Cheap Talk in the Decentralized Market for Lemons*

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October, 2008
(Job Market Paper)

Abstract

This paper develops a decentralized market structure where sellers have private information about the quality of goods (adverse selection) and strategically transmit information to buyers through non-binding and costless advertisement (cheap talk). I demonstrate that cheap talk can be informative and thus mitigate information friction in the market. The key insight is that cheap-talk messages can serve as an instrument that creates endogenous market segmentation, and the incentives of agents can be aligned in a way that sellers partially reveal their qualities and buyers compensate for such behaviors.

JEL Classification Numbers: C72, D82, D83.

Keywords : Adverse Selection, Decentralized Market, Cheap Talk.

1 Introduction

Since Akerlof (1970), many market and institutional features have been identified as sources to mitigate or resolve the inefficiency due to adverse selection. There are two primary approaches: signaling from informed agents (for example, warranties for durable goods, licensing for credence goods, and schooling in the labor market), and screening from uninformed agents (for example, discounting for early purchase in the airline industry and deductibles in the insurance industry). One common feature of the two approaches is that they rely on *credible* and/or *payoff-relevant* devices. For example, warranties and discounting for early purchase are effective only when firms commit to those policies. Also, schooling can signal the abilities of workers because it is more costly for less able workers. Those devices are used to sort different types by directly affecting the incentives of informed agents.

This paper examines whether a *non-binding* and *costless* device can alleviate the "lemons" problem, friction caused by private information about the quality of goods. The incentives of

*I am deeply indebted to my advisors, George Mailath and Andrew Postlewaite, and Philipp Kircher for many insightful discussions. I am also grateful to Jan Eeckhout, Steven Matthews, and seminar participants in Penn Economic Theory club.

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agents to overcome the inefficiency due to information asymmetry are independent of the nature of such devices. The difficulty with a non-binding and costless device is that it is hard to provide an incentive for informed agents to reveal their private information. By definition, deviation is costless and there is no explicit way to punish deviators.

The existing literature has been negative on the possibility to overcome the lemons problem through a non-binding and costless device. In his classic papers (1970, 1974), Nelson argued that qualities of experience goods cannot be revealed through costless advertisements. He reasoned that sellers' advertisements are unverifiable and thus cannot be punished, and then all sellers will claim to have high-quality goods. His reasoning has motivated many researchers (including him) to find several payoff-relevant mechanisms.[1]

I develop a decentralized market structure that incorporates adverse selection, cheap talk, and search friction. In the model, sellers own a unit of an indivisible good whose quality is not observable to buyers (adverse selection). Sellers announce messages, but those messages have no intrinsic meaning (cheap talk). Each buyer selects a seller based on announced messages (search friction) and makes a take-it-or-leave-it offer to the seller. Due to the nature of cheap-talk messages, there always exists an equilibrium in which they play no role. I show that there exists another equilibrium in which cheap-talk messages are informative about the quality of goods.

The key component in the model generating a non-trivial role of cheap-talk communication is that buyers condition on cheap-talk messages for their searches. This creates an incentive for sellers to affect search intensities of buyers by strategically transmitting their private information. An informative equilibrium exists when the incentives of both sides are well-aligned. The search behavior of buyers must provide an incentive for sellers to reveal their private information. Conversely, the information transmission behavior of sellers must provide an incentive for buyers to follow their equilibrium search behavior.

To see how the incentives of agents can be aligned in my model, consider the two-quality case (low-quality (lemon) and high-quality), whose equilibrium structure is depicted in Figure 1. In equilibrium, all high-quality sellers announce H. Among low-quality (lemon) sellers, some announce H, while the others announce L. On the long side of the market, relatively more buyers submit bids to sellers with message L. The remaining buyers go to sellers with message H. This is an equilibrium structure because market agents face the following trade-offs. In equilibrium, the trade-offs of buyers and lemon sellers are exactly matched, i.e., they are indifferent between H and L.

1. Lemon sellers: probability of trading vs. sale price

   Message L attracts more buyers, but the sale price tends to be low because the quality is revealed to be low. To the contrary, message H induces buyers to bid high (because the average quality is high), but there is a high probability that no buyer shows up.

[1] His own suggestions were repeated purchases (1970), and costly advertisements (1974). Kihlstrom and Riordan (1984) refined Nelson's idea on costly advertisement. Another prominent device employed in the literature is price (or pricing schedule). Wolinsky (1983), Bagwell and Riordan (1991), and Taylor (1999) showed that prices can act as signals of quality in various contexts. Milgrom and Roberts (1986) considered both price and costly advertisement.

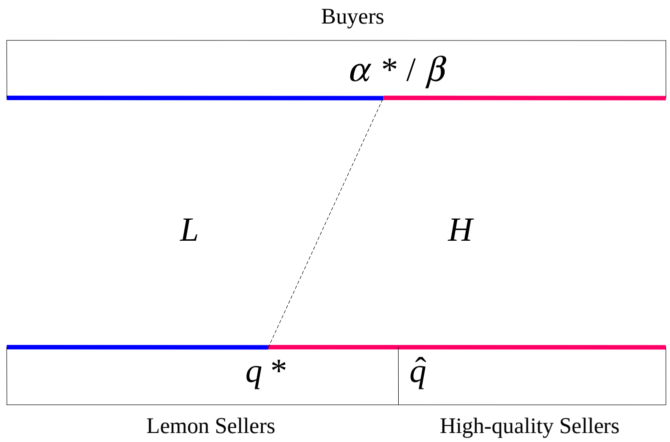

Figure 1: Measure q^* of lemon sellers announce L, and all other sellers announce H. Measure α^* of buyers select sellers with message L, while the remaining buyers trade with sellers with message H.

2. Buyers: competition among buyers vs. quality uncertainty

   In terms of quality uncertainty, it is safe to bid to sellers with message L. But there is more competition among buyers, which lowers the probability of winning and drives up the winning bid. To the contrary, there are fewer buyers interested in goods with H, but there is a positive probability that the goods have a low quality.

In this equilibrium, cheap-talk messages convey information about the quality of goods by serving as an instrument that creates endogenous market segmentation. The reduction of information asymmetry in the market is manifest in the fact that high-quality goods (that do not trade without cheap-talk communication) trade as well.

Typically, qualities cannot be fully revealed. To see this, suppose a fully revealing equilibrium exists. Now there is no quality uncertainty with both messages, and so buyers are not more willing to trade with low message sellers. This eliminates the incentives of lemon sellers to reveal their quality, which unravels a fully revealing equilibrium. This is a stark contrast to the signaling models. Partial separation between different types is one of many possibilities in many signaling models, while it is necessary in my cheap-talk model.

Whether cheap talk can be informative or not relies on the market environment. There are two primary requirements. First, the market should not be too thick, that is, the ratio of buyers to sellers should not be too large. If it is, the high message attracts many buyers, and so low-quality sellers have too small an incentive to reveal their quality. Second, the social surplus in trading high-quality goods should not be sufficiently greater than that of low-quality goods. If it is, buyers are sufficiently more willing to trade with high-quality sellers. Then low-quality sellers have great incentives to pretend that they have high-quality goods.

How much information can be transmitted depends on the market thickness. As the market becomes thin (as the ratio of buyers to sellers in the market gets smaller), the probability of trading becomes more valuable than sale prices. Sellers have a greater incentive to attract more buyers by revealing their quality. This allows cheap-talk messages to convey more information. In the limit as the ratio of buyers to sellers tends to zero, quality uncertainty can be fully resolved.

The remainder of the paper is organized as follows. The next section reviews related literature. Section 3 studies the two-quality case in detail. Section 4 and 5 consider the case where there is a continuum of qualities. Section 4 focuses on the environment where quality uncertainty plays a crucial role in the market and so the role of cheap-talk communication is highlighted. Section 5 supplements Section 4 by providing some results on the general continuum quality case, and by studying another extreme case where buyers' values are independent of sellers' costs (the constant value case). I conclude in Section 6.

2 Related Literature

2.1 Cheap Talk in Bargaining

Farrel and Gibbons (1989) and Matthews and Postlewaite (1989) studied whether cheap talk can be informative in a bilateral bargaining situation where each party has private information about their values. They showed that allowing for cheap-talk communication enlarges the set of equilibria in a double auction.

Different from their results, if the seller possesses private information about the quality of the good, cheap talk cannot be informative in a bilateral setting. In the standard problem, there is no instrument that can provide an incentive for the lemon seller to reveal her quality.

2.2 Directed Search Literature

Methodologically, this paper belongs to the growing directed search literature. In short, my model is a directed search model with non-binding communication and interdependent values. The combination of the two features generates a unique insight that is absent in the previous models. With price commitment or constant values,[2] the search behaviors of uninformed agents are determined solely by the trade-off between competition among uninformed agents and the deterministic gains in matches.[3] My model introduces risk to the considerations of uninformed agents. This unique feature is highlighted by comparing the two extreme cases in my model, the constant surplus case (Section 4) and the constant value case (Section 5.5).

[2] The classic directed search models (Peters (1991), Moen (1997), and Burdett, Shi, and Wright (2001)) considered the model with price posting and constant value. Riordan (1985) and Inderst and Müller (2002) studied the case of price posting and interdependent values, while Menzio (2007) studied the case of communication and constant value.

[3] With interdependent values, if sellers commit to prices, qualities are fully revealed through prices, and so the gains of uninformed agents are again deterministic (See Inderst and Müller (2002)).

2.3 Endogenous Market Segmentation

The main idea of this paper is that a payoff-irrelevant device (cheap talk) can be used to create endogenous market segmentation. A similar idea has been employed in other contexts. Mailath, Samuelson, and Shaked (2000) considered the labor market where both workers and firms search for each other. They showed that "color" can create inequality endogenously. Firms search "green" workers because they are more likely to acquire skills than "red" workers. On the other hand, "green" workers are more willing to acquire skills than "red" workers because it takes less time for them to be matched with firms and so their return on skill investment is higher. Fang (2001) considered an economy where the informational free-riding problem is so severe that a socially efficient technology cannot be adopted. He showed that in such a situation "social activity" can emerge as an endogenous signaling instrument. Firms pay more to workers who perform a seemingly irrelevant "social activity" because those workers are more likely to acquire new skills. On the other hand, skilled workers are more willing to do such "social activity" because they expect higher wages from firms.

2.4 Bargaining with Interdependent Values

This paper is also related to the bargaining literature with interdependent values. Evans (1989) and Vincent (1989) are early references. Deneckere and Liang (2006) provided a general characterization on the problem. The main insight of these papers is that it depends on the amount of uncertainty whether the interdependency of valuations with costs yields a fundamentally different outcome than that of the constant value case. More precisely, they show that as the bargaining friction becomes negligible, delay disappears, as in the Coase conjecture, if quality uncertainty is small (in the sense that the average valuation of the buyer is greater than the highest cost of the seller). If quality uncertainty is large, then there is a real-time delay, and partial separation between different types occurs over time. The equilibrium behavior of my paper in the two type case is similar to that behavior.

Recently, Hörner and Vielle (2006) studied the setting in which a seller with a unit of an indivisible good faces a sequence of buyers. They demonstrated how detrimental to allocations it could be that the seller has a strong signaling device (public offers). One of my results - that cheap talk cannot play any role if trading surplus of high-quality goods is sufficiently larger than that of low-quality goods - is related to their point. The key in my result is that low-quality sellers may have too great an incentive to mimic high-quality sellers.

2.5 The Role of Non-binding List prices

In some markets (for example, housing, used cars, and online posting sites), sellers post prices (list prices) but those prices are often non-binding. Nevertheless, correlations between list prices and allocations have been observed: sale prices are typically lower than list prices, and the higher the list price is, the sale price is higher, the number of interested buyers is smaller, and the duration on the market is longer.[4]

Despite the fact that many empirical efforts have been made to identify the determinants of list prices, there has been only one theoretical explanation for those facts, which was provided by Arnold (1999), Chen and Rosenthal (1996), and Yavas, A., and S. Yang (1995). They focus on the fact that sale prices are typically lower than list prices and postulate that list prices are ceiling prices that sellers commit to accept. They show that the relationship between list prices, sale prices, and durations on the markets can be generated in their models. The crucial idea is that if a seller commits to a low list price, buyers expect a greater gain in case their valuations turn out to be high, and so they are more interested in the property.

My model provides an alternative theory for such markets (once cheap-talk messages are replaced with non-binding list prices). Relative to the previous works, my model emphasizes the information transmission role of list prices. Interestingly, my model predicts the stylized facts listed above, and provides an intuitive reason for them. The higher the list price is, the more uncertain the quality of the good is, and so there are less interested buyers. This prolongs the duration of the good on the market, but once there is an interested buyer, he bids high and so the sale price is higher.

3 The Two-Quality Case

3.1 Environment

In a market for an indivisible good, there is a continuum of sellers with measure 1 and a continuum of buyers with measure $\beta > 0$. All buyers are homogenous, while there are two types of sellers. Measure $\widehat{q} \in (0,1)$ of sellers possess a unit of low-quality good (lemon), while measure $1-\widehat{q}$ of sellers own a unit of high-quality good. A unit of low-quality good costs c_L (or reservation utility) to a seller, and yields utility v_L to a buyer. The corresponding values for a unit of high-quality good are c_H ($> c_L$) and v_H ($\geq v_L$). Quality of a good is private information to each seller. Trading is always socially desirable ($v_H > c_H$ and $v_L > c_L$). I assume that the social surplus in trading is independent of quality, that is, $v_H - c_H = v_L - c_L$. This assumption enables me to focus on quality uncertainty, which is the central issue of this paper. I later explain how the equilibrium outcomes change when I relax this assumption.

The market proceeds as follows.

1. Each seller announces either H or L.

   - In equilibrium, there are at most two distinct submarkets induced by cheap-talk messages. Therefore, the two-message restriction has no loss of generality.

[4] See, for example, Horowitz (1992) and Merlo and Ortalo-Magné (2004) for the housing market, and Farmer and Stango (2004) for the online used computer market.

2. Each buyer chooses between H and L.

   - The equilibrium outcomes do not change whether buyers observe measures of sellers announcing each message or not. But if buyers observe them, the equilibrium behavior following a deviation of a positive measure of sellers should be specified, which is rather cumbersome. In the following, for simplicity, I assume that buyers choose a message without observing measures of each group.

3. Each buyer randomly select one seller with the chosen message.

   - I use the "urn-ball" matching technology. The probability π_k that a seller is matched with k buyers follows a Poisson distribution. Formally,

   $$\pi_k(\lambda) = \frac{\lambda^k}{k!e^{\lambda}}, k = 0, 1, ...,$$

   where λ is the ratio of buyers to sellers. To see how this is derived, suppose there are λN buyers and N sellers, and each buyer selects a seller with equal probability. As N tends to infinity, by the Poisson convergence theorem (see, for example, Billingsley (1995), Theorem 23.2.), the probability that a seller is matched with k buyers converges to $\pi_k(\lambda)$.

4. Each buyer makes a take-it-or-leave-it offer to the matched seller, without observing how many competitors he is facing.

   - Buyers have beliefs over the number of competitors. The urn-ball matching technology is particularly tractable in my setup, because the probability that a buyer is competing with k other buyers is also equal to $\pi_k(\lambda)$.[5]
   - No qualitative result, but some quantitative results, in this paper depend on the modeling choice that buyers do not observe the number of competitors.
   - I assume that buyers use the same bidding strategy. This is natural because buyers are anonymous as well as homogeneous in my setup.

5. Sellers who are matched with at least one buyer decide whether to accept the highest offer or not. If a seller accepts an offer b, then her utility is $b - c$ where c is her cost. If a seller was not matched with any buyer or the highest bid is lower than her cost, her utility is 0. If a buyer wins the auction with bid b, his utility is $v - b$ where v is the buyer valuation of the good the matched seller possesses. All agents maximize their expected utility.

