On the Informed Seller Problem: Optimal Information Disclosure

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Abstract

We consider a revenue maximizing seller who, before proposing a mechanism to sell her object(s), observes a vector of signals correlated with buyers' valuations. Each buyer knows only the signal that the seller observes about him but not the signals she observes about other buyers. The seller therefore has information about buyers' valuations that is not common knowledge. How will the seller disclose this information if her goal is to maximize revenue? We analyze the scenario where the seller chooses how to disclose her information and then chooses a revenue maximizing mechanism. We allow for very general disclosure policies, that can be random, public, private, or any mixture of these possibilities. Through the disclosure of information privately the seller can create correlation in buyers' types, which then consist of valuations plus beliefs. For the standard independent private values model we show that information revelation is irrelevant: irrespective of the disclosure policy an optimal mechanism for this informed seller generates expected revenue that is equal to her maximal revenue under full information disclosure. We also consider a more general allocation environment, allowing for interdependent, for common values, and for multiple items. There disclosure policies do matter and we show that the best the seller can do is to release no information at all. This result is opposite from the celebrated linkage principle. *Keywords: mechanism design, informed principal, information disclosure, correlated information, optimal auctions. JEL Classification Codes: C72, D44, D82.*

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1. Introduction

The literature on revenue maximizing auctions[1] is a cornerstone of auction theory. A crucial assumption in this literature is that the seller does not have any information that is unknown to the buyers. However, this assumption often fails in practice: a seller typically has superior information about how much objects are worth, and about participants' willingness to pay. In this paper we consider a seller that has superior information about participants' willingness to pay. As in the work on revenue maximizing auctions, the seller has the power to choose any auction to sell her objects. However, here we also allow the seller to disclose information to the buyers before the auction. We are interested in evaluating the impact of information disclosure on the revenue that the seller can generate, and in characterizing optimal information disclosure policies.

A striking example of a seller having superior information about buyers than their competitors have, is a sale of a company. A typical company sale includes a pre-auction stage where prospective buyers submit to the seller non-binding bids as an expression of interest, ("indicative bidding"). After indicative bidding, the seller knows a lot more about the various buyers' willingness to pay, than their competitors do. Based on these indicative bids, the seller screens who should take part in the auction,[2] and chooses a reservation price. Hansen (2001) suggests that these expressions of interest are close to how much bidders are actually willing to pay.

Similar phenomena occur in block trading.[3] Prior to sale, institutional investors submit *indications of interest*, also known as IOI's. IOI's are very common and typically contain information about how many shares, and at what price the particular buyer is willing to purchase. Another common instance where the seller may have superior information to the buyers are sales of real estate. There the seller typically observes how many times a buyer visits the house, how carefully he examines everything, whether he comes to the house with his/her architect or not, and as in the case of company sales, preliminary statements about how much a buyer would be willing to pay at the auction. All this information is correlated with a buyer's willingness to pay. The seller may then know more about the competitors of a buyer, than a buyer himself. The same can be true when sales (or purchases) are repeated. Consider, for instance, purchases by government agencies. In such situations the government agency may have interacted in the past more often with some of the bidders, and thus have superior information about their costs than the participants at the current auction have.

In this paper we look at a seller who has information about individual buyers that is not commonly known. First the seller decides how much information to disclose to the buyers

[1] The seminal papers are Myerson (1981) and Riley and Samuelson (1981).

[2] Given what we know from auction theory, see for instance, Bulow and Klemperer (1996), it may appear suboptimal for a seller to want to screen out buyers. For a justification see Hansen (2001) or Milgrom (2004).

[3] Trade of large amount of shares, usually traded off the floor.

and then designs an auction to sell the object(s). Formally this is an informed principal problem with multiple agents, with the additional step that the seller can influence the buyers perceptions about their competitors by disclosing information. How can a seller put all this information to best use? Should she release this information publicly or privately? How much should she disclose and to whom? We address these questions both in the standard independent private value setup as well as in a general model that allows for interdependent values, for common values, and for multiple goods. In both set-ups, buyers' (initial) information is statistically independent from one another.

Very broadly speaking, the seller has three options: (1) no information disclosure, (2) some partial, possibly private, information disclosure, and (3) full information disclosure. Each of these alternative information disclosure policies has very different implications regarding what buyers know at the stage where they are about to participate in the auction, which in turn, may lead to big differences in achievable revenue. Of particular interest may be the possibilities of releasing information privately. Then, a buyer's beliefs about another buyer are private information, because they depend on the information that he has received from the seller which is not publicly available. Hence, when a seller has information available that is not commonly known, she can use it to affect the buyer's beliefs and create correlation in types,[4] even though buyers' initial information was statistically independent. Can then the seller create correlations that allow her to reduce the buyers' information rents? Can she eliminate the distortions of incomplete information by using mechanisms of the Cremer-McLean (1988) type? What is the optimal information disclosure policy, given that the seller will be also subsequently choosing an optimal mechanism? These are the questions we address.

What information disclosure policies are optimal is not clear a priori. A potential rationale for no information revelation is that more information makes more deviations for the buyers feasible, making, in some sense, the incentive constraints harder to satisfy.[5] However, in quite few instances the reverse is true, and full information disclosure is the optimal policy. This is the case, for instance, in Milgrom and Weber (1982), who discover the celebrated "linkage principle," and in Eső and Szentes (2006). Also, the possibility of private information revelation maybe strictly preferred to all public information disclosure policies. An environment where this can be true is analyzed by Harstad and Mares (2003).

In this paper we establish that in the standard independent private value, (IPV), model information revelation is irrelevant: *irrespective* of the disclosure policy, an optimal mechanism for this informed seller generates expected revenue that is equal to her revenue under full information disclosure. This is despite the fact that the seller can create correlation in buyers' types, which consist then of *valuations plus beliefs*. However, in the general model, where values can be interdependent, or common, the seller's expected revenue depends on which disclosure policy she employs. Then, the question that arises is which one is the

[4] "Types" then consist of valuations plus beliefs.

[5] An environment where this reasoning applies is studied in Myerson (1986).

best. In response to this question, we show that the best information disclosure policy is to release *no information at all.* A by-product of our analysis is that unless the seller observes information that tells her exactly what is the valuation of a buyer, full surplus extraction is *not* possible, irrespective of how sophisticated disclosure policies she employs.

There are three main forces behind our results. First, disclosure policies, irrespective of how sophisticated they are, essentially have no impact on a buyer's information rents. The reason for this is that a buyer can still "mimic" the behavior of the same set of valuations, as in the case where all the information that the seller has were public. Moreover, the expected payments that a seller can extract from "selling" to the buyers (agents), information about their competitors are always equal to zero. This is because there is a common prior, which implies that the side contracts written between the seller and a buyer that have positive expected value for the seller have negative expected value for the buyer, so they are never accepted. The second force is the fact that the seller is not penalized from having private information. Formally, this tells us that the seller's incentive constraints are not binding. This happens because the seller's information is non-exclusive: what the seller knows about a buyer, is also known to that particular buyer himself. This is also true in the interdependent value model, which is somewhat more surprising. The third force is related to how disclosure policies affect the set of incentive compatible mechanisms. In the case of independent private values disclosure policies do not enlarge the set of incentive compatible mechanisms in any relevant way. The reason is that in the IPV case, even when beliefs are part of buyers' types, an optimal Bayesian incentive compatible mechanism is also dominant strategy incentive compatible. This is not true in general, however. In general, disclosure policies affect the set of incentive compatible mechanisms, which becomes largest when the seller discloses no information at all. This is the reason why in the general case, the best that the seller can do is to disclose no information at all.

This paper shows that when the seller has the ability to use her information to design optimal mechanisms, the scope of information disclosure is limited. In the case of independent private values we have an Information Irrelevance Theorem, saying that if the seller can design optimal mechanisms, all that matters is the payoff relevant part of type, not the belief-relevant part. This is in contrast to what happens with exogenously fixed auction rules, where the belief-relevant part of type has a significant impact on equilibrium behavior. This is demonstrated in Bergemann and Välimäki (2006). This result may be viewed as surprising. However, it can be also viewed as a direct consequence of the structure of optimal auctions in the IPV case. There the optimal mechanism is dominant strategy incentive incentive compatible, which implies that even when the seller releases information the mechanism will be incentive compatible and optimal. To see, though, why this logic is incomplete and misses important forces, note that precisely those arguments *hold* even in an environment where the buyers' and seller's beliefs differ, and/or the buyers do not use Bayes' rule to update beliefs. However, our analysis indicates that the information irrelevance result *fails* under either of these conditions. Indeed, then the seller can do really well by designing disclosure policies that allow her to write very lucrative side-contracts, which the buyers are happy to voluntarily accept.[6]

For general mechanism design problems we show that information disclosure may matter, and the best that the seller can do is to disclose no information at all. This finding may be viewed as somewhat surprising, given that we allow for interdependent values, which is also the case in Milgrom and Weber (1982), who prove the opposite result. However, there are two important differences between Milgrom and Weber (1982) and this paper. In Milgrom and Weber (1982) the auction rules are fixed, and the buyers' (prior) information is affiliated. Here the seller can choose the auction she uses, and the prior information that buyers have is statistically independent. We now give a more comprehensive account of some related work.

This paper is related to the work on *mechanism design by an informed principal.*[7] The first paper in this literature is Myerson (1983), which formulates the important idea of inscrutable mechanisms. Two other seminal contributions are Maskin and Tirole (1990) and (1992).[8] Other than Myerson (1983), two papers allow for multiple agents: the paper by Tisljar (2003), under the restriction that buyers do not have private information, and the paper of Mylovanov (2005) that examines a private value quasilinear environment. In all the aforementioned papers the principal's information pertains to his/her preferences and is exclusive. In this paper the seller has information that is not commonly known, but each buyer knows the information that the seller has about himself, hence the seller's information is non-exclusive.

This paper is also related to the literature on *mechanism design with endogenous types.*[9] In Obara (2006) agents' types are endogenous. Types are affected by actions *agents* choose before participating in the mechanism. He investigates under which circumstances the actions chosen generate an environment where the designer can extract full surplus from the agents. In the papers by Bergemann and Pesendorfer (2007), Eső and Szentes (2006), Ganuza (2004), Ganuza and Penalva (2006), the endogeneity of information is directly controlled by the mechanism designer, or the seller. More specifically, the seller controls the technology that generates *the value estimates* for the buyers. Valuations are *private*, but in order for a buyer to assess how much the good is worth to him, he must receive information from the seller. Those papers consider such problems under different assumptions about the technology that the seller has to influence valuations. With the exception of Eső and

[6]For the case of non-bayesian updating, or non-common priors, an example where the seller can generate strictly higher revenue for each possible information that she may have, than the one generated by full disclosure (the Myerson (1981) benchmark) is available from the author upon request.

[7]There are also papers that study the choices of an informed seller within *specific* classes of mechanisms. See for instance Cai, Riley and Yi (2006) and Jullien and Mariotti (2006).

[8]We will be discussing extensively the methods and the results of those papers in subsection 2.1.1.

[9]For an excellent survey of this topic, as well as of other important topics about information in mechanism design, see Bergemann and Välimäki (2006).

Szentes (2006),[10] in those papers at an optimum the seller discloses less information than is efficient: more information increases efficiency which has a positive effect on revenue, but also increases information rents which has a negative effect on revenue. As opposed to these papers, in this paper for the case of private values buyers *know* their valuations, and the seller influences *the belief-relevant* part of their type. Another difference is that in this paper we examine also interdependent and common value environments. Moreover, here the seller's information is private. Our question is not how the seller can best influence the buyers' own estimates in order to boost revenue, as is the focus of Bergemann and Pesendorfer (2007), Ganuza (2004), and Eső and Szentes (2006), but rather, how can the seller influence a buyer's perceptions about his competitors in order to increase revenue.

Our findings are intimately related to the recent work on *mechanism design with statistically correlated information* by Neeman (2004), and by Heifetz and Neeman (2006). The classic papers in that literature are Cremer and McLean (1988), and McAfee and Reny (1992). They establish that when agents are risk neutral, even very mild correlation in types renders private information worthless and allows the designer to extract full surplus. In those papers a type of an agent consists of a parameter that affects payoffs. In this paper full surplus extraction is typically not possible, because as in Neeman (2004), and in Heifetz and Neeman (2006), beliefs are also part of the types. When this is the case, typically, there are many valuations that have the same beliefs, therefore the knowledge of beliefs does not make the exact inference of valuations possible.

This work relates also to the large literature of *information revelation in auctions*. Perry and Reny (1999) show that the linkage principle of Milgrom and Weber (1982) may fail when there are multiple objects. Ottaviani and Prat (2001) show that the linkage principle holds in a non-linear pricing model where the monopolist first discloses information and then chooses an optimal schedule. Recently, Board (2006) allows for the possibility that information released changes the winner and shows that in this case the linkage principal fails when the number of buyers is two. With the exception of Ottaviani and Prat (2001), the aforementioned papers examine the effect of information disclosure for a given auction procedure.[11] Also by contrast with these papers where buyers' information is affiliated, in this paper the buyers' initial information is statistically independent.

Initial information is also statistically independent in Landsberger, Rubinstein, Zamir and Wolfstetter (2001), and in Kaplan and Zamir (2002). Landsberger, Rubinstein, Zamir and Wolfstetter (2001) show that when the seller knows the ranking of buyers' valuations and

[10] Eső and Szentes (2006) show that the inefficiency created by having too little information, can be eliminated by allowing the seller to charge a fee for the information she provides. In that paper the seller simultaneously chooses the mechanism and how much information to disclose. Then, her incentives to disclose information are completely aligned with efficiency because her rents are tied to how big the pie will be. They establish that an optimal disclosure policy for the seller is to provide as much information as possible.

