

Dynamic Pricing in the Presence of Social Learning*

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Abstract

This paper considers a monopolist selling a new experience good over time to many buyers. Buyers learn from their own private experiences (*individual learning*) as well as by observing other buyers' experiences (*social learning*). Individual learning generates *ex post heterogeneity*, which plays an important role under some specification of learning. When learning is through good news signals, there is a tradeoff between exploitation and exploration for the monopolist and experimentation is terminated too early. Nonetheless, when learning is through bad news signals, efficient experimentation level is achieved since only the homogeneous unsure buyers purchase the experience good. We also describe how the interplay of individual and social learning affects the equilibrium price. In particular, this causes the instantaneous price reaction to the arrival of good news signals to be ambiguous.

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Contents

1	Introduction	3
2	Model Setting	7
2.1	Belief Updating	8
2.2	Strategies and Payoffs	10
2.3	Symmetric Markov Perfect Equilibrium	12
3	Equilibrium in the Good News Case	13
3.1	Socially Efficient Allocation	13
3.2	Characterizing Equilibrium for $n = 2$	16
3.2.1	Niche Market vs. Mass Market	18
3.2.2	Equilibrium in the Individual Learning Phase	18
3.2.3	Equilibrium in the Aggregate Learning Phase	19
3.2.4	Equilibrium Price Path	25
3.3	Efficiency	27
4	Equilibrium in the Bad News Case	28
4.1	Socially Efficient Allocation	29
4.2	Equilibrium	30
5	Conclusion	32
	Appendix	34

1 Introduction

Consider a monopolist selling a new experience good over time to many buyers. Buyers learn from their own private experiences (*individual learning*) as well as by observing other buyers' experiences (*social learning*). Take the market for new drugs as a motivating example. Some drugs enter the market with substantial uncertainty about their product characteristics.¹ Patients' valuations are interdependent under such circumstances: the effectiveness of a new drug depends not only on each patient's individual attribute (*idiosyncratic uncertainty*) but also on the unknown product characteristic (*aggregate uncertainty*). Patients can learn from other patients about the aggregate uncertainty but not the idiosyncratic uncertainty. The success of a new drug for one patient is a good news signal of the product characteristic but it does not necessarily mean that the drug would be successful for other patients as well.

Suppose the monopolist observes each patient's past actions and outcomes, how will she price strategically in such an environment? Without success of the new drug, everyone becomes increasingly pessimistic. In order to keep patients buying the new drug, the price has to be reduced. How will the monopolist react when a success arrives? Will strategic pricing achieve an efficient allocation? The interdependent value setting leads to *ex post heterogeneity* even though the buyers are *ex ante homogeneous*. This *ex post heterogeneity* has significant implications for both the price path and efficiency when the monopolist is not allowed to price discriminate among the buyers.

We model dynamic monopoly pricing as an infinite-horizon, continuous-time process. There is a finite number of *ex ante* identical buyers. At each instant of time, the monopolist offers a price for the experience good. Each buyer then decides to buy either one unit of the experience good or one unit of another good of known characteristic. The experience good may generate publicly observable lump-sum payoffs (e.g. curing the disease) depending on the product characteristic and the individual attribute. Both the product characteristic and the individual attribute are binary. For tractability, we assume the arrival of lump-sum payoffs immediately resolves both aggregate uncertainty and the idiosyncratic uncertainty of the receiver. As a result, there is a simple dichotomy of the learning process: in the *aggregate learning phase*, the uncertainty about the product characteristic has not been resolved while in the *individual learning phase*, it is common knowledge that the product characteristic is high and *ex post heterogeneity* exists (some buyers are sure about their individual attributes while others still unsure).

If there is a single buyer in the market, the optimal price is set such that the buyer is indifferent between purchasing the experience good and taking the outside option (i.e., purchasing the good of

¹Although the F.D.A. conducts an extensive period of pre-launch testing in the pharmaceutical industry, there are a few exceptions. For example, dietary supplements do not need to be pre-approved by the F.D.A. before entering the market. There is also a "hurry-up mechanism" which allows approval of a drug that has not yet been proved effective in thorough clinical trials but has shown promise that it might benefit patients with life-threatening diseases. The cancer drug Avastin is a recent example (New York Times (2010)).

known characteristic). The buyer's continuation value is independent of her posterior beliefs. Hence, the buyer makes myopic decisions since learning is not valuable.² The introduction of another buyer generates a failure of this property. In the absence of price discrimination, if the monopolist wishes to sell to two different buyers, the optimal price is set to make the more pessimistic buyer indifferent between the alternatives. Buyers will not behave myopically since there is a future benefit from free riding. By taking the outside option for a small amount of time, a buyer can extract rents if the experimenter does not receive any lump-sum payoffs during that period. Then the free rider becomes more optimistic about the experience good and pays less than what she is willing to pay.

A symmetric Markov perfect equilibrium is solved when there are two buyers. In both the aggregate learning phase and the individual learning phase, the equilibrium purchasing behavior is characterized by a cutoff in the unsure buyer's posterior belief. Each unsure buyer purchases the experience good above the cutoff and takes the outside option below the cutoff. By comparing cutoffs in different learning phases, we can distinguish a mass market from a niche market. In a mass market, the monopolist always continues to sell to both buyers after the arrival of lump-sum payoffs; while in a niche market, if the first lump-sum payoff arrives too late, the price is set such that only the sure buyer purchases the experience good. When experimentation (i.e., the unsure buyer purchases the experience good) occurs in the individual learning phase, the optimal price is determined the same as the one-buyer case. Although the unsure buyer is indifferent between the alternatives, the sure buyer is receiving rents since she is more optimistic about the experience good than the unsure buyer.

If the arrival of the first lump-sum payoff immediately terminates experimentation, price jumps after the arrival. Otherwise, the instantaneous price reaction to the arrival of the first lump-sum payoff is ambiguous. This comes from two opposing effects on the unsure buyer's reservation value. On the one hand, the unsure buyer becomes more optimistic since the arrival of good news signal increases the probability of receiving lump-sum payoffs. This *informational effect* raises the unsure buyer's reservation value. On the other hand, the unsure buyer also loses the chance of becoming the first sure buyer to extract rents. The loss of rents generates a *continuation value effect*, which lowers the unsure buyer's reservation value. The latter effect is driven by ex post heterogeneity. If buyers' valuations are perfectly correlated, there is no such effect and the equilibrium price always goes up after the arrival of the first lump-sum payoff. If buyers' valuations are interdependent, with the same priors, the equilibrium price might drop or jump depending on the arrival time of the first lump-sum payoff.

We also discuss the efficiency property of the dynamic monopoly pricing. If buyers' valuations are perfectly correlated, efficiency can be achieved since the monopolist is able to fully internalize

²In a dynamic duopoly pricing model (e.g., Bergemann and Välimäki (1996)), learning determines the future competition positions of different sellers. The buyer generally is not making myopic decisions since her continuation value varies with posterior beliefs. But if one seller's price is fixed to zero, the unsure buyer's optimal decisions become purely myopic in the framework of Bergemann and Välimäki (1996).

social surplus by subsidizing experimentation. Nonetheless, if buyers' valuations are interdependent, the equilibrium experimentation is lower than the socially efficient level. This is also due to the existence of ex post heterogeneity which implies that the monopolist faces two different types of buyers in the individual learning phase. The sure buyers are willing to pay more than the unsure buyers. The absence of price discrimination leads to a tradeoff between exploitation and exploration for the monopolist. In equilibrium, experimentation is terminated too early. The inefficiency in the individual learning phase also causes a leakage of social surplus, which reduces the monopolist's incentives to subsidize experimentation in the aggregate learning phase. As a result, the equilibrium experimentation is inadequate in the aggregate learning phase as well.

Until now, we have been dealing with the case where the experience good generates lump-sum payoffs (it is called *good news case* in the paper). Instead of generating lump-sum payoffs, the good might generate random lump-sum damages (it is called *bad news case* in the paper). Side effects of new drugs are examples of bad news signals. We also characterize a symmetric Markov perfect equilibrium for the bad news case. It turns out that in the bad news case, the equilibrium is always efficient. The key insight is that although buyers become heterogeneous in the individual learning phase, the buyers who have received lump-sum damages will never purchase the experience good. The potential buyers are those unsure ones who are ex post homogeneous in a symmetric equilibrium.

The good and bad news cases also differ in the equilibrium value for the unsure buyers. In the aggregate learning phase, the equilibrium value is determined to deter the following "one-shot" deviations: taking the outside option for a small amount of time and then switching back to purchase the experience good. In the good news case, because of the future benefit from free riding, each unsure buyer receives a value higher than the outside option to deter deviations. However, in the bad news case, no extra subsidy is needed since the deviation of an unsure buyer makes the deviator more pessimistic.

Technically, the difficulty of our analysis mainly comes from the presence of multi-dimensional beliefs: the posterior belief about the product characteristic and the posterior beliefs about the individual attributes are all needed to describe the market. However, given the priors, the posterior about the product characteristic can be expressed as a function of posteriors about the individual attributes conditional on the product characteristic is high. In a symmetric Markov perfect equilibrium, on equilibrium path, we are able to use one posterior as a sufficient state variable for all the posteriors. But off equilibrium path, the "one-shot" deviations lead to heterogeneous posterior beliefs about the individual attributes. Even under that circumstance, the problem can be transformed in a way such that one state variable is sufficient to express the value functions. The benefits of this method are twofold: all value functions can be explicitly derived by solving ordinary differential equations; and we can still apply traditional value matching and smooth pasting conditions to characterize the optimal stopping decisions.

Related Literature

Bergemann and Välimäki (1996) and Felli and Harris (1996) are two early papers analyzing the impact of price competition on experimentation. They show that if there is only individual learning, the dynamic duopoly competition with vertically differentiated products can achieve efficiency. However, Bergemann and Välimäki (2000) shows that in the presence of social learning, the dynamic duopoly competition cannot achieve efficiency. Bergemann and Välimäki (2002) and Bonatti (2009) allow *ex ante heterogeneity* in the sense that buyers are different in their willingness to pay.³ Both papers assume a continuum of buyers. At each instant of time, an individual buyer only makes a myopic optimal choice and strategic interactions between the buyers don't exist.

Bergemann and Välimäki (2006) also considers a dynamic monopoly pricing problem, but with a continuum of buyers and independent valuations. The difference in crucial modelling assumptions leads to investigate different properties of equilibrium price path. The framework of a continuum of buyers makes it impossible to discuss the impact of a single good news signal on price. Instead, Bergemann and Välimäki (2006) is more concerned about whether price would always go down or eventually go up in equilibrium. Bose, Orosel, Ottaviani, and Versterlund (2006) and Bose, Orosel, Ottaviani, and Versterlund (2008) develop another way of modelling dynamic monopoly pricing under social learning. Their model is closer to the herding literature: each short-lived buyer makes a purchasing decision in a pre-determined sequence. In contrast, in our model, all buyers are long-lived and are making purchasing decisions repeatedly.

This paper is also closely connected to the continuous-time strategic experimentation literature. A nonexhaustive list of related papers includes Bolton and Harris (1999), Keller and Rady (1999, 2010), and Keller, Rady, and Cripps (2005).⁴ The analysis of our model setting is greatly simplified by the use of exponential bandits, building on Keller, Rady, and Cripps (2005). Most of the papers in the strategic experimentation literature assume a common value environment. This enables us to use a uni-dimensional posterior belief as the unique state variable to characterize the value functions. By considering interdependent values, we introduce multi-dimensional posterior beliefs and show that the dimensionality of the problem can be reduced by expressing one posterior as a function of other posteriors.

In addition to the theoretical body of work, there are a few empirical studies attempting to quantify the importance of learning considerations on consumers' dynamic purchasing behavior. However, most of the existing works have exclusively focused on modelling the individual consumer behavior and analyzing the impact of idiosyncratic uncertainty (see, e.g., Akerberg (2003), Craw-

³Villas-Boas (2004) also investigates a duopoly model with *ex ante heterogeneity* along a location. It considers a two-period model and is mainly concerned about consumer loyalty, i.e., whether in the second period, buyers return to the seller they bought from in the first period.

⁴The strategic experimentation framework is also used as a building block to investigate broader issues. For example, Strulovici (2010) investigates voting in a strategic experimentation environment; Bergemann and Hege (2005), Hörner and Samuelson (2009) and Bonatti and Hörner (2009) consider moral hazard problems when effort affects speed of learning.

ford and Shum (2005), Erdem and Keane (1996) and so on). One exception is Ching (2009), which includes both individual learning and social learning. Ching (2009) is based on the pass of the Hatch-Waxman Act in 1984. This act eliminates the clinical trial study requirements for approving generic drugs and encourages more entries of generic drugs that have uncertain product qualities. It is shown that both individual learning and social learning are needed to explain the slow diffusion of generic drugs into the market.

The remainder of this paper is organized as follows. Section 2 introduces the model and defines the solution concept. Section 3 and section 4 solve a symmetric Markov perfect equilibrium and discuss efficiency of the equilibrium for the good news case and the bad news case, respectively. Section 5 concludes the paper.

2 Model Setting

Time $t \in [0, +\infty)$ is continuous. The market consists of $n \geq 2$ buyers indexed by $i = 1, 2, \dots, n$ and one monopolist, who are all risk-neutral with common discount rate $r > 0$. The monopolist with a zero marginal cost of production sells a *risky product* with unknown value to the buyers. At each point of time, a buyer can either buy one unit of the risky product or take a safe outside option/product.

If a buyer purchases the safe product, she receives a known deterministic flow payoff $s > 0$.⁵ The value of the risky product to a buyer $i = 1, \dots, n$ depends on both an intrinsic characteristic of the product (*aggregate uncertainty*) and the quality of the match between the product and that buyer (*idiosyncratic uncertainty*). The match between buyer i and the risky product is either *relevant* or *irrelevant*. If the match is irrelevant ($\kappa_i = 0$), the risky product yields a constant flow payoff $\xi_f \geq 0$. For a relevant ($\kappa_i = 1$) match, in addition to the flow payoffs, the risky product generates lump-sum payoffs ξ_l which arrive according to a Poisson process with intensity λ . The arrival of lump-sum payoffs is independent across buyers. Nature chooses the characteristic of the risky product λ randomly from two alternatives: $\lambda = \lambda_H$ (*high*) and $\lambda = \lambda_L = 0$ (*low*). The common prior that λ is high is: $q_0 = \Pr(\lambda = \lambda_H)$. After choosing the characteristic λ , nature chooses randomly and independently the individual match quality for each buyer. With probability $\rho_0 > 0$, $\kappa_i = 1$. Both the characteristic and the match qualities are initially unobservable to all players (seller and buyers), but the parameters λ_H , ξ_f , ξ_l , ρ_0 and q_0 are common knowledge.

We consider two cases in the above setting. In the *good news case*, $\xi_l > 0$ and the arrival of lump-sum payoffs makes the risky product more attractive than the safe one. We assume the risky product is superior to the safe one only when the buyers can receive lump-sum payoffs:

Assumption 1 (*Good News Case*) *In the good news case, $\xi_l > 0$ and $\xi_f < s < \xi_f + \lambda_H \xi_l$.*

⁵Alternatively, we can assume the flow payoff is random but drawn from a commonly known distribution with expectation $s > 0$.

In the *bad news case*, $\xi_l < 0$ and the arrival of lump-sum payoffs makes the risky product less attractive than the safe one. We impose the requirement that the risky product is superior to the safe one only when the buyers cannot receive lump-sum payoffs:

Assumption 2 (*Bad News Case*) *In the bad news case, $\xi_l < 0$ and $\xi_f > s > \xi_f + \lambda_H \xi_l$.*

All players observe each buyer's past actions and outcomes. As a result, both the seller and the buyers hold common posterior beliefs about the aggregate characteristic and any given buyer's match quality. In both cases, if one buyer receives a lump-sum payoff from the risky product, every player immediately knows that that buyer's match is relevant and the characteristic of the product is high since $\lambda_L = 0$. The non-arrival of lump-sum payoffs may be due to either a low characteristic or an irrelevant match. Social learning is important because it provides additional information about the characteristic of the risky product even if the buyers' match qualities are drawn independently. Although the assumption $\lambda_L = 0$ seems to be a little restrictive at the first sight, the current model is rich enough to include the extreme cases of common value ($\rho_0 = 1, q_0 < 1$) and independent values ($q_0 = 1, \rho_0 < 1$).

At each instant of time t , the monopolist first announces a price based on the previous history and then each buyer decides which product to purchase conditional on the previous history and the announced price. It is assumed that the monopolist cannot price discriminate and so charges the same price to all buyers.

2.1 Belief Updating

Denote by N_{it} the total number of lump-sum payoffs received by buyer i before time t . Let P_t be the price charged by the monopolist at time t . Set $a_{it} = 1$ if buyer i purchases the risky product at time t ; $a_{it} = 0$ if buyer i purchases the safe product at time t .

A *feasible outcome* is a triple of $(\{a_i, N_i\}_{i=1}^n, P)$ where i) $N_i : \mathbb{R}_+ \rightarrow \mathbb{N}_+$ is a right continuous and non-decreasing function; and ii) $a_i : \mathbb{R}_+ \rightarrow \{0, 1\}$ and $P : \mathbb{R}_+ \rightarrow \mathbb{R}$ are functions right continuous at all times when N_i is continuous for each i . There are two public histories of interest at time t . A pre- t public history is defined as:

$$h_{t-} \triangleq (\{a_{i\tau}, N_{i\tau}\}_{i=1}^n, P_\tau)_{0 \leq \tau < t};$$

and a time- t public history is defined as: $h_t \triangleq (h_{t-}, P_t)$.

Posterior beliefs are given by:

$$q_t \triangleq \Pr[\lambda_H \mid h_{t-}] \quad \text{and} \quad \rho_{it} \triangleq \Pr[\kappa_i = 1 \mid \lambda_H, h_{t-}]$$

such that the posterior belief of receiving lump-sum payoffs is given by

$$\Pr[\lambda\kappa_i = \lambda_H \mid h_{t-}] = \rho_{it}q_t.$$

Given a pair of priors (ρ_0, q_0) , the posteriors $(\rho_{1t}, \dots, \rho_{nt}, q_t)$ evolve on feasible outcomes according to Bayes' rule. A buyer i who has not received any lump-sum payoff before time t expects an arrival of lump-sum payoffs from the risky product with rate $\lambda_H a_{it} \rho_{it} q_t$. If a lump-sum payoff is received, ρ_{it} immediately jumps to 1; otherwise, ρ_{it} obeys the following differential equation at those times t when a_{it} is right continuous:⁶

$$\dot{\rho}_{it} = -\lambda_H a_{it} \rho_{it} (1 - \rho_{it}). \quad (1)$$

If no buyer has received a lump-sum payoff, then with an expected arrival rate $\lambda_H q_t \sum_{i=1}^n a_{it} \rho_{it}$, some buyer receives a lump-sum payoff and q_t jumps to 1. Otherwise, q_t obeys the following differential equation at those times when a_{it} is right continuous for $\forall i$:

$$\dot{q}_t = -\lambda_H q_t (1 - q_t) \sum_{i=1}^n a_{it} \rho_{it}. \quad (2)$$

The posterior belief q can be expressed as a function of ρ_i 's. When no buyer has received a lump-sum payoff for a length of time t , denote $x_{it} \triangleq \rho_0 e^{-\lambda_H \int_0^t a_{i\tau} d\tau} + 1 - \rho_0$ to be the probability that an unsure buyer i has not received lump-sum payoffs conditional on λ_H . By Bayes' rule

$$q_t = \frac{q_0 \prod_{i=1}^n x_{it}}{q_0 \prod_{i=1}^n x_{it} + 1 - q_0}. \quad (3)$$

From equation (1),

$$\rho_{it} = \frac{\rho_0 e^{-\lambda_H \int_0^t a_{i\tau} d\tau}}{x_{it}} \implies 1 - \rho_{it} = \frac{1 - \rho_0}{x_{it}}. \quad (4)$$

Substitute (4) into (3) yields:

$$q_t = \frac{q_0 (1 - \rho_0)^n}{q_0 (1 - \rho_0)^n + (1 - q_0) \prod_{i=1}^n (1 - \rho_{it})}. \quad (5)$$

Notice that equation (5) also holds when at least one buyer has received lump-sum payoffs. In that situation, at least one of the ρ_{it} 's is one and q_t is also one. After long history of no realization of

⁶If buyer i has not received good news within time t and $t+h$, then the posterior belief $\rho_{i,t+h}$ could be written as:

$$\rho_{i,t+h} = \frac{\rho_{it} e^{-\lambda_H \int_0^h a_{i,t+\tau} d\tau}}{\rho_{it} e^{-\lambda_H \int_0^h a_{i,t+\tau} d\tau} + 1 - \rho_{it}}.$$

Since $a_{i\tau}$ is right continuous with respect to time at time t , there exists some $\bar{h} > 0$ such that $a_{i,t+\tau} = a_{i,t}$ for all $\tau \leq \bar{h}$. Hence by definition,

$$\dot{\rho}_{it} = \lim_{h \rightarrow 0} \frac{\rho_{i,t+h} - \rho_{i,t}}{h} = -\lambda_H a_{it} \rho_{it} (1 - \rho_{it}).$$

\dot{q}_t is derived similarly.

lump-sum payoffs, the posteriors ρ_{it} would converge to zero while q_t would not. This reflects the fact that ρ_{it} is a conditional probability and q_t is bounded below by $q_0(1 - \rho_0)^n$.

