\quad + \left[\frac{1}{q_{01}} U'\left(\tau Y_1 - b_{01}\right) + \frac{1}{q_{02}} \beta E V_2'\left(b_{02}, Y_2\right)\right] \varepsilon \\
&> U\left(\tau Y_1 - b_{01}\right) + \beta E V_2\left(b_{02}, Y_2\right)
\end{aligned}
$$

Hence if the second inequality is satisfied so it is the first but not vicersa. Hence the variation reduces the probability of default (rollover crisis) at $t = 1$ because (31) is less likely to hold than (30). Then we have that consumption in the first period is larger and so the variation increases utility, a contradiction. *Q.E.D.*

B.2 Dilution problem absent rollover risk

Proposition 3. *In the three period economy, if there is no rollover risk and there are no shocks in $t = 1$ then the optimal solution must have $b_{02} = 0$ if the probability of default in $t = 2$ is positive.*[31]

Proof of Proposition 3. It is helpful to use a primal approach to solve for the equilibrium outcome. Without rollover risk and uncertainty at $t = 0$, we can consider the following programming problem:

$$
\max_{b_{01}, b_{02}, b_{12}, \delta_1, \delta_2} U\left(G_0\right) + \beta \mathbb{E}_0 \left\{\delta_1 \left[U\left(G_1\right) + \beta\left(\delta_2 U\left(G_2\right) + \left(1 - \delta_2\right) \underline{V}_2\right)\right] + \left(1 - \delta_1\right) \underline{V}_1\right\} \tag{32}
$$

subject to budget constraints

$$
\begin{aligned}
G_0 + D_0 &\leq q_{01} b_{01} + q_{02} b_{02} + \tau Y_0 \\
G_1 + b_{01} &\leq q_{12} b_{12} + \tau Y_1 \\
G_2 + b_{02} + b_{12} &\leq \tau Y_2
\end{aligned}
$$

the pricing equations

$$
\begin{aligned}
q_{01} &= m, \quad q_{02} = m q_{12}, \\
q_{12} &= \mathbb{E}_1\left[m_{12} \delta_2\right],
\end{aligned}
$$

[31] A sufficient condition for this is that β / m is sufficiently low or D_0 sufficiently large.

the "default" constraints

$$\begin{aligned} U(G_1) + \beta \mathbb{E}_1 U(G_2) &\geq \underline{V}_1 \\ U(G_2) &\geq \underline{V}_2 \end{aligned}$$

and the "debt-dilution" constraint

$$U(G_1) + \beta \mathbb{E}_1 U(G_2) \geq V_1(b_{01}, b_{02}) \tag{33}$$

It is clear that a fundamental equilibrium outcome solves the above problem and the converse is also true.

We now show that short term debt is desirable because it relaxes the debt-dilution constraint (33). To this end, consider a relaxed version of (32) in which we drop the debt-dilution constraint (33). Notice that such relaxed problem has a continuum of solutions since the split between long and short term debt issued in period zero is undetermined. Let $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$ be a solution to this relaxed programming problem. The optimality condition for b_{12} for this relaxed problem is

$$\begin{aligned} 0 \;=\; & m \frac{\partial q_{12}}{\partial b_2} b_{02}^* U'(G_0^*) + \\ & \left(q_{12}^* + \frac{\partial q_{12}}{\partial b_2} b_{12}^* \right) U'(\tau Y_1 - b_{01}^* + q_{12}^* b_{12}^*) - \int_{\mathcal{Y}_2(b_{02}^*, b_{12}^*)} U'(\tau Y_2 - b_{02}^* - b_{12}^*) \, d\mu_{Y2} \end{aligned} \tag{34}$$

where $\mathcal{Y}_2(b_{02}, b_{12}) \equiv \{Y_2 : U(\tau Y_2 - (b_{02} + b_{12})) \geq \underline{V}_2\}$. This implies that if $b_{02}^* = 0$ then the government at $t = 0$ can achieve the value of this relaxed problem in the more constrained problem (32). To see this it is sufficient to check that the dilution constraint is met at $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$. To this end notice that starting at (b_{01}^*, b_{02}^*) in period $t = 1$ the optimal b_{12} chosen by period 1 government is such that

$$0 = \left(q_{12}^* + \frac{\partial q_{12}}{\partial b_2} b_{12} \right) U'(\tau Y_1 - b_{01}^* + q_{12} b_{12}) - \int_{\mathcal{Y}_2(b_{02}^*, b_{12})} U'(\tau Y_2 - b_{02}^* - b_{12}) \, d\mu_{Y2} \tag{35}$$

where $q_{12} = m \Pr(U(\tau Y_2 - (b_{02}^* + b_{12}^*)) \geq \underline{V}_2)$. Hence the allocation $\{b_{01}^*, b_{02}^*, b_{12}^*, \delta_1^*, \delta_2^*\}$ satisfies the debt-dilution constraint if and only if it satisfies (35) with $b_{12} = b_{12}^*$ and $q_{12} = q_{12}^*$. Now, from (34) and $b_{02}^* = 0$ it follows that (35) is satisfied. Hence the solution to the relaxed problem can be implemented when $b_{02}^* = 0$.