[11] A somewhat intermediate approach is Kremer and Skrzypacz (2004). There the release of information from the seller is though which "standard" auction procedure she chooses.

she makes it common knowledge, a first price auction raises higher revenue than in the case where the seller does not disclose this information. Kaplan and Zamir (2002) consider again information revelation in *first price* auctions, but allow for more general *public* disclosure policies than Landsberger, Rubinstein, Zamir and Wolfstetter (2001) allow. There are three main differences from our paper and these. First in Landsberger, Rubinstein, Zamir and Wolfstetter (2001) and in Kaplan and Zamir (2002) the information that the seller receives is correlated *across* buyers. Because of this correlation, information disclosure creates correlation in the payoff relevant part of types, their *valuations*. Second, in those papers information disclosure is public, whereas we allow the seller to employ public, as well as, *private* disclosure policies. Finally, they consider first price auctions, whereas our seller designs revenue maximizing mechanisms.

In Section 2 we describe and analyze the problem sketched earlier in the well-known independent private value set-up. Section 3 analyzes the problem in a much richer allocation setup that allows for interdependent or common values. Some remarks about the applicability of our findings in cases where the mechanism designer is interested in efficient, instead of revenue maximizing, mechanisms are offered in Section 4. There we also discuss the case where the seller's information is statistically correlated across buyers. We finish with a brief summary and some final remarks in Section 5. Proofs not found in the main text are in the Appendix.

2. The Basic Environment: Independent, Private Values

A risk neutral seller, indexed by 0, owns a unit of an indivisible object, and faces I risk neutral buyers. The seller's valuation for the object is zero, whereas that of buyer i is distributed on a set $V_i = [a_i, b_i]$ according to a continuous, and strictly positive density f_i. A buyer's valuation v_i is private and independently distributed across buyers. We use $f(v) = \times_{i \in I} f_i(v_i)$, where $v \in V = \times_{i \in I} V_i$ and $f_{-i}(v_{-i}) = \times_{\substack{j \in I \\ j \neq i}} f_j(v_j)$.

There are two "phases."

- At *phase 1* the seller observes privately a vector of signals $s = (s_1, ..., s_I)$ and decides what information to disclose to the buyers.
- At *phase 2* the seller chooses an auction procedure, a mechanism, to sell the object.

Phase 1 captures, in some sense, some previous interaction between the seller and the buyers. Such an interaction could be a previous auction, or a process during which the seller collected some information about some, (or all), of the buyers.

Seller's Information: The vector of signals s is taken to be exogenous. We use S_i to denote the set of signals that the seller can observe about buyer i; $S \equiv \times_{i \in I} S_i$ to denote the set of vectors of signals about all buyers, and $S_{-i} = \times_{i \in I} S_i$ to denote the set of vectors of signals about all buyers, but i. For simplicity, we take the set S_i to be finite. No result, however, hinges on this simplification. Let $\pi_i(s_i)$ denote the probability that the seller observes signal s_i about i, $i \in I$. These probabilities are common knowledge. We assume that $s_i's$ are independently distributed from one another, and use $\pi(s) = \times_{i\in I}\pi_i(s_i)$ to denote the joint distribution. Moreover, s_i is statistically independent from v_j, for all $i, j \in I$.

Each signal s_i is potentially informative about buyer $i's$ valuation, v_i. The posterior of v_i conditional on s_i is denoted by $f_i(v_i\,|s_i)$, which is assumed to be strictly positive and continuous on $V_i(s_i) = [\underline{v}_i(s_i), \bar{v}_i(s_i)]$.[12] Each buyer knows only the signal that the seller obtained about himself, but not the signals that the seller observes about the other buyers. For our results here, as well as in the more general model considered later, it does not make any difference whether the seller's information is verifiable or not.

Information Disclosure: The seller at phase 1 observes the vector of signals and chooses how much information to disclose to the buyers. An *information disclosure policy*[13] is a mapping from the vector of signals observed by the seller to a vector of messages, one for each buyer, that is, $D : s \to \Delta(\Lambda)$, where $\Lambda := \times_{i\in I}\Lambda_i$, and where Λ_i is the set of messages that the seller can send to buyer i. The vector of messages revealed is $\lambda = (\lambda_1, \lambda_2, ..., \lambda_I)$, with $\lambda_i \in \Lambda_i$, $i \in I$. We use $d(\lambda\,|s)$ to denote the probability that the disclosure policy reveals λ given s. The disclosure policy is common knowledge, but each buyer observes *only* λ_i.

Types: A type of a player consists of whatever he knows, that is not common knowledge. Here both the buyers and the seller have private information. Buyer $i's$ beliefs about j are determined from the prior and the disclosure policy conditional on s_i and λ_i. Because s_i and λ_i are privately observed, $i's$ beliefs about $-i$ are not known to the other buyers and they become part of buyer *$i's$ type*. Buyers $i's$ type consists of a payoff-relevant part, v_i, and a belief-relevant part, s_i, λ_i. The set of types of buyer i is denoted by Θ_i, and it consists of triplets of the form v_i, s_i, λ_i, where $v_i \in V_i(s_i)$ and $\lambda_i \in \Lambda_i(s_i)$, where $s_i \in S_i$. The set $\Lambda_i(s_i)$ contains all the messages that buyer i receives with strictly positive probability, when the signal that the seller has observed about him is s_i. In other words, a message λ_i is in $\Lambda_i(s_i)$ if $d(\lambda_i, \tilde{\lambda}_{-i}\,|s_i, \tilde{s}_{-i}) > 0$, for some $\tilde{s}_{-i} \in S_{-i}$ and $\tilde{\lambda}_{-i} \in \Lambda_{-i}$. The set of types for the seller, denoted by Θ_0, consists of vectors $\theta_0 = (s, \lambda)$, where $s \in S$ and $d(\lambda\,|s) > 0$. A type profile $\theta \in \Theta$, where $\Theta = \times_{i\in I}\Theta_i \times \Theta_0$, is given by

$$(\theta_i; \theta_{-i}; \theta_0) = \left(\underbrace{v_i, s_i, \lambda_i}_{\text{buyer } i\text{'s}} \; ; \; \underbrace{v_{-i}, s_{-i}, \lambda_{-i}}_{\text{buyers } -i} \; ; \; \underbrace{s_i, s_{-i}, \lambda_i, \lambda_{-i}}_{\text{seller}} \right).$$

[12] In Appendix B we discuss how the analysis can proceed when $f_i(v_i\,|s_i)$ is not strictly positive on all $V_i(s_i) = [\underline{v}_i(s_i), \bar{v}_i(s_i)]$. Continuity is also not essential for the results. Details can be found in Skreta (2007).

[13] The disclosure policy captures how the seller's strategy maps what the seller observes, to what she reports. At an equilibrium this map is understood by all. Alternatively, we could have the seller choose d before she sees s. For our analysis this difference turns out to be immaterial.

Often for convenience, we will write: $\theta = (\theta_i; \theta_{-i}; \theta_0) = (v, s, \lambda)$. We also use v for v_i, v_{-i}, s for s_i, s_{-i}, and λ for λ_i, λ_{-i}.

Information Disclosure Policies & Types - Examples:

Disclosure Policy	Type of buyer i
fully revealing: $\lambda_i = s$, for all i and $s \in S$	v_i
no information revelation $\lambda_i = s_i$, for all i	v_i, s_i
"partial" information revelation $\lambda_i = \begin{cases} s_{-i} \text{ w. pr } 0.5 \\ \hat{s}_{-i} \text{ w. pr } 0.5 \end{cases}$	v_i, s_i, λ_i

In case that the disclosure policy is fully revealing everyone has observed the same amount of information, and the seller's and buyer $i's$ beliefs about buyers $-i$ coincide and are common knowledge. In this case, buyer i's type consists merely of v_i. In the case of no information revelation, buyer $i's$ posterior about $-i$ is equal to the prior. In that case, $i's$ type is the couple v_i, s_i. For other disclosure policies, $i's$ type is the triplet v_i, s_i, λ_i. Summarizing, the information disclosure policy affects buyers types through affecting their beliefs about each other. Types then become *correlated*, since different types of a buyer have different beliefs about θ_{-i}.

Summarizing, we are considering a problem where the mechanism designer has private information that is correlated with buyers' willingness to pay. This information can be used to affect agents' beliefs about each other. Agents' beliefs become part of their type, which then consists by the valuation, and a belief-relevant part. Put differently, the mechanism designer can influence *agents' types* through the disclosure of information. Our objective is to evaluate the role of information disclosure on the performance of revenue maximizing mechanisms. Our seller here is informed, so formally, the problem we are about to analyze belongs in the class of *informed principal* problems.

2.1 Analysis of Seller's Mechanism Choice Problem

2.1.1 Mechanism Design by an Informed Principal: Summary and Interpretation

The mechanism selection game is usually modeled as a three stage game between a principal and the agent.[14] At stage one the principal proposes a mechanism, (a game form), at stage two the agent(s) accept or reject the mechanism. If they all accept, the mechanism is played at stage three, otherwise all get their outside option. At stage three the beliefs of the agents are updated to account for what they infer from the mechanism that the principal proposed at stage one. Also, the principal updates her beliefs about the agents types after observing that they all participated. Obviously this kind of game can have a plethora of equilibria. Moreover, its analysis can be quite challenging. However, there is a number

[14] See Maskin and Tirole (1990) and (1992).

of general lessons obtained by the earlier works by Roger Myerson, Eric Maskin and Jean Tirole. First, at equilibria of such three stage games all types of the principal offer the same schedule.

The second lesson, is that in the case of private values,[15] first studied by Maskin and Tirole (1990), equilibrium mechanisms guarantee the principal at least as high payoffs as she can obtain in the complete information case where her information were commonly known.[16] We call this private value case as $PV_{\text{Informed\_Principal}}$, to stress that the term "private values" refers to how the principal's information affects the agents' payoffs, and that this notion is completely distinct from the notion of the IPV setting. With private values, $PV_{\text{Informed\_Principal}}$, equilibria are Strongly Unconstrained Pareto Optimal, ($SUPO$),[17] and the equilibrium allocations correspond to the Walrasian equilibrium allocations of a fictitious exchange economy, where different types of the principal trade slack on the incentive and the participation constraints of the agents. Typically, the informed principal in a private value environment does strictly better than an uninformed one, however as Maskin and Tirole (1990) show in a single agent environment, and Mylovanov (2005) confirms for the case of multiple agents, in the case quasilinear preferences the principal is just as well off, when she has private information and when she does not.

In the case of common values,[18] denoted as $CV_{\text{Informed\_Principal}}$, typically the complete information allocation is not incentive compatible for the principal, hence she cannot always achieve her full information payoff. With common values, $CV_{\text{Informed\_Principal}}$, the principal can always achieve the payoff of the least costly separating equilibrium, what Maskin and Tirole (1992) refer to, as the Rothchild, Stigliz and Wilson, (RSW), allocation. This equilibrium can be thought as the "closest" to complete information subject to incentive compatibility constraints.[19]

[15] In the terminology of Maskin and Tirole (1990) and (1992), the "private values"case refers to the situation where the principal's information does not enter directly in the agents' payoff functions.

[16] Here we follow Maskin and Tirole (1990) and call "complete information," the case where there is no uncertainty about the principal, even though the agent can have private information.

[17] An allocation is $SUPO$, from the point of view of different types of the principal for given beliefs (of the agent), if there exists no other feasible allocation, (satisfying the constraints for the agent), even for different beliefs, that Pareto dominated it, (Maskin and Tirole (1990)).

[18] "Common values," $CV_{\text{Informed\_Principal}}$, refers to the case where the principal's information enters directly the payoff functions of the agents, For example, in the case of a seller of a used car the quality of the car enters directly the payoff function of the buyers. Similarly, an insurance company selling flood insurance, may have superior information about the probability of flooding compared to the agents, and the knowledge of these probabilities has a direct effect on the agents' payoffs. For more examples, and further discussions on the distinction between the case of private, and common values, see Maskin and Tirole (1990), and Maskin and Tirole (1992).

[19] To see this, note that at a separating equilibrium the agent infers the principal's type by the mechanism that she proposes, so at the point that the agent is about to participate he faces no uncertainty about the type of the principal. This implies that the incentive and the participation constraints for the agent have to hold pointwise for all types of the principal exactly as in the complete information case. What is different from that case, is that the allocation has to be incentive compatible for the principal. In contrast to the

The problem we consider here is somewhat distinct, in the sense that it belongs in the *common value class*, that is $CV_{\text{Informed\_Principal}}$, because the seller's information enters directly in the payoffs of the buyers, since it determines their beliefs about their opponents. However, the seller in our case, *can* achieve her complete information payoff. This is always true in a $PV_{\text{Informed\_Principal}}$ environment, but can fail with $CV_{\text{Informed\_Principal}}$.

We follow a somewhat an indirect approach, which is possible because of the beautiful insights from the path-breaking papers of Myerson (1983) and Maskin and Tirole (1990 and 1992). Once we are done with our somewhat "reduced form analysis," we will argue why the allocations we obtain correspond to equilibrium allocations of the three stage game sketched earlier. However, we will not be very explicit about all the details that are similar to the ones in the previous literature.

Let's start by abstracting for the moment from who has power to choose the mechanism. As usual the word mechanism refers to the game whose outcome will determine the probability distribution over who will obtain the object and the payments.

By the *revelation principle*[20] we can obtain all the equilibrium feasible allocations, by looking at equilibrium allocations of direct revelation mechanisms, that satisfy incentive, IC, participation, PC, and resource constraints, RES, for all buyers and the seller.

A *direct revelation mechanism, (DRM)*, $M = (p, x)$ consists of an *assignment rule* $p : \Theta \longrightarrow \Delta(I)$ and a *payment rule* $x : \Theta \longrightarrow \mathbb{R}^I$. The assignment rule specifies a probability distribution over the set of buyers given a vector of reports. We denote by $p_i(\theta)$ the probability that i obtains the good when the vector of reports is θ. Similarly, $x_i(\theta)$ denotes the expected payment incurred by i, given θ.