A nice property about equation (5) is that it only depends on ρ_{it} 's and does not explicitly depend on those purchasing decisions or time t . An intuitive explanation is that since we have n signals (each player receives one overall signal) but $n + 1$ beliefs, one belief must be redundant and could be expressed as a function of other n beliefs. In the critical history when nobody has received lump-sum payoffs, the updating of q is determined by how fast learning is for each buyer i . Given prior ρ_0 , ρ_{it} is sufficient to encode all information about how fast learning is for buyer i until time t . Therefore, we are able to express q_t as a function of $\boldsymbol{\rho}_t \triangleq (\rho_{1t}, \dots, \rho_{nt})$ for a given pair of priors (ρ_0, q_0) .

2.2 Strategies and Payoffs

Throughout the paper, we focus on symmetric Markov perfect equilibria. The natural state variables include posterior about aggregate uncertainty q and posteriors about idiosyncratic uncertainty $\boldsymbol{\rho}$. Equation (5) shows that q is a function of $\boldsymbol{\rho}$. Hence, given a pair of priors (ρ_0, q_0) , it suffices to use posterior beliefs $\boldsymbol{\rho}_t$ as state variables. We always require $\boldsymbol{\rho}_t$ to be feasible in the sense that

$$\boldsymbol{\rho}_t \in \Sigma = \{\boldsymbol{\rho} \in [0, 1]^n : \text{either } \rho_i = 1 \text{ or } \rho_i \leq \rho_0 \text{ all for } i\}.$$

Mild regularity conditions have to be imposed on Markovian strategies to guarantee feasibility of the induced outcomes.

Purchasing Decision Given a pair of priors (ρ_0, q_0) , buyer i 's acceptance policy is a function of states $\boldsymbol{\rho}$ and price P

$$\alpha_i : \Sigma \times \mathbb{R} \rightarrow \{0, 1\}.$$
⁷

In the discrete time model, the purchasing strategy satisfies the cutoff property: given belief $\boldsymbol{\rho}$, buyer i would purchase the risky product if price is lower than certain cutoff and vice versa.⁸ We naturally require the Markov strategy α_i satisfies the cutoff property as well and the cutoff price is left-continuous in beliefs:⁹

Definition 1 (*Admissible purchasing strategies*) For each buyer i , the purchasing strategy α_i is admissible if

⁷More accurately, the strategy should be written as $\alpha_i(\boldsymbol{\rho}, P; \rho_0, q_0)$. Throughout the paper, (ρ_0, q_0) would always be dropped since no confusion is caused.

⁸The cutoff property is easy to prove in the discrete time model since the value from purchasing the risky product always increases if the price becomes lower and vice versa.

⁹Definitions 1 and 2 are sufficient for the history to be right continuous in time, which induces a well-defined updating rule for beliefs. Most of the papers in continuous time game literature either explicitly or implicitly make such kind of assumptions (see, e.g., Keller and Rady (2010)).

- α_i satisfies the cutoff property in price P : for each $\boldsymbol{\rho} \in \Sigma$, there exists a cutoff price $\underline{P}_i(\boldsymbol{\rho})$ such that

$$\alpha_i(\boldsymbol{\rho}, P) = \begin{cases} 1 & \text{if } P \leq \underline{P}_i(\boldsymbol{\rho}) \\ 0 & \text{otherwise} \end{cases}$$

and

- the cutoff price \underline{P}_i is left-continuous in beliefs: for each $\boldsymbol{\rho} \in \Sigma$ and $\delta > 0$, there exists some $\epsilon > 0$ s.t.

$$|\underline{P}_i(\boldsymbol{\rho}') - \underline{P}_i(\boldsymbol{\rho})| \leq \delta$$

for all feasible $\boldsymbol{\rho}' \leq \boldsymbol{\rho}$ satisfying $\|\boldsymbol{\rho}' - \boldsymbol{\rho}\| \leq \epsilon$.¹⁰

Buyer i 's incremental utility associated with purchasing decision a_{it} is given by

$$du_{it} = a_{it}\xi_l dN_{it} + a_{it}(\xi_f - P_t)dt + (1 - a_{it})sdt.$$

Since lump-sum payoffs arrives with rate $\rho_{it}q_t\lambda_H$, the expected flow of utility is

$$a_{it}\rho_{it}q_t\lambda_H\xi_l + a_{it}(\xi_f - P_t) + (1 - a_{it})s.$$

The choice of a_{it} not only affects flow utility but also affects how beliefs ρ_{it} and q_t are updated. Given beliefs $\boldsymbol{\rho} \in \Sigma$, monopolist's strategy P and other buyers' strategies α_{-i} , buyer i 's value (sum of normalized expected discounted utility) from purchasing strategy α_i is

$$U_i(\alpha_i, P, \alpha_{-i}; \boldsymbol{\rho}) = \mathbb{E} \int r e^{-rt} \{ \alpha_i(\boldsymbol{\rho}_t, P_t) (\rho_{it}q(\boldsymbol{\rho}_t)\lambda_H\xi_l + \xi_f - P_t) + (1 - \alpha_i(\boldsymbol{\rho}_t, P_t))s \} dt$$

where the expectation is taken over $\{\boldsymbol{\rho}_t\}_{t=0}^{\infty}$ with $\boldsymbol{\rho}_0 = \boldsymbol{\rho}$ and $q(\boldsymbol{\rho}_t)$ is given by equation (5).

Pricing Decision Given a pair of priors (ρ_0, q_0) , the monopolist's price is a function of states $\boldsymbol{\rho}$

$$P : \Sigma \rightarrow \mathbb{R}.$$

Admissibility of pricing function P not only requires P to be left continuous in beliefs but also includes a technical condition. The technical condition restricts how cutoff price \underline{P}_i and actual price P would change as beliefs change when $\underline{P}_i = P$.

Definition 2 (*Admissible pricing strategy*) Given feasible purchasing strategies $\{\alpha_i\}_{i=1}^n$, the pricing strategy P is admissible if it satisfies:

¹⁰We write $(x_1, \dots, x_n) \leq (y_1, \dots, y_n)$ if $x_i \leq y_i$ for $i = 1, \dots, n$ and $\|\cdot\|$ is the Euclidean norm.

- for each $\boldsymbol{\rho} \in \Sigma$ and $\delta > 0$, there exists some $\epsilon > 0$ s.t.

$$|P(\boldsymbol{\rho}') - P(\boldsymbol{\rho})| \leq \delta$$

for all feasible $\boldsymbol{\rho}' \leq \boldsymbol{\rho}$ satisfying $\|\boldsymbol{\rho}' - \boldsymbol{\rho}\| \leq \epsilon$ and

- for each buyer i , if $P(\boldsymbol{\rho}) \leq \underline{P}_i(\boldsymbol{\rho})$, then there exists some $\epsilon' > 0$ such that $P(\boldsymbol{\rho}') \leq \underline{P}_i(\boldsymbol{\rho}')$ for all feasible $\boldsymbol{\rho}'$ such that $\|\boldsymbol{\rho}' - \boldsymbol{\rho}\| \leq \epsilon'$.

Given buyers' strategies $\{\alpha_i\}_{i=1}^n$, the flow profits associated with price P_t is given by

$$\sum_{i=1}^n \alpha_i(\boldsymbol{\rho}_t, P_t) P_t.$$

The choice of P_t affects not only flow profits but also the purchasing decisions and so how beliefs are updated. Given beliefs $\boldsymbol{\rho}$ and buyers' strategies $\{\alpha_i\}_{i=1}^n$, the monopolist's value (sum of normalized expected discounted profits) from the pricing policy P is

$$J(P, \alpha; \boldsymbol{\rho}) = \mathbb{E} \int r e^{-rt} \sum_{i=1}^n \alpha_i(\boldsymbol{\rho}_t, P(\boldsymbol{\rho}_t)) P(\boldsymbol{\rho}_t) dt$$

where the expectation is taken over $\{\boldsymbol{\rho}_t\}_{t=0}^{\infty}$ with $\boldsymbol{\rho}_0 = \boldsymbol{\rho}$.

2.3 Symmetric Markov Perfect Equilibrium

We consider Markov perfect equilibrium in symmetric strategies. The formal definition of our solution concept is the following:

Definition 3 *Given a pair of priors (ρ_0, q_0) , a collection of admissible strategies $\{P^*, \alpha^*\}$ is a Markov perfect equilibrium if for all i , feasible beliefs $\boldsymbol{\rho}$ and all admissible strategies \tilde{P} and $\tilde{\alpha}_i$:*

$$J(P^*, \alpha^*; \boldsymbol{\rho}) \geq J(\tilde{P}, \tilde{\alpha}_i; \boldsymbol{\rho})$$

and

$$U_i(\alpha_i^*, P^*, \alpha_{-i}^*; \boldsymbol{\rho}) \geq U_i(\tilde{\alpha}_i, P^*, \alpha_{-i}^*; \boldsymbol{\rho}).$$

Moreover, $\{P^*, \alpha^*\}$ is symmetric if for all permutations $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$,

$$P(\tilde{\boldsymbol{\rho}}) = P(\boldsymbol{\rho})$$

where $\tilde{\rho}_i = \rho_{\pi^{-1}(i)}$ and

$$\alpha_i(\boldsymbol{\rho}, P) = \alpha_{\pi(i)}(\tilde{\boldsymbol{\rho}}, P).$$

3 Equilibrium in the Good News Case

In the good news case, $\xi_l > 0$ and the arrival of lump-sum payoff makes the risky product more favorable to the receiver of this payoff. In this section, we normalize $\xi_f = 0$ and $\xi_l = v > 0$. Assumption 1 implies $g \triangleq \lambda_H v > s > 0$.

Since the arrival of one lump-sum payoff immediately resolves aggregate uncertainty, there are only two situations to consider: an *aggregate learning phase*, where the aggregate uncertainty has not been resolved, and an *individual learning phase*, where the aggregate uncertainty has been resolved. In the individual learning phase, an unsure buyer just needs to learn her individual match quality and for such a buyer i , without the arrival of lump-sum payoff, posterior belief ρ_i is updated according to equation (1).

In the aggregate learning phase, both individual learning and social learning exist. If unsure buyers behave symmetrically, they share the same posterior belief ρ , and belief q about λ_H is given by equation (5):

$$q = \frac{(1 - \rho_0)^n q_0}{(1 - \rho_0)^n q_0 + (1 - \rho)^n (1 - q_0)}. \quad (6)$$

Given a pair of priors (ρ_0, q_0) , when unsure buyers behave symmetrically, their posterior belief ρ can be used as a sufficient state variable.

3.1 Socially Efficient Allocation

Before solving for a symmetric Markov perfect equilibrium, we first solve for the socially efficient allocation. The linear utility function enables us to obtain the efficient allocation policy by solving a specific multi-armed bandit problem where payoffs are given by the aggregate surplus.

Given the priors ρ_0 and q_0 , the socially efficient allocation is characterized by a cutoff strategy in posterior belief ρ . There are two cutoffs ρ_I^e and ρ_A^e for the individual learning phase and the aggregate learning phase, respectively. In the individual (aggregate) learning phase, it is optimal for the social planner to keep the unsure buyers experimenting until belief drops to ρ_I^e (ρ_A^e) and no lump-sum payoff has been received before that. A backward procedure is used to solve for the socially efficient allocation. We first characterize the socially efficient allocation in the individual learning phase and then use the optimal social surplus function in the individual learning phase to solve the cooperative problem in the aggregate learning phase.

Socially Efficient Allocation in the Individual Learning Phase. In the individual learning phase, suppose k buyers have received good news; then it is socially optimal for them to keep purchasing the risky product by assumption 2 and the social surplus function is

$$\Omega_k(\rho) = kg + (n - k)W(\rho)$$

where

$$W(\rho) = \sup_{\alpha \in \{0,1\}} \mathbb{E} \int_{t=0}^{\infty} r e^{-rt} [\alpha \rho_t g + (1 - \alpha) s] dt$$

is the optimal value for an unsure buyer with posterior belief ρ .

Since the unsure buyer is facing a standard two-armed bandit problem, previous research (see Keller, Rady, and Cripps (2005)) has characterized the optimal cutoff and value function W . If $k \geq 1$ buyers are sure to receive lump-sum payoffs, it is efficient for the remaining $n - k$ unsure buyers to stop purchasing the risky product once the posterior belief ρ reaches

$$\rho_I^e = \frac{rs}{(r + \lambda_H)g - \lambda_H s}$$

and still no lump-sum payoff has been received. Since in the individual learning phase, the aggregate uncertainty has been resolved ($q = 1$), the efficient cutoff ρ_I^e does not depend on the priors ρ_0 and q_0 . The value function for a buyer with posterior belief ρ is

$$W(\rho) = \max \left\{ s, g\rho + \frac{\lambda_H s}{r + \lambda_H} \left(\frac{rs}{(r + \lambda_H)(g - s)} \right)^{r/\lambda_H} (1 - \rho) \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H} \right\}. \quad (7)$$

Efficiency in the Aggregate Learning Phase. In the aggregate learning phase, the socially efficient allocation solves the symmetric cooperative problem (see claim 2 in the appendix):

$$\Omega_A(\rho) = \sup_{\alpha(\cdot) \in \{0,1\}} \mathbb{E} \left\{ \int_{t=0}^h r e^{-rt} n [\alpha(\rho_t) \rho_t q(\rho_t) g + (1 - \alpha(\rho_t)) s] dt + e^{-rh} \Omega(\rho_h | \alpha) \right\}$$

where

$$\begin{aligned} \mathbb{E} \Omega(\rho_h | \alpha) = & q \sum_{k=1}^n \binom{n}{k} \rho^k \left(1 - e^{-\lambda_H \int_0^h \alpha_t dt} \right)^k \left(\rho e^{-\lambda_H \int_0^h \alpha_t dt} + 1 - \rho \right)^{n-k} \Omega_k(\rho_h) \\ & + \left[q \left(\rho e^{-\lambda_H \int_0^h \alpha_t dt} + 1 - \rho \right)^n + 1 - q \right] \Omega_A(\rho_h) \end{aligned}$$

and

$$\rho_h = \frac{\rho e^{-\lambda_H \int_0^h \alpha_t dt}}{\rho e^{-\lambda_H \int_0^h \alpha_t dt} + 1 - \rho}.$$

The Hamilton-Jacobi-Bellman equation (HJB equation hereafter) for the above problem is

$$r\Omega_A(\rho) = \max \left\{ rns, rn\rho q(\rho)g + n\rho q(\rho)\lambda_H(\Omega_1(\rho) - \Omega_A(\rho)) - \lambda_H\rho(1 - \rho)\Omega'_A(\rho) \right\} \quad (8)$$

where $\Omega_1(\rho) = g + (n - 1)W(\rho)$ is the social surplus when one buyer receives lump-sum payoff.

The first part of the maximand corresponds to using the safe product, the second to the risky product. The effect of using the risky product for the social planner can be decomposed into three

elements: i) the (normalized) expected payoff rate $rn\rho q(\rho)g$, ii) the jump of the value function to $\Omega_1(\cdot)$ if one buyer receives a lump-sum payoff, which occurs at rate $n\lambda_H$ with probability $pq(\rho)$, and iii) the effect of Bayesian updating on the value function when no lump-sum payoff is received. When no lump-sum payoff is received, both ρ and q are updated. The updating of q is implicitly incorporated since q is a function of ρ .

The optimal cutoff ρ_A^e is pinned down by solving the following differential equation:

$$r\Omega_A(\rho) = rn\rho q(\rho)g + n\rho q(\rho)\lambda_H(\Omega_1(\rho) - \Omega_A(\rho)) - \lambda_H\rho(1 - \rho)\Omega'_A(\rho), \quad (9)$$

with boundary conditions:

$$\Omega_A(\rho_A^e) = ns \quad (\text{value matching condition}) \quad \text{and} \quad \Omega'_A(\rho_A^e) = 0 \quad (\text{smooth pasting condition}).$$

Substitute the two boundary conditions into differential equation (8) and immediately we see that the cutoff ρ_A^e should satisfy

$$rn\rho q(\rho)g + n\rho q(\rho)\lambda_H\Omega_1(\rho) = (r + n\rho q(\rho)\lambda_H)ns. \quad (10)$$

In the appendix, we show that equation (10) implies a unique solution ρ_A^e for a given pair of priors (ρ_0, q_0) . The socially efficient allocation in the aggregate learning phase can be characterized as follows:

Proposition 1 (*Characterize socially efficient allocation*) *For any posteriors (ρ, q) , it is socially efficient to purchase the risky product in the aggregate learning phase if and only if*

$$\rho q > \frac{rs}{(r + \lambda_H)g + (n - 1)\lambda_H W(\rho) - n\lambda_H s}.$$

When the aggregate uncertainty is resolved, it is always socially efficient for the unsure buyers to continue experimentation until the posterior reaches ρ_I^e .

