ASSET-PRICE CHANNELS AND MACROECONOMIC FLUCTUATIONS

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Abstract. We estimate a dynamic general equilibrium model that features heterogenous agents, incomplete risk sharing, and collateralized debts. We obtain three empirical findings. First, the average rate of return on each asset in excess of the average loan rate, called the "excess return", is quantitatively important on first order. Second, the collateral constraint, through the asset-price channel, plays a critical role in amplifying and propagating aggregate fluctuations. Third, shocks to the household's patience factor and housing demand are important in generating hump-shaped impulse responses through distributional effects of net worths between the household and the entrepreneur.

I. Introduction

This paper presents an empirical study of asset-price channels through which the borrowing constraint amplifies aggregate fluctuations. These channels are characterized by the excess returns of assets over the loan rate. We evaluate the empirical importance of the first-order excess returns. We show how the borrowing constraint and excess returns propagate various shocks through endogenous responses of asset prices.

Following Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Iacoviello (2005), and Christiano, Motto, and Rostagno (2008), we study an economy that consists of two types of agents: households and entrepreneurs. The representative household consumes a homogeneous good, land services (housing), and leisure and supplies labor and loanable funds in competitive markets. The representative entrepreneur consumes and produces the homogeneous good. Production of the good requires labor, capital, and land (commercial structures). To finance consumption, production, and

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capital accumulation, the entrepreneur borrows loanable funds from households subject to a collateral constraint. The borrowing limit is determined by the present value of commercial structures and the accumulated capital stock. Land and capital are used both as collateral and as production inputs.[1]

Our key findings are as follows. First, our model generates a steady-state excess return on assets over the loan rate because the household is more patient than the entrepreneur so that the credit constraint is binding; with the binding constraint, borrowers assign a positive value to the existing loans, giving rise to the excess return. In this sense, the steady-state excess return measures the tightness of the collateral constraint. With our estimated magnitude of the excess return (about 5% per annum), we find important interactions between the asset prices and the borrowing limit and such interactions generate a multiplier effect that amplify and propagate several important business cycle shocks. Since the excess return arises from the household's patience factor, we provide a micro foundation for the patience factor. We show that our benchmark model is equivalent to one with heterogeneous households who face idiosyncratic income shocks that are persistent and uninsurable and thus have a precautionary saving incentive.

Second, our estimate indicates that the borrowing capacity is constrained by one third of the value of the entrepreneur's assets. Such a borrowing limit plays a critical role in two aspects. On one hand, a *permanent* increase in the borrowing limit raises the steady-state level of output, investment, and asset prices; it generates distributional effects that raises the household's consumption relative to the entrepreneur's. On the other hand, it amplifies the dynamic responses of aggregate consumption, investment, and output to various shocks. Raising the borrowing limit increases the entrepreneur's debt burden and thus the leverage ratio in the long run, which in turn amplifies the short-run dynamic responses of housing prices and excess returns. These dynamic interactions between collateral limits and asset prices are one of the key features of our model that enable us to obtain the empirically important hump-shaped and persistent responses of output, consumption, investment, and housing prices in response to various shocks.

Third, a *transitory* increase in the borrowing constraint, modeled as a stationary financial shock, raises the borrowing limit temporarily for any given asset value and

[1]The land asset in our model is a general proxy for any fixed-supply asset or an asset that grows at a much slower rate than the capital. As Davis and Heathcote (2007) show, land grows at a very slow rate and land prices are the main driving force of housing prices observed in the U.S. We therefore interchange the terms "land" and "housing" in this paper as does Kocherlakota (2008).

thus enables the entrepreneur to borrow more. Demand for land and investment both rise, so do the prices of land and capital. The rise in asset prices further relaxes the borrowing constraint and leads to more investment in land and capital. This transitory change in the borrowing constraint leads to hump-shaped responses of consumption, investment, and output and generates positive comovement between macroeconomic variables. While our estimate suggests that this shock explains a small fraction of output fluctuations, it is possible that our sample period is too short to reflect the full magnitude of financial shocks in periods like the recent financial crisis.

Fourth, our empirical results show that there is a single shock driving most of the fluctuations in each financial variable: for the price of capital, it is the shock to investment-specific technology growth; for the land price, it is the housing demand shock; for the excess return, it is also the housing demand shock. In contrast, no single shock dominates in driving the fluctuations in aggregate quantities. The short-run output fluctuations are mostly accounted for by four shocks: the patience shock, the shock to neutral technology growth, the housing demand shock, and the labor supply shock. The short-run investment fluctuations are driven mostly by the patience shock and the housing demand shock, while long-run investment fluctuations are driven by the shock to investment-specific technology growth. The consumption fluctuations are primarily driven by the shock to labor supply and the shock to neutral technology growth.

The idea that indebtedness and borrowing constraints play a critical role in amplifying business cycles can be traced back at least to Fisher (1933). Recent studies emphasize the interactions between asset prices and the borrowing capacity in propagating business cycle fluctuations and can be classified into two strands of literature. One strand of literature builds on the pioneering work by Townsend (1979) and Gale and Hellwig (1985) and focuses on the costly state verification problem caused by asymmetric information between creditors and debtors. Examples includes Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley, Marimon, and Quadrini (2004), Gertler, Gilchrist, and Natalucci (2007), and Christiano, Motto, and Rostagno (2008). In this class of models, the loans are priced to take into account the default risk and the debtors optimally choose the amount to borrow, taking the loan rate as given. Thus, although there is a positive an external financing premium (i.e., the difference between the loan rate and the risk-free interest rate) because of the cost of monitoring, there is no steady-state excess return on assets over and above the loan rate because no borrowers are constrained in equilibrium. The other strand of literature focuses on the costly contract enforcement problem (i.e., the problem of controlling over assets). Examples include Kiyotaki and Moore (1997), Kiyotaki (1998), Krishnamurthy (2003), Cordoba and Ripoll (2004), and Iacoviello (2005).[2] In this class of models, the borrowing capacity is constrained by the value of the collateral assets. In equilibrium, the borrowing constraint is binding and thus borrowers assign a positive value to existing loans, giving rise to a steady-state excess return. The binding borrowing constraints and the associated excess return help amplify business cycle shocks. But how important such a propagation mechanism is remains an empirical question. Our results suggest that the propagation mechanism through dynamic interactions between asset prices and borrowing limits is quantitatively important.

In the rest of the paper, we present our economic model and its implications. In Section IV, we provide intuition for how excess returns can be an important asset price channel and explain its importance using a micro-founded analysis. In Section V, we discuss our econometric methodology and our empirical results in detail.

II. The Model

The economy is populated by two types of agents, households and entrepreneurs, with a continuum of each type with measure one. There are four types of commodities: labor, goods, land, and a risk-free bond. Goods production requires labor, capital, and land as inputs. The output can be used for consumption (by both types of agents) and capital investment (by the entrepreneurs). Each household has preferences over goods, land services (i.e., housing), and leisure, while each entrepreneur consumes goods only.

II.1. **The representative household.**

The household's preferences are represented by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\left\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\right\}, \tag{1}$$

where C_{ht} denotes goods consumption, L_{ht} denotes housing consumption, and N_{ht} denotes labor hours. The parameter $\beta\in(0,1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence, and the term E is an expectation operator. The terms A_t, φ_t, and ψ_t are preference shocks. We assume that the intertemporal preference shock A_t follows the stochastic process

$$A_t = A_{t-1}(1+\lambda_{at}),\quad \ln\lambda_{at}=(1-\rho_a)\ln\bar{\lambda}_a+\rho_a\ln\lambda_{a,t-1}+\varepsilon_{at}, \tag{2}$$

[2]Open economy extensions of this class of models include Aoki, Benigno, and Kiyotaki (2007) and Mendoza (2008), among others.

where $\bar{\lambda}_a > 0$ is a constant, $\rho_a \in (-1, 1)$ is the persistence parameter, and ε_{at} is an i.i.d. white noise process with a zero mean and a finite variance σ_a^2. The housing preference shock φ_t follows the stationary process

$$\ln \varphi_t = (1 - \rho_\varphi) \ln \bar{\varphi} + \rho_\varphi \ln \varphi_{t-1} + \varepsilon_{\varphi t}, \tag{3}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\varphi \in (-1, 1)$ measures the persistence of the shock, and $\varepsilon_{\varphi t})$ is the white noise innovation with a zero mean and a finite variance σ_φ^2. The labor supply shock ψ_t follows the stationary process

$$\ln \psi_t = (1 - \rho_\psi) \ln \bar{\psi} + \rho_\psi \ln \psi_{t-1} + \varepsilon_{\psi t}, \tag{4}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\psi \in (-1, 1)$ measures the persistence, and $\varepsilon_{\psi t})$ is the white noise innovation with a zero mean and a finite variance σ_ψ^2.

Denote by q_{lt} the relative price of housing (in consumption units), R_t the gross real risk-free rate, and w_t the real wage. Further, denote by S_t the household's purchase in period t of the risk-free bond that pays off one unit of consumption good in all states of nature in period $t+1$. In period 0, the household begins with $L_{h0} > 0$ units of housing and $S_0 > 0$ units of the risk-free bond. The flow of funds constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht} - L_{h,t-1}) + \frac{S_t}{R_t} \leq w_t N_{ht} + S_{t-1}. \tag{5}$$

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (1) subject to (5) and the borrowing constraint $S_t \geq -\bar{S}$ for some large number $\bar{S}$.

Denote by μ_{ht} the Lagrangian multiplier for the flow-of-funds constraint (5). The first order conditions for the household's optimizing problem are given by

$$\begin{aligned}
\mu_{ht} &= A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E_t} \frac{\beta \gamma_\mathrm{h}}{\mathrm{C_{h,t+1}} - \gamma_\mathrm{h} \mathrm{C_{ht}}} (1 + \lambda_\mathrm{a,t+1}) \right], && (6) \\
w_t &= \frac{A_t}{\mu_{ht}} \psi_t, && (7) \\
q_{lt} &= \beta \mathrm{E_t} \frac{\mu_\mathrm{h,t+1}}{\mu_\mathrm{ht}} \mathrm{q_{l,t+1}} + \frac{\mathrm{A_t} \varphi_\mathrm{t}}{\mu_\mathrm{ht} \mathrm{L_{ht}}}, && (8) \\
\frac{1}{R_t} &= \beta \mathrm{E_t} \frac{\mu_\mathrm{h,t+1}}{\mu_\mathrm{ht}}. && (9)
\end{aligned}$$

Equation (6) equates the marginal utility of income and of consumption; Equation (7) equates the real wage and the marginal rate of substitution (MRS) between leisure and income; Equation (8) equates the current relative price of land to the marginal benefit of purchasing an extra unit of land, which consists of the current utility benefits (i.e., the MRS between housing and consumption) and the land's discounted future resale value; and Equation (9) is the standard Euler equation for the risk-free bond.