What makes an informed principal problem special, is precisely the fact that the person in charge of choosing (p, x) has private information. One consequence of that is that the seller must be also submitting reports in the mechanism. However, the most important implication, is that the choice of the schedule (p, x) itself can be revealing information to the buyers about the seller's information. This can happen because different types of the seller may prefer different $(p, x)'s$. This information release changes the buyers beliefs, and in order for (p, x) to be feasible, it must satisfy the incentive and the participation constraints of the buyers with respect to their posterior beliefs, obtained after they see the mechanism chosen by the seller.[21]

private value case, $PV_{\text{Informed\_Principal}}$, with $CV_{\text{Informed\_Principal}}$, the incentive constraints for the principal are typically binding. The reason is the same as in the "lemons problem." Exactly as a seller of a used car will try to convince the buyer that the car is of the highest possible quality by offering the same terms as the highest quality type, the principal will try to convince the agent that her information is the most favorable one.

[20] See, for instance Myerson (1979), or Myerson (1981).

[21] Relating to this point, Yilankaya (1999) shows in a bilateral trade setting that, (holding the beliefs of the buyer constant), some types of the seller prefer a double auction, whereas other types prefer a posted price. At an optimum, however, the buyer understands which types of the seller prefer the double auction, which overturns its optimality.

The *inscrutability principle* due to Myerson (1983), argues that it is without any loss to restrict attention to incentive feasible allocations, where all types of the individual in charge of choosing the mechanism, (the seller in our case), choose the same schedule. In other words, by the *inscrutability principle,* for all θ_0 the seller must be choosing the same schedule $p_i(\theta_i;\theta_{-i};\theta_0)$ and $x_i(\theta_i;\theta_{-i};\theta_0)$. Then, the choice of the schedule itself does not reveal any additional information. To see why, consider a situation where when $\theta_0 \in \Theta^A$, the seller chooses (p^A, x^A), and when $\theta_0 \in \Theta^B$, the seller chooses (p^B, x^B). In order for this scenario to be equilibrium feasible, it must be the case that (p^A, x^A), (respectively (p^B, x^B)), satisfies all the constraints for the buyers given their beliefs that the seller's type belongs in Θ^A, (respectively Θ^B). But then a schedule, call it (p^Γ, x^Γ), that is equal to (p^A, x^A) when $\theta_0 \in \Theta^A$, and is equal to (p^B, x^B), when $\theta_0 \in \Theta^B$, will satisfy all the constraints for the buyers, who now perceive that $\theta_0 \in \Theta^A \cup \Theta^B$. The buyers, upon observing (p^Γ, x^Γ), do not learn anything that they do not already know about the seller. From these considerations, we can conclude that the set of feasible schedules (p, x) is weakly larger when all types of the seller choose the same schedule. For this reason, in what follows we restrict attention to cases where all types of the seller offer the same direct revelation mechanism, (inscrutability). In may be worth reminding the reader that our analysis of the mechanism selection phase is for a given information disclosure policy $d(\lambda\,|s\,)$.

2.1.2 Mechanism Choice by Our Informed Seller for a Given Disclosure Policy

Fix an information disclosure policy $d(\lambda\,|s\,)$ and consider a *DRM* p and x, (revelation principle). By the inscrutability principle all types of the seller choose the same (p, x), which implies that after being confronted with (p, x), the buyers beliefs about the seller are equal to the beliefs after the disclosure policy has released information, but *before* the mechanism is proposed. The mechanism (p, x) does not release any new information.

We let $P_i(v_i, s_i, \lambda_i) = E_{v_{-i},s_{-i},\lambda_{-i}}[p_i(v, s, \lambda)\,|s_i, \lambda_i]$ denote the expected probability that i obtains the good, and $X_i(v_i, s_i, \lambda_i) = E_{v_{-i},s_{-i},\lambda_{-i}}[x_i(v, s, \lambda)\,|s_i, \lambda_i]$ denote the expected payment that he incurs, when his type is v_i, s_i, λ_i given the *DRM* under consideration, with respect to $i's$ beliefs *before* (p, x) is proposed.

The constraints for each *buyer i,* with $i \in I$, are given by:

IC$_i$: $P_i(v_i, s_i, \lambda_i)v_i - X_i(v_i, s_i, \lambda_i) \ \geq\ P_i(v_i', s_i', \lambda_i')v_i - X_i(v_i', s_i', \lambda_i')$, for all $\theta_i, \theta_i' \in \Theta_i$.

PC$_i$: $P_i(v_i, s_i, \lambda_i)v_i - X_i(v_i, s_i, \lambda_i) \geq 0$, for all $\theta_i \in \Theta_i$.

The constraints for the *seller* are given by

IC$_S$: $\int_{V(s)} \Sigma_{i\in I}[\Sigma_{\lambda\in\Lambda} d(\lambda\,|s\,)x_i(v, s, \lambda)dF(v\,|s\,) \geq \int_{V(s)} \Sigma_{i\in I}[\Sigma_{\lambda\in\Lambda} d(\lambda\,|s\,)x_i(v, \hat{s}, \hat{\lambda})dF(v\,|s\,)$, for all $\theta_0, \hat{\theta}_0 \in \Theta_0$.

PC$_S$: $\int_{V(s)} \Sigma_{i\in I}[\Sigma_{\lambda\in\Lambda} d(\lambda\,|s\,)x_i(v, s, \lambda)dF(v\,|s\,) \geq 0$, for all $\theta_0 \in \Theta_0$.

Resource Constraints are given by

$$\textbf{RES} : p_i(v, s, \lambda) \in [0, 1] \text{ and } \Sigma_{i \in I} p_i(v, s, \lambda) \leq 1, \text{ for all } \theta \in \Theta.$$

We move on to investigate consequences of these constraints.

Implications of Feasibility

We first investigate consequences of *incentive compatibility* for buyer i, $i \in I$. Let

$$u_i(v, s, \lambda) = p_i(v, s, \lambda)v_i - x_i(v, s, \lambda), \tag{1}$$

denote buyer $i's$ payoff given a mechanism p, x and v, s, λ. The expected payoff of type $\theta_i = v_i, s_i, \lambda_i$ of buyer i at a truth-telling equilibrium of a direct revelation mechanism is given by

$$U_i(v_i, s_i, \lambda_i) \equiv E_{v_{-i}, s_{-i}, \lambda_{-i}}[u_i(v, s, \lambda) \,|s_i, \lambda_i].$$

By the Envelope Theorem[22] we have that

$$\frac{\partial U_i(v_i, s_i, \lambda_i)}{\partial v_i} = P_i(v_i, s_i, \lambda_i), \tag{2}$$

where P is bounded, so it is integrable, that is

$$U_i(v_i, s_i, \lambda_i) = \int_{\underline{v}_i(s_i)}^{v_i} P_i(t_i, s_i, \lambda_i) dt_i + U_i(\underline{v}_i(s_i), s_i, \lambda_i), \text{ for } v_i \in V_i(s_i). \tag{3}$$

Using standard arguments, we can obtain the following *necessary* conditions for feasibility.

Lemma 1 *If a mechanism (p, x) satisfies IC_i, PC_i and RES for all buyers $i \in I$, then the following must be true:*

$U_i(\underline{v}_i(s_i), s_i, \lambda_i) \geq 0$ *for all* $i \in I$ *and* s_i, λ_i
$P_i(v_i, s_i, \lambda_i)$ *is increasing in* v_i, *for all* $i \in I$ *and* s_i, λ_i
$U_i(v_i, s_i, \lambda_i) = \int_{\underline{v}_i(s_i)}^{v_i} P_i(t_i, s_i, \lambda_i) dt_i + U_i(\underline{v}_i(s_i), s_i, \lambda_i)$, *for all* i *and* $\theta_i = v_i, s_i, \lambda_i$
$p_i(v, s, \lambda) \geq 0$ *and* $\Sigma_{i \in I} p_i(v, s, \lambda) \leq 1$, *for all* v, s, λ.

Now we will employ Lemma 1 in order to obtain a useful rewriting of the seller's expected revenue. In order to do so, we need a few additional pieces of notation. Let $J_i(v_i, s_i)$ denote the virtual surplus for buyer i conditional on s_i which is given by

$$J_i(v_i, s_i) = v_i - \frac{1 - F_i(v_i \,|s_i\,)}{f_i(v_i \,|s_i\,)}, \tag{4}$$

[22]For general Envelope Theorems applicable here, see Milgrom and Segal (2002).

also let $t_i(v,s,\lambda)$ denote the information premium paid by buyer i, when the vector of reports are v,s,λ, which is defined by

$$-t_i(v,s,\lambda) \equiv u_i(v,s,\lambda) - \int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i,v_{-i},s,\lambda)dt_i - u_i(\underline{v}_i(s_i),v_{-i},s,\lambda), \tag{5}$$

and finally, let

$$\underline{U}_i(s,\lambda) \equiv \int_{V(s)} u_i(\underline{v}_i(s_i),v_{-i},s,\lambda)dF(v\,|s\,), \tag{6}$$

denote the expected payoff that accrues to i from the *seller's* perspective, given a mechanism p,x, when his realized valuation is $\underline{v}_i(s_i)$, and when the seller's type is s,λ. Valuation $\underline{v}_i(s_i)$ is the lowest possible valuation for i given s_i.

The seller's revenue when her type is $s,\lambda = (s_i,s_{-i},\lambda_i,\lambda_{-i})$, given a disclosure policy d, and when she employs a mechanism (p,x), is given by:

$$\int_{V(s)} \Sigma_{i\in I} x_i(v,s,\lambda)dF(v\,|s\,). \tag{7}$$

Now *ignoring* the seller's constraints, and by using Lemma 1, we establish in Proposition 1 that the seller's revenue when her type is s,λ can be rewritten as

$$\int_{V(s)} \Sigma_{i\in I} p_i(v,s,\lambda)J_i(v_i,s_i)f(v\,|s\,)dv + \underbrace{\int_{V(s)} \Sigma_{i\in I} t_i(v,s,\lambda)f(v\,|s\,)dv}_{\text{information premium}} - \Sigma_{i\in I}\underline{U}_i(s,\lambda). \tag{8}$$

The first and the last term of the expression in (8) are standard. The new term is the middle one, and it consists of the sum of extra payments that the seller may be able to extract for "selling" information to the buyers about their competitors.

Proposition 1 *Fix a disclosure policy d. The seller's expected revenue when her type is s,λ from a mechanism p,x that satisfies IC_i for all $i \in I$, is given by (8).*

Proof. With the help of (1), (7) can be rewritten as

$$\int_{V(s)} \Sigma_{i\in I}[p_i(v,s,\lambda)v_i - u_i(v,s,\lambda)]dF(v\,|s\,). \tag{9}$$

Then, using (5), (9) can be rewritten as

$$\int_{V(s)} \Sigma_{i\in I}\left[p_i(v,s,\lambda)v_i - \int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i,v_{-i},s,\lambda)dt_i - u_i(\underline{v}_i(s_i),v_{-i},s,\lambda) + t_i(v,s,\lambda)\right] dF(v\,|s\,).$$

Using standard arguments the above expression can be rewritten as in (8). ■

The proof of Proposition 1 highlights how one can obtain an expression of the seller's expected revenue in situations where the seller's and buyer $i's$ beliefs about the valuations of $-i$ are different. The use of (5) allows us to circumvent the complications that come from the discrepancies in beliefs. The proof of the corresponding result in Myerson (1981), (Lemma 3), relies on the fact that the beliefs of the seller and the beliefs of i about $-i$ coincide. See equation 4.10 in that paper. All the arguments of the proof of Proposition 1 are also applicable in cases where the differences in the seller's and the buyers' beliefs result from the lack of a common prior.[23]

We now proceed to establish a further property of incentive compatible mechanisms for some fixed disclosure policy d. We show that at a mechanism that is incentive compatible for the buyers, the ex-ante, (that is before the seller observes s), expected information premium must be zero. This is done in two parts. First, in Lemma 2, we show that if a mechanism is incentive compatible, then it must be the case that from the perspective of buyer i, the expected information premium, must be zero for all types of i, that is for all, v_i, s_i, λ_i. Second, with the help of Lemma 2, we establish in Proposition 2 that the same must be true from the *ex-ante* perspective of the seller.

Lemma 2 *Consider a mechanism (p, x) that satisfies IC_i. Then, the expected information premium for buyer i, given s_i, λ_i, must be zero, that is*

$$E_{v_{-i},s_{-i},\lambda_{-i}}\left[t_i(v, s, \lambda)\,|s_i, \lambda_i\right] = 0.$$

The $t_i's$ can be interpreted as a side-contract that the seller can write with the buyers. Then, the result in Lemma 2 can be understood as saying that the expected value of a side contract for the buyers must be zero. The next result establishes that the same must be true from the seller's perspective.

Proposition 2 *For every information disclosure policy, and every feasible mechanism, given this disclosure policy, the* ***ex-ante*** *expected information premium from the seller's perspective is zero, that is*

$$\Sigma_{s\in S}\pi(s)\int_{V(s)}\Sigma_{\lambda\in\Lambda}d(\lambda\,|s)t_i(v, s, \lambda)f(v\,|s)dv = 0. \tag{10}$$

Returning to our side-contract interpretation, Lemma 2 and Proposition 2 can be understood in the framework of the following coin-tossing example. There are two individuals tossing two different coins. Each individual observes the outcome of their coin. There is a principal that observes both outcomes of the coins and discloses information as follows. Each vector of coin outcomes is mapped to a vector of private reports - one to each individual

[23] In the problem considered in this paper, even though we start with a common prior that is common knowledge, after the seller releases information, buyers may not any more know the beliefs of their competitors.

about the realized outcome of the other person's coin. More specifically, let H_i denotes the outcome "heads" of coin i, and T_i denote the outcome tails of coin i. The principal sends to each individual i a message about the realization of $j's$ coin for $i, j = 1, 2$; the messages are $\hat{H}_j, \hat{T}_j$. For instance, $d(\hat{T}_2, \hat{T}_1 \,|H_1, H_2)$ denotes the probability that each agent receives the information that the other agent's coin turned out to be tails, T, when actually both coins are heads, H. The principal offers contingent payments, $t_i's$, that can depend both on the vector of the realized coin outcomes and on the vector of messages send to the individuals. It is relatively easy to see, that if contingent payments have positive expected value for the principal, they have negative expected value for at least one of the agents. Hence the only contracts willingly accepted have, at most, zero expected value for the principal. For the same reasons as in this example, namely because there is a common prior and agents form posterior beliefs consistent with Bayes' rule, it is not possible for the seller's information premium to have strictly positive expected value.