   - In other words, each seller runs a first-price auction with unknown reservation price and unknown number of bidders.

[5] This directly comes from the conditional independence property of Poisson distribution. For an elementary exposition, see Satterthwaite and Shneyerov (2007), p. 173.

3.2 Submarket Analysis

Each message induces a submarket that consists of sellers who announce the message, and buyers who submit bids to them. I first solve for the equilibrium outcomes in a submarket. Each submarket is characterized by (q, λ) where q is the proportion of lemon sellers and λ is the ratio of buyers to sellers in the submarket. The absolute measures of buyers and sellers do not affect the equilibrium outcome, because my matching technology exhibits constant return to scale.

3.2.1 Buyers' Expected Payoff

I make three observations to facilitate the analysis. First, buyers' bids are not deterministic. If so, a buyer is strictly better off by bidding slightly higher than the equilibrium bid. His expected payment increases slightly, but he always wins the auction. From now on, I represent buyers' symmetric bidding strategies by a probability distribution function F over R_+. By the same argument as before, F has no atom. Second, letting $\underline{b}$ be the minimum of the support of F, $\underline{b}$ is equal to the offer of the monopsonist who is facing a seller who has a low-quality good with probability q. A buyer who bids $\underline{b}$ wins the auction only when he is the only bidder, and so behaves like a monopsonist. Third, $\underline{b}$ is either c_L or c_H, because the monopsonist has no reason to offer more.

Let $M(q)$ be the expected payoff of the monopsonist. Then

$$M(q) = \max\{q(v_L - c_L), E_q[v] - c_H\},$$

where $E_q[v] = qv_L + (1-q)v_H$. M is decreasing first and increasing later in q. For q small, the monopsonist endures the risk of paying a high price for the low-quality good, and so offer c_H. Since higher q means a higher probability of trading a low-quality good whose valuation is lower than c_H, M is decreasing in q. For q large, the monopsonist makes a safer offer, c_L. In this case, M is increasing in q, because higher q means a higher probability of trading.

In equilibrium, buyers are indifferent over bids in the support of F. Therefore I can immediately calculate buyers' expected payoff. Let $U(q, \lambda)$ be the expected payoff of buyers in a submarket with $U(q, \lambda)$. Then

$$U(q, \lambda) = \pi_0 M(q) = \pi_0 \max\{q(v_L - c_L), E_q[v] - c_H\}.$$

That is, buyers' expected payoff is equal to the probability that they are the only bidder times the expected payoff of the monopsonist. The effects of λ and q are separated. Therefore U inherits all the properties of M. U is strictly decreasing in λ. This is natural because λ measures the level of competition among buyers in a submarket.

3.2.2 Sellers' Expected Payoffs

To calculate sellers' expected payoffs, F should be derived. I find F from buyers' indifference over bids in the support of F. The expected payoff of a buyer bidding b is

$$q\sum_{k=0}^{\infty}\pi_k F(b)^k (v_L - b) = q\pi_0 e^{\lambda F(b)} (v_L - b) \text{ if } b \in [c_L, c_H),$$

and

$$\sum_{k=0}^{\infty}\pi_k F(b)^k (E_q[v] - b) = \pi_0 e^{\lambda F(b)} (E_q[v] - b) \text{ if } b \geq c_H.$$

If $\underline{b} = c_L$ and $b \in [c_L, c_H)$ are in the support of F, then

$$F(b) = F(c_L) + \frac{1}{\lambda}\ln\left(\frac{v_L - c_L}{v_L - b}\right).$$

Similarly, if c_H and $b \geq c_H$ are in the support of F, then

$$F(b) = F(c_H) + \frac{1}{\lambda}\ln\left(\frac{E_q[v] - c_H}{E_q[v] - b}\right).$$

Let $\overline{b}$ be the maximum of the support of F. There are three cases.

(1) $\underline{b} = c_L$ and $\overline{b} < c_H$.

This is the case in which only lemons trade. From the previous result,

$$F(b) = \frac{1}{\lambda}\ln\left(\frac{v_L - c_L}{v_L - b}\right), \text{ for } b \in \left[\underline{b}, \overline{b}\right].$$

For $\overline{b} < c_H$,

$$E_q[v] - c_H \leq U(q, \lambda) = \pi_0 q (v_L - c_L).$$

This inequality can be interpreted as the incentive compatibility condition of buyers, because the left-hand side is the maximum deviation payoff of buyers (by deviating to c_H).

(2) $\underline{b} = c_L$ and $\overline{b} > c_H$.

In this case, lemons fully trade, while high-quality goods partially trade (there may not be trade even if a seller is matched with buyers). Let $\left[\underline{b}, \overline{b}_L\right] \cup \left[\underline{b}_H, \overline{b}\right]$ be the support of F where $\overline{b}_L < c_H$. Then $\underline{b}_H = c_H$, and

$$F(b) = \begin{cases} \frac{1}{\lambda}\ln\left(\frac{v_L - c_L}{v_L - b}\right), & \text{if } b \in \left[c_L, \overline{b}_L\right], \\ \frac{1}{\lambda}\ln\left(\frac{\widehat{q}(v_L - c_L)}{E[v] - b}\right), & \text{if } b \in \left[c_H, \overline{b}\right]. \end{cases}$$

For $\underline{b} = c_L$ and $\overline{b}_L < c_H$,

$$\pi_0 q (v_L - c_L) < E_q[v] - c_H < q (v_L - c_L).$$

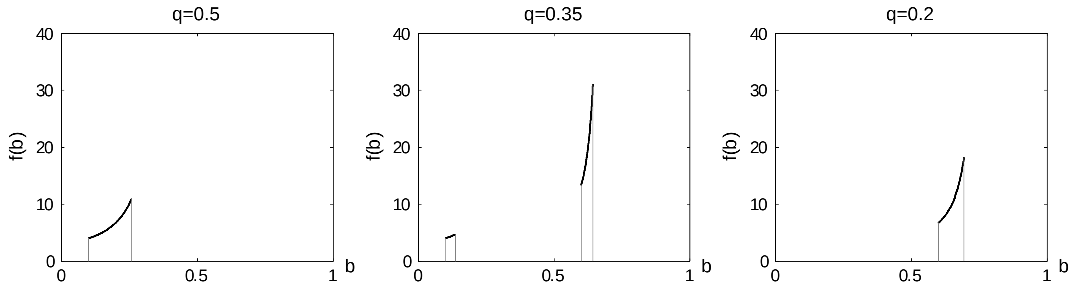

Figure 2: Buyers' bidding strategies represented by density functions over a real line for $c_L = 0.1, v_L = 0.35, c_H = 0.6, v_H = 0.85$ and $\beta = 1$.

(3) $\underline{b} = c_H$.

This is the case in which trade occurs whenever a seller is matched with at least one buyer. From the preliminary result,

$$F(b) = \frac{1}{\lambda} \ln \left(\frac{E_q[v] - c_H}{E_q[v] - b} \right), b \in \left[\underline{b}, \overline{b}\right].$$

The condition for this case is that the optimal offer of the monopsonist is equal to c_H. Therefore,

$$q(v_L - c_L) \leq U(q, \lambda) = E_q[v] - c_H.$$

Let $V_L(q, \lambda)$ and $V_H(q, \lambda)$ be the expected payoffs of lemon sellers and high-quality sellers, respectively. Then,

$$\begin{aligned} V_L(q, \lambda) &= \sum_{k=1}^{\infty} \pi_k \int_{c_L}^{\overline{b}} (b - c_L)\, dF^k(b), \text{ and} \\ V_H(q, \lambda) &= \sum_{k=1}^{\infty} \pi_k \int_{c_H}^{\max\{\overline{b}, c_H\}} (b - c_H)\, dF^k(b). \end{aligned}$$

Applying the previous results on F,

(1) $E_q[v] - c_H \leq \pi_0 q(v_L - c_L)$

$$\begin{aligned} V_H(q, \lambda) &= 0, \text{ and} \\ V_L(q, \lambda) &= (1 - \pi_0 - \lambda\pi_0)(v_L - c_L). \end{aligned}$$

(2) $\pi_0 q(v_L - c_L) < E_q[v] - c_H < q(v_L - c_L)$

$$
\begin{aligned}
V_H(q,\lambda) &= \left(E_q[v]-c_L\right)-\pi_0 q\left(v_L-c_L\right)-\pi_0\frac{q\left(v_L-c_L\right)}{E_q[v]-c_H}\left(c_H-c_L\right) \\
&\quad -\pi_0\left(E_q[v]-c_H\right)\ln\frac{E_q[v]-c_H}{\pi_0 q\left(v_L-c_L\right)}, \text{ and} \\
V_L(q,\lambda) &= \left(v_L-c_L\right)-\frac{E_q[v]-c_H}{q}-\pi_0\left(v_L-c_L\right)\ln\left(\frac{q\left(v_L-c_L\right)}{E_q[v]-c_H}\right) \\
&\quad +\left(\left(E_q[v]-c_H\right)-\pi_0 q\left(v_L-c_L\right)\right)\frac{E_q[v]-c_L}{E_q[v]-c_H} \\
&\quad -\pi_0\left(E_q[v]-c_H\right)\ln\frac{E_q[v]-c_H}{q\pi_0\left(v_L-c_L\right)}.
\end{aligned}
$$

(3) $q\left(v_L-c_L\right)\leq E_q[v]-c_H$

$$
\begin{aligned}
V_H(q,\lambda) &= \left(1-\pi_0-\lambda\pi_0\right)\left(E_q[v]-c_H\right), \text{and} \\
V_L(q,\lambda) &= \left(1-\pi_0-\lambda\pi_0\right)\left(E_q[v]-c_H\right)+\left(1-\pi_0\right)\left(c_H-c_L\right).
\end{aligned}
$$

Unlike buyers' expected payoff, the effects of q and λ on sellers' expected payoffs are intertwined. To see this, consider the case where $E_q[v]-c_H<q\left(v_L-c_L\right)$. If λ is sufficiently high, high-quality goods partially trade (Case (2)), while if λ is close to 0, only lemons trade (Case (1)).

Both V_H and V_L are increasing in λ, and decreasing in q. More buyer competition (higher λ) increases the probability of trading for sellers and drives up the winning bid. Higher q implies lower average quality of goods, which leads buyers to bid lower. When only lemons trade, V_H and V_L are constant in q. This is because conditional on that only lemons trade, buyers' bidding strategy is independent of q.

3.3 Equilibrium Characterization

Subsequently, I assume that only lemons trade without cheap-talk messages, that is, $E_{\widehat{q}}[v]-c_H<\widehat{q}\pi_0\left(v_L-c_L\right)$. This is the environment where the role of non-binding communication can be highlighted. In addition, I focus on the case in which high-quality sellers always announce H. This is without loss of generality. First, if both types of sellers announce both messages, then the equilibrium is "babbling", that is, messages do not convey any information. Second, in equilibrium, it never happens that all lemon sellers announce L, while high-quality sellers announce both messages. I call the submarket induced by L "the lemons submarket" and the submarket induced by H "the high-quality submarket".

I use the following notations.

$$
\widetilde{E}_q[v]\equiv\frac{\widehat{q}-q}{1-q}v_L+\frac{1-\widehat{q}}{1-q}v_H \text{ for } q\leq\widehat{q},
$$

and

$$\lambda_L(\alpha, q) \equiv \frac{\alpha}{q},\ \lambda_H(\alpha, q) \equiv \frac{\beta - \alpha}{1 - q}.$$

$\widetilde{E}_q[v]$ is the expected buyer valuation of the goods in the high-quality submarket when measure q of lemon sellers announce L. $\lambda_L(\alpha, q)$ and $\lambda_H(\alpha, q)$ are the ratio of buyers to sellers in the lemons submarket and in the high-quality submarket, respectively, when measure q of lemon sellers and measure α of buyers are in the lemons submarket, and all other agents are in the high-quality submarket.

3.3.1 Buyers' Indifference Function

Suppose measure $q \in (0, \widehat{q}]$ of lemon sellers announce L. For each q, let $B(q)$ be the measure of buyers such that buyers are indifferent between the two submarkets if measure $B(q)$ of buyers participate in the lemons submarket. Formally, let $B(q)$ be the value such that

$$U(1, \lambda_L(B(q), q)) = U\left(\frac{\widehat{q} - q}{1 - q}, \lambda_H(B(q), q)\right).$$

Applying the previous results,

$$\frac{1}{e^{\lambda_L(B(q),q)}}(v_L - c_L) = \frac{1}{e^{\lambda_H(B(q),q)}} \max\left\{\widetilde{E}_q[v] - c_H, \frac{\widehat{q} - q}{1 - q}(v_L - c_L)\right\}.$$

Arranging terms,

$$B(q) = \beta q + q(1 - q)\ln\left(\frac{v_L - c_L}{\max\left\{\widetilde{E}_q[v] - c_H, \frac{\widehat{q}-q}{1-q}(v_L - c_L)\right\}}\right).$$

Let $\overline{q} \in (0, \widehat{q})$ be the value such that

$$\frac{\widehat{q} - \overline{q}}{1 - \overline{q}}(v_L - c_L) = \widetilde{E}_q[v] - c_H.$$

First, consider the case where $q \in [\overline{q}, \widehat{q}]$. In this case, all buyers bid at least c_H, and so both types of goods fully trade. On this interval, B is not necessarily increasing in q. To see this, fix $q \in [\overline{q}, \widehat{q}]$ and $B(q)$. As q increases, λ_L decreases (relatively less competition among buyers), which makes participating in the lemons submarket more attractive. On the other hand, in the high-quality submarket, competition become more severe (higher λ_H) but the average quality in the submarket improves (higher $\widetilde{E}_q[v]$). When the quality improvement effect outweighs that of competition, B is decreasing. For instance, since

$$B'(\widehat{q}) = \beta - \frac{\widehat{q}(v_H - v_L)}{(v_H - c_H)},$$

if β is small, B is decreasing at around $\widehat{q}$.

Now consider the case where $q < \overline{q}$. In this case, high-quality goods never fully trade. At $q < \overline{q}$, B is strictly increasing in q, for $q < \overline{q}$. This is because unlike the previous case, buyers do not benefit from the improvement of average quality. When high-quality goods partially trade, it is not because buyers are willing to offer c_H, but because competition pushes up buyers' bids higher than c_H. Formally,

$$\begin{aligned} B'(q) &= \beta + (1-q)\ln\left(\frac{1-q}{\widehat{q}-q}\right) + q\left(\frac{(1-\widehat{q})}{(\widehat{q}-q)} - \ln\left(1+\frac{1-\widehat{q}}{\widehat{q}-q}\right)\right) \\ &> \beta + (1-q)\ln\left(\frac{1-q}{\widehat{q}-q}\right). \end{aligned}$$

3.3.2 Sellers' Indifference Function

Now suppose measure $\alpha \in (0, \beta)$ of buyers participate in the lemons submarket. Let $S(\alpha) \in [0, \widehat{q}]$ be the measure of lemon sellers such that lemon sellers are indifferent between the two submarkets if measure $S(\alpha)$ of lemon sellers join the lemons submarket. Formally,

$$V_L(1, \lambda_L(\alpha, S(\alpha))) = V_L\left(\frac{\widehat{q} - S(\alpha)}{1 - S(\alpha)}, \lambda_H(\alpha, S(\alpha))\right).$$

I let $S(\alpha) = 0$ if lemon sellers always prefer the high-quality submarket to the low-quality submarket, and let $S(\alpha) = 1$ if the opposite is true.

