Positive Online Feedback, Trust Accumulation, and Efficiency*

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Abstract

Most feedback on eBay is positive. Based on the idea that buyer's feedback conveys information about seller's honesty, this paper shows that the overwhelmingly positive feedback indicated by empirical evidence may be an outcome of an equilibrium with a low productive efficiency. Specifically, I show that a seller who is currently not trusted by buyers will provide products with low values, but will also set lower prices to signal the product quality and accumulate positive ratings. After gets negative ratings, the seller does not want to be stuck with a bad record and would prefer to give up some potential flow payoff for a good reputation, since buyers only trust sellers with a sufficient number of positive ratings and are only willing to pay high prices to those sellers. Those periods during which sellers sell low quality products at low prices are viewed as "trust accumulation periods." This paper further shows that more positive ratings are usually companied with less efficient production, in which case the seller needs to undergo a lengthier trust accumulation period.

Keywords*: Online Feedback, Fairness, Signalling, Trust Accumulation.*
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1 Introduction

Most online transactions happen between strangers[1]. In an online transaction, buyers are usually not able to observe product quality when the transaction take place, which leads to an information asymmetry. According to Akerlof's well-known 1970 paper, due to asymmetric information, sellers with high quality products will exit the market. Therefore, to decrease the asymmetry and build trust between sellers and buyers, feedback systems in which buyers are allowed to rate and comment on each transaction are widely used.

Empirical evidence shows that feedback on eBay is overwhelmingly positive: Resnick and Zeckhauser (2003) find that 99.1% of the ratings are positive based on their data; in addition, Cabral and Hortacsu (2010) show that in their sample, a seller gets 4.9 negative and 1,625 positive ratings on average. This paper endeavors to answer the following question: Does the transactional feedback provide enough incentives for the seller to sell high quality products? The answer is negative. This paper shows that the overwhelmingly positive feedback indicated by empirical evidence may be an outcome of an equilibrium in which more low quality products are transacted.

To understand the online transaction story, we first need to understand when buyers rate positively and what information is conveyed by positive ratings. In their empirical studies, Jin and Kato (2006) find that feedback on actual quality is rare, and feedback mainly conveys information about the seller's inclination to cheat. [2] Buyers are not required to leave ratings after each transaction, so free riding is possible when leaving feedback is cost-positive; hence our assumptions about buyers' rating incentives should also be consistent with the fact that sometimes buyers are willing to rate transactions even when there are positive rating costs. With these concerns in mind, this paper draws its assumptions about buyer rating incentives on Rabin (1993) who incorporates the idea of "fairness" into game theory. According to psychological evidence that people are kind to those who are kind to them, whereas hurt those who hurt them, Rabin assumes that people care about "fairness" and derive utilities from reciprocity or retaliation behaviors. With this idea, if a seller is thought to be honest, he is regarded as being "nice" and may receive a positive rating; but if a seller is perceived to have cheated, he is regarded as being "malicious" and may receive a negative rating. This idea

[1]Resnick and Zeckhauser(2003) show that 89% of all buyer-seller pairs conducted only one transaction during the five-month period data on eBay.

[2]Jin and Kato (2006) study online auctions in eBay, in which the signals from a seller are usually the descriptions of products. Besides the original auction format, there is another form of online transaction on eBay, called "Buy it Now". With the "Buy it Now" format, sellers set prices before transactions, so prices can be also regarded as signals sellers send to buyers. In this paper we study the "Buy it Now" format and discuss the role of prices in online transactions.

is also consistent with the fact that sometimes buyers are willing to post ratings regardless of the incentive for free riding. When the rating cost is strictly positive, a buyer may not rate if he thinks the seller is *moderately* kind or *moderately* bad to him; but is triggered to rate when he thinks the seller is *very* kind or *very* bad to him[3].

This paper studies online transactions by using repeated games played between a long-lived seller and many short-lived buyers. In addition, since many online transaction websites, such as eBay and Amazon, provide easy access for new sellers[4], it is assumed that the seller can discard his name at any time and re-enter the community with a new identity at no cost. To induce a notion of productive efficiency, I assume that there are two technologies the seller can choose to use at the beginning of each period: A high technology which generates high quality products and a low technology which generates low quality products. The high technology is costly but more productively efficient in the sense that it yields more expected surplus. Buyers do not observe the technology, so the seller has a possibility to cheat.

The first contribution of this paper is to provide empirically testable results in terms of the dynamic relationship between seller reputation and prices. Most existing empirical studies which discuss the relationship between transaction prices and seller reputation mainly focus on how reputation affects prices by analyzing cross section data. However, the concern that price may affect rating implies that there is an endogeneity problem, and it is not surprising that the effects of feedback on price are usually vague. Dellarocas (2003) summarizes some main conclusions of related empirical studies and shows that the effects of feedback on price are ambiguous, since different studies focus on different components of eBay's complex feedback profile and often reach different conclusions. In addition, Cabral and Hortacsu (2010) show that even though a 1% increase in the percentage of negative feedback correlates with

[3]There are alternative assumptions for buyer rating incentives and we do not necessarily need the fairness assumptions. The results of this paper are robust to the alternative assumptions which satisfy the property that ratings depend on buyer's *expectation* about seller's honesty conditional on the realized product values. People may instead argue that ratings do not depend on the expectation but on the realized consumer's surplus, i.e., the product value minus the price; and rating is positive only when this surplus is positive. However, Resnick and Zeckerhauser (2003) find that items which do not match the description are more likely to receive neutral rather than negative feedback, reflecting that buyers may have thought discrepancies are honest mistakes on the part of sellers.

In addition, the case in which buyers always rate can be viewed as a special case in this paper. Resnick and Zeckhauser (2003) also examine the question of why some buyers leave feedback regardless of free riding, and suggest three intuitive explanations: The buyer may do it as part of some quasi-civic duty, as a courtesy, or because they expect reciprocity from the seller in the future. Though this explains why sometimes buyers do not free ride, explanations do not assess what information is conveyed through the buyer's feedback. In fact, for example, if we assume that buyers rate because they regard rating as part of a quasi-civic duty, and they rate positively when they think sellers do not cheat but negatively when they think sellers do cheat, then the main results of this paper still hold.

[4]For example, only an email account and a credit card are required to be a new seller on eBay.

a 7.5% decrease in price, the estimates have a relatively low level of statistical significance. Cabral and Hortacsu also state that it is hard to get clear results using cross section data.

In this paper, it is predicted that after bad ratings are recorded, prices will decrease, whereupon good ratings will accumulate. With a bad rating record, the seller is not trusted by buyers and buyers will only accept low prices from him. Then the seller is willing to give buyers some surplus in order to improve their ratings and reputation, because he is aware that only with a sufficient number of positive ratings will buyers trust and be willing to pay high prices to him. Notice that this prediction depends crucially on the possibility that a seller is able to accumulate positive ratings by setting low prices and being honest. We call those periods in which sellers provide low quality products, set lower prices, and get positive ratings "trust accumulation periods."

In addition, since the rules of online transactions allow the seller to discard his name and re-enter the community with a clean record at no cost, the seller may discard his name after receiving bad ratings. However, given the cost to induce buyers to give positive ratings is small, in *any* equilibrium a seller with a new name must set low prices and accumulate good ratings for a certain period of time. A seller with a new name needs to experience many periods with low flow payoffs to convince buyers to accept higher prices.

The main result of this paper is that negative ratings usually appear less often when the production is less efficient. With this result, the overwhelmingly positive feedback found in empirical studies per se may be an outcome of an equilibrium with a low productive efficiency. When production is less efficient, the seller provides high quality products less frequently, so buyers are cautious to accept high prices; hence the seller needs to spend more time accumulating trust in order to convince buyers to accept high prices. As we discuss above, in the trust accumulation periods, though the seller provides low quality products, he almost always gets positive ratings. This breaks the positive correlation between good ratings and high product quality.

In Section 5, I discuss the case when there are two types of seller: The first is the normal type who makes choices between two technologies; the other is the commitment type who always uses the high technology. As we argued above, if feedback conveys information about seller's honesty, the normal type can always get positive ratings by setting low prices when he produces low quality products. For this reason, product quality cannot be identified by public rating records. As a result, a seller's type may not be revealed in the long run when monitoring is imperfect, which contrasts with results found by Cripps, Mailath, and Samuelson (2004). Two examples are provided to show that seller's type may or may not be revealed in the long run when monitoring is imperfect.

The rest of the paper is organized as follows. The model setup is in Section 2. Section 3 characterizes equilibrium and describes how trust is built. Section 4 studies the relationship between positive feedback and productive efficiency. Section 5 discusses the variation with a commitment type of seller. Section 6 concludes.

2 The model

There are a long-lived seller and many short-lived buyers in a community with an infinite horizon. The discount factor for the seller is δ. There are two technologies the seller can choose to use at the beginning of each period: the high technology H and the low technology L. By using technology i, $i = H, L$, the distribution of the product value v is $F_i(v)$, $i = H, L$. Let $q^i = E_i[v]$. Assume $F_L(v)$ is distributed on $[0, 1]$, $F_H(v)$ is distributed on $[\alpha, 1]$ with $\alpha > 0$, and that the distributions satisfy MLRP, that is, $f_H(v)/f_L(v)$ is increasing in v. Let $\xi = F_L(\alpha)$.

There is a cost c^i to use i, with $c^H = c$, $c^L = 0$, and $q^H - c > q^L > 0$. The product is non-storable. The technology is not observable by buyers, but in each period the buyer can observe the ratings of the seller left by all the previous buyers. Each rating can be either positive (+) or negative (−). Define $\Theta = \cup_{l=0,\ldots,\infty}\Theta^l$ as the set of rating records, where $\Theta^l = \{+, -\}^l$ is the set of rating records with length l, and $\Theta^0 = \{E\}$ denotes the empty record (E). A $r \in \Theta$ is called a rating record.

In each period, the seller chooses the technology and sets a price; then the buyer decides whether to buy based on the price and the seller's rating record. We assume that the buyer can observe *all* the rating history but only the *current* price. After the transaction takes place, the buyer observes the product value v and then is allowed to rate once. Define the record transition functions τ_+ and τ_- as a mapping from Θ to Θ, where $\tau_+(r)$ is the new record of the seller at the beginning of the next period if the buyer rates "+"; and $\tau_-(r)$ if the buyer rates "−". The stage game is described as in Figure 1.

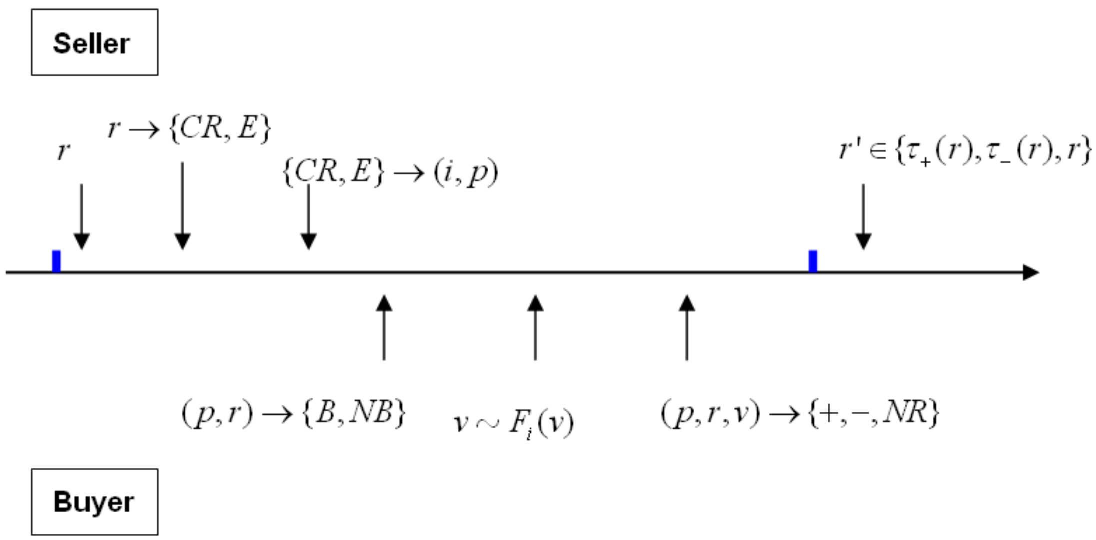

Figure 1: Time line

The buyer does not observe calendar time, but has a common prior belief Ψ over the time in which they are likely to enter the game after observing r. We assume that $\Psi(t = 0|r = E) > 0$. The strategy of the buyer is: (1) purchasing strategy, which is a mapping from prices and rating records to purchasing decisions:

$$s: \ \mathbb{R}^{+} \times \Theta \rightarrow \{0, 1\}$$

where $s(p, r) = 1$ if the buyer purchases and $s(p, r) = 0$ if he does not; and (2) rating strategy:

$$\phi: \ \mathbb{R}^{+} \times \Theta \times [0, 1] \rightarrow \{+, -, NR\}$$

which is a mapping from prices, rating records, and product values (v) to rating actions. Denote ϕ^{+} and ϕ^{-} as the probability of rating positively and negatively, respectively.