As we have mentioned earlier, a disclosure policy induces discrepancies in beliefs. It may also induce correlations in types even in an environment where prior information is statistically independent. From Proposition 2 we concluded that from the *ex-ante* perspective, *irrespective* from the disclosure policy, the ex-ante expected information premium is zero. Hence exploiting the discrepancy in beliefs, is not beneficial for the seller at least from the ex-ante perspective. From (10) we can also conclude that the seller's *ex-ante* expected revenue, given a strategy that induces a disclosure policy d, and given a mechanism p, x that is incentive compatible for the buyers given d, is given by:

$$\Sigma_{s\in S}\pi(s)\Sigma_{\lambda\in\Lambda}d(\lambda\,|s)\left[\int_{V(s)}\Sigma_{i\in I}p_i(v,s,\lambda)J_i(v_i,s_i)f(v\,|s)dv-\Sigma_{i\in I}\underline{U}_i(s,\lambda)\right]. \tag{11}$$

We now proceed to characterize the mechanism p, x that our informed seller will choose given an information disclosure policy $d(\lambda\,|s)$.

Obtaining a Solution of the Informed Seller's Problem via a Relaxed Program

Instead of deriving implications of the incentive compatibility constraints for the seller, we look for mechanisms that maximize (11) for *each* type of the seller $\theta_0 = s, \lambda$, and check whether this schedule $p_i(v, s, \lambda), x_i(v, s, \lambda)$ is indeed incentive compatible for the seller. This is analogous to the approach in Maskin and Tirole (1990) who characterize $SUPO$ allocations, (Strong Unconstrained Pareto Optimal allocations). "Unconstrained" refers to the fact that the principal's constraints are not taken into account.

Here we consider schedules, where for each type of the seller $\theta_0 = s, \lambda$, (1) the allocation rule p solves Program S, which is given by

$$max_p \int_{V(s)} \Sigma_{i\in I} p_i(v,s,\lambda) J_i(v_i,s_i) f(v\,|s) dv - \Sigma_{i\in I} \underline{U}_i(s,\lambda)$$

$$(RES)\ p_i(v,s,\lambda) \geq 0 \text{ and } \Sigma_{i\in I} p_i(v,s,\lambda) \leq 1 \text{ for each } v,s,\lambda$$
$$(IC)\ P_i(v_i,s_i,\lambda_i) \text{ increasing in } v_i, \text{ for all } s_i,\lambda_i \text{ and all } i\in I$$
$$(PC)\ \underline{U}_i(s,\lambda) \geq 0 \text{ for all } s,\lambda \text{ and all } i\in I$$

and, (2), the payment rule is constructed from the allocation rule as follows: $x_i(v,s,\lambda) = p_i(v,s,\lambda)v_i - \int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i,v_{-i},s,\lambda)dt_i$.

The first set of constraints of Program S are resource constraints, the second set are necessary conditions following from incentive compatibility for the buyers, and the third set capture the participation constraints for the buyers. Notice, that a solution of Program S may depend on the disclosure policy, because the disclosure policy affects buyers' beliefs, which, in turn, determine the set of assignment rules that satisfy the property that $P_i(v_i,s_i,\lambda_i)$ is increasing in v_i for all i. Also, note that Program S ignores the term on information premium. Thus, it is as if we separate the seller's problem into two parts: one consisting of the virtual surplus and the term on the information premium.

We first establish that the value of Program S is independent from the disclosure policy. In particular, we show that irrespective of the information disclosure policy, a solution of this problem, call it p^*, x^*, coincides with a mechanism chosen by the seller when s is commonly known, call it p^s, x^s, that is

$$\begin{aligned} p^*(v,s,\lambda) &= p^s(v) \\ x^*(v,s,\lambda) &= x^s(v). \end{aligned} \tag{12}$$

Then we establish that the schedule in (12) is incentive compatible for the seller, thus showing that for the informed seller problem at hand, the incentive constraints for the seller are not binding.

Remark 1 *In what follows we are assuming that for all $i \in I$, $J_i(v_i,s_i) = v_i - \frac{1-F_i(v_i|s_i)}{f_i(v_i|s_i)}$ is strictly increasing in v_i. If not, we can "iron" $J_i(v_i,s_i)$, and proceed as in Myerson (1981).*[24]

Proposition 3 *For each s, a solution of Program S is independent from the disclosure policy, and it coincides with a solution of the seller's problem when s is common knowledge.*

As we have mentioned earlier, even though the expression in (8) looks almost identical to the standard one, it does not rely on the coincidence of the seller's and the buyers' beliefs, and in addition it does not require the seller to know anything about the beliefs that the buyers have. From Proposition 3 we have that the allocation rule that maximizes (8), (to

[24] As is shown, in Skreta (2007), we can iron virtual surpluses also in environments where f_i fails to be continuous and strictly positive.

be precise the first term of (8)), depends only in the seller's beliefs and is dominant strategy incentive compatible. This implies that the seller does not need to know the buyers' beliefs in order to check incentive compatibility either.

We now proceed to establish that a mechanism $p(v, \lambda, s)$, $x(v, \lambda, s)$, where for each s it coincides with a solution of Program S, p^*, x^* is incentive compatible for the seller.

Lemma 3 *A mechanism such that, for each s, it coincides with p^*, x^*, described in (12), satisfies participation and incentive constraints for the seller for every disclosure policy.*

Proof. By the definition of p^*, x^* it follows that the seller's expected revenue is always non-negative, hence it satisfies the seller's participation constraints. If the seller misreports $\hat{s}$, instead of s, then the mechanism will set $p_i^*(v, \hat{s}, \lambda) = 1$, for i with the highest $J_i(v_i, \hat{s}_i)$, for each v. This is clearly worse then giving the object to the buyer with the true highest virtual surplus, namely $J_i(v_i, s_i)$. Also, by definition p^*, x^*, is independent from λ, and hence from the disclosure policy. From the independence of p^*, x^* from λ we can also see that misreporting about λ cannot be strictly beneficial. Clearly, it is then in the seller's best interest to report his information truthfully. ■

We now state and prove our main result for this baseline model.

Theorem 1 *Fix a disclosure policy. At every optimal mechanism given this disclosure policy, the seller's expected revenue at each state s is equal to the expected revenue of a revenue maximizing mechanism when s is common knowledge.*

Proof. First define

$$R(p, x \,|s\,) \equiv \int_{V(s)} \Sigma_{i\in I}\Sigma_{\lambda\in\Lambda} c(\lambda \,|s\,) p_i(v, s, \lambda) J_i(v_i, s_i) f(v \,|s\,) dv.$$

Step 1: Schedule p^*, x^* is also ex-ante optimal

Let p^E and x^E denote a mechanism that given $d(\lambda \,|s\,)$ maximizes *ex-ante* expected revenue for the seller, ignoring the seller's constraints. Recalling (11), p^E and x^E solves:

$$\Sigma_{s\in S}\pi(s)\Sigma_{\lambda\in\Lambda} d(\lambda \,|s\,) \left[\int_{V(s)} \Sigma_{i\in I} p_i(v, s, \lambda) J_i(v_i, s_i) f(v \,|s\,) dv - \Sigma_{i\in I} \underline{U}_i(s, \lambda) \right],$$

(RES) $p_i(v, s, \lambda) \geq 0$ and $\Sigma_{i\in I} p_i(v, s, \lambda) \leq 1$ for each v, s, λ
(IC) $P_i(v_i, s_i, \lambda_i)$ increasing in v_i, for all s_i, λ_i and all $i \in I$
(PC) $\underline{U}_i(s, \lambda) \geq 0$ for all s, λ and all $i \in I$

We call this problem Program E.

Since p^*, x^* also satisfies the constraints of Program E, by the definition of p^E, x^E we immediately have that

$$\Sigma_{s\in S}\pi(s) \cdot R(p^E, x^E \,|s\,) \geq \Sigma_{s\in S}\pi(s) \cdot R(p^*, x^* \,|s\,). \tag{13}$$

Now p^*, x^* is optimal for each s, λ, and p^E, x^E is feasible for Program S, (the constraints of Program S and Program E coincide), then it must be the case that *for each* s :

$$R(p^*, x^* \,|s\,) \geq R(p^E, x^E \,|s\,). \tag{14}$$

Because (14) holds for each s, it follows that it also holds in expectation over all possible s, namely,

$$\Sigma_{s\in S}\pi(s) \cdot R(p^*, x^* \,|s\,) \geq \Sigma_{s\in S}\pi(s) \cdot R(p^E, x^E \,|s\,). \tag{15}$$

From (13) and (15) it follows that

$$\Sigma_{s\in S}\pi(s) \cdot R(p^*, x^* \,|s\,) = \Sigma_{s\in S}\pi(s) \cdot R(p^E, x^E \,|s\,). \tag{16}$$

Step 2: Schedule p^*, x^* is a solution to the informed seller problem

Let p^I, x^I denote a solution for our informed seller given some disclosure policy $d(\lambda \,|s\,)$. From Lemma 3 we know that p^*, x^*, described in (12) is feasible for the informed seller for every disclosure policy. Moreover, the revenue generated for each s, λ by p^*, x^* is independent from the disclosure policy and is feasible irrespective of the beliefs of the buyers. From this it follows, that if p^I, x^I is a solution to the informed seller problem for some disclosure policy $d(\lambda \,|s\,)$, then it must be the case that for each s, λ the seller's revenue at p^I, x^I is as least as high as at p^*, x^* for all s, λ, that is,

$$\begin{aligned} &\int_{V(s)} \Sigma_{i\in I}\left[p_i^I(v,s,\lambda)J_i(v_i,s_i) + t_i^I(v,s,\lambda)\right] f(v\,|s\,)dv - \Sigma_{i\in I}\underline{U}_i(s,\lambda) \\ \geq \; &\int_{V(s)} \Sigma_{i\in I}p_i^*(v,s,\lambda)J_i(v_i,s_i)f(v\,|s\,)dv. \end{aligned}$$

(Each type of seller has the option of using p^*, x^*, irrespective of the disclosure policy.)[25]

Now suppose that revenue is *strictly higher* for some $\hat{s}, \hat{\lambda}$, then by taking expectations over all s, λ, and by recalling that from the ex-ante perspective the information premium is zero, namely equation (10), we get that

$$\Sigma_{s\in S}\pi(s) \cdot R(p^I, x^I \,|\hat{s}\,) > \Sigma_{s\in S}\pi(s) \cdot R(p^*, x^* \,|\hat{s}\,),$$

which combined with (16), contradicts the ex-ante optimality of p^E, x^E. From this, it follows that for all s, λ

$$\begin{aligned} &\int_{V(s)} \Sigma_{i\in I}\left[p_i^I(v,s,\lambda)J_i(v_i,s_i) + t_i^I(v,s,\lambda)\right] f(v\,|s\,)dv - \Sigma_{i\in I}\underline{U}_i(s,\lambda) \\ = \; &\int_{V(s)} \Sigma_{i\in I}p_i^*(v,s,\lambda)J_i(v_i,s_i)f(v\,|s\,)dv. \end{aligned}$$

[25] This is analogous to the analysis of private values in Maskin and Tirole (1990), where the informed principal can at least guarantee her full information payoff.

Hence p^*, x^* is a solution to the informed seller problem irrespective of the disclosure policy. ■

Let us now recapitulate the steps we followed to solve our informed principal problem and offer connections with the pre-existing literature. We first derived necessary conditions for menus (p, x) that satisfy the constraints for the *buyers*. Then we solved an artificial problem, where each type of the seller chooses the menu that is best for her, ignoring all her constraints. The schedule p^*, x^* turned out to satisfy the incentive constraints for the seller when the buyers know the seller's type. Hence p^*, x^* is *safe* in the terminology of Myerson (1983). By construction, it is also interim efficient, ($SUPO$ in the terminology of Maskin and Tirole (1990)), which means that it is a *strong solution,* in the sense of Myerson (1983). As argued in the works of Maskin and Tirole (1990 and 1992), strong solutions are equilibria of their three-stage game, and they survive all common refining criteria.

Corollary 1 *The schedule* p^*, x^* *is a* ***strong solution****, in the sense of Myerson (1983).*

In Theorem 1 we established that any mechanism chosen by the informed seller must generate at each s, λ exactly the same revenue that is generated by the mechanism that is optimal when s is commonly known, that is the mechanism that guarantees her full information payoff. We can then conclude, that an optimal disclosure policy for the seller is to always reveal all the information that she has.

Corollary 2 *Assuming that an informed seller discloses all the information that she has, is without any loss in terms of expected revenue: full transparency is optimal.*

We have established an information irrelevance result. Despite the fact that the seller can use her private information to create correlations in the buyers' types, all disclosure policies are equivalent in terms of revenue generation, and the seller can do no better, compared to the case where her information is publicly available to all the buyers. Next, we summarize the forces present behind this result

2.1.3 Discussion

Here we highlight three important forces behind our result, and discuss which of the them are present in more general mechanism design problems where, for instance, values can be interdependent or common, and, which ones are more specific to this independent private value scenario. The key forces of the result are three. First, disclosure policies, irrespective of how sophisticated they are, they do not eliminate information rents. Second, the seller's incentive constraints are not binding, and finally when looking at the relaxed program for the seller, namely Program S, its solution is dominant strategy incentive compatible. As we discuss below, the first two forces are present in more general mechanism design problems, however, the last one is more particular to this simple independent private value problem.