Proof. In the appendix. ■

Given the priors, the unique pair of efficient cutoffs $(\rho_A^e(\rho_0, q_0), q_A^e(\rho_0, q_0))$ is determined by equations

$$q_A^e = \frac{(1 - \rho_0)^n q_0}{(1 - \rho_0)^n q_0 + (1 - \rho_A^e)^n (1 - q_0)} \quad (11)$$

and

$$\rho_A^e = \frac{rs}{\rho_A^e [(r + \lambda_H)g + (n - 1)\lambda_H W(\rho_A^e) - n\lambda_H s]}, \quad (12)$$

where $W(\cdot)$ is given by equation (7). Figure 1 is an illustration about how we can use equations (11) and (12) to determine the efficient cutoffs in the aggregate learning phase. Equation (12) is

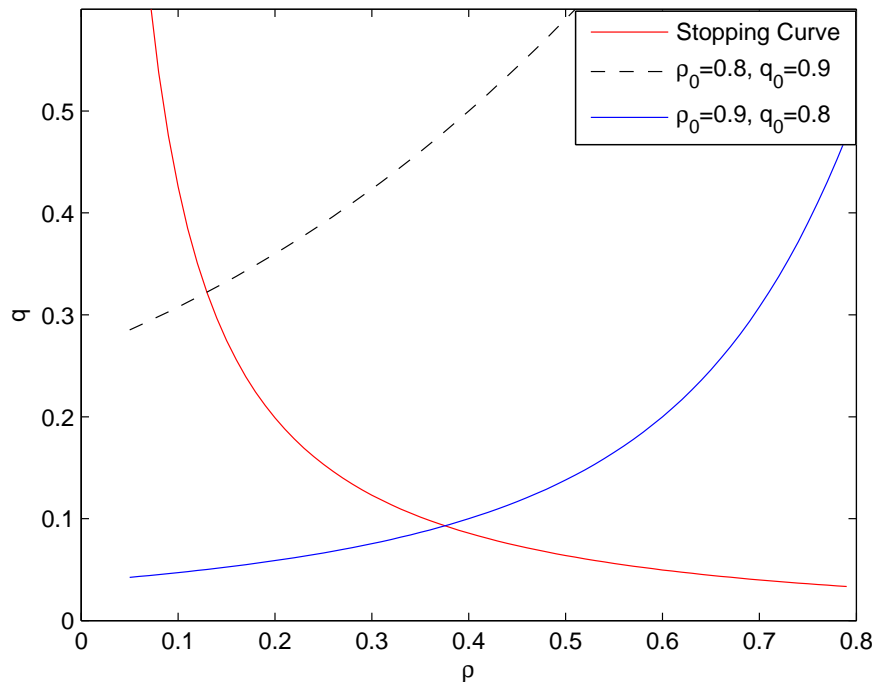


Figure 1: Solutions to the Cooperative Problem with Two Players

a stopping curve because it consists of all pairs of stopping cutoffs (ρ_A^e, q_A^e) and this equation is independent of priors (ρ_0, q_0) . Equation (11) describes how ρ and q evolve jointly over time starting from ρ_0 and q_0 . This equation indeed depends on priors.

Unlike the individual learning phase, the cutoff ρ_A^e does depend on the priors (ρ_0, q_0) . We formulate the problem so that ρ is the unique state variable in order to avoid solving partial differential equations. But the actual optimal stopping decision depends not only on belief ρ but also on q . For a fixed ρ_0 , a higher q_0 means that the society can afford to experiment more and thus the efficient cutoff ρ_A^e should be lower. The multi-dimensionality of beliefs implies a stationary optimal stopping curve described by equation (12). For a fixed pair of priors (ρ_0, q_0) , a two-dimensional optimal stopping problem is transformed into a one-dimensional one by expressing q as a function of ρ . As a result, we are able to apply traditional value matching and smooth pasting conditions to solve our optimal stopping problems.

3.2 Characterizing Equilibrium for $n = 2$

In the two-buyer case, there are three situations to consider. When the aggregate uncertainty is not resolved, denote U_A as the value function for each unsure buyer; J_A as the value function for the monopolist. When one buyer has received lump-sum payoffs, denote U_I as the value function

for the unsure buyer; V_I as the value function for the sure buyer; J_I as the value function for the monopolist. When both buyers have received lump-sum payoffs, denote V_2 as the value function for the sure buyers; J_2 as the value function for the monopolist.

For $\zeta = A, I$, denote α_ζ^0 (α_ζ^1) as strategy for the sure (unsure) buyers. Let P_ζ be the price charged by the monopolist. Then definition 3 implies that a triple of $(P_\zeta, \alpha_\zeta^0, \alpha_\zeta^1)$ is a symmetric Markov perfect equilibrium if the following conditions are satisfied:

- for $\zeta = I$, $\alpha_\zeta^0 = 1$ if $P \leq g - s$ and $= 0$ otherwise;
- for $\zeta = A$, the unsure buyers choose acceptance policy α_ζ^1 to maximize:

$$U_\zeta(\rho) = \sup_{\alpha_\zeta^1} \mathbb{E} \left\{ \int_{t=0}^{\tau} r e^{-rt} [\alpha_\zeta^1(\rho_t q_\zeta(\rho_t) g - P_\zeta(\rho_t)) + (1 - \alpha_\zeta^1) s] dt + e^{-r\tau} \left(\frac{1}{2} V_I(\rho_\tau) + \frac{1}{2} U_I(\rho_\tau) \right) \right\}$$

and given α_ζ^1 , the monopolist choose price $P_\zeta(\rho_t)$ to maximize

$$J_\zeta(\rho) = \sup_{P_\zeta(\cdot)} \mathbb{E} \left\{ \int_{t=0}^{\tau} 2r e^{-rt} \alpha_\zeta^0(P_\zeta(\rho_t)) dt + e^{-r\tau} J_I(\rho_\tau) \right\},$$

where τ is the first (possibly infinite) time at which a new unsure buyer receives good news;

- for $\zeta = I$, the unsure buyer chooses acceptance policy α_ζ^1 to maximize:

$$U_\zeta(\rho) = \sup_{\alpha_\zeta^1} \mathbb{E} \left\{ \int_{t=0}^{\tau} r e^{-rt} [\alpha_\zeta^1(\rho_t q_\zeta(\rho_t) g - P_\zeta(\rho_t)) + (1 - \alpha_\zeta^1) s] dt + e^{-r\tau} V_2(\rho_\tau) \right\}$$

and given $(\alpha_\zeta^0, \alpha_\zeta^1)$, the monopolist choose price $P_\zeta(\rho_t)$ to maximize

$$J_\zeta(\rho) = \sup_{P_\zeta} \mathbb{E} \left\{ \int_{t=0}^{\tau} r e^{-rt} [\alpha_\zeta^0(P_\zeta(\rho_t)) + \alpha_\zeta^1(\rho_t, P_\zeta(\rho_t))] dt + e^{-r\tau} J_2(\rho_\tau) \right\};$$

- beliefs update according to Bayes' rule: ρ_t satisfies the law of motion, i.e. equation (1); $q_\zeta(\rho_t) \equiv 1$ for $\zeta = I$ and $q_\zeta(\rho_t)$ is given by equation (6) for $\zeta = A$;
- when both buyers have received received lump-sum payoffs, the price is $g - s$ such that $J_2 \equiv 2(g - s)$ and $V_2 \equiv s$.

First, it is straightforward to see that the sure buyers always buy the risky product if the price is lower than $g - s$ and not buy otherwise. Second, when both unsure buyers purchase the risky product, the conditional probability that any given unsure buyer becomes good is simply 1/2, since the two unsure buyers' payoff distributions are identical. Finally, if all of the buyers turn out to be good, it is optimal for the monopolist charging price $g - s$ to extract all of the surplus.

Admissible strategies imply that on equilibrium path, for $\zeta = A, I$, i) the unsure buyer's purchasing decision is characterized by a cutoff price $\underline{P}_\zeta(\rho_t)$, at which the unsure buyer is indifferent between the risky and safe products:

$$\alpha_\zeta^1(\rho_t, P_t) = \begin{cases} 1 & \text{if } P_t \leq \underline{P}_\zeta(\rho_t) \\ 0 & \text{otherwise} \end{cases}$$

and ii) the monopolist charges price $\underline{P}_\zeta(\rho_t)$ to keep selling to the unsure buyers until the belief ρ_t reaches a certain cutoff.

3.2.1 Niche Market vs. Mass Market

As in the social planner's problem, the equilibrium purchasing behavior can be characterized by two cutoffs ρ_A^* and ρ_I^* . If one buyer has received lump-sum payoffs, the monopolist stops selling to the unsure buyer and only serves the sure buyer when posterior belief of the unsure buyer is below ρ_I^* . If no buyer has received lump-sum payoffs, the price is kept falling over time to keep both unsure buyers experimenting until posterior ρ reaches ρ_A^* . After that, both buyers purchase the safe product.

The efficient cutoff in the individual learning phase ρ_I^e is always smaller than the efficient cutoff in the aggregate learning phase ρ_A^e for any pair of priors (ρ_0, q_0) . Under strategic interactions, this may not be true. For a fixed pair of priors (ρ_0, q_0) , it turns out that ρ_I^* could be either smaller or larger than ρ_A^* . We can distinguish a mass market from a niche market by comparing these two cutoffs.

Definition 4 (*Niche market and mass market*).

1. The market is niche if the cutoffs determined by (ρ_0, q_0) satisfy: $\rho_A^* \leq \rho_I^*$, and
2. The market is mass if the cutoffs determined by (ρ_0, q_0) satisfy: $\rho_A^* > \rho_I^*$.

In a mass market, the arrival of good news never terminates experimentation while in a niche market, experimentation is shut down by the arrival of the first lump-sum payoff at $\rho \leq \rho_I^*$. Obviously, whether a mass or niche market appears in equilibrium depends on the the priors, which in turn determines the relative importance of social learning and individual learning. We may expect that experimentation would continue after the first arrival of lump-sum payoffs if the individual learning component is quite important and vice versa.

3.2.2 Equilibrium in the Individual Learning Phase

A backward procedure is used to characterize ρ_I^* and ρ_A^* . In the individual learning phase, the equilibrium cutoff ρ_I^* and the various value functions are provided in the following proposition.

Proposition 2 Fix a symmetric Markov perfect equilibrium; in the history such that the aggregate uncertainty is resolved, the unsure buyer purchases the risky product if and only if her posterior belief ρ is larger than

$$\rho_I^* \triangleq \frac{r(g+s)}{2rg + \lambda_H(g-s)}.$$

The equilibrium price is $P_I(\rho) = g\rho - s$ and the unsure buyer receives value $U_I(\rho) \equiv s$; the sure buyer receives value

$$V_I(\rho) = \max \left\{ s, s + g(1-\rho) \left(1 - \left[\frac{(1-\rho)\rho_I^*}{\rho(1-\rho_I^*)} \right]^{r/\lambda_H} \right) \right\}; \quad (13)$$

and the monopolist receives value

$$J_I(\rho) = \begin{cases} 2(g\rho - s) + (g + s - 2g\rho_I^*) \frac{1-\rho}{1-\rho_I^*} \left[\frac{(1-\rho)\rho_I^*}{\rho(1-\rho_I^*)} \right]^{r/\lambda_H} & \text{if } \rho > \rho_I^* \\ g - s & \text{otherwise.} \end{cases}$$

Proof. In the appendix. ■

It is straightforward to see that equilibrium cutoff ρ_I^* is strictly larger than the efficient cutoff ρ_I^e . This is because *ex post* heterogeneity generates a tradeoff between exploitation and exploration for the monopolist. The absence of price discrimination increases the opportunity cost for the monopolist to keep the unsure buyers experimenting. The temptation to charge a high price and extract the full surplus from the sure buyer causes an early termination of experimentation. Another remark is that the unsure buyer is making a myopic choice in the individual learning phase since there is no learning value attached to the purchasing behavior (the unsure buyer always receives value s).

3.2.3 Equilibrium in the Aggregate Learning Phase

Now consider the situation where none of the buyers have received lump-sum payoffs yet. Assume that the posterior belief ρ is large enough that both buyers purchase the risky product in equilibrium. To characterize the equilibrium price and cutoff, we proceed as follows. First, we use the incentive compatibility constraint to derive the value function of the experimenting buyers. Second, we derive expressions of equilibrium price and monopolist's value function based on the experimenting buyers' value function derived in the first step. Finally, we apply value matching and smooth pasting conditions (see, e.g., Dixit (1993)) to pin down the equilibrium cutoff.

To keep both unsure buyers experimenting, the unsure buyer's value should be such that i) each buyer has an incentive to participate (i.e. the value is larger than the outside option s); ii) each buyer should not benefit from the following deviations: stopping experimentation for a very small amount of time and then switching back to the specified equilibrium behavior.

The deviations described in constraint ii) are similar to one-shot deviations in discrete time models. Formally, it implies that for any $\rho > \rho_A^*$, there exists \bar{h} such that for all $h \leq \bar{h}$,

$$U_A(\rho) \geq \hat{U}(\rho; h) = \int_{t=0}^h r e^{-rt} s dt + \rho q (1 - e^{-\lambda_H h}) e^{-rh} U_I(\rho) + [1 - \rho q (1 - e^{-\lambda_H h})] e^{-rh} U(\rho, \rho_h) \quad (14)$$

where $\hat{U}(\rho; h)$ denotes the value for a deviator who deviates for h length of time. The deviator receives a deterministic payoff s within the h length of time. After the deviation, with probability $\rho q (1 - e^{-\lambda_H h})$, the non-deviator has received lump-sum payoffs and the continuation value for the deviator is $U_I(\rho) = s$; with the complementary probability, the non-deviator has not received lump-sum payoffs and the two unsure buyers become asymmetric. The non-deviator ρ_h is more pessimistic than the deviator ρ . We use $U(\rho, \rho_h)$ to denote the value for buyer ρ if both ρ and ρ_h buyers are unsure. When the non-deviator has not received lump-sum payoffs within h length of time, the posterior belief is such that $\rho_h = \frac{\rho e^{-\lambda_H h}}{\rho e^{-\lambda_H h} + (1 - \rho)}$. Obviously, equation (14) is a constraint tighter than the participation constraint since $U_I(\rho) = s$ and $U(\rho, \rho_h) \geq s$.

The most important technical result in this paper is to evaluate $\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h}$. The result is given by lemma 1 in the appendix. Here we just provide a sketch of the proof.

Sketch of the proof for lemma 1. The main difficulty of the proof is to evaluate the off-equilibrium-path value function $U(\rho, \rho_h)$. First notice that $\rho > \rho_A^*$ means that it is optimal for the monopolist to sell to both unsure buyers on equilibrium path. Then, for h sufficiently small, it is still optimal for the monopolist to sell to both unsure buyers after an h -deviation.

In other words, given a sufficiently small h , we can find some \bar{h}' such that for all $h' \leq \bar{h}'$, we have:

$$\begin{aligned} U(\rho, \rho_h) = \mathbb{E} \int_{t=0}^{h'} r e^{-rt} (\rho_t q_t g - \tilde{P}_t) dt \\ + \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} V_I(\rho_{h+h'}) + \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} s \\ + [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) - \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-rh'} U(\rho_{h'}, \rho_{h+h'}). \quad (15) \end{aligned}$$

In the above expression, ρ_t is the belief for the deviator and starts from $\rho_0 = \rho$; \tilde{q}_h is the belief about the product characteristic after an h -deviations: $\tilde{q}_h = \frac{q_0 (1 - \rho_0)^2}{q_0 (1 - \rho_0)^2 + (1 - q_0) (1 - \rho) (1 - \rho_h)}$; and \tilde{P}_t is the off-equilibrium-path price set by the monopolist after an h -deviation. Obviously, the value function $U(\rho, \rho_h)$ depends on the off-equilibrium-path price and cannot be evaluated directly.

Meanwhile, notice the non-deviator's value can be expressed as:

$$\begin{aligned}
U(\rho_h, \rho) = \mathbb{E} \int_{t=0}^{h'} r e^{-rt} (\rho'_t q_t g - \tilde{P}_t) dt \\
+ \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} s + \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} V_I(\rho_{h'}) \\
+ [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) - \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-rh'} U(\rho_{h+h'}, \rho_{h'}), \quad (16)
\end{aligned}$$

where ρ'_t is the belief for the deviator and starts from $\rho'_0 = \rho_h$.

The key step is to decompose $U(\rho, \rho_h)$ as:

$$U(\rho, \rho_h) = U(\rho_h, \rho) + (U(\rho, \rho_h) - U(\rho_h, \rho)).$$

The reason to do this decomposition is that the off-equilibrium-path price is cancelled when we subtract $U(\rho_h, \rho)$ from $U(\rho, \rho_h)$ and hence $Z(\rho, \rho_h) \triangleq U(\rho, \rho_h) - U(\rho_h, \rho)$ is independent of the off-equilibrium path price \tilde{P} .

Buyer ρ_h 's value $U(\rho_h, \rho)$ can be computed without using the off-equilibrium price. If the non-deviator has not received lump-sum payoffs during an h -deviation, she becomes more pessimistic than the deviator. If the monopolist wants to sell to both buyers, the optimal price is set according to the reservation value of the more pessimistic buyer. An expression of $U(\rho_h, \rho)$ can be derived from the ρ_h buyer's incentive compatibility constraint. In the appendix, we show this implies a first-order ordinary differential equation for $U(\rho_h, \rho)$, which can be solved by imposing the boundary condition that $U(\rho_h, \rho_h) = U_A(\rho_h)$.

Second, given any $t < h'$, notice equations (15) and (16) also hold for posteriors $(\rho(t), \rho_h(t))$ where

$$\rho(t) = \frac{\rho e^{-\lambda_H t}}{\rho e^{-\lambda_H t} + (1 - \rho)}, \quad \text{and} \quad \rho_h(t) = \frac{\rho_h e^{-\lambda_H t}}{\rho_h e^{-\lambda_H t} + (1 - \rho_h)}.$$

Redefine

$$Z(t) = Z(\rho(t), \rho_h(t)) = U(\rho(t), \rho_h(t)) - U(\rho_h(t), \rho(t))$$

to be a function of time t . A first-order ordinary differential equation about $Z(t)$ can be obtained by subtracting equation (16) from equation (15) and letting the length of time interval converge to zero. Solving the ordinary differential equation, the expression for $Z(\rho, \rho_h)$ can be recovered by substituting time t as functions of $\rho(t)$ and $\rho_h(t)$. The boundary condition is such that $Z = 0$ once ρ_h reaches ρ_A^* .

After $U(\rho, \rho_h)$ is evaluated, $\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h}$ can be computed by taking limits. ■

Lemma 2 in the appendix implies that in equilibrium, a profit-maximizing monopolist should always make the incentive constraints to be “binding” in the sense that $\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} = 0$. Lemma 1 and lemma 2 together gives an important characterization of the on-equilibrium-path value function U_A :

Proposition 3 Fix any pair of priors such that ρ_A^* is the equilibrium cutoff in the aggregate learning phase. In a mass market, given any $\rho > \rho_A^*$, a necessary condition for the unsure buyers to keep experimentation is that the value $U_A(\rho)$ satisfies differential equation

$$0 = 2(r + \lambda_H \rho q)(U_A(\rho) - s) + \lambda_H \rho(1 - \rho)U'_A(\rho) + (r + \lambda_H \rho)g(1 - \rho)q \left(\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_A^*)} \right)^{r/\lambda_H} - \lambda_H g \rho(1 - \rho)q \\ - \left[\frac{r + \lambda_H \rho_A^*}{1 - \rho_A^*} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \lambda_H \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H}. \quad (17)$$

In a niche market, given any $\rho > \rho_A^*$, a necessary condition for the unsure buyers to keep experimentation is that the value $U_A(\rho)$ satisfies differential equation

$$0 = 2(r + \lambda_H \rho q)(U_A(\rho) - s) + \lambda_H \rho(1 - \rho)U'_A(\rho) \\ + \frac{r \lambda_H g}{r + \lambda_H} \frac{(1 - \rho)^2 q \rho_A^*}{1 - \rho_A^*} \left(\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right)^{r/\lambda_H} - \frac{r g}{r + \lambda_H} \lambda_H \rho(1 - \rho)q \quad (18)$$

for $\rho \leq \rho_I^*$; and differential equation

$$0 = 2(r + \lambda_H \rho q)(U_A(\rho) - s) + \lambda_H \rho(1 - \rho)U'_A(\rho) + (r + \lambda_H \rho)g(1 - \rho)q \left(\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)} \right)^{r/\lambda_H} - \lambda_H g \rho(1 - \rho)q \\ - r \left[\frac{r + \lambda_H + \lambda_H \rho_I^*}{(r + \lambda_H)(1 - \rho_I^*)} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \frac{\lambda_H}{r + \lambda_H} \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H} \quad (19)$$

for $\rho > \rho_I^*$.