Suppose that the buyer will buy the product when the expected value of the product conditional on (r, p) is no less than the price offered by the seller. At this moment, the belief of the buyer that the technology is i is denoted as $\lambda(i|r, p)$. We assume that the buyer has reciprocity concerns when he rates the seller. Following the idea of kindness function from Rabin (1993), let $\kappa_{SB}(p, i) = k \cdot (q^{i} - p)$ be the kindness the seller to the buyer when the seller uses technology i and offers price p. In addition, the kindness the buyer to the seller (κ_{BS}) is assumed to be 1 if the buyer rates $+$, -1 if he rates $-$, and 0 if he does not rate.

Let c_r be the rating cost to the buyer. Assume that the buyer's utility function after purchasing is:

$$U(p,v,\phi;r) = v - p - c_r(\phi^+ + \phi^-) + \beta E\left[\kappa_{SB}(p,i)|p,v,r\right] \cdot \kappa_{BS}(\phi^+,\phi^-)$$

where $\beta > 0$ is the parameter representing how much reciprocity concern the buyer has. Notice that the buyer cannot observe i, so he will take the expectation based on the information he has, i.e., (p,v,r). Denote the belief of buyer that the technology is i at this moment as $\pi(i|p,v,r)$.

If the buyer rates positively, his utility is $v - p - c_r + \beta E\left[\kappa_{SB}(p,i)|p,v,r\right]$; if he rates negatively, his utility is $v - p - c_r - \beta E\left[\kappa_{SB}(p,i)|p,v,r\right]$; and if he does not rate, his utility is $v - p$. Therefore, the buyer will rate positively if $E\left[\kappa_{SB}(p,i)|p,v,r\right] \geq c_r/\beta$; not rate if $-c_r/\beta \leq E\left[\kappa_{SB}(p,i)|p,v,r\right] < c_r/\beta$; and rate negatively if $E\left[\kappa_{SB}(p,i)|p,v,r\right] < -c_r/\beta$. Let $a = c_r/k\beta$. Notice that if the buyer knows the technology is i for sure, he will rate positively if $p \leq q^i - a$, negatively if $p > q^i + a$, and not rate if $p \in (q^i - a, q^i + a]$. We can regard the parameter a as the cost for a seller to induce a positive rating.

Since the buyer only purchases the product if $p \leq q^i$ when he knows i exactly, he does rate negatively when he knows i exactly in equilibrium. In addition, we assume $f_L(\alpha)/f_H(\alpha) \leq 1$. This property as well as the linear kindness function guarantee that the seller will not be negatively rated when he uses the high technology[5]. This provides a clean base for the following discussion.

At the beginning of each period, the seller can decide whether to keep his current rating record (CR) or discard and replace it with an empty one (E). We also refer to the replacement as "free name change" as it is costless. The name change is not detectable by the buyers, so the seller can always pretend to be a new seller[6]. The seller observes calendar time so his private information includes the name change decisions, technologies, prices, and ratings he gets in all periods. Denote the seller's private history at the beginning of period t by $z_t \in Z_t$, where

$$z_t \in Z_t \quad = \quad \left[\{CR, E\} \times \{H, L\} \times \mathbb{R}^+ \times \{B, NB\} \times \{+, -, NR\}\right]^t$$

[5] The buyer only purchases the product when $\lambda(H|r,p)q^H + (1 - \lambda(H|r,p))q^L \geq p$. After observing v, the buyer's belief is updated to

$$\pi(H|p,v,r) = \frac{\lambda(H|r,p)}{\lambda(H|r,p) + (1 - \lambda(H|r,p))\frac{f_L(v)}{f_H(v)}}$$

If the seller uses the high technology, $v \geq \alpha$, so $\pi(H|p,v,r) \geq \lambda(H|r,p)$. With linear kindness function this implies that $E\left[\kappa_{SB}(p,i)|p,v,r\right] \geq 0$.

[6] In this paper, I assume that the seller always chooses to keep the current record (CR) if he is indifferent between CR and E.

In period t, let the seller's behavior strategy be: (1) name change:

$$x_t: \ Z_t \to \{CR, E\}$$

and (2) price and product:

$$\sigma_t: \ Z_t \times \{CR, E\} \to \triangle(\{H, L\} \times \mathbb{R})$$

Let $\{\mathcal{Z}_t\}_{t=0}^{\infty}$ denote the filtration on $[\{CR, E\} \times \{H, L\} \times \mathbb{R}^+ \times \{B, NB\} \times \{+, -, NR\}]^{\infty}$ induced by the private history of the seller. A strategy profile $\{x, \sigma, s, \phi\}$ induces a probability measure M over $\Omega_0 = [\{CR, E\} \times \{H, L\} \times \mathbb{R}^+ \times \{B, NB\} \times \{+, -, NR\}]^{\infty}$. Every r_t is measurable with respect to $\mathcal{Z}_t$, but does not necessarily have length t since one name change is allowed in each period.

Since the buyer is playing the public strategy, the seller will have a public strategy as a best reply. Therefore, we restrict our attention to Markov strategies. Let $W(r)$ be the continuation payoff of the seller if he keeps the current record r in the current period. Let $V(r) = \max\{W(r), V(E)\}$ be the continuation payoff of the seller, since the seller will discard his current record and replace it with a clean one if $W(r) < V(E)$. More specifically,

$$\begin{aligned} W(r) &= \max_{p,i} \big\{ s(p,r)\{p - c^i + \delta V(\tau_+(r)) E[\phi^+(p,v,r)|i] + \delta V(\tau_-(r)) E[\phi^-(p,v,r)|i] \\ &\quad + \delta V(r) \big[1 - E[\phi^-(p,v,r)|i] - E[\phi^+(p,v,r)|i]\big] \} + (1 - s(p,r)) \delta V(r) \big\} \end{aligned}$$

The seller does not observe v, so he will take the expectation of the buyer's rating conditional on the technology he uses. Specifically, given (p, r),

$$\begin{aligned} E[\phi^+(p,v,r)|i] &= Pr\{E[k(q^i - p)|p,v,r] \geq c_r/\beta\} \\ E[\phi^-(p,v,r)|i] &= Pr\{E[k(q^i - p)|p,v,r] < -c_r/\beta\} \end{aligned}$$

The equilibrium notion used here is stationary Markov Perfect Equilibrium. Now let us define an equilibrium.

Definition (Equilibrium)**.** $(x, \sigma; s, \phi; \lambda; \pi)$ is an equilibrium if (x, σ) and (s, ϕ) are best responses to each other, given the belief (λ, π); and (λ, π) is consistent with Bayes' rule given $(x, \sigma; s, \phi)$ on-equilibrium path.

Here we are looking for the equilibria in which $V(\tau_+(r)) \geq V(r)$ at any $r \in R$, that is, a positive rating is always valuable. Since the product is non-storable, the assumption that

$q^L > 0$ implies that in each period it is always optimal for the seller to offer a low and positive price such that the buyer will accept the price.

Lemma 1. *In any equilibrium where* $V(\tau_+(r)) \geq V(r)$ *at any* r*,* $s(p,r) = 1$ *for any* r*.*

Proof. Firstly, given a price which induces $s(p,r) = 0$, the seller will get $\delta V(r)$. However, note that $s(q^L, r) = 1$, that is, no matter what the buyer's belief is, the buyer is going to buy the product when $p = q^L$ at r; and with any belief, $p = q^L$ will not trigger a negative rating. So by using L and setting $p = q^L$, the seller will get:

$$q^L + \delta V(\tau_+(r)) E[\phi^+(p,v,r)|L] + \delta V(r)(1 - E[\phi^+(p,v,r)|L])$$

which is strictly greater than $V(r)$ if $V(\tau_+(r)) \geq V(r)$. Therefore, given any r, using the low technology and setting $p = q^L$ strictly dominates any p inducing $s(p,r) = 0$. □

This lemma implies we can only focus on the following problem for the seller:

$$W(r) = \max_{p,i} w(p,i;r)$$

where

$$\begin{aligned} w(p,i;r) &= p - c^i + \delta\{V(\tau_+(r))E[\phi^+(p,v,r)|i] + V(\tau_-(r))E[\phi^-(p,v,r)|i] \\ &\quad + V(r)[1 - E[\phi^+(p,v,r)|i] - E[\phi^-(p,v,r)|i]]\} \end{aligned}$$

Then let us start to discuss the equilibrium.

3 Equilibrium characterization

3.1 Continuation payoff

We start to characterize equilibrium by showing the upper and lower bounds of continuation payoff $V(r)$ in any equilibrium. We will use these bounds to discuss the incentive compatibility conditions for equilibrium characterization.

The lower bound is obvious, since the seller can use the low technology each period and set prices at q^L, $V(r) \geq \frac{q^L}{1-\delta}$. Lemma 2 shows the upper bound. From Lemma 2, we have $V(r) \leq \frac{q^H - c}{1-\delta}$, implying that seller's continuation payoff can not exceed the surplus generated by the most efficient technology.

Lemma 2. *In any equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ for any r, we have $V(r) \leq \frac{q^H - c}{1-\delta}$ for any $r \in \Theta$.*

Proof. Suppose not, i.e., in an equilibrium there exists some $r^0 \in R$, such that $V(r^0) > \frac{q^H-c}{1-\delta}$. Then following the strategy profile in that equilibrium, there must exist $r^1 \in R$, reachable from r^0 with positive probability such that: (1) The flow payoff at r^1 is strictly greater than $q^H - c$; and (2) $V(r^1) \geq V(r^0) > \frac{q^H-c}{1-\delta}$.

Since at r^1 the flow payoff is strictly greater than $q^H - c$, the seller must use the low technology with positive probability but sell it with a price higher than $q^H - c$. This implies that both technologies are used with strictly positive probabilities. Denote the price as $p(r^1)$. If $p(r^1) > q^H - a$, then $V(r^1) = p(r^1) - c + \delta V(r^1)$ in equilibrium, since the seller feels indifferent between using technology H and L. But this implies $V(r^1) = \frac{p(r^1)-c}{1-\delta} \leq \frac{q^H-c}{1-\delta}$ since $p(r^1) \leq q^H$, which contradicts $V(r^1) > \frac{q^H-c}{1-\delta}$.

