Reputation in Repeated Settlement Bargaining*†

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June 2008

Abstract

A long-lived player encounters disputes with a sequence of short-lived players, each at a time. The long-lived player has private information about his liability towards each claim. Each pair of long- and short-lived players attempt to resolve their dispute via negotiation but, should there be a disagreement, an unbiased yet imperfect third party is called upon to make a judgement. The short-lived players learn about the informed player through two channels: observed behavior of the informed player ("soft" information) and, if any, verdicts of the third party ("hard" information). The interplay between these two sources of information generates new, interesting equilibrium dynamics that feature reputation *building*. The predictions of the model explain a number of empirical puzzles identified in litigation and enable us to discuss several related policy issues.

*Preliminary draft; comments welcome

†This paper has benefited from conversations with a number of our colleagues, including Yeon-Koo Che, Jeff Ely, Drew Fudenberg, Srihari Govindan, Ayca Kaya, Jon Levin, Hamid Sabourian, Larry Samuelson, Klaus Schmidt and Andy Skrzypacz. Hyung-Ho Moon provided valuable research assistance. Qingmin Liu thanks the hospitality of the Cowles Foundation at Yale University.

1 Introduction

1.1 Motivation and overview

Recently, Merck and Pfizer, two of the largest pharmaceutical manufacturers, have been involved in a series of high-profile litigations surrounding their painkillers' alleged link to increased risk of blood clots, heart attacks and strokes.[1] Despite the close similarity of their cases, the two firms have adopted contrasting approaches to the litigations. Merck committed from the outset the policy of contesting every case in court and, following a number of successful trial outcomes, are now on the verge of striking a mass settlement.[2] In sharp contrast, Pfizer are attempting to settle its disputes before taking any of them to court.[3] How do we reconcile these differences in behavior?

Product liability litigations such as the above drugs cases in fact represent one of many important applications of dispute resolutions that share several distinguishing characteristics. First, many transactions and dispute resolutions feature a large player and a pool of small players. For example, a landlord routinely contests with tenants over the amount of deposit to be returned and a firm often faces labor-wage disputes with its employees, while a chain of restaurants and other firms producing goods with safety attributes risk causing damages to their customers. Second, these disputes are rarely resolved by the two sides alone. Amid the deadlock of a labor-wage dispute, the parties often turn to a third party, such as an arbitrator or even a court. When traders disagree on the quality of goods or the terms of a deal, they hire an expert to make an assessment on their behalf. Even when transactions are conducted smoothly without outside intervention, the third party is usually in the shadow of the interaction.

Economic significance of such disputes are often strikingly large. A prominent example is found in securities class actions, where an underwriter or auditor representing multiple initial public offerings (IPOs) faces claims from shareholders on the basis of fraudulent financial practices behind large price

[1] The drugs in dispute are Vioxx for Merck and Bextra and Celebrex for Pfizer. They all belong to the same class of painkillers known as COX-2 inhibitors.

[2] Source: New York Times, http://topics.nytimes.com/top/news/health/diseasesconditions andhealthtopics/vioxx_drug.

[3] Source: Wall Street Journal (May 2, 2008).

declines.[4] In the year of 2007 alone, the US experienced 177 federal class action securities fraud claims with a total disclosure loss of 151 billion dollars.[5] Medical malpractice litigations offer another important case in point. In the US, the claimed damages involved in such suits totaled 28.7 billion dollars in 2004 and, around the same period, the corresponding amount was about 2.4 billion euros in Italy.[6] Observers of these events have indeed voiced reputation concerns of the *repeat* players. However, a rigorous game-theoretic framework is still absent.

In this paper, we develop a reputation model of repeated settlement bargaining in the presence of a third party, or an "expert". A critical aspect of this model is that expert verdicts are informative but nonetheless imperfect. Learning arises from two sources: the informed player's equilibrium actions and the decisions made by the expert, if any. We interpret the first of these two sources as "soft" information and the second as "hard" information. The interplay of these two sources of information is indeed the key innovative feature of our reputation model that generates new, interesting equilibrium dynamics. To our surprise, it enables us to explain many empirical observations and provide theoretical foundations for analyzing a number of related policy issues.