First, observe that unlike B, S is strictly increasing at α if $S(\alpha) \in (0, \widehat{q})$. As more buyers join the lemons submarket, the lemons submarket always becomes more attractive than the high-quality submarket. Therefore for lemon sellers to be indifferent between the two submarkets, more sellers should join the lemons submarket.

There are three critical values of α, $0 < \beta_1 < \beta_2 < \beta_3 < \beta$. β_2 and β_3 are defined to be the values that satisfy $S(\beta_2) = \overline{q}$ and $S(\beta_3) = \widehat{q}$. These two values are well-defined in interior because sellers' payoffs are equal to 0 if the measure of buyers is negligible. $\beta_2 < \beta_3$ comes from the fact that S is strictly increasing. β_1 is the value that satisfies

$$\frac{1}{e^{\lambda_H(\beta_1, S(\beta_1))}} \frac{\widehat{q} - S(\beta_1)}{1 - S(\beta_1)} (v_L - c_L) = \widetilde{E}_{S(\beta_1)}[v] - c_H.$$

This value is also well-defined because the right-hand side is strictly greater (smaller) than the left-hand side if β is close to β_2 (0), and the right-hand side increases faster than the left-hand side.

By construction, if $\alpha > \beta_3$, then all lemon sellers prefer staying in the lemons submarket ($S(\alpha) = \widehat{q}$). If $\alpha \in [\beta_2, \beta_3]$ then $S(\alpha) \in [\overline{q}, \widehat{q}]$ and so both types of goods are fully traded in the high-quality submarket. If $\alpha \in [\beta_1, \beta_2]$ then high-quality goods partially trade in the high-quality submarket. If $\alpha < \beta_1$ then only lemons trade in the high-quality submarket.

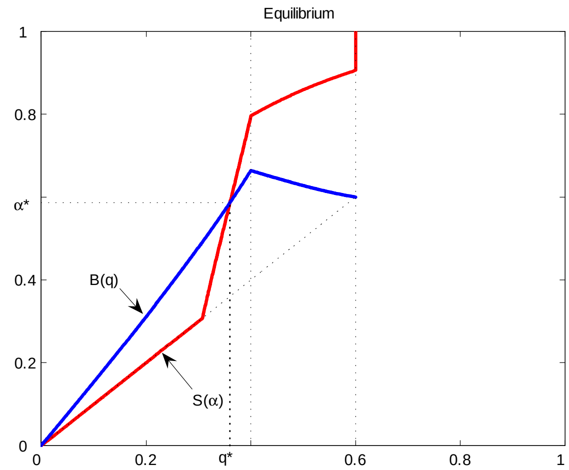

Figure 3: For parameter values $c_L = 0.1, v_L = 0.35, c_H = 0.6, v_H = 0.85, \widehat{q} = 0.6$ and $\beta = 1$.

The case where $\alpha < \beta_1$ needs emphasis. In this case,

$$v_L - \frac{(1 + \lambda_L(\alpha, S(\alpha)))}{e^{\lambda_L(\alpha, S(\alpha))}}(v_L - c_L) = v_L - \frac{(1 + \lambda_H(\alpha, S(\alpha)))}{e^{\lambda_H(\alpha, S(\alpha))}}(v_L - c_L).$$

This equation holds only when $\lambda_L(\alpha, S(\alpha)) = \lambda_H(\alpha, S(\alpha))$, which implies $S(\alpha) = \alpha/\beta$. Intuitively, when only lemons trade in both submarkets, buyers' bidding strategies are independent of the proportion of lemon sellers. Therefore for lemon sellers to be indifferent between the two submarkets, the arrival rates of buyers should be identical.

3.3.3 Equilibrium

An equilibrium is characterized by (α^*, q^*) such that $\alpha^* = B(q^*)$ and $q^* = S(\alpha^*)$ or by q^* that is a fixed point of a function $S(B(\cdot))$.

Proposition 1 *When the social surplus in trading is independent of quality, and only lemons trade without cheap-talk messages, there always exists an informative equilibrium. In such an equilibrium, high-quality goods trade with positive probability in the high-quality submarket.*

This is because $q < S(B(q))$ for $q \in \left(0, \widetilde{\beta}/\beta\right)$, while $q > S(B(q))$ for q close to $\widehat{q}$.

1. Suppose measure ε of lemon sellers announce L. Then the proportion of lemon sellers in the high-quality submarket is still so high that only lemons trade in the high-quality submarket. Given that, for buyers to be indifferent between the two submarkets, λ_H should be lower than λ_L (buyers have a positive probability to meet high-quality sellers in the high-quality submarket). But then lemon sellers prefer the lemons submarket. As shown before, for lemon

sellers to be indifferent between the two submarkets when only lemons trade in the high-quality submarket, $\lambda_H = \lambda_L$.

2. Now suppose all lemon sellers announce L. Then buyers are indifferent between the two submarkets only when the level of competition (the ratio of buyers to sellers) is identical in both submarkets. But then lemon sellers strictly prefer the high-quality submarket.

It depends on the measure of buyers, β, whether high-quality goods fully trade in the high-quality submarket or not. Formally, full trade occurs in the high-quality submarket if $S\left(B\left(\overline{q}\right)\right) \geq \overline{q}$, which is equivalent to

$$e^{\beta} \leq \left(\frac{1-\overline{q}}{\widehat{q}-\overline{q}}\right)^{1+\overline{q}} \left[\frac{1}{(v_L - c_L)} - \ln\left(\frac{1-\overline{q}}{\widehat{q}-\overline{q}}\right)\left(\frac{\widehat{q}-\overline{q}}{1-\overline{q}}\right)\right].$$

Therefore high-quality goods fully trade in the high-quality submarket when β is sufficiently small. Intuitively, if β is large, it is less likely that sellers do not meet any buyer in the high-quality submarket, which increase lemon sellers' incentive to join the high-quality submarket. Then the proportion of lemon sellers in the high-quality submarket is so large that full trade cannot occur. Conversely, if β is small, lemon sellers have less incentive to join the high-quality submarket whose ratio of buyers to sellers is small. Therefore, they should be compensated through higher sale prices, which happens when buyers bid more than c_H.

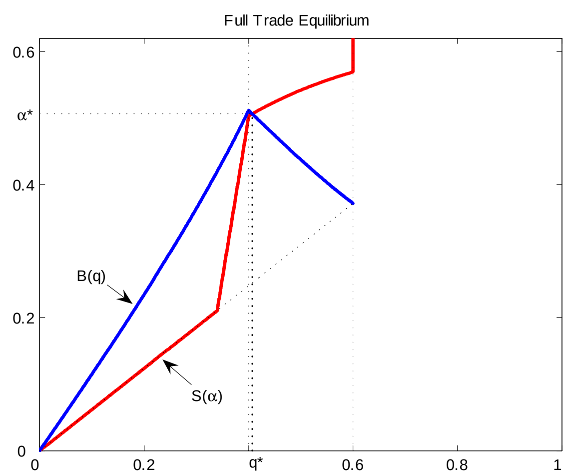

Figure 4: Full trade occurs for $\beta = 0.62$ and the same parameter values as in Figure 3.

Does cheap-talk communication enhance the efficiency in the market? First of all, it is straightforward that both types of sellers are better off. Now high-quality sellers achieve a positive payoff. Lemon sellers benefit from more severe competition among buyers in the lemons submarket. Since lemon sellers are indifferent between two submarkets, they are unambiguously better off.

In the two-quality case, it is non-trivial whether buyers are better off or not. Consider buyers in the lemons submarket. On the one hand, they face no quality uncertainty, which increases their expected payoff. On the other hand, they face more severe competition and should bid more on average. Formally, buyers are better off if and only if

$$\frac{\widehat{q}}{e^{\beta}}\left(v_L - c_L\right) \leq \frac{1}{e^{B(q^*)/q^*}}\left(v_L - c_L\right) \Leftrightarrow \frac{\widehat{q}}{e^{\beta}} \leq \frac{1}{e^{B(q^*)/q^*}}.$$

Buyers are better off only when the market is sufficiently thin (the ratio of buyers to sellers is sufficiently small). This ambiguous result does not hold when there are a continuum of qualities. With a continuum of qualities, buyers are strictly better off when cheap talk is informative. Furthermore, the more information is transmitted through cheap-talk communication, the better off buyers are.

3.4 Relaxing Constant Surplus Assumption

Now I discuss what happens if I relax the constant surplus assumption.

Lower trading surplus with high-quality goods

Suppose $v_L - c_L > v_H - c_H$, and all lemon sellers announce L (full separation). In this case, for buyers to be indifferent between the two submarkets, $\lambda_L > \lambda_H$. Then lemon sellers have less incentive to join the high-quality submarket than in the constant surplus case.[6] If $v_L - c_L$ is sufficiently larger than $v_H - c_H$, the incentive to move to the high-quality submarket disappears, and so the full separation state persists. More precisely, there are two kinds of fully separating equilibria.

(1) Fully separating equilibrium without trade in the high-quality submarket

This happens when

$$\frac{1}{e^{\beta/\widehat{q}}}\left(v_L - c_L\right) \geq \left(v_H - c_H\right),$$

and

$$v_L - \frac{1}{e^{\beta/\widehat{q}}}\left(v_L - c_L\right) \leq c_H.$$

The first inequality is buyers' incentive compatibility condition that they should prefer the lemons submarket, even though they can extract the full trading surplus from high-quality sellers. The second inequality is high-quality sellers' incentive compatibility condition. It states that the maximum bid in the lemons submarket (the left-hand side) should be less than the cost of high-quality sellers. Since the first inequality implies the second one, a fully separating equilibrium without trade in the high-quality submarket exists if and only if

$$\frac{1}{e^{\beta/\widehat{q}}}\left(v_L - c_L\right) \geq \left(v_H - c_H\right).$$

[6] In the constant surplus case, under the full separation, lemon sellers prefer the high-quality market, because buyers bid higher in the high-quality market, but the probabilities of meeting buyers are identical in both submarkets.

That is, this equilibrium exists when β is sufficiently small. The intuition behind this result is as follows. Suppose β is close to 0, and sellers are fully separated. Then buyers strictly prefer the lemons submarket to the high-quality submarket because their expected payoff is close to $v_L - c_L$, which is greater than $v_H - c_H$. Lemon sellers obviously do not deviate to the high-quality submarket. High-quality sellers also have no incentive to deviate, because buyers bid lower than c_H in the lemons submarket.

(2) Fully separating equilibrium with trade in the high-quality submarket

Let $\pi_0 = 1/e^{\lambda_L}$ and $\pi_0' = 1/e^{\lambda_H}$. A fully separating equilibrium exists if and only if

$$\pi_0 (v_L - c_L) = \pi_0' (v_H - c_H),$$

$$(1 - \pi_0 - \lambda \pi_0)(v_L - c_L) \geq \left(1 - \pi_0' - \lambda' \pi_0'\right)(v_H - c_H) + \left(1 - \pi_0'\right)(c_H - c_L),$$

and

$$\begin{aligned} & (1 - \pi_0) v_L + \pi_0 c_L - c_H - \pi_0 (v_L - c_H) \ln \frac{v_L - c_H}{\pi_0 (v_L - c_L)} \\ \leq \quad & \left(1 - \pi_0' - \lambda \pi_0'\right)(v_H - c_H). \end{aligned}$$

The first condition is buyers' indifference between the two submarkets. The two inequalities are the incentive compatibility conditions for lemon sellers, and high-quality sellers, respectively. These conditions hold when

$$\frac{1}{e^{\beta/\widehat{q}}} (v_L - c_L) \leq (v_H - c_H),$$

and v_L is sufficiently close to v_H. The latter guarantees that λ_L is sufficiently higher than λ_H, and the former (β is sufficiently large) ensures that trade occurs in the high-quality submarket as well.

Of particular interest is the constant value case where $v_H = v_L$. In this case, equilibrium is always fully separating, whether trade occurs in the high-quality submarket or not. This is not a special feature of the two-quality case. In Section 5, I show that when buyers' values are independent of sellers' costs, a fully revealing equilibrium exists even when there is a continuum of qualities.

Higher trading surplus with high-quality goods

Now suppose $v_L - c_L < v_H - c_H$. Under the full separation, $\lambda_L < \lambda_H$. Therefore lemon sellers have a greater incentive to join the high-quality submarket than the constant surplus case. This guarantees that $\widehat{q} > S(B(\widehat{q}))$. However, if $v_H - c_H$ is sufficiently larger than $v_L - c_L$, lemon sellers may have so high an incentive to join the high-quality submarket, and thus an informative equilibrium may not exist. That is, in this case, the condition that $\varepsilon < S(B(\varepsilon))$ for ε sufficiently small may be violated. To see this, suppose $v_H - c_H$ is sufficiently larger than $v_L - c_L$, and so $E_q[v] - c_H \geq v_L - c_L$. Since $v_L - c_L < \widetilde{E}_q[v] - c_H$ for any q, $\lambda_L(q, B(q)) < \lambda_H(q, B(q))$ for any q. But then lemon sellers always prefer the high-quality submarket to the lemons submarket. Hence there cannot exist an informative equilibrium. In Section 5, I show that this insight generalizes into the case with a continuum of qualities.

3.5 More Finite Qualities

The insights from the two-quality case are transparent to the general finite quality case. Figure 5 shows an example of equilibrium when there are three qualities of goods. The high two submarkets consist of different two qualities, whose proportions determine the amount of quality uncertainty and thus the level of buyer competition in the submarket. The general characterization for the finite quality case is, however, quite involved. I turn my attention to a continuum quality case from the next section.

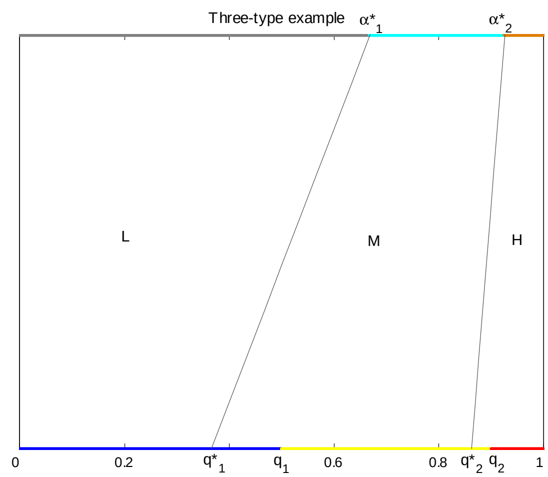

Figure 5: For parameter values, $c_L = 0, v_L = 1/8, c_M = 3/8, v_M = 4/8, c_H = 6/8, v_H = 7/8, q_1 = 0.5, q_2 = 0.9$, and $\beta = 1$. Measue q_1^* of low-quality sellers announce L, all other low-quality sellers and measue $(q_2^* - q_1)$ of middle-quality sellers announce M, and all other remaining sellers announce H. On the long-side of the market, measure α_1^*, $\alpha_2^* - \alpha_1^*$, and $1 - \alpha_2^*$ of buyers submit bids to sellers with L, M, and H, respectively.

4 A Continuum Quality: Constant Surplus Case

4.1 Environment

Now there is a continuum of qualities distributed uniformly over $[0, 1]$. A unit of $q \in [0, 1]$-quality good costs $c(q)$ to a seller and yields utility $v(q)$ to a buyer. This section studies the case where $c(q) = q$ and $v(q) = q + \Delta$ for some $\Delta > 0$. This parametric assumption incorporates two simplifications. First, the trading surplus is independent of quality, that is, $v(q) - c(q) = \Delta$ for all $q \in [0, 1]$. Second, the distribution of seller's costs (reservation utilities) is uniform. These properties enable me to focus on quality uncertainty aspect of the problem. In addition, they allow me to apply a recursive method in characterizing the set of equilibria and thus provide more understanding in the equilibrium structure.