Disclosure Policies Create Correlation in Types, but do not Eliminate Information Rents

From our analysis it follows that information disclosure policies do not affect the information rents that each type of a buyer enjoys, which are always equal to the ones obtained when the seller's information is fully disclosed. Recall that buyer $i's$ expected payoff given a disclosure policy d and a mechanism that is incentive compatible given d, is given by

$$U_i(v_i, s_i, \lambda_i) = \int_{\underline{v}_i(s_i)}^{v_i} P_i(t_i, s_i, \lambda_i)dt_i + U_i(\underline{v}_i(s_i), s_i, \lambda_i), \text{ for } v_i \in V_i(s_i). \tag{17}$$

From (17) we have that $U_i(v_i, s_i, \lambda_i) \geq 0$, for all $v_i \in V_i(s_i)$. Moreover, if $v_i^{\min}(s_i)$ is the lowest type for which $P_i(t_i, s_i, \lambda_i) > 0$, then for $v_i \in [v_i^{\min}(s_i), \bar{v}_i(s_i)]$ we have that $U_i(v_i, s_i, \lambda_i) > 0$. Therefore, irrespective of the sophistication of the information disclosure policies employed by the seller, for types that receive the object with strictly positive probability, full surplus extraction from the buyer i's perspective is not possible, unless he has no private information, which can happen when s_i fully reveals $i's$ valuation to the seller. This observation may sound a bit surprising, since for most information disclosure policies, buyers types become correlated. However, full surplus extraction is impossible because more than one payoff relevant types have the same beliefs. Here the set of payoff relevant types, that is the $v_i's$, that are associated with the same probability distribution over belief-relevant parts, namely s_i, λ_i, belong in the set $V_i(s_i) = [\underline{v}_i(s_i), \bar{v}_i(s_i)]$. Then, irrespective of the disclosure policy, all $v_i's$ in $V_i(s_i)$ can mimic each other exactly as in the case where s_i were commonly known. Hence the expected information rent is $\int_{\underline{v}_i(s_i)}^{v_i} P_i(t_i, s_i, \lambda_i)dt_i + U_i(\underline{v}_i(s_i), s_i, \lambda_i)$. This was first observed by Zvika Neeman (2004).

However, this observation on its own, does not immediately imply that disclosure policies do not affect information rents. The reason is that expected information rent in (17) is calculated with $i's$ beliefs and these beliefs may differ from the seller's. What is relevant for the seller's is "how often" $u_i(v, s, \lambda) > \int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i, v_{-i}, s, \lambda)dt_i + u_i(\underline{v}_i(s_i), v_{-i}, s, \lambda)$ versus the reverse, from her perspective. In Lemma 2 we established that from a buyer's perspective the expected difference, (slack), of $u_i(v, s, \lambda)$ and $\int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i, v_{-i}, s, \lambda)dt_i + u_i(\underline{v}_i(s_i), v_{-i}, s, \lambda)$, namely the $t_i(v, s, \lambda)$, must be zero. We then proceeded to establish in Proposition 2 that the same is true from the seller's perspective. From these considerations it follows that irrespective from the disclosure policy the expected "slack" is always the same and equal to zero. This result is due to the fact that we are in a consistent model with a common prior, and buyers form beliefs according to Bayes' rule. This feature will be also present in all models where there is a common prior and initial information is statistically independent, irrespective of whether values are private or interdependent.

The Seller's Constraints are Not Binding

From our analysis it became clear that the seller's incentive constraints were not binding. This feature is due to the fact that the information that the seller has is semi-private, in the sense that it is *not exclusive*, (remember that buyer i knows the signal that the seller has about himself). This force is present in more general mechanism design problems, like the interdependent value environment that we consider in the next Section

Disclosure Policies do not Affect the Value of Program S, *because its Solution is Dominant Strategy Incentive Compatible*

The reason behind this result is that in this baseline, IPV model a solution of Program S is dominant strategy incentive compatible. This feature is not unique to this problem. Mookherjee and Reichelstein (1992) describe environments where an optimal Bayesian incentive compatible mechanism is dominant strategy incentive compatible. However, it is not true in more general mechanism design problems.

This last reason, is what makes all information disclosure policies equivalent in the environment we have examined so far. As we will show in the next section, in more general mechanism design problems, where a solution of Program S is not dominant strategy implementable, information disclosure policies will matter, because they affect the set of incentive compatible mechanisms, which in turn affect the value of Program S. For those cases we show that the value of Program S is maximized when the seller discloses no information at all.

3. General Environment: Interdependent Values, Non-linear Payoffs, Multiple Goods.

Here we examine revenue maximizing mechanisms in more general environments that allow for interdependent or common values and for multiple objects. There is a risk neutral seller who faces I risk-neutral buyers. Let Z denote the set of allocations, that is the set of all possible assignment of the good(s).[26] Both Z and I are finite natural numbers. Buyer i's payoff from allocation z is denoted by $u_i^z(v_i, v_{-i})$ and it depends on v_i, and on the valuations of all the other buyers v_{-i}. Values are therefore *interdependent*. We assume that, for all $i \in I$, $u_i^z(\cdot, v_{-i})$ is *increasing*, *convex* and *differentiable* for each z and v_{-i}.[27] A buyer's payoff from not participating in the mechanism is taken to be zero. We also normalize the seller's payoff from all $z \in Z$ to be zero.

As in the baseline model, the $v_i's$ are statistically independent from one another. Also, the structure of the signals that the seller observes, the definitions of the disclosure policy, as well as the statements of incentive compatible mechanisms, are all as in the baseline model. For the same reasons as there, we will appeal to the revelation and the inscrutability

[26] In the case that the seller has $1 \leq N < \infty$ objects for sale, an allocation z is an assignment of the objects to the buyers and to the seller. It is a vector with N components, where each component stands for an object and it specifies who gets it, therefore the set of possible allocations is finite, and given by $Z \subseteq [I \cup \{0\}]^N$. Note that the formulation is very flexible and it allows, for the goods to be heterogeneous, substitutes for some buyers, whereas complements for other buyers and for externalities.

[27] This model shares common features to the one considered by Figueroa and Skreta (2007).

principals.

We only need to adjust very slightly the definition of direct revelation mechanisms. Here the *assignment rule* $p : \Theta \longrightarrow \Delta(Z)$ specifies the probability of each allocation for a given vector of reports. We denote by $p^z(\theta)$ the probability that allocation z is implemented when the vector of reports is θ.

With some abuse of notation, we now let

$$\begin{aligned} u_i(v,s,\lambda) &= \sum_{z\in Z} p^z(v,s,\lambda)u_i^z(v_i,v_{-i}) - x_i(v,s,\lambda), \\ U_i(v_i,s_i,\lambda_i) &= E_{v_{-i},s_{-i},\lambda_{-i}}\left[\sum_{z\in Z} p^z(v,s,\lambda)u_i^z(v_i,v_{-i}) - x_i(v,s,\lambda)\,|s_i,\lambda_i\right], \end{aligned}$$

and

$$U_i(v_i,s_i,\lambda_i) = \max_{v_i'} E_{v_{-i},s_{-i},\lambda_{-i}}[\sum_{z\in Z} p^z(v_i',v_{-i},s,\lambda)u_i^z(v_i,v_{-i}) - x_i(v_i',v_{-i},s,\lambda)\,|s_i,\lambda_i],$$

stand for their analogs in this more general environment. Notice that $U_i(v_i,s_i,\lambda_i)$ is convex, since it is a maximum of convex functions.

Using analogous arguments as in the baseline model, we establish a result parallel to Lemma 1. In the present context, the incentive constraints translate into the requirement that the derivative of U_i, namely,

$$P_i(v_i,s_i,\lambda_i) \equiv E_{v_{-i},s_{-i},\lambda_{-i}}[\sum_{z\in Z} p^z(v,s,\lambda)\frac{\partial u_i^z(v_i,v_{-i})}{\partial v_i}\,|s_i,\lambda_i],$$

evaluated at the true type is weakly increasing.[28]

Similarly, as in the baseline model, we define virtual surpluses. The concept now of virtual surplus is allocation-specific, because an allocation maybe affecting all buyers, and it is given by

$$J_z(v,s) \equiv \Sigma_{i\in I}\left[u_i^z(v_i,v_{-i}) - \frac{1-F_i(v_i\,|s_i)}{f_i(v_i\,|s_i)}\frac{\partial u_i^z(v_i,v_{-i})}{\partial v_i}\right].$$

The seller's revenue, when her type is s, λ, is given by:

$$\int_{V(s)} \Sigma_{i\in I} x_i(v,s,\lambda)dF(v\,|s). \tag{18}$$

Then using similar arguments as in the baseline case, (18), can be rewritten as

$$\int_{V(s)} \sum_{z\in Z} p^z(v,s,\lambda)J_z(v,s)f(v\,|s)dv + \underbrace{\int_{V(s)} \Sigma_{i\in I} t_i(v,s,\lambda)f(v\,|s)dv}_{\text{expected information premium}} - \Sigma_{i\in I}\underline{U}_i(s,\lambda), \tag{19}$$

[28] More precisely, this holds for a selection from its subgradient, which is single valued almost surely.

where here $t_i(v,s,\lambda)$ and $\underline{U}_i(s,\lambda)$ stand for

$$-t_i(v,s,\lambda) \equiv u_i(v,s,\lambda) - \int_{\underline{v}_i(s_i)}^{v_i} \sum_{z\in Z} p^z(t_i,v_{-i},s,\lambda)\frac{\partial u_i^z(t_i,v_{-i})}{\partial v_i}dt_i - u_i(\underline{v}_i(s_i),v_{-i},s,\lambda) \quad (20)$$

and

$$\underline{U}_i(s,\lambda) \equiv \int_{V(s)} u_i(\underline{v}_i(s_i),v_{-i},s,\lambda)dF(v\,|s)\underline{U}_i(s). \quad (21)$$

Now because Lemma 2 and Proposition 2 apply directly to this more general environment, at a mechanism (p,x) that satisfies IC_i we have that

$$E_{v_{-i},s_{-i},\lambda_{-i}}\left[t_i(v,s,\lambda)\,|s_i,\lambda_i\right] = 0, \quad (22)$$

and

$$\Sigma_{s\in S}\pi(s)\int_{V(s)}\Sigma_{\lambda\in\Lambda}d(\lambda\,|s)t_i(v,s,\lambda)f(v\,|s)dv = 0. \quad (23)$$

Fix a mechanism that is incentive compatible for the buyers, then from (22) and (23) we obtain that the seller's *ex-ante* expected revenue at is given by:

$$\Sigma_{s\in S}\pi(s)\Sigma_{\lambda\in\Lambda}d(\lambda\,|s)\left[\int_{V(s)}\Sigma_{z\in Z}p^z(v,s,\lambda)J_z(v,s)dF(v\,|s) - \Sigma_{i\in I}\underline{U}_i(s,\lambda)\right]. \quad (24)$$

As before, we ignore the seller's incentive constraints and look for mechanisms that maximize (24) for each s,λ and check whether this schedule $p_i(v,s,\lambda),x_i(v,s,\lambda)$ is indeed incentive compatible for the seller. In particular, we consider schedules, where for each s, (1) the allocation rule p solves **Program** S_G, which is given by

$$max_p\int_{V(s)}\Sigma_{z\in Z}p^z(v,s,\lambda)J_z(v,s)dF(v\,|s) - \Sigma_{i\in I}\underline{U}_i(s,\lambda) \quad (25)$$

(RES) $p_i(v,s,\lambda)\geq 0$ and $\Sigma_{i\in I}p_i(v,s,\lambda)\leq 1$ for each v,s,λ
(IC) $P_i(v_i,s_i,\lambda_i)$ increasing in v_i, for all s_i,λ_i and all $i\in I$
(PC) $\underline{U}_i(s,\lambda)\geq 0$ for all s,λ and all $i\in I$

and (2) the payment rule is constructed from the allocation rule as follows: $x_i(v,s,\lambda) = \Sigma_{z\in Z}p^z(v,s,\lambda)u_i^z(v_i,v_{-i}) - \int_{\underline{v}_i(s_i)}^{v_i}\Sigma_{z\in Z}p^z(t_i,v_{-i},s,\lambda)\frac{\partial u_i^z(t_i,v_{-i})}{\partial v_i}dt_i$.

Solving Program S_G is straightforward if pointwise optimization of the objective function leads to a feasible solution. In this case the solution is dominant strategy incentive compatible, and for all practical purposes we are essentially back to the baseline case. However, for this problem pointwise optimization generally fails[29] to generate a feasible

[29] Note that in this problem incentive compatibility may be binding even if virtual surpluses are monotonic. For a discussion of this point see Figueroa and Skreta (2007b). There, it is also established that when pointwise optimization fails to lead to an incentive compatible allocation, a solution maybe randomizing between different allocations.

mechanism. When this is the case, a solution will most likely not be dominant strategy implementable. Then, a solution of Program S_G may depend on the disclosure policy. The reason is that disclosure policies affect buyers' beliefs, which in turn determine the set of assignment rules that satisfy the property that $P_i(v_i, s_i, \lambda_i)$ is increasing in v_i, for all i. In other words, disclosure policies affect the set of incentive compatible mechanisms. In Appendix C we present an example where whether the pointwise optimum is feasible or not, actually depends on the disclosure policy. This implies that the maximized value of Program S_G depends on the disclosure policy.[30] Given this observation, the question that naturally arises is what disclosure policies are best, in the sense of maximizing the value of Program S_G. We address this next. We establish that the value of Program S_G is maximized when the seller discloses no information. The reason is that any mechanism that is incentive compatible under any alternative disclosure policy, is also incentive compatible under the "no information disclosure" policy.