The ordinary differential equations in proposition 3 can be solved by using observation 1 in the appendix. In a mass market, for any $\rho > \rho_A^*$, the value function $U_A(\rho)$ is given by

$$U_A(\rho) = s + \frac{\lambda_H}{2r + \lambda_H} g \rho(1 - \rho)q - g(1 - \rho)q \left[\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)} \right]^{r/\lambda_H} \\ + \left[\frac{r + \lambda_H \rho_A^*}{r(1 - \rho_A^*)} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \frac{\lambda_H}{r} \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H} \\ + C(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{2r/\lambda_H}. \quad (20)$$

In a niche market, for any $\rho_A^* < \rho \leq \rho_I^*$, the value function $U_A(\rho)$ is given by

$$U_A(\rho) = s + \frac{r \lambda_H}{(2r + \lambda_H)(r + \lambda_H)} g \rho(1 - \rho)q - \frac{\lambda_H g}{r + \lambda_H} \frac{\rho_A^* (1 - \rho)^2 q}{1 - \rho_A^*} \left(\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right)^{r/\lambda_H} \\ + D(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{2r/\lambda_H}; \quad (21)$$

and for $\rho > \rho_I^*$, the value function $U_A(\rho)$ is given by¹¹

$$\begin{aligned}
U_A(\rho) &= s + \frac{\lambda_H}{2r + \lambda_H} g \rho (1 - \rho) q - g (1 - \rho) q \left[\frac{(1 - \rho) \rho_I^*}{\rho (1 - \rho_I^*)} \right]^{r/\lambda_H} \\
&+ \left[\frac{r + \lambda_H + \lambda_H \rho_I^*}{(r + \lambda_H)(1 - \rho_I^*)} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \frac{\lambda_H}{r + \lambda_H} \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g (1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H} \\
&+ \left(D - \frac{2\lambda_H g}{2r + \lambda_H} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{1+2r/\lambda_H} \right) (1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{2r/\lambda_H}. \tag{22}
\end{aligned}$$

The constants C and D are pinned down by boundary condition $U_A(\rho_A^*) = s$. Since the value U_A depends on posteriors, there is learning value attached to purchasing behavior and the unsure buyer is not making a myopic choice. The monopolist has to provide extra subsidy to deter deviations since the deviator gains by becoming more optimistic: $U_A(\rho) > s$.¹²

Proposition 3 is only a necessary condition to deter “one-shot” deviations. In the appendix, we prove the sufficiency of this result: given the on-equilibrium-path value function $U_A(\rho)$ and off-equilibrium-path value function $U(\rho, \rho_h)$, it is not optimal for an experimenting buyer to deviate for any $h > 0$ length of time.

Denote the equilibrium price in the aggregate learning phase to be $P_A(\rho)$. Then, the value for a buyer from purchasing the risky product can be characterized by the following HJB equation:

$$rU_A(\rho) = r(\rho q(\rho)g - P_A(\rho)) + \lambda_H \rho q(\rho)(U_I(\rho) - U_A(\rho)) + \lambda_H \rho q(\rho)(V_I(\rho) - U_A(\rho)) - \lambda_H \rho (1 - \rho)U_A'(\rho) \tag{23}$$

where $q(\rho) = \frac{q_0(1-\rho_0)^2}{q_0(1-\rho_0)^2 + (1-q_0)(1-\rho)^2}$, $U_I(\rho) \equiv s$, and $V_I(\rho)$ is given by equation (13).

Meanwhile, by selling the products, the monopolist’s value can be characterized as follows:

$$rJ_A(\rho) = 2rP_A(\rho) + 2\rho q(\rho)\lambda_H(J_I(\rho) - J_A(\rho)) - \lambda_H \rho (1 - \rho)J_A'(\rho). \tag{24}$$

where $J_I(\rho)$ is given by proposition 2.

Equations (23) and (24) are value functions if both unsure buyers purchase the risky product. The RHS of equation (23) can be decomposed into the following four elements: i) the expected payoff rate from purchasing the risky product $r(\rho q(\rho)g - P_A(\rho))$; ii) the jump of the value function to V_I if a given buyer receives a lump-sum payoff; iii) the drop of the value function to $U_I \equiv s$ if the other buyer receive a lump-sum payoff; and iv) the effect of Bayesian updating on the value function when no lump-sum is received. Equation (24) could be interpreted similarly.

The on-equilibrium-path price $P_A(\rho)$ can be derived from the on-equilibrium-path value function

¹¹The undetermined coefficient in the differential equation is chosen such that $U_A(\rho)$ is continuous at ρ_I^* .

¹²The inequality $U_A(\rho) > s$ also rules out the existence of mixed strategy equilibria. Certainly, the monopolist never has incentives to mix. The buyers will not mix in equilibrium because mixing implies they are indifferent between the alternatives: $U_A(\rho) = s$. But this would cause deviations.

$U_A(\rho)$. It is straightforward to show: in a mass market,

$$P_A(\rho) = \rho q(\rho)g - s + \frac{\lambda_H}{2r + \lambda_H} g \rho (1 - \rho) q(\rho) + C q(\rho) (1 - \rho)^2 \left(\frac{1 - \rho}{\rho}\right)^{2r/\lambda_H} \quad (25)$$

for $\rho > \rho_A^*$; in a niche market

$$P_A(\rho) = \rho q(\rho)g - s - \frac{\lambda_H}{2r + \lambda_H} g \rho (1 - \rho) q(\rho) + D q(\rho) (1 - \rho)^2 \left(\frac{1 - \rho}{\rho}\right)^{2r/\lambda_H} \quad (26)$$

for $\rho_A^* < \rho \leq \rho_I^*$, and

$$P_A(\rho) = \rho q(\rho)g - s + \frac{\lambda_H}{2r + \lambda_H} g \rho (1 - \rho) q(\rho) + \left(D - \frac{2\lambda_H g}{2r + \lambda_H} \left(\frac{\rho_I^*}{1 - \rho_I^*}\right)^{1+2r/\lambda_H}\right) q(\rho) (1 - \rho)^2 \left(\frac{1 - \rho}{\rho}\right)^{2r/\lambda_H} \quad (27)$$

for $\rho > \rho_I^*$. In the above equations, C and D are constants determined in equations (20) and (21). Notice in equations (26) and (27), the signs in front of term $\frac{\lambda_H}{2r+\lambda_H} g \rho (1 - \rho) q(\rho)$ are different. This reflects the change in continuation value when ρ drops below ρ_I^* . By proposition 2, for $\rho \leq \rho_I^*$, upon the arrival of the first lump-sum payoff, the monopolist immediately shut down experimentation and charge price $g - s$. This greatly reduces the unsure buyers' incentives to experiment. However, it is easy to check that in a niche market, the price $P_A(\rho)$ is still continuous at ρ_I^* .

We substitute the price expression $P_A(\rho)$ into equation (24) and characterize the equilibrium cutoff ρ_A^* by applying value matching and smooth pasting conditions:

$$U_A(\rho_A^*) = s, \quad J_A(\rho_A^*) = 0, \quad J'_A(\rho_A^*) = 0.$$

Proposition 4 (*Characterize the symmetric Markov perfect equilibrium*) *In the aggregate learning phase, the unsure buyers purchase the risky product under posterior beliefs (ρ, q) if and only if*

$$\rho q > \frac{rs}{rg + \lambda_H(V_I(\rho) + J_I(\rho)) - \lambda_H s}.$$

A mass market appears if and only if

$$\frac{1 - q_0}{q_0(1 - \rho_0)^2} > \frac{g}{(1 - \rho_I^*)s}. \quad (28)$$

Moreover, for all $\rho_0 < 1$ and $q_0 < 1$, the symmetric Markov perfect equilibrium is inefficient so that experimentation is terminated too early.

Proof. In the appendix. ■

The unique equilibrium cutoff ρ_A^* is characterized by equation

$$\rho q(\rho) = \frac{rs}{rg + \lambda_H(V_I(\rho) + J_I(\rho)) - \lambda_H s}. \quad (29)$$

It is straightforward to show the equilibrium is inefficient by comparing the efficient stopping curve with the equilibrium stopping curve. The result is quite intuitive. The inefficiency in the individual learning phase causes a leakage of social surplus for the monopolist, which reduces the monopolist's incentives to subsidize experimentation in the aggregate learning phase. Therefore, the equilibrium experimentation is terminated too early in the aggregate learning phase as well.

There are two remarks about proposition 4. First, it is straightforward to check that at ρ_A^* , the smooth pasting condition for $U_A(\cdot)$ is also satisfied: $U'_A(\rho_A^*) = 0$. Explicitly, the monopolist is solving an optimal stopping problem given the price she has to charge in order to keep the unsure buyers experiment. Implicitly, given the equilibrium pricing strategy $P_A(\cdot)$, the unsure buyers are facing an optimal stopping problem as well. At the equilibrium cutoff, the smooth pasting condition for $U_A(\cdot)$ should also be satisfied. This fact is useful when we discuss efficiency for any $n \geq 2$ buyers because it enables us to characterize equilibrium cutoff without solving value functions. Second, the appearance of a mass or niche market depends on the relative importance of social learning and individual learning. Given q_0 , when ρ_0 goes up, the monopolist has higher incentives to keep the remaining unsure buyer experimenting. A mass market is more likely to appear as a result.

3.2.4 Equilibrium Price Path

After solving the equilibrium cutoff ρ_A^* , the constants C and D in equations (20) and (21) can be solved from the value matching condition and the expression for equilibrium price can be derived. Figure 2 depicts different price paths in a symmetric Markov perfect equilibrium depending on how many buyers have received lump-sum payoffs. Notice that in the figure, when the first lump-sum arrives, there is an instantaneous drop in price in order to encourage the buyer who remains unsure to experiment.

The negative response of the monopoly price to the arrival of good news seems to be surprising. In a mass market, the equilibrium prices are given by

$$P_A(\rho) = \rho q(\rho)g - s + \frac{\lambda_H}{2r + \lambda_H} g \rho (1 - \rho) q(\rho) + C q(\rho) (1 - \rho)^2 \left(\frac{1 - \rho}{\rho}\right)^{2r/\lambda_H}$$

and $P_I(\rho) = \rho g - s$. In general the price before the arrival of good news might be either larger or smaller than the price after the arrival of good news. The arrival of good news brings two opposite effects on the reservation value of the buyer who remains unsure. There is a positive informational effect: the arrival of good news reveals the product characteristic is high and hence makes the unsure buyer more optimistic about the unconditional probability of receiving lump-sum payoffs ($\rho q(\rho)g$ increases to ρg). However, there is another negative continuation value effect: for the buyer who remains unsure, she loses the chance of becoming the first sure buyer to extract rents.¹³ The price

¹³When both buyers are unsure, the continuation value for the buyer who first receives a good news signal is $V_I(\rho) \geq s$; when only one buyer is unsure, the continuation value is just s .

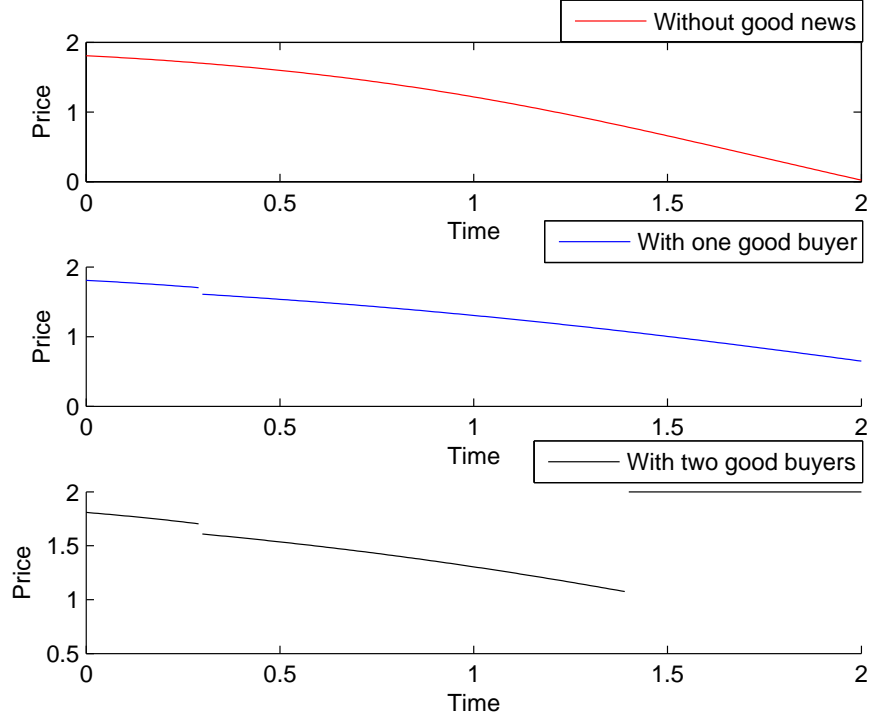


Figure 2: Equilibrium Price Dynamics

has to be lower to compensate the loss of rents if the monopolist wishes to sell to the unsure buyer. The continuation value effect is captured by the term $\frac{\lambda_H}{2r+\lambda_H} g\rho(1-\rho)q(\rho) + Cq(1-\rho)^2 \left(\frac{1-\rho}{\rho}\right)^{2r/\lambda_H} > 0$.¹⁴

Figure 3 describes a situation where with the same priors, the price might either drop or jump depending on the arrival time of the first lump-sum payoff. This figure indicates: at early days of the market, the continuation value effect is more likely to dominate and at late days of the market, the informational effect is more likely to dominate. From the equilibrium price expressions, the comparison of these two effects is closely related to posteriors ρ and q . Given ρ , as q increases (buyers' valuations become more independent), the informational effect becomes smaller and continuation value effect eventually dominates. On the other hand, for a fixed q , as ρ increases (buyers' valuations become more correlated), the informational value effect becomes larger and eventually dominates.

¹⁴The two buyer case is quite extreme in the sense that once the remaining unsure buyer finds out she is good, her value immediately drops to s . This will not happen if there are $n > 2$ buyers. But the reduction in the continuation value also leads to a continuation value effect at smaller magnitude.

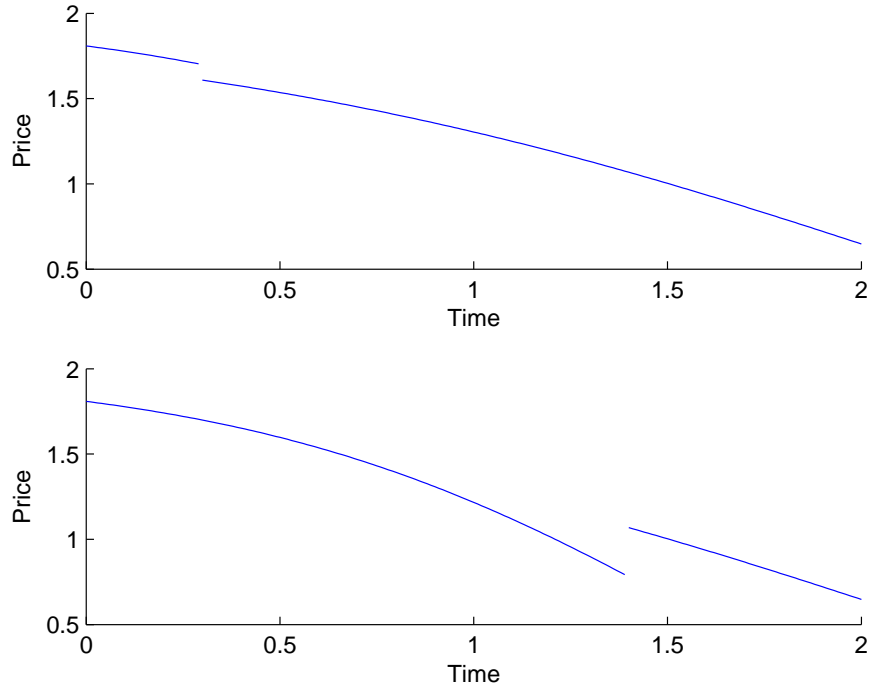


Figure 3: Instantaneous Price Response to the First Arrival of Good News

3.3 Efficiency

This section discusses the efficiency property of the symmetric Markov perfect equilibrium for n buyers. We first investigate the extreme case of common value ($\rho = 1$) and then compare this result to the general interdependent case.

Purely Common Valuations. Under this special case, buyers are *ex post homogeneous*. In other words, if one buyer receives a lump-sum payoff, then it is common knowledge that all buyers are able to receive lump-sum payoffs and the monopolist should immediately raise price to $g - s$ to extract all of the surplus.

In the aggregate learning phase, similarly the monopolist should set a price such that i) each experimenting buyer has an incentive to participate (i.e. the buyer's value is larger than the outside option); ii) it is not optimal for each experimenting buyer to have "one-shot" deviations. The common value assumption simplifies the analysis of the "one-shot deviation" problem since the deviator always has the same posterior belief as the buyers who have not deviated. It turns out that under the common value case, restrictions i) and ii) coincide and the strategic equilibrium is always efficient.

Proposition 5 *Under the common value case ($\rho = 1$), the unsure buyers will always receive value s in equilibrium and the symmetric Markov perfect equilibrium is efficient.*

Proof. In the appendix. ■

The intuitive explanation for the efficiency result in the common value case is that the ex post homogeneity means the monopolist does not need to face the tradeoff between exploitation and exploration. This enables the monopoly to completely internalize the social surplus and overcome the free riding problem by subsidizing experimentation.

Interdependent Valuations. Since ex post heterogeneity exists in the general interdependent case, it will be natural to guess that the inefficiency result in proposition 4 could be extended to general n case. An induction argument could be used to avoid solving every value function explicitly.

Theorem 1 *Consider a market with any $n \geq 2$ buyers. The symmetric Markov perfect equilibrium is inefficient in both the aggregate learning and individual learning phases if $\rho_0 < 1$ and $q_0 < 1$. Moreover, the equilibrium experimentation will always be terminated too early.*

Proof. In the appendix. ■

We are in a position to summarize the roles played by ex post heterogeneity. First, in the aggregate learning phase, ex post heterogeneity means there is a future benefit for the deviator by becoming more optimistic than non-deviators. The monopolist has to provide extra subsidy to deter deviations. In the common value case, such future benefit does not exist and there is no need to provide extra subsidy. Second, in the individual learning phase, ex post heterogeneity implies that the receivers of lump-sum payoffs are more optimistic than the unsure buyers. If the monopolist wishes to serve all buyers, the sure buyers could extract rents. This generates a loss of rents for the buyers who remain unsure upon the arrival of the first lump-sum payoff. The reduction in continuation values leads to an ambiguous price reaction to the arrival of the first lump-sum payoff. On the contrary, in the common value case, the equilibrium value for the buyers is always the same as the outside option and there is no such continuation value effect. Hence, upon the arrival of the first lump-sum payoff, the instantaneous reaction of equilibrium price is always to go up. Finally, ex post heterogeneity generates a tradeoff between exploitation and exploration for the monopolist. Equilibrium experimentation is lower than the socially efficient level as we have seen in the two-buyer case. On the other hand, in the common value case, there is no ex post heterogeneity and the monopolist is able to fully internalize social surplus.

4 Equilibrium in the Bad News Case

In the bad news case, the arrival of lump-sum payoffs (we call them lump-sum damages afterwards) would immediately reveal that the risky product is unsuitable for the buyer. Denote $\xi_f = A$ and $\xi_l = -B < 0$. Condition $A - \lambda_H B < s < A$ is imposed such that the risky product is superior to the safe one only when the buyers cannot receive lump-sum damages.