If $p(r^1) \leq q^H - a$, we have

$$V(r^1) \leq p(r_1^1) - c + \delta V(\tau_+(r^1))$$

and then

$$V(r^0) \leq V(r^1) \leq q^H - c + \delta V(\tau_+(r^1))$$

From this inequality we know $V(\tau_+(r^1)) > \frac{q^H-c}{1-\delta}$ since $V(r^0) > \frac{q^H-c}{1-\delta}$. Let $r^2 = \tau_+(r_1)$. Since $V(r^2) > \frac{q^H-c}{1-\delta}$, following the same discussion for r^0, we will get $V(r^2) \leq q^H - c + \delta V(\tau_+(r^2))$ and then

$$V(r^0) \leq V(r^1) \leq q^H - c + \delta V(\tau_+(r_1)) \leq (1+\delta)(q^H - c) + \delta^2 V(\tau_+(r^2))$$

Repeat this process many times and we will have

$$V(r^0) \leq \frac{1-\delta^t}{1-\delta}(q^H - c) + \delta^t V(\tau_+(r_t))$$

Since $V(r)$ is bounded by $\frac{q^H}{1-\delta}$, the limit exists and then $V(r^0) \leq \frac{q^H-c}{1-\delta}$, a contradiction. □

Now let us look at the seller's incentive compatibility conditions. Suppose at (r, p) the buyer believes that the seller uses technology H with probability 1. By shirking to use technology L but still setting the price at p, the seller is able to get a flow payoff c in the current period, but the cost for cheating is that he may receive a bad rating and get lower future payoffs. Notice that $[\sup_r V(r) - \inf_r V(r)]$ is the largest loss in the future payoff, which is less than $\frac{1}{1-\delta}(q^H - q^L - c)$ by Lemma 2. Therefore, when the cost to use the high technology is too large relatively to $\frac{1}{1-\delta}(q^H - q^L - c)$, in any equilibrium the IC conditions can never be satisfied and only the low technology is used on the equilibrium path.

Proposition 1. *When $(1 + \frac{\xi\delta}{1-\delta})c > \frac{\xi\delta}{1-\delta}(q^H - q^L)$, in any equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ for any r, the seller only uses the low technology at any $r \in \Theta$ on-equilibrium path.*

Proof. Suppose not, i.e, when $(1+\frac{\xi\delta}{1-\delta})c > \frac{\xi\delta}{1-\delta}(q^H - q^L)$, there exists an equilibrium in which at some r, the high technology is used with a positive probability. The (IC) conditions for using the high technology require at least one of the following:

$$\begin{aligned} c &\leq \delta\xi[V(\tau_+(r)) - V(r)] \\ c &\leq \delta\xi[V(\tau_+(r)) - V(\tau_-(r))] \\ c &\leq \delta\xi[V(r) - V(\tau_-(r))] \end{aligned}$$

From Lemma 2, we have

$$c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c)$$

which leads to a contradiction. □

In the rest part of this paper, let us focus on the case when $(1 + \frac{\xi\delta}{1-\delta})c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L)$, since we care about the equilibrium at which technology H is used.

3.2 Trust accumulation

In this part I show how trust is built in equilibrium. If buyers believe that the seller only uses the low technology at r, the highest price buyers will accept is q^L. We say that buyers trust the seller at r if buyers believe that the seller uses the high technology with a positive probability and will accept a price greater than q^L at r.

In an equilibrium, there may exist some records on the equilibrium path at which the seller is not trusted by buyers. With such a record, the seller has two choices. First, the seller may set the price at q^L. However, since the buyer will not rate if $p = q^L$ when he believes that the seller only uses the low technology, the seller is stuck with the current record by setting $p = q^L$. Second, the seller may offer a price lower than q^L, giving up some flow payoff but expecting a good rating and more profit in the future. In this case the optimal price is $q^L - a$, since a price higher than $q^L - a$ does not induce a good rating and a price lower than $q^L - a$ is strictly dominated by $q^L - a$. The following lemma formalizes these arguments.

Lemma 3. *In any equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ for any r, for a record r on the* *equilibrium path, if (1)* $(p^L, L) \in arg\max_{(p,i)} w(p,i;r)$*, and (2)* $(p^L, H) \notin arg\max_{(p,i)} w(p,i;r)$*, we have either* $p^L = q^L - a$ *or* $p^L = q^L$*.*

Proof. Since $(p^L, L) \in arg\max_{(p,i)} w(p,i;r)$ and $(p^L, H) \notin arg\max_{(p,i)} w(p,i;r)$, the buyer believes that the seller uses the low technology with probability 1 at r when p^L is offered by the seller; thus $p^L \leq q^L$. Then notice that any (L, p) with $p < q^L - a$ is dominated by $(L, q^L - a)$ since by offering $p = q^L - a$, the flow payoff is greater and the future stage does not change given any belief. This implies $p^L \in [q^L - a, q^L]$. Next we claim that $p^L \notin (q^L - a, q^L)$. For any $p \in (q^L - a, q^L)$, if $(L, p) \in arg\max f(p,i;r)$, we have $V(r) = p + \delta V(r)$, and then $V(r) < \frac{q^L}{1-\delta}$, a contradiction. □

Notice that in any equilibrium, at some record r which is on the equilibrium path, if the low technology is used with strictly positive probability and $p^L(r) = q^L$, then we have $V(r) = q^L + \delta V(r)$, implying $V(r) = \frac{q^L}{1-\delta}$. Therefore, from Lemma 3 we concludes that in any equilibrium with $V(E) > \frac{q^L}{1-\delta}$, if at r the seller uses the low technology with probability 1, then $p(r) = q^L - a$ and the seller will get a positive rating for sure. Generally speaking, the periods in which the seller uses the low technology but sets lower prices to induce good ratings are viewed as "*trust accumulation periods*". When buyers do not trust the seller, they do not accept high prices, leading the seller to give up using the high technology; but the seller would like to give up some flow payoffs to accumulate good ratings.

When we allow free name change, a new seller's name is usually not trusted by the buyers. The reason is that it is hard to punish a seller with a new name when he cheats, since when we allow free name change the seller's worst continuation payoff is bounded below by $V(E)$. The following proposition states that in any equilibrium the seller with a new name will not be trusted at least in the first $n^* - 1$ periods.

Proposition 2. *Given* $(1 + \frac{\xi\delta}{1-\delta})c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L)$*, in any equilibrium satisfying* $V(\tau_+(r)) \geq V(r)$ *for any* r*, there exist at least* $n^* - 1$ *periods in which the seller uses the low technology on the equilibrium path starting from* E*, where* n^* *is the least natural number satisfying*

$$c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c + a)(1 - \delta^{n^*})$$

Proof. Let r be the first rating record at which the high technology is used with positive probability on some equilibrium path starting from E. Suppose there are n periods before r is reached on this path. Note that From Lemma 3, if $r \neq E$, we have $p = q^L - a$ in any period before r is reached. Then

$$V(E) = (1 - \delta^n) \cdot \frac{q^L - a}{1-\delta} + \delta^n V(r)$$

At r, the (IC) condition and the fact that any continuation payoff is bounded below by $V(E)$ imply that either $c \leq \xi\delta(V(r) - V(E))$ or $c \leq \xi\delta(V(\tau_+(r)) - V(E))$. If $p^H(r) > q^H - a$, the IC condition requires $c \leq \xi\delta(V(r) - V(E))$. Then plug $V(E) = (1-\delta^n) \cdot \frac{q^L - a}{1-\delta} + \delta^n V(r)$ into the inequality, we have

$$c \leq \xi\delta(1-\delta^n)[V(r) - \frac{q^L - a}{1-\delta}] \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c + a)(1-\delta^n)$$

Let n^* be the least natural number satisfying $c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c + a)(1-\delta^n)$. So if $p^H(r) > q^H - a$, there exist at least n^* periods in which only the low technology is used.

If $p^H(r) \leq q^H - a$, the IC condition requires $c \leq \xi\delta(V(\tau_+(r)) - V(E))$. Then plug $V(E) = (1-\delta^n) \cdot \frac{q^L - a}{1-\delta} + \delta^n V(r)$ into the inequality, and using the fact that $V(r) = p^H(r) - c + \delta V(\tau_+(r))$, we have

$$\delta^{n+1} \leq \frac{[V(\tau_+(r)) - \frac{q^L - a}{1-\delta}] - \frac{c}{\xi\delta}}{[V(\tau_+(r)) - \frac{q^L - a}{1-\delta}] + \frac{1}{\delta}(p^H(r) - c - q^L + a)}$$

Notice that $p^H(r) \leq q^H$, $V(\tau_+(r)) \leq \frac{q^H - c}{1-\delta}$, and the (IC) condition for using the high technology requires $p^H(r) - c \geq q^L - a$. Plug these inequalities into the expression above, we have

$$\delta^{n+1} \leq \frac{q^H - q^L - c + a - (1-\delta)\frac{c}{\xi\delta}}{q^H - q^L - c + a}$$

and then $c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c + a)(1 - \delta^{n+1})$. Since n^* is the least natural number satisfying $c \leq \frac{\xi\delta}{1-\delta}(q^H - q^L - c + a)(1-\delta^n)$, then if $p^H(r) \leq q^H - a$, there exist at least $n^* - 1$ periods in which only the low technology is used.

In sum, there exist at least $n^* - 1$ periods in which only the low technology is used in either case. □

Proposition 2 shows a necessary condition for *any* equilibrium. A seller with a new name needs to experience at least $n^* - 1$ trust accumulation periods starting from E. With those periods, it may not be profitable for the seller to cheat one time, destroy trust, and then replace the stained name with a clean one. The low flow payoffs in those trust accumulation periods can be regarded as the punishment for discarding a name.

In different equilibria, the number of trust accumulation periods the seller will undergo when he starts from E may vary. Roughly speaking, if there are a large number of trust accumulation periods starting from a new name, the seller may not discard his name when he gets negative ratings. With a bad rating record, the seller only discards his name if it is faster for him to get trust from buyers starting from a clean name than keeping the current name. Since both the number of trust accumulation periods and equilibrium prices vary in different equilibria, it is hard to give a general prediction for the time when the seller changes his name.

Due to free name change, there is an inefficiency in terms of production, since at least in the first $n^* - 1$ periods, the seller only uses the low technology. Because the high technology yields more expected surplus, we want the seller to use the high technology as much as possible. Here we define productive efficiency formally. Given an equilibrium strategy profile $\{x, \sigma, s, \phi\}$, let

$$\rho = (1-\delta)\sum_{t=0}^{\infty}\delta^t E\big[\int_p \sigma_t(H,p)dp\big]$$

be the discounted sum of the expected probability that the seller uses the high technology in all periods. An equilibrium which has a greater ρ is regarded as being more productively efficient.

Definition (Productive Efficiency)**.** Given the primitives, an equilibrium is more productively efficient than another if ρ is greater.

In an equilibrium, if there are exactly $n^* - 1$ periods in which the low technology is used on the equilibrium path, according to Proposition 2, it is the most efficient equilibrium. When a is small, we are able to get such an equilibrium. Define $g: R \to \{0, 1, ..., N, ...\}$ such that $g(r) = n$ if the most recent n ratings of record r are all positive but the $(n+1)$th recent rating is either negative or not existing. The following corollary describes one of the most efficient equilibria.

Corollary 1. *When $a \leq \frac{\delta^{n*}(q^H - q^L - c)}{1-\delta^{n^*}}$, there exists an equilibrium at which the seller only uses the low technology on the equilibrium path in exactly the first $n^* - 1$ periods.*

Proof. We construct the equilibrium as follows:

(1) When $g(r) < n^* - 1$, the seller uses the low technology at r, offers price $p(r) = q^L - a$, and the buyer will buy the product and rate positively, holding the belief that $\lambda(H|r,p) = 0$ for any p and $\pi(H|r,p,v) = 0$ for any (p,v).

(2) When $g(r) = n^* - 1$, the seller uses the high technology at r, offers $p^H(r) = q^L - a + c$. Then the buyer holds the belief that $\lambda(H|r,p) = 0$ if $p \neq q^L - a + c$, $\lambda(H|r,p) = 1$ if $p = q^L - a + c$; $\pi(H|r,p,v) = 0$ if $p \neq q^L - a + c$ or $v < \xi$, $\pi(H|p,v,r) = 1$ if $p = q^L - a + c$ and $v \geq \xi$. The buyer will buy the product and rate positively.

(3) When $g(r) \geq n^*$, the seller uses the high technology at r, offers $p^H(r) = q^H$. Then the buyer holds the belief that $\lambda(H|r,p) = 0$ if $p > q^H$, $\lambda(H|r,p) = 1$ if $p \leq q^H$. For any $p \leq q^H$, the buyer believes that $\pi(H|r,p,q) = 0$ if $v < \xi$, $\pi(H|r,p,q) = 1$ if $v \geq \xi$. The buyer will buy the product and not rate.