The final step in the proof is to show that $b_{02}^* = 0$ is necessary when the solution to (32) is such that there are defaults in $t = 2$ in some states. Note that (34) and (35) can be jointly satisfied if and only if $\frac{\partial q_{12}}{\partial b_2} b_{02}^* = 0$ so at least one of the following conditions must be satisfied: i) $b_{02}^* = 0$, ii) no default at $t = 2$ so that $\partial q_{12}/\partial b_2 = 0$. Hence if there are defaults in $t = 2$ then it must be that $b_{02}^* = 0$. *Q.E.D.*

In presence of rollover risk, the government has an incentive to reduce the probability of being in the crisis zone next period. As we argued in the text, the government has two lever to do this: reduce its indebtedness and lengthen the maturity of its debt. As this proposition show, in the absence of fundamental default risk, the government will first lengthen its maturity to the maximal extent before it starts to reduce the debt it issues. Lengthening the maturity of debt is, in fact, costless while deleveraging has a cost. By continuity, we expect the government to use the maturity lever more than the issuance lever if his ultimate objective is to reduce the likelihood of being in the crisis zone next period. This provides a justification to our identification strategy because maturity choices will be more responsive to fluctuations in rollover risk relative to debt issuance.

C Data Appendix

Term structure of German interest rates: Information on the term structure of ZCBs for German federal government securities in obtained from the Bundesbank online database. We collect monthly data on the parameters of the Nelson and Siegel (1987) and Svensson (1994) model, and we generate nominal bond yields for all maturities between $n = 1$ and $n = 20$ quarters. We convert these monthly series at a quarterly frequency using simple averages. These series are available for the period 1973:Q1-2013:Q4.

Italian real gross domestic product: OECD Quarterly National Accounts, GDP expenditure approach, US dollars, volume estimates (reference year 2010), 1960:Q1-2014:Q4.

Debt to output ratio: OECD Quarterly Public Sector Debt, Total gross debt of the central government, percentage of annualized GDP (current prices), 2000:Q1-2014:Q4.

Interest rate spreads: Yields differentials between an Italian and a German ZCB with a residual maturity of five years. Nominal yields on Italian bonds are obtained through Bloomberg [details]. Nominal yields on the corresponding German bonds is obtained from the Bundesbank online database.

Duration of outstanding Italian government bonds: We use detailed information on outstanding bonds issued by the Italian central government to construct an indicator of debt duration for the 2008:Q1-2014:Q4 period. We collect outstanding principal and coupon payments to bondholders for every maturity n in every quarter t $(C_{n,t})$ using data from the Italian treasury at http://www.dt.tesoro.it/en/debito_pubblico/dati_statistici/scadenze_titoli_suddivise_anno/index.html. The indicator of debt duration that we consider is

$$\text{duration}_t = \sum_{n=1}^{N} n \frac{C_{n,t}}{V_t},$$

where $V_t = \sum_{n=1}^{N} C_{n,t}$ is the ZCB-equivalent outstanding face value of the bonds issued. This indicator, the weighted average of the times of principal and coupon repayments, maps exactly to $\frac{1}{\lambda'}$ in our model.

D Rollover Risk and Public Debt Management

Our identification builds on the hypothesis that Treasury departments would respond to heightened rollover risk by actively lengthening the maturity of their debt. While the literature on this topic is limited, previous cross-country studies have shown that the maturity of new issuances in emerging markets typically shortens around default crises (Broner, Lorenzoni, and Schmukler, 2013; Arellano and Ramanarayanan, 2012), and examples of governments extending the life of their debt in turbulent times are not well documented. This section details some of these examples. Using a narrative approach, we analyze two historical episodes where governments responded to heightened rollover risk by lengthening the maturity of public debt.

D.1 Italy in the early 1980s

Two main factors at the beginning of the 1980s contributed to place the Italian government at risk of a roll-over crisis. First, the average residual maturity of government debt was at its historical low, going from a peak value of 9.2 years in 1972 to 1.1 years in 1980.[32] At that time, the Italian government needed to refinance the entire stock of debt, roughly 60% of GDP, within the span of a year. Second, and in an effort to increase the independence of the central bank, a major institutional reform freed the Bank of Italy from the obligation of buying unsold public debt in auctions.[33]

[32] This decline was due to the chronic inflation of the 1970s which discouraged investors from holding long duration bonds that were unprotected from inflation risk, see Pagano (1988).