II.2. **The representative entrepreneur.** The entrepreneur has a utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\left[\log(C_{et}-\gamma_e C_{e,t-1})\right], \tag{10}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The entrepreneur produces goods using capital, labor, and land as inputs. The production function is given by

$$Y_t = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{et}^{1-\alpha}, \tag{11}$$

where Y_t denotes output of the goods, K_{t-1}, N_{et}, and $L_{e,t-1}$ denote the inputs of capital, labor, and land, and the parameters $\alpha \in (0,1)$ and $\phi \in (0,1)$ measure the output elasticities of these production factors. We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p \nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \quad \ln\lambda_{zt} = (1-\rho_z)\ln\bar{\lambda}_z + \rho_z \ln\lambda_{z,t-1} + \varepsilon_{zt}, \tag{12}$$

and the transitory component follows the stochastic process

$$\ln\nu_{zt} = \rho_{\nu_z}\ln\nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{13}$$

In these expressions, $\bar{\lambda}_z$ is mean growth rate of Z_t^p and ρ_z and ρ_{ν_z} are the persistence parameters. The innovations ε_{zt} and $\varepsilon_{\nu_z t}$ are i.i.d. white noise processes that are mutually independent, each with a zero mean and a finite variance given by σ_z^2 and $\sigma_{\nu_z}^2$, respectively.

The entrepreneur is endowed with K_0 units of initial capital stock. Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-\bar{\lambda}_I\right)^2\right]I_t, \tag{14}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the flow of funds constraint

$$C_{et} + q_{lt}(L_{et}-L_{e,t-1}) + B_{t-1} = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{et} + \frac{B_t}{R_t}, \tag{15}$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. Following Greenwood, Hercowitz, and Krusell (1997), we interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p \nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1-\rho_q) \ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt}, \tag{16}$$

and the transitory component μ_t follows the stochastic process

$$\ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}. \tag{17}$$

In these expressions, $\bar{\lambda}_q$ is mean growth rate of Q_t^p and ρ_q and ρ_{ν_q} are the persistence parameters. The innovations ε_{qt} and $\varepsilon_{\nu_q t}$ are i.i.d. white noise processes that are mutually independent, each with a zero means and a finite variance given by σ_q^2 and $\sigma_{\nu_q}^2$, respectively. In the spirit of Kiyotaki and Moore (1997), we assume that the entrepreneur can choose to default on the debt payment in each period. If the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital. Since it is costly to liquidate the seized land and capital stock, the creditor can recoup up to a fraction θ_t of the total value of the seized assets. Thus, the amount of debt that the entrepreneur can issue is bounded above by the maximum amount that the creditor can recoup in the event that the debt cannot be repaid. Specifically, we follow Kiyotaki and Moore (1997) and Aoki, Benigno, and Kiyotaki (2007) and assume that the entrepreneur faces the collateral constraint

$$B_t \leq \theta_t E_t[q_{l,t+1} L_{et} + q_{k,t+1} K_t], \tag{18}$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units and $\theta_t \in (0,1)$ is fraction of total seized assets that the creditor can obtain in the event of liquidation.[3] We allow θ_t to be time-varying and interpret it as a liquidity shock that reflects the uncertainty in the tightness of the credit market. We assume that the liquidity shock θ_t follows the stochastic process

$$\ln \theta_t = (1-\rho_\theta) \ln \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{19}$$

where $\bar{\theta}$ is the steady-state value of θ_t, $\rho_\theta \in (0,1)$ is the persistence parameter, and $\varepsilon_{\theta t}$ is an i.i.d. white noise innovation with a zero mean and a finite variance σ_θ^2.

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (10) subject to (11)-(18). Denote by μ_{et} the Lagrangian multiplier for the flow-of-funds constraint (15),

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[3]Since the price of new capital is $1/Q_t$, the Tobin's q in this model is given by $q_{kt}Q_t$, which is the ratio of the value of installed capital to the price of new capital.

μ_{kt} the multiplier for the capital accumulation equation (14), and μ_{bt} the multiplier for the borrowing constraint (18). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{20}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\begin{aligned}
\mu_{et} &= \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E_t} \frac{\beta \gamma_\mathrm{e}}{\mathrm{C_{e,t+1}} - \gamma_\mathrm{e} \mathrm{C_{et}}}, && (21)\\
w_t &= (1-\alpha) Y_t / N_{et}, && (22)\\
\frac{1}{Q_t} &= q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \right] \\
&\quad + \beta \Omega \mathrm{E_t} \frac{\mu_{\mathrm{e,t+1}}}{\mu_{\mathrm{et}}} \mathrm{q_{k,t+1}} \left(\frac{\mathrm{I_{t+1}}}{\mathrm{I_t}} - \bar{\lambda}_\mathrm{I} \right) \left(\frac{\mathrm{I_{t+1}}}{\mathrm{I_t}} \right)^2, && (23)\\
q_{kt} &= \beta \mathrm{E_t} \frac{\mu_{\mathrm{e,t+1}}}{\mu_{\mathrm{et}}} \left[\alpha (1-\phi) \frac{\mathrm{Y_{t+1}}}{\mathrm{K_t}} + \mathrm{q_{k,t+1}} (1-\delta) \right] + \frac{\mu_{\mathrm{bt}}}{\mu_{\mathrm{et}}} \theta_\mathrm{t} \mathrm{E_t} \mathrm{q_{k,t+1}}, && (24)\\
q_{lt} &= \beta \mathrm{E_t} \frac{\mu_{\mathrm{e,t+1}}}{\mu_{\mathrm{et}}} \left[\alpha \phi \frac{\mathrm{Y_{t+1}}}{\mathrm{L_{et}}} + \mathrm{q_{l,t+1}} \right] + \frac{\mu_{\mathrm{bt}}}{\mu_{\mathrm{et}}} \theta_\mathrm{t} \mathrm{E_t} \mathrm{q_{l,t+1}}, && (25)\\
\frac{1}{R_t} &= \beta \mathrm{E_t} \frac{\mu_{\mathrm{e,t+1}}}{\mu_{\mathrm{et}}} + \frac{\mu_{\mathrm{bt}}}{\mu_{\mathrm{et}}}. && (26)
\end{aligned}$$

Equation (21) equates the marginal utility of income to the marginal utility of consumption since consumption is the numéraire; Equation (22) is the labor demand equation which equates the real wage to the marginal product of labor; Equation (23) is the investment Euler equation, which equates the cost of purchasing an additional unit of investment good and the benefit of having an extra unit of new capital, where the benefit includes the shadow value of the installed capital net of adjustment costs and the present value of the saved future adjustment costs; Equation (24) is the capital Euler equation, which equates the shadow price of capital to the present value of future marginal product of capital and the resale value of the un-depreciated capital, plus the value of capital as a collateral asset for borrowing; Equation (25) is the land Euler equation, which equates the price of the land to the present value of the future marginal product of land and the resale value, plus the value of land as a collateral asset for borrowing; Equation (26) is the bond Euler equation for the entrepreneur, which reveals that the borrowing constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution.

II.3. **Market clearing conditions and equilibrium.** In a competitive equilibrium, the markets for goods, labor, land, and the risk-free bond all clear. Goods market clearing implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{27}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. Labor market clearing implies that labor demand equals labor supply, that is,

$$N_{et} = N_{ht} \equiv N_t. \tag{28}$$

Land market clearing implies that

$$L_{ht} + L_{et} = \bar{L}, \tag{29}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, bond market clearing implies that

$$S_t = B_t. \tag{30}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{ht}, C_{et}, I_t, N_t, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur and (ii) all markets clear.

II.4. **Stationary equilibrium dynamics and steady state.** We are interested in studying the fluctuations around the balanced growth path. For this purpose, we focus on a stationary equilibrium by appropriately transforming the growing variables. Specifically, we make the following transformations of the variables

$$\tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t\Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t\Gamma_t}, \quad \tilde{B}_t \equiv \frac{B_t}{\Gamma_t},$$
$$\tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \quad \tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t, \quad \tilde{q}_{lt} \equiv \frac{q_{lt}}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt}Q_t, \tag{31}$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha}]^{\frac{1}{1-(1-\phi)\alpha}}$. In Appendix B, we describe the stationary equilibrium and derive the log-linearized equilibrium conditions around the steady state for solving the model. To solve the log-linearized equilibrium system requires the input of several key steady-state values. These include the shadow value of the loanable funds $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, the ratio of commercial real-estate to aggregate output $\frac{\tilde{q}_l L_e}{\tilde{Y}}$, the ratio of residential land to commercial real estate $\frac{L_h}{L_e}$, the ratio of loanable funds to output $\frac{\tilde{B}}{\tilde{Y}}$, the capital-output ratio $\frac{\tilde{K}}{\tilde{Y}}$, and the "big ratios" $\frac{\tilde{C}_h}{\tilde{Y}}$, $\frac{\tilde{C}_e}{\tilde{Y}}$, and $\frac{\tilde{I}}{\tilde{Y}}$. The model implies a set of restrictions between these steady-state ratios and the parameters and we will use these restrictions along with the first moments of selected time series in the data to sharpen our priors and to help identify a subset of the parameters in our estimation.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady state value of $g_{\gamma t}$ and $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (A4) for the household and (A10) (described in the Appendix) to obtain

$$\frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \quad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta\bar{\lambda}_a}{g_\gamma}. \tag{32}$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

To get the ratio of commercial real estate to output, we use the land Euler equation (A9) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (A5), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (32). In particular, we have

$$\frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}}. \tag{33}$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (A12) and then solve for the capital-output ratio using the capital Euler equation (A8). Specifically, we have

$$\frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k}, \tag{34}$$

$$\frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a\bar{\theta} + 1 - \delta)\right]^{-1} \beta\alpha(1-\phi), \tag{35}$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (A7). The investment-output ratio is then given by

$$\frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a\bar{\theta} + 1 - \delta)}. \tag{36}$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (33) and (35), the binding borrowing constraint (A16) implies that

$$\frac{\tilde{B}}{\tilde{Y}} = \bar{\theta} g_\gamma \frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}}{\bar{\lambda}_q}\frac{\tilde{K}}{\tilde{Y}}. \tag{37}$$

The entrepreneur's flow-of-funds constraint (A15) implies that

$$\frac{\tilde{C}_e}{\tilde{Y}} = \alpha - \frac{\tilde{I}}{\tilde{Y}} - \frac{1-\beta(1+\bar{\lambda}_a)}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}. \tag{38}$$

The aggregate resource constraint (A13) then implies that

$$\frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}. \tag{39}$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e, the housing demand equation) (A3) and the definition for the marginal utility (A1) to obtain

$$\frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)}, \tag{40}$$

where the steady-state risk-free rate is given by (32).