It is easy to check that the (IC) conditions holds. The last thing we need to make sure is that $V(E) \geq \frac{q^L}{1-\delta}$. Following the strategy profile, $V(E) = (1-\delta^{n^*}) \cdot \frac{q^L - a}{1-\delta} + \delta^{n^*} \cdot \frac{q^H - c}{1-\delta}$, and $V(E) \geq \frac{q^L}{1-\delta}$ will be guaranteed if $a \leq \frac{\delta^{n^*}(q^H - q^L - c)}{1-\delta^{n^*}}$. □

In the equilibrium constructed in Corollary 1, the seller uses the low technology in the first $n^* - 1$ periods (i.e., from period 0 to period $n^* - 2$), and then starts to use the high technology. In period $n^* - 1$, the seller uses technology H but the price is low, such that the flow payoff in period $n^* - 1$ is still $q^L - a$[7]. In period n^*, the price rises to q^H and prices stay at q^H in all the following periods. In this equilibrium, the seller will not get negative ratings on equilibrium path. On an off-equilibrium path, if the seller gets a negative rating, he will be punished by n^* periods of low flow payoffs.

In this equilibrium, once the seller gets a negative rating, his continuation payoff drops to $V(E)$, the smallest continuation payoff in this equilibrium. Intuitively, to prevent the seller from shirking, a negative rating needs to reduce the payoff significantly no matter how many good ratings the seller has before. To see this, consider a special case in which $a = 0$. The IC condition for the seller to use the high technology is $c \leq \delta\xi[V(\tau_+(r)) - V(\tau_-(r))]$. With a good rating record, $V(\tau_+(r))$ may be close to $\frac{q^H}{1-\delta}$; hence to provide the incentive for using the high technology, $V(\tau_-(r))$ need to be bounded above no matter how many positive ratings r contains.

The empirical studies by Cabral and Hortacsu (2010) show that when a seller first receives negative feedback, his weekly sales rate drops from a positive 5% to a negative 8%, which indicates that trust built by many positive ratings can be easily destroyed by a negative rating. If the buyer believes that a negative rating hurts the seller a lot, i.e., if he has enough power to punish the seller if the seller shirk, he tends to trust the seller and is willing to pay high prices. This provides the incentive for the seller to use the costly high technology. Therefore, an efficient rating system should always emphasize the most recent performance of sellers.

Finally, since a can be regarded as the cost to induce a good rating, when a is large, the seller loses the incentive to accumulate trust. The following corollary shows that when a is large, the seller only uses the low technology at any r on-equilibrium path.

Corollary 2. *If $a > \frac{\delta^{(n^*-1)}(q^H - q^L - c)}{1-\delta^{(n^*-1)}}$, then the seller only uses the low technology at any $r \in \Theta$ on-equilibrium path in any equilibrium.*

[7] In period $n^* - 1$, the flow payoff is not necessarily $q^L - a$. It can be a little bit greater than $q^L - a$, depending on the buyer's belief λ. However, in period $n^* - 1$, the flow payoff must be lower than $q^H - c$; otherwise, the incentive compatibility conditions are not satisfied.

Proof. According to Proposition 2, there are at least n^*-1 periods in which the low technology is used starting from E. From Lemma 3, in each of those periods, the price is either q^L or $q^L - a$. Given $a > 0$, if at E the price is $q^L - a$, then the flow payoff in the next period must be $q^L - a$, otherwise, the seller is stuck and $V(E) = q^L - a + \delta \cdot \frac{q^L}{1-\delta} < \frac{q^L}{1-\delta}$, a contradiction. The same logic applies to the following $n^* - 3$ periods. This implies that $q^L \leq (1-\delta)V(E) \leq [1-\delta^{(n^*-1)}](q^L - a) + \delta^{(n^*-1)}(q^H - c)$. By straightforward calculation we have $a \leq \frac{\delta^{(n^*-1)}(q^H-q^L-c)}{1-\delta^{(n^*-1)}}$, which violates the assumption. This implies that at E the price must be q^L, and the seller is stuck in the rating record E and only the low technology is used. □

4 Efficiency

This section discusses the relationship between the number of negative ratings and efficiency. I begin by showing why the rating profile is overwhelmingly positive. Intuitively, if a seller is not trusted by buyers at some rating record, he is not able cheat since buyers refuse to pay high prices. Then the seller will set low prices in the following periods to accumulate good ratings. As a result, in those periods he only gets positive ratings. If buyers believe that the seller provides high quality product with positive probabilities at a rating record r, in equilibrium the incentive compatibility condition for the seller to produce high value product must be satisfied. If the incentive compatibility conditions are satisfied with strict inequality, i.e., the flow payoff from shirking is strictly less than the loss in the discounted continuation payoff, the seller is not willing to shirk. The seller only shirks and gets negative ratings with positive probabilities when the incentive compatibility conditions are satisfied with equality, i.e., when the seller feels indifferent between using technology H and L.

In an equilibrium, let $\bar{V}$ be the supremum of $V(r)$, given r is on-equilibrium path. Lemma 4 states in an equilibrium, there are at least $\bar{n}$ positive ratings between two negative ratings.

Lemma 4. *Suppose $c < \frac{\delta\xi}{1-\delta}(q^H - q^L - c)$. Then in an equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ at any r, there are at least $\bar{n}$ positive ratings between two negative ratings on any equilibrium path, where $\bar{n}$ is the smallest n satisfying*

$$c \leq \frac{1}{1+\delta^{\bar{n}+1}} \cdot \frac{\delta\xi}{1-\delta}[(1-\delta)\bar{V} - q^L + a] \tag{1}$$

The formal proof is shown in the appendix and the intuition is provided as follows. Suppose the seller gets a negative rating in period t with record r_t and the next negative rating appears in period $t+n$ with record r_{t+n}. As we discuss above, if the seller gets a negative ratings at r_t, then the incentive compatibility condition at r_t must be satisfied with equality, i.e., either $c = \delta\xi[V(\tau_+(r_t)) - V(\tau_-(r_t))]$ or $c = \delta\xi[V(r_t) - V(\tau_-(r_t))]$. This implies that a negative rating reduces the continuation payoff by $c/\delta\xi$. If $c/\delta\xi$ is large, $V(\tau_-(r_t))$ almost hits the lower bound of the continuation payoff.

At r_{t+n}, we have either $c = \delta\xi[V(\tau_+(r_{t+n}))-V(\tau_-(r_{t+n}))]$ or $c = \delta\xi[V(r_{t+n})-V(\tau_-(r_{t+n}))]$. Since $V(\tau_-(r_{t+n}))$ is bounded below by $\frac{q^L}{1-\delta}$, either $V(\tau_+(r_{t+n}))$ or $V(r_{t+n})$ needs to be large enough to satisfy the equalities. This implies that there are a number of periods between $\tau_-(r_t)$ and r_{t+n} on the path to allow the continuation payoff to rise from $V(\tau_-(r_t))$, which is close to the lower bound of continuation payoff, to a high continuation payoff $V(r_{t+n})$ (or $V(\tau_+(r_{t+n})))$, since $V(\tau_-(r_t)) \geq (1-\delta^{n-1})\frac{q^L-a}{1-\delta} + \delta^{n-1}V(r_{t+n})$.

Lemma 4 helps us to compare different equilibria given the same primitives. From (1), when $\bar{V}$ is low $\bar{n}$ is large, implying that when the continuation payoff is small, negative ratings may appear less frequently. The basic intuition is illustrated by the example shown in Figure 2.

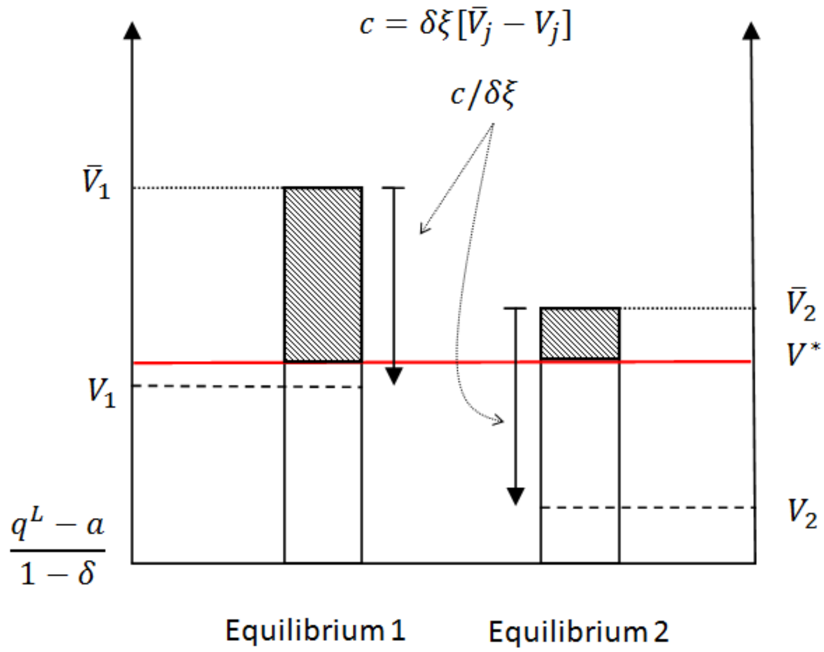

Figure 2

Given the primitives, in equilibrium j, $j = 1, 2$ as shown in Figure 2, when the rating record r is good such that the continuation payoff is almost $\bar{V}_j$, a negative rating reduces the continuation payoff by $c/\delta\xi$ to V_j, with $V_1 > V_2$. Let V^* be the continuation payoff decided by $c = \delta\xi[V^* - \frac{q^L}{1-\delta}]$. When $V_j < V^*$, $c > \delta\xi[V_j - \frac{q^L}{1-\delta}] \geq \delta\xi[V_j - V_j(E)]$, the incentive compatibility conditions for the seller to use the high technology do not satisfy, and buyers do not believe that the seller uses the high technology in the next a few periods. Therefore, in those periods, the seller uses the low technology and sets the price at $q^L - a$ to accumulate good ratings.

We claim that in equilibrium 2, the seller needs to experience more trust accumulation periods. Suppose that in equilibrium j, the number of the trust accumulation periods is n_j. The lowest n_2 must satisfy $(1-\delta)V_2 = (1-\delta^{n_2})(q^L - a) + \delta^{n_2}(1-\delta)V^*$, and the highest n_1 must satisfy $(1-\delta)V_1 = (1-\delta^{n_1})(q^L - a) + \delta^{n_1}(1-\delta)\bar{V}_1$. When

$$\frac{(1-\delta)V_2 - (q^L - a)}{(1-\delta)V^* - (q^L - a)} < \frac{(1-\delta)V_1 - (q^L - a)}{(1-\delta)\bar{V}_1 - (q^L - a)}$$

as shown in Figure 2, we have $\delta^{n_1} < \delta^{n_2}$ and $n_2 > n_1$.

In equilibrium 2, negative ratings appear less often because the seller is stuck in the trust accumulation periods most of the time. Since the seller only uses the low technology in trust accumulation periods, in this equilibrium the productive efficiency is low. Roughly speaking, given a record r, the continuation payoff is small when the seller uses the high technology less frequently in the future. Based on these arguments, we are able to relate efficiency to the appearance of negative ratings.

The following result implies that the first negative rating may appear more quickly if ρ is higher. The function $N_0(\cdot)$ shown below is uniform for all equilibria.

Proposition 3. *Given the primitives, there exists a decreasing function $N_0(\cdot)$, such that in any equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ at any $r \in \Theta$, the first negative rating starting from E arrives at least after $N_0(\rho)$ periods.*

In addition, given an equilibrium, when the calendar time is t, let

$$\rho_t = (1-\delta)\sum_{t'=t+1}^{\infty} \delta^{t'-t-1} E\Big[\int_p \sigma_{t'}(H,p)dp \Big| \mathcal{Z}_t\Big]$$

be the discounted sum of the expected probabilities of the seller to use the high technology in all periods after t, which is viewed as a $\mathcal{Z}_t$-measurable random variable. ρ_t denotes the expected efficiency after period t in an equilibrium. For each equilibrium, the following proposition states that given the seller gets a negative rating in period t, the next negative rating may arrive more quickly when the expected productive efficiency is higher.