The market proceeds as in the previous section except that I do not specify the set of messages ex ante. As is common in the cheap talk literature, what matter is neither the form of messages nor the cardinality of the set of messages, but the amount of information transmission. From now on, I focus on how many distinct submarkets are sustainable in equilibrium, without explicitly specifying what message set is needed.[7]

Definition 1 *An equilibrium with n submarkets (n messages) is characterized by a strictly increasing sequence* $\{q_0 = 0, q_1, ..., q_n = 1\}$ *and a sequence* $\{\lambda_1, ..., \lambda_n\}$ *such that*

(1) (Sellers' optimality) if $q \in [q_{k-1}, q_k]$ *then q-quality seller prefers joining the k-th submarket (by announcing the k-th message) to joining other submarkets,*$\forall k = 1, ..., n$*,*[8]

(2) (Buyers' optimality) if $\lambda_k > 0$*, buyers weakly prefer joining the k-th submarket (submitting bids to sellers with the k-th message) than joining other submarkets, and*

(3) (Market clearing) $\beta = \sum_{k=1}^{n} (q_k - q_{k-1}) \lambda_k$*.*

Remark 1 This is a reduced-form definition of market equilibrium. I did not impose the optimality conditions on buyers' bidding strategies and sellers' acceptance strategies. In addition, I did not explicitly require buyers' beliefs about sellers' qualities in each submarket to be consistent. They are straightforward, and so omitted.

Remark 2 The definition can be generalized for the case with infinitely many submarkets, and for the case with a continuum of submarkets. But it requires unnecessarily substantial investment in notations.

4.2 Submarket Analysis

I first solve for the submarket outcomes. Suppose a submarket is populated by sellers in $\left[\underline{q}, \overline{q}\right]$ and its ratio of buyers to sellers is given by $\lambda > 0$. Let F be buyers' symmetric bidding strategy in this submarket, and $\left[\underline{b}, \overline{b}\right]$ be the support of F.

4.2.1 Buyers' Expected Payoff

As in the two-quality case, $\underline{b}$ is equal to the offer of the monopsonist who is facing a seller whose quality is uniformly distributed over $\left[\underline{q}, \overline{q}\right]$. Let $U\left(\underline{q}, \overline{q}, \lambda\right)$ be the expected payoff of buyers in a

[7]An alternative is to start with a sufficiently rich set of messages, for example, a unit interval in the real line. If there is an equilibrium with n submarkets, one can partition the unit interval into n distinct subintervals, and require agents in each submarket to randomize over each subinterval.

[8]I focus on the equilibrium in which the set of qualities in the same submarket is convex. There may exist an equilibrium in which this is not true. This can happen only when some high-quality goods never trade in any submarket.

submarket and $M\left(\underline{q},\overline{q}\right)$ be the expected payoff of the monopsonist. Then as in the two-quality case,

$$U\left(\underline{q},\overline{q},\lambda\right)=\pi_0 M\left(\underline{q},\overline{q}\right).$$

U is again strictly decreasing in λ, and inherits all the properties of $M\left(\underline{q},\overline{q}\right)$.

First, $\underline{b}=\min\left\{\overline{q},\underline{q}+\Delta\right\}$. Intuitively, the marginal benefit of the monopsonist's increasing b is $b+\Delta$, while the corresponding marginal cost is $\left(b-\underline{q}\right)+b$: $b-\underline{q}$ is the marginal increase of payment to all of the seller types who would accept the previous offer, and b is the gross payment to the seller type who newly accepts the offer. The marginal benefit and cost match when $b=\underline{q}+\Delta$, but if $\overline{q}<\underline{q}+\Delta$ then the monopsonist has no reason to bid more than $\overline{q}$.

Second, by direct calculation,

$$M\left(\underline{q},\overline{q}\right)=\begin{cases}\Delta-\frac{\overline{q}-\underline{q}}{2}, & \text{if } \overline{q}-\underline{q}\leq\Delta,\\ \frac{\Delta^2}{2\left(\overline{q}-\underline{q}\right)}, & \text{otherwise.}\end{cases}$$

Notice that the expected payoff of the monopsonist depends on only $\overline{q}-\underline{q}$, which can be interpreted as the measure of quality uncertainty in the current setting. This is the sense in which the parametric assumption in this section underscores the quality uncertainty aspect of the problem. It is natural that M is strictly decreasing in the amount of quality uncertainty $(\overline{q}-\underline{q})$.

4.2.2 Sellers' Expected Payoffs

F is again derived from buyers' indifference over $\left[\underline{b},\overline{b}\right]$. If $\overline{q}-\underline{q}\leq\Delta$ then $\underline{b}=\overline{q}$ and so trade occurs whenever a seller is matched with at least one buyer. Buyers' expected payoff by bidding $b\geq\underline{b}$ is

$$\sum_{k=0}^{\infty}\pi_k F\left(b\right)^k\left(\int_{\underline{q}}^{\overline{q}}\frac{\left(v\left(q\right)-b\right)}{\overline{q}-\underline{q}}dq\right)=\pi_0 e^{\lambda F(b)}\left(E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right]-b\right),$$

where

$$E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right]=\frac{1}{\overline{q}-\underline{q}}\int_{\underline{q}}^{\overline{q}}v\left(q\right)dq=\frac{\underline{q}+\overline{q}}{2}+\Delta.$$

Since buyers are indifferent over $\left[\underline{b},\overline{b}\right]$,

$$e^{\lambda F(b)}=\frac{E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right]-\overline{q}}{E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right]-b}.$$

Using $F\left(\overline{b}\right)=1$, I find that

$$\overline{b}=\left(1-\pi_0\right)E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right]+\pi_0\overline{q}.$$

Now suppose $\overline{q}-\underline{q}>\Delta$. In this case, $\underline{b}=\underline{q}+\Delta<\overline{q}$, and so trade may not occur even if a seller is matched with buyers. Buyers' expected payoff by bidding $b \geq \underline{b}$ is

$$\pi_0 e^{\lambda F(b)} \frac{b-\underline{q}}{\overline{q}-\underline{q}} \left(E_{\underline{q},b}\left[v\left(q'\right)\right] - b \right), \text{ if } b \leq \overline{q},$$

and

$$\pi_0 e^{\lambda F(b)} \left(E_{\underline{q},\overline{q}}\left[v\left(q'\right)\right] - b \right), \text{ if } b > \overline{q}.$$

Since buyers are indifferent over all bids in $\left[\underline{b}, \overline{b}\right]$,

$$e^{\lambda F(b)} = \begin{cases} \frac{\Delta\left(E_{\underline{q},\underline{q}+\Delta}[v(q')] - \left(\underline{q}+\Delta\right)\right)}{\left(b-\underline{q}\right)\left(E_{\underline{q},b}[v(q')]-b\right)}, & \text{if } b \leq \overline{q}, \\ \frac{\Delta\left(E_{\underline{q},\underline{q}+\Delta}[v(q')] - \left(\underline{q}+\Delta\right)\right)}{E_{\underline{q},\overline{q}}[v(q')]-b}, & \text{if } b > \overline{q}. \end{cases}$$

Regarding $\overline{b}$, there are two possibilities, $\overline{b} \geq \overline{q}$ and $\overline{b} < \overline{q}$. Using $F\left(\overline{b}\right) = 1$, I find that

$$\overline{b} = \left\{ \begin{array}{cc} \Delta + \frac{\underline{q}+\overline{q}}{2} - \pi_0 \frac{\Delta^2/2}{\overline{q}-\underline{q}} & \leq \\ \underline{q} + \Delta\left(1+\sqrt{1-\pi_0}\right) & > \end{array} \right\} \overline{q} \text{ if } \left\{ \begin{array}{l} \overline{q} - \underline{q} \leq \Delta\left(1+\sqrt{1-\pi_0}\right), \\ \overline{q} - \underline{q} > \Delta\left(1+\sqrt{1-\pi_0}\right). \end{array} \right.$$

From F, I can calculate the expected payoffs of sellers. Though calculation is not particularly hard, the form of $V\left(q; \underline{q}, \overline{q}, \lambda\right)$ is unnecessarily complicated. Below, I present only the expected payoffs of the boundary sellers, $\underline{q}$ and $\overline{q}$. Due to the single-crossing property in Lemma 1, only these payoffs are necessary for further analysis. Let $z = \overline{q} - \underline{q}$. There are three cases according to z.

(1) If $z \leq \Delta$ then

$$\begin{aligned} V\left(\underline{q}; \underline{q}, \overline{q}, \lambda\right) &= \Delta\left(1-\pi_0-\lambda\pi_0\right) + \frac{z}{2}\left(1-\pi_0+\lambda\pi_0\right), \text{ and} \\ V\left(\overline{q}; \underline{q}, \overline{q}, \lambda\right) &= \Delta\left(1-\pi_0-\lambda\pi_0\right) + \frac{z}{2}\left(1-\pi_0-\lambda\pi_0\right). \end{aligned}$$

(2) If $\Delta < z < \Delta\left(1+\sqrt{1-\pi_0}\right)$ then

$$\begin{aligned} V\left(\underline{q}; \underline{q}, \overline{q}, \lambda\right) &= \frac{z}{2} + \left(1-\pi_0\right)\Delta - \pi_0 \frac{\Delta^2}{2z} \\ &\quad - \pi_0 \frac{\Delta}{2} \ln \frac{z}{(2\Delta - z)} - \pi_0 \frac{\Delta^2}{2z} \ln \frac{z\left(2\Delta - z\right)}{\pi_0 \Delta^2}, \text{ and} \\ V\left(\overline{q}; \underline{q}, \overline{q}, \lambda\right) &= \Delta - \frac{z}{2} - \pi_0 \frac{\Delta^2}{2z} - \pi_0 \frac{\Delta^2}{2z} \ln \frac{z\left(2\Delta - z\right)}{\pi_0 \Delta^2}. \end{aligned}$$

(3) If $z \geq \Delta\left(1+\sqrt{1-\pi_0}\right)$ then

$$\begin{aligned} V\left(\underline{q}; \underline{q}, \overline{q}, \lambda\right) &= \Delta\left(1-\pi_0+\sqrt{1-\pi_0}\right) - \pi_0 \frac{\Delta}{2} \ln \frac{\left(1+\sqrt{1-\pi_0}\right)}{\left(1-\sqrt{1-\pi_0}\right)}, \text{ and} \\ V\left(\overline{q}; \underline{q}, \overline{q}, \lambda\right) &= 0. \end{aligned}$$

Similarly to $U\left(\underline{q},\overline{q},\lambda\right)$, both $V\left(\underline{q};\underline{q},\overline{q},\lambda\right)$ and $V\left(\overline{q};\underline{q},\overline{q},\lambda\right)$ are functions of only λ and $\overline{q}-\underline{q}$.[9] Both are increasing in λ. $V\left(\underline{q},\underline{q},\overline{q},\lambda\right)$ is increasing in $\overline{q}-\underline{q}$, while $V\left(\overline{q},\underline{q},\overline{q},\lambda\right)$ is not monotone in $\overline{q}-\underline{q}$.

4.3 Necessary and Sufficient Conditions for an Equilibrium

Unlike the two-quality case, it is complicated to directly characterize equilibrium with a continuum of qualities. I find tractable conditions that are necessary and sufficient for an equilibrium.

The following lemma shows that I can restrict my attention to the expected payoffs of the boundary sellers between two adjacent submarkets.

Lemma 1 *(Single Crossing Property) Suppose* $[q_1,q_2]$ *and* $[q_2,q_3]$ *form separate submarkets with the ratio of buyers to sellers,* λ *and* λ'*, respectively, and buyers are indifferent between the two submarkets. Sellers whose qualities are below (above)* q_2 *prefer the submarket with* $[q_1,q_2]$ *(*$[q_2,q_3]$*) to the submarket with* $[q_2,q_3]$ *(*$[q_1,q_2]$*), if and only if*

(1) q_2 *is indifferent between the two submarkets, and*

(2) $\lambda>\lambda'$*.*

Proof. See Appendix. ■

The intuition behind this result is the same as that of the common single crossing property based on the trade-off between the probability of trading and transaction prices. The lower the cost is, the more willingly a seller is to trade the good. Therefore if q_2-quality seller is indifferent between the two submarkets that have different levels of buyer competition and different bidding behaviors of buyers, sellers whose qualities are lower (higher) than q_2 prefers the submarket with relatively more buyers (with relatively higher transaction prices).

Corollary 1 $\{q_0=0,q_1,...,q_n=1\}$ *and* $\{\lambda_1,...,\lambda_n\}$ *constitute an equilibrium with* n *submarkets if and only if*

(1) (Boundary Sellers' Indifference) q_k*-quality seller is indifferent between* k*-th and* $(k+1)$*-th submarkets,* $k=1,...,n-1$*,*

(2) (Buyers' Indifference) buyers are indifferent over all active submarkets ($\lambda_k>0$*), and weakly prefer active submarkets to inactive submarkets (*$\lambda_k=0$*), and*

(3) (Monotone Market Arrangement) $\lambda_k>\lambda_{k+1}$ *if* $\lambda_k>0$*, and* $\lambda_k=0$*, if* $\lambda_{k-1}=0,k=1,...,n-1$*,*

(4) (Market Clearing Condition) $\beta=\sum_{k=1}^{n}\left(q_k-q_{k-1}\right)\lambda_k$*.*

[9] More generally,

$$V\left(q;\underline{q},\overline{q},\lambda\right)=V\left(q-\underline{q};0,\overline{q}-\underline{q},\lambda\right).$$

Qualifications in (2) and (3) are due to the possibility of inactive submarkets (open submarkets with no trade).

(2) and (3) together imply the following monotone market arrangement for sellers. Suppose in equilibrium, buyers are indifferent over all submarkets. Then $\lambda_1 \geq \lambda_2 ... \geq \lambda_n$ is equivalent to

$$q_2 - q_1 \leq q_3 - q_2 \leq ... \leq q_n - q_{n-1}.$$

That is, quality uncertainty is higher in high-quality submarkets than in low-quality submarkets. This is the analogue to the fact that there is a positive amount of quality uncertainty in the high-quality submarket in the two-quality case. These relationships highlight how the incentives for both sides are aligned in my model.

4.4 Partial Equilibrium Analysis

The analysis from now on proceeds as follows. First, in this subsection, I suppose buyers' equilibrium utility $u \in (0, \Delta)$ is known, and find the set of $\{q_0 = 0, q_1, ..., q_n = 1\}$ and $\{\lambda_1, ..., \lambda_n\}$ that are consistent with u. In other words, I find a partition of sellers and the corresponding ratios of buyers to sellers with which buyers get the same utility u in every submarket.[10] Second, in the next subsection, I endogenize u by imposing the market clearing condition, $\beta = \sum_{k=1}^{n} (q_k - q_{k-1}) \lambda_k$. Subsequently, let $z_k = q_k - q_{k-1}, k = 1, ..., n$.

Preliminaries

To facilitate the analysis, I introduce some functions. Let $\lambda(z, u)$ be the value such that $u = U(0, z, \lambda(z, u))$. In words, $\lambda(z, u)$ is the tightness (the ratio of buyers to sellers) that is required to guarantee buyers utility u when quality uncertainty is z in a submarket. $\lambda(z, u)$ is well-defined for $z \leq \Delta^2/2u$ if $u < \Delta/2$, and for $z \leq 2(\Delta - u)$ if $u \geq \Delta/2$. For later use, let $\overline{z}(u)$ be equal to $\Delta^2/2u$ if $u < \Delta/2$, and be equal to $2(\Delta - u)$ if $u \geq \Delta/2$. $\lambda(z, u)$ is strictly decreasing in both z and u, because U is strictly decreasing in z and λ. Intuitively, as quality uncertainty increases, buyer competition should be reduced to ensure buyers the same utility u. Similarly, for a fixed amount of quality uncertainty, buyer competition should be reduced to deliver more utility to buyers.