Proposition 4 *All mechanisms that are feasible for Program S_G under every disclosure policy, are also feasible for Program S_G under the "no information" disclosure policy, hence the "no information" disclosure policy dominates any other information disclosure policy.*

From Proposition 4 it follows that the value of Program S_G is maximized when the seller discloses no information. We call this disclosure policy d^*. Suppose that the seller uses d^*, and let $p^{**}(., s), x^{**}(., s)$ denote a solution of Program S_G, when the seller's information is s. We remove the argument of λ from p^{**}, x^{**}, because the vector v, s summarizes all relevant information, when the seller discloses no information. In the Theorem that follows we establish that the mechanism p^{**}, x^{**}, where for each s it is equal to $p^{**}(., s), x^{**}(., s)$, is a solution to our informed seller's problem.

Theorem 2 *In the general case, an informed seller will maximize revenue by disclosing no information, and by employing a mechanism p^{**}, x^{**}, where for each s it solves Program S_G.*

Proof. First we need to establish that p^{**}, x^{**} is feasible for our informed seller problem. Given that p^{**}, x^{**} is a solution of Program S_G for each s, λ, it satisfies resource constraints, as well as all the constraints for the buyers. Now, it is easy to see that it satisfies the participation constraints of the seller, since the seller can always guarantee zero by not assigning the objects and not charging anything. Hence the value of Program S_G is positive for each s, λ. Then, we just need to establish incentive compatibility for the seller. This is

[30] The example in Appendix C demonstrates this point in an interdependent value setting. However, the same can occur also in a private value environment when pointwise optimization of S_G fails to lead to an incentive compatible mechanism. There an optimum can involve randomizations, and in those cases, a disclosure policy can relax the incentive constraints by allowing for less distortionary randomizations.

also immediate, since p^{**}, x^{**} solves Program S_G for each s, and hence we have that

$$\int_{V(s)} \Sigma_{z\in Z} p^{**}(v,s) J_z(v,s) dF(v\,|s\,) \geq \int_{V(s)} \Sigma_{z\in Z} p^{**}(v,\hat{s}) J_z(v,s) dF(v\,|s\,).$$

By misreporting the seller can only select a schedule that maximizes a "wrong "objective function.

We now examine a further property of the informed seller problem. It is not hard to see, that a mechanism where $t_i(v,s,\lambda) < 0$ for some v,s,λ is not feasible, because a seller of type s,λ, strictly benefits by setting $t_i(v,s,\lambda) = 0$. Hence it must be the case that $t_i(v,s,\lambda) \geq 0$, for all v,s,λ. Given this observation, and Proposition 2, it follows that a solution to the informed seller's problem it must in fact be the case that $t_i(v,s,\lambda) = 0$ for all v,s,λ.

With this in hand, the best that our informed seller can hope for, is a mechanism, where for each s it solves Program S_G. As we just argued, such a mechanism is indeed feasible for the informed seller problem. Given that the value of Program S_G is maximal, under the no information disclosure policy, d^*, we will now argue that the mechanism p^{**}, x^{**} is a solution to our informed seller problem.

To see this note that the allocation induced by the mechanism p^{**}, x^{**} is $SUPO$ in the terminology of Maskin and Tirole (1990). It is *UPO* for the beliefs equal to the prior,[31] and there is no other *UPO,* even for other posterior beliefs, that is better than p^{**}, x^{**}. Note however, that p^{**}, x^{**} need not be the only solution of Program S_G given the no information disclosure policy. When there is more than one solution, all of them are $SUPO$.

Following exactly analogous arguments as in Maskin and Tirole (1990), one can show that p^{**}, x^{**} is an equilibrium of the three stage game we described earlier earlier. Remember that all stages of this game take place phase 2, for the given disclosure policy that is chosen at phase 1. At phase 1: the seller discloses no information. At phase 2, stage one, the seller proposes p^{**}, x^{**}, at stage two, the buyers accept or reject p^{**}, x^{**}, and finally, at stage three, the buyers and the seller report their types truthfully. Given that p^{**}, x^{**} satisfies IC for both the buyers and the seller, at stage three they will find it optimal to reveal their information truthfully. Now since p^{**}, x^{**} satisfies also PC for the buyers, they will all find it optimal to participate at stage 2. Now at stage 1 suppose that the seller proposes an out-of equilibrium mechanism. *No matter what* the buyers will infer from that, the alternative schedule cannot be strictly better than p^{**}, x^{**}, no matter which types chose to deviate. This is because for all types s, $p^{**}(v,s), x^{**}(v,s)$, (given beliefs determined by the no information disclosure policy), gives type s of the seller the highest expected revenue that she can hope for. Hence no alternative $\tilde{p}, \tilde{x}$ makes s strictly better-off than $p^{**}(v,s), x^{**}(v,s)$. Therefore p^{**}, x^{**} is an equilibrium allocation. ■

Before discussing to what degree our findings extend further than the situations examined so far, let us compare this more general case to the IPV one. We saw that a schedule p, x with the property that for each $\theta_0 = s, \lambda$, it coincides with a solution of S_G, is feasible

[31] These are the beliefs of the buyers at phase two, after the seller discloses no information at phase one.

for the informed seller problem. Hence, as we have already pointed out in the baseline IPV model, the incentive constraints of the seller do not bind irrespective of the disclosure policy, even in this interdependent value environment. This is because the seller's information is non-exclusive.[32] Also, as in the baseline case, the seller can achieve her complete information payoff. This is obtained by the solution of program S_G under the full information disclosure policy. However, here the seller here can typically do strictly better than that. In fact we showed that the value of Program S_G is maximal under the no information disclosure policy. We now move on to discuss a few extensions.

4. Extensions

Our findings extend in a straightforward manner to a number of other mechanism design problems.

Efficient Mechanisms: If the interest is in designing efficient, instead of revenue maximizing, mechanisms, then the previous analysis can be very easily modified. All that one needs to do, is to replace virtual surpluses with actual surpluses.

Multidimensional Types: Mechanism design problems with multidimensional types are notoriously difficult, however, finding the optimal information disclosure policy, is not harder compared to case of single dimensional types. All the steps described in the proof of Proposition 4 go through in this case as well. Unfortunately, the standard difficulties in solving the analog of Program S_G are still present.

Buyers' (Prior) Information Correlated: Suppose, like we have so far, that the information that the seller observes about the buyers is statistically independent across them, but now, the ex-ante information of the buyers is statistically correlated. Then, when the conditions of Cremer and McLean (1988), or of McAfee and Reny (1992) are satisfied, the seller can extract full surplus, exactly as in those papers, so there is no further scope for information disclosure. If we think of types consisting of a payoff relevant part, and a belief relevant part, as in the work of Neeman (2004), and Heifetz and Neeman (2006) our results go through in the following sense. The correlation in the prior information can be exploited using "Cremer-McLean" type of lotteries, to infer the buyers' valuations. When beliefs are part of types, however, the inference will typically not be perfect, so instead of learning the buyers valuations, as in CM, the seller will have a sharper estimate of where they lie. After that initial step, we are essentially back to the case of statistically independent private information, with the priors replaced by the posteriors.

[32] However, when *the seller's information is exclusive,* as, for instance, in the case where the seller's valuation is private information, the IPV case, and the interdependent value case differ dramatically. Mylovanov (2005) demonstrates in an IPV setup that the seller can achieve as much revenue as an uniformed one. However, related work by the author, Skreta (2007b), demonstrates that in the interdependent values case, the incentive constraints for an informed seller are binding, and hence she achieves *less* than an uninformed one.

Seller's Information Correlated Across Buyers: For the cases where the seller observes information that is statistically correlated *across* buyers, we conjecture that the seller will be able to use this information to her advantage, even in the baseline IPV case, and of course in the case of interdependent values. The reason is that there the release of information will create correlation in the payoff-relevant part of the buyers' types. The papers by Landsberger, Rubinstein, Zamir and Wolfstetter (2001), and by Kaplan and Zamir (2002), demonstrate this in the framework of first price auctions and public disclosure policies.

5. Concluding Remarks

This paper studies revenue maximizing auctions by an informed seller. The seller has information about various buyers that is not known to all buyers participating in the auction. Before the auction, the seller can disclose any amount of information she sees fit. After the disclosure of information the seller chooses a revenue maximizing mechanism. At that point we have an informed seller problem.

For the classical single object independent private values environment we establish an information irrelevance theorem. Release of information correlated with buyers' valuations has no effect on the expected revenue generated by a revenue maximizing mechanism. This is despite the fact that the seller can create correlation in types. In more general private value or interdependent value environments, however, information disclosure matters. There we establish that, in general, the best that the seller can do is to disclose no information at all.

This finding is opposite from the "linkage principal," but it is consistent with the common practice of sellers in auctions of companies who release as little information as possible. Hansen (2001) states this as one of the key four stylized facts of corporate auctions. In his words, "Sellers restrict the flow of information to bidders." As noted in the introduction, in sales of companies sellers have substantial information about various bidders' characteristics from the pre-auction selection stage.

The main message of our analysis is that for a seller who has the power to choose the rules of trade based on the information that she has, sophisticated disclosure policies do not pay. Simply revealing nothing is optimal. In some special cases, like in the classical independent private value paradigm, our information irrelevance result allows us to conclude that anything between no and full information disclosure is optimal. It would of interest to investigate these questions within the framework of particular auction mechanisms. We leave this for future work.

We close with a few final points to help relate our findings to the pre-existing literature on informed principal. First, even though our informed seller problem, (both in the baseline IPV case, and in the general case), falls in the common value category,[33] $CV_{\text{Informed\_Principal}}$,

[33] In the terminology of Maskin and Tirole (1990) and (1992).

it has a private value, $PV_{\text{Informed\_Principal}}$, flavor in that the seller can always achieve her complete information payoff. Second, in our general model the informed principal can do *strictly* better than an uniformed one, even though we are in a quasilinear setup. This is because the seller's private information relaxes the incentive constraints of the buyers. This finding is in contrast to the analysis of Maskin and Tirole (1990) and Mylovanov (2005), who consider $PV_{\text{Informed\_Principal}}$ environments, and show that in the quasilinear case the principal is indifferent between having private information and not. Finally, in our informed principal problem, the principal's information is correlated with the information of the agents. This is also the case in Cella (2007), who considers a single agent, private value, $PV_{\text{Informed\_Principal}}$, quasilinear environment, where as in this paper, the informed principal is sometimes strictly better-off than an uninformed one. The forces present are different, however, since in our set-up the asymmetric information between the buyers and the seller appears only if there is more than one buyer.

6. Appendix A: Omitted Proofs

Proof of Lemma 2

By definition we have that at a truth telling equilibrium it must be the case that

$$U_i(v_i, s_i, \lambda_i) = E_{v_{-i}, s_{-i}, \lambda_{-i}} \left[u_i(v, s, \lambda) \,|\, s_i, \lambda_i \right]. \tag{26}$$

Now observe that (3) can be rewritten as

$$U_i(v_i, s_i, \lambda_i) = E_{v_{-i}, s_{-i}, \lambda_{-i}} \left[\int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i, v_{-i}, s, \lambda) dt_i + u_i(\underline{v}_i(s_i), v_{-i}, s, \lambda) \,|\, s_i, \lambda_i \right]. \tag{27}$$

Combining (26) and (27) we obtain that

$$E_{v_{-i}, s_{-i}, \lambda_{-i}} \left[u_i(v, s, \lambda) - \int_{\underline{v}_i(s_i)}^{v_i} p_i(t_i, v_{-i}, s, \lambda) dt_i - u_i(\underline{v}_i(s_i), v_{-i}, s, \lambda) \,|\, s_i, \lambda_i \right] = 0. \tag{28}$$

With the help of (5), (28) can be rewritten as $E_{v_{-i}, s_{-i}, \lambda_{-i}} \left[t_i(v, s, \lambda) \,|\, s_i, \lambda_i \right] = 0.$ ■

Proof of Proposition 2

Recall from Lemma 2, that for a mechanism that satisfies IC_i it must hold that

$$E_{v_{-i}, s_{-i}, \lambda_{-i}} \left[t_i(v, s, \lambda) \,|\, s_i, \lambda_i \right] = 0, \tag{29}$$

which can be rewritten as

$$\Sigma_{\lambda_{-i} \in \Lambda_{-i}} \Sigma_{s_{-i} \in S_{-i}} \int_{V_{-i}(s_{-i})} t_i(v, s, \lambda) \frac{\pi_i(s_i)\pi_{-i}(s_{-i}) d(\lambda \,|\, s)}{\pi_i(s_i) \Sigma_{s_{-i}} \pi_{-i}(s_{-i}) \Sigma_{\lambda_{-i}} d(\lambda \,|\, s)} f_{-i}(v_{-i} \,|\, s_{-i}) dv_{-i} = 0. \tag{30}$$

Now (30) implies that

$$\Sigma_{\lambda_{-i}\in\Lambda_{-i}}\Sigma_{s_{-i}\in S_{-i}}\pi_i(s_i)\pi_{-i}(s_{-i})\int_{V_{-i}(s_{-i})} t_i(v,s,\lambda)d(\lambda\,|s)f_{-i}(v_{-i}\,|s_{-i})dv_{-i}=0$$

for all v_i, λ_i and s_i. Adding over all v_i, λ_i, s_i we get (10). ■

Proof of Proposition 3

The expression $\Sigma_{i\in I}p_i(v,s,\lambda)J_i(v_i,s_i)$ is maximized pointwise, that is for each vector (v,s,λ), by setting

$$\begin{aligned} p_i^*(v,s,\lambda) &= 1 \text{ if } J_i(v_i,s_i)\geq J_j(v_j,s_j),\ \text{all } j\neq i \text{ and } J_i(v_i,s_i)\geq 0 \\ p_i^*(v,s,\lambda) &= 0\text{, otherwise.} \end{aligned} \tag{31}$$

As usual, ties can be broken arbitrarily.