Taking the ratio between (40) and (33) results in the solution

$$\frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1 - \beta - \beta\bar{\lambda}_a\bar{\theta})}{\beta\alpha\phi g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)} \frac{\tilde{C}_h}{\tilde{Y}}. \tag{41}$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (A2) and the labor demand equation (A6) to get

$$N = \frac{(1 - \alpha)g_\gamma(1 - \gamma_h/R)}{\bar{\psi}(g_\gamma - \gamma_h)} \frac{\tilde{Y}}{\tilde{C}_h}. \tag{42}$$

III. Parameter estimation

We fit our model to quarterly time series data. The data that we use include the real land price, the inverse of quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), real per capita nonfinancial business debt, and per capita hours worked (as a fraction of total time endowment). The sample period covers 1975:Q1 through 2008:Q4. The data on investment-specific technology is needed to get the sizes of standard deviations of investment technology in line with those in Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Fisher (2006); it helps assess how important investment-specific technology shocks are relative to neutral technology shocks. We describe the details of these time series in Appendix A.

III.1. **Prior distributions.** We estimate all the structural parameters subject to the restriction that the model's steady-state equilibrium under the estimated parameters match the first moments of 8 variables in the U.S. data. Specifically, we restrict the parameters such that the model implies that (1) the average labor income share is about 70%; (2) the average business loan rate is about 4% per annual; (3) the capital-output ratio is on average about 1.35 at the annual frequency, as in the data for the sample period 1950-2007; (4) the investment-capital ratio is on average about 0.148 at the annual frequency, as in the data for the sample period 1950-2007; (5) the average land-output ratio is about 0.65 at the annual frequency, as in the data for the period 1987-2007; (6) the average debt-output ratio is about 0.63 at the annual frequency, as in the data for the sample period 1952:Q1-2008:Q2; (7) the average housing-output ratio is about 1.1416 at the annual frequency, as in the data for the period 1950-2007; and (8) the average market hours is about 20% of time endowment.[4] While these restrictions make our estimation a more challenging task than an unconstrained optimization procedure, it helps identify the model's parameters.

This point is illustrated by Table 1. We assume agnostic prior distributions for the first set of parameters summarized in the vector $\Psi_1 = \{\gamma_h, \gamma_e, \Omega, \bar{\lambda}_q, g_\gamma, \rho_i, \sigma_i\}$, where ρ_i and σ_i are the persistence parameters and the standard deviations of the 8 shocks. The prior distributions for the second set of parameters summarized in the vector $\Psi_2 = \{\beta, \bar{\lambda}_a, \bar{\varphi}, \alpha, \phi, \delta\}$, and $\bar{\theta}$, however, are obtained from simulations based on the prior distributions of the first set of parameters with the moment restrictions imposed. As is evident in the Table 1, these moment restrictions help identify the second set of structural parameters. In particular, the table shows that the restrictions from the first moments impose very tight bounds on the probability intervals for the prior distributions of the parameters such as α, ϕ, β, and $\bar{\theta}$.[5]

The priors for the habit persistence parameters γ_h and γ_e follow the Beta distribution with the shape parameters given by $a = 1$ and $b = 2$. Thus, we assign positive density to $\gamma_h = \gamma_e = 0$ and let the probability density decline linearly as the value of γ_h (or γ_e) increases from 0 to 1. With these shape parameters, the lower probability bound (5%) for γ_h and γ_e is 0.0256 and the upper probability bound (95%) is 0.7761. In other words, with 90% probability, our priors for the habit persistence parameters lie in the interval between 0.0256 and 0.7761. This interval covers most of the calibrated values for the

[4]Since we have a closed-economy model with no government spending, we measure private domestic output by the sum of personal consumption expenditures and the gross private domestic investment, both provided by the BEA through Haver Analytics. Capital stock corresponds to the net stocks of private non-residential fixed assets and housing stock corresponds to the net stocks of private non-corporate residential fixed assets, also provided by the BEA. Given our output series, we compute the debt-output ratio by using the debt outstanding in the nonfinancial business sector taken from the Flow of Funds table provided by the Federal Reserve Board of Governors. To compute the land-output ratio, we use the time series of the nominal value of land input and the nominal value of output in the private non-farm business sector for the period 1987-2007, both provided by the BLS.

[5]Even with some deep parameters well identified, the posterior density function is very non-Gaussian and has many local peaks. We randomly simulate one million starting points and select the converged result that gives the highest posterior density. Among these starting points, many converge to the point that has the highest peak.

habit persistence parameter used in the literature (for example, Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum, and Evans (2005)). We assume that the priors for the shock persistent parameters also follow the Beta distribution with the same shape parameters $a = 1$ and $b = 2$, as shown in Table 2.

The priors for the investment adjustment cost parameter Ω follow the Gamma distribution with the shape parameter $a = 1$ and the "rate parameter" $b = 0.5$ so that the probability density at $\Omega = 0$ is positive and finite and we allow positive densities for large values of Ω. As shown in Table 1, the 90% probability interval for the priors of Ω covers values from 0.1 to 6, which includes most of the values estimated in the DSGE literature (e.g., Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Liu, Waggoner, and Zha (2009)).

The priors for the steady-state growth rate of the investment-specific technology and for per capita output follow the Gamma distribution with the 90% probability interval covering the range between 0.1 and 1.5, corresponding to annual growth rates between 0.4% and 6%. This wide range reflects our agnostic priors on these parameters.

The prior distributions for the second set of parameters in Ψ_2 are simulated. In particular, given the prior distributions for the first set of parameters in Ψ_1, we draw the parameters in Ψ_1 and compute the implied values of the parameters in Ψ_2 with the first-moment restrictions imposed. Since the relation between the parameters in Ψ_2 and those in Ψ_1 are highly nonlinear, the distributions for the simulated parameters are unknown. Yet, we can obtain the 90% probability intervals, which are shown in the lower panel of Table 1. Evidently, the restrictions from the data's first moments put very tight bounds on the prior distributions for these parameters.

Finally, as shown in Table 2, we assume that the standard deviations of the shocks follow the Inverse Gamma distribution with the 5%-95% probability interval given by $[0.0005, 0.2]$. We have examined the sensitivity of our estimates by extending the upper bound of this interval to 5 and find that the results are not sensitive.

III.2. **Posterior estimates.** In the right columns of Tables 1 and 2, we report the posterior-peak estimates of the parameters.

Our posterior estimates suggest that habit persistence is modestly important, with the entrepreneur's habit parameter slightly larger than the household's (0.64 v. 0.40). Our estimate of the investment-adjustment cost parameter is 0.29, much smaller than those obtained in the literature (e.g., Christiano, Eichenbaum, and Evans (2005) obtain an estimate of Ω at around 2.5 and Smets and Wouters (2007) report an estimate larger than 5). Since land is fixed in supply and the land input is complementary to the capital input, the adjustment in investment is contained even with a small adjustment cost parameter.

Our estimated growth rate of the investment-specific technology is higher than that calibrated by Greenwood, Hercowitz, and Krusell (1997) (5% v. 3.2% per annum), mainly because Greenwood, Hercowitz, and Krusell (1997) use a shorter sample that stops at 1990 and the quality-adjusted relative price of equipment, software, and consumer durables has declined substantially since the early 1990s. Our estimated growth rate of output is about 2% per annum, which is similar to the average growth rate of real per capita GDP in the U.S. for the postwar period.

The estimated value of the patience factor is 0.011, which, as we show below, is large enough to help the model generate large and persistent fluctuations in aggregate quantities and asset prices. The estimated average load-to-value ratio is $\bar{\theta} = 0.32$. The estimates of other parameters in the vector Ψ_2 also seem plausible.

Table 2 shows the estimates of the shock persistence and standard deviations. The patience shock, the housing demand shock, the labor supply shock, and the financial shock are all persistent with AR(1) coefficients above 0.9. The estimates of the standard deviations reveal that the patience shock is dominant in size (with a standard deviation of 0.126), followed by the housing demand shock (with a standard deviation of 0.06). The financial shock also has a fairly sizable standard deviation (0.013). Other shock have relatively small standard deviations.

IV. Understanding the Asset-Price Channels of the Model

In this section, we provide intuition of how, unlike Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2008), a positive first-order excess return exists in our model. We first point out that the excess return arises because the household is more patient than the entrepreneur. We then present a micro foundation that gives rise to the excess return. In particular, we establish that our benchmark model with patient households is equivalent to a model with heterogeneous households who face idiosyncratic income shocks that are persistent and uninsurable. Finally, we use impulse responses to illustrate the importance of the first-order excess return in propagating the shocks through the asset-price channels.

IV.1. **First-order excess returns.** We organize this subsection in two steps. First, we show that even if the entrepreneur faces a borrowing constraint, the steady-state return on a productive asset is g_γ/β, where β is the entrepreneur's discount factor for the future consumption stream and g_γ is the steady-state gross growth rate of consumption. We then show how the loan rate in steady state depends on the household's patience. Because the household's patience parameter $\bar{\lambda}_a$ is tied to the entrepreneur's borrowing constraint, a positive excess return exists when the entrepreneur faces the borrowing limit.

The entrepreneur has two types of assets: land and capital. Each asset can be intuitively thought of as a tree bearing fruits and growing at a gross rate of g_γ. The entrepreneur can trade a portion of the tree in the market and the return on this tree depends on the price of a unit of tree as well as the marginal product (fruit) of the remaining tree. In steady state, it should be g_γ/β. To see if this intuition works in the model when the entrepreneur faces the borrowing constraint, we first derive the expected return on each of these assets. We begin with the return on land.

Suppose the entrepreneur purchases one unit of land at the price q_{lt} in period t. Since he can pledge a fraction θ_t of the present value of the land as collateral, the net out-of-pocket payment (i.e., the down payment) for the land purchase is given by

$$u_t \equiv q_{lt} - \theta_t \mathrm{E}_t \frac{q_{l,t+1}}{R_t}, \tag{43}$$

where R_t is the risk-free loan rate. The land is used for period-$t+1$ production and yields $\phi\alpha Y_{t+1}/L_{et}$ units of extra output. In addition, the entrepreneur can keep the remaining value of the land in period $t+1$ after repaying the debt so that the total payoff from the land is $\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}$. The return on the land from period t to $t+1$ is thus given by

$$R_{l,t+1} = \frac{\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}}{q_{lt} - \theta_t \mathrm{E}_t \frac{q_{l,t+1}}{R_t}}. \tag{44}$$

We can similarly derive the return on capital, which is given by

$$R_{k,t+1} = \frac{\phi\alpha Y_{t+1}/K_t + q_{k,t+1}(1-\delta) - \theta_t \mathrm{E}_t q_{k,t+1}}{q_{kt} - \theta_t \mathrm{E}_t \frac{q_{k,t+1}}{R_t}}. \tag{45}$$

Using the bond Euler equation (26) for the entrepreneur, we can rewrite the land Euler equation (25) and the capital Euler equation (24) in terms of the asset returns. In particular, the land and capital Euler equations can be rewritten as

$$1 = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R_{j,t+1}, \quad j \in \{l, k\}. \tag{46}$$

In the steady-state equilibrium, consumption grows at the rate g_λ. Thus, with log-utility in our model, (46) implies that $R_j = g_\lambda/\beta$.