[33] Starting from 1975, the Bank of Italy was required to act as a residual buyer of all the public debt that was unsold in the auctions. This resulted in a massive increase in the share of public debt held by the Bank

The short duration of government debt coupled with the loss of central bank financing meant that the Italian government had to use primary markets to refinance its maturing debt. However, these markets were not well developed at the time, and private demand of government bonds was weak and volatile (Campanaro and Vittas, 2004). Panel (a) of Figure A-1 reports statistics regarding the placement of Italian treasuries during the 1981-1982 period. The solid line plots the ratio between the demand of bonds by private operators and the target set by the Treasury at auctions of public debt. This ratio averaged only 0.65 over this period, with a standard deviation of 0.25. The dashed line reports the ratio between the quantity sold and the Treasury's target. Until July 1981, this ratio was equal to 1 because of the statutory requirement for the Central Bank to buy unsold bonds. Following the reform of the Central Bank, though, the Treasury became exposed to variation in the private demand of bonds.

The possibility of a debt crisis became evident in the last quarter of 1982. On the auction of October 15th, private demand covered only 46% of the Treasury's needs, and the Central Bank did not to purchase the unsold bonds. The Treasury was thus forced to use the overdraft account it had with the Bank of Italy to cover its financing needs, reaching the statutory limit.[34] This led to a budgetary crisis, which further depressed private demand out of fears of a debt restructuring.[35] While the Parliament later voted a law that allowed a temporal overshoot of the overdraft account (Scarpelli, 2001), these events exposed to policymakers the risks implicit in refinancing large amounts of debt in short periods of time.

The response of the Italian government to these events is consistent with our identification strategy. As documented in Alesina, Prati, and Tabellini (1989) and in Scarpelli (2001), the Treasury actively pursued a policy to extend the life of its public debt. Financial innovation was the main tool used for this purpose, with the introduction of new types of bonds whose interest payments were indexed to the prevailing nominal rate. These *Certificati di Credito del Tesoro* (CCT) were palatable to investors because they offered protection from inflation risk, and at the same time they had longer maturity than the *Buoni Ordinari del Tesoro* (BOT), the prevailing form of bond financing at the time.[36] Panel (b) of Figure

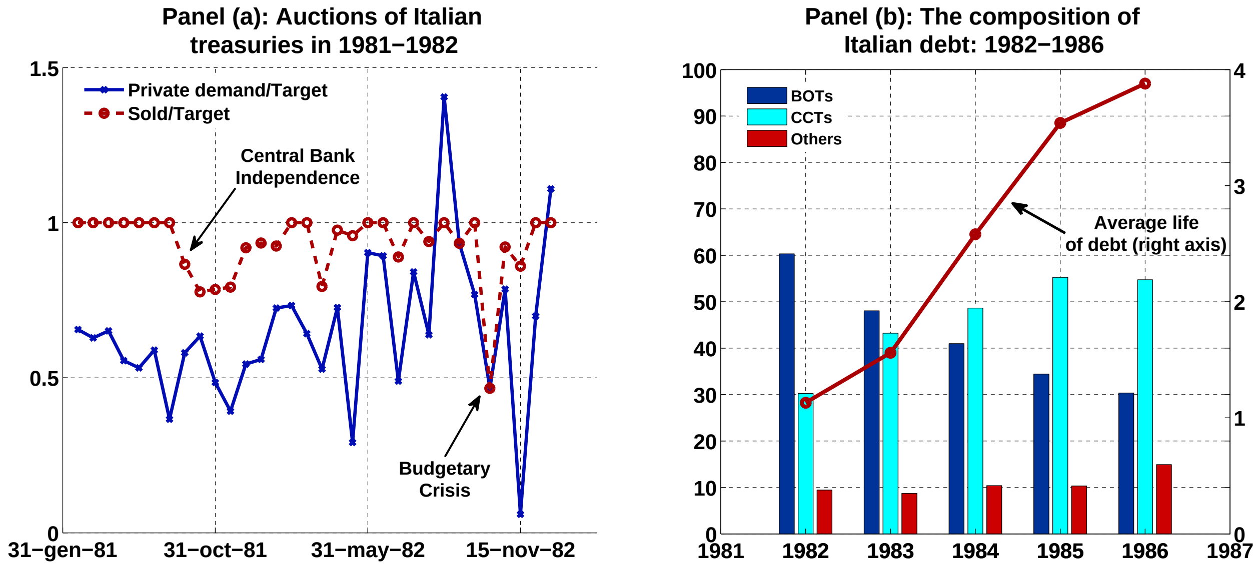
of Italy, reaching a maximum of 40% in 1976. See Tabellini (1988) for a discussion of the historical context underlying the "divorce" between the Bank of Italy and the Italian Treasury.

[34]This account allowed the Italian Treasury to directly borrow from the Bank of Italy up to a limit of 14% of the expenditures budgeted for the current year.

[35]These fears were not without motivations. Rino Formica, ministry of Finance at the time, publicly called for an "agreement" that would allow the Treasury to reimburse only part of its debt. Beniamino Andreatta, ministry of the Treasury, strongly opposed this view. This controversy, known in the public debate as "lite delle comari", eventually led to the fall of the Italian government on December 1st 1982.