Proposition 4. *In an equilibrium satisfying $V(\tau_+(r)) \geq V(r)$ at any $r \in \Theta$, suppose the* *seller gets a negative rating in period t, and let $\bar{n}_t$ be the number of positive ratings before the next negative rating appears. Then there exists a decreasing function $N(\cdot)$ such that $\bar{n}_t \geq N(\rho_t)$.*

These results show that online feedback in terms of product quality is necessary to encourage sellers to sell high quality products. Feedback on transactions does not provide sufficient incentives for sellers to provide high quality products.

5 Discussion

5.1 With a commitment type of seller

So far we assume that the seller can choose the technology at the beginning of each period. In this part we add another type of seller into the model: A seller who always uses the high technology (the commitment type). In this section, we are not trying to generally characterize equilibrium, but trying to use examples to illustrate if feedback mainly conveys information about seller's inclination to cheat, then the type of the seller may not be revealed in the long run. This result contrasts with results found by Cripps, Mailath, and Samuelson (2004). Because the opportunistic seller can get positive ratings by setting low prices when he produces low value products, product quality as well as the type of seller may not be identified by the public rating records in the long run.

5.1.1 Setup

There are two types of long-lived seller, one always uses the high technology (the commitment type), and the other can choose to use either the high or the low technology at the beginning of each period (the normal type). The prior belief is that the seller is the commitment type with probability μ_0 and is the normal type with probability $1-\mu_0$. Let

$$\begin{aligned} \tilde{x}_t &: \quad Z_t \to \{CR, E\} \\ \hat{x}_t &: \quad Z_t \to \{CR, E\} \end{aligned}$$

be the name change strategy in period t for each type of seller, respectively, where the tilde denotes the normal type and the hat denotes the commitment type. Following this notation, define

$$\tilde{\sigma}_t \quad : \quad Z_t \times \{CR, E\} \to \triangle(\{H, L\} \times \mathbb{R})$$
$$\hat{\sigma}_t \quad : \quad Z_t \times \{CR, E\} \to \triangle(\mathbb{R})$$

as the pricing and production strategy in period t, respectively. Note that the commitment type seller also needs to pay a cost c to produce the high quality product. Given μ_0, a strategy profile $(\tilde{\sigma}, \tilde{x}; \hat{\sigma}, \hat{x}, s, \phi)$ induces a probability measure M over $\Omega = \{n, cm\} \times [\{CR, E\} \times \{H, L\} \times \mathbb{R}^+ \times \{B, NB\} \times \{+, -, NR\}]^\infty$. The strategy profile $(\hat{\sigma}, \hat{x}, s, \phi)$ (resp., $(\tilde{\sigma}, \tilde{x}, s, \phi)$) determines a probability measure $\hat{M}$ (resp., $\tilde{M}$) over Ω when the seller is the commitment (resp., normal) type.

We assume that the short-lived buyers do not observe calendar time but have a common prior belief Ψ over the time in which they are likely to enter the game. As before, assume that $\Psi(t = 0|r = E) > 0$. With this assumption, buyer's information set must be in Θ. Let r_t be the public rating record in period t. Apparently, r_t is measurable with respect to $\mathcal{Z}_t$. Note that t may not be the length of the record since we allow for name change.

At the beginning of period t, the buyer holds the belief that the seller is the commitment type with probability $\mu_t(r_t)$, and that technology is i with probability $\lambda(i|p, r_t)$ after he observes the price offered by the seller. He will decide whether to accept the offer or not based on (p, r_t). If the transaction takes place, the buyer updates his belief and thinks the technology is i with probability $\pi(i|p, v, r_t)$ after he observes the product value v, and then evaluates $E[k(q^i - p)|p, v, r_t]$ and rates the transaction.

Following the literature, with the commitment type of seller, we call μ "reputation" of the seller. As before, since the buyer is playing public strategy, the seller will have a public strategy as a best reply. So we restrict attention to Markov strategies.

Definition (Equilibrium)**.** $(x, \sigma; s, \phi; \mu; \lambda; \pi)$ is an equilibrium if (x, σ) and (s, ϕ) are best responses to each other, given the belief (μ, λ, π); and (μ, λ, π) is consistent with Bayes' rule given $(x, \sigma; s, \phi)$ on-equilibrium path.

5.1.2 Trust accumulation

First we have a similar result to Lemma 3, which claims that if the seller uses the low technology and separates from the commitment type in prices, then he will set the price at $q^L - a$.

Lemma 5. *In any equilibrium, at a r on-equilibrium path, if the normal type of seller only uses technology L and sets the price at $p \notin supp\hat{\sigma}(r)$, then $p = q^L - a$ if $\tilde{V}(E) > \frac{q^L}{1-\delta}$.*

Proof. Since $p \notin supp\hat{\sigma}(r)$ and the normal type of seller only uses the low technology, the buyer believes that the seller uses the low technology with probability 1 with (p, r), thus $p \leq q^L$. Notice that any price less than $q^L - a$ is strictly dominated by $q^L - a$ with any belief, since the flow payoff is less and the future record is always $\tau_+(r)$. If $p \in (q^L - a, q^L]$, the buyer will not rate, and since p is optimal for the seller, we have $\tilde{V}(r) = p + \delta\tilde{V}(r)$, implying $\tilde{V}(r) = \frac{p}{1-\delta} \leq \frac{q^L}{1-\delta}$. This is not true since for any r, $\tilde{V}(r)$ is bounded below by $\tilde{V}(E) > \frac{q^L}{1-\delta}$. □

With $p = q^L - a$, the seller is going to get a positive rating for sure. This implies that separation periods act as "trust accumulation periods". If the seller uses the low technology, note that with the existence of commitment type, a normal type of seller is always able to mimic; and if he decides not to, he gives up some potential flow payoff for some future profit and must be rewarded in the future. This rewarding could be either from mimicking the commitment type in the future, or from the high surplus of using the high technology when he is trusted by buyers and has no incentive to shirk.

The idea is different from the case without the commitment type. Without the commitment type, before the trust is built, the buyer will not accept any price greater than q^L, and the seller is "forced" to set low prices. With the commitment type, price has a signalling effect during the trust accumulation periods: The seller who uses the low technology sets a different price from the seller who uses the high technology.

5.1.3 An equilibrium

In this subsection, we show an equilibrium at which the price signalling effect always exists. We show the existence of the equilibrium by construction. In the equilibrium constructed, strategy only depends on the ratings in the most recent N periods. In the equilibrium, the normal type of seller uses technology L if $g(r) < N$, separating itself from the commitment type in prices by setting $p^L(r) = q^L - a$, and accumulating positive ratings. When $g(r) \geq N$, the normal type of seller starts to use the high technology, and offers $p(r) = q^H$. The equilibrium is described as follows.

Example 1. Given $c < \frac{\xi\delta}{1-\delta}(q^H - q^L - c)$, there exist $a \geq 0$, N and p_n, $n = 0, ..., N-1$ such that the strategy profile described below is an equilibrium:

(1) When $g(r) = n < N$, the normal type of seller uses the low technology at r, offers price $p^L(r) = q^L - a$. The commitment type of seller offers a price $p^H(r) = p_n$. The buyer will buy the product and rate positively, holding the belief that $\lambda(H|r, p \neq p_n) = 0$, $\pi(H|r, p, v) = 1$ if $p = p_n$ and $v \geq \xi$, and $\pi(H|r, p, v) = 0$ otherwise.

(2) When $g(r) \geq N$, the normal type of seller uses the high technology at r, offers $p^H(r) = q^H$. Then the buyer holds the belief that $\lambda(H|r,p) = 0$ if $p > q^H$, $\lambda(H|r,p) = 1$ if $p \leq q^H$. For any $p \leq q^H$, the buyer believes that $\pi(H|p,v,r) = 0$ if $v < \xi$, $\pi(H|p,v,r) = 1$ if $v \geq \xi$. The buyer will buy the product and rate positively.

(3) The seller will not discard his record at any record.

The formal proof is shown in the appendix. When $g(r) \geq N$, to prevent mimicking we need to provide the incentive for the opportunistic seller to use technology H, hence the punishment for shirking and using technology L and then mimicking the commitment type needs to be large enough, i.e, $\tilde{V}(\tau_+(r)) - \tilde{V}(\tau_-(r))$ needs to be large enough. Here we let $\tilde{V}(\tau_+(r))$ be the highest continuation payoff $\frac{q^H - c}{1-\delta}$ when $g(r) \geq N$ and $\tilde{V}(\tau_-(r))$ be the lowest continuation payoff $\tilde{V}(E)$. This provides the largest punishment for shirking then mimicking.

When $g(r) < N$, since the opportunistic seller does not use the high technology, to prevent the normal type of seller from mimicking, we require $p^H(r)$ to be low. In this equilibrium, $p^H(r)$ may be greater than $q^L - a$ but not too much. When N is large, this implies that the commitment type will undergo a long time of negative flow profit if $p^H(r) - c < 0$. An implication here is that given the current proportion of commitment type is small, it is very hard to attract more commitment type of seller to enter the market[8].

It is obvious that $\mu_t = \mu_0$ at any t in this equilibrium. As we discuss before, this is because the product quality is not identified by public signals, since the opportunistic seller can use low prices to avoid bad ratings when he produces low quality products. Next let us add a disturbance into the model and check whether similar equilibrium exists.

5.1.4 With a disturbance

Now we add a disturbance into the model. Let

$$\phi^i_-(p,r) = pr\{E_i[k(q^i - p)|p,v,r] + \varepsilon < -c_r/\beta|i\}$$

be the probability of getting a negative rating, given the record r, price p, and technology i, where ε is a random variable which denotes the disturbance when the buyer evaluates the kindness of the seller to him. Similarly, let

$$\phi^i_{NR}(p,r) = pr\{-c_r/\beta \leq E_i[k(q^i - p)|p,v,r] + \varepsilon < c_r/\beta|i\}$$

[8] When μ_0 is small, pooling may not be supported as an equilibrium result in the first a few periods, since the pooling price p is bounded above by $q^H\mu + q^L(1-\mu)$. For a simple example, let us suppose $a = 0$ and consider the record E. A necessary condition for pooling at E is $p - q^L \geq \delta\xi[\tilde{V}(\tau_+(E)) - \tilde{V}(E)]$, but the upper bound for the left hand side goes to zero if $\mu_0 \to 0$.

and

$$\phi^i_+(p,r) = pr\{E_i[k(q^i - p)|p,v,r] + \varepsilon \geq c_r/\beta|i\}$$

be the probability of getting no rating and a positive rating, respectively.

Consider ε as a small negative disturbance. Denote the CDF (PDF) of ε as $F_\varepsilon(y)$ ($f_\varepsilon(y)$) with $y \in (-\infty, 0]$. And let

$$\chi(y) = \begin{cases} 1 & if\ y = 0 \\ 0 & if\ y < 0 \end{cases}$$

We say that $\varepsilon \to 0$ if $F_\varepsilon(y) \to \chi(y)$ pointwisely.

With the disturbance, no matter how kind the seller is to the buyer, he cannot guarantee himself a positive rating. Now let us consider a disturbance such as an unexpected package loss. The probability of package loss is independent of the seller's strategy. If the buyer will rate negatively for sure when he does not receive the package, the probability of getting a negative rating is independent of the kindness the seller is willing to show to the buyer. Specifically, assume

$$\varepsilon = \begin{cases} 0 & with\ prob\ 1 - \eta \\ -\infty & with\ prob\ \eta \end{cases}$$

With this assumption the disturbance is kind of "uniform" to the seller. When $\varepsilon = -\infty$ happens, the seller will get a negative rating. In this case, let us construct an equilibrium with the similar structure to Example 1.