On the other hand, the loan rate R_t is determined by the household's intertemporal Euler equation (9). It follows that in steady state, $R = \frac{g_\gamma}{\beta(1+\bar{\lambda}_a)}$, where $\bar{\lambda}_a > 0$ measures the extent to which the household is more patient than the entrepreneur. Thus, the steady state excess return, denoted by $R_j^e = R_j - R$ for $j \in \{l, k\}$, is always positive.[6]

To see how a positive first-order excess return is related to the entrepreneur's borrowing limit, one should note from (32) that

$$\frac{\beta\bar{\lambda}_a}{g_\gamma} = \frac{\tilde{\mu}_b}{\tilde{\mu}_e}.$$

Thus, the borrowing constraint is binding (i.e., $\tilde{\mu}_b > 0$) if and only if the household is more patient than the entrepreneur (i.e., $\bar{\lambda}_a > 0$).

This result carries over to the dynamics of excess returns. Denote by $R^e_{j,t+1} \equiv R_{j,t+1} - R_t$ the excess return for asset $j \in \{l, k\}$. By combining the bond Euler equation (26) and the asset-pricing equation (46), we obtain

$$\beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R^e_{j,t+1} = \frac{\mu_{bt}}{\mu_{et}} R_t, \quad j \in \{l, k\}. \tag{47}$$

As in the standard asset-pricing model, the mean excess return depends on the asset's riskiness measured by the covariance between the return and the marginal utility. But unlike the standard model, the excess return here in our model contains a first-order term that is positive if and only if the borrowing constraint is binding (i.e., $\mu_{bt} > 0$).

The first-order excess return represents a key distinction between our model with costly contract enforcement in the spirit of Kiyotaki and Moore (1997) and the model with costly state verification studied by Bernanke, Gertler, and Gilchrist (1999) and empirically evaluated by Christiano, Motto, and Rostagno (2008). In Bernanke, Gertler, and Gilchrist (1999), the intermediary sets the loan rate to break even, taking into account of the default risk. The entrepreneurs optimize the loan amount, taking the loan rate as given. In equilibrium, no entrepreneurs are borrowing-constrained and there is no first-order excess return. In our model, however, the entrepreneur is always constrained in the loan market because the household is more patient and their extra savings lower the equilibrium loan rate. In the dynamic equilibrium with fluctuations around the steady state, the entrepreneur's consumption under the binding borrowing constraint is lower than that in an otherwise identical economy without the borrowing constraint. Thus, the entrepreneur's marginal utility of consumption is higher and it requires a positive excess return to convince the entrepreneur to invest in capital and land.

[6] To a first-order approximation, the expected returns on land and on capital are the same.

IV.2. **Precautionary savings motive: A micro foundation for the patience factor.** A crucial assumption in our model is that the household is more patient than the entrepreneurs, so that their saving depresses the loan rate and giving rise to a binding borrowing constraint and a first-order excess return. We now argue that assumption on the patience factor is not arbitrary or a mere convenient modeling device. Being more patient is a rational choice by the households in an environment with persistent and uninsurable idiosyncratic income shocks. We make this argument in the context of the life-cycle model below.

Consider an economy in which there are two types of agents, households and entrepreneurs. For each type in period t, there are agents born in periods $t-m$ for $m = 0, 1, 2, ... + \infty$. We use $x_{t,t-m}(i)$ to denote the value of x in period t of the i^{th} agent who is born in period $t-m$. A living individual faces a constant death probability $\varkappa$ in subsequent periods regardless of the age. By the law of large number, a $\varkappa$ fraction of each type of agents die in each period. We assume that a measure $\varkappa$ of each type of agents are born in each period. Upon death, the remaining net worth for each type of agents is transferred and evenly distributed to the new born agents of the same type.

In period t, a typical household i born in $t-m$ has the expected discounted utility[7]

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \log c_{t+\tau,t-m}(i) + \varphi_t \log \ell_{h,h,t+\tau,t-m}(i) - \psi_t n_{t+\tau,t-m}(i) \right\}, \tag{48}$$

where $\beta = \bar{\beta}(1-\varkappa)$ and $\bar{\beta}$ is a discounting factor for each individual. The flow of funds constraint of household i is

$$c_{t,t-m}(i) + q_t[\ell_{h,t+1,t-m}(i) - \ell_{h,t,t-m}(i)] + \frac{s_{t,t-m-1}(i)}{R_t} \leq w_t e_{t,t-m}(i) n_{t,t-m}(i) + s_{t-1,t-m-1}(i). \tag{49}$$

The household's wage is w_t and her effective labor supply $e_{t,t-m}(i)n_{t,t-m}(i)$. We assume $e_{t,t-m}(i)$ is idiosyncratic and evolves according to

$$e_{t+1,t-m}(i) = e_{t,t-m}(i)\varepsilon_{t+1,t-m}(i) \tag{50}$$

where $\varepsilon_{t+1,t-m}(i)$ is a random shock to the household's labor income. This shock is assumed to be independent across time and across households with $E\varepsilon_{t+1,t-m}(i) = 1$. The time-varying distribution function $F_t(\varepsilon)$ is the same for all agents in period t. Furthermore, we assume that each newly born household is endowed with the same initial conditions – that is, $e_{t,t-1}(i) = 1$, $s_{t,t}(i) = \tilde{S}_t$, and $\ell_{h,t,t}(i) = \tilde{L}_{h,t}$.

[7]To make our derivation tractable, we assume that there is no habit. The conclusion, however, does not depend on this assumption.

With this setup, one can easily show that

Proposition 1. In this economy, the aggregate intertemporal Euler equation and the aggregate household budget constraint are given by

$$\frac{1}{R_t} = \beta E_t \frac{C_{ht}}{C_{h,t+1}}(1+\lambda_{a,t+1}), \tag{51}$$

$$\begin{aligned} C_{h,t} + q_t[L_{h,t+1} - L_{h,t}] + \frac{S_t}{R_t} &= w_t \varkappa \sum_{m=0}^{\infty} (1-\varkappa) \int_0^1 e_{t,t-m}(i) n_{t,t-m}(i) di + S_t \\ &= w_t N_{h,t} + S_{t-1}, \end{aligned} \tag{52}$$

where $1 + \lambda_{a,t+1} = E_i \frac{1}{\varepsilon_{t+1,t+1-m}(i)}$. Further, if $\varepsilon(i)$ follows the log-normal distribution, then we have

$$1 + \lambda_{at} = \exp\left(\frac{1}{2}\sigma^2_{\varepsilon t}\right),$$

where $\sigma^2_{\varepsilon t} = Var_i(\ln(\varepsilon_{t,t-m}(i))$ denotes the cross-sectional dispersion of the idiosyncratic income shocks.

We prove the proposition in the Appendix.

Proposition 1 reveals that Equations (51) and (52) are exactly the same as Equations (9) (when $\gamma_h = 0$) and (5). Since $E_i \varepsilon_{t,t-m}(i) = 1$, we have $E_i \frac{1}{\varepsilon_{t,t-m}(i)} > 1$ by Jensen's inequality and thus $\lambda_{at} > 0$.

In period t, a typical entrepreneur j, who is born in period $t-m$, has the expected discounted utility

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \log c_{t+\tau,t-m}(j). \tag{53}$$

Notice that at the disaggregate level, both entrepreneurs and households are assumed to have the same discounting factor.

Entrepreneurs have no labor endowment but have access to a constant-returns-to-scale technology for producing consumption good from land, capital, and labor. Following Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2008), we assume that there is a common neutral technology shock and an i.i.d. idiosyncratic shock to the production of an individual entrepreneur. With this assumption, one can show that the intertemporal Euler equation in aggregation is the same as Equation (26). Thus, even when an individual household and an individual entrepreneur have the same discounting factor β, the aggregate Euler equations imply that the household acts as though she were more patient than the entrepreneur at the aggregate level.

IV.3. **The importance of the credit constraint.** To illustrate the quantitative importance of the credit constraint, we plot the impulse responses of several key macroeconomic variables in the benchmark model with estimated parameters and an alternative model with otherwise identical parameter values except for a lower value of the average patience factor ($\bar{\lambda}_a$). Since a smaller patience factor implies a lower excess return in the steady state, the alternative economy represents one with a less severe credit constraint. The differences of the impulse responses in these two economies capture the importance of the credit constraint. We focus on the patience shock and the housing demand shock, which, as we will show in Section V, are both important sources of macroeconomic fluctuations.

Figures 1 and 2 display the impulse responses of the aggregate quantities and asset prices following the patience shock and the housing demand shock, respectively. In each panel, we display the impulse response in the benchmark model with our estimated parameters (the solid line) and the alternative model with a low steady-state excess return and thus low levels of credit frictions (the dashed line). In particular, we set the steady-state excess return to be 10% of that in the benchmark model while we keep all other parameters at their estimated levels. The figures reveal that the credit constraint is important for macroeconomic fluctuations. Through the asset-price channels, the credit constraint substantially magnifies the responses of output, investment, and consumption of the two types of agents and makes these responses more persistent. The land price and the capital price are also magnified by the credit constraint.

V. Quantitative results

In this section, we present and discuss our main quantitative results based on our estimation of the model.

V.1. **Variance decomposition.** Tables 3 and 4 display the forecast variance decompositions for aggregate quantities and asset prices at various forecasting horizons. The tables reveal that, at the business cycle frequency (between 8 and 32 quarters), fluctuations in aggregate output are primarily driven by 4 shocks: the intertemporal preference shock ("Patience"), the shock to the growth rate of neutral technology ("Ngrowth"), the shock to the housing demand ("Housing"), and the shock to the labor supply ("Labor"). Taken together, these shocks account for more than 90% of output fluctuations at these forecasting horizons.

The patience shock accounts for about $10 - 20\%$ of short-run output fluctuations and its effect diminishes over time. It is tempting to interpret our patience shock as an intertemporal wedge that captures distortions in the consumption-savings decisions in the sense of Chari, Kehoe, and McGrattan (2007). But we find this interpretation oversimplifying. Unlike the representative agent model studied by Chari, Kehoe, and McGrattan (2007), our model with heterogeneous agents contains a natural source of intertemporal distortion that comes from the borrowing constraint and the first-order excess returns on assets. Although the explicit intertemporal shock (i.e., the preference shock) explains a modest fraction of output fluctuations, the intertemporal distortion through the credit frictions can propagate and amplify several other important shocks to generate business cycle fluctuations, as we discuss in the next section.

The shock to the housing demand accounts for about 25% of output fluctuations at the business cycle frequency. This shock is important because, as shown in Table 4, it accounts for about 90% of the land price fluctuations and, through the asset-price channel that we discuss in the next section, fluctuations in the land price can lead to important output fluctuations.

The labor supply shock accounts for about 20% of the output fluctuations. It captures the wedge between the household's intra-temporal marginal rate of substitution and the marginal product of labor (i.e., the "labor wedge") emphasized by Chari, Kehoe, and McGrattan (2007). The permanent shock to the neutral technology is the most important driving force for output fluctuations in the medium and long run. In contrast, transitory shocks to the neutral technology and the 2 biased technology shocks do not seem to be important for output fluctuations. Finally, perhaps surprisingly, the financial shock explains a very small fraction of output variances. In this sense, exogenous shocks to the tightness of the borrowing constraint does not generate large fluctuations, while the endogenous first-order excess returns are the main source of amplification and propagation in the model.