[36]Indexed securities like CCT are not subject to refinancing and rollover problem but are essentially equal to short term debt for the incentive to generate ex-post inflation because any effort to generate ex-post

Figure A-1: **Rollover risk and public debt management: Italy in the early 1980s**



Notes: The statistics in Panel (a) are constructed using data from Bank of Italy, Supplements to the Statistical Bulletin-Financial Markets. The statistics in Panel (b) are constructed using data from the Italian Treasury. The bar indicates the percentage of a particular class of bonds over total outstanding debt. The line is the average life of outstanding debt (reported in years on the right axis.

A-1 reports the composition of the outstanding Italian debt (bars) along with its residual average life during the 1982-1986 period. We can see that the Treasury quickly replaced BOTs with CCTs as the main source of public financing. The efforts of the Treasury were successful in increasing the duration of outstanding debt, with its residual average life more than tripling within the span of four years.

inflation will not reduce the real value of debt. See Missale and Blanchard (1994).

E The Lenders' Stochastic Discount Factor

We now derive some results concerning the lenders' stochastic discount factor, and describe in more details the calibration of θ_1. Let $q_t^{*,n}$ be the log price of a non-defaultable ZCB maturing in n periods. These bond prices satisfy the recursion

$$\exp\{q_t^{*,n}\} = \mathbb{E}_t[M_{t,t+1} \exp\{q_{t+1}^{*,n-1}\}],$$

where $M_{t,t+1}$ is defined in the system (13), and the initial condition is $q_t^0 = 0$. Ang and Piazzesi (2003) show that $\{q_t^{*,n}\}$ are linear functions of the state variable χ_t,

$$q_t^{*,n} = A_n + B_n \chi_t,$$

where A_n and B_n satisfy the recursion

$$\begin{aligned} B_{n+1} &= -\tau_1 + B_n \phi^*, \\ A_{n+1} &= -\tau_0 + A_n + B_n \mu^* + \frac{1}{2} B_n^2 \sigma_\chi^2, \end{aligned} \tag{36}$$

with $A_0 = B_0 = 0$, $\phi^* = [\phi - \sigma_\chi^2 \kappa_1]$ and $\mu^* = [\mu(1-\phi) - \sigma_\chi^2 \kappa_0]$. In order to avoid the divergence of B_n for large n, we restrict θ_1 to satisfy $|\phi^*| < 1$.

E.1 Results from the pricing model

E.1.1 The risk free rate

By definition, log-yields on a bond maturing next quarter equal $y_t^1 = -q_t^{*,1}$. In the model, those are equal to

$$y_t^1 = \tau_0 + \tau_1 \chi_t. \tag{37}$$

The mean and variance of y_t^1 can then be easily derived as a function of deep model parameters

$$\mathbb{E}[y_t^1] = \tau_0 + \tau_1 \mu \qquad \mathrm{var}[y_t^1] = \tau_1^2 \frac{\sigma_\chi^2}{(1-\phi^2)}. \tag{38}$$

E.1.2 Expected excess returns

By definition, holding period excess log returns on a ZCB maturing in n periods equal $rx^n_{t+1} = q^{*,n-1}_{t+1} - q^{*,n}_t + q^{*,1}_t$. Substituting the expression for log prices, we can rewrite it as

$$rx^n_{t+1} = \underbrace{[A_{n-1} + B_{n-1}\mu(1-\phi) - A_n + A_1]}_{\tilde{A}_n} + \underbrace{[B_{n-1}\phi - B_n + B_1]}_{\tilde{B}_n}\chi_t + B_{n-1}\varepsilon_{\chi,t+1}, \tag{39}$$

where A_j and B_j are defined in (36). Taking conditional expectations on both sides, we obtain

$$\mathbb{E}_t[rx^n_{t+1}] = \tilde{A}_n + \tilde{B}_n\chi_t, \tag{40}$$

which verifies that expected excess log returns are linear functions of χ_t.

E.2 Calibration of θ_1

We use the data on the term structure of German's interest rates to construct time series for realized excess log returns and the log forward rates for $n = 4, 8, 12, 16, 20$. Table A-1 reports summary statistics on yields and realized excess log returns as a function of n.

Table A-1: **Summary statistics: yields and holding period returns**

	Mean	Standard deviation	Sharpe Ratio
$y^1_t - \text{infl}_t$	2.16	1.93	
$y^{20}_t - \text{infl}_t$	2.94	1.72	
rx^4_{t+1}	0.21	2.05	0.11
rx^8_{t+1}	0.94	4.22	0.22
rx^{12}_{t+1}	1.54	6.08	0.25
rx^{16}_{t+1}	2.02	7.70	0.26
rx^{20}_{t+1}	2.40	9.14	0.26

Notes: The sample period is 1973:Q1-2013:Q4. The inflation rate is the year-on-year percentage change in the German CPI index, obtained from OECD Main Economic Indicators*. Variables are reported as annualized percentages.*

We can verify that the yield curve slopes up on average: yields on 5 years bonds are, on average, 80 basis points higher than yields on bonds maturing next quarter. We can also see that long term bonds earn a positive excess return on average. For example, holding a 5 year bond and selling it off next quarter earns, on average, an annualized premium of 2.40% relative to investing the same amount of money in a bond that matures next quarter. Excess returns on long term bonds increase monotonically with n, and so does their Sharpe ratio.