Next, let $W_L(z, u) = V(0; 0, z, \lambda(z, u))$ and $W_U(z, u) = V(z; 0, z, \lambda(z, u))$. $W_L(z, u)$ ($W_U(z, u)$) is the expected payoff of the lower (upper) boundary seller when buyers get utility u in a submarket with quality uncertainty z. After arranging terms,

$$W_L(z, u) = \begin{cases} \Delta + \frac{z}{2} - u - u\frac{2z}{(2\Delta - z)} + u \ln \frac{2u}{(2\Delta - z)}, & \text{if } z \leq \Delta, \\ \frac{z}{2} + \Delta - \frac{2zu}{\Delta} - u - \frac{zu}{\Delta} \ln \frac{z}{(2\Delta - z)} + u \ln \frac{2u}{(2\Delta - z)}, & \text{if } z \in (\Delta, 2(\Delta - u)), \\ \Delta\left(1 - \frac{2zu}{\Delta^2} + \sqrt{1 - \frac{2zu}{\Delta^2}}\right) - \frac{zu}{\Delta} \ln \frac{\left(1 + \sqrt{1 - 2zu/\Delta^2}\right)^2}{2zu/\Delta^2} & \text{if } z \geq 2(\Delta - u), \end{cases}$$

[10] Buyers may strictly prefer some submarkets to others. I call such equilibrium "partially indifferent equilibrium" and characterize the set of such equilibria in Appendix A.

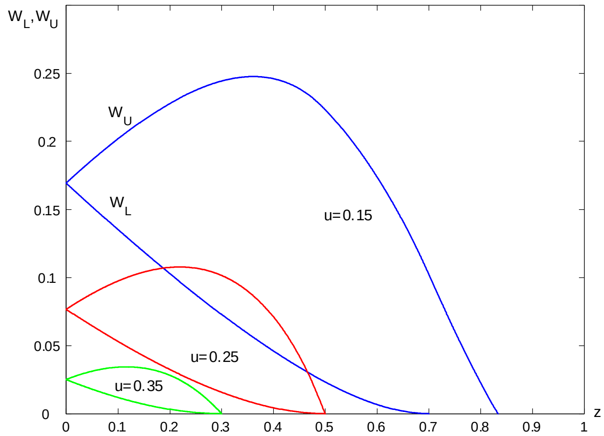

Figure 6: W_L and W_U as functions of z for different values of u.

and

$$W_U(z,u) = \begin{cases} \Delta - \frac{z}{2} - u + u \ln \frac{2u}{(2\Delta - z)}, & \text{if } z \leq 2(\Delta - u), \\ 0, & \text{if } z \geq 2(\Delta - u). \end{cases}$$

Figure 6 shows sample paths of $W_L(\cdot, u)$ and $W_U(\cdot, u)$. First consider $W_U(\cdot, u)$. As quality uncertainty increases, buyer competition in the submarket should be reduced to guarantee buyers utility u. In addition, from the upper boundary seller's perspective, higher z means lower $\underline{q}$ for fixed $\overline{q}$ and so the lower average quality. This makes buyers bid lower. Both of these effects lower the expected payoff of the upper boundary seller.

Now consider $W_L(\cdot, u)$. As quality uncertainty increases, $\lambda(z, u)$ decreases, which reduces the lower boundary seller's expected payoff. However, from the lower boundary seller's perspective, a higher z means a higher $\overline{q}$ for fixed $\underline{q}$ and so a greater average quality. Since buyers bid higher, this offsets the first effect. For z small, the second effect dominates, while the first effect does for z sufficiently large. Overall, $W_L(\cdot, u)$ is increasing first and decreasing later.

Both W_L and W_U are decreasing in u. For a fixed amount of quality uncertainty, buyer competition should be reduced to yield a higher utility to buyers. This effect reduces both W_L and W_U.

Recursive Method

Suppose z_1 is given. Since the upper boundary seller in the first submarket should be indifferent between the first and the second submarkets, z_2 is determined so that $W_U(z_1, u) = W_L(z_2, u)$. In the same way, I can find $z_3, z_4, ...$ This process stops once $z_1 + ... + z_n \geq 1$ for some n. If $z_1 + ... + z_n = 1$, then the sequence $\{q_0, ..., q_n\}$ and $\{\lambda_1, ..., \lambda_n\}$ such that $q_k - q_{k-1} = z_k$ and

$\lambda_k = \lambda(z_k, u)$ constitute a partial equilibrium for u.

For more systematic analysis, I define the following function. Let $\gamma_+(\cdot, u) : [0, \overline{z}(u)] \to [0, \overline{z}(u)]$ so that $\gamma_+(0, u) > 0$, $W_U(z, u) = W_L(\gamma_+(z, u), u)$. In words, $\gamma_+(z, u)$ is the amount of quality uncertainty in the next submarket, when quality uncertainty is z in some submarket. By immediate extension, let $\gamma_+^k(z, u) = \gamma_+\left(\gamma_+^{k-1}(z, u), u\right)$ for $k \geq 1$ where $\gamma_+^0(z, u) = z$.

Lemma 2 *(1) $\gamma_+^k(\cdot, u)$ is continuous.*

(2) $\gamma_+^k(\cdot, u)$ is strictly increasing on $[0, 2(\Delta - u))$, and constant on $[2(\Delta - u), \overline{z}(u)]$.

(3) $\gamma_+^k(z, \cdot)$ is continuous and strictly decreasing.

Proof. See Appendix. ■

Partial equilibrium with one submarket

A partial equilibrium with one submarket (in which a positive measure of buyers participate in the market) exists if and only if $\overline{z}(u) > 1$. Figure 7 shows such equilibria for different u's. In the left panel, the one-message equilibrium is the unique equilibrium, while there exists a two-message equilibrium in the right panel. To see this, consider $z + \gamma_+(z)$. In the left panel, $z + \gamma_+(z)$ is greater than 1 for any z, and thus there does not exist a two-message equilibrium. In the right panel, $z + \gamma_+(z)$ is smaller than 1 if z is close 0, while it is greater than 1 if z is large (for example, when $\gamma_+(z) = 1$). Since γ_+ is continuous, there exists z^* such that $z^* + \gamma_+(z^*) = 1$. In general, one-message equilibrium is unique if and only if $\gamma_+(0, u) \geq 1$, which holds when u is sufficiently small (See Figure 6).

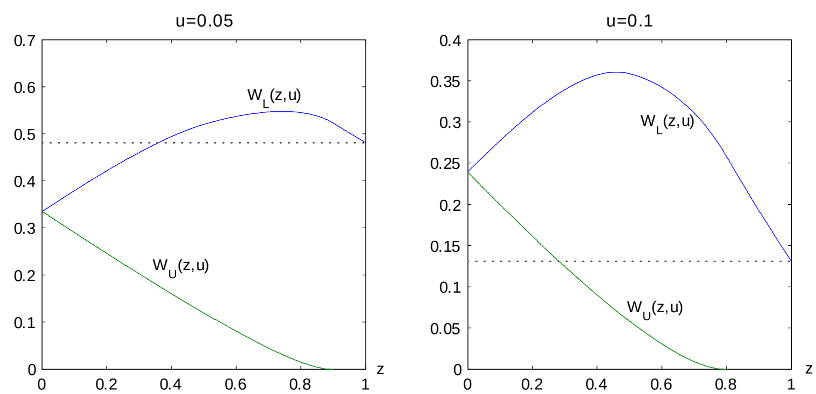

Figure 7: One-message partial equilibrium for $\Delta = 0.5$.

Partial equilibrium with n submarkets

Consider an equilibrium with n submarkets, $\{z_1, ..., z_n\}$. By the incentive compatibilities of the boundary sellers of each submarket, $z_k = \gamma_+^{k-1}(z_1, u)$ for all k. Since $\sum_{k=1}^{n} \gamma_+^{k-1}(z, u)$ is strictly increasing and $z_1 + ... + z_n = 1$, a necessary and sufficient condition for an n-message partial equilibrium to exist is

$$\sum_{k=1}^{n} \gamma_+^{k-1}(0, u) < 1 < \sum_{k=1}^{n} \gamma_+^{k-1}(\overline{z}(u), u) = n \cdot \overline{z}(u).$$

Lemma 3 *An n-message equilibrium exists if and only if $u \in (\underline{u}_n, \overline{u}_n)$, where $\underline{u}_n$ be the value such that $\sum_{k=1}^{n} \gamma_+^{k-1}(0, \underline{u}_n) = 1$, and $\underline{u}_n$ be the value such that $\overline{z}(\underline{u}_n) = 1/n$.*

Proof. This comes from the fact that both $\sum_{k=1}^{n} \gamma_+^{k-1}(0, u)$ and $n \cdot \overline{z}(u)$ is strictly decreasing. The former follows from (3) in Lemma 2, and the latter is by the definition of $\overline{z}(u)$. ■

The set of partial equilibria

Given u, let $\overline{N}(u)$ be the smallest integer such that $\sum_{k=1}^{\overline{N}(u)+1} \gamma_+^{k-1}(0, u) \geq 1$, and let $\underline{N}(u)$ be the smallest integer that is strictly greater than $1/\overline{z}(u)$.

Proposition 2 *(The set of partial equilibria) Given u, $\underline{N}(u) \leq \overline{N}(u)$, and for any n between $\underline{N}(u)$ and $\overline{N}(u)$ (including both), there exists a unique n-message partial equilibrium.*

Proof. Since $\gamma_+(\cdot, u)$ is increasing and $\gamma_+(\overline{z}(u)) = \overline{z}(u)$, $\gamma_+(z) \leq \overline{z}(u), \forall z \in [0, \overline{z}(u)]$. Therefore,

$$1 \leq \sum_{k=1}^{\overline{N}(u)+1} \gamma_+^{k-1}(0, u) = 0 + \sum_{k=2}^{\overline{N}(u)+1} \gamma_+^{k-1}(0, u) < \sum_{k=2}^{\overline{N}(u)+1} \overline{z}(u) = \overline{N}(u) \cdot \overline{z}(u).$$

The strict inequality is due to the fact that $W_U(0, u) > 0 = W_L(\overline{z}(u), u)$, and so $\gamma_+(z) < \overline{z}(u)$. By the definition of $\underline{N}(u)$, $\overline{N}(u) \geq \underline{N}(u)$.

Now suppose $\underline{N}(u) \leq n \leq \overline{N}(u)$. Then

$$\sum_{k=1}^{n} \gamma_+^{k-1}(0, u) \leq \sum_{k=1}^{\overline{N}(u)} \gamma_+^{k-1}(0, u) < 1 < \sum_{k=1}^{\underline{N}(u)} \overline{z}(u) \leq \sum_{k=1}^{n} \gamma_+^{k-1}(\overline{z}(u), u).$$

Since $\sum_{k=1}^{n} \gamma_+^{k-1}(\cdot, u)$ is continuous and strictly increasing, there exists a unique z^n such that $\sum_{k=1}^{n} \gamma_+^{k-1}(z^n, u) = 1$. ■

Figure 8 shows how $\underline{N}(u)$ and $\overline{N}(u)$ vary as u changes. Both are step functions whose jump sizes are always equal to 1, and increase without bound as u tends to Δ. Intuitively, to ensure a high

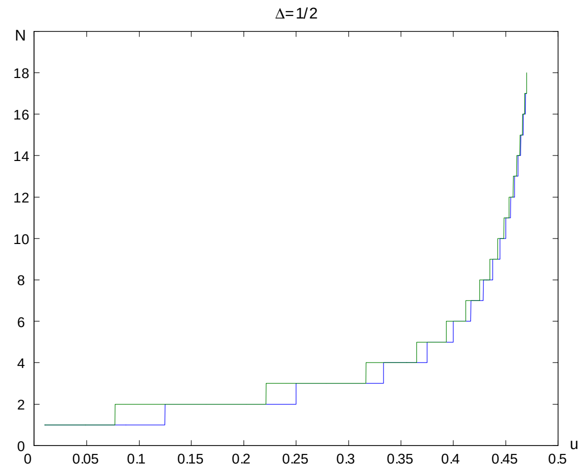

Figure 8: The upper and lower bounds of the possible number of submarkets.

utility to buyers, quality uncertainty should be small enough, and so there should be sufficiently many submarkets.

Lemma 4 *The jump size of* $\underline{N}(u)$ *and* $\overline{N}(u)$ *is always equal to* 1*. For* $u < \Delta$*, they are finite, but as* u *tends to* Δ*, they increase without bound.*

Proof. The jump size of $\underline{N}(u)$ is 1 by the definition of $\underline{N}(u)$. The result for $\overline{N}(u)$ comes from (3) in Lemma 2. For $u < \Delta$, $\gamma_+(0, u) < z_k$ for all $k > 1$, and so $N(u) < 1/\gamma_+(0) + 1$. The last result follows from the construction of $\underline{N}(u)$ and the fact that $\overline{N}(u) \geq \underline{N}(u)$. ■

4.5 General Equilibrium Analysis

The partial equilibrium analysis showed that for each $u \in (\underline{u}_n, \overline{u}_n)$, there exists a unique n-message partial equilibrium, $\{z_1, ..., z_n\}$ and $\{\lambda_1, ..., \lambda_n\}$. Let $\beta_n(u) = \sum_{k=1}^{n} z_k \lambda_k$.

The following proposition shows that if $u \in (\underline{u}_{n+1}, \overline{u}_n)$ then $\beta_{n+1}(u) > \beta_n(u)$.

Proposition 3 *Given* u*, if there exists two partial equilibria with different numbers of submarkets, the total measure of buyers is greater in the equilibrium with more submarkets than in the other equilibrium.*

Proof. See Appendix. ■

Intuitively, there is less quality uncertainty with $n+1$ submarkets than with n submarkets. Therefore for buyers to get the same utility, there should be relatively more buyers with more submarkets. This implies the following fact (See Figure 9).

Corollary 2 *Buyers are better off when there are more submarkets.*

Now let $\overline{\beta}_n$ be the value such that

$$\overline{\beta}_n = \sup_{u \in (\underline{u}_n, \overline{u}_n)} \beta_n(u).$$

It is immediate that $\overline{\beta}_1 = \infty$, because one-message (babbling) equilibrium always exists.

Proposition 4 *(1) For $n > 1$, there exists $\overline{\beta}_n < \infty$ such that an equilibrium with n submarkets exists if and only if $\beta < \overline{\beta}_n$ ($\beta \leq \overline{\beta}_n$ if the supremum is achieved). Therefore for there to exist an informative equilibrium, β should be not too large.*

(2) As n tends to infinity, $\overline{\beta}_n$ converges to 0. Therefore there can exist many submarkets if and only if β is sufficiently small.

Proof. I use the following fact.

$$\beta_n(u) = \sum_{k=1}^{n} z_k \cdot \lambda(z_k, u) \leq \ln\left(\frac{\Delta}{u}\right) \sum_{k=1}^{n} z_k = \ln\left(\frac{\Delta}{u}\right).$$

$\overline{\beta}_n < \infty$ because multiple submarkets exist only when $W_U(0, u) > W_L(1, u)$ and so u is bounded away from 0. $(z_1, ..., z_n)$ is continuous in u, and $\beta_n(u)$ converges to 0 as u tends to $\overline{u}_n$. This establishes the first result.