4.1 Socially Efficient Allocation

As in the previous section, we discuss socially efficient allocation separately in the individual learning and aggregate learning phases. Different from the good news case, large priors (ρ_0, q_0) mean the probability of receiving lump-sum damages is high and discourage the group of buyers to take the risky product. Therefore, instead of solving an optimal stopping problem (i.e., terminating experimentation when belief reaches a certain cutoff), in the bad news case, we solve an optimal starting problem, i.e., beginning experimentation when belief is lower than a certain cutoff.

Socially Efficient Allocation in the Individual Learning Phase. In the individual learning phase, suppose k buyers have received lump-sum damages, the social surplus function could be written as (the sure buyers will take the safe product and receive s for sure)

$$\Omega_k(\rho) = ks + (n - k)W(\rho)$$

where

$$W(\rho) = \sup_{\alpha \in \{0,1\}} \mathbb{E} \int_{t=0}^{\infty} r e^{-rt} [\alpha(A - \lambda_H \rho_t B) + (1 - \alpha)s] dt$$

defines the optimal control problem for the unsure buyer. The corresponding HJB equation is

$$W(\rho) = \max \left\{ s, A - \lambda_H \rho B + \frac{1}{r} [\lambda_H \rho (s - W(\rho)) - \lambda_H \rho (1 - \rho) W'(\rho)] \right\}. \quad (30)$$

Solve the optimal starting problem defined by equation (30) and we get the following result:

Proposition 6 *In the individual learning phase, if $k \geq 1$ buyers are sure to receive lump-sum damages, it is socially efficient for those k buyers to always purchase the safe product. For the remaining $n - k$ unsure buyers, it is socially efficient to begin experimentation if and only if*

$$\rho \leq \rho_I^e = \frac{(r + \lambda_H)(A - s)}{\lambda_H A + r \lambda_H B - \lambda_H s}.$$

The value functions for a typical buyer with posterior belief ρ is given by:

$$W(\rho) = \max \left\{ s, A - \frac{\lambda_H A + r \lambda_H B - \lambda_H s}{r + \lambda_H} \rho \right\}.$$

Socially Efficient Allocation in the Aggregate Learning Phase. In the aggregate learning phase, we similarly write down the HJB equation as:

$$\Omega_A(\rho) = \max \left\{ ns, n(A - \lambda_H \rho q(\rho) B) + \frac{1}{r} [\lambda_H n \rho q(\rho) (\Omega_1(\rho) - \Omega_A(\rho)) - \lambda_H \rho (1 - \rho) \Omega'_A(\rho)] \right\}. \quad (31)$$

The optimal starting problem (31) is solved by solving differential equation

$$(r + \lambda_H n \rho q) \Omega_A(\rho) = rn(A - \lambda_H \rho q B) + \lambda_H n \rho q [(n - 1)W(\rho) + s] - \lambda_H \rho (1 - \rho) \Omega'_A(\rho), \quad (32)$$

with boundary condition $\Omega_A(\rho_A^e) = ns$.¹⁵

The socially efficient allocation in the aggregate learning phase is characterized by the following proposition:

Proposition 7 *Given any $q_0 < 1$, there exists a unique $\rho_A^e(q_0) > \rho_I^e$ ($\rho_A^e(q_0)$ could be one) such that it is socially efficient to start experimentation in the aggregate learning phase if and only if $\rho \leq \rho_A^e(q_0)$.*

Proof. In the appendix. ■

4.2 Equilibrium

In any symmetric equilibrium, buyers can be divided into two groups: sure buyers and unsure buyers. Let α_k^0 (α_k^1) be strategy for the sure (unsure) buyers where subscript k indicates the number of buyers who have received lump-sum damages. Let V_k , U_k and J_k be value functions for the sure buyers, the unsure buyers and the monopolist respectively when k buyers have received lump-sum damages. Finally, let P_k denote the price charged by the monopolist. Definition 3 implies that the triple of $(P_k, \alpha_k^0, \alpha_k^1)$ is a symmetric Markov perfect equilibrium if:

- $\alpha_k^0 = 1$ if $P \leq A - \lambda_H B - s$ and $= 0$ otherwise;
- for any $k < n$, given P_k , the unsure buyers choose acceptance policy α_k^1 to maximize:

$$\begin{aligned} U_k(\rho) &= \sup_{\alpha_k^1} \mathbb{E} \int_{t=0}^{\tau} r e^{-rt} [\alpha_k^1 (A - \rho_t q_k(\rho_t) \lambda_H B - P_k(\rho_t)) + (1 - \alpha_k^1) s] dt \\ &+ e^{-r\tau} \left(\frac{1}{n-k} V_{k+1}(\rho_\tau) + \frac{n-k-1}{n-k} U_{k+1}(\rho_\tau) \right) \end{aligned}$$

where τ is the first (possibly infinite) time at which a new unsure buyer receives good news;

- given (α_k^0, α_k^1) , the monopolist choose price $P_k(\rho_t)$ to maximize

$$J_k(\rho) = \sup_{P_k} \mathbb{E} \left\{ \int_{t=0}^{\tau} r e^{-rt} [k \alpha_k^0 (P_k(\rho_t)) + (n-k) \alpha_k^1(\rho_t, P_k(\rho_t))] dt + e^{-r\tau} J_{k+1}(\rho_\tau) \right\}$$

¹⁵Notice that $W(\rho)$ is not continuously differentiable at ρ_I^e (smoothing pasting condition is no longer satisfied). But it is Lipschitz continuous and hence the solution to above boundary value problem is still unique.

- beliefs update according to Bayes' rule: ρ_t satisfies the law of motion, i.e. equation (1); $q_k(\rho_t) \equiv 1$ for $k \geq 1$ and $q_k(\rho_t)$ is given by equation (6) for $k = 0$;
- for $k = n$, the monopolist will not serve any buyer such that $J_n \equiv 0$ and $V_n \equiv s$.

First, it is straightforward to see that the sure buyers will buy the risky product if the price is lower than $A - \lambda_H B - s$ and not buy otherwise. Second, the assumption $A - \lambda_H B - s < 0$ implies that selling to the sure buyers is purely losing money. Hence, a profit-maximizing monopolist should never set price lower than $A - \lambda_H B - s$ in order to sell to the sure buyers. This also implies that $V_k \equiv s$. Third, when $n - k$ unsure buyers purchase the risky product, the conditional probability that any given unsure buyer receives lump-sum damages is simply $1/(n - k)$, since the $n - k$ unsure buyers' payoff distributions are identical. Finally and most importantly, the cutoff strategy for the monopolist means that she will *start* selling to the unsure buyers if the belief ρ is lower than a certain cutoff. Once the monopolist starts to sell to the unsure buyers, she will continue to sell as long as no lump-sum damage is received.

In a symmetric Markov perfect equilibrium, when experimentation takes place on equilibrium path, the monopolist also has to charge a price such that both participation constraint and no profitable one-shot deviation constraint are satisfied. In the bad news case, it turns out that the "one-shot" deviations don't impose more restrictions than the participation constraint.

Claim 1 *In equilibrium, the most pessimistic unsure buyer's value is always s .*

Claim 1 implies that the on-equilibrium-path value for each unsure buyer is always s since they are equally pessimistic. This is different from proposition 3 in the good news case. In the good news case, a one-shot deviation makes the non-deviators more pessimistic if they haven't received any lump-sum payoffs during the deviation period. As a result, once a deviation happens, the price charged by the monopolist is lower than what the deviator is willing to pay. The deviator can benefit from the deviation and thus the equilibrium value for the experimenting buyers has to be larger than s to deter deviations. However, in the bad news case, a one-shot deviation makes the deviator more pessimistic. After the deviation, if the monopolist wishes to serve all unsure buyers, the optimal price is determined by what the deviator is willing to pay; if the monopolist does not wish to serve all unsure buyers, the deviator is the first buyer to be excluded. In both cases, the deviator cannot gain more than the outside option after the deviation. Therefore, setting the on-equilibrium-path value to be s is enough to deter deviations.

The equilibrium price path could be derived from claim 1: in the individual learning phase, the monopolist would charge $P_I(\rho) = A - \lambda_H \rho B - s$ and in the aggregate learning phase, the monopolist would charge $P_A(\rho) = A - \lambda_H \rho q(\rho) B - s$. The arrival of first lump-sum damage will unanimously lead to a drop in price if $q_0 < 1$ but the subsequent arrival of lump-sum damages

will not have any impact on price. The negative response in price to the first arrival of lump-sum damages can be explained by the fact that there is no continuation value effect from claim 1. The informational effect associated with the first arrival of lump-sum damages always discourages unsure buyers to experiment and reduces the price. But the subsequent arrival of bad news reveals no more information to the remaining unsure buyers and hence will not affect the price at all. Solve the monopolist’s optimal starting problem and we get the following theorem:

Theorem 2 *Consider a market with $n \geq 2$ buyers. The symmetric Markov perfect equilibrium is efficient in both the aggregate learning and individual learning phases.*

Proof. In the appendix. ■

The above theorem is very intuitive: different from the good news model, there is no tradeoff between exploitation and exploration in the individual learning phase because the buyers who have received lump-sum damages will never purchase the risky product. As a result, although buyers become ex post heterogeneous, the buyers who will purchase the risky product are always the unsure buyers, who are ex post homogeneous in a symmetric equilibrium. Hence, the equilibrium is always efficient in the individual learning phase. The efficiency in the aggregate learning phase is a little surprising. It seems that the monopolist cannot fully internalize social surplus since the unsure buyers can benefit from social learning by switching to the safe product. The intuition turns out to be incorrect. In the good news case, the society benefits from the arrival of good news but the receivers of the lump-sum payoffs pay less than what they are willing to pay. In other words, they can “steal” some of the social surplus from the monopolist. On the contrary, in the bad news case, the society benefits from the *non-arrival* of the bad news. The unsure buyers cannot “steal” social surplus from the monopolist when no lump-sum damages have been received.

5 Conclusion

By combining aggregate and idiosyncratic uncertainty, this paper relaxes the usual common value assumption made in the social learning literature (see, i.e., Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992) and Rosenberg, Solan, and Vieille (2007)).¹⁶ We consider a dynamic monopoly pricing environment where the monopolist cannot price discriminate among the buyers. The interdependence of buyers’ valuations generates ex post heterogeneity. If the monopolist wishes to serve multiple buyers, the optimal price is set to make the most pessimistic buyer indifferent between the alternatives. In the good news case, this has significant implications both on equilibrium path and off equilibrium path. On equilibrium path, the receivers of lump-sum payoffs become more optimistic than the non-receivers. This implies: i) the arrival of the first good news

¹⁶An exception is Murto and Välimäki (2009), which considers interdependent value in an observational learning setting.

signal generates a continuation value effect, which might lead to an instantaneous drop in price; and ii) the monopolist faces different buyers after the arrival of lump-sum payoffs and the absence of price discrimination leads to an inefficient equilibrium. On the contrary, if there is perfect payoff correlation among the buyers, the arrival of the first good news signal always leads to a jump in price and the equilibrium is efficient.

There is another subtle off-equilibrium-path implication. By taking the outside option, each buyer can extract rents if she becomes more optimistic than other buyers. This generates a future benefits from free riding. If the monopolist wishes to serve multiple unsure buyers, each unsure buyer receives a value higher than the outside option to deter deviations. If there is perfect payoff correlation among the buyers, there is no need to provide such an extra subsidy.

However, in the bad news case, the above implications do not exist. There are two reasons. On equilibrium path, the receivers of lump-sum damages immediately take the outside option and the buyers who stay in the experience good market are still ex post homogeneous. Off equilibrium path, a buyer cannot benefit from deviations because the deviator becomes more pessimistic after a deviation.

There are several extensions to consider in the future. For tractability, we have assumed the arrival of lump-sum payoffs immediately resolves the aggregate uncertainty and the idiosyncratic uncertainty of the receiver. It is possible to consider a model where the arrival of lump-sum payoffs cannot immediately resolves the aggregate uncertainty or the idiosyncratic uncertainty of the receiver. For example, we may assume lump-sum payoffs arrive at another Poisson rate when the product characteristic is low. As long as ex post heterogeneity exists, the resulting equilibrium would be inefficient as well.

A natural extension of the current model is to consider a dynamic duopoly pricing environment. This issue is partially investigated by Bergemann and Välimäki (2002), which considers a model with a continuum of buyers such that buyers are choosing according to their myopic preferences at each instant of time. It would be interesting to consider a model with a finite number of buyers such that each buyer's choice has non-trivial effects on learning and future prices.

Appendix

A General Solution to Linear First Order Ordinary Differential Equations

The following observation is widely used throughout the paper to solve linear first order ordinary differential equations.

Observation 1 *Given that f and g are continuous functions on an interval I , the ordinary differential equation $y' + f(x)y = g(x)$ has general solution*

$$y(x) = \frac{H(x)}{h(x)}$$

where $h(x) = e^{R(x)}$, $R(x)$ is an antiderivative of $f(x)$ on I and $H(x)$ is an antiderivative of $h(x)g(x)$ on I .

Proof. Multiply both sides of differential equation $y' + f(x)y = g(x)$ by $h(x)$. Then the original differential equation becomes

$$\frac{d}{dx}(h(x)y(x)) = h(x)g(x).$$

After integration, it is straightforward to see that the general solution is $y(x) = \frac{H(x)}{h(x)}$. ■

B Proofs of Results from Section 3

B.1 Proof of Proposition 1

Proof. Before proving the proposition, we first show the socially optimal allocation is indeed symmetric.

Claim 2 *The socially optimal allocation is symmetric when buyers are homogeneous.*

Proof. For any posteriors ρ , denote the social surplus to be $\Omega(\rho)$. The social planner's problem can be written as:

$$\Omega(\rho) = \sup_{\alpha(\cdot) \in \{0,1\}^n} \mathbb{E} \left\{ \int_{t=0}^h r e^{-rt} \sum_{i=1}^n [\alpha_i(\rho_t) \rho_{it} q(\rho_t) g + (1 - \alpha_i(\rho_t)) s] dt + e^{-rh} \Omega(\rho_h | \alpha) \right\}.$$

Consider any $\tilde{\rho}$ which is a permutation of ρ . The social surplus should be the same: $\Omega(\rho) = \Omega(\tilde{\rho})$ since the strategies α can be permuted as well. When buyers are homogeneous with the same prior ρ_0 , denote $\rho_0 = (\rho_0, \dots, \rho_0)$. Then from the HJB equation, it is socially optimal for buyer i to purchase the risky product if and only if:

$$r \rho_0 q_0 g + \rho_0 q_0 \lambda_H (\Omega_1(\rho_0) - \Omega(\rho_0)) - \lambda_H \rho (1 - \rho) \frac{\partial \Omega(\rho_0)}{\partial \rho_i} > r s.$$

Since $\Omega(\rho) = \Omega(\tilde{\rho})$, for any $j \neq i$, we can switch i and j without affecting the partial derivatives. In other words, the partial derivatives are identical: $\frac{\partial \Omega(\rho_0)}{\partial \rho_i} = \frac{\partial \Omega(\rho_0)}{\partial \rho_j}$. Therefore, it is socially optimal for buyer i to purchase the risky product if and only if it is also optimal for j to purchase, which implies the socially optimal allocation is symmetric. ■

Notice in equation

$$r n \rho q(\rho) g + n \rho q(\rho) \lambda_H \Omega_1(\rho) = (r + n \rho q(\rho) \lambda_H) n s, \quad (33)$$

$\Omega_1(\cdot)$ is a piece-wise function since $W(\cdot)$ is a piece-wise function. The next result claims that ρ_A^e is always larger than ρ_I^e such that $\Omega_1(\rho_A^e) > (n-1)s + g$.

Claim 3 *Beginning from any combination of ρ_0 and $q_0 < 1$, the efficient cutoff in the aggregate learning phase will always be larger than the efficient cutoff in the individual learning phase: $\rho_A^e > \rho_I^e$.*

Proof. We first substitute the expression $\Omega_1(\rho) = g + (n-1)W(\rho)$ into equation (33) and get

$$rn\rho q(\rho)g + n\rho q(\rho)\lambda_H[g + (n-1)W(\rho)] = (r + n\rho q(\rho)\lambda_H)ns. \quad (34)$$

By contradiction, assume $\rho_A^e \leq \rho_I^e$ and then from the definition of $W(\cdot)$, it must be $W(\rho_A^e) = s$. Equation (34) gives us a cutoff $\tilde{\rho}_A^e$ satisfying

$$\tilde{\rho}_A^e q(\tilde{\rho}_A^e) = \rho_I^e = \frac{rs}{(r + \lambda_H)g - \lambda_H s}.$$

As $q(\tilde{\rho}_A^e) < 1$, the above equation implies that: $\tilde{\rho}_A^e > \rho_I^e$, which contradicts assumption $\rho_A^e \leq \rho_I^e$. Therefore, it must be true that $\rho_A^e > \rho_I^e$ and thus $W(\rho_A^e) > s$. ■

From claim 3, ρ_A^e should satisfy equation (34) where $q(\rho_A^e)$ is given by equation (6). Given the priors, the efficient cutoffs $(\rho_A^e(\rho_0, q_0), q_A^e(\rho_0, q_0))$ can be solved jointly:

$$q_A^e = \frac{rs}{\rho_A^e[(r + \lambda_H)g + (n-1)\lambda_H W(\rho_A^e) - n\lambda_H s]}. \quad (35)$$

$$q_A^e = \frac{(1 - \rho_0)^n q_0}{(1 - \rho_0)^n q_0 + (1 - \rho_A^e)^n (1 - q_0)}. \quad (36)$$

Clearly, $W(\rho_A^e)$ is increasing in ρ_A^e and thus q_A^e is decreasing in ρ_A^e from equation (35). Equation (36) describes how ρ and q evolve jointly over time: since both ρ and q decrease over time, q_A^e is increasing in ρ_A^e . Hence the intersection of equations (36) and (35) is unique. Equation (35) describes the stopping curve such that it is socially efficient to keep experimentation if

$$\rho q > \frac{rs}{(r + \lambda_H)g + (n-1)\lambda_H W(\rho_A^e) - n\lambda_H s}.$$

Finally, we still have to check that it is indeed the case that $\rho_A^e > \rho_I^e$. Notice that ρ_A^e is decreasing in q_A^e on the stopping curve. If $q = 1$, it is easy to check the unique cutoff ρ_A^e is the same as $\rho_I^e = \frac{rs}{(r + \lambda_H)g - \lambda_H s}$. And for $q_A^e < 1$, we should have $\rho_A^e > \rho_I^e$. ■

B.2 Proof of Proposition 2

Proof. In the individual learning phase, denote ρ to be the common posterior belief about the unsure buyer's idiosyncratic uncertainty. The equilibrium purchasing decision of the unsure buyer is characterized by cutoff ρ_I^* .

Denote $P_I(\rho)$ as the price set by the monopolist for $\rho > \rho_I^*$. Then, the value function for the unsure buyer satisfies

$$rU_I(\rho) = r(g\rho - P_I(\rho)) + \rho\lambda_H(s - U_I(\rho)) - \lambda_H\rho(1 - \rho)U_I'(\rho).$$

Participation constraint implies that $U_I(\rho)$ should be no less than s , the value from taking the outside option. Obviously, a profit-maximizing monopolist will always set prices such that the two values are exactly the same, or equivalently, $P_I(\rho) = g\rho - s$.