Example 2. Assume ε as above. Given $c < \frac{\xi\delta(1-\eta)}{1-\delta(1-\eta)}(q^H - q^L - c)$, there exist $a \geq 0$, N and p_n, $n = 0, ..., N-1$ such that the strategy profile described below is an equilibrium:

(1) When $g(r) = n < N$, the normal type of seller uses the low technology at r, offers price $p^L(r) = q^L - a$. In addition, the commitment type of seller offers a price $p^H(r) = p_n$. The buyer will buy the product, rating positively if $\varepsilon = 0$, holding the belief that $\lambda(H|r, p \neq p_n) = 0$, $\pi(H|r,p,v) = 1$ if $p = p_n$ and $v \geq \xi$, and $\pi(H|r,p,v) = 0$ otherwise.

(2) When $g(r) \geq N$, the normal type of seller uses the high technology at r, offers $p^H(r) = q^H$. Then the buyer holds the belief that $\lambda(H|r,p) = 0$ if $p > q^H$, $\lambda(H|r,p) = 1$ if $p \leq q^H$. For any $p \leq q^H$, $\pi(H|p,v,r) = 0$ if $v < \xi$, $\pi(H|p,v,r) = 1$ if $v \geq \xi$. The buyer will buy the product, rating positively if $\varepsilon = 0$.

(3) The seller will not discard his record at any r.

The proof is shown in the appendix. In this equilibrium, following the strategy profile, in each period both types of seller have a probability η to get a negative rating. As a result, the belief of the buyer that the seller is the commitment type does not change in all periods, so $\mu_t(r) = \mu_0$ for any r.

Cripps, Mailath, and Samuelson (2004) suggest that with imperfect monitoring the true type of player will be revealed in the long run, but here we do not necessarily have this result, because the distribution of public signals does not satisfy the assumption of identification[9] The combination of different i ($i = H, L$) and p may induce the same distribution of the public rating history.

However, the next example shows an equilibrium at which $\mu_t \to 0$ $\tilde{M}$-almost surely. This comes from the fact that the distribution of ε always induces different distributions of ratings for the two types given any r.

Example 3. Suppose the probability distribution function of ε satisfies $f_\varepsilon(y) > 0$ for any $y \in (-\infty, 0)$, $f'_\varepsilon(y) > 0$ and $\lim_{y \to 0^-} f_\varepsilon(y) = \infty$. Then there exists some distribution of ε, with which there exist an equilibrium such that $\mu_t \to 0$ $\tilde{M}$-almost surely, but $x_t = CR$ in all periods. Formally, the strategy profile has the following structure:

(1) When $g(r) = n < N$, the normal type of seller uses the low technology at r, offers price $p_n^L \leq q^L - a$. In addition, the commitment type of seller offers a price $p_n^H \in (p_n^L, q^H - a]$. The buyer holds the belief that $\lambda(H|r, p \neq p_n^H) = 0$, $\pi(H|r, p, v) = 1$ if $v \geq \xi$ and $p = p_n^H$, and $\pi(H|r, p, v) = 0$ otherwise.

(2) When $g(r) \geq N$, the normal type of seller uses the high technology at r, offers $\tilde{p}_N$ and the commitment type of seller set price at $\bar{p}_N$, with $\bar{p}_N < \tilde{p}_N$. Then the buyer holds the belief that $\lambda(H|r, p) = 0$ if $p > q^H$, $\lambda(H|r, p) = 1$ if $p \leq q^H$; and for any p, $\pi(H|p, v, r) = 0$ if $v < \xi$, $\pi(H|p, v, r) = 1$ if $v \geq \xi$.

(3) The seller will not discard his record at any $r \in R$.

No name change So far we assume that the entry for a new name is free. If we do not allow for name change[10], it is easy to check that the strategy profiles described in the examples above are still equilibria. When name change is not allowed, following the standard arguments in reputation literature, μ_t is a M-almost sure convergent sequence. The following corollary claims that if μ_t is convergent to some positive number almost surely, then the two types of seller will have the same distribution of ratings almost surely. If the commitment type of seller gets positive ratings most of time, this result implies that the normal type of seller also gets positive ratings most of time.

[9] See Assumption 2 in in Cripps, Mailath, Samuelson (2004).

[10] In some online transaction website, the government issued ID is required for a new seller to register. An example is www.taobao.com, which is the most popular online transaction website in China.

Remark 1. If name discarding is not allowed, then in any equilibrium, we have

$$\lim_{t\to\infty} \mu_t \left| \sum_{i=1,2} \int \tilde{\sigma}(r_t)(i,p)\phi^i(p,r_t)dp - \int_p \hat{\sigma}(r_t)(p)\phi^i(p,r_t)dp \right| = 0$$

$\tilde{M}$-almost surely.

6 Conclusion and future extensions

Based on the idea that buyer feedback conveys information about seller honesty, this paper begins by showing how trust is built by feedback in terms of transactions. In equilibrium, the periods in which the seller uses the low technology but sets low prices act as "trust accumulation periods," since the seller almost always gets good ratings in those periods. The existence of the trust accumulation periods leads to the overwhelmingly positive rating profile shown in the existing empirical literature.

However, since good ratings can be induced by low product quality together with low prices, positive feedback fails to imply a high productive efficiency. I show that when the production is less productively efficient, negative ratings may appear less often. In addition, due to the same reason, when there is a commitment-type seller who always provides high quality products, the seller's type may not be identified by the public rating history in the long run when monitoring is imperfect.

There are some implications for effective online feedback systems. First, to induce a more efficient equilibrium, the posted feedback statistics should emphasize the most recent ratings from buyers. Second, feedback in terms of product quality is necessary to induce a high productively efficient equilibrium if buyer feedback mainly conveys information about seller honesty.

One possible future study could be to test the empirical predictions implied by this paper. One prediction is that after bad ratings are recorded, prices decrease but positive ratings accumulate. Theoretical extensions include checking the robustness of the main results with different variations, such as analyzing the auction format.

Appendix

Proof for Lemma 4

Proof. Firstly, in any equilibrium, at some r on equilibrium path, if the seller uses technology H or L with probability 1, then the probability of getting negative rating is zero, since $p \leq q^i$. The negative ratings can only appear when the seller uses both technologies with strictly positive probabilities and offers $p(r) > q^L + a$; i.e., both $(L, p(r)) \in arg\max w(p, i; r)$ and $(H, p(r)) \in arg\max w(p, i; r)$. This implies either

$$p(r) - c + \delta V(\tau_+(r)) = p(r) + \delta\xi V(\tau_-(r)) + \delta(1-\xi)V(\tau_+(r))$$

or

$$p(r) - c + \delta V(r) = p(r) + \delta\xi V(\tau_-(r)) + \delta(1-\xi)V(r)$$

i.e., either $c = \delta\xi[V(\tau_+(r)) - V(\tau_-(r))]$ or $c = \delta\xi[V(r) - V(\tau_-(r))]$.

Let r^1 be a record at which the seller uses both technologies with strictly positive probabilities. Given a rating path starting from $\tau_-(r^1)$, let r^2 be the first record at which the seller uses both technologies with strictly positive probabilities. Notice that at any record on the path from $\tau_-(r^1)$ to r^2, given the seller uses technology H (L) with probability 1, the buyer must rate at any record other than r^2, otherwise the seller will be stuck in that record and we cannot find such a r^2. This also implies that all the ratings between $\tau_-(r^1)$ and r^2 must be positive, because the seller will not get negative ratings given the seller uses technology H or L with probability 1. Since the ratings are all positive, the seller will not discard name at any record on the path from $\tau_-(r^1)$ to r^2, given $V(\tau_+(r)) \geq V(r)$ at any r.

The IC conditions require either $c = \delta\xi[V(\tau_+(r^j)) - V(\tau_-(r^j))]$ or $c = \delta\xi[V(r^j) - V(\tau_-(r^j))]$, $j = 1, 2$. Firstly let us suppose $c = \delta\xi[V(\tau_+(r^1)) - V(\tau_-(r^1))]$ and $c = \delta\xi[V(\tau_+(r^2)) - V(\tau_-(r^2))]$, and the proof for other cases are quite similar. From $c = \delta\xi[V(\tau_+(r^2)) - V(\tau_-(r^2))]$,

$$\begin{aligned} V(\tau_+(r^2)) &= \frac{c}{\delta\xi} + V(\tau_-(r^2)) \geq \frac{c}{\delta\xi} + V(E) \\ &\geq \frac{c}{\delta\xi} + \frac{q^L - a}{1-\delta} \end{aligned}$$

Suppose there are n periods between $\tau_-(r^1)$ and r^2. Notice $V(\tau_-(r^1)) \geq (1-\delta^{n+1})\frac{q^L-a}{1-\delta} +$

$\delta^{n+1}V(\tau_+(r^2))$. Thus

$$\begin{aligned} V(\tau_+(r^1)) - \frac{c}{\delta\xi} &= V(\tau_-(r^1)) \geq (1-\delta^{n+1})\frac{q^L-a}{1-\delta} + \delta^{n+1}V(\tau_+(r^2)) \\ &\geq (1-\delta^{n+1})\frac{q^L-a}{1-\delta} + \delta^{n+1}\left[\frac{c}{\delta\xi} + \frac{q^L-a}{1-\delta}\right] \\ &= \delta^{n+1}\frac{c}{\delta\xi} + \frac{1}{1-\delta}(q^L-a) \end{aligned}$$

And then

$$(1+\delta^{n+1})\frac{c}{\delta\xi} \leq [V(\tau_+(r^1)) - \frac{q^L-a}{1-\delta}] \leq \bar{V} - \frac{q^L-a}{1-\delta}$$

i.e.,

$$c \leq \frac{1}{1+\delta^{n+1}} \cdot \frac{\delta\xi}{1-\delta}[(1-\delta)\bar{V} - q^L + a]$$

The right hand side is increasing in n, so there exists some $\bar{n}$ such that the inequality holds when $n \geq \bar{n}$.

Here we discuss the case when the IC conditions are $c = \delta\xi[V(\tau_+(r^1)) - V(\tau_-(r^1))]$ and $c = \delta\xi[V(\tau_+(r^2)) - V(\tau_-(r^2))]$. If the IC conditions requires other equalities, following the similar steps, we can get similar inequalities as (1). The result shown in the lemma is got by summarizing all the inequalities. □

Proof for Proposition 3

Proof. First, note that for any t, given the strategy profile, the expected flow payoff of the seller in period t is $\int_p[(\sigma_t(H,p)+\sigma_t(L,p))p - \sigma_t(H,p)\cdot c]dp$. Since the buyer only purchases the product when the price does not exceed the expected product value, for any p in the support of σ_t,

$$p \leq \frac{\sigma_t(H,p)}{\sigma_t(H,p)+\sigma_t(L,p)} \cdot q^H + \frac{\sigma_t(L,p)}{\sigma_t(H,p)+\sigma_t(L,p)} \cdot q^L$$

thus the flow payoff is less than $\int_p[\sigma_t(H,p)(q^H-c) + (1-\sigma_t(H,p))\cdot q^L]dp$. Therefore,

$$\begin{aligned} V(E) &= \sum_{t'=0}^{\infty}\delta^{t'}E\Big[\int_p[(\sigma_{t'}(H,p)+\sigma_{t'}(L,p))p - \sigma_{t'}(H,p)\cdot c]dp\Big|\mathcal{Z}_t\Big] \\ &\leq \sum_{t'=t}^{\infty}\delta^{t'}E\Big[\int_p[\sigma_{t'}(H,p)(q^H-c) + (1-\sigma_{t'}(H,p))\cdot q^L]dp\Big|\mathcal{Z}_t\Big] \\ &= \frac{1}{1-\delta}\big[\rho(q^H-c) + (1-\rho)q^L\big] \end{aligned}$$

Now consider the record E and suppose the calendar time is 0. Given an equilibrium path, suppose the next period in which the seller gets a negative rating with a strictly positive probability is period n_0. Denote the record in period n_0 as r_{n_0}. The incentive compatibility condition at r_{n_0} is either $c = \delta\xi[V(\tau_+(r_{n_0})) - V(\tau_-(r_{n_0}))]$ or $c = \delta\xi[V(r_{n_0}) - V(\tau_-(r_{n_0}))]$. Let us take the case in which $c = \delta\xi[V(\tau_+(r_{n_0})) - V(\tau_-(r_{n_0}))]$ as an example and the the other case follows the similar proof.

