Preferences with Uncertain Temptations

Eddie Dekel and Bart Lipman

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Modeling Temptation

- What preferences, and corresponding representations, deviate from "standard" preferences "only" by allowing for temptation (of various kinds)?
- Build on DLR [2001] (see also corrigendum DLRS [2006]) and Gul–Pesendorfer [2001]. Decision theory literature originates with Kreps [1988]. Temptation representation builds on Strotz [1955].
  - DLR – general preferences over subsets with a representation using uncertain positive and (difficult to interpret) "negative" preferences
  - GP
    - Specific form of temptation modeled using a *costly-self-control* representation. Also has some interpretational difficulties.
    - A limiting ("discontinuous") *overwhelming* temptation representation.

1. Models of uncertain-temptation with costly self control lie between DLR and GP
   - Capture natural forms of temptation that are ruled out by GP.
   - What subset of DLR allows "only" for temptation?
   - What further natural restrictions can be characterized?
2. Models of uncertain and overwhelming temptation can then be obtained limits of models in 1. But there is a (surprising – for us –) additional relationship.

Preferences

B, finite set of consumption bundles.
$\Delta(B)$, probability distributions on B.
X, menus, closed nonempty subsets of $\Delta(B)$.
$\succ$ a preference relation on X
$V : X \to \mathbf{R}$, a representation of preferences

Representations (DLR)

- DLR: additive EU representation:

$$V(x) = \int \max_{\beta \in x} w_s(\beta) \mathrm{d}F(s) - \int \max_{\beta \in x} v_s(\beta) \mathrm{d}G(s)$$

where each w_i and each v_j is EU.
(Axioms: Weak order, Continuity, Independence, (Finiteness))

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- Flexibility-driven preferences (Kreps):

$$V(x) = \int \max_{\beta \in x} w_s(\beta) \mathrm{d}F(s)$$

  (Axioms: Add monotonicity: $x \subset x'$ implies $x' \succeq x$.)

Representations (GP)

- Gul–Pesendorfer [2001] consider temptation as reason for preferring smaller sets and characterize preferences that allow for certain temptations

$$
\begin{aligned}
V(x) &= \max_{\beta \in x}[u(\beta) + v(\beta)] - \max_{\beta \in x} v(\beta) \\
&= \max_{\beta \in x} w(\beta) - \max_{\beta \in x} v(\beta) \\
&= \max_{\beta \in x} [u - c(\beta, x)]
\end{aligned}
$$

where $w = u + v$, $c(\beta, x) = \max_{\beta' \in x} v(\beta') - v(\beta)$
(Axioms: Add Set Betweenness $x \succeq y \implies x \succeq x \cup y \succeq y$.)

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  (Axioms: Add Set Betweenness $x \succeq y \implies x \succeq x \cup y \succeq y$.)
- Interpretation:
  u: commitment utility as $V(\{\beta\}) = u(\beta)$
  v: temptation utility (measures cost of self–control)
  $u + v$?

Representations (GP / Strotz)

- GP also consider overwhelming temptation

$$\begin{aligned} & \lim_{k\to\infty} \max_{\beta\in x}[u(\beta) + kv(\beta)] - k\max_{\beta\in x} v(\beta) \\ = & \max_{\beta\in\arg\max_{\gamma\in x} v(\gamma)} u(\beta) \end{aligned}$$

These are not Hausdorff continuous. (Think of x_n converging to x that has flat surface orthogonal to v.)
(Axioms: Weakens continuity; o/w the same as GP costly self control.)
Denote $B_v(x) = \arg\max_{\gamma\in x} v(\gamma)$ so $V(x) = \max_{\beta\in B_v(x)} u(\beta)$

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  These are not Hausdorff continuous. (Think of x_n converging to x that has flat surface orthogonal to v.)
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  Denote $B_v(x) = \arg\max_{\gamma\in x} v(\gamma)$ so $V(x) = \max_{\beta\in B_v(x)} u(\beta)$
- We want to allow for uncertainty (Random Strotz, RS)

$$V(x) = \int \max_{\beta\in B_v(x)} u(\beta)\, \mathrm{d}F(v)$$

Representations

- What is the class of temptation-driven preferences?
- Are there other special cases of interest?
- How do they relate?