The second result is because n is large only when u is sufficiently close to Δ (Lemma 4). ■

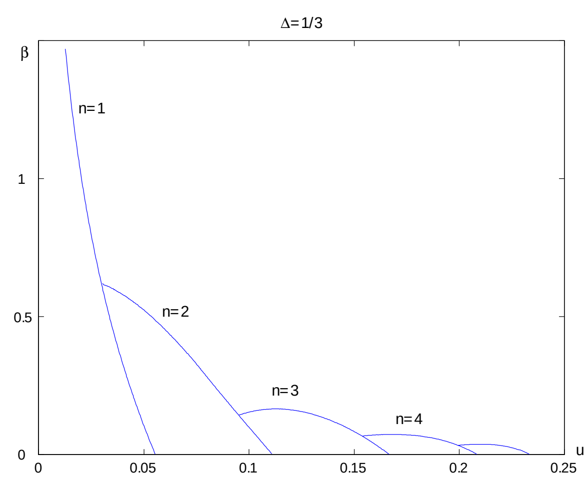

Figure 9: The set of general equilibria.

Figure 9 shows the possible number of submarkets and buyers' equilibrium expected utility for each β. For smaller β, there can exist more submarkets and so more information can be transmitted through cheap-talk messages. The intuition behind this result is as follows. When β is small, sellers have a great incentive to attract more buyers, which is possible only by revealing their qualities. Conversely, when β is large, low-quality sellers have a higher incentive to pretend to have a higher quality.

Remark 3 The numerical example suggests much sharper results regarding the behavior of $\overline{\beta}_n$: $\overline{\beta}_n$ is strictly decreasing in n. This implies that for any β, there exists $N(\beta)$ such that there exists an equilibrium with n submarkets if and only if $n \leq N(\beta)$. This is a consistent finding in the numerical analyses I have performed. Unfortunately, I cannot establish this result analytically. There are two prominent approaches to this problem. One is to consider the difference equation derived from the game, as in the cheap talk literature (See Crawford and Sobel (1982)). The other is to apply the Lagrangian method, and compare $\overline{\beta}_n$'s. The difficulty in the first approach is that different from Crawford and Sobel, my problem should deal with the two-dimensional difference equation. Furthermore, the monotonicity does not necessarily hold in my problem, and so I cannot apply Tarski's fixed point theorem. The difficulty in the second approach is that while it is possible to derive the conditions for $\overline{\beta}_n$ for fixed n, it is quite involved to compare $\overline{\beta}_n$ and $\overline{\beta}_{n+1}$.

5 A Continuum Quality: Varying Surplus Cases

This section supplements the previous section by studying varying surplus cases. I first solve for the submarket outcomes for the general continuum quality case. Then I study a linear example and provide a necessary and sufficient condition of the relationship between v and c for cheap talk to be informative. Last, I analyze another extreme case where buyers' values independent of sellers' costs.

5.1 Environment

Like in the previous section, there are a continuum of qualities distributed uniformly over $[0, 1]$. A unit of $q \in [0, 1]$-quality good costs $c(q)$ to a seller and yields utility $v(q)$ to a buyer. Unlike in the previous section, I do not impose parametric assumptions on c and v, but use the following conditions.

Assumption 1 *c and v are continuously differentiable. c is strictly increasing and v is increasing.*

Assumption 2 *There exists $\Delta > 0$ such that $v(q) - c(q) \geq \Delta$ for all $q \in [0, 1]$.*

Assumption 3 *(Regularity) For any q',*

$$\int_{q'}^{q''} \left(v(q) - c\left(q''\right)\right) dq \text{ is strictly quasi-concave in } q''.$$

Assumption 3 ensures that buyers' symmetric mixed bidding strategy is unique and has a convex support. For $q' \in [0,1]$, let $r(q')$ be the value such that

$$r(q') = \arg\max_{q'' \in [0,1]} \int_{q'}^{q''} \left(v(q) - c(q'')\right) dq.$$

$r(q')$ is the highest quality the monopsonist is willing to trade when the seller's quality is known to be greater than q'. $r(q')$ corresponds to $q' + \Delta$ in the previous section.

5.2 Submarket Analysis

The analysis for the submarket outcomes proceeds as in the constant surplus case. I use the same notations as in the previous section.

The minimum of the support of F is equal to the offer of the monopsonist who is facing a seller whose quality is uniformly distributed over $\left[\underline{q}, \overline{q}\right]$. Therefore $\underline{b} = \min\left\{c(\overline{q}), c\left(r\left(\underline{q}\right)\right)\right\}$. Then

$$\begin{aligned} U\left(\underline{q}, \overline{q}, \lambda\right) &= \pi_0 M\left(\underline{q}, \overline{q}\right) \\ &= \pi_0 \frac{\min\left\{r\left(\underline{q}\right), \overline{q}\right\} - \underline{q}}{\overline{q} - \underline{q}} \left(E_{\underline{q}, \min\left\{r\left(\underline{q}\right), \overline{q}\right\}}\left[v(q)\right] - c\left(\min\left\{r\left(\underline{q}\right), \overline{q}\right\}\right)\right). \end{aligned}$$

To interpret this expression, fix $\underline{q}$ and λ. When $\overline{q}$ is close to $\underline{q}$, quality uncertainty in the submarket is small and so all qualities fully trade $\left(\underline{b} = c(\overline{q}) < c\left(r\left(\underline{q}\right)\right)\right)$. As $\overline{q}$ increases, buyers bid more aggressively to increase the probability of trading, which lowers their payoffs. On the other hand, the average quality of goods improves, which increases buyers' payoff. Whether $U\left(\underline{q}, \overline{q}, \lambda\right)$ is increasing in $\overline{q}$ or not depends on the relative importance of these two effects. In the constant surplus case, the former effect always dominates the latter. In general, $U\left(\underline{q}, \overline{q}, \lambda\right)$ is increasing in $\overline{q}$ if v increases sufficiently faster than c. For example, when $v(q) = \sigma q + \Delta$ and $c(q) = q$, then $U\left(\underline{q}, \overline{q}, \lambda\right)$ is increasing in $\overline{q}$ if and only if $\sigma \geq 2$.

If quality uncertainty is sufficiently large and v does not increase sufficiently faster than c, then $\underline{b} = c\left(r\left(\underline{q}\right)\right) < c(\overline{q})$. In this case, further increase of $\overline{q}$ always lowers buyers' expected payoff. This is because the probability of buyers' meeting sellers whose qualities are lower than $r\left(\underline{q}\right)$ decreases. The fractional term in $U\left(\underline{q}, \overline{q}, \lambda\right)$ shows this effect.

F and sellers' expected payoffs are calculated in the same way as in the previous section. Let $s\left(\underline{q}, \lambda\right)$ be the value such that

$$c\left(s\left(\underline{q}, \lambda\right)\right) = E_{\underline{q}, s\left(\underline{q}, \lambda\right)}\left[v(q)\right] - \pi_0 \frac{r\left(\underline{q}\right) - \underline{q}}{s\left(\underline{q}, \lambda\right) - \underline{q}} \left(E_{\underline{q}, r\left(\underline{q}\right)}\left[v(q)\right] - c\left(r\left(\underline{q}\right)\right)\right).$$

$s\left(\underline{q}, \lambda\right)$ is a generalization of $\underline{q} + \Delta\left(1 + \sqrt{1 - \pi_0}\right)$ in the previous section. To see how $s\left(\underline{q}, \lambda\right)$ varies as λ changes, consider the two extreme cases, $\lambda = 0$ and $\lambda = \infty$. In the former case, $s\left(\underline{q}, \lambda\right) = r\left(\underline{q}\right)$, while in the latter case, $c\left(s\left(\underline{q}, \lambda\right)\right) = E_{\underline{q}, s\left(\underline{q}, \lambda\right)}\left[v(q)\right]$.

(1) $\overline{q} \leq r\left(\underline{q}\right)$.

$$
\begin{aligned}
V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= E_{\underline{q},\overline{q}}\left[v\left(q\right)\right]-c\left(\underline{q}\right)-\left(1+\lambda+\frac{c\left(\overline{q}\right)-c\left(\underline{q}\right)}{M\left(\underline{q},\overline{q}\right)}\right)U\left(\underline{q},\overline{q},\lambda\right), \\
V\left(\overline{q};\underline{q},\overline{q},\lambda\right) &= E_{\underline{q},\overline{q}}\left[v\left(q\right)\right]-c\left(\overline{q}\right)-\left(1+\lambda\right)U\left(\underline{q},\overline{q},\lambda\right).
\end{aligned}
$$

(2) $r\left(\underline{q}\right) < \overline{q} < s\left(\underline{q},\lambda\right)$.

$$
\begin{aligned}
V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= E_{\underline{q},r\left(\underline{q}\right)}\left[v\left(q\right)\right]-c\left(\underline{q}\right) \\
&\quad -U\left(\underline{q},\overline{q},\lambda\right)\left(1+\ln\frac{E_{\underline{q},\overline{q}}\left[v\left(q\right)\right]-c\left(\overline{q}\right)}{U\left(\underline{q},\overline{q},\lambda\right)}+\frac{\left(c\left(r\left(\underline{q}\right)\right)-c\left(\underline{q}\right)\right)}{M\left(\underline{q},\overline{q}\right)}\right) \\
&\quad -U\left(\underline{q},\overline{q},\lambda\right)\int_{c\left(\underline{q}\right)}^{c\left(\overline{q}\right)}\left(\frac{\overline{q}-\underline{q}}{c^{-1}\left(b\right)-\underline{q}}\frac{1}{E_{\underline{q},c^{-1}\left(b\right)}\left[v\left(q\right)\right]-b}\right)db,
\end{aligned}
$$

$$
V\left(\overline{q};\underline{q},\overline{q},\lambda\right)=E_{\underline{q},r\left(\underline{q}\right)}\left[v\left(q\right)\right]-c\left(\overline{q}\right)-U\left(\underline{q},\overline{q},\lambda\right)\left(1+\ln\frac{E_{\underline{q},\overline{q}}\left[v\left(q\right)\right]-c\left(\overline{q}\right)}{U\left(\underline{q},\overline{q},\lambda\right)}\right).
$$

(3) $\overline{q} \geq s\left(\underline{q},\lambda\right)$.

$$
\begin{aligned}
V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= E_{\underline{q},s\left(\underline{q},\lambda\right)}\left[v\left(q\right)\right]-c\left(\underline{q}\right)-U\left(\underline{q},\overline{q},\lambda\right)\frac{\overline{q}-\underline{q}}{s\left(\underline{q},\lambda\right)-\underline{q}} \\
&\quad -U\left(\underline{q},\overline{q},\lambda\right)\int_{c\left(\underline{q}\right)}^{\overline{b}}\left(\frac{\overline{q}-\underline{q}}{c^{-1}\left(b\right)-\underline{q}}\frac{1}{E_{\underline{q},c^{-1}\left(b\right)}\left[v\left(q\right)\right]-b}\right)db, \\
V\left(\overline{q};\underline{q},\overline{q},\lambda\right) &= 0.
\end{aligned}
$$

5.3 Linear Example

As a concrete example that departs from the constant surplus assumption, I consider the case in which $c\left(q\right)=q$ and $v\left(q\right)=\sigma q+\Delta$ where $\Delta>1$ and $1\leq\sigma<2$.[11] In addition, I restrict attention to an equilibrium with two submarkets, low-quality submarket and high-quality submarket. Let λ_L and λ_H be the ratios of buyers to sellers in the low-quality submarket and in the high-quality submarket, respectively. In addition, let $\pi_{L,0}=1/e^{\lambda_L}$ and $\pi_{H,0}=1/e^{\lambda_H}$.

Buyers' expected payoffs in each submarket are

$$
\begin{aligned}
U\left(0,q,\lambda_L\right) &= \pi_{L,0}\left(\sigma\frac{q}{2}+\Delta-q\right), \\
U\left(q,1,\lambda_H\right) &= \pi_{H,0}\left(\sigma\frac{q+1}{2}+\Delta-1\right).
\end{aligned}
$$

[11] $\Delta>1$ ensures full trade in every submarket.

The expected payoffs of the boundary seller in each submarket are

$$V(q;0,q,\lambda_L) = \sigma\frac{q}{2} + \Delta - q - (1+\lambda_L)\,U(0,q,\lambda_L),$$

and

$$V(q;q,1,\lambda_H) = \sigma\frac{q+1}{2} + \Delta - q - (1+\lambda_H)\,U(q,1,\lambda_H) - \pi_{H,0}(1-q).$$

In equilibrium, buyers and the boundary seller are indifferent between the two submarkets. That is, a two-message equilibrium is characterized by (q^*, α^*) such that

$$U\left(0, q^*, \frac{\alpha^*}{q^*}\right) = U\left(q^*, 1, \frac{\beta-\alpha^*}{1-q^*}\right)$$

and

$$V\left(q^*; 0, q^*, \frac{\alpha^*}{q^*}\right) = V\left(q^*; q^*, 1, \frac{\beta-\alpha^*}{1-q^*}\right).$$

Figure 10 shows q^* as a function of σ for different values of β. As σ increases, q^* decreases. The intuition behind this pattern is as follows. As σ increases, for fixed q^*, the high-quality submarket becomes more attractive to buyers than the low-quality submarket. The only way to recover buyers' indifference between the two submarkets is that q^* decreases, so that quality uncertainty in the high-quality submarket increases, while that of the low-quality submarket decreases.

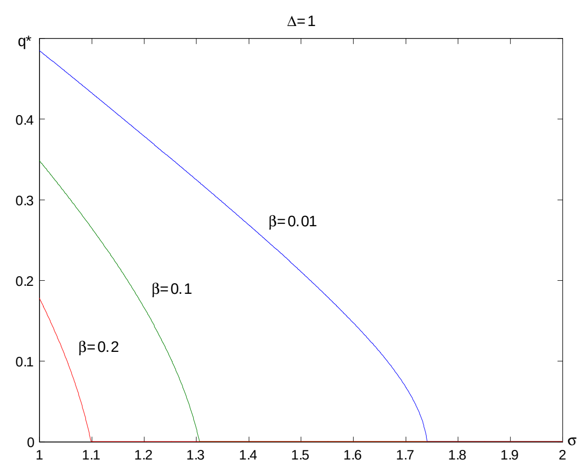

Figure 10: The boundary quality, q^*, between the two submarket as a function of σ.

If σ is sufficiently large, then sellers have a great incentive to join the high-quality submarket, and thus there cannot exist two submarkets (this is when $q^* = 0$). The cutoff value of σ depends on β. This is because, as in the previous section, sellers have less incentive to join the high-quality submarket when β is small.

5.4 No Role for Cheap Talk

The previous example suggests the possibility that there cannot exist an informative equilibrium if v increases sufficiently faster than c with respect to q. The following result shows that it is indeed the case in general.

Theorem 1 *There exists an informative equilibrium for some β, if and only if*

$$M(0,q) > M(q,1)\,,\ \textit{for some } q \in (0,1)\,.$$

In other words, one-message equilibrium is the unique equilibrium independent of β if and only if $M(0,q) \leq M(q,1)$ for all $q \in (0,1)$.

Proof. See Appendix. ■

For an easy interpretation of this result, suppose $r(0) = 1$ so that $r(q) = 1$ for all q. Then the condition where there cannot exist an informative equilibrium is equivalent to

$$E_{0,q}\left[v\left(q'\right)\right] - c(q) \leq E_{q,1}\left[v\left(q'\right)\right] - c(1)\,, \forall q \in (0,1)\,.$$

As q increases, $E_{0,q}[v(q')]\,, c(q)$, and $E_{q,1}[v(q')]$ increase, but the increase of $E_{0,q}[v(q')]$ tends to be slower than that $E_{q,1}[v(q')]$. Since the inequality holds for q close to 1 for sure, in the regular case (for instance, when v and c are linear), the condition shrinks to

$$v(0) - c(0) \leq E_{0,1}\left[v\left(q'\right)\right] - c(1)\,.$$

Roughly, the left-hand side is buyers' expected payoff when quality uncertainty is minimized subject to sellers' incentive compatibilities, while the right-hand side is buyers' expected payoff when the potential trading surplus is maximized. Then the condition states that whether cheap talk can be informative about the quality of goods depends on the relative importance between the amount of quality uncertainty and the amount of trading surplus. Returning back to the linear example where $c(q) = q$ and $v(q) = \sigma q + \Delta$, one can show that the inequality holds if and only if $\sigma \geq 2$.