First we claim that the ratings from period 0 to $n_0 - 1$ must be all positives. Since from period 0 to $n_0 - 1$, the seller gets negative ratings with probability zero, the only case we need to rule out is that the seller does not get any rating in some period. If it is true, then the seller is stuck in that record forever, and we are not able to find a period in which the seller gets a negative rating with a strictly positive probability.

Since $c = \delta\xi[V(\tau_+(r_{n_0})) - V(\tau_-(r_{n_0}))]$ and $V(\tau_-(r_{n_0})) \geq V(E)$, $V(\tau_+(r_{n_0})) \geq c/\delta\xi + V(E)$. Note that $V(E) \geq (1 - \delta^{n_0+2})\frac{q^L - a}{1-\delta} + \delta^{n_0+2} V(\tau_+(r_{t_0}))$. Therefore, $V(E) \geq (1 - \delta^{n_0+2})\frac{q^L-a}{1-\delta} + \delta^{n_0+2}[c/\delta\xi + V(E)]$, and then $V(E) \geq \frac{q^L-a}{1-\delta} + \frac{\delta^{n_0+2}}{1-\delta^{n_0+2}} \cdot \frac{c}{\delta\xi}$. With $V(E) \leq \frac{1}{1-\delta}\big[\rho(q^H - c) + (1 - \rho)q^L\big]$, we have

$$\frac{q^L - a}{1 - \delta} + \frac{\delta^{n_0+2}}{1 - \delta^{n_0+2}} \cdot \frac{c}{\delta\xi} \leq \frac{1}{1 - \delta}\big[\rho(q^H - c) + (1 - \rho)q^L\big]$$

Let $N_0(\rho)$ be the smallest n_0 satisfying the above inequality given ρ. The left hand side is decreasing in n_0 and the right hand side is increasing in ρ, so $N_0(\rho)$ is decreasing in ρ. □

Proof for Proposition 4

Proof. First, note that for any t, given the strategy profile, the expected flow payoff of the seller in period t is $\int_p [(\sigma_t(H,p) + \sigma_t(L,p))p - \sigma_t(H,p) \cdot c]dp$. Since the buyer only purchases the product when the price does not exceed the expected product value, for any p in the support of σ_t,

$$p \leq \frac{\sigma_t(H,p)}{\sigma_t(H,p) + \sigma_t(L,p)} \cdot q^H + \frac{\sigma_t(L,p)}{\sigma_t(H,p) + \sigma_t(L,p)} \cdot q^L$$

thus the flow payoff is less than $\int_p [\sigma_t(H,p)(q^H - c) + (1-\sigma_t(H,p)) \cdot q^L]dp$. Therefore,

$$\begin{aligned} E[V(r_{t+1})|\mathcal{Z}_t] &= \sum_{t'=t+1}^{\infty} \delta^{t'-t-1} E\Big[\int_p [(\sigma_{t'}(H,p) + \sigma_{t'}(L,p))p - \sigma_{t'}(H,p) \cdot c]dp \Big| \mathcal{Z}_t\Big] \\ &\leq \sum_{t'=t+1}^{\infty} \delta^{t'-t-1} E\Big[\int_p [\sigma_{t'}(H,p)(q^H - c) + (1-\sigma_{t'}(H,p)) \cdot q^L]dp \Big| \mathcal{Z}_t\Big] \\ &= \frac{1}{1-\delta}\big[\rho_t(q^H - c) + (1-\rho_t)q^L\big] \end{aligned}$$

Given an equilibrium path, suppose that the seller gets a negative rating in period t with the record r_t, and the next period in which the seller gets a negative rating with a strictly positive probability is period $t+n$. First we claim that the ratings from period $t+1$ to $t+n-1$ must be all positives. Since from period $t+1$ to $t+n-1$, the seller gets negative ratings with probability zero, the only case we need to rule out is that the seller does not get any rating in some period. If it is true, then the seller is stuck in that record forever, and we are not able to find a period in which the seller gets a negative rating with strictly positive probability.

The IC conditions in this two periods must be satisfied with equalities, i.e., at r_t, we have either $c = \delta\xi[V(\tau_+(r_t)) - V(\tau_-(r_t))]$ or $c = \delta\xi[V(r_t) - V(\tau_-(r_t))]$, and at r_{t+n}, we also have either $c = \delta\xi[V(\tau_+(r_{t+n})) - V(\tau_-(r_{t+n}))]$ or $c = \delta\xi[V(r_{t+n}) - V(\tau_-(r_{t+n}))]$. In the following discussion, we take the case in which $c = \delta\xi[V(\tau_+(r_t)) - V(\tau_-(r_t))]$ and $c = \delta\xi[V(\tau_+(r_{t+n})) - V(\tau_-(r_{t+n}))]$ as an example, and all other cases follow the similar proof.

Given r_t, $E[V(r_{t+1})|r_t] \leq \frac{1}{1-\delta}\big[\rho_t(r_t)(q^H - c) + (1-\rho_t(r_t))q^L\big]$. Since in this case,

$$E[V(r_{t+1})|r_t] = \int_p [\sigma_t(H,p) + (1-\xi)\sigma_t(L,p)]dp \cdot V(\tau_+(r_t)) + \int_p \xi\sigma_t(L,p)dp \cdot V(\tau_-(r_t))$$

We have

$$\begin{aligned} V(\tau_+(r_t)) &= E[V(r_{t+1})|r_t] + \int_p \xi\sigma_t(L,p)dp \cdot [V(\tau_+(r_t)) - V(\tau_-(r_t))] \\ &\leq E[V(r_{t+1})|r_t] + \xi[V(\tau_+(r_t)) - V(\tau_-(r_t))] \\ &\leq \frac{1}{1-\delta}\big[\rho_t(r_t)(q^H - c) + (1-\rho_t(r_t))q^L\big] + c/\delta \end{aligned} \tag{2}$$

Let $n_e(\rho_t)$ be the least natural number n_e satisfying

$$(1-\delta)V(E) \geq (1-\delta^{n_e})(q^L - a) + \delta^{n_e}\big[\rho_t(r_t)(q^H - c) + (1-\rho_t(r_t))q^L\big] + c(1-\delta)/\delta$$

Obviously, $n_e(\rho_t)$ is increasing in ρ_t. Then from (2),

$$(1-\delta)V(E) \geq (1-\delta^{n_e})(q^L - a) + \delta^{n_e}(1-\delta)V(\tau_+(r_t))$$

From the IC condition at r_{t+n},

$$\begin{aligned} V(\tau_+(r_{t+n})) &= \frac{c}{\delta\xi} + V(\tau_-(r_{t+n})) \geq \frac{c}{\delta\xi} + V(E) \\ &\geq \frac{c}{\delta\xi} + (1-\delta^{n_e})\frac{q^L - a}{1-\delta} + \delta^{n_e} V(\tau_+(r_t)) \end{aligned}$$

Notice $V(\tau_-(r_t)) \geq (1-\delta^n)\frac{q^L-a}{1-\delta} + \delta^n V(\tau_+(r_{t+n}))$. Thus

$$\begin{aligned} V(\tau_+(r_t)) - \frac{c}{\delta\xi} &= V(\tau_-(r_t)) \geq (1-\delta^n)\frac{q^L-a}{1-\delta} + \delta^n V(\tau_+(r_{t+n})) \\ &\geq (1-\delta^n)\frac{q^L-a}{1-\delta} + \delta^n\left[\frac{c}{\delta\xi} + (1-\delta^{n_e})\frac{q^L-a}{1-\delta} + \delta^{n_e}V(\tau_+(r_t))\right] \\ &= \delta^n\frac{c}{\delta\xi} + \frac{1-\delta^{n+n_e}}{1-\delta}(q^L-a) + \delta^{n+n_e}V(\tau_+(r_t)) \end{aligned}$$

And then

$$(1+\delta^n)\frac{c}{\delta\xi} \leq (1-\delta^{n+n_e})[V(\tau_+(r_t)) - \frac{q^L-a}{1-\delta}]$$

Therefore,

$$c \leq \frac{1-\delta^{n+n_e(r_t)}}{1+\delta^n} \cdot \frac{\delta\xi}{1-\delta}[\rho_t(r_t)(q^H - c) + (1-\rho_t(r_t))q^L - (q^L-a) + c\cdot\frac{1-\delta}{\delta}]$$

Let N be the least natural number satisfying the expression above. Since this is a necessary condition, we have $\bar{n}_t(r_t) \geq N$. The right hand side is increasing both in ρ_t and in n. Apparently, N is decreasing in ρ_t. □

Proof for Example 1

Firstly let us construct the continuation payoffs and then check the incentive compatibility conditions. Following the strategy profile, firstly notice that: (1) for any r such that $g(r)=0$, $\tilde{V}(r) = \tilde{V}(E)$; (2) at any two records with the same $g(r)$ the seller will have the same continuation payoffs; (3) at any record such that $g(r) > N$, the seller will have the same continuation payoff.

Following she strategy profile,

$$\begin{aligned}\tilde{V}(E) &= (q^L - a) + +\delta\tilde{V}(\tau_+(E)) \\ &= (q^L - a)\cdot\frac{1-\delta^N}{1-\delta} + \delta^T\tilde{V}(r^N)\end{aligned}$$

where $g(r^N) = N$. Therefore,

$$\begin{aligned}(1-\delta)\tilde{V}(E) &= (1-\delta^N)(q^L - a) + \delta^N(q^H - c) \\ (1-\delta)\tilde{V}(r^N) &= (q^H - c)\end{aligned}$$

And for any r^n such that $g(r^n) = n < N$,

$$(1-\delta)\tilde{V}(r^n) = (q^L - a)\cdot(1-\delta^{N-n}) + \delta^{N-n}\cdot(q^H - c)$$

To check the incentive compatibility conditions, when $g(r) \geq N$, by straightforward calculation we need

$$c \leq \delta\xi[\tilde{V}(r^N) - \tilde{V}(E)]$$

This can be satisfied when N is large enough given $c < \frac{\xi\delta}{1-\delta}(q^H - q^L - c)$. Pick some N satisfying the expression above. Now let us fix N and check the incentive when $g(r) < N$. Choose p_n such that

$$\begin{aligned}p_n - (q^L - a) &\leq \delta\xi[\tilde{V}(r^{n+1}) - \tilde{V}(E)] \\ p_n - c &\leq q^L - a\end{aligned}$$

are satisfied. When a is close enough to zero, we can find p_n which is greater than $q^L + a$ such that the seller will receive a negative rating with probability ξ if he sets price at p_n but uses the low technology. So far we have checked all the incentive compatibility conditions. The last thing we need to check is $V(E) \geq \frac{q^L}{1-\delta}$. This is true when $a \leq \frac{\delta^N}{1-\delta^N}(q^H - q^L - c)$, so the strategy profile is an equilibrium when a is small enough.

Proof for Example 2

Firstly let us construct the continuation payoffs and then check the incentive compatibility conditions. Following the strategy profile, firstly notice that: (1) for any r such that $g(r) = 0$, $\tilde{V}(r) = \tilde{V}(E)$; (2) at any two records with the same $g(r)$ the seller will have the same continuation payoffs; (3) at any record such that $g(r) > N$, the seller will have the same continuation payoff.