$\int \max_{\beta \in x} w(\beta)\, \mathrm{d}F(w) - \int \max_{\beta \in x} v(\beta)\, \mathrm{d}G(v)$	
$\vdots$	
more general temptation models?	
$\int [\max_{\beta \in x} (u+v)(\beta) - \max_{\beta \in x} v(\beta)] \mathrm{d}F(v)$	$\to \int \max_{\beta \in B_v(x)} u(\beta)\, \mathrm{d}F(v)$
$\max_{\beta \in x} (u+v)(\beta) - \max_{\beta \in x} v(\beta)$	$\to \max_{\beta \in B_v(x)} u(\beta)$

Results

- The connection between random Strotz representations (uncertain overwhelming temptation) and GP (costly self control) is much closer than the limit results suggest.

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Any preferences having a random GP representation has a random Strotz representation.

- That is, if $\int [\max_{\beta \in x} (u+v)(\beta) - \max_{\beta \in x} v(\beta)] \mathrm{d}F(v)$ represents preferences, then there exists $\hat{F}$ such that

$$\int [\max_{\beta \in x} (u+v)(\beta) - \max_{\beta \in x} v(\beta)] \mathrm{d}F(v) \approx \int \max_{\beta \in B_w(x)} u(\beta)\, \mathrm{d}\hat{F}(w)$$

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- Converse? Not complete...
  - Any random Strotz preference is the limit of uncertain-costly self control preferences. But (when) is it equivalent to such a preference?
  - Need to rule out preferences that are not Hausdorff continuous.
  - **Conjecture**: *Any continuous random Strotz preference has a random GP representation.*

Example/Interpretation

Consider the preferences $\{\alpha\} \succ \{\alpha, \beta\} \succ \{\beta\}$. Any individual with a costly temptation preferences will behave just like an individual with (suitable) random Strotz preferences when choosing subsets. Now consider their second-period behavior, i.e., their choice from sets. Specifically assume one of these decision makers is given the set $\{\alpha, \beta\}$. In the costly temptation model the set of (u, v) pairs is partitioned (generically) into those that select α and suffer from having to resist the temptation, and those that give in to temptation and choose β. In the random Strotz model each such person will choose α and β with positive probability. Moreover, each (u, v) will correspond to random Strotz preferences that (generically) have different probabilities.

Constructive proof

Geometry

$$\max_{\beta \in x} \left(u(\beta) + v\left(\beta\right) \right) - \max_{\beta \in x} v(\beta) \approx \int_0^1 \max_{\beta \in B_{v+su}(x)} u\left(\beta\right) \mathrm{d}\left(s\right)$$

Constructive proof

Algebra

$$\max_{\beta\in x}\left(u(\beta)+v\left(\beta\right)\right)-\max_{\beta\in x}v(\beta)\approx\int_0^1\max_{\beta\in B_{v+su}(x)}u\left(\beta\right)\mathrm{d}\left(s\right)$$

Proof.