The monopolist's problem is to choose between charging a low price $g\rho - s$ to keep experimentation and charging a high price $g - s$ to only serve the sure buyer. Obviously, this is an optimal stopping problem with HJB equation

$$rJ_I(\rho) = \max \{r(g - s), 2r(g\rho - s) + \rho\lambda_H(2(g - s) - J_I(\rho)) - \lambda_H\rho(1 - \rho)J_I'(\rho)\}. \quad (37)$$

On the RHS of equation (37), $g - s$ is the value if the monopolist only sells to the good buyer by charging $g - s$; if the monopolist decides to continue experimentation, she not only receives instantaneous revenue from selling to both buyers $2(g\rho - s)$ but also may receive a future value of $2(g - s)$ if the unsure buyer receives lump-sum payoffs. From the value matching and smooth pasting conditions, it is straightforward to find out the equilibrium cutoff as

$$\rho_I^* = \frac{r(g + s)}{2rg + \lambda_H(g - s)}.$$

The equilibrium value function $J_I(\rho)$ could be solved as:

$$J_I(\rho) = \begin{cases} 2(g\rho - s) + (g + s - 2g\rho_I^*) \frac{1-\rho}{1-\rho_I^*} \left[\frac{(1-\rho)\rho_I^*}{(1-\rho_I^*)\rho} \right]^{r/\lambda_H} & \text{if } \rho > \rho_I^* \\ g - s & \text{otherwise.} \end{cases}$$

For the sure buyer, she only needs to pay $P_I(\rho) = g\rho - s < g - s$ before ρ reaches ρ_I^* but has to pay $g - s$ afterwards. The value function for this buyer is given by differential equation

$$rV_I(\rho) = r(g(1 - \rho) + s) + \rho\lambda_H(s - V_I(\rho)) - \lambda_H\rho(1 - \rho)V_I'(\rho) \quad (38)$$

for $\rho > \rho_I^* = \frac{r(g+s)}{2rg+\lambda_H(g-s)}$ and $V_I(\rho) = s$ for $\rho \leq \rho_I^* = \frac{r(g+s)}{2rg+\lambda_H(g-s)}$. Equation (38) is an ordinary differential equation with boundary condition: $V_I(\rho_I^*) = s$. This gives us the expression of $V_I(\rho)$ in the proposition. ■

B.3 Characterize $\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h}$

Lemma 1 Fix a pair of priors (ρ_0, q_0) such that ρ_A^* is the equilibrium cutoff in the aggregate learning phase. In a mass market, for any $\rho > \rho_A^*$,

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} &= 2(r + \lambda_H\rho q)(U_A(\rho) - s) + \lambda_H\rho(1 - \rho)U_A'(\rho) \\ &\quad + (r + \lambda_H\rho)g(1 - \rho)q \left(\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)} \right)^{r/\lambda_H} - \lambda_H g \rho (1 - \rho) q \\ &\quad - \left[\frac{r + \lambda_H\rho_A^*}{1 - \rho_A^*} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \lambda_H \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H}. \end{aligned} \quad (39)$$

In a niche market, for $\rho_A^* < \rho \leq \rho_I^*$,

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} &= 2(r + \lambda_H\rho q)(U_A(\rho) - s) + \lambda_H\rho(1 - \rho)U_A'(\rho) \\ &\quad - \frac{rg}{r + \lambda_H} \lambda_H \rho (1 - \rho) q + \frac{r\lambda_H g}{r + \lambda_H} \frac{\rho_A^* (1 - \rho)^2 q}{1 - \rho_A^*} \left(\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right)^{r/\lambda_H}; \end{aligned} \quad (40)$$

and for $\rho > \rho_I^*$,

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} &= 2(r + \lambda_H \rho q)(U_A(\rho) - s) + \lambda_H \rho(1 - \rho)U_A'(\rho) \\ &\quad + (r + \lambda_H \rho)g(1 - \rho)q \left(\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)} \right)^{r/\lambda_H} - \lambda_H g \rho(1 - \rho)q \\ &\quad - r \left[\frac{r + \lambda_H + \lambda_H \rho_I^*}{(r + \lambda_H)(1 - \rho_I^*)} \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} - \frac{\lambda_H}{r + \lambda_H} \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+r/\lambda_H} \right] g(1 - \rho)^2 q \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H}. \end{aligned} \quad (41)$$

Proof. First notice that if $\lim_{h \rightarrow 0} \frac{U_A(\rho) - U(\rho, \rho_h)}{h}$ exists, $\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h}$ can be written as:

$$\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} = (r + \lambda_H \rho q(\rho))(U_A(\rho) - s) + \lim_{h \rightarrow 0} \frac{U_A(\rho) - U(\rho, \rho_h)}{h}. \quad (42)$$

Therefore the main issue is to evaluate $U(\rho, \rho_h)$ for $\rho > \rho_h$. We proceed in the following steps:

1. Decompose off-equilibrium-path value function

Fix $h > 0$ to be sufficiently small and we know the monopolist will still sell to both buyers after an h -deviation since her strategy is admissible. Therefore, there exists \bar{h}' such that for all $h' \leq \bar{h}'$, we have:

$$\begin{aligned} U(\rho, \rho_h) &= \mathbb{E} \int_{t=0}^{h'} r e^{-rt} (\rho_t q_t g - \tilde{P}_t) dt \\ &\quad + \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} V_I(\rho_{h+h'}) + \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} s \\ &\quad + [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) - \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-rh'} U(\rho_{h'}, \rho_{h+h'}). \end{aligned} \quad (43)$$

In the above expression, ρ_t is the belief for the deviator and starts from $\rho_0 = \rho$; \tilde{q}_h is the belief about the product characteristic after an h -deviations: $\tilde{q}_h = \frac{q_0(1-\rho_0)^2}{q_0(1-\rho_0)^2 + (1-q_0)(1-\rho)(1-\rho_h)}$; and \tilde{P}_t is the off-equilibrium-path price set by the monopolist after an h -deviation.

By purchasing the risky product, the non-deviator gets value

$$\begin{aligned} U(\rho_h, \rho) &= \mathbb{E} \int_{t=0}^{h'} r e^{-rt} (\rho'_t q_t g - \tilde{P}_t) dt \\ &\quad + \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} s + \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} V_I(\rho_{h'}) \\ &\quad + [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) - \rho_h \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-rh'} U(\rho_{h+h'}, \rho_{h'}), \end{aligned} \quad (44)$$

where ρ'_t is the belief for the deviator and starts from ρ_h .

Obviously, the off-equilibrium-path value function $U(\rho, \rho_h)$ can be decomposed as

$$U(\rho, \rho_h) = U(\rho_h, \rho) + Z(\rho, \rho_h)$$

where $Z(\rho, \rho_h) = U(\rho, \rho_h) - U(\rho_h, \rho)$.

The fact that the ρ_h buyer purchases the risky product means that it is not profitable for her

to have “one-shot” deviations:

$$U(\rho_h, \rho) \geq \tilde{U}(h') = \int_{t=0}^{h'} r e^{-rt} s dt + \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-r h'} s + [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-r h'} U(\rho_h, \rho_{h'}). \quad (45)$$

Since the ρ_h buyer is more pessimistic about the probability of receiving lump-sum payoffs, the optimal off-equilibrium-path price \tilde{P} is set such that the ρ_h buyer has incentives to experiment.

Denote $\tilde{U}(\rho; \rho_h)$ as $U(\rho_h, \rho)$ for a fixed ρ_h since ρ_h does not change in the expression of $\tilde{U}(h')$. The fact that

$$\lim_{h' \rightarrow 0} \frac{U(\rho_h, \rho) - \tilde{U}(h')}{h'} = (r + \lambda_H \rho \tilde{q}_h) \tilde{U}(\rho; \rho_h) - (r + \lambda_H \rho \tilde{q}_h) s + \lambda_H \rho (1 - \rho) \tilde{U}'(\rho; \rho_h)$$

is left-continuous in ρ and ρ_h implies that in equilibrium, the following equation is satisfied:¹⁷

$$\lim_{h' \rightarrow 0} \frac{U(\rho_h, \rho) - \tilde{U}(h')}{h'} = 0.$$

Thus we derive an ordinary differential equation for $\tilde{U}(\rho; \rho_h)$

$$(r + \lambda_H \rho \tilde{q}_h) \tilde{U}(\rho; \rho_h) = (r + \lambda_H \rho \tilde{q}_h) s - \lambda_H \rho (1 - \rho) \tilde{U}'(\rho; \rho_h) \quad (46)$$

where the expression for \tilde{q}_h is provided by equation (5)

$$\tilde{q}_h(\rho) = \frac{q_0(1 - \rho_0)^2}{q_0(1 - \rho_0)^2 + (1 - q_0)(1 - \rho)(1 - \rho_h)}.$$

The off-equilibrium-path value function $U(\rho, \rho_h)$ can be further decomposed as:

$$U(\rho, \rho_h) = \tilde{U}(\rho; \rho_h) + Z(\rho, \rho_h).$$

2. Solve for the off-equilibrium-path value function $\tilde{U}(\rho; \rho_h)$.

Equation (46) is an ordinary differential equation with general solution:

$$\tilde{U}(\rho; \rho_h) = s + C_h \times (1 - \rho) \tilde{q}_h \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H}.$$

When $\rho = \rho_h$, the two buyers are identical and it goes back to the equilibrium path: $\tilde{U}(\rho_h; \rho_h) = U_A(\rho_h)$. This boundary condition implies:

$$C_h = \frac{U_A(\rho_h) - s}{(1 - \rho_h) \tilde{q}_h \left(\frac{1 - \rho_h}{\rho_h} \right)^{r/\lambda_H}}; \quad (47)$$

¹⁷The proof is similar to the proof of lemma 2. If it is strictly larger than zero, we can find a neighborhood of beliefs to increase price $\tilde{P}(\rho, \rho_h, \tilde{q}_h)$ but the buyers will still purchase the risky product. This constitutes a profitable deviation for the monopolist.

where q_h satisfies: $q_h = \frac{q_0(1-\rho_0)^2}{q_0(1-\rho_0)^2 + (1-q_0)(1-\rho_h)^2}$.

Since on equilibrium path, experimentation stops at ρ_A^* , the unsure buyer receives a value less than the outside ($U_A(\rho) < s$) for $\rho < \rho_A^*$. Equation (47) implies that the non-deviator's posterior will never be lower than ρ_A^* no matter how large h is. In other words, the monopolist always stops selling to both buyers if $(\rho, \rho_h) = (f(\rho_A^*; h), \rho_A^*)$, where

$$f(\rho_A^*; h) = \frac{\rho_A^*}{\rho_A^* + e^{-\lambda_H h}(1 - \rho_A^*)}$$

corresponds to the deviator's posterior when the non-deviator's posterior drops to ρ_A^* .

3. Solve for the off-equilibrium-path value function $Z(\rho, \rho_h)$.

Denote

$$Z(t) = Z(\rho(t), \rho_h(t)) = U(\rho(t), \rho_h(t)) - U(\rho_h(t), \rho(t))$$

where $\rho(t)$ and $\rho_h(t)$ are posterior beliefs after t length of time beginning from ρ and ρ_h (given that no lump-sum payoff is received during this period). The posteriors can be expressed as:

$$\rho(t) = \frac{\rho e^{-\lambda_H t}}{\rho e^{-\lambda_H t} + (1 - \rho)}, \quad \rho_h(t) = \frac{\rho_h e^{-\lambda_H t}}{\rho_h e^{-\lambda_H t} + (1 - \rho_h)},$$

and

$$\tilde{q}_h(t) = \frac{q_0(1 - \rho_0)^2}{q_0(1 - \rho_0)^2 + (1 - q_0)(1 - \rho(t))(1 - \rho_h(t))}.$$

Given any $t < h'$, notice equations (43) and (44) also hold for posteriors $(\rho(t), \rho_h(t))$. Subtract equation (44) from (43) yields:

$$\begin{aligned} Z(t) = & \mathbb{E} \int_0^{h''} r e^{-r\tau} (\rho_\tau q_\tau g - \rho'_\tau q_\tau g) d\tau \\ & + e^{-rh''} (1 - e^{-\lambda_H h''}) \{ \rho(t) \tilde{q}_h(t) [V_I(\rho_h(t + h'')) - s] + \rho_h(t) \tilde{q}_h(t) [s - V_I(\rho(t + h''))] \} \\ & + e^{-rh''} \left[1 - \rho(t) \tilde{q}_h(t) (1 - e^{-\lambda_H h''}) - \rho_h(t) \tilde{q}_h(t) (1 - e^{-\lambda_H h''}) \right] Z(t + h''). \end{aligned} \quad (48)$$

Let h'' go to 0 and we get an ordinary differential equation about $Z(t)$:

$$(r + \lambda_H \rho(t) \tilde{q}_h(t) + \lambda_H \rho_h(t) \tilde{q}_h(t)) Z(t) - \dot{Z}(t) = H(t) \quad (49)$$

where

$$H(t) = r(\rho(t) - \rho_h(t)) \tilde{q}_h(t) g + \lambda_H \rho(t) \tilde{q}_h(t) (V_I(\rho_h(t)) - s) - \lambda_H \rho_h(t) \tilde{q}_h(t) (V_I(\rho(t)) - s).$$

In a *mass market*, both $\rho(t)$ and $\rho_h(t)$ are larger than ρ_I^* . In that case,

$$V_I(\rho) = s + g(1 - \rho) \left(1 - \left[\frac{(1 - \rho) \rho_I^*}{\rho(1 - \rho_I^*)} \right]^{r/\lambda_H} \right)$$

and

$$H(t) = r(\rho(t) - \rho_h(t))\tilde{q}_h(t)g + \lambda_H \rho(t)\tilde{q}_h(t)g(1 - \rho_h(t))(1 - [\frac{(1 - \rho_h(t))\rho_I^*}{\rho_h(t)(1 - \rho_I^*)}]^{r/\lambda_H}) \\ - \lambda_H \rho_h(t)\tilde{q}_h(t)g(1 - \rho(t))(1 - [\frac{(1 - \rho(t))\rho_I^*}{\rho(t)(1 - \rho_I^*)}]^{r/\lambda_H}).$$

The solution to differential equation (49) is

$$Z(t) = (\rho(t) - \rho_h(t))\tilde{q}_h(t)g - [(1 - \rho_h(t))(\frac{1 - \rho_h(t)}{\rho_h(t)})^{r/\lambda_H} - (1 - \rho(t))(\frac{1 - \rho(t)}{\rho(t)})^{r/\lambda_H}]\tilde{q}_h(t)g(\frac{\rho_I^*}{1 - \rho_I^*})^{r/\lambda_H} \\ + Ce^{rt}(1 - \rho(t))(1 - \rho_h(t))\tilde{q}_h(t). \quad (50)$$

From the expressions of $\rho(t)$ and $\rho_h(t)$, time t can be inversely expressed as either

$$-\frac{1}{\lambda_H} \log[\frac{(1 - \rho)\rho(t)}{\rho(1 - \rho(t))}] \quad \text{or} \quad -\frac{1}{\lambda_H} \log[\frac{(1 - \rho_h)\rho_h(t)}{\rho_h(1 - \rho_h(t))}].$$

Therefore, e^{rt} can be expressed as either

$$(\frac{\rho(1 - \rho(t))}{(1 - \rho)\rho(t)})^{r/\lambda_H} \quad \text{or} \quad (\frac{\rho_h(1 - \rho_h(t))}{(1 - \rho_h)\rho_h(t)})^{r/\lambda_H}.$$

As a result, $Ce^{rt}(1 - \rho(t))(1 - \rho_h(t))\tilde{q}_h(t)$ can be written as:

$$\tilde{D}_1(1 - \rho(t))(1 - \rho_h(t))\tilde{q}_h(t)(\frac{1 - \rho_h(t)}{\rho_h(t)})^{r/\lambda_H} + \tilde{D}_2(1 - \rho(t))(1 - \rho_h(t))\tilde{q}_h(t)(\frac{1 - \rho(t)}{\rho(t)})^{r/\lambda_H}.$$

When the two buyers are identical, there should be no difference in the values: $Z(\rho(t), \rho_h(t)) = 0$ for $\rho(t) = \rho_h(t)$. This implies $\tilde{D}_1 = -\tilde{D}_2 = D_h$. Drop the time index t to transform $Z(t)$ back into $Z(\rho, \rho_h)$:

$$Z(\rho, \rho_h) = (\rho - \rho_h)\tilde{q}_h g - [(1 - \rho_h)(\frac{1 - \rho_h}{\rho_h})^{r/\lambda_H} - (1 - \rho)(\frac{1 - \rho}{\rho})^{r/\lambda_H}]\tilde{q}_h g(\frac{\rho_I^*}{1 - \rho_I^*})^{r/\lambda_H} \\ + D_h(1 - \rho)(1 - \rho_h)\tilde{q}_h[(\frac{1 - \rho_h}{\rho_h})^{r/\lambda_H} - (\frac{1 - \rho}{\rho})^{r/\lambda_H}]. \quad (51)$$

Observe that: after the non-deviator stops purchasing the risky product, the deviator always receives the outside option. This implies a boundary condition for $Z(\rho, \rho_h)$: $Z(f(\rho_A^*; h), \rho_A^*) = 0$, which gives us an expression for D_h

$$D_h = -\frac{(e^{\lambda_H h} - 1)g}{1 - e^{-rh}}(\frac{\rho_A^*}{1 - \rho_A^*})^{1+r/\lambda_H} + \frac{[1 + (e^{\lambda_H h} - 1)\rho_A^* - e^{-rh}]g}{(1 - \rho_A^*)(1 - e^{-rh})}(\frac{\rho_I^*}{1 - \rho_I^*})^{r/\lambda_H}. \quad (52)$$

As a result, $U(\rho, \rho_h)$ has the expression

$$\begin{aligned}
U(\rho, \rho_h) = & s + (\rho - \rho_h)\tilde{q}_h g + C_h(1 - \rho)\tilde{q}_h \left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H} \\
& - \left[(1 - \rho_h)\left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H} - (1 - \rho)\left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H} \right] \tilde{q}_h g \left(\frac{\rho_I^*}{1 - \rho_I^*}\right)^{r/\lambda_H} \\
& + D_h(1 - \rho)(1 - \rho_h)\tilde{q}_h \left[\left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H} - \left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H} \right], \quad (53)
\end{aligned}$$

where C_h and D_h are given by equations (47) and (52), respectively.