Then by the law of motion,

$$\begin{aligned}\tilde{V}(E) &= (q^L - a) + \delta\eta\tilde{V}(E) + \delta(1-\eta)\tilde{V}(\tau_+(E)) \\ &= [(q^L - a) + \delta\eta\tilde{V}(E)] \cdot \frac{1 - [\delta(1-\eta)]^N}{1 - \delta(1-\eta)} + [\delta(1-\eta)]^N \tilde{V}(r^N)\end{aligned}$$

where $g(r^N) = N$. In addition, since at r^N the normal type of seller starts to use the high technology until a negative rating is realized unexpectedly, we have

$$\tilde{V}(r^N) = q^H - c + \delta\eta\tilde{V}(E) + \delta(1-\eta)\tilde{V}(r^N)$$

From the two equations above we get

$$\begin{aligned}(1-\delta)\tilde{V}(E) &= (q^L - a) \cdot [1 - [\delta(1-\eta)]^N] + (q^H - c) \cdot [\delta(1-\eta)]^N \\ (1-\delta)\tilde{V}(r^N) &= (q^L - a) \cdot \frac{\delta\eta[1 - [\delta(1-\eta)]^N]}{1 - \delta(1-\eta)} + (q^H - c) \cdot \frac{1 - \delta + \delta\eta[\delta(1-\eta)]^N}{1 - \delta(1-\eta)}\end{aligned}$$

For any r^n such that $g(r^n) = n < N$, we have

$$\tilde{V}(r^n) = [(q^L - a) + \delta\eta\tilde{V}(E)] \cdot \frac{1 - [\delta(1-\eta)]^{N-n}}{1 - \delta(1-\eta)} + [\delta(1-\eta)]^{N-n}\tilde{V}(r^N)$$

so we get all the values for continuation payoff by plugging $\tilde{V}(E)$ and $\tilde{V}(r^N)$ into the expression above.

To check the incentive compatibility conditions, when $g(r) \geq N$, by straightforward calculation, we need

$$c \leq \delta\xi(1-\eta)[\tilde{V}(r^N) - \tilde{V}(E)]$$

This can be satisfied when N is large enough given $c < \frac{\xi\delta(1-\eta)}{1-\delta(1-\eta)}(q^H - q^L - c)$. Pick some N satisfying the expression above. Now let us fix N and check the incentive when $g(r) < N$. Let p_n be some positive number satisfying:

$$\begin{aligned}p_n - q^L + a &\leq \delta\xi(1-\eta)[\tilde{V}(r^{n+1}) - \tilde{V}(E)] \\ p_n - c &\leq q^L - a\end{aligned}$$

When a is close enough to zero, we can find p_n which is greater than $q^L + a$ such that the seller will receive a negative rating with probability ξ if he sets price at p_n but uses the low technology. So far we have checked all the incentive compatibility conditions. The last thing we need to check is $V(E) \geq \frac{q^L}{1-\delta}$. This is true when $a \leq \frac{[\delta(1-\eta)]^N}{1-[\delta(1-\eta)]^N}(q^H - q^L - c)$, so the strategy profile is an equilibrium when a is small enough.

Proof for Example 3

From the strategy profile, for any two records r, r' which are on-equilibrium path, if $g(r) = g(r')$, then $\tilde{V}(r) = \tilde{V}(r')$. Let r^n denote a rating record with $g(r^n) = n$. Given a small a, let us assume that $\tilde{V}(E) = \tilde{V}(r^0) > \frac{q^L}{1-\delta}$ and show it is true later.

First, if the strategy profile is an equilibrium, we have $p_n^L \leq q^L - a$. Following the stragegy profile, when $n < N$, the normal type of seller separates in prices from the commitment type, so $p_n^L \leq q^L$. If $p_n^L \in (q^L - a, q^L]$, then the buyer will not rate. Since (L, p_n^L) is optimal for the seller, we have

$$\tilde{V}(r^n) = p_n^L + \delta\left[[1 - \phi_-^L(p, r^n)]\tilde{V}(r^n) + \phi_-^L(p, r^n)\tilde{V}(r^0)\right]$$

implying $\tilde{V}(r^n) \leq \frac{p_n^L}{1-\delta} \leq \frac{q^L}{1-\delta}$, but this contradicts $\tilde{V}(r^n) \geq \tilde{V}(E) > \frac{q^L}{1-\delta}$. Therefore, $p_n^L \leq q^L - a$.

Now let us follow the strategy profile and get p_n^i and $\tilde{V}(r^n)$, where $g(r^n) = n$ for $n = 0, ..., N$. Since $p_n^L \leq q^L - a,$. Following the strategy profile, if it is an equilibrium, for the normal type of seller,

$$\begin{aligned}
\tilde{V}(r^0) &= \max_{p \leq q^L - a} \left\{ p + \delta\left[[1 - \phi_-^L(p, r^0)]\tilde{V}(r^1) + \phi_-^L(p, r^0)\tilde{V}(r^0)\right] \right\} \\
\tilde{V}(r^1) &= \max_{p \leq q^L - a} \left\{ p + \delta\left[\phi_+^L(p, r^1)\tilde{V}(r^2) + \phi_{NR}^L(p, r^1)\tilde{V}(r^1) + \phi_-^L(p, r^1)\tilde{V}(r^0)\right] \right\} \\
&\dots \\
\tilde{V}(r^N) &= \max_{p \leq q^H} \left\{ p - c + \delta\left[\phi_+^H(p, r^N)\tilde{V}(r^N) + \phi_-^H(p, r^N)\tilde{V}(r^0)\right] \right\}
\end{aligned}$$

Given $\tilde{V}(r^n)$, $n = 0, ..., N$, we are going to get p_n^L as functions of $\{\tilde{V}(r^0), ..., \tilde{V}(r^N)\}$ Plug those fictions into the expressions above and solve these equations, we will have all the values for p_n^L and $\tilde{V}(r^n)$, $n = 0, ..., N$.

When $g(r) \geq N$, by straightforward calculation, the IC condition for using the high technology requires

$$c \leq \delta[\phi_-^L(p_N^H, r^N) - \phi_-^H(p_N^H, r^N)][\tilde{V}(r^N) - \tilde{V}(E)]$$

When $\varepsilon \to 0$, $(1-\delta)\tilde{V}(r^N) \to q^H - c$, $(1-\delta)\tilde{V}(r^n) \to (1-\delta^n)(q^L - a) + \delta^n(q^H - c)$, and $\phi^L_-(p^H_N, r^N) - \phi^H_-(p^H_N, r^N)$ is closed to ξ. So the right hand side must be within a small neighborhood of $RHS(N) = \delta\xi\frac{1-\delta^N}{1-\delta}[q^H - c - q^L + a]$. That is to say, for any $\eta > 0$, we can find some disturbance of ε such that

$$\delta[\phi^L_-(p^H_N, r^N) - \phi^H_-(p^H_N, r^N)][\tilde{V}(r^N) - \tilde{V}(E)] \in [RHS(N) - \eta, RHS(N) + \eta]$$

Since $RHS(N)$ is strictly increasing in n, for η small enough, we can always find some N^* satisfying $c \leq RHS(N^*) - \eta$, but $c > RHS(N^* - 1) + \eta$. Therefore, the seller will use the high technology when $N \geq N^*$ but only uses the low technology if $N < N^*$.

Now we have pinned down N^* and next let us show that the seller has no incentive to mimic the commitment type when $N < N^*$. The IC condition at r^n requires $p^H_n - p^L_n \leq \delta[\phi^L_-(p^H_n, r^n) - \phi^H_-(p^H_n, r^n)][\tilde{V}(r^{n+1}) - \tilde{V}(r^0)]$. With $\varepsilon \to 0$, the right hand side is closed to $\delta\xi[\tilde{V}(r^{n+1}) - \tilde{V}(r^0)] > 0$, implying that we can always find $c + p^L_n > p^H_n > p^L_n$ to satisfy the inequality.

When $\varepsilon \to 0$, all p^L_n are very close to $q^L - a$. When a is small enough, we have $\tilde{V}(r^0) > \frac{q^L}{1-\delta}$. So far we have shown that the strategy profile is an equilibrium. In this equilibrium, the seller has no incentive to discard his name after getting negative ratings, since given any record, the continuation payoff is always $\tilde{V}(E)$ after a positive rating. Finally, to show that $\mu_t \to 0$ $\tilde{M}$-almost surely, it is enough to show that at any record the commitment type has a strictly higher probability to get a positive rating. Next let us complete the proof by showing this.

Suppose not, at some r the commitment type gets a positive rating with less probability. If $g(r) < N^*$, then $\phi^H_-(p^H_n, r^n) > \phi^L_-(p^L_n, r^n)$, implying $F_\varepsilon[-k(q^H - p^H_n + a)] > F_\varepsilon[-k(q^L - p^L_n + a)]$ and $p^H_n - p^L_n > q^H - q^L$, but this contradicts $c + p^L_n > p^H_n > p^L_n$. If $g(r) \geq N^*$, the first order conditions for the commitment type and the normal type are:

$$\begin{aligned} 1 &= \delta\frac{\partial\phi^H_-(\hat{p}^{N^*}, r^{N^*})}{\partial p}[\hat{V}(r^{N^*}) - \hat{V}(E)] \\ 1 &= \delta\frac{\partial\phi^H_-(\tilde{p}^{N^*}, r^{N^*})}{\partial p}[\tilde{V}(r^{N^*}) - \tilde{V}(E)] \end{aligned}$$

When $\varepsilon \to 0$, both $(1-\delta)\hat{V}(r^{N^*})$ and $(1-\delta)\tilde{V}(r^{N^*})$ are close to $q^H - c$, but $\hat{V}(E)$ is strictly small than $\tilde{V}(E)$, given $p^H_n - p^L_n < c$ and p^H_n can be arbitrarily close to p^L_n. Since $f'_\varepsilon(y) > 0$ for any y, we have $\hat{p}^{N^*} < \tilde{p}^{N^*}$, so $\phi^H_-(\hat{p}^{N^*}, r^{N^*}) < \phi^H_-(\tilde{p}^{N^*}, r^{N^*})$.

Proof for Remark 1

This proof follows exactly the same steps as in Lemma 1 of Cripps, Mailath, Samuelson (2004). Firstly, when name discarding is not allowed, μ_t is a supermartingale with respect to the filtration $\{\mathcal{Z}_t\}_t$ and measure $\tilde{M}$, so $\{\mu(r_t)\}_t$ is convergent almost surely under $\tilde{M}$. By Bayes' rule, if the rating the seller gets in period t is j, where $j = +, -, NR$, we have

$$\mu(r_t, j) = \frac{\mu(r_t)\int_p \hat{\sigma}(r_t)(p)\phi_j^H(p, r_t)dp}{\mu(r_t)\int_p \hat{\sigma}(r_t)(p)\phi_j^H(p, r_t)dp + (1-\mu(r_t))\cdot\sum_{i=H,L}\int_p \tilde{\sigma}(r_t)(i,p)\phi_j^i(r_l, p)dp}$$

Denote the denominator by A, then we have

$$\frac{\mu(r_t, j)}{\mu(r_t)}\cdot A = \int_p \hat{\sigma}(r_t)(p)\phi_j^H(p, r_t)dp$$

Similarly,

$$\frac{1-\mu(r_t, j)}{1-\mu(r_t)}\cdot A = \sum_{i=H,L}\int_p \tilde{\sigma}(r_t)(i,p)\phi_j^i(r_l, p)dp$$

Notice that $0 < A < 1$, and take the difference for the above two equations,

$$|\mu(r_t, j) - \mu(r_t)| \geq \mu(r_t)(1-\mu(r_t))\left|\sum_{i=H,L}\int_p \tilde{\sigma}(r_t)(i,p)\phi_j^i(r_l, p)dp - \int_p \hat{\sigma}(r_t)(p)\phi_j^H(p, r_t)dp\right|$$

Since $\{\mu(r_t)\}_t$ is convergent almost surely under $\tilde{M}$, the left hand side of the inequality converges to zero almost surely under $\tilde{M}$. Therefore,

$$\lim_{t\to\infty}\mu(r_t)(1-\mu(r_t))\left|\sum_{i=H,L}\int_p \tilde{\sigma}(r_t)(i,p)\phi_j^i(r_l, p)dp - \int_p \hat{\sigma}(r_t)(p)\phi_j^H(p, r_t)dp\right| = 0$$

almost surely under $\tilde{M}$. Note that $\mu(r_t)$ does not converge to 1 $\tilde{M}$- almost surely, so we have the result in the remark.

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