Fix some x. For $s\in[0,1]$, choose $\beta^*\left(s\right)\in\arg\max_{\beta\in B_{v+su}(x)}u\left(\beta\right)$.
Let $U\left(s\right)=\max_{\hat{s}\in[0,1]}\left[v\left(\beta^*\left(\hat{s}\right)\right)+su\left(\beta^*\left(\hat{s}\right)\right)\right]$
$=v\left(\beta^*\left(s\right)\right)+su\left(\beta^*\left(s\right)\right)$.
From the envelope theorem, $U'\left(s\right)=u\left(\beta^*\left(s\right)\right)$.
So $V_{RS}\left(x\right)=\int_0^1 u\left(\beta^*\left(s\right)\right)\mathrm{d}s=\int_0^1 U'\left(s\right)\mathrm{d}s=U\left(1\right)-U\left(0\right)$.
$U\left(1\right)=v\left(\beta^*\left(1\right)\right)+u\left(\beta^*\left(1\right)\right)$ where $\beta^*\left(1\right)\in B_{v+u}\left(x\right)$, so
$U\left(1\right)=\max_{\beta\in x}\left(u\left(\beta\right)+v\left(\beta\right)\right)$.
Similarly $U\left(0\right)=\max_{\beta\in x}v\left(\beta\right)$.
So
$U\left(1\right)-U\left(0\right)=\max_{\beta\in x}\left(u\left(\beta\right)+v\left(\beta\right)\right)-\max_{\beta\in x}v\left(\beta\right)=V_{GP}\left(x\right)$. □

Converse

- Given $\int \max_{\beta \in B_w(x)} u(\beta)\, \mathrm{d}F(w)$ can we find $\hat{F}$ s.t. $\int [\max_{\beta \in x} (u+v)(\beta) - \max_{\beta \in x} v(\beta)] \mathrm{d}\hat{F}(v)$ represents the same preferences?

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- If F is uniform with support $[u+\alpha v, v]$ then renormalize (let $\hat{v} = v/\alpha$).
- For convex combinations of uniform get random GP.
- If F has continuously differentiable density with support $[u+v, v]$ also get random GP.
- What if F is a singular distribution on $[u+v, v]$? Don't know (yet).

Converse (cont'd)

General continuously differentiable densities

Proof.

As before $V_{RS}(x) = \int_0^1 u(\beta^*(s))\, f(s)\, \mathrm{d}s = \int_0^1 U'(s)\, f(s)\, \mathrm{d}s =$ $U(s)\, f(s)\big|_0^1 - \int_0^1 U(s)\, f'(s)\, \mathrm{d}s$. This is DLR additive EU.

- If $f' > 0$ then $V_{RS}(x) =$
  $f(1)\max(u+v) - f(0)\max v - \int_0^1 f'(s)\max(su+v)\,\mathrm{d}s =$
  $\left(f(1) - \int_0^1 f'(s)\,\mathrm{d}s\right)(\max(u+v) - \max v) +$
  $\int_0^1 f'(s)(1-s)\left(\max\left(u + \frac{su+v}{1-s}\right) - \max\left(\frac{su+v}{1-s}\right)\right)\mathrm{d}s.$
- If $f' \leq 0$ then it has one EU preference with negative weight. In an earlier paper we showed this is an uncertain costly temptation where the uncertainty is about the strength of temptation (for *finite* additive EU) $\sum_i [\max_{\beta\in x}(u + k_i v)(\beta) - \max_{\beta\in x} k_i v(\beta)]$. (Axiom: Neg. SB: $x \succeq y \Rightarrow x \cup y \succeq y$.)

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- **Conjecture**: If F has continuous density but violates the above condition on the support for a set of w's with positive measure then preferences are discontinuous.
- **Proof:** Cylinder and perturbations.
- "**Conclusion"**: Continuous random Strotz and random GP are indistinguishable based on preferences over menus. Continuous random Strotz are those with the support condition above.

Alternative proof and axioms:

- Any continuous random Strotz preference satisfies the DLR axioms plus weak set betweenness: $\{\alpha\} \succeq \{\beta\}$ for all $\alpha \in x$, $\beta \in y$ $\Rightarrow x \succeq x \cup y \succeq y$. In our earlier paper we conjectured that this exactly characterizes the uncertain costly temptation representation when there is a finite additive EU representation $(\Sigma \max_{\beta \in x} w_i(\beta) - \Sigma \max_{\beta \in x} v_i(\beta))$. John Stovall proved this conjecture, and we are now extending this to the infinite case. This would establish that any continuous random Strotz representation has a random GP representation.