It is immediate to see that this allocation rule satisfies resource constraints. Because virtual valuations are strictly increasing in v_i, one can show following standard arguments that $p_i^*(v,s,\lambda)$ is increasing in v_i, which of course implies that its expectation, $P_i^*(v_i,s_i,\lambda_i)$, is increasing in v_i as well. Moreover, given the allocation rule in (31) we have that $\underline{U}_i(s,\lambda)=0$ for all s,λ and all $i\in I$. Therefore all constraints of Program S are satisfied.

By the definition of p^* and x^* it follows immediately that it coincides with a mechanism that the seller would choose when s is common knowledge, that is, when she has no private information. Hence (12) holds. ■

Proof of Proposition 4

Take a disclosure policy $d(\lambda\,|s)$ and let $p(v,\lambda,x)$ and $x(v,\lambda,x)$ denote an optimal mechanism given this disclosure policy. We'd like to show that if this mechanism is incentive compatible given $d(\lambda\,|s)$, it is also incentive compatible when the seller discloses no information at all.

If $p(v,\lambda,x)$ and $x(v,\lambda,x)$ is incentive compatible given $d(\lambda\,|s)$ then it must be the case that

$$P_i(v_i,\lambda_i,s_i)=E_{v_{-i},s_{-i},\lambda_{-i}}\left[\sum_{z\in Z}p^z(v,s,\lambda)\frac{\partial u_i^z(v_i,v_{-i})}{\partial v_i}\,|\lambda_i,s_i\right] \tag{32}$$

is increasing in v_i for *each* λ_i, s_i.

When the seller discloses no information, then

$$P_i(v_i,s_i)=E_{v_{-i},s_{-i}}\left[\sum_{z\in Z}p^z(v,s)\frac{\partial u_i^z(v_i,v_{-i})}{\partial v_i}\,|s_i\right].$$

We will now show that irrespective of the information disclosure policy, it holds that

$$P_i(v_i,s_i)=\Sigma_{\lambda_i}prob(\lambda_i\,|s_i)P(v_i,\lambda_i,s_i). \tag{33}$$

Observe that $P_i(v_i,s_i)$, and $P_i(v_i,\lambda_i,s_i)$ can be respectively written as

$$P_i(v_i,s_i)=\Sigma_{s_{-i}\in S_{-i}}\pi(s_{-i})\int_{V_{-i}(s_{-i})}\sum_{z\in Z}p^z(v,s)\frac{\partial u_i^z(v_i,v_{-i})}{\partial v_i}f_{-i}(v_{-i}\,|s_{-i})dv_{-i}$$

$$P_i(v_i, \lambda_i, s_i) = \Sigma_{\lambda_{-i}\in\Lambda_{-i}} \Sigma_{s_{-i}\in S_{-i}} \int_{V_{-i}(s_{-i})} \sum_{z\in Z} p^z(v, s, \lambda) \frac{\partial u_i^z(v)}{\partial v_i} \frac{f_{-i}(v_{-i}\,|s_{-i})\pi(s_i)\pi(s_{-i})d(\lambda\,|s)}{\pi(s_i)\Sigma_{s_{-i}}\pi_{-i}(s_{-i})\Sigma_{\lambda_{-i}}d(\lambda\,|s)} dv_{-i}.$$

Define $p^z(v, s) = \Sigma_{\lambda\in\Lambda} p^z(v, s, \lambda)d(\lambda\,|s)$. Then, (33) follows from the following considerations:

$$\begin{aligned}
& \Sigma_{\lambda_i} prob(\lambda_i\,|s_i)P(v_i, \lambda_i, s_i) \\
= \; & \Sigma_{\lambda_i}\Sigma_{s_{-i}}\pi_{-i}(s_{-i})\Sigma_{\lambda_{-i}}d_i(\lambda\,|s)\Sigma_{\lambda_{-i}}\Sigma_{s_{-i}} \int_{V_{-i}(s_{-i})} \sum_{z\in Z} p^z(v, s, \lambda) \frac{\partial u_i^z(v)}{\partial v_i} \frac{f_{-i}(v_{-i}\,|s_{-i})\pi_{-i}(s_{-i})d(\lambda\,|s)}{\Sigma_{s_{-i}}\pi_{-i}(s_{-i})\left[\Sigma_{\lambda_{-i}}d(\lambda\,|s)\right]} dv_{-i} \\
= \; & \Sigma_{\lambda_i}\Sigma_{\lambda_{-i}\in\Lambda_{-i}}\Sigma_{s_{-i}\in S_{-i}} \int_{V_{-i}(s_{-i})} \pi_{-i}(s_{-i})d(\lambda\,|s) \sum_{z\in Z} p^z(v, s, \lambda) \frac{\partial u_i^z(v)}{\partial v_i} f_{-i}(v_{-i}\,|s_{-i})dv_{-i} \\
= \; & \Sigma_{s_{-i}\in S_{-i}}\pi_{-i}(s_{-i}) \int_{V_{-i}(s_{-i})} \Sigma_{\lambda\in\Lambda} \sum_{z\in Z} p^z(v, s, \lambda)d(\lambda\,|s) \frac{\partial u_i^z(v)}{\partial v_i} f_{-i}(v_{-i}\,|s_{-i})dv_{-i} \\
= \; & \Sigma_{s_{-i}\in S_{-i}}\pi_{-i}(s_{-i}) \int_{V_{-i}(s_{-i})} \sum_{z\in Z} \Sigma_{\lambda\in\Lambda} p^z(v, s, \lambda)d(\lambda\,|s) \frac{\partial u_i^z(v)}{\partial v_i} f_{-i}(v_{-i}\,|s_{-i})dv_{-i} \\
= \; & \Sigma_{s_{-i}\in S_{-i}}\pi_{-i}(s_{-i}) \int_{V_{-i}(s_{-i})} \sum_{z\in Z} p^z(v, s) \frac{\partial u_i^z(v)}{\partial v_i} f_{-i}(v_{-i}\,|s_{-i})dv_{-i} \\
= \; & P_i(v_i, s_i).
\end{aligned}$$

Now, from (32) we know that $P(v_i, \lambda_i, s_i)$ is increasing in v_i for each λ_i,and s_i. Then from (33) we can immediately conclude that $P_i(v_i, s_i)$ is increasing in v_i for each s_i, which implies that the given mechanism is also incentive compatible for the no information disclosure policy. ■

7. Appendix B: Convexifying the Space of Valuations.

Our analysis can be easily modified to allow for the possibility that $f_i(v_i\,|s_i)$ is not strictly positive in all of $V_i(s_i)$.

We will now show that it is without loss of generality to include all $v_i's$ in the convex hull of $V_i(s_i)$, that is $\bar{V}_i(s_i)$. The reason is that the set of feasible mechanisms does not get smaller when we include $v_i's$ in the convex hull of $V_i(s_i)$ that are occurring with probability zero. Our extended type space, which we denote by $\bar{\Theta}_i$, consists of triplets v_i, s_i, λ_i, where $v_i \in \bar{V}_i(s_i)$, $\lambda_i \in \Lambda_i(s_i)$ for some $s_i \in S_i$. In the following Lemma we establish that any schedule feasible on Θ_i, $i \in I$, can be appropriately redefined on $\bar{\Theta}_i$ and remain feasible.

Lemma 4 *Take a mechanism that satisfies IC_i and PC_i on Θ_i. Then this mechanism can be extended on $\bar{\Theta}_i$, in way such that the resulting extended mechanism satisfies IC_i and PC_i on all of $\bar{\Theta}_i$.*

Proof. Take a mechanism p, x that satisfies IC_i and PC_i on Θ_i. In what follows we will take $V_i(s_i)$ to be a closed set.[34] Now, consider a $v_i \in \bar{V}_i(s_i) \backslash V_i(s_i)$ and define $v_i^L(v_i) = \max\{v_i' \in V_i(s_i) : v_i' \leq v_i\}$ and $v_i^H(v_i) = \min\{v_i' \in V_i(s_i) : v_i' \geq v_i\}$, (these maxima and minima exist because $V_i(s_i)$ is closed). Now let $v_i^{Ind}(v_i) \in [v_i^L(v_i), v_i^H(v_i)]$ denote the type for which the following is true:

$$P_i(v_i^H(v_i), s_i, \lambda_i)v_i^{Ind}(v_i) - X_i(v_i^H(v_i), s_i, \lambda_i) = P_i(v_i^L(v_i), s_i, \lambda_i)v_i^{Ind}(v_i) - X_i(v_i^L(v_i), s_i, \lambda_i). \tag{34}$$

By the incentive compatibility of p, x it holds that

$$\begin{aligned} P_i(v_i^H(v_i), s_i, \lambda_i)v_i^H(v_i) - X_i(v_i^H(v_i), s_i, \lambda_i) &\geq P_i(v_i^L(v_i), s_i, \lambda_i)v_i^H(v_i) - X_i(v_i^L(v_i), s_i, \lambda_i) \text{ and} \\ P_i(v_i^H(v_i), s_i, \lambda_i)v_i^L(v_i) - X_i(v_i^H(v_i), s_i, \lambda_i) &\leq P_i(v_i^L(v_i), s_i, \lambda_i)v_i^L(v_i) - X_i(v_i^L(v_i), s_i, \lambda_i), \end{aligned}$$

hence a type that satisfies (34) exists by continuity.

Now consider the following extension of p, x, call it p^E, x^E on $\bar{\Theta}$

$$\begin{aligned} p_i^E(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= p_i(\tilde{v}_i(v_i), s_i, \lambda_i; \tilde{v}_{-i}(v_{-i}), s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) \\ x_i^E(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= x_i(\tilde{v}_i(v_i), s_i, \lambda_i; \tilde{v}_{-i}(v_{-i}), s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}), \end{aligned}$$

where

$$\tilde{v}_i(v_i) = \begin{cases} v_i \text{ if } v_i \in V_i(s_i) \\ v_i^L(v_i) \text{ if } v_i \in \bar{V}_i(s_i) \backslash V_i(s_i) \text{ and } v_i \leq v_i^{Ind}(v_i) \\ v_i^H(v_i) \text{ if } v_i \in \bar{V}_i(s_i) \backslash V_i(s_i) \text{ and } v_i > v_i^{Ind}(v_i) \end{cases}$$

and

$$\tilde{v}_{-i}(v_{-i}) = (\tilde{v}_1(v_1), ..., \tilde{v}_{i-1}(v_{i-1}), \tilde{v}_{i+1}(v_{i+1}), ..., \tilde{v}_i(v_i)). \tag{35}$$

Note that for $v_i \in V_i(s_i)$ and $v_{-i} \in V_{-i}(s_{-i})$ we have that

$$\begin{aligned} p_i^E(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= p_i(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) \\ x_i^E(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= x_i(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}). \end{aligned}$$

Fix a $v_{-i} \in V_{-i}(s_{-i})$, (so that this vector of valuations that arises with strictly positive probability). It is easy to see that the "real options" that buyer i can choose from are the same in both mechanisms, because for fixed v_{-i}, s_i, s_{-i} and λ_i, λ_{-i}, the menus

[34] If $V_i(s_i)$ is not closed, it is very easy to show that p, x can be extended on vectors of types that include $v_i's$ on the closures of $V_i(s_i)$, but not on $V_i(s_i)$ itself. For such vectors of types set

$$\begin{aligned} p_i^c(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= p_i(v_i^c(v_i), s_i, \lambda_i; v_{-i}^c(v_{-i}), s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) \\ x_i^c(v_i, s_i, \lambda_i; v_{-i}, s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) &= x_i(v_i^c(v_i), s_i, \lambda_i; v_{-i}^c(v_{-i}), s_{-i}, \lambda_{-i}; s_i, s_{-i}, \lambda_i, \lambda_{-i}) \end{aligned}$$

where $v_i^c(v_i)$ is closest type type to v_i, (in the usual sense), on the closure of $V_i(s_i)$, and $v_{-i}^c(v_{-i}) = (v_1^c(v_1), ..., v_{i-1}^c(v_{i-1}), v_{i+1}^c(v_{i+1}), ..., v_i^c(v_i))$. It is straightforward to establish that the mechanism p^c, x^c satisfies IC_i and PC_i for all i, given that p, x does.