We consider a long-lived player (e.g. defendant) who is in dispute with a sequence of short-lived players (e.g. plaintiffs). The long-lived player has private information regarding his liability towards each claim and, hence, a compensation to each short-lived player. He is either "good" or "bad", with the bad type being more likely to be liable for each claim. For instance, an underwriter privately knows whether or not he failed to disclose some information relevant for share price decline and a landlord has better knowledge of whether the damage is due to the tenant's negligence or the poor condition of his own building.

In each of infinitely many periods, a new short-lived player enters the game with a claim and observes the public history. The two parties first attempt to negotiate a settlement themselves. The short-lived player makes

[4]For instance, Alexander [1] notices that "two prominent investment banking firms stated in their own prospectuses that in 1986 they were involved in 60 and 73 lawsuits, respectively, over public offerings they had underwritten" (Alexander [1], p.558). Also, see Palmrose [22].

[5]Source: Stanford law school securities class action clearinghouse, http://securities.stanford.edu.

[6]Source: OECD [21].

a take-it-or-leave-it settlement demand offer. If the demand is accepted, the short-lived player receives the corresponding amount from the long-lived player and leaves the game. But, in the event of a disagreement, when the long-lived player rejects the demand, an expert is called upon to make a decision on their behalf. An expert verdict is publicly observable, and so are the details of an agreed settlement.

Expert resolution is costly to the parties and, most importantly, the third party is imperfect. The "quality" of the expert is measured by the probability $q > \frac{1}{2}$ with which he correctly rules a liable (or non-liable) long-lived player to be indeed liable (or not liable). Moreover, it is assumed that the expert quality is fixed over time. That is, we do not allow for the expert to learn about the true type of the long-lived player. This last assumption is reasonable in a number of contexts, for instance, when there is a pool of many experts with average quality q and past evidence/verdicts cannot be used to influence the current case because of either physical or legal constraints.

We construct a Markov perfect Bayesian equilibrium that displays the following dynamics: the bad type attempts to gradually build up his reputation when it is low, but he can successfully do so only with a probability strictly less than 1. Reputation may move up or down and also, with a strictly positive probability, the bad type will reveal himself and hence fail to build reputation. It is worth noting that the bad type reveals himself only when he voluntarily gives up reputation building; the hard information from the expert, due to its imperfectness, can never lead to full revelation. We also characterize the exact payoff gain from reputation in all Markov equilibria. Surprisingly, for low prior beliefs, the benefit is minimal, in sharp contrast to the result of Fudenberg and Levine [10].

The interplay between soft and hard information generates equilibrium dynamics that are characterized by two threshold levels of reputation:[7]

When reputation is above the upper threshold, the short-lived player makes a low settlement demand and both types of the long-lived player accept it for sure. This is where the full benefits of successful reputation building are reaped by the bad long-lived player. There is no learning on the part of the short-lived players in this region.

In the intermediate reputation region, the short-lived players make a large settlement demand that the good type cannot accept; the bad type mimics

[7] We also demonstrate that this threshold property indeed applies to all Markov equilibria.

the good type and also reject settlement demands for sure. Rejection leads to an arrival of hard information which, in expectation, will reduce reputation. But, with the prospect of a high continuation payoff (or a low expected compensation payment) in the neighboring high reputation region, the bad type is reluctant to accomodate the large demand of the short-lived players. Here, learning occurs but only through hard information since nothing can be learned about the long-lived player's type from the pooling behavior.

When reputation is below the lower threshold, the long-lived player faces a slim prospect of a high payoff as the high reputation region is far away, that is, many pieces of good luck (favorable expert verdicts) are needed to reach the high reputation region. Thus, the long-lived player is willing to accommodate the short-lived players' demands. In equilibrium, he builds reputation by playing a mixed strategy and hard information arrives only occasionally. Here, the impact of hard information is reduced in the presence of soft informaiton and, in fact, soft information can sometimes revert the adverse effects of hard information. We fully characterize an open interval within the low reputation region where a brave rejection of the demand enhances reputation even after an unfavorable verdict from the expert.