Table A-2 reports the results of the C-P regressions. The top panel reports OLS estimates of equation (16), where $\overline{rx}_{t+1}$ are realized excess log returns averaged across $n = 4, 8, 12, 16, 20$ and the vector $\mathbf{f}_t$ includes the risk free rate and the log forward rates for these five maturities. The bottom panel reports the individual bond regressions of equation (17).

Table A-2: **C-P regressions**

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	R^2
Estimates of equation (16)	-0.002	-1.65	5.00	-21.70	47.20	-45.18	16.53	0.12
	(-0.27)	(-2.89)	(2.92)	(-2.10)	(1.58)	(-1.19)	(0.95)	
		a_n		b_n		R^2		
Estimates of equation (17)	4	-0.001		0.46		0.20		
		(-2.06)		(5.48)				
	8	-0.000		0.77		0.13		
		(-0.37)		(4.92)				
	12	0.000		1.02		0.11		
		(0.14)		(4.60)				
	16	0.001		1.27		0.11		
		(0.30)		(4.55)				
	20	0.001		1.48		0.11		
		(0.34)		(4.56)				

Notes: The sample period is 1973:Q1-2013:Q4. Robust $t-$statistics in parenthesis.

Differently from the analysis of Cochrane and Piazzesi (2005) on U.S. data, the estimated vector $\hat{\gamma}$ is not "tent" shaped. However, we confirm using German data the finding that a single linear combination of log forward rates has predictive power for excess log returns, and that the sensitivity of the latter to this factor (the estimated b_n's) increases with the maturity of the bonds.

The parameters in θ_1 are chosen as follows. First, we set τ_0 and τ_1 so that the model implied mean and standard deviation of y_t^1, defined in the equations (38), match the sample statistics reported in Table A-1.[37] The remaining parameters are obtained using

[37] This sets τ_0 and τ_1 implicitly as functions of μ, ϕ and σ_χ.

the method of simulated moments. The set of empirical targets contains two distinct sets of moments. The first set includes the results of the second stage regression reported in Table A-2 for an $n = 20$ bond, specifically the point estimates for a_{20}, b_{20}, and the standard deviation of the OLS residuals. In the second set of moments we include the parameters of an AR(1) model estimated on the first stage factor, $\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$. We choose the remaining parameters in θ_1 in order to minimize a weighted distance between these sets of moments and the corresponding statistics computed in model simulated data. The weighting matrix is diagonal, with the inverse of each sample moment (in absolute value) on the main diagonal. The model implied statistics are computed on a long simulation ($T = 5000$) from the model. In simulations, we add small measurement errors to the forward rates in order to avoid multicollinearity when estimating the Cochrane and Piazzesi (2005) first stage regression.

F Numerical Solution

Let $\mathbf{S} = [B, \lambda, y, \chi, \pi]$ be the vector collecting the model's state variables. Before explaining the numerical solution, it is convenient to simplify the objects in the recursive equilibrium. In the model set up, we argued that since bond prices depend on the current default decision, they must depend on the inherited portfolio of debt, (B, λ). Since we are only interested in characterizing the equilibrium outcome along the equilibrium path, it is convenient to restrict attention to what we refer to the *fundamental pricing schedule*, $q_n^{\text{fund}}(s, B', \lambda')$ defined in (7). Note that such schedule solves the following functional equation:

$$q_n^{\text{fund}}(s, B', \lambda') = \mathbb{E}\left\{M(s_1, s_1')\,\delta(\mathbf{S}')\,q_{n-1}^{\text{fund}}(B'', \lambda''; s')\,|\mathbf{S}\right\} \text{ for } n \geq 1$$

where we again adopt the convention that $q_0^{\text{fund}} = 1$.

Moreover, to save on notation, we let $q(s, B', \lambda'|\lambda)$ be the fundamental value of a portfolio of ZCBs with decay parameter λ given the realization s for the exogenous state, and given the government's choices for the new portfolio is $[B', \lambda']$.[38] In this reformulation of the problem, B is the face value of outstanding debt. The price of this portfolio of ZCBs can be written as

$$q(s, B', \lambda'|\lambda) = \mathbb{E}\left\{M(s_1, s_1')\,\delta(\mathbf{S}')\left[\lambda + (1-\lambda)q(s, B'', \lambda''|\lambda)\right]|\mathbf{S}\right\}, \tag{41}$$

where $B'' = B'(s', B', \lambda')$ and $\lambda'' = \lambda'(s', B', \lambda')$.