5.5 The Constant Value Case

This subsection considers the constant value case, that is, $v(q) = v$ for all $q \in [0,1]$. For simplicity, I assume that $r(0) = 1$ so that full trade occurs in every submarket.

I first characterize the partial equilibrium in which buyers get utility $u \in (0, v - c(0))$. In a submarket with $\left[\underline{q}, \overline{q}\right]$, for buyers to get u,

$$u = \frac{1}{e^{\lambda}}\left(v - c\left(\overline{q}\right)\right).$$

Notice that λ is determined by u and $\overline{q}$, not $\underline{q}$. Let $\lambda(q,u)$ be the tightness that is required to ensure buyers utility u in a submarket whose highest quality is $\overline{q}$. That is, $\lambda(q,u) = \ln((v - c(q))/u)$. For fixed u, $\lambda(q,u)$ is well-defined only when $c(q) \leq v - u$. To simplify the notation, I assume that for fixed u, if $c(q) > v - u$, then such q-quality seller fully reveals her quality. This is without loss of generality, because such qualities do not trade in equilibrium.

Buyers' bidding strategy F in each submarket is given by

$$F(b) = \frac{1}{\lambda(\overline{q},u)} \ln \frac{v - c(\overline{q})}{v - b}, b \in \left[c(\overline{q}), \overline{b}\right].$$

From F, I can calculate the expected payoff of sellers in a submarket.

Lemma 5 *Given u, when $c(q) \leq v - u$, the expected payoff of q-quality seller in a submarket whose highest quality is q' is if $q \leq q'$ then*

$$W\left(q; q', u\right) = v - c(q) - (1 + \lambda(q,u))u,$$

and if $q > q'$ then

$$W\left(q; q', u\right) = v - c(q) - \left(1 + \lambda\left(q', u\right)\right)u - \pi_0\left(q', u\right)\left(c\left(q'\right) - c(q)\right),$$

where $\pi_0(q,u) = 1/e^{\lambda(q,u)}$.

Sellers are indifferent over all submarkets whose highest qualities are lower than their own qualities. Among submarkets whose highest qualities are higher than their own qualities, sellers prefer the submarket whose highest quality is lowest. To see the latter, observe that

$$V\left(q; q', u\right) - V(q; q, u) = -\left(\frac{c\left(q'\right) - c(q)}{v - c\left(q'\right)} - \ln\left(1 + \frac{c\left(q'\right) - c(q)}{v - c\left(q'\right)}\right)\right)u.$$

Since $x - \ln(1 + x)$ is increasing for $x > 0$, $V\left(q; q', u\right)$ is strictly decreasing in q'. This finding leads to the following result.

Proposition 5 *Every partial equilibrium is characterized by a cutoff quality $q^* \in [0,1]$ such that all qualities above q^* are fully revealed and all qualities below q^* form one submarket. If $q^* = 0$ then all qualities are fully revealed, whereas if $q^* = 1$ then no information is transmitted in equilibrium.*

Proof. By Lemma 5 and the subsequent discussion, it is an equilibrium that all qualities above q^* are fully revealed and all qualities below q^* form one submarket. To show that this is the only equilibrium structure, suppose there exists $[q_1, q_2]$ such that $0 < q_1 < q_2$, qualities in $[q_1, q_2]$ form a submarket (not fully revealed), and trade occurs in the submarket. The expected payoff of q_1-quality seller is then $V(q_1; q_2, u)$. This payoff is smaller than $V(q_1; q', u)$ for all $q' \leq q_1$, and so sellers whose qualities are close to q_1 deviate. Hence there cannot exist such an interval. ∎

To find the set of general equilibria, fix u and q^*. Let

$$\begin{aligned}\beta(u,q^*) &= q^*\lambda(q^*,u)+\int_{q^*}^{1} q\lambda(q,u)\,dq \\ &= q^*\ln\left(\frac{v-c(q^*)}{u}\right)+\int_{q^*}^{1}\max\left\{\ln\left(\frac{v-c(q)}{u}\right),0\right\}dq \\ &= \int_0^1 \max\left\{\ln\frac{v-\max\{c(q^*),c(q)\}}{u},0\right\}dq.\end{aligned}$$

$\beta(u,q^*)$ is decreasing in both u and q^*. The intuition behind this result is the same as in the constant surplus case.

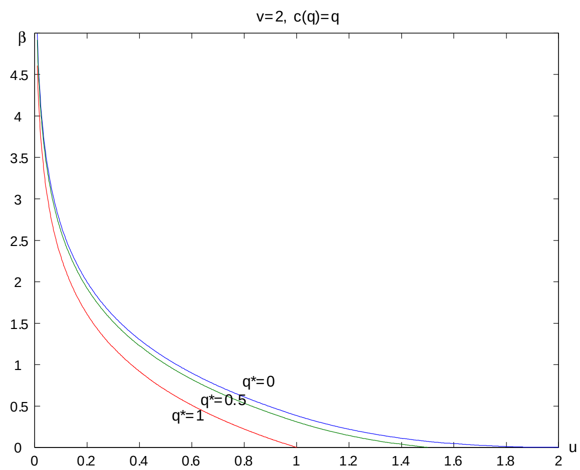

Figure 11: The set of general equilibria in the constant values case.

Proposition 6 *For fixed $\beta > 0$, there exists a continuum of equilibria. Each equilibrium differs in the cutoff value $q^* \in [0,1]$ such that all qualities above q^* are fully revealed and all qualities below q^* form one submarket. Buyers' expected payoff, u^*, is determined so that*

$$\beta = \int_0^1 \max\left\{\ln\frac{v-\max\{c(q^*),c(q)\}}{u},0\right\}dq.$$

Comparison to Menzio (2007)

Menzio studied the constant value case in the labor market context. In his setup, with the urn ball matching technology, a two-message equilibrium exists if and only if β is neither too large nor too small, and there cannot exist more than two submarkets.[12]

[12] He considered more general class of matching technologies. He showed that if the inverse of job-finding probability is concave, then there can exist more than two submarkets.

The crucial difference from mine is allocation mechanism. In his model, a seller (firm) selects one buyer (worker) if she is matched with more than one buyer, and they engage in an alternating offer bargaining game. The consequence of this is that ex post payoffs of sellers are independent of λ, the ratio of buyers to sellers in a submarket. λ affects only the probability of sellers being matched. In my model, λ affects not only the probability of trading, but also buyers' bidding behavior, and thereby ex post payoffs of sellers. This translates into low-quality sellers' having a greater incentive to reveal their qualities than in Menzio's, which ultimately yield the different equilibrium behavior.

It is not feasible to compare my setup to Menzio's beyond the constant value case. The difficulty is that it is not known how one can generalize an alternating offer bargaining for the interdependent value case. The best known result in bargaining with interdependent values is only for the case where the uninformed player makes all the offers (Deneckere and Liang (2006)). But that case yields a trivial result - cheap talk can never be informative- whether values are constant or interdependent with costs.

6 Conclusion

I developed a decentralized market structure where cheap talk can play a non-trivial role. Cheap-talk messages can serve as an instrument that creates endogenous market segmentation. Whether cheap talk can be informative depends on whether the incentives of market agents can be well-aligned in a way that each side provides an incentive for the other side. In my model where sellers have private information concerning quality uncertainty, sellers provide an incentive for buyers by partially revealing their private information, while buyers provide an incentive for sellers by controlling their search intensity.

Appendix A: Partially Indifferent Equilibrium

Appendix A studies the equilibrium in which buyers strictly prefer some submarkets to others in the constant surplus case.

I first consider the case with two submarkets. Suppose z_1 and z_2 are the measures of sellers in each submarket. Since buyers strictly prefer the first submarket to the second submarket, $z_2 > \overline{z}(u)$. Then u should be less than $\Delta/2$, and so $\overline{z}(u) = \Delta^2/(2u)$. Otherwise, by the incentive compatibility, $z_1 \geq \overline{z}(u)$, and so the market is empty of buyers. In addition, $\gamma_-(\overline{z}(u), u) \leq z_1 < \overline{z}(u)$. The first inequality guarantees that the lower boundary seller in the second submarket is indifferent between the two submarkets. The second inequality is for a positive measure of buyers. Since $z_1 + z_2 = 1$, this type of equilibrium exists if and only if $\gamma_-(\overline{z}(u), u) + \overline{z}(u) < 1$. There are a continuum of such equilibria, because $(z_1, 1 - z_1)$ is an equilibrium for any $z_1 \in [\gamma_-(\overline{z}(u), u), 1 - \overline{z}(u)]$.

In general, partially different equilibrium exists if and only if $u < \Delta/2$ and $2(\Delta - u) + \Delta^2/(2u) < 1$. For $u \geq \Delta/2$, such equilibrium does not exist because if $z_n > \overline{z}(u)$ then by the incentive compatibility of the boundary sellers, $z_k \geq \overline{z}(u)$, and so all submarkets are empty of buyers. When $\overline{z}(u) + \gamma_-(\overline{z}(u), u) \geq 1$, it is because buyers should be indifferent over all submarkets. For $u < \Delta/2$, if $\overline{z}(u) + \gamma_-(\overline{z}(u), u) < 1$, as shown for the two-message equilibrium, there exists a partially different equilibrium. Subsequently, I assume that $u < \Delta/2$ and $2(\Delta - u) + \Delta^2/(2u) < 1$.

I make the following assumption.

Assumption 4 *Whenever* $\{z_1, ..., z_n\}$ *is a partial equilibrium,* $z_{k+1} \geq z_k, k = 1, ..., n-1$.

This assumption requires the monotone arrangement even for inactive submarkets. This is without loss of generality because whenever there exists an equilibrium that does not satisfy this assumption, there exists another equilibrium that has the same number of submarkets, yields the same outcomes, and satisfies this assumption.

Define $\gamma_- (\cdot, u) : \left[0, \Delta^2/ (2u)\right] \rightarrow [0, 2(\Delta - u)]$ by $W_U\left(\gamma_- (z, u), u\right) = W_L (z, u)$. This is the inverse of $\gamma_+ (\cdot, u)$. Let $\overline{n} (u)$ be the largest integer such that $\gamma_-^{\overline{n}(u)-1} (\overline{z} (u), u)$ is well-defined and $\sum_{k=0}^{\overline{n}(u)-1} \gamma_-^k (\overline{z} (u), u) < 1$.

Proposition 7 *Suppose* $u < \Delta/2$ *and* $\overline{z} (u) + \gamma_- (\overline{z} (u), u) < 1$. *There exists a partially indifferent equilibrium with* n *submarkets in which buyers strictly prefer the other submarkets to the last submarket if and only if* $2 \leq n \leq \overline{n} (u)$. *If exists, there are a continuum of such equilibria.*

Proof. Fix n, and consider z such that $\gamma_- (\overline{z} (u), u) = 2(\Delta - u) \leq z \leq \overline{z} (u)$, and $\sum_{k=1}^{n-1} \gamma_-^{k-1} (z, u) + \overline{z} (u) < 1$. There exists such n if and only if $2 \leq n \leq \overline{n} (u)$, and if exists, the set of such z is a subset of $[2(\Delta - u), \overline{z} (u)]$. Now consider a sequence $\left\{\gamma_-^{n-2} (z, u), \gamma_-^{n-3} (z, u), ..., \gamma_-^1 (z, u), z, 1 - \sum_{k=1}^{n-1} \gamma_-^{k-1} (z, u)\right\}$. By construction, this is an equilibrium. ■

If the measure of sellers in the last submarket is greater than $2\overline{z} (u)$, then there exists an equilibrium with more than $\overline{n} (u)$ submarkets. The following result shows that even considering all the possibilities, the number of submarkets cannot be greater than $\overline{N} (u)$. One implication of this result is that the results on the general equilibrium in Section 4 do not change by the existence of partially indifferent equilibria.

Proposition 8 *There does not exist an equilibrium with more than* $\overline{N} (u)$ *submarkets, whether buyers are indifferent over all submarkets or not.*

Proof. Suppose there exists an equilibrium with n submarkets in which buyers are not indifferent over all submarkets. Because of Assumption 4, there exists $m < n$ such that $z_k > \overline{z} (u)$ if and only if $k > m$. By the incentive compatibility condition of the lower boundary sellers, $z_m \geq \gamma_- (\overline{z} (u), u)$, and $z_k = \gamma_-^{m-k} (z_m, u), k = 1, ..., m-1$. Since $0 < z_1$, $\gamma_+^{k-1} (0, u) < \gamma_+^{k-1} (z_1, u) = z_k$ and

$$\sum_{k=1}^{m} \gamma_+^{k-1} (0, u) < \sum_{k=1}^{m} z_k < 1 - (n - m) \overline{z} (u).$$

Now notice that

$$\sum_{k=1}^{n} \gamma_+^{k-1} (0, u) < \sum_{k=1}^{m} \gamma_+^{k-1} (0, u) + (n - m) \overline{z} (u) < 1.$$

By the definition of $\overline{N} (u)$, $n \leq \overline{N} (u)$.

■

Appendix B: Omitted Proofs

Proof of Lemma 1: I prove the result for the general continuum quality case.

Let F_1 and F_2 be buyers' bidding strategies in each submarket, and let $\left[\underline{b}_1, \bar{b}_1\right]$ and $\left[\underline{b}_2, \bar{b}_2\right]$ be the supports of F_1 and F_2, respectively. The result is obvious if $\bar{b}_1 \leq c(q_2)$. From now on, suppose $\bar{b}_1 > c(q_2)$.

Observe that $\bar{b}_1 \leq \bar{b}_2$. This is because $U(q_1, q_2, \lambda) = U\left(q_2, q_3, \lambda'\right)$, and so

$$\bar{b}_1 = E_{q_1,q_2}\left[v(q)\right] - U(q_1, q_2, \lambda) \leq v(q_2) - U\left(q_2, q_3, \lambda'\right) \leq \bar{b}_2.$$

($\Rightarrow$) It is straightforward that q_2-quality seller is indifferent between the two submarkets. I first show that if $\lambda = \lambda'$ then F' first-order stochastically dominates F, which implies that sellers strictly prefer the submarket with $[q_2, q_3]$. To show this, fix $b \in \left[\underline{b}_2, \bar{b}_1\right]$. If $b \geq c(q_3)$ then

$$e^{\lambda F(b)} = \frac{M(q_1, q_2)}{E_{q_1,q_2}\left[v(q)\right] - b}, \text{ and } e^{\lambda' F'(b)} = \frac{M(q_2, q_3)}{E_{q_2,q_3}\left[v(q)\right] - b}.$$

Since $E_{q_1,q_2}\left[v(q)\right] \leq E_{q_2,q_3}\left[v(q)\right]$, and $M(q_1, q_2) = M(q_2, q_3)$ (because $U(q_1, q_2, \lambda) = U\left(q_2, q_3, \lambda'\right)$ and $\lambda = \lambda'$), $F(b) \geq F'(b)$. If $b < c(q_3)$ then

$$e^{\lambda F(b)} = \frac{M(q_1, q_2)}{E_{q_1,q_2}\left[v(q)\right] - b}, \text{ and } e^{\lambda' F''(b)} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{M(q_2, q_3)}{E_{q_2,c^{-1}(b)}\left[v(q)\right] - b}.$$

$F(b) > F'(b)$ follows if

$$\frac{c^{-1}(b) - q_2}{q_3 - q_2}\left(E_{q_2,c^{-1}(b)}\left[v(q)\right] - b\right) > E_{q_1,q_2}\left[v(q)\right] - b.$$

If $\underline{b}_2 \geq c(q_3)$ then the condition is vacuously satisfied. Suppose $\underline{b}_2 = c(r(q_2)) < c(q_3)$. When $b = \underline{b}_2$,

$$\frac{c^{-1}(b) - q_2}{q_3 - q_2}\left(E_{q_2,c^{-1}(b)}\left[v(q)\right] - b\right) = M(q_2, q_3) = M(q_1, q_2) > E_{q_1,q_2}\left[v(q)\right] - c(r(q_2)).$$

Observe that as b increases from $\underline{b}_2 = c(r(q_2))$ to $c(q_3)$, the left-hand side decreases more slowly than the right-hand side. Therefore the inequality holds for any $b \in [\underline{b}_2, c(q_3))$.