Table 3 also shows that consumption fluctuations are mainly driven by the permanent neutral technology shock and the labor supply shock while investment fluctuations are driven by the housing demand shock and the patience shock in the short run and by the permanent biased technology shock in the long run.

Table 4 shows that the capital price fluctuations are mainly driven by the permanent biased technology shock while the land price fluctuations are predominantly driven by the housing demand shock.

V.2. **Historical decomposition.** In this section, we present the historical decomposition of several key variables to examine the sources of macroeconomic fluctuations in our model.

[–To be completed–]

VI. Expansion of the borrowing capacity: Policy implications

In our model, there is no market failure. The binding borrowing constraint is a consequence of the relative patience of the households, not a manifestation of inefficiency. So there is no room for welfare-improving policy intervention. But one can think of the consequences of stabilizing policies in the framework. Here we focus on variations in one particular dimension of the model that might capture the consequences of a policy intervention that aims at expanding the borrowing capacity. In the context of our model, we can think of the policy as one that raises the level of θ_t, which can be a permanent or a transitory change. We now examine the consequences of this type of policy intervention on macroeconomic stability both in the short run and in the long run.

VI.1. **Permanent expansion of the borrowing capacity.** We begin with the case in which $\bar{\theta}$ is raised to a higher level, representing a permanent expansion of the borrowing capacity. The rise in $\bar{\theta}$ enables the entrepreneurs to borrow more at any given value of their assets and helps raise the steady-state level of output, as shown in Figure 3. As the value of $\bar{\theta}$ doubles (from 0.32 to 0.64), the level of output rises by about 2%. The increase in $\bar{\theta}$ raises the loan demand and also the net worth of the household. Thus, household consumption increases with $\bar{\theta}$. Since the increase in leverage reduces the net worth of the entrepreneur, entrepreneur consumption decreases with $\bar{\theta}$. Thus, while the expansion of borrowing capacity raises steady state output, it does not lead to a Pareto improvement in the long run.

In the short run, the permanent increase in the borrowing capacity can also affect the magnitude and persistence of business cycle fluctuations. Figure 4 shows the impulse responses of several key macroeconomic variables to the patience shock and compares the responses in the benchmark economy with estimated parameters (the solid line) and an alternative economy with a higher borrowing capacity (the dashed line). In particular, the alternative economy has a $\bar{\theta}$ twice as large as that in the benchmark economy. The figure reveals that the permanent expansion of the borrowing capacity destabilizes the economy. The peak response of output to the patience shock rises from 0.0069 to 0.0112, an increase of about 60%. The responses of consumption, investment, and asset prices are also substantially magnified and are more persistent.

VI.2. **Transitory expansion of the borrowing capacity.** We now consider a transitory expansion of the borrowing capacity, that is, a temporary rise in θ_t before reverting to its original level. This is equivalent to a transitory financial shock in our model. Since the average borrowing capacity remains the same, there is no long-run effect of such transitory shocks. Figure 5 shows the impulse responses of the macroeconomic variables to a one-standard-deviation financial shock. The figure reveals that the shock raises output and investment, both rising persistently before reaching their peaks in about 4 quarters. The shock raises the asset prices as well, with the responses of the land price larger and more persistent than the capital price. The expansion in the borrowing capacity has also important distributional effects on consumption. After a slight initial dip below the steady state, household consumption rises persistently before reaching the peak 16 quarters after the impact period. In contrast, entrepreneur consumption rises initially (with a slight hump) and the declines persistently. The distributional effects are a consequence of the increased leverage. The entrepreneur consumption rises initially because the expansion in the borrowing capacity enables the entrepreneur to acquire more land, capital, and labor for production; the increased demand for land and capital drives up the asset prices, so the entrepreneur's net worth goes up in the short run. In the long run, entrepreneur consumption falls because the entrepreneur is more leveraged and the debt repayment reduces the entrepreneur's net worth. The rise in the household consumption reflects the wealth effect from the increased household net worth.

To summarize, we find that a permanent expansion of the entrepreneur's borrowing capacity raises the steady-state level of output but destabilizes macroeconomic fluctuations in the short run. A temporary expansion of the borrowing capacity generates persistent increases in output, investment, and asset prices, while it also generates distributional effects that favors the household at the expense of the entrepreneur's consumption.

VII. Conclusion

We have estimated a DSGE model with heterogeneous agents and collateral constraints. Our preliminary estimation suggests that the collateral constraint in the spirit of Kiyotaki and Moore (1997) plays an important role in amplifying several sources of business cycle shocks. As the household is more patient than the entrepreneur, the collateral constraint is binding. The binding constraint gives rise to a first-order excess return on assets, which is empirically important. With the binding collateral constraint, interactions between asset prices and the debts generate a multiplier effect that propagates shocks. We show that the tightness of the collateral constraint depends on the magnitude of the patience factor, which can arise from precautionary savings motive in a model with heterogeneous households facing persistent and uninsurable income risks.

Based on our estimated model, we have examined the macroeconomic and distributional consequences of changes in the steady-state loan-to-value ratio. We interpret exogenous changes in the loan-to-value ratio as some forms of government intervention that expands the borrowing capacity for entrepreneurs with given assets. We find that a permanent expansion of the borrowing capacity raises the steady-state level of aggregate output and increases the household consumption relative to the entrepreneur consumption. The expansion of borrowing capacity also increases the sensitivity of macroeconomic variables to several important business cycle shocks. This finding is consistent with the empirical observation that financially more liberalized economies on average have higher mean growth rates and also experience more volatile business cycles (Ranciere, Tornell, and Westermann, 2008).

While this is still an incomplete and preliminary work, our results suggest that credit frictions in the form of collateral constraints are potentially important for understanding business cycles. Our findings lend support to the view expressed by Fisher (1933), who argues that changes in the leverage position (i.e., indebtedness) is an important source of business cycle fluctuations. In particular, our findings suggest that overly indebtedness can be a destabilizing force. We do not provide a theory of why individual entrepreneurs would want to incur excessive debts. Interestingly, Fisher (1933) has articulated a plausible source of the overly indebtedness. In reality, he notes that individuals borrow to invest in projects that they believe can yield adequate returns. They base this belief on, for example, signals of high potential values of the project such as new patents for innovations. These potentials do not always materialize and in the event they do not, the indebted investor needs to de-leverage, which depresses the asset value and triggers a multiplier effect that reduces aggregate activity. One can imagine that such scenario described by Fisher (1933) can be incorporated in our model as news shocks to productivity. Whether or not news shocks are important for understanding macroeconomic fluctuations would be an empirical issue that can be studied within our general framework.

Appendix A. Data Description

Appendix B. Some detailed derivations

In this section, we describe the stationary equilibrium conditions and derive the log-linearized conditions for solving the model.

B.1. **Stationary equilibrium.** The stationary equilibrium is the solution to the following system of equations:

$$
\begin{aligned}
\tilde{\mu}_{ht} &= \frac{1}{\tilde{C}_{ht}-\gamma_h\tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E_t}\frac{\beta\gamma_{\mathrm{h}}}{\tilde{\mathrm{C}}_{\mathrm{h,t+1}}\Gamma_{\mathrm{t+1}}/\Gamma_{\mathrm{t}}-\gamma_{\mathrm{h}}\tilde{\mathrm{C}}_{\mathrm{ht}}}(1+\lambda_{\mathrm{a,t+1}}), && \text{(A1)}\\
\tilde{w}_t &= \frac{\psi_t}{\tilde{\mu}_{ht}}, && \text{(A2)}\\
\tilde{q}_{lt} &= \beta\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{h,t+1}}}{\tilde{\mu}_{\mathrm{ht}}}(1+\lambda_{\mathrm{a,t+1}})\tilde{\mathrm{q}}_{\mathrm{l,t+1}} + \frac{\varphi_{\mathrm{t}}}{\tilde{\mu}_{\mathrm{ht}}\mathrm{L_{ht}}}, && \text{(A3)}\\
\frac{1}{R_t} &= \beta\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{h,t+1}}}{\tilde{\mu}_{\mathrm{ht}}}\frac{\Gamma_{\mathrm{t}}}{\Gamma_{\mathrm{t+1}}}(1+\lambda_{\mathrm{a,t+1}}). && \text{(A4)}\\
\tilde{\mu}_{et} &= \frac{1}{\tilde{C}_{et}-\gamma_e\tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E_t}\frac{\beta\gamma_{\mathrm{e}}}{\tilde{\mathrm{C}}_{\mathrm{e,t+1}}\Gamma_{\mathrm{t+1}}/\Gamma_{\mathrm{t}}-\gamma_{\mathrm{e}}\tilde{\mathrm{C}}_{\mathrm{et}}}, && \text{(A5)}\\
\tilde{w}_t &= (1-\alpha)\tilde{Y}_t/N_t, && \text{(A6)}\\
1 &= \tilde{q}_{kt}\left[1-\frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}-\bar{\lambda}_I\right)^2 - \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}-\bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\right]\\
&\quad +\beta\Omega\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{e,t+1}}}{\tilde{\mu}_{\mathrm{et}}}\frac{\mathrm{Q_t}\Gamma_{\mathrm{t}}}{\mathrm{Q_{t+1}}\Gamma_{\mathrm{t+1}}}\tilde{\mathrm{q}}_{\mathrm{k,t+1}}\left(\frac{\tilde{\mathrm{I}}_{\mathrm{t+1}}}{\tilde{\mathrm{I}}_{\mathrm{t}}}\frac{\mathrm{Q_{t+1}}\Gamma_{\mathrm{t+1}}}{\mathrm{Q_t}\Gamma_{\mathrm{t}}}-\bar{\lambda}_{\mathrm{I}}\right)\left(\frac{\tilde{\mathrm{I}}_{\mathrm{t+1}}}{\tilde{\mathrm{I}}_{\mathrm{t}}}\frac{\mathrm{Q_{t+1}}\Gamma_{\mathrm{t+1}}}{\mathrm{Q_t}\Gamma_{\mathrm{t}}}\right)^2, && \text{(A7)}\\
\tilde{q}_{kt} &= \beta\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{e,t+1}}}{\tilde{\mu}_{\mathrm{et}}}\left[\alpha(1-\phi)\frac{\tilde{\mathrm{Y}}_{\mathrm{t+1}}}{\tilde{\mathrm{K}}_{\mathrm{t}}}+\tilde{\mathrm{q}}_{\mathrm{k,t+1}}\frac{\mathrm{Q_t}\Gamma_{\mathrm{t}}}{\mathrm{Q_{t+1}}\Gamma_{\mathrm{t+1}}}(1-\delta)\right]+\frac{\tilde{\mu}_{\mathrm{bt}}}{\tilde{\mu}_{\mathrm{et}}}\theta_{\mathrm{t}}\mathrm{E_t}\tilde{\mathrm{q}}_{\mathrm{k,t+1}}\frac{\mathrm{Q_t}}{\mathrm{Q_{t+1}}}, && \text{(A8)}\\
\tilde{q}_{lt} &= \beta\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{e,t+1}}}{\tilde{\mu}_{\mathrm{et}}}\left[\alpha\phi\frac{\tilde{\mathrm{Y}}_{\mathrm{t+1}}}{\mathrm{L_{et}}}+\tilde{\mathrm{q}}_{\mathrm{l,t+1}}\right]+\frac{\tilde{\mu}_{\mathrm{bt}}}{\tilde{\mu}_{\mathrm{et}}}\theta_{\mathrm{t}}\mathrm{E_t}\tilde{\mathrm{q}}_{\mathrm{l,t+1}}\frac{\Gamma_{\mathrm{t+1}}}{\Gamma_{\mathrm{t}}}, && \text{(A9)}\\
\frac{1}{R_t} &= \beta\mathrm{E_t}\frac{\tilde{\mu}_{\mathrm{e,t+1}}}{\tilde{\mu}_{\mathrm{et}}}\frac{\Gamma_{\mathrm{t}}}{\Gamma_{\mathrm{t+1}}}+\frac{\tilde{\mu}_{\mathrm{bt}}}{\tilde{\mu}_{\mathrm{et}}}. && \text{(A10)}\\
\tilde{Y}_t &= \left(\frac{Z_tQ_t}{Z_{t-1}Q_{t-1}}\right)^{-\frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}}[L_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}]^{\alpha}N_t^{1-\alpha}, && \text{(A11)}\\
\tilde{K}_t &= (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t}+\left[1-\frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}-\bar{\lambda}_I\right)^2\right]\tilde{I}_t, && \text{(A12)}\\
\tilde{Y}_t &= \tilde{C}_{ht}+\tilde{C}_{et}+\tilde{I}_t, && \text{(A13)}
\end{aligned}
$$