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- **Conclusion**: Continuous random Strotz and random GP are indistinguishable based on preferences over menus. Both correspond to the DLR axioms plus weak set betweenness.

Additive EU (DLR) satisfying weak set betweenness has a random GP representation.

Proof

$$V(x) = \int \max_{\beta \in x} w(\beta) F dw - \int \max_{\beta \in x} v(\beta) G dv$$

$$u(\beta) = \int w(\beta) F(dw) - \int v(\beta) G(dv)$$

We can write any w and v as a sum of u and some f orthogonal to u. In addition to the two usual degrees of freedom on the affine utility functions w and v we also have an extra degree of freedom because F and G are not normalized. So we can renormalize to get the weight on f to be 1, so $w = \theta_w u + f$, as well as $w \cdot \mathbf{1} = 0$ and $w \cdot w = 1$, and similarly for v. Hence also $u \cdot \mathbf{1} = f \cdot \mathbf{1} = 0$.

Assumption: For now we only deal with the case where for all w and v in the support we can use the same f: $w = \theta_w u + \alpha_w f$, and similarly for v, for some f orthogonal to u.

$$\begin{aligned} V(x) &= \int_\theta \max_{\beta \in x}[\theta u + f] F(d\theta) - \int_\theta \max_{\beta \in x}[\theta u + f] G(d\theta) \\ &= \int_0^1 \max_{\beta \in x}[F^{-1}(t) u + f] dt - \int_0^1 \max_{\beta \in x}[G^{-1}(t) u + f] dt \end{aligned}$$

Let $q(t) = F^{-1}(t) - G^{-1}(t)$.

$$\begin{aligned} V(x) &= \int_0^1 \max[q(t) u + G^{-1}(t) u + f] dt - \int_0^1 \max[G^{-1}(t) u + f] dt \\ &= \int_0^1 q(t) \left\{ \max[u + v(t)] - \max v(t) \right\} \, dt \end{aligned}$$

where $v(t) = (1/q(t))[G^{-1}(t)u + f]$ if $q(t) \neq 0$ (anything o/w).
By construction $\int_\theta [\theta u + f] (F - G)(d\theta) = u$ so $\int_\theta \theta (F - G)(d\theta) = 1$, so $E(F) - E(G) = 1$, and $\int_0^1 q(t)\, dt = 1$.

- Need $q(t) \geq 0$. This holds if F FOSD G, and this is implied by the axiom.

Assume the preferences satisfy weak set betweenness. Fix any α in the interior of the simplex and any $\hat{\theta}$ and let

$$\beta = \alpha + \varepsilon \left(\frac{\hat{\theta}}{f \cdot f} f - u \right), \ \beta^* = \beta + \frac{\hat{\varepsilon}}{f \cdot f} f$$

$$u(\alpha) > u(\beta) = u(\beta^*)$$

So by weak set betweenness:

$$V(\{\alpha, \beta, \beta^*\}) \leq V(\{\alpha, \beta\}).$$

Also

θ	$\succ_\theta$ Ranking	Gain from β^*
$\theta > \hat{\theta} + \frac{\hat{\varepsilon}}{\varepsilon}$	$\alpha \succ \beta^* \succ \beta$	0
$\left[\hat{\theta}, \hat{\theta} + \frac{\hat{\varepsilon}}{\varepsilon}\right]$	$\beta^* \succ \alpha \succ \beta$	$\hat{\varepsilon} + \varepsilon\left(\hat{\theta} - \theta\right)$
$\theta < \hat{\theta}$	$\beta^* \succ \beta \succ \alpha$	$\hat{\varepsilon}$

So

$$\int_{\theta<\hat{\theta}} \hat{\varepsilon} H(d\theta) + \int_{\hat{\theta}\le\theta<(\hat{\varepsilon}/\varepsilon)+\hat{\theta}} [\hat{\varepsilon} + \varepsilon(\hat{\theta} - \theta)] H(d\theta) \le 0$$