With this notation, we can then rewrite the decision problem for the government using three simple sub-problems. We define the value of repaying the debt conditional on lenders rolling over the debt, $V_{\text{roll}}^R(\mathbf{S})$, as follows

$$V_{\text{roll}}^R(\mathbf{S}) = \max_{B', \lambda'}\left\{U(\tau Y - \lambda B + \Delta) + \beta\mathbb{E}[V(B', \lambda', s')|\mathbf{S}]\right\}, \tag{42}$$

where

$$\Delta = q(s, B', \lambda'|\lambda')\,B' - q(s, B', \lambda'|\lambda)\,(1-\lambda)B.$$

The value of repaying conditional on lenders not rolling over the debt, $V_{\text{no roll}}^R(\mathbf{S})$, is

$$V_{\text{no roll}}^R(\mathbf{S}) = \left\{U(\tau Y - \lambda B) + \beta\mathbb{E}[V(B(1-\lambda), \lambda, s')|\mathbf{S}]\right\}, \tag{43}$$

[38]Note that we need to price an arbitrary λ portfolio, given government choices $[B', \lambda']$, in order to know the market value of the portfolio repurchased by the government. See Sanchez, Sapriza, and Yurdagul (2015) for a discussion of this issue.

while the value of defaulting, $V^D(y,\chi)$, is

$$V^D(y,\chi) = \left\{U(\tau \exp\{y\}[1-d(y)]) + \beta\{\psi \mathbb{E}[V(0,\overline{\lambda},y',\chi',\pi')|\mathbf{S}] + (1-\psi)\mathbb{E}[V^D(y',\chi')|\mathbf{S}]\}\right\}. \tag{44}$$

Note that $V^D(.)$ does not depend on π because this process is assumed to be iid. The value function of the government can then be written as

$$V(\xi,\mathbf{S}) = \begin{cases} V^R_{\text{roll}}(\mathbf{S}) & \text{if } V^R_{\text{no roll}}(\mathbf{S}) \geq V^D(y,\chi) \\ V^R_{\text{roll}}(\mathbf{S}) & \text{if } V^R_{\text{no roll}}(\mathbf{S}) < V^D(y,\chi) \text{ and } \xi = 0 \\ V^D(y,\chi) & \text{if } V^R_{\text{no roll}}(\mathbf{S}) < V^D(y,\chi) \text{ and } \xi = 1 \end{cases}$$

This value function, its associated policy functions and the fundamental pricing function are enough to determine the equilibrium outcome path. This is because the equilibrium price of a portfolio of bond on path is either zero - in the case of a fundamental default or a rollover crisis - or it is equal to the fundamental value defined in (41) if there is repayment in the current period.

The numerical solution of the model consists in approximating the fundamental pricing schedule q, and the value functions $\{V^R_{\text{roll}}(\mathbf{S}), V^R_{\text{no roll}}(\mathbf{S}), V^D(y,\chi)\}$.

The inverse duration for the debt portfolio, λ, is assumed to be a discrete variable from the set $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_N\}$. The value functions are approximated using piece-wise smooth functions. Specifically, $V^R_{\text{roll}}(.)$, is approximated as follows,

$$V^R_{\text{roll}}(\lambda_j, \tilde{\mathbf{S}}) = {\gamma^R_{\text{roll},\lambda_j}}' \mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [B,y,\chi,\pi]$ is a realization of state variables that excludes λ, $\gamma^R_{\text{roll},\lambda_j}$ is a vector of coefficients and $\mathbf{T}(.)$ is a vector collecting Chebyshev's polynomials. The value of repaying conditional on the lenders not rolling over the debt, and the value of defaulting are defined in a similar fashion, and we denote by $\gamma^R_{\text{no roll},\lambda_j}$ and γ^D the coefficients that parametrize those values. The pricing schedule q is approximated on a grid.

We index the numerical solution by $[\Gamma, q]$, with $\Gamma = \{[\gamma^R_{\text{roll},\lambda_j}, \gamma^R_{\text{no roll},\lambda_j}]_j, \gamma^D\}$ collecting the coefficients parametrizing the value functions. The numerical solution is obtained via value function iteration. Specifically, the algorithm is as follows:

Step 0: Defining the state space and the polynomials. Specify the set of values in Λ. Set upper and lower bounds for the state variables $\tilde{\mathbf{S}} = [B,y,\chi,\pi]$. Given these bounds, construct a Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(.)$ following Judd, Maliar, Maliar, and Valero (2014). Let $\tilde{\mathcal{S}}$ denote the set of points for the state variables $\tilde{\mathbf{S}}$ in this grid. These grid points and the Chebyshev's polynomials are used in the approximation of the value functions. For each exogenous state variable, specify points within the upper and lower bounds defined above, and generate a second grid $\mathcal{S}^{\text{exog}}$ using tensor multiplications.