In a *niche market*, there are three cases to consider:

- If both $\rho(t)$ and $\rho_h(t)$ are smaller than ρ_I^* , then both $V_I(\rho(t))$ and $V_I(\rho_h(t))$ are s and hence $H(t) = r(\rho(t) - \rho_h(t))\tilde{q}_h(t)g$. It is straightforward to check that the solution to differential equation (49) is:

$$Z(t) = \frac{rg}{r + \lambda_H}(\rho(t) - \rho_h(t))\tilde{q}_h(t) + Ce^{rt}(1 - \rho(t))(1 - \rho_h(t))\tilde{q}_h(t). \quad (54)$$

Drop time index t and apply the boundary condition $Z(f(\rho_A^*; h), \rho_A^*) = 0$, then we get

$$Z_3(\rho, \rho_h) = \frac{rg}{r + \lambda_H}(\rho - \rho_h)\tilde{q}_h + D_{h3}(1 - \rho)(1 - \rho_h)\tilde{q}_h \left[\left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H} - \left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H} \right] \quad (55)$$

where

$$D_{h3} = -\frac{rg}{r + \lambda_H} \frac{e^{\lambda_H h} - 1}{1 - e^{-rh}} \left(\frac{\rho_A^*}{1 - \rho_A^*}\right)^{1+r/\lambda_H}.$$

- If $\rho(t) > \rho_I^*$ and $\rho_h(t) \leq \rho_I^*$, then

$$H(t) = r(\rho(t) - \rho_h(t))\tilde{q}_h(t)g - \lambda_H \rho_h(t)\tilde{q}_h(t)g(1 - \rho(t)) \left(1 - \left[\frac{(1 - \rho(t))\rho_I^*}{\rho(t)(1 - \rho_I^*)}\right]^{r/\lambda_H}\right).$$

Similarly, we solve Z as:

$$\begin{aligned}
Z_2(\rho, \rho_h) = & \frac{rg}{r + \lambda_H}(\rho - \rho_h)\tilde{q}_h - \frac{\lambda_H g}{r + \lambda_H} \rho_h(1 - \rho)\tilde{q}_h + \rho_h(1 - \rho)\tilde{q}_h g \left[\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)}\right]^{r/\lambda_H} \\
& + D_{h2}(1 - \rho)(1 - \rho_h)\tilde{q}_h \left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H}. \quad (56)
\end{aligned}$$

D_{h2} is determined such that Z_2 and Z_3 coincide when $\rho = \rho_I^*$. This gives us

$$D_{h2} = -\frac{rg}{r + \lambda_H} \left[(e^{(r+\lambda_H)h} - e^{rh}) \left(\frac{\rho_A^*}{1 - \rho_A^*}\right)^{1+r/\lambda_H} + e^{-\lambda_H h} \left(\frac{\rho_I^*}{1 - \rho_I^*}\right)^{1+r/\lambda_H} \right].$$

- If both $\rho(t)$ and $\rho_h(t)$ are larger than ρ_I^* , we have already solved

$$\begin{aligned}
Z_1(\rho, \rho_h) &= (\rho - \rho_h)\tilde{q}_h g - [(1 - \rho_h)\left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H} - (1 - \rho)\left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H}]\tilde{q}_h g \left(\frac{\rho_I^*}{1 - \rho_I^*}\right)^{r/\lambda_H} \\
&\quad + D_{h1}(1 - \rho)(1 - \rho_h)\tilde{q}_h \left[\left(\frac{1 - \rho_h}{\rho_h}\right)^{r/\lambda_H} - \left(\frac{1 - \rho}{\rho}\right)^{r/\lambda_H}\right]. \quad (57)
\end{aligned}$$

D_{h1} is determined such that Z_1 and Z_2 coincide when $\rho_h = \rho_I^*$:

$$D_{h1} = \left[\frac{1}{\rho_I^*} + \frac{(r + \lambda_H)e^{-rh} - \lambda_H - re^{-(r+\lambda_H)h}}{(r + \lambda_H)(1 - e^{-rh})} + \frac{r(e^{\lambda_H h} - 1)}{(r + \lambda_H)(1 - e^{-rh})} \right] \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{1+r/\lambda_H} - D_{h3}.$$

After solving $U(\rho, \rho_h)$, $\lim_{h \rightarrow 0} \frac{U_A(\rho) - U(\rho, \rho_h)}{h}$ can be evaluated by taking limits. In a niche market, we have to consider $\rho \leq \rho_I^*$ and $\rho < \rho_I^*$, separately. Substitute the results into equation (42) and we get the equations stated in lemma 1. ■

B.4 “Binding” Incentive Constraint

Lemma 2 Fix a pair of priors (ρ_0, q_0) such that ρ_A^* is the equilibrium cutoff in the aggregate learning phase. For $\rho > \rho_A^*$, we must have:

$$\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} = 0.$$

Proof. First, it is obvious that

$$\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} \geq 0$$

since $U_A(\rho) \geq \hat{U}(\rho; h)$ for $h \leq \bar{h}$. Suppose by contradiction that there exists ρ_1 such that

$$F(\rho_1) = \lim_{h \rightarrow 0} \frac{U_A(\rho_1) - \hat{U}(\rho_1; h)}{h} = c > 0.$$

From lemma 1, $F(\rho)$ is left continuous in ρ , which implies that if $F(\rho_1) = c > 0$, then there exists h^\dagger and ϵ_1 such that for all $h < h^\dagger$ and $\rho_1 - \epsilon_1 < \rho' < \rho_1$,

$$U_A(\rho') - \hat{U}(\rho'; h) > hc/2.$$

Choose ϵ_2 to satisfy

$$\rho_1 - \epsilon = \frac{\rho_1 e^{-\lambda_H h^\dagger}}{\rho_1 e^{-\lambda_H h^\dagger} + (1 - \rho_1)}$$

and define $\hat{\epsilon} = \min\{\epsilon_1, \epsilon_2\}$. Now define a new pricing strategy such that

$$\tilde{P}_A(\rho) = \begin{cases} P_A(\rho) + \frac{a}{2} & \text{if } \rho_1 - \hat{\epsilon} < \rho \leq \rho_1 \\ P_A(\rho) & \text{otherwise.} \end{cases}$$

Obviously, the unsure buyer will still purchase the risky product since

$$U_A(\rho') - \hat{U}(\rho'; h) > hc/2.$$

But the monopolist can obtain a higher profit and hence this constitutes a profitable deviation for the monopolist. Therefore, it is impossible to have

$$\lim_{h \rightarrow 0} \frac{U_A(\rho) - \hat{U}(\rho; h)}{h} > 0$$

in equilibrium. ■

B.5 Sufficiency of Proposition 3

Lemma 3 *The value functions derived are sufficient to deter one-shot deviations in the sense that:*

1. *given the expressions of on-equilibrium path value function $U_A(\rho)$ and off equilibrium path value function $U(\rho, \rho_h)$, it is not optimal for an experimenting buyer to deviate for any $h > 0$ length of time on equilibrium path;¹⁸*
2. *given the off equilibrium path value function $U(\rho_h, \rho)$, the ρ_h buyer will not deviate for any h' length of time.*

Proof. To show there is no profitable one-shot deviation on equilibrium path, we consider mass market and niche market separately.

In a mass market, $\rho > \rho_I^*$. Then the value function for a deviator with deviation length $h > 0$ is given by:

$$\hat{U}(\rho; h) = \int_{t=0}^h re^{-rt} s dt + \rho q(1 - e^{-\lambda_H h})e^{-rh} s + [1 - \rho q(1 - e^{-\lambda_H h})]e^{-rh} U(\rho, \rho_h)$$

where $U(\rho, \rho_h)$ satisfies equation (53).

By rearranging terms,

$$\hat{U}(\rho; h) - s = e^{-rh}[1 - \rho q(1 - e^{-\lambda_H h})](U(\rho, \rho_h) - s). \quad (58)$$

By Bayes' rule,

$$\rho_h = \frac{\rho e^{-\lambda_H h}}{1 - \rho(1 - e^{-\lambda_H h})} \quad \text{and} \quad \tilde{q}_h = \frac{q[1 - \rho(1 - e^{-\lambda_H h})]}{1 - \rho q(1 - e^{-\lambda_H h})},$$

such that,

$$\rho_h \tilde{q}_h = \frac{\rho q e^{-\lambda_H h}}{1 - \rho q(1 - e^{-\lambda_H h})}.$$

¹⁸Given the one-shot deviations are not optimal, it is not optimal either for a deviator to have a double-deviation since the deviator weakly prefers purchasing the risky product after an h -deviation (strictly prefers when the monopolist sells to both buyers). This logic can be extended such that any deviation with countably many deviating times is not optimal. Admissibility of strategies requires the deviating times to be at most countable. Therefore, the non-profitability of one-shot deviations basically rules out all deviations allowed in our model.

We can directly evaluate $U_A(\rho) - \hat{U}(\rho; h)$ and get

$$U_A(\rho) - \hat{U}(\rho; h) = \left[\frac{\lambda_H(1 - e^{-(2r+\lambda_H)h})}{2r + \lambda_H} - e^{-rh}(1 - e^{-\lambda_H h}) \right] g\rho(1 - \rho)q \\ + (e^{\lambda_H h} - 1 - \frac{\lambda_H(1 - e^{-rh})}{r}) \left[\left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{r/\lambda_H} - \left(\frac{\rho_I^*}{1 - \rho_I^*} \right)^{r/\lambda_H} \right] gq(1 - \rho)^2 \frac{\rho_A^*}{1 - \rho_A^*} \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H}.$$

The next step is to show both

$$S(h) \triangleq \frac{\lambda_H(1 - e^{-(2r+\lambda_H)h})}{2r + \lambda_H} - e^{-rh}(1 - e^{-\lambda_H h})$$

and

$$T(h) \triangleq (e^{\lambda_H h} - 1 - \frac{\lambda_H(1 - e^{-rh})}{r})$$

are larger than zero for $h \geq 0$. Notice $S(0) = 0$ and

$$S'(h) = \lambda_H e^{-(2r+\lambda_H)h} + r e^{-rh}(1 - e^{-\lambda_H h}) - \lambda_H e^{-(r+\lambda_H)h} = e^{-rh} [\lambda_H x^{\frac{r+\lambda_H}{\lambda_H}} + r - (r + \lambda_H)x]$$

where $x = e^{-\lambda_H h} \in (0, 1]$. For $x \leq 1$, $\lambda_H x^{\frac{r+\lambda_H}{\lambda_H}} + r - (r + \lambda_H)x$ is decreasing in x and $\lambda_H x^{\frac{r+\lambda_H}{\lambda_H}} + r - (r + \lambda_H)x$ achieves its minimum at $h = 0$. Hence, $S'(h) \geq 0$ and $S(h)$ also achieves its minimum of zero at $h = 0$. As a result, $S(h) \geq 0$ for all $h \geq 0$. Similarly, it can be shown that $T(0) = 0$, $T'(0) = 0$ and $T''(h) > 0$. Therefore, $T'(h) \geq 0$ for all $h \geq 0$ and $T(h) \geq 0$ as well.

In a mass market, $\rho_A^* > \rho_I^*$. Hence, $S(h) \geq 0$ and $T(h) \geq 0$ imply that $U_A(\rho) - \hat{U}(\rho; h) \geq 0$: there is no profitable one-shot deviation for any $h \geq 0$.

In a niche market, we also have to consider the following three cases.

- Case 1: $\rho \leq \rho_I^*$.

Then, it is straightforward to show

$$\hat{U}(\rho; h) - s = \left[\frac{r\lambda_H e^{-(2r+\lambda_H)h}}{(2r + \lambda_H)(r + \lambda_H)} + \frac{r e^{-rh}(1 - e^{-\lambda_H h})}{r + \lambda_H} \right] g\rho(1 - \rho)q \\ - \frac{[e^{-rh}\lambda_H + r(e^{\lambda_H h} - 1)]}{r + \lambda_H} \frac{g(1 - \rho)^2 q \rho_A^*}{1 - \rho_A^*} \left[\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right]^{r/\lambda_H} \\ + Dq(1 - \rho)^2 \left(\frac{1 - \rho}{\rho} \right)^{2r/\lambda_H} \quad (59)$$

and

$$U_A(\rho) - s = \frac{r\lambda_H}{(2r + \lambda_H)(r + \lambda_H)} g\rho(1 - \rho)q - \frac{\lambda_H g}{r + \lambda_H} \frac{(1 - \rho)^2 q \rho_A^*}{1 - \rho_A^*} \left[\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right]^{r/\lambda_H} \\ + Dq(1 - \rho)^2 \left(\frac{1 - \rho}{\rho} \right)^{2r/\lambda_H}. \quad (60)$$

In order to show $\hat{U}(\rho; h) \leq U(\rho)$, it suffices to prove for all $h \geq 0$, $S(h) \geq 0$ and $T(h) \geq 0$, which have been shown already. Therefore, $\hat{U}(\rho; h) \leq U(\rho)$ for all $h \geq 0$ and $\rho \leq \rho_I^*$.

- Case 2, $\rho > \rho_I^*$ and $\rho_h > \rho_I^*$.

$$\begin{aligned}
U_A(\rho) - \hat{U}(\rho; h) &= \left[\frac{\lambda_H(1 - e^{-(2r+\lambda_H)h})}{2r + \lambda_H} - e^{-rh}(1 - e^{-\lambda_H h}) \right] g\rho(1 - \rho)q \\
&\quad + \left(\frac{r(e^{\lambda_H h} - 1) - \lambda_H(1 - e^{-rh})}{r + \lambda_H} \right) \left[\frac{(1 - \rho)\rho_A^*}{\rho(1 - \rho_A^*)} \right]^{1+r/\lambda_H} g\rho(1 - \rho)q \\
&- \left[\frac{(r + \lambda_H)e^{-rh} - \lambda_H - re^{-(r+\lambda_H)h}}{(r + \lambda_H)} + \frac{r(e^{\lambda_H h} - 1) - \lambda_H(1 - e^{-rh})}{r + \lambda_H} \right] \left[\frac{(1 - \rho)\rho_I^*}{\rho(1 - \rho_I^*)} \right]^{1+r/\lambda_H} g\rho(1 - \rho)q.
\end{aligned}$$

Notice $\rho_h > \rho_I^*$ implies that $[\frac{(1-\rho)\rho_I^*}{\rho(1-\rho_I^*)}]^{1+r/\lambda_H} < (e^{-\lambda_H h})^{1+r/\lambda_H}$. Hence, $U_A(\rho) - \hat{U}(\rho; h) \geq 0$ if

$$S(h) + \frac{rT(h)}{r + \lambda_H} \left(\left[\frac{(1 - \rho_A^*)\rho_I^*}{\rho_A^*(1 - \rho_I^*)} \right]^{1+r/\lambda_H} - 1 \right) - \frac{(r + \lambda_H)e^{-rh} - \lambda_H - re^{-(r+\lambda_H)h}}{(r + \lambda_H)} \geq 0.$$

We have shown that $T(h) \geq 0$ and $T(h) = O(h^2)$ since $T'(h) = 0$ and $T''(h) > 0$. It is straightforward to check that $X(h) \triangleq S(h) - \frac{rT(h)}{r+\lambda_H} - \frac{(r+\lambda_H)e^{-rh} - \lambda_H - re^{-(r+\lambda_H)h}}{(r+\lambda_H)} = O(h^3)$ since $X(0), X'(0)$ and $X''(0)$ are all zero. This implies that there exists some \bar{h} such that the sign of $U_A(\rho) - \hat{U}(\rho; h)$ is positive for all $h \leq \bar{h}$.¹⁹ Now suppose $U_A(\rho) - \hat{U}(\rho; h) < 0$ for some h . Since $U_A(\rho) - \hat{U}(\rho; \bar{h}) \geq 0$, it must be the case that $U_A(\rho') - \hat{U}(\rho'; h - \bar{h}) < 0$ where $\rho' = \frac{\rho e^{-\lambda_H \bar{h}}}{\rho e^{-\lambda_H \bar{h}} + 1 - \rho}$. Then eventually, we will find some $\hat{\rho}$ such that $U_A(\hat{\rho}) - \hat{U}(\hat{\rho}; \bar{h}) < 0$, which leads to a contradiction. Hence we have $U_A(\rho) - \hat{U}(\rho; \bar{h}) \geq 0$ for all h .

- Case 3, $\rho > \rho_I^*$ but $\rho_h \leq \rho_I^*$. In this case, if there is a profitable one-shot deviation, using the argument in case 2, we can find a profitable one-shot deviation beginning from some ρ sufficiently close to ρ_I^* . But this is impossible since the value matching conditions and the fact there is no profitable one-shot deviation in case 1 imply there cannot be profitable one-shot deviations when ρ is close to ρ_I^* .

To show the ρ_h buyer will not deviate for any h' length of time, first notice that it suffices to consider $h' \leq h$ because the first part of the proof already implies that it is not optimal to deviate any longer once h' exceeds h . The value associated with an h' -deviation is provided by:

$$\tilde{U}(h') = \int_{t=0}^{h'} r e^{-rt} s dt + \rho \tilde{q}_h (1 - e^{-\lambda_H h'}) e^{-rh'} s + [1 - \rho \tilde{q}_h (1 - e^{-\lambda_H h'})] e^{-rh'} U(\rho_h, \rho_{h'}, \hat{q}_{h+h'}).$$

Given

$$U(\rho_h, \rho) = s + C_h \times (1 - \rho) \tilde{q}_h \left(\frac{1 - \rho}{\rho} \right)^{r/\lambda_H},$$

it is straightforward to show: $U(\rho_h, \rho) \geq \tilde{U}(h')$ for all $h' \leq h$. ■

¹⁹The bound \bar{h} is uniform for all ρ since the comparison of $T(h)$ and $X(h)$ does not depend on ρ .

B.6 Proof of Proposition 4

Proof. In a niche market, $U_A(\rho_A^*) = s$ and equation (21) imply that

$$D(1 - \rho_A^*)^2 = g\rho_A^*(1 - \rho_A^*)$$

and

$$D = \frac{\lambda_H}{2r + \lambda_H} \left(\frac{\rho_A^*}{1 - \rho_A^*} \right)^{1+2r/\lambda_H}.$$

Substitute above expression into equation (26) yields

$$P_A(\rho_A^*) = \rho_A^* q(\rho_A^*) g - s.$$

Then boundary conditions

$$J_A(\rho_A^*) = 0 \quad \text{and} \quad J'_A(\rho_A^*) = 0$$

immediately imply that ρ_A^* should satisfy equation

$$\rho q(\rho) = \frac{rs}{rg + \lambda_H g - \lambda_H s} = \frac{rs}{rg + \lambda_H (V_I(\rho) + J_I(\rho)) - \lambda_H s}.$$

In a mass market, similarly we get ρ_A^* should also satisfy

$$\rho q(\rho) = \frac{rs}{rg + \lambda_H (V_I(\rho) + J_I(\rho)) - \lambda_H s}.$$

Thus, the equilibrium cutoff ρ_A^* is characterized by equation (29) no matter whether it is a mass or niche market. Since $\rho q(\rho)$, $V_I(\rho)$ and $J_I(\rho)$ are all increasing in ρ , the solution to the above equation is unique given a pair of priors (ρ_0, q_0) .

Furthermore, mass market appears ($\rho_A^* > \rho_I^*$) if and only if

$$\rho_I^* q(\rho_I^*) < \frac{rs}{rg + \lambda_H (V_I(\rho_I^*) + J_I(\rho_I^*)) - \lambda_H s}$$

or equivalently,

$$\frac{q_0(1 - \rho_0)^2}{q_0(1 - \rho_0)^2 + (1 - q_0)(1 - \rho_I^*)^2} < \frac{\rho_I^e}{\rho_I^*}.$$

Rearrange terms and we get the condition in the proposition.

From proposition 1, the efficient cutoff ρ_A^e is characterized by equation

$$\rho q(\rho) = \frac{rs}{(r + \lambda_H)g + \lambda_H W(\rho) - 2\lambda_H s}.$$

First, $J_I(\rho) + V_I(\rho) + s$ which represents the total equilibrium surplus in the individual learning phase must be strictly less than the socially optimal surplus $\Omega_1(\rho) = g + W(\rho)$ for any $\rho > \rho_I^e$ since equilibrium is inefficient in the individual learning phase. Hence,

$$rg + \lambda_H (V_I(\rho) + J_I(\rho)) - \lambda_H s < (r + \lambda_H)g + \lambda_H W(\rho) - 2\lambda_H s. \quad (61)$$

Second, it cannot be the case that $\rho_A^* \leq \rho_I^e$ for $q_0 < 1$. Otherwise, $V_I(\rho_A^*) = s$, $J_I(\rho_A^*) = g - s$ and $V_I(\rho_A^*) + J_I(\rho_A^*) = g$. Therefore,

$$\rho_A^* \times q(\rho_A^*) = \rho_I^e = \frac{rs}{rg + \lambda_H (g - s)}. \quad (62)$$

The above equation implies that $\rho_A^* > \rho_I^e$ which contradicts the assumption.