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$$\int_{\theta<\hat{\theta}} F(d\theta) \le \int_{\theta<\hat{\theta}} G(d\theta)$$

This holds for all $\hat{\theta}$ at which G has no mass, hence everywhere and F FOSD G. $\square$

- GP show that with set betweenness weakening Hausdorff continuity to upper-semi continuity allows for either the costly temptation or the Strotz representation. Using the above we see this characterizes uniform or degenerate random Strotz. With weak set betweenness instead does this give all random Strotz?

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- The uppersemicontinuity corresponds to the max in $\max_{\beta \in B_v(x)} u$. Does dropping this and leaving just VN-M continuity correspond to random Strotz with arbitrary tie breaking?

A different kind of temptation is ruled out by these models. Assume that a menu being considered contains broccoli, vanilla ice cream or a brownie (but not any combination). *Both* ice cream and brownies are tempting, so whatever item is selected one suffers a cost from the temptation. A corresponding representation (without uncertainty) is $\max\left(u+\sum v_j\right)-\sum \max v_j$ and was characterized by DLR (who also characterized the case with uncertainty). Does this have a different and more appealing class of representation that also avoids the interpretational difficulties with second-period $\left(u+\sum v_j\right)$ choice?

THE END

Examples

Multiple temptations

- Broccoli, chocolate, and potato chips.
  Plausible ordering:

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- Interpretations:
  1. Uncertainty about temptation
  2. Two snacks harder to resist than one

Multiple-temptations example

Uncertain-temptation representation

	u	v_1	v_2
b	3	2	2
c	0	0	6
p	0	6	0

$$V_1(x) = \frac{1}{2}\sum_{i=1}^{2}\left(\max_{\beta\in x}\left(u(\beta)+v_i\left(\beta\right)\right) - \max_{\beta\in x} v_i(\beta)\right)$$

Multiple-temptations example

Uncertain-temptation representation

	u	v_1	v_2
b	3	2	2
c	0	0	6
p	0	6	0

  $$V_1(x) = \frac{1}{2}\sum_{i=1}^{2}\left(\max_{\beta\in x}\left(u(\beta)+v_i(\beta)\right) - \max_{\beta\in x} v_i(\beta)\right)$$

- $$\{b\} \succ \{b,c\},\{b,p\} \succ \{b,c,p\}$$

x	b	$\{b,c\}$	c	$\{b,c,p\}$	$\{c,p\}$
V	3	$\frac{3+0}{2}$	0	0	0

Uncertain-temptation representation

$$V_{UT}(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + v_i\left(\beta\right)\right) - \max_{\beta \in x} v_i(\beta) \right)$$

(Axioms: Stovall: Weak Set Betweenness: $\{\alpha\} \succeq \{\beta\}$ $\forall \alpha \in x, \beta \in y \Rightarrow x \succeq x \cup y \succeq y$.)

Multiple-temptations example

Joint-temptation representation

-

	u	v_1	v_2
b	3	2	2
c	0	0	6
p	0	6	0

$$V_2(x) = \max_{\beta \in x} \left(u(\beta) + \sum_{i=1}^{2} v_i(\beta) \right) - \sum_{i=1}^{2} \max_{\beta \in x} v_i(\beta)$$

$$\{b\} \succ \{b,c\}, \{b,p\} \succ \{b,c,p\}$$

x	b	$\{b,c\}$	c	$\{b,c,p\}$	$\{c,p\}$
V	3	1	0	-5	-6

Joint-temptation representation

$$V_{JT}(x) = \max_{\beta \in x}\left(u(\beta) + \sum_i v_i(\beta)\right) - \sum_i \max_{\beta \in x} v_i(\beta)$$

(Axiom: Positive set betweenness: $x \succeq y \implies x \succeq x \cup y$.)