Step 1: Update value functions. Start with a guess for the value and pricing functions, (Γ^c, q^c). For each $\mathbf{S}^i \in \Lambda \times \tilde{\mathcal{S}}$, update the value functions using the definitions in equation (42)-(44). Denote by Γ^u the updated guess, and by $[r^R_{roll}, r^R_{noroll}, r^D]$ the distance between the initial guess and its update using the sup-norm.

Step 2: Update pricing function. For each exogenous state s^i in $\mathcal{S}^{\text{exo}}$, and for each $(B^{i'}, \lambda^{i'}, \lambda^i) \in \mathcal{B} \times \Lambda \times \Lambda$, evaluate the right hand side of equation (41) using (Γ^u, q^c). Denote by $\hat{q}^u(B^{i'}, \lambda^{i'}; s^i|\lambda^i)$ this value, and by r^Q the distance between q^c and $\hat{q}^u$ under the sup norm. Update the pricing schedule as

$$q^u(.) = \theta \hat{q}^u(.) + (1-\theta) q^c(.) \qquad \theta \in (0,1).$$

Step 3: Iteration. If $\max\{r^R_{roll}, r^R_{noroll}, r^D\} \leq 10^{-5}$ and $r^Q \leq 10^{-3}$, stop the algorithm. If not, set (Γ^u, q^u) as the new guess, and repeat Step 1-2. □

Regarding the specifics of the algorithm, we generate $\tilde{\mathcal{S}}$ using an anisotropic Smolyak grid of $\mu = 6$ in the B dimension and $\mu = 3$ on the other dimensions. The upper and lower bound for B are $[0, 10]$, while the upper and lower bounds for $s = (y, \chi, \pi)$ are equal to +/- 3 times the standard deviation of these stochastic processes. For the construction of $\mathcal{S}^{\text{exog}}$, we consider 9 equally spaced points within the previously defined bounds for y, χ and π. The grid for λ contains 15 equally spaced values within the interval []. This interval implies a range of $+/-$ two years around an average observed duration of 6.75 years, the Italian pre-crisis level. The grid for debt choices over which the pricing function is defined, $\mathcal{B}$, consists of 1000 equally spaced points between $[0, 10]$.

When iterating over the value and pricing functions, we compute expectations over future outcomes using Gauss-Hermite quadrature, with $n = 5$ sample points on each random variable. The smoothing parameter for the updating of the pricing schedule is set at $\theta = 0.05$.

Things to say: i) How we simulate the model; ii) Some indications of accuracy; iii) The adjustment cost business should go in the main text.

In the numerical solution, we introduce a small cost for adjusting debt maturity,

$$\alpha \left(\frac{4}{\lambda'} - \overline{d} \right)^2 .$$

We set $\overline{d} = 7$, and $\alpha = 0.001$. We introduce this adjustment cost for two purposes. First, it ameliorates the convergence properties of the algorithm as it breaks down indifference in region of the state space where default risk and risk premia on long term bonds are small.[39] Second, we make sure that in this region of the state space the maturity choice is consistent with the pre-crisis level of the weighted average life of Italian outstanding debt.

[39]Maturity choices in the model are not determined absent default risk and with risk neutral lenders.

G Details of the Counterfactual Experiment

This section details the counterfactual experiment of Section 6.1. First, we explain how we use the particle filter to extract information on the state vector given data on detrended real GDP, the estimated χ_t series, interest rate spreads and the data counterpart of λ'. Second, we discuss how the retrieved state vector is used to generate the main counterfactual of the paper.

G.1 Particle Filtering

The model has five state variables $\mathbf{S}_t = [B_t, \lambda_t, y_t, \chi_t, \pi_t]$. The vector $\mathbf{Y}_t$ collects the observable for quarter t. The state-space representation is

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t). \end{aligned}$$

The first equation is the measurement equation, with η_t being a vector of Gaussian measurement errors with variance-covariance matrix equal to Σ. The second equation is the transition equation, describing the law of motion for the model's state variables. The vector ε_t collects the innovations to the structural shocks y_t, χ_t and π_t. The functions $g(.)$ and $f(.)$ are generated using the numerical procedure described in Appendix F.

Let $\mathbf{Y}^t = [\mathbf{Y}_1, \ldots, \mathbf{Y}_t]$, and denote by $p(\mathbf{S}_t|\mathbf{Y}^t)$ the conditional distribution of the state vector given observations up to period t. Although the conditional density of $\mathbf{Y}_t$ given $\mathbf{S}_t$ is known and Gaussian, there is no analytical expression for the density $p(\mathbf{S}_t|\mathbf{Y}^t)$. We use the particle filter to approximate this density for each t. The approximation is done via a set of pairs $\{\mathbf{S}_t^i, \tilde{w}_t^i\}_{i=1}^N$, in the sense that

$$\frac{1}{N}\sum_{i=1}^{N} f(\mathbf{S}_t^i)\tilde{w}_t^i \overset{\text{a.s.}}{\rightarrow} \mathbb{E}[f(\mathbf{S}_t)|\mathbf{Y}^t],$$

and it is used the (mean) trajectory of the state vector over the sample. We refer to $\mathbf{S}_t^i$ as a particle and to $\tilde{w}_t^i$ as its weight. The algorithm used to approximate $\{p(\mathbf{S}_t|\mathbf{Y}^t)\}_t$ builds on **?**, and it goes as follows

Step 0: Initialization. Set $t = 1$. Initialize $\{\mathbf{S}_0^i, \tilde{w}_0^i\}_{i=1}^N$ from the ergodic distribution of the model and set $\tilde{w}_0^i = 1 \; \forall i$.