$$\bar{L} = L_{ht} + L_{et}, \tag{A14}$$

$$\alpha\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(L_{et} - L_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} - \frac{\tilde{B}_t}{R_t}, \tag{A15}$$

$$\tilde{B}_t = \theta_t E_t\left[\tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}L_{et} + \tilde{q}_{k,t+1}\tilde{K}_t\frac{Q_t}{Q_{t+1}}\right]. \tag{A16}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

B.2. **Log-linearized equilibrium system.** Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (A1)-(A16) around the steady state. Define the constants $\Omega_h \equiv (g_\gamma - \beta(1+\bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv (g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$. The log-linearized equilibrium conditions are given by

$$\begin{aligned}\Omega_h\hat{\mu}_{ht} &= -[g_\gamma^2 + \gamma_h^2\beta(1+\bar{\lambda}_a)]\hat{C}_{ht} + g_\gamma\gamma_h(\hat{C}_{h,t-1} - \hat{g}_{\gamma t}) \\ &\quad -\beta\bar{\lambda}_a\gamma_h(g_\gamma - \gamma_h)\mathrm{E}_t\hat{\lambda}_{a,t+1} + \beta(1+\bar{\lambda}_a)g_\gamma\gamma_h\mathrm{E}_t(\hat{C}_{h,t+1} + \hat{g}_{\gamma,t+1}),\end{aligned} \tag{A17}$$

$$\hat{w}_t + \hat{\mu}_{ht} = \hat{\psi}_t, \tag{A18}$$

$$\begin{aligned}\hat{q}_{lt} + \hat{\mu}_{ht} &= \beta(1+\bar{\lambda}_a)\mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}\right] \\ &\quad +[1-\beta(1+\bar{\lambda}_a)](\hat{\varphi}_t - \hat{L}_{ht}) + \beta\bar{\lambda}_a\mathrm{E}_t\hat{\lambda}_{a,t+1},\end{aligned} \tag{A19}$$

$$\hat{\mu}_{ht} - \hat{R}_t = \mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1} - \hat{g}_{\gamma,t+1}\right], \tag{A20}$$

$$\Omega_e\hat{\mu}_{et} = -(g_\gamma^2 + \beta\gamma_e^2)\hat{C}_{e,t} + g_\gamma\gamma_e(\hat{C}_{e,t-1} - \hat{g}_{\gamma t}) + \beta g_\gamma\gamma_e\mathrm{E}_t(\hat{C}_{e,t+1} + \hat{g}_{\gamma,t+1}), \tag{A21}$$

$$\hat{w}_t = \hat{Y}_t - \hat{N}_t, \tag{A22}$$

$$\begin{aligned}\hat{q}_{kt} &= (1+\beta)\Omega\lambda_k^2\hat{I}_t - \Omega\lambda_k^2\hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\ &\quad -\beta\Omega\lambda_k^2\mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}],\end{aligned} \tag{A23}$$

$$\begin{aligned}\hat{q}_{kt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}(\hat{\mu}_{bt} + \hat{\theta}_t) + \frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1} - \hat{g}_{\gamma,t+1}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\right)\mathrm{E}_t\hat{\mu}_{\mathrm{e,t+1}} \\ &\quad +\frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1}) + \beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{\mathrm{Y}}_{\mathrm{t+1}} - \hat{\mathrm{K}}_{\mathrm{t}}),\end{aligned} \tag{A24}$$

$$\begin{aligned}\hat{q}_{lt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}(\hat{\theta}_t + \hat{\mu}_{bt}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\theta\right)\mathrm{E}_t\hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{g}_{\gamma,t+1}) \\ &\quad +\beta\mathrm{E}_t\hat{q}_{l,t+1} + (1-\beta-\beta\bar{\lambda}_a\bar{\theta})\mathrm{E}_t[\hat{\mathrm{Y}}_{\mathrm{t+1}} - \hat{\mathrm{L}}_{\mathrm{et}}],\end{aligned} \tag{A25}$$

$$\hat{\mu}_{et} - \hat{R}_t = \frac{1}{1+\bar{\lambda}_a}\left[\mathrm{E}_t(\hat{\mu}_{\mathrm{e,t+1}} - \hat{\mathrm{g}}_{\gamma,\mathrm{t+1}}) + \bar{\lambda}_{\mathrm{a}}\hat{\mu}_{\mathrm{bt}}\right], \tag{A26}$$

$$\hat{Y}_t = \alpha\phi\hat{L}_{e,t-1} + \alpha(1-\phi)\hat{K}_{t-1} + (1-\alpha)\hat{N}_t - \frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}[\hat{g}_{zt} + \hat{g}_{qt}], \tag{A27}$$

$$\hat{K}_t = \frac{1-\delta}{\lambda_k}[\hat{K}_{t-1} - \hat{g}_{\gamma t} - \hat{g}_{qt}] + \left(1 - \frac{1-\delta}{\lambda_k}\right)\hat{I}_t, \tag{A28}$$

$$\hat{Y}_t = \frac{\tilde{C}_h}{\tilde{Y}}\hat{C}_{ht} + \frac{C_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t, \tag{A29}$$

$$0 = \frac{L_h}{\bar{L}}\hat{L}_{ht} + \frac{L_e}{\bar{L}}\hat{L}_{et}, \tag{A30}$$

$$\alpha\hat{Y}_t = \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t + \frac{\tilde{q}_l L_e}{\tilde{Y}}(\hat{L}_{et} - \hat{L}_{e,t-1}) + \frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t - \hat{R}_t), \tag{A31}$$

$$\begin{aligned}\hat{B}_t &= \hat{\theta}_t + g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1}) \\ &\quad + \left(1 - g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\right)\mathrm{E}_t(\hat{q}_{k,t+1} + \hat{K}_t - \hat{g}_{q,t+1}).\end{aligned} \tag{A32}$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{\nu}_{zt} - \hat{v}_{z,t-1}, \tag{A33}$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt} + \hat{\nu}_{qt} - \hat{v}_{q,t-1}, \tag{A34}$$

$$\hat{g}_{\gamma t} = \frac{1}{(1-(1-\phi)\alpha)}\hat{g}_{zt} + \frac{(1-\phi)\alpha}{(1-(1-\phi)\alpha)}\hat{g}_{qt}. \tag{A35}$$

The technology shocks follow the processes

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{z,t-1} + \hat{\varepsilon}_{zt}, \tag{A36}$$

$$\hat{\nu}_{zt} = \rho_{\nu_z}\hat{\nu}_{z,t-1} + \hat{\varepsilon}_{\nu_z t}, \tag{A37}$$

$$\hat{\lambda}_{qt} = \rho_q\hat{\lambda}_{q,t-1} + \hat{\varepsilon}_{qt}, \tag{A38}$$

$$\hat{\nu}_{qt} = \rho_{\nu_q}\hat{\nu}_{q,t-1} + \hat{\varepsilon}_{\nu_q t}. \tag{A39}$$

$$\tag{A40}$$

There preference shocks follow the processes

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{a,t-1} + \hat{\varepsilon}_{at}, \tag{A41}$$

$$\hat{\varphi}_t = \rho_\varphi\hat{\varphi}_{t-1} + \hat{\varepsilon}_{\varphi t}, \tag{A42}$$

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1} + \hat{\varepsilon}_{\psi t}. \tag{A43}$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \hat{\varepsilon}_{\theta t}. \tag{A44}$$

We solve the 19 equations (A17)-(A35) for the 19 unknowns in the vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]'.$$

The state variables consist of the predetermined variables and the exogenous forcing processes summarized in the vector

$$s_t = [\hat{C}_{h,t-1}, \hat{C}_{e,t-1}, \hat{I}_{t-1}, \hat{L}_{e,t-1}, \hat{K}_{t-1}, \hat{B}_{t-1}, \hat{\lambda}_{zt}, \hat{\nu}_t, \hat{\lambda}_{qt}, \hat{\mu}_t, \hat{\lambda}_{at}, \hat{\varphi}_t, \hat{\psi}_t, \hat{\theta}_t]'$$

We use the gensys code to solve the model.

Table 1. Prior Distributions and Posterior Modes of Structural Parameters

	Prior					Posterior
Parameter	Distribution	a	b	Low	High	Mode
γ_h	Beta(a,b)	1.00	2.00	0.0256	0.7761	0.400
γ_e	Beta(a,b)	1.00	2.00	0.0256	0.7761	0.643
Ω	Gamma(a,b)	1.00	0.50	0.1027	5.9940	0.2873
$100(\bar{\lambda}_q - 1)$	Gamma(a,b)	1.86	3.01	0.100	1.500	1.250
$100(g_\gamma - 1)$	Gamma(a,b)	1.86	3.01	0.100	1.500	0.500
β	Simulated			0.9800	0.9860	0.9843
$\bar{\lambda}_a$	Simulated			0.0051	0.0255	0.0109
$\bar{\varphi}$	Simulated			0.0	0.0569	0.0318
α	Simulated			0.3	0.3	0.3000
ϕ	Simulated			0.1075	0.1085	0.1082
δ	Simulated			0.0134	0.0332	0.0201
$\bar{\theta}$	Simulated			0.3142	0.3176	0.3171

Note: "Low" and "High" denote the bounds of the 5%-95% probability interval for the prior distributions.