Since $W(\cdot)$ is a strictly increasing function for $\rho > \rho_I^e$, inequality (61) implies that $\rho_A^* > \rho_A^e$. ■

B.7 Proof of Proposition 5

Proof. Given the monopoly price $P_A(q)$ (notice $\rho \equiv 1$ and we should switch to use q as the state variable), the value function for a representative unsure buyer can be written as

$$rU_A(q) = r(gq - P_A(q)) + nq\lambda_H(s - U_A(q)) - n\lambda_Hq(1 - q)U'_A(q). \quad (63)$$

Participation constraint implies that $U_A(q) \geq s$ and there is also an incentive compatibility constraint which means “one-shot deviations” are not profitable:

$$U_A(q) \geq \hat{U}(q; h) = \int_{t=0}^h re^{-rt} s dt + e^{-rh} q(1 - e^{-(n-1)\lambda_H h})s + e^{-rh}(1 - q + qe^{-(n-1)\lambda_H h})U_A(qh)$$

for any $h > 0$ where $q_h = \frac{qe^{-(n-1)\lambda_H h}}{1 - q + qe^{-(n-1)\lambda_H h}}$. The incentive compatibility constraint is tighter than the participation constraint and should be binding in equilibrium since $U(q; h) \geq s$. Let h go to zero and the incentive constraint implies $U_A(\cdot)$ should satisfy the following differential equation:

$$U_A(q) = s + \frac{n-1}{r} [q\lambda_H(s - U_A(q)) - (n-1)\lambda_Hq(1 - q)U'_A(q)]$$

for $q \geq q_A^*$. The general solution is

$$U_A(q) = s + D_A(1 - q)\left(\frac{1 - q}{q}\right)^{r/((n-1)\lambda_H)}$$

such that $D_A \geq 0$.

On the other hand, given price $P_A(\rho)$, the monopolist’s value function is given by:

$$rJ_A(q) = nrP_A(q) + nq\lambda_H(n(g - s) - J_A(q)) - n\lambda_Hq(1 - q)J'_A(q). \quad (64)$$

At the optimal stopping cutoff q_A^* , value matching and smooth pasting conditions are satisfied:

$$U_A(q_A^*) = s, \quad J_A(q_A^*) = 0 \quad \text{and} \quad J'_A(q_A^*) = 0. \quad (65)$$

Boundary conditions (65) imply that $U_A(q_A^*) = s$ for some $q_A^* < 1$. As a consequence, it must be the case that $D_A = 0$ and $U_A(q) \equiv s$. From equation (63), the equilibrium price is $P_A(q) = gq - s$. Substitute the price expression into equation (64) yields

$$rJ_A(q) = nr(gq - s) + nq\lambda_H(n(g - s) - J_A(q)) - n\lambda_Hq(1 - q)J'_A(q).$$

This is an ordinary differential equation with boundary conditions

$$J_A(q_A^*) = 0 \quad \text{and} \quad J'_A(q_A^*) = 0.$$

It is easy to solve q_A^* as:

$$q_A^* = q_A^e = \frac{rs}{n\lambda_H(g - s) + rg}.$$

Therefore, the Markov perfect equilibrium is efficient. ■

B.8 Proof of Theorem 1

Proof. In the individual learning phase, denote ρ_k^* to be the equilibrium cutoff on beliefs such that the monopolist would stop experimentation when $k \geq 1$ buyers have received lump-sum payoffs. Let V_k , U_k and J_k be value functions for the sure buyers, the unsure buyers and the monopolist respectively when k buyers have received lump-sum payoffs. Finally, let P_k denote the price charged by the monopolist. From a backward procedure, it could be shown that:

Lemma 4 *The equilibrium cutoffs should satisfy*

$$\rho_k^* = \frac{nrs + kr(g-s)}{nrg + (n-k)\lambda_H(g-s)}$$

and

$$\rho_1^e < \rho_k^* < \rho_{k+1}^*$$

for all $1 \leq k \leq n-2$.

Proof. If all of the n buyers turn out to be good, then naturally it is optimal for the monopolist to charge $g-s$ and fully extract the total surplus. If all but one buyers have already received lump-sum payoffs, the monopolist faces the same tradeoff between stopping and keeping experimentation as discussed in the $n=2$ case. The monopolist has to charge $g\rho-s$ to keep experimentation and her value function in that case could be written as:

$$(r + \rho\lambda_H)J_{n-1}(\rho) = nrg(g\rho-s) + n\rho\lambda_H(g-s) - \lambda_H\rho(1-\rho)J'_{n-1}(\rho);$$

with boundary conditions

$$J_{n-1}(\rho_{n-1}^*) = (n-1)(g-s) \quad \text{and} \quad J'_{n-1}(\rho_{n-1}^*) = 0.$$

It is straightforward to see that:

$$\rho_{n-1}^* = \frac{rs + (n-1)rg}{\lambda_H(g-s) + nrg}$$

and

$$J_{n-1}(\rho) = \max \left\{ (n-1)(g-s), n(g\rho-s) + [(n-1)g + s - n\rho\rho_{n-1}^*] \frac{1-\rho}{1-\rho_{n-1}^*} \left[\frac{(1-\rho)\rho_{n-1}^*}{(1-\rho_{n-1}^*)\rho} \right]^{r/\lambda_H} \right\}.$$

Meanwhile, the value for the sure buyers is given by:

$$V_{n-1}(\rho) = \max \left\{ s, s + g(1-\rho) \left(1 - \left[\frac{(1-\rho)\rho_{n-1}^*}{\rho(1-\rho_{n-1}^*)} \right]^{r/\lambda_H} \right) \right\}.$$

If all but two buyers have received lump-sum payoffs, the value function for the monopolist becomes:

$$J_{n-2}(\rho) = \max \left\{ (n-2)(g-s), nP_{n-2}(\rho) + \frac{2\rho\lambda_H}{r} [J_{n-1}(\rho) - J_{n-2}(\rho)] - \frac{\lambda_H\rho(1-\rho)}{r} J'_{n-2}(\rho) \right\}. \quad (66)$$

The price P_{n-2} is set such that the unsure buyers have incentive to take experimentation:

$$rP_{n-2}(\rho) = r(\rho g - U_{n-2}(\rho)) + \lambda_H \rho(s - U_{n-2}(\rho)) + \lambda_H \rho(V_{n-1}(\rho) - U_{n-2}(\rho)) - \lambda_H \rho(1 - \rho)U'_{n-2}(\rho).$$

Value matching and smooth pasting conditions mean that at the optimal cutoff ρ_{n-2}^* ,

$$U_{n-2}(\rho_{n-2}^*) = s, U'_{n-2}(\rho_{n-2}^*) = 0, J_{n-2}(\rho_{n-2}^*) = (n-2)(g-s) \text{ and } J'_{n-2}(\rho_{n-2}^*) = 0.$$

The above equations imply ρ_{n-2}^* satisfies equation

$$(n-2)(g-s) = n \left\{ \rho_{n-2}^* g - s + \frac{\rho_{n-2}^* \lambda_H}{r} [V_{n-1}(\rho_{n-2}^*) - s] \right\} + \frac{2\rho_{n-2}^* \lambda_H}{r} [J_{n-1}(\rho_{n-2}^*) - (n-2)(g-s)].$$

If $\rho_{n-2}^* > \rho_{n-1}^*$, then $V_{n-1}(\rho_{n-2}^*) > s$ and $J_{n-1}(\rho_{n-2}^*) > (n-1)(g-s)$. But this implies

$$\begin{aligned} (n-2)(g-s) &> n(\rho_{n-2}^* g - s) + \frac{2\rho_{n-2}^* \lambda_H}{r}(g-s) \\ &\implies \rho_{n-2}^* < \frac{2rs + (n-2)rg}{2\lambda_H(g-s) + nrg} < \rho_{n-1}^* = \frac{rs + (n-1)rg}{\lambda_H(g-s) + nrg}. \end{aligned}$$

Contradiction! Hence, it must be the case that $\rho_{n-2}^* \leq \rho_{n-1}^*$ such that $V_{n-1}(\rho_{n-2}^*) = s$ and $J_{n-1}(\rho_{n-2}^*) = (n-1)(g-s)$. Then it is straightforward to see

$$\rho_{n-2}^* = \frac{2rs + (n-2)rg}{2\lambda_H(g-s) + nrg}.$$

For general $1 \leq j \leq n-1$, assume

$$\rho_k^* = \frac{nrs + kr(g-s)}{nrg + (n-k)\lambda_H(g-s)}$$

for $k \geq j+1$. At ρ_j^* ,

$$j(g-s) = n \left[(\rho_j^* g - s) + \frac{\lambda_H \rho_j^*}{r} (V_{j+1}(\rho_j^*) - s) \right] + \frac{(n-j)\lambda_H \rho_j^*}{r} [J_{j+1}(\rho_j^*) - j(g-s)].$$

It is similar to show by contradiction that it is impossible to have $\rho_j^* > \rho_{j+1}^*$ and hence it must be the case

$$\rho_j^* = \frac{nrs + jr(g-s)}{nrg + (n-j)\lambda_H(g-s)}.$$

Standard induction argument then implies that for all $1 \leq k \leq n-1$, we would have

$$\rho_k^* = \frac{nrs + kr(g-s)}{nrg + (n-k)\lambda_H(g-s)}$$

and furthermore, it is trivial to see

$$\rho_I^e < \rho_k^* < \rho_{k+1}^*$$

for all $1 \leq k \leq n-2$. ■

Lemma 4 means the equilibrium is not efficient in the individual learning phase. From the boundary conditions, the equilibrium cutoff ρ_A^* in the aggregate learning phase should satisfy

$$\rho_A^* q(\rho_A^*) = \frac{rs}{rg + \lambda_H [V_1(\rho_A^*) + J_1(\rho_A^*) + (n-1)U_1(\rho_A^*)] - n\lambda_H s}.$$

It is immediate to see

$$rg + \lambda_H [V_1(\rho) + J_1(\rho) + (n-1)U_1(\rho)] - n\lambda_H s < (r + \lambda_H)g + \lambda_H(n-1)W(\rho) - n\lambda_H s$$

since the equilibrium in the individual learning phase is inefficient:

$$V_1(\rho) + J_1(\rho) + (n-1)U_1(\rho) < g + (n-1)W(\rho) = \Omega_1(\rho)$$

for $\rho > \rho_I^e$. This together with the fact that $\rho_A^* > \rho_I^e$ guarantees that $\rho_A^* > \rho_A^e$. ■

C Proofs of Results from Section 4

C.1 Proof of Proposition 7

Proof. Notice

$$\frac{r}{\lambda_H} \log\left(\frac{\rho}{1-\rho}\right) + \log\left(\frac{q_0(1-\rho_0)^n + (1-q_0)(1-\rho)^n}{(1-\rho)^n}\right)$$

is an antiderivative of $\frac{r+\lambda_H n \rho q}{\lambda_H \rho(1-\rho)}$. From observation 1, we solve differential equation (32) as:

$$\Omega_A(\rho) = \frac{\int h(x) \frac{rn[A-\lambda_H x q(x)B] + \lambda_H n x q(x)[(n-1)W(x)+s]}{\lambda_H x(1-x)} dx}{h(\rho)}$$

where

$$h(\rho) = \left(\frac{\rho}{1-\rho}\right)^{r/\lambda_H} \frac{q_0(1-\rho_0)^n + (1-q_0)(1-\rho)^n}{(1-\rho)^n}.$$

Notice that $W(\cdot)$ is a piece-wise function and hence we have to first determine the relationship between ρ_I^e and ρ_A^e .

Lemma 5 *Given any $q_0 < 1$, the efficient cutoff for starting experimentation in the aggregate learning phase is larger than the efficient cutoff in the individual learning phase: $\rho_A^e > \rho_I^e$.*

Proof. For $\rho \leq \rho_I^e$,

$$W(\rho) = A - \frac{\lambda_H A + r\lambda_H B - \lambda_H s}{r + \lambda_H} \rho$$

and $\Omega_A(\rho)$ can be computed using integration by parts:

$$\Omega_A(\rho) = \frac{\int h(x) \frac{rn[A-\lambda_H x q(x)B] + \lambda_H n x q(x)[(n-1)W(x)+s]}{\lambda_H x(1-x)} dx}{h(\rho)} = n \left[A - \frac{\lambda_H}{r + \lambda_H} \rho q(rB + A - s) \right] + \frac{C}{h(\rho)}.$$

Since 0 is included in the domain of $\Omega_A(\cdot)$, the constant term C must be 0 to guarantee $\Omega_A(\cdot)$ is bounded away from infinity. Therefore,

$$\Omega_A(\rho) = n \left[A - \frac{\lambda_H}{r + \lambda_H} \rho q(rB + A - s) \right].$$

Suppose on the contrary, we have $\rho_A^e \leq \rho_I^e$, then ρ_A^e should satisfy

$$n \left[A - \frac{\lambda_H}{r + \lambda_H} \rho_A^e q(\rho_A^e) (rB + A - s) \right] = ns \implies \rho_A^e q(\rho_A^e) = \rho_I^e.$$

This leads to a contradiction since $q < 1$. ■

For $\rho > \rho_I^e$, $W(\rho) = s$ and by observation 1,

$$\Omega_A(\rho) = \frac{\int_{\rho_I^e}^{\rho} h(x) \frac{rn[A - \lambda_H x q(x) B] + \lambda_H n^2 x q(x) s}{\lambda_H x (1-x)} dx + C}{h(\rho)}.$$

The constant C is set such that $\Omega_A(\rho)$ is continuous at ρ_I^e :

$$C = h(\rho_I^e) n \left[A - \frac{\lambda_H}{r + \lambda_H} \rho_I^e q(\rho_I^e) (rB + A - s) \right] > 0.$$

At the efficient starting cutoff $\rho_A^e(q_0)$, $\Omega_A(\rho_A^e; q_0) = ns$. Substitute the expression of $\Omega_A(\rho)$ into the above equation:

$$C - h(\rho_I^e) ns + \int_{\rho_I^e}^{\rho_A^e} h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} dx = 0.$$

Notice

$$C - h(\rho_I^e) ns = h(\rho_I^e) n \left[A - s - \frac{\lambda_H}{r + \lambda_H} \rho_I^e q(\rho_I^e) (rB + A - s) \right] > 0$$

doesn't depend on ρ_A^e . Therefore, if an interior solution $\rho_A^e(q_0)$ indeed exists, it must be the case

$$\int_{\rho_I^e}^{\rho_A^e} h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} dx < 0$$

and hence $A - \lambda_H \rho_A^e q_0 B - s < 0$. Suppose for a given q_0 , there exist two cutoffs ρ_1 and $\rho_2 > \rho_1$. Then we have

$$\int_{\rho_I^e}^{\rho_1} h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} dx = \int_{\rho_I^e}^{\rho_2} h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} dx$$

which is impossible since

$$h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} < 0$$

for $x \in (\rho_1, \rho_2)$. Therefore, if there exists some ρ_A^e such that $\Omega_A(\rho_A^e; q_0) = ns$, such ρ_A^e must be unique. When there does not exist ρ_A^e such that

$$C - h(\rho_I^e) ns + \int_{\rho_I^e}^{\rho_A^e} h(x) \frac{rn[A - \lambda_H x q(x) B - s]}{\lambda_H x (1-x)} dx = 0,$$

just set $\rho_A^e = 1$ since it is always beneficial to take experimentation under such q_0 . To summarize, for any q_0 , there is a unique $\rho_A^e(q_0)$ such that it is socially efficient to start experimentation if and only if $\rho \leq \rho_A^e(q_0)$. ■

C.2 Proof of Theorem 2

Proof. When k buyers have already received lump-sum damages, the monopolist would choose to sell if:

$$J_k(\rho) = (n-k)(A - \lambda_H \rho B - s) + \frac{1}{r} [(n-k)\lambda_H \rho (J_{k+1}(\rho) - J_k(\rho)) - \lambda_H \rho (1-\rho) J'_k(\rho)] \geq 0.$$

Induction argument can be used to solve equilibrium cutoffs. First,

$$J_{n-1}(\rho) = A - s - \frac{\lambda_H(A - s + rB)}{r + \lambda_H} \rho \geq 0$$

if and only if $\rho \leq \rho_{n-1}^* = \rho_I^e$. We can guess that

$$J_k(\rho) = (n-k) \left[A - s - \frac{\lambda_H(A - s + rB)}{r + \lambda_H} \rho \right].$$

Suppose this is true for $j = k+1, \dots, n-1$, then from

$$J_k(\rho) = (n-k)(A - \lambda_H \rho B - s) + \frac{1}{r} [(n-k)\lambda_H \rho (J_{k+1}(\rho) - J_k(\rho)) - \lambda_H \rho (1-\rho) J'_k(\rho)]$$

it is easy to get

$$J_k(\rho) = (n-k) \left[A - s - \frac{\lambda_H(A - s + rB)}{r + \lambda_H} \rho \right].$$

Our guess hence is true by induction.

Obviously,

$$J_k(\rho) = (n-k) \left[A - s - \frac{\lambda_H(A - s + rB)}{r + \lambda_H} \rho \right] \geq 0$$

if and only if $\rho \geq \rho_I^e$ for all $k \geq 1$. Therefore, the symmetric Markov perfect equilibrium is efficient in the individual learning phase. In the aggregate learning phase, for $\rho \leq \rho_I^e$, the monopolist's value function is

$$J_A(\rho) = n(A - \lambda_H \rho q B - s) + \frac{1}{r} [n\lambda_H \rho q (J_1(\rho) - J_A(\rho)) - \lambda_H \rho (1-\rho) J'_A(\rho)].$$

The solution to the above differentiable equation is given by:

$$J_A(\rho) = n(A - s) - n\rho q(\rho) \frac{\lambda_H}{r + \lambda_H} (A - s + rB).$$

It is easy to check that for any $q < 1$, $J_A(\rho) > 0$ for all $\rho \leq \rho_I^e$ and hence the equilibrium cutoff must be $\rho > \rho_I^e$. For $\rho > \rho_I^e$,

$$J_A(\rho) = n[A - \lambda_H \rho q B - s] - \frac{1}{r} [n\lambda_H \rho q J_A(\rho) + \lambda_H \rho (1-\rho) J'_A(\rho)].$$

Math observation 1 implies

$$J_A(\rho) = \frac{\int_{\rho_I^e}^{\rho} h(x) \frac{rn(A - \lambda_H x q(x) B - s)}{\lambda_H x(1-x)} dx + D}{h(\rho)}$$

where

$$h(\rho) = \left(\frac{\rho}{1-\rho}\right)^{r/\lambda_H} \frac{q_0(1-\rho_0)^n + (1-q_0)(1-\rho)^n}{(1-\rho)^n}.$$

D is chosen such that $J_A(\cdot)$ is continuous at ρ_I^e and it is immediate to see that $D = C - h(\rho_I^e)ns$, where C is the constant given in the proof of proposition 7. Notice that

$$\int_{\rho_I^e}^{\rho} h(x) \frac{rn(A - \lambda_H x q(x)B - s)}{\lambda_H x(1-x)} dx = \int_{\rho_I^e}^{\rho} h(x) \frac{rn(A - \lambda_H x q(x)B) + \lambda_H n^2 x q(x)s}{\lambda_H x(1-x)} dx - ns(h(\rho) - h(\rho_I^e)).$$

As a result, $J_A(\rho) = \Omega_A(\rho) - ns$.

For a fixed q_0 , the monopolist starts selling her product as long as $J_A(\rho_0; q_0) \geq 0$ which implies that the equilibrium cutoff $\rho_A^*(q_0)$ must be the same as $\rho_A^e(q_0)$. Therefore, the symmetric Markov perfect equilibrium is efficient in the aggregate learning phase as well. ■

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