Step 1: Prediction. For each $i = 1, \ldots, N$, simulate a particle $\mathbf{S}^i_{t|t-1}$ given $\mathbf{S}^i_{t-1}$ following the procedure described in Appendix F.

Step 2: Filtering. Assign to each particle $\mathbf{S}^i_{t|t-1}$ the particle weight

$$w^i_t = p(\mathbf{Y}_t | \mathbf{S}^i_{t|t-1}; \Sigma)\tilde{w}^i_{t-1}.$$

Step 3: Resampling. Rescale the particles $\{w^i_t\}$ so that they add up to unity, and denote these rescaled values by $\{\tilde{w}^i_t\}$. Sample N values for the state vector with replacement from $\{\mathbf{S}^i_{t|t-1}, \tilde{w}^i_t\}^N_{i=1}$, and denote these draws by $\{\mathbf{S}^i_t\}_i$. Set $\tilde{w}^i_t = 1\ \forall i$. If $t < T$, set $t = t + 1$ and go to Step 1. If not, stop. □

Regarding the tuning of the filter, we set $N = 10000$. The matrix $\mathbf{\Sigma}$ is diagonal. We set the diagonal elements as follows. We compute the sample variance of the observables for the 1999:Q1-2012:Q2 period. For detrended real GDP and the $\{\hat{\chi}_t\}$ we set the measurement errors equal to 10% of their sample variance. For interest rate spreads and the debt duration series, the variance of the measurement errors is set to 5% of their sample variance. We choose smaller measurement errors for these latter series because of two reasons. First, the empirical counterparts to y_t and χ_t are noisy because they have been estimated. Second, our identification strategy suggests that interest rate spreads and debt duration will be particularly informative about $\{\pi_t\}$, the underlying unobservable we wish to estimate.

Here we should add i) Effective particles, ii) all the state variables (point estimate and errors bands).

G.2 Counterfactual Experiment

We now discuss how we use the approximation to $\{p(\mathbf{S}_t|\mathbf{Y}^t)\}^{2012:Q2}_{t=2008:Q1}$ along with the structural model to generate the decomposition presented in Figure x.

Let $s^{\text{data}}_t = y^{20,\text{ita}}_t - y^{20,\text{ger}}_t$ be the interest rate spread at time t, and let $\hat{s}^{\text{model}}_t$ be

$$\hat{s}^{\text{model}}_t = \sum_{i=1}^{N} g_{\text{spread}}(\mathbf{S}^i_t)\tilde{w}^i_t,$$

where $g_{\text{spread}}(.)$ is the implicit relation between interest rate spreads and the state variables in the model. The measurement error component in Figure x is defined as $s^{\text{data}}_t - \hat{s}^{\text{model}}_t$.

In order to construct the fundamental and the non-fundamental components, we generate a counterfactual spread $\hat{s}^{\text{fund}}_t$ by simulation. We proceed as follows. Let $t = 1$. Set

$\pi_1^i = 0$ for every i, and let $\mathbf{S}_1^{i,\text{fund}}$ be the state vector with $\pi_1^i = 0$. For each i, feed the model with $\mathbf{S}_1^{i,\text{fund}}$, and define $s_1^{i,\text{fund}}$ to be the model implied counterfactual. We define $\mathbf{S}_2^{i,\text{fund}}$ be the updated state vector, where χ_2^i and y_2^i are consistent with their values in $\{\mathbf{S}_2^i, \tilde{w}_2^i\}$, $\pi_2^i = 0$, and the endogenous state variables are the one implied by the model's law of motion $f(.)$. We then repeat this procedure for each $t = 2, \ldots, T$.

Given $\{s_t^{i,\text{fund}}\}_{i \in N, t \in T}$, we next construct, for each t

$$\hat{s}_t^{\text{fund}} = \frac{1}{N} \sum_{i=1}^{N} s_t^{i,\text{fund}} \tilde{w}_t^i \approx \mathbb{E}[s_t^{\text{fund}} | \mathbf{Y}^t].$$

This is the (average) interest rate spread implied by the model under the assumption that $\{\pi_t\}$ was identically zero over the 2008:Q1-2012:Q2 period, and it is the fundamental component of interest rate spreads in Figure x. The non-fundamental component of the spreads is then defined as $\hat{s}_t^{\text{model}} - \hat{s}_t^{\text{fund}}$ for each t.