Table 2. Prior Distributions and Posterior Modes of Shock Parameters

	Prior					Posterior
Parameter	Distribution	a	b	Low	High	Mode
ρ_a	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9149
ρ_z	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.4286
ρ_{ν_z}	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.4389
ρ_q	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.7380
ρ_{ν_q}	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.6021
ρ_φ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9998
ρ_ψ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9842
ρ_θ	Beta(a,b)	1.0000	2.0000	0.0256	0.7761	0.9814
σ_a	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.1259
σ_z	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0003
σ_{ν_z}	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0004
σ_q	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0030
σ_{ν_q}	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0035
σ_φ	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0625
σ_ψ	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0070
σ_θ	Inverse Gamma(a,b)	0.5943	0.0011	0.0005	0.2000	0.0129

Note: "Low" and "High" denote the bounds of the 5%-95% probability interval for the prior distributions.

Table 3. Forecast Error Variance Decomposition for Aggregate Quantities

Horizon	Patience	Ngrowth	Nlevel	Bgrowth	Blevel	Housing	Labor	Financial
				Output				
4Q	22.3269	14.4750	7.0389	3.3182	0.5897	29.0590	20.8181	2.3741
8Q	19.2229	21.0221	3.7512	1.7742	0.2988	29.5674	21.9627	2.4007
16Q	13.5771	29.4414	2.3272	2.7363	0.1890	25.8225	23.9051	2.0014
32Q	8.7294	40.3615	1.5364	4.9189	0.1232	18.5813	24.3761	1.3732
				Investment				
4Q	40.2661	2.6957	4.8279	1.8303	1.9818	38.0776	7.2110	3.1096
8Q	35.3476	4.9471	2.8716	5.5500	1.1340	39.1008	7.8288	3.2201
16Q	27.2783	6.8208	2.1006	17.8547	0.9147	34.3479	7.9286	2.7544
32Q	22.8417	8.4017	1.7207	28.0810	0.7631	28.4345	7.4885	2.2688
				Consumption				
4Q	4.4949	47.9935	4.9197	0.2371	0.1738	0.1971	41.9595	0.0243
8Q	2.0440	52.0420	2.3753	1.0834	0.1094	2.4615	39.7125	0.1718
16Q	3.4132	51.9448	1.2430	0.6390	0.0828	7.0669	35.0960	0.5143
32Q	2.9037	56.1710	0.6674	2.9272	0.0358	6.7266	30.0932	0.4752

Note: Columns $2-9$ correspond to the shocks: the intertemporal preference shock (Patience), the permanent shock to neutral technology (Ngrowth), the transitory shock to neutral technology (Nlevel), the permanent shock to biased technology (Bgrowth), the transitory shock to biased technology (Blevel), the housing demand shock (Housing), the labor supply shock (Labor), and the financial shock (Financial).

Table 4. Forecast Error Variance Decomposition for Asset Prices

Horizon	Patience	Ngrowth	Nlevel	Bgrowth	Blevel	Housing	Labor	Financial
Capital price								
4Q	9.7650	0.3043	6.6364	73.2255	1.6753	6.5135	1.4690	0.4111
8Q	4.2220	0.1443	2.8773	88.2831	0.8367	2.8230	0.6347	0.1789
16Q	1.6622	0.0566	1.1261	95.3977	0.3292	1.1090	0.2487	0.0705
32Q	0.7203	0.0245	0.4877	98.0059	0.1426	0.4807	0.1078	0.0306
Land price								
4Q	5.8548	1.2335	0.3596	0.0070	0.0103	91.3092	1.2127	0.0129
8Q	4.7886	1.4929	0.1908	0.0334	0.0065	92.2562	1.2087	0.0228
16Q	3.5171	1.8523	0.1058	0.0250	0.0046	93.1698	1.2821	0.0433
32Q	2.0720	2.3474	0.0593	0.1236	0.0024	94.0833	1.2691	0.0428
Excess Return								
4Q	0.9805	0.0222	0.5362	0.1612	0.6567	92.0455	0.0884	5.5093
8Q	0.9251	0.1311	0.5283	0.5603	0.6914	91.9810	0.0952	5.0875
16Q	1.5853	0.3650	0.5748	2.0160	0.5937	90.2515	0.1892	4.4244
32Q	1.8709	0.6008	0.6191	4.0389	0.4901	88.3429	0.2740	3.7634

Note: Columns $2-9$ correspond to the shocks: the intertemporal preference shock (Patience), the permanent shock to neutral technology (Ngrowth), the transitory shock to neutral technology (Nlevel), the permanent shock to biased technology (Bgrowth), the transitory shock to biased technology (Blevel), the housing demand shock (Housing), the labor supply shock (Labor), and the financial shock (Financial).

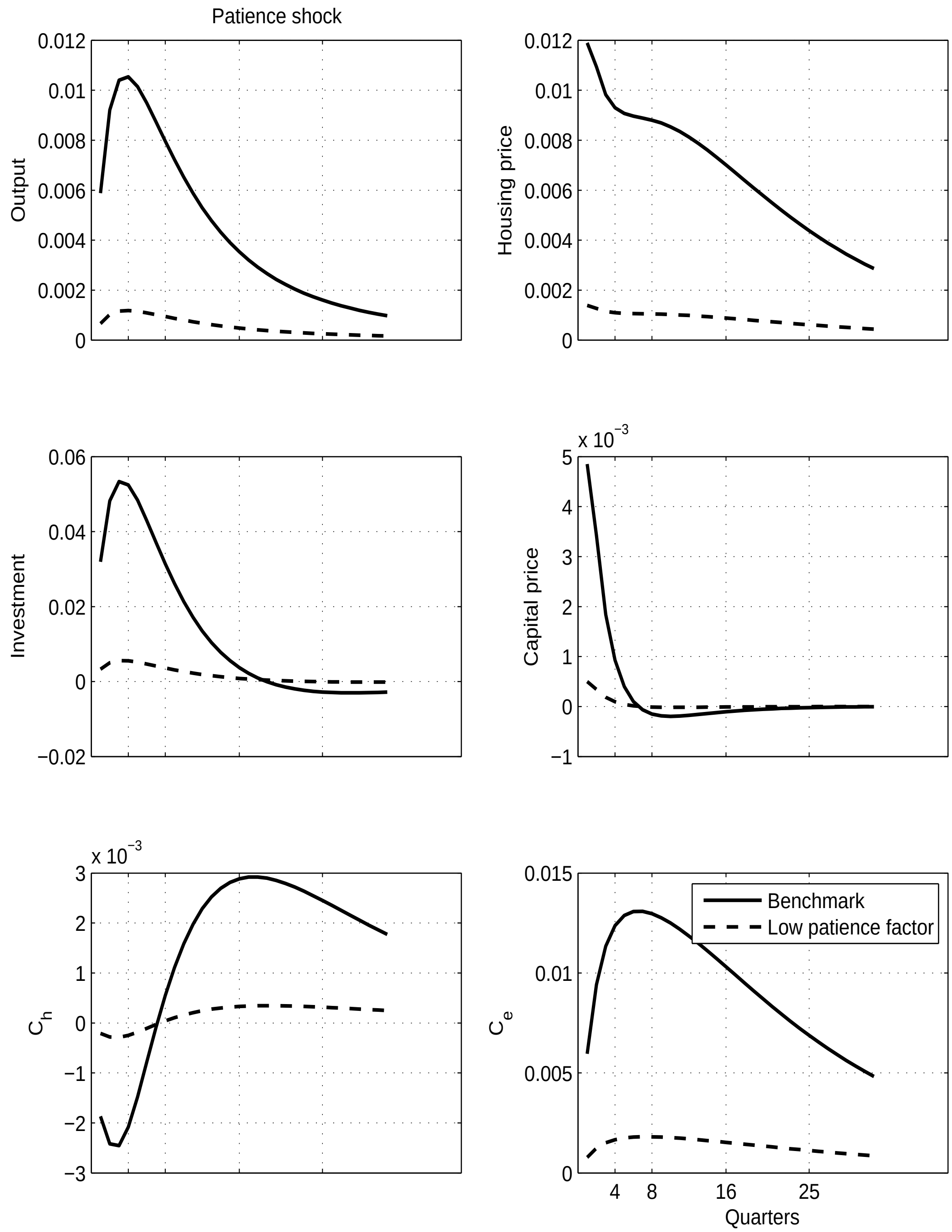

FIGURE 1. Impulse responses to a patience shock.

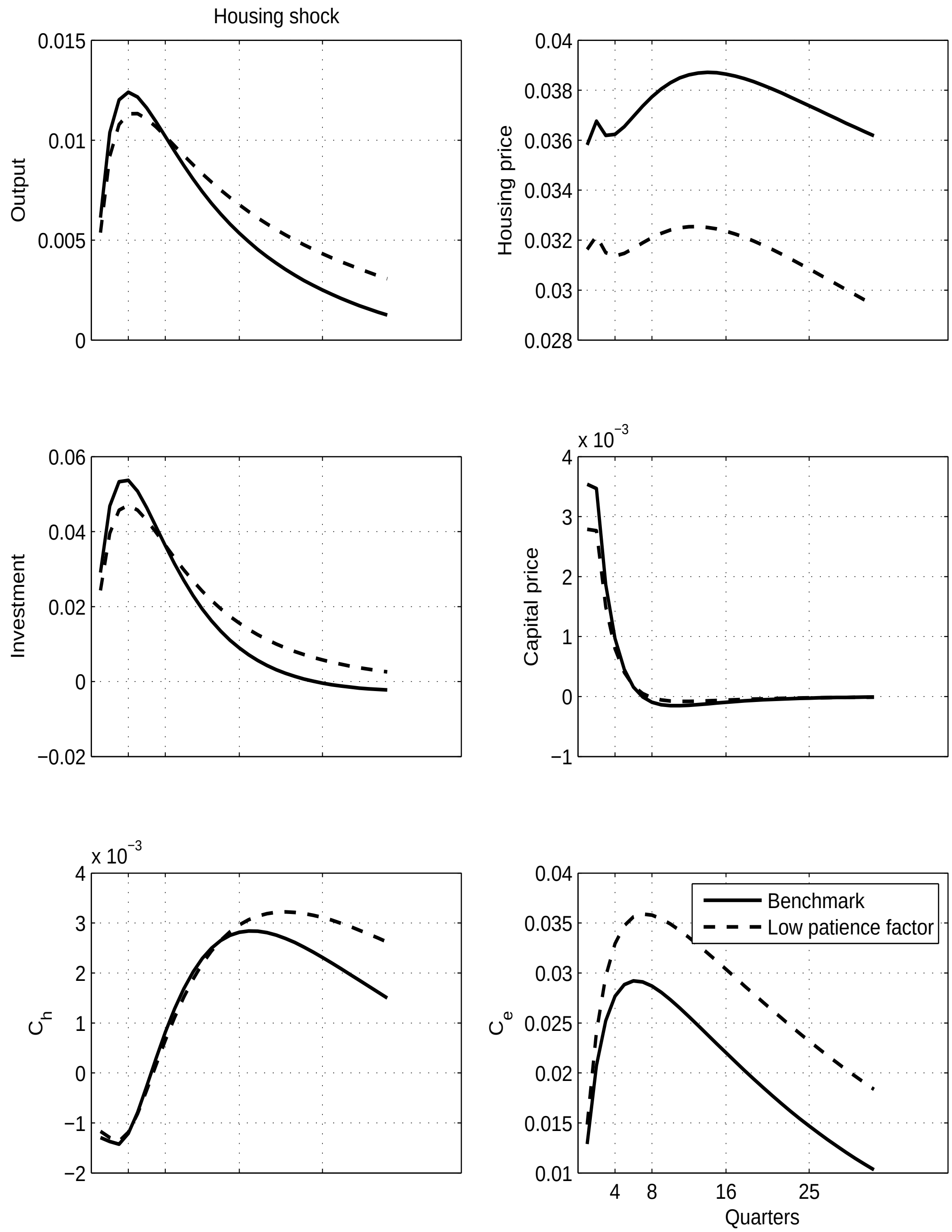

FIGURE 2. Impulse responses to a housing demand shock.