$\{p_i(v_i,...), x_i(v_i,..)\}_{v_i \in V_i(s_i)}$ and $\{p_i^E(v_i,...), x_i^E(v_i,...)\}_{v_i \in \bar{V}_i(s_i)}$ coincide. This follows by the definition of p_i^E, x_i^E. For a $v_{-i} \in \bar{V}_{-i}(s_{-i}) \backslash V_{-i}(s_{-i})$, the menu $\{p_i^E(v_i,...), x_i^E(v_i,...)\}_{v_i \in \bar{V}_i(s_i)}$ is actually equal to a menu for a $v_{-i} \in V_{-i}(s_{-i})$, namely $\tilde{v}_{-i}(v_{-i})$ defined in (35), so in this case too $\{p_i^E(v_i,...), x_i^E(v_i,...\}_{v_i \in \bar{V}_i(s_i)}$ is a menu that is identical to a menu $\{p_i(v_i,...), x_i(v_i,...)\}_{v_i \in V_i(s_i)}$. Hence in extending p, x on $\bar{\Theta}$ no new "real" options have been added for buyer i. Given the fact that there are no new options, the feasibility of p^E, x^E on Θ follows immediately from the feasibility of p, x on Θ, since they coincide on those types. The feasibility of p^E, x^E on $\bar{\Theta}$ can be easily verified by its definition with the help of (34). ■

8. Appendix C : An Example where the Value of Program S_G depends on the Disclosure Policy

Suppose that the seller faces two buyers 1 and 2 and that there are two possible allocations z_A and z_B. Both $v_i's$, $i = 1, 2$ are uniformly distributed on $[0, 1]$. The payoffs that accrue to each buyer when the realized v is (v_1, v_2) are given by

$$u_1^{z_A}(v_1, v_2) = \begin{cases} v_1 + 2 \text{ when } v_2 > 0.5 \\ e^{v_1} \text{ when } v_2 \leq 0.5 \end{cases}, \text{ and } u_2^{z_A}(v_1, v_2) = 0 \text{ for all } v_1, v_2$$

$$u_1^{z_B}(v_1, v_2) = \begin{cases} e^{v_1} \text{ when } v_2 \leq 0.5 \\ v_1 + 2 \text{ when } v_2 > 0.5 \end{cases}, \text{ and } u_2^{z_B}(v_1, v_2) = 0 \text{ for all } v_1, v_2.$$

Note that these payoff functions satisfy all the conditions described in Section 3. The virtual valuations of each of these two allocations are given by

$$J^{z_B}(v_1, v_2) = \begin{cases} 2v_1 + 1 \text{ when } v_2 > 0.5 \\ 2e^{v_1} - v_1 e^{v_1} \text{ when } v_2 \leq 0.5 \end{cases}$$

and

$$J^{z_B}(v_1, v_2) = \begin{cases} 2e^{v_1} - v_1 e^{v_1} \text{ when } v_2 > 0.5 \\ 2v_1 + 1 \text{ when } v_2 \leq 0.5 \end{cases}.$$

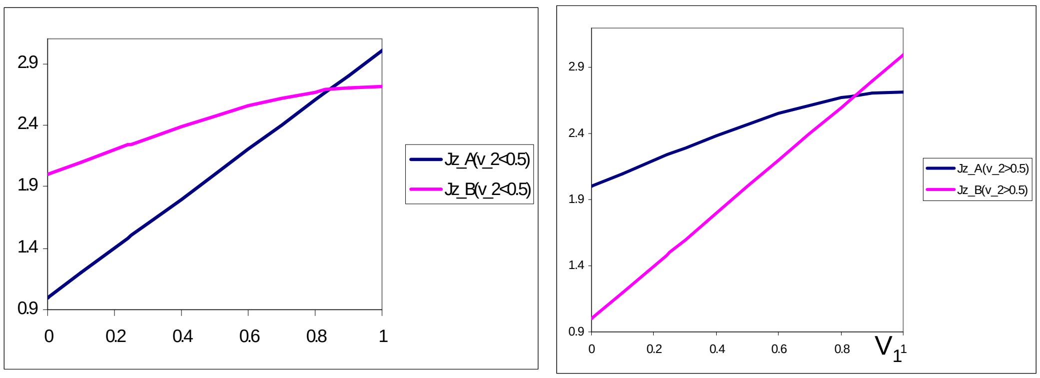

Figure 1: When v_2<0.5 Pointwise optimal assignment dictates to assign probability one to allocation z_A for v_1<0.83, and probability one to allocation z_B when v_1>0.83. The reverse is true when v_2=0.5.

As can be easily seen from Figure 1, when $v_2 < 0.5$, pointwise optimization dictates to assign probability one to allocation z_A when $v_1 < 0.83$, and probability one to allocation z_B when $v_1 > 0.83$. The reverse is true when $v_2 \geq 0.5$. Now we will argue that whether this allocation rule is feasible or not, depends on the disclosure policy that the seller uses.

Suppose that the seller observes no signal for buyer 1, whereas she observes the valuation of buyer 2.

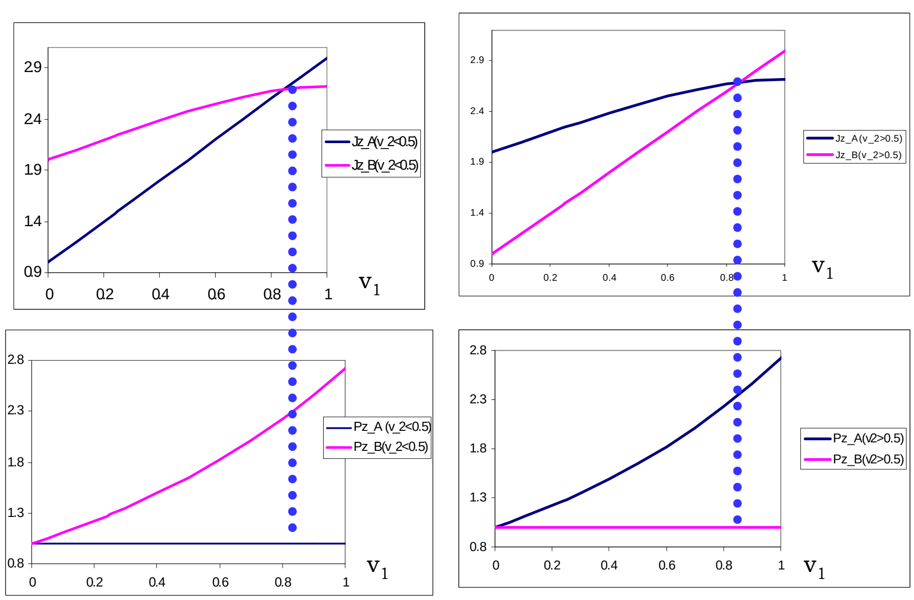

Figure 2: When the seller discloses to buyer 1 whether v_2 is above or below 0.5, then the Pointwise optimal assignment violates incentive compatibility at (about) v_1 =0.83 in both ranges of values of v_2.

From Figure 2 one can see that when the seller discloses to buyer 1 whether v_2 is above or below 0.5, (or for that matter any information that allows buyer 1 to conclude whether v_2 is above or below 0.5), then the pointwise-optimal assignment violates incentive compatibility at $v_1 = 0.83$ in both ranges of values of v_2, because P_1 fails to be increasing. For instance, when λ_1 contains the information that $v_2 < 0.5$, then we have that

$$P_1(v_1, s_1, \lambda_1) = \begin{cases} P^{z_B}(v_1) \text{ for } v_1 < 0.83 \\ P^{z_A}(v_1) \text{ for } v_1 \geq 0.83 \end{cases},$$

and $P^{z_B}(v_1) > P^{z_A}(v_1)$ for all $v_1 \in (0, 1]$, violating the requirement that P_1 is increasing in v_1. An analogous situation occurs when λ_1 contains the information that $v_2 \geq 0.5$.

However, when the seller discloses no information to buyer 1, then the pointwise optimal assignment is feasible, because then $P_1(v_1, s_1, \lambda_1)$ is the average over the case of $v_2 < 0.5$ and the case of $v_2 > 0.5$, and it is increasing in v_1. This can be seen in Figure 3.

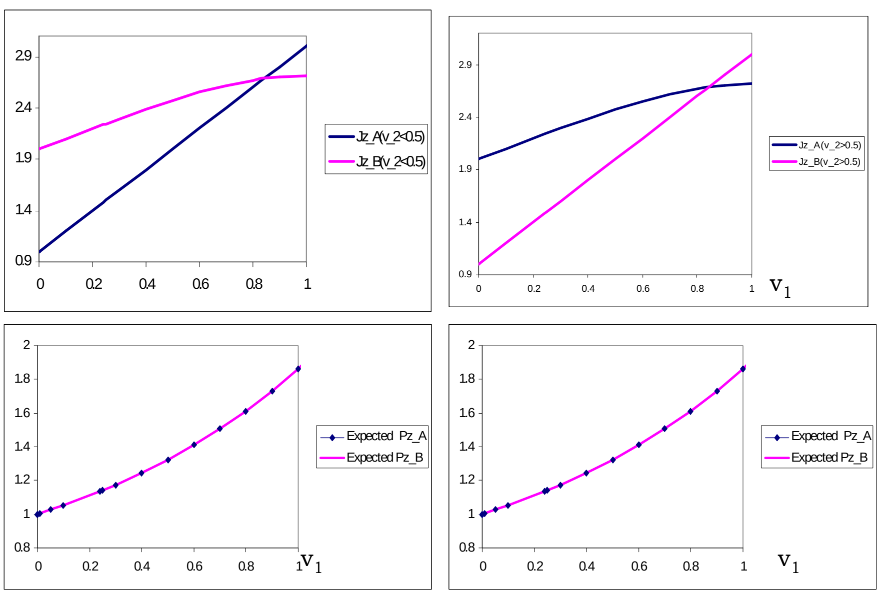

Figure 3: When the seller discloses NO INFORMATION to buyer 1, then the Pointwise optimal assignment is incentive compatible!

References

[1] Akerlof, G. (1970): "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics* 84(3):488–500.

[2] Bergemann, D., and M. Pesendorfer (2007): "Information Structures in Optimal Auctions," *Journal of Economic Theory*, forthcoming.

[3] Bergemann, D. and J. Valimaki (2006): "Information in Mechanism Design." In Richard Blundell, Whitney Newey and Torsten Persson, eds., Proceedings of the 9th World Congress of the Econometric Society. Cambridge: Cambridge University Press, forthcoming.

[4] Board, S. (2006): "Revealing Information in Auctions: The Allocation Effect," *Economic Theory*, forthcoming.

[5] Bulow, J. and P. Klemperer (1996): "Auctions vs. Negotiations," *American Economic Review*, 86, No. 1 (March 1996):180-194.

[6] Cai H., J. Riley, and Yi L. (2006): "Reserve Price Signaling," *Journal of Economic Theory*, forthcoming.

[7] Cella, M. (2007):"Informed Principal with Correlation," mimeo University of Milan.

[8] Cremer, J., and R. McLean (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56, 1247-1258.

[9] Eso, P., and B. Szentes (2006): "Optimal Information Disclosure in Auctions and the Handicap Auction," *Review of Economic Studies*, forthcoming.

[10] Figueroa, N. and V. Skreta (2007): "The Role of Outside Options in Auction Design," mimeo.

[11] Figueroa, N. and V. Skreta (2007b): "A Note on Optimal Auctions," mimeo.

[12] Ganuza, J-J. (2004): "Ignorance Promotes Competition: An Auction Model with Endogenous Private Valuations," *Rand Journal of Economics*, 35, 583-598.

[13] Ganuza, J-J. and J.S. Penalva (2006): "On Information and Competition in Private Value Auctions," mimeo.

[14] Hansen, R. G. (2001): "Auctions of Companies," *Economic Inquiry,* Vol 39, 1, 30-43.

[15] Harstad, R. and V. Mares (2003): "Private Information Revelation in Common-Value Auctions," *Journal of Economic Theory*, 109, 264-282

[16] Heifetz, A. and Z. Neeman, (2006): "On the Generic (Im)Possibility of Full Surplus Extraction in Mechanism Design," *Econometrica*, 2006, 74, 213-233.

[17] Jullien, B. and T. Mariotti, (2006): "Auction and the Informed Seller Problem," *Games and Economic Behavior,* 56, 2, 225-258.

[18] Kaplan, T.R. and S. Zamir (2002): "The Strategic Use of Seller Information in Private-Value Auctions," mimeo.

[19] Kremer, I. and Skrzypacz, A. (2004): "Auction Selection by an Informed Seller," mimeo Stanford University.

[20] Landsberger, M., Rubinstein, J., Zamir S. and E. Wolfstetter (2001): "First Price Auctions When the Ranking of Valuations is Common Knowledge," *Review of Economic Design* 6, 461-480.

[21] Maskin, E. and J. Tirole, (1990): "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values," *Econometrica,* 58, 379-409.

[22] Maskin, E. and J. Tirole (1992): "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica*, 60: 1-42.

[23] McAfee P. and P. Reny, (1992): "Correlated Information and Mechanism Design," *Econometrica*, 60, 395-421.

[24] Milgrom, P. (2004): "Putting Auction Theory to Work," Cambridge University Press.

[25] Milgrom, P. and I. Segal, (2002): "Envelope Theorems for Arbitraty Choice Sets," *Econometrica*, 70, 583-601.

[26] Milgrom, P. and R.Weber (1982): "A Theory of Auctions and Competitive Bidding", *Econometrica*, 50, 1089-1122.

[27] Mookherjee, D. and S. Reichelstein, (1992): "Dominant strategy implementation of Bayesian incentive compatible allocation rules," *Journal of Economic Theory*, 56(2), 378-399.

[28] Myerson, R. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica* 47, 61-73.

[29] Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research* 6, 58-73.

[30] Myerson, R. (1983): "Mechanism Design by an Informed Principal," *Econometrica* 51, 1767-1798.

[31] Myerson, R. (1986): "Multistage Games with Communication." *Econometrica* 54, 323– 358.

[32] Mylovanov, T. (2005): "The Informed Principal Problem with Independent Private Values and Quasilinear Preferences," mimeo University of Bonn.

[33] Neeman, Z. (2004): "The Relevance of Private Information in Mechanism Design," *Journal of Economic Theory*, 117, 55-77.

[34] Obara, I. (2006): "The Full Surplus Extraction Theorem with Hidden Actions," mimeo UCLA.

[35] Ottaviani, M. and A. Prat,(2001): "The Value of Public Information in Monopoly," *Econometrica*, 69(6), 1673-1683.

[36] Perry, M., and P. Reny (1999): "On the Failure of the Linkage Principle in Multi-Unit Auctions," *Econometrica*, 67(4), 895-900.

[37] Quesada, L. (2005): "Collusion as an Informed Principal Problem," mimeo University of Wisconsin- Madison.

[38] Riley, J. G. and W. F. Samuelson (1981): "Optimal Auctions," *American Economic Review, 71, 381-392.*

[39] Skreta, V. (2006): "Mechanism design for arbitrary type spaces," *Economics Letters*, 91(2), 293-299.

[40] Skreta, V. (2007): "Optimal Auctions with General Distributions: A Technical Note," mimeo.

[41] Skreta, V. (2007b): "Auction Design by an Informed Seller: Interdependent Values," in progress.

[42] Tisljar, R. (2002): "Mechanism Design by an Informed Principal - Pure-Strategy Equilibria for a Common Value Model," mimeo University of Bonn.

[43] Tisljar, R. (2003): "Optimal Trading Mechanisms for an Informed Seller," *Economics Letters,* 81, 1–8.

[44] Yilankaya, O. (1999): "A Note on the Seller's Optimal Mechanism in Bilateral Trade with two-sided Incomplete Information," *Journal of Economic Theory* 87, 125-143.