Now suppose $\lambda < \lambda'$. Since q_2-type can mimic q_1-type,

$$V(q_2; q_1, q_2, \lambda) \geq V(q_1; q_1, q_2, \lambda) - (1 - \pi_0)(c(q_2) - c(q_1)).$$

Since $c(q_1) < c(q_2) < \underline{b}_2$,

$$V(q_2; q_2, q_3, \lambda) = V(q_1; q_2, q_3, \lambda) - (c(q_2) - c(q_1))\left(1 - \pi_0'\right).$$

Then

$$V(q_1; q_2, q_3, \lambda) - V(q_1; q_1, q_2, \lambda) \geq (c(q_2) - c(q_1))\left(\pi_0 - \pi_0'\right) > 0.$$

This contradicts the supposition that no seller deviates.

($\Leftarrow$) I first show that $\pi_0' > \pi_0 e^{\lambda F(c(q_2))}$, that is, the probability of trading of q_2-quality seller in the $[q_1, q_2]$ submarket, $1 - \pi_0 e^{\lambda F(c(q_2))}$, is greater than that in the $[q_2, q_3]$ submarket, $1 - \pi_0'$. Suppose not. Then

$$\pi_0' e^{\lambda' F'(b)} = \pi_0' \leq \pi_0 e^{\lambda F(b)} \text{ for } b \leq \underline{b}_2,$$

and

$$\begin{aligned} \pi_0' e^{\lambda' F'(b)} &= \frac{\pi_0' M(q_2, q_3)}{E_{q_2,q_3}[v(q)] - b} = \frac{U(q_2, q_3, \lambda')}{E_{q_2,q_3}[v(q)] - b} \\ &\leq \pi_0 e^{\lambda F(b)} = \frac{U(q_1, q_2, \lambda')}{E_{q_1,q_2}[v(q)] - b}, \forall b \in \left[c(q_3), \bar{b}_1\right]. \end{aligned}$$

In addition, for $b \in (\underline{b}_2, c(q_3))$,

$$\pi_0' e^{\lambda' F'(b)} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{\pi_0' M(q_2, q_3)}{E_{q_2,c^{-1}(b)}[v(q)] - b} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{U(q_2, q_3, \lambda')}{E_{q_2,c^{-1}(b)}[v(q)] - b},$$

and

$$\pi_0 e^{\lambda F(b)} = \frac{U(q_1, q_2, \lambda)}{E_{q_1,q_2}[v(q)] - b}.$$

$\pi_0' e^{\lambda' F'(b)} < \pi_0 e^{\lambda F(b)}$ follows from the fact that $\pi_0' e^{\lambda' F'(\underline{b}_2)} = \pi_0' \leq \pi_0 e^{\lambda F(c(q_2))}$, and as b increases, $\pi_0' e^{\lambda' F'(b)}$ increases more slowly than $\pi_0 e^{\lambda F(b)}$. Therefore $\pi_0' e^{\lambda' F(b)}$ first-order stochastically dominates $\pi_0 e^{\lambda F(b)}$. This implies that

$$\begin{aligned} V(q_2; q_2, q_3, \lambda) &= \int_{\underline{b}'}^{\bar{b}'} \max\{b - c(q_2), 0\} \, d\left(\pi_0' e^{\lambda' F'(b)}\right) \\ &> \int_{\underline{b}}^{\bar{b}} \max\{b - c(q_2), 0\} \, d\left(\pi_0 e^{\lambda F(b)}\right) = V(q_2; q_1, q_2, \lambda), \end{aligned}$$

which is a contradiction.

(i) $q < q_2$

Since q-type seller can mimic q_2-type seller,

$$V(q; q_1, q_2, \lambda) \geq V(q_2; q_1, q_2, \lambda) + (c(q_2) - c(q))\left(1 - \pi_0 e^{\lambda F(c(q_2))}\right).$$

In addition, since $\underline{b}_2 > c(q_2)$, both q-type and q_2-type seller trade in the $[q_2, q_3]$ submarket whenever there is a matched buyer. Hence,

$$V(q; q_2, q_3, \lambda) = V(q_2; q_2, q_3, \lambda) + (c(q_2) - c(q))\left(1 - \pi_0'\right).$$

Then

$$V(q; q_1, q_2, \lambda) - V(q; q_2, q_3, \lambda) \geq (c(q_2) - c(q))\left(\pi_0' - \pi_0 e^{\lambda F(q_2)}\right) > 0.$$

(ii) $q > q_2$

The result is obvious if $q \geq \bar{b}_1$. Consider the case where $q < \bar{b}_1$.

(ii-1) $\pi_0' > \pi_0 e^{\lambda F(c(q))}$

Since q-type seller can mimic q_2-type seller,

$$V(q; q_2, q_3, \lambda) \geq V(q_2; q_2, q_3, \lambda) - (c(q) - c(q_2))\left(1 - \pi_0'\right).$$

In addition,

$$\begin{aligned}
V(q;q_1,q_2,\lambda) &= \sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(b-c(q))\,dF^k(b)\\
&= \sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(b-c(q_2))\,dF^k(b)-\sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(c(q)-c(q_2))\,dF^k(b)\\
&= \sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{\bar b_1}(b-c(q_2))\,dF^k(b)-\sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{c(q)}(b-c(q_2))\,dF^k(b)\\
&\quad -\sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(c(q)-c(q_2))\,dF^k(b)\\
&= V(q_2;q_1,q_2,\lambda)-\sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{\bar b}\min\{b-c(q_2),c(q)-c(q_2)\}\,dF^k(b)\\
&\le V(q_2;q_1,q_2,\lambda)-(c(q)-c(q_2))\left(1-\pi_0e^{\lambda F(c(q))}\right).
\end{aligned}$$

Hence

$$V(q;q_2,q_3,\lambda)-V(q;q_1,q_2,\lambda)\ge(c(q)-c(q_2))\left(\pi_0'-\pi_0e^{\lambda F(c(q))}\right)>0.$$

(ii-2) $\pi_0'\le\pi_0e^{\lambda F(c(q))}$

In this case, similarly to the argument for $\pi_0'>\pi_0e^{\lambda F(c(q_2))}$, one can show that $\pi_0'e^{\lambda' F(b)}$ first-order stochastically dominates $\pi_0e^{\lambda F(b)}$ for $b\ge c(q)$. Therefore $V(q;q_2,q_3,\lambda)>V(q;q_1,q_2,\lambda)$. **Q.E.D.**

Proof of Lemma 2: (1), (2), and the continuity of $\gamma_+^k(z,\cdot)$ come from the properties of $W_L(\cdot,u)$ and $W_U(\cdot,u)$.

To show that $\gamma_+^k(z,\cdot)$ is strictly decreasing, first consider $k=1$.

$$\frac{\partial\gamma_+(z,u)}{\partial u}=-\frac{\partial W_U(z,u)/\partial u-\partial W_L\left(\gamma_+(z,u),u\right)/\partial u}{-\partial W_L\left(\gamma_+(z,u),u\right)/\partial z}.$$

The denominator is negative because $W_L(\cdot,u)$ is strictly decreasing at $\gamma_+(0,u)$, and so it is strictly decreasing at any $z>\gamma_+(0,u)$. By calculation, I also find that

$$\frac{\partial W_U(z,u)}{\partial u}-\frac{\partial W_L\left(\gamma_+(z,u),u\right)}{\partial u}>0,\ z\in(0,\overline{z}(u)).$$

Therefore $\gamma_+^k(z,\cdot)$ is strictly decreasing if $k=1$.

Now suppose $\gamma_+^m(z,\cdot)$ is strictly decreasing in u for $m=1,...,k-1$. Then

$$\begin{aligned}
\frac{\partial\gamma_+^k(z,u)}{\partial u} &= -\frac{\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial u-\partial W_L\left(\gamma_+^k(z,u),u\right)/\partial u}{-\partial W_L\left(\gamma_+(z,u),u\right)/\partial z}\\
&\quad -\frac{\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial z\cdot\partial\gamma_+^{k-1}(z,u)/\partial u}{-\partial W_L\left(\gamma_+(z,u),u\right)/\partial z}.
\end{aligned}$$

The first line is negative by the same argument as before. The second one is also negative because $\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial z, \partial\gamma_+^{k-1}(z,u)/\partial u, \partial W_L(\gamma_+(z,u),u)/\partial z < 0$. **Q.E.D.**

Proof of Proposition 3: Given u, suppose $\{z_1,...,z_m\}$ and $\{z_1',...,z_{m-1}'\}$ are partial equilibria. Let $f(z,u) = z\cdot\lambda(z,u)$. Then $f(\cdot,u)$ is strictly concave.

First notice that $z_2 < z_1'$, and $z_{k+1} \le z_k', k = 2,...,m-1$, because $z_1 > 0$, and $z_1 + ... + z_m = z_1' + ... + z_{m-1}' = 1$. From the strict concavity of f, $f(z_k') \le f(z_{k+1}) + f'(z_{k+1})(z_k' - z_{k+1}), k = 1,...,m-1$, with strict inequality for $k = 1$. Then

$$\begin{aligned}\sum_{k=1}^{m-1} f(z_k') &< \sum_{k=1}^{m-1}\left[f(z_{k+1}) + f'(z_{k+1})(z_k' - z_{k+1})\right] \\ &= \sum_{k=2}^{m} f(z_k) + \sum_{k=1}^{m-1} f'(z_{k+1})(z_k' - z_{k+1}) \\ &< \sum_{k=2}^{m} f(z_k) + \sum_{k=1}^{m-1} f'(z_1)(z_k' - z_{k+1}) = \sum_{k=2}^{m} f(z_k) + z_1 f'(z_1) < \sum_{k=1}^{m} f(z_k).\end{aligned}$$

The last inequality comes from the fact that $f(0) = 0$ and f is strictly concave. This establishes the result because the total measure of buyers is given by

$$\beta_n = z_1\lambda(z_1,u) + ... + z_n\lambda(z_n,u) = \sum_{k=1}^{n} f(z_k,u).$$

Q.E.D.

Proof of Theorem 1: ($\Rightarrow$) Suppose $M(0,q) \le M(q,1)$ for all $q \in (0,1)$. I first show that there cannot exist an equilibrium with two submarkets. Suppose a two-message equilibrium exists and let q^* be the boundary seller between the two submarket. Then for some $\lambda, \lambda' \ge 0$, $U(0,q^*,\lambda) \ge U(q^*,1,\lambda')$.[13] Since $M(0,q^*) \le M(q^*,1)$ and $U(0,q^*,\lambda) = M(0,q^*)/e^{\lambda} \ge U(q^*,1,\lambda')/e^{\lambda'}$, for $U(0,q^*,\lambda) \ge U(q^*,1,\lambda')$, $\lambda \le \lambda'$. That is, there should be less buyer competition in the second submarket. In this case, sellers in the first submarket prefer the second submarket (see the sufficiency proof of Lemma 1).

Now consider an equilibrium with $n\,(>2)$ submarkets, $\{q_1,...,q_n\}$ and $\{\lambda_1,...,\lambda_n\}$. By the same argument as before, $M(q_{k-1},q_k) > M(q_k,q_{k+1})$ for all k such that $\lambda_k > 0$. It is enough to show that if $M(0,q_1) \le M(q_1,1)$ then there exists at least one $k > 1$ such that $M(0,q_1) \le M(q_k,q_{k+1})$. I get the result by applying the following claim inductively.

Claim: For $q_1 < q_2 < q_3$, $M(q_1,q_3)$ cannot be strictly greater than both $M(q_1,q_2)$ and $M(q_2,q_3)$.

Proof: There are three cases. (1) $r(q_1) \le q_2$.

$$\begin{aligned}M(q_1,q_3) &= \frac{r(q_1) - q_1}{q_3 - q_1}\left(E_{q_1,r(q_1)}\left[v(q')\right] - c(r(q_1))\right) \\ &\le \frac{r(q_1) - q_1}{q_2 - q_1}\left(E_{q_1,r(q_1)}\left[v(q')\right] - c(r(q_1))\right) = M(q_1,q_2).\end{aligned}$$

[13] The weak inequality is used because there may be no trade in the submarket with $[q^*,1]$.

(2) $r(q_1) \geq q_3$.

$$M(q_1, q_3) = E_{q_1,q_3}\left[v\left(q'\right)\right] - c(q_3) \leq E_{q_2,q_3}\left[v\left(q'\right)\right] - c(q_3) = M(q_2, q_3).$$

(3) $r(q_1) \in (q_2, q_3)$

$$\begin{aligned}
& (q_2 - q_1) M(q_1, q_2) + (q_3 - q_2) M(q_2, q_3) \\
= & \int_{q_1}^{q_2} (v(q) - c(q_2))\, dq + \max_{q' \in [q_2, q_3]} \int_{q_2}^{q'} \left(v(q) - c\left(q'\right)\right) dq \\
\geq & \int_{q_1}^{q_2} (v(q) - c(q_2))\, dq + \int_{q_2}^{r(q_1)} \left(v(q) - c\left(q'\right)\right) dq \\
= & \int_{q_1}^{r(q_1)} (v(q) - c(q_2))\, dq = (q_3 - q_1) M(q_1, q_3).
\end{aligned}$$

Since $M(q_1, q_3)$ is less than or equal to a weighted average of $M(q_1, q_2)$ and $M(q_2, q_3)$, it cannot be the case that $M(q_1, q_3) \geq M(q_1, q_2), M(q_2, q_3)$. **Q.E.D.**

($\Leftarrow$) Suppose for some $q^* \in (0,1)$, $M(0, q^*) > M(q^*, 1)$. Let $\underline{\lambda}$ be the value such that

$$U(0, q^*, \underline{\lambda}) = \frac{1}{e^{\underline{\lambda}}} M(0, q^*) = M(q^*, 1).$$

By the strict inequality, $\underline{\lambda} > 0$ and then $V(q^*; 0, q^*, \underline{\lambda}) \geq V(q^*; q^*, 1, 0) = 0$. Now given $\lambda \geq \underline{\lambda}$, let $\lambda' (< \lambda)$ be the value such that $U(0, q^*, \lambda) = U\left(q^*, 1, \lambda'\right)$. If λ is sufficiently large, then λ' is also sufficiently large and for such λ and λ',

$$V(q^*; 0, q^*, \lambda) < v(q^*) - c(q^*) \leq E_{q^*, \min\{r(q^*), 1\}}\left[v\left(q'\right)\right] - c(q^*) \approx V\left(q^*; q^*, 1, \lambda'\right).$$

Since $V(q^*; 0, q^*, \cdot)$ and $V(q^*; q^*, 1, \cdot)$ are continuous, there exists $\lambda^* > 0$ such that $V(q^*; 0, q^*, \lambda^*) = V\left(q^*; q^*, 1, \lambda'(\lambda^*)\right)$. This establishes a two-message equilibrium for $\beta = q^* \lambda^* + (1 - q^*) \lambda'(\lambda^*) > 0$. **Q.E.D.**

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