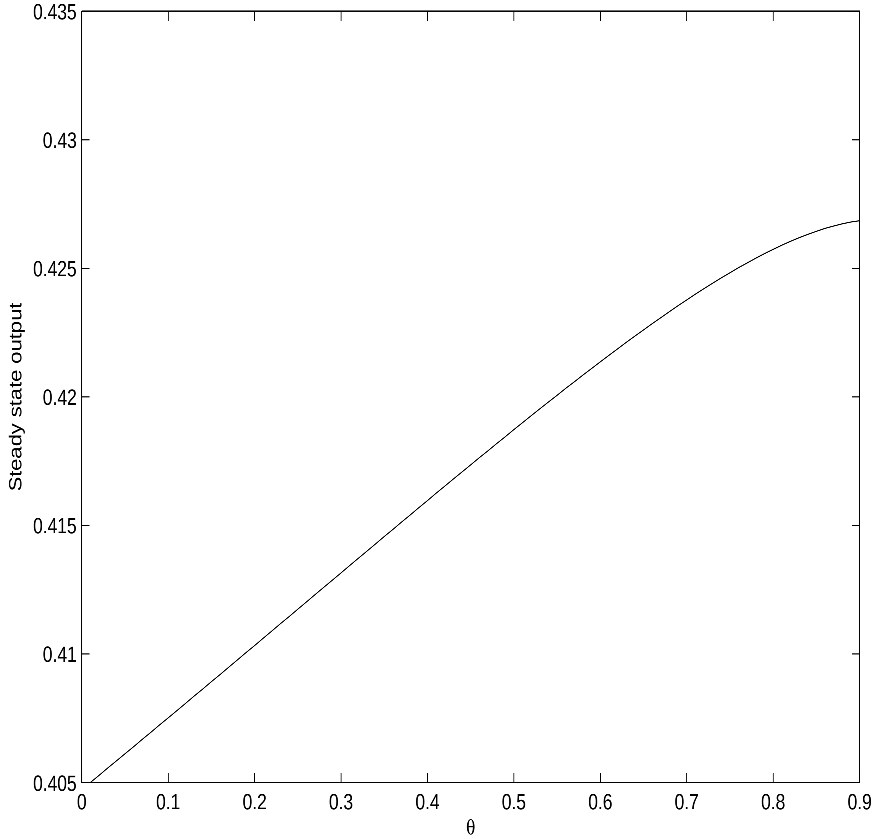

Figure 3. Effects of raising borrowing capacity on steady-state output.

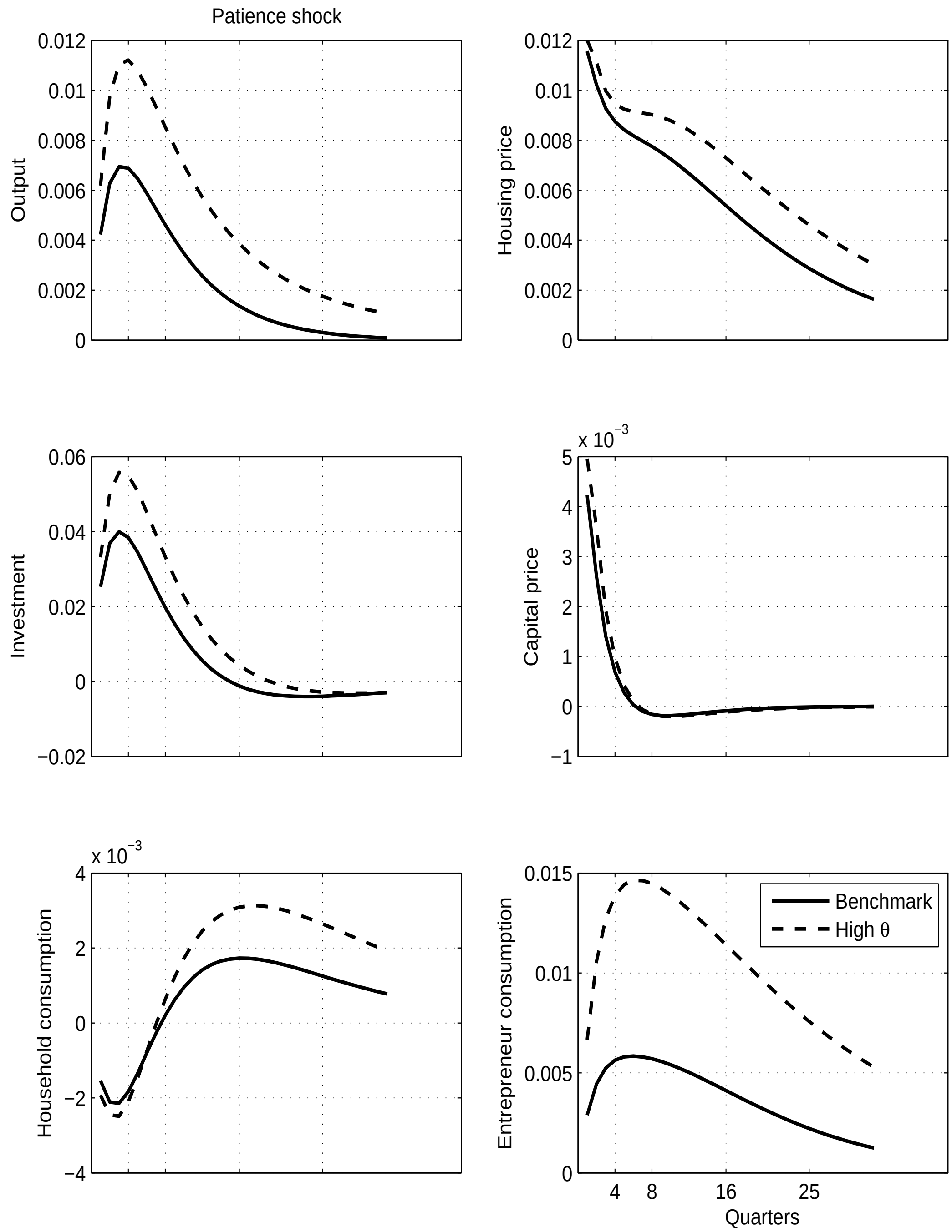

FIGURE 4. Impulse responses to a patience shock.

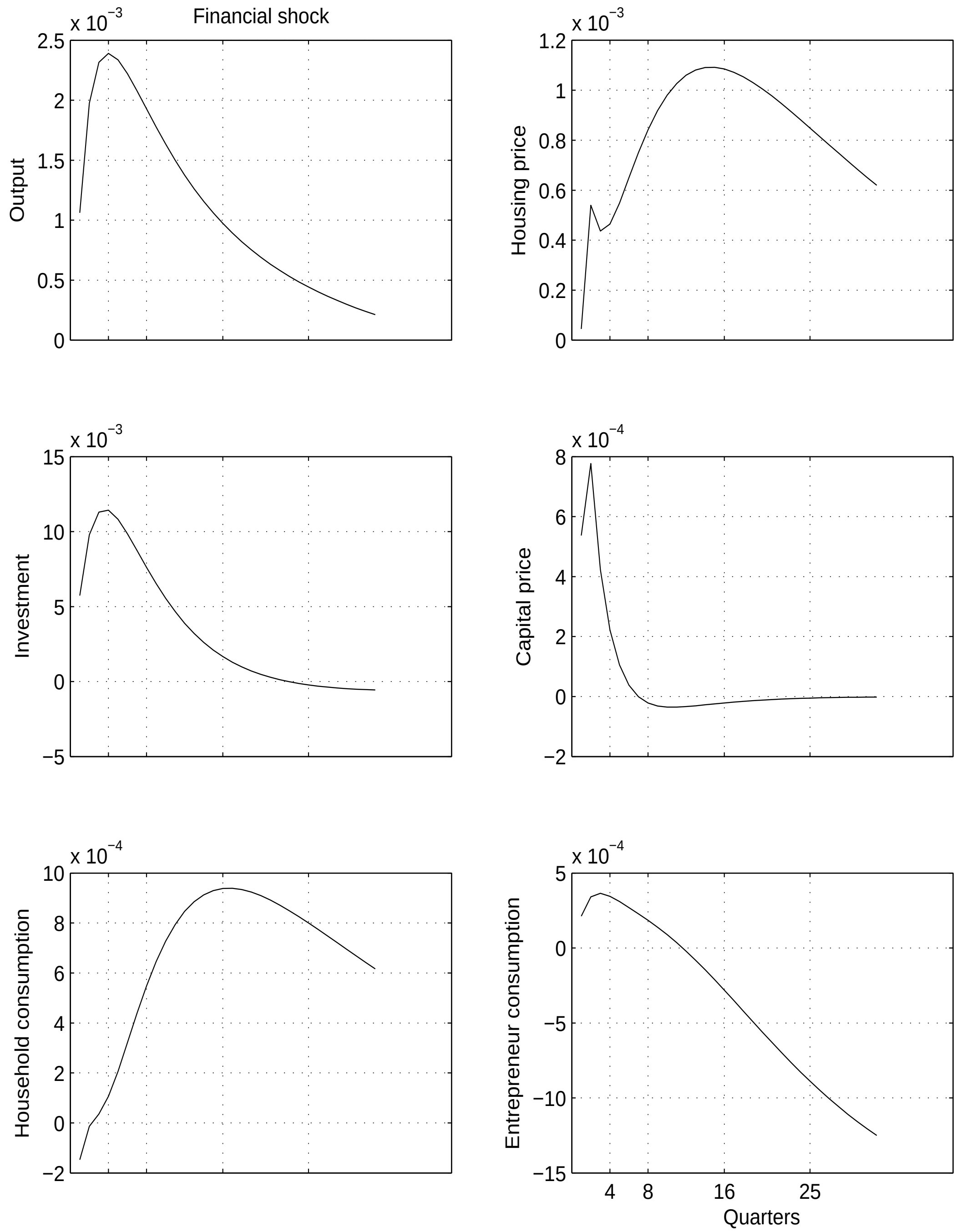

FIGURE 5. Impulse responses to a financial shock.

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