Compromise example

- Broccoli, frozen yogurt, and ice cream.

$$\{b, y\} \succ \{y\} \text{ and } \{b, i, y\} \succ \{b, i\}$$

Compromise example

- Broccoli, frozen yogurt, and ice cream.

$$\{b, y\} \succ \{y\} \text{ and } \{b, i, y\} \succ \{b, i\}$$

- Inconsistent with $V(x) = \max_{\beta \in x} w(\beta) - \max_{\beta \in x} v(\beta)$:
  $\{b, y\} \succ \{y\}$ implies $w(b) > w(y)$. So consider $\{b, i, y\}$. w max can't be at y. Hence adding y to $\{b, i\}$ can only increase v max, so $\{b, i, y\} \preceq \{b, i\}$
  (Violates set betweenness and independence.)

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  (Violates set betweenness and independence.)
- Possible interpretation: Maybe I won't be tempted, so prefer to have b available. Maybe I will be tempted, so prefer y as compromise.

Compromise example

Uncertain-strength-of-temptation representation

-

	u	v
b	6	0
i	0	8
y	4	6

$$V_3(x) = \left(\max_{\beta \in x} \left(u(\beta) + v\left(\beta\right) \right) - \max_{\beta \in x} v(\beta) \right) + \max_{\beta \in x} u\left(\beta\right)$$

Compromise example

Uncertain-strength-of-temptation representation

	u	v
b	6	0
i	0	8
y	4	6

  $$V_3(x) = \left(\max_{\beta \in x} \left(u(\beta) + v\left(\beta\right)\right) - \max_{\beta \in x} v(\beta) \right) + \max_{\beta \in x} u\left(\beta\right)$$

- $$\{b, y\} \succ \{y\} \text{ and } \{b, i, y\} \succ \{b, i\}$$

x	b	$\{b, y\}$	$\{b, i, y\}$	y	$\{b, i\}$	i
V	12	$4 + 6$	$4 + 6$	8	$0 + 6$	0

Uncertain-strength-of-temptation representation

$$V_{UST}(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + k_i v(\beta) \right) - k_i \max_{\beta \in x} v(\beta) \right)$$

(Axiom: Negative set betweenness: $x \succeq y \implies x \cup y \succ y$.)

Special case of uncertain temptation

Temptation representation

$$V_T(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + \sum_j v_{ij}(\beta) \right) - \sum_j \max_{\beta \in x} v_{ij}(\beta) \right)$$

(Axioms: (i) DFC: For all x, there is $\alpha \in x$ such that $\{\alpha\} \succeq x$. (ii) AIC...)

Summary of costly temptation results

$$V(x) = \sum_{i=1}^{I} \max_{\beta \in x} w_i(\beta) - \sum_{j=1}^{J} \max_{\beta \in x} v_j(\beta) \quad \text{[WO, CONT., IND, FINITE]}$$

$$V_T(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + \sum_j v_{ij}(\beta) \right) - \sum_j \max_{\beta \in x} v_{ij}(\beta) \right) \quad \text{[+DFC...}$$

$$V_{JT}(x) = \max_{\beta \in x} \left(u(\beta) + \sum_i v_i(\beta) \right) - \sum_i \max_{\beta \in x} v_i(\beta) \quad \text{[+PSB]}$$

$$V_{UT}(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + v_i(\beta) \right) - \max_{\beta \in x} v_i(\beta) \right) \quad \text{[+WSB]}$$

$$V_{UST}(x) = \sum_i p_i \left(\max_{\beta \in x} \left(u(\beta) + k_i v_i(\beta) \right) - k_i \max_{\beta \in x} v_i(\beta) \right) \quad \text{[+NSB]}$$

GP: $V(x) = \max_{\beta \in x} w_1(\beta) - \max_{\beta \in x} v_1(\beta)$ [+SB]