1.2 Contributions

Theory of reputation Our results enrich the adverse selection theory of reputation initiated by Kreps and Wilson [18], Milgrom and Roberts [20] and Kreps, Milgrom, Roberts and Wilson [17] and later developed by, among many others, Fudenberg and Levine [10] and Mailath and Samuelson [19].[8] In standard reputation models, no players actually *build* reputation in equilibrium; the privately informed player starts pooling with another type from the very beginning of the game and so "reputation springs to life".[9] Furthermore, even though reputation can increase the equilibrium payoff, reputation can always be built. In many applications, these features are not completely realistic. By introducing the interplay between soft and hard information, we show non-degenerate reputation building dynamics in which reputation can be built and maintained but not always.

Bar-Isaac [2] considers soft and hard information in a model where the quality of a monopolist is imperfectly revealed in each period via an exoge-

[8] See also repeated bargaining models with incomplete information studied by Hart and Tirole [12], Schmidt [23] and others.

[9] See Mailath and Samuelson [19] for related commens on standard reputation models.

nous signal should he decide to produce. In contrast to our paper, the focus here is on the learning behavior of the firm under various assumptions about the degree of private information that it possesses compared to the market. We, on the other hand, analyze the dynamics and value of reputation building.

We also note that external signals have been introduced in the dynamic signaling literature (see, for instance, Kremer and Skryzpacz [16] and the references therein). In these models, the arrival of hard information is entirely exogenous and, moreover, the game ends right after a signal. In our repeated model the arrival of hard information is optional to the long-lived player, and we can therefore explore his manipulation of such signals.

Do the merits matter? Our repeated settlement bargaining model enable us to piece together several empirical observations identified by legal scholars, regarding the relationship between merit and settlement outcome. Alexander [1], now a classic in this field, studies securities class action lawsuits involving underwriters behind similar claims of fraud in computer-related IPOs.[10] She finds that, beyond very few exceptions, "the cases settled at an apparent 'going rate' of approximately one quarter of the potential damages...a strong case in this group appears to have been worth no more than a weak one" (Alexander [1], p.500). Thus, the *merit* of a case, or "the parties' estimates of the strength of the case" (Alexander [1], p500), does not appear to matter for settlement, an observation that cannot be satisfactorily explained by existing, static economic models of litigation. Our results are however consistent with it; this is exactly what happens in the low reputation region where settlements occur and, moreover, the *amount* of settlement is constant over this interval of merits.

We also clarify the puzzle. Although the settlement amount, conditional on agreement, is independent of merit, settlement is nevertheless meritorious in that the settlement *rate* (i.e. the likelihood of settlement) is strictly decreasing in merit over the low reputation region. This is confirmed by Studdert and Mello [26] who find that, in medical malpractice litigations, merits do indeed affect the settlement rate. Furthermore, it is observed that cases favoring neither party, or "close calls", are more likely to go to court (see Palmrose [22] and Studdert and Mello [26]). Such cases can be interpreted as corresponding to reputation levels over, or close to, the intermediate region

[10] See Choi [7] for a survey on securities class actions.

in our equilibrium, where the bad defendant rejects plaintiffs' demands and proceed to trial for sure, or with a very high probability. Here, the conflict between the defendant's long-run interest and the plaintiff's short-run interest leads to the low settlement rate. The short-lived plaintiffs demand a relatively high compensation based on his estimate of the case's strength. However, the long-lived defendant is forward looking; the high reputation region is within reach and thus he will only accept a low settlement demand, an amount less than what the plaintiffs are willing to offer.

Policy implications An important policy relevant question in litigation is how the settlement rate is affected by the precision of the court, which is captured by expert quality q in our model. We show that, as q goes to 1, the low reputation region is completely squeezed out by the intermediate region, while the high reputation region shrinks (yet remains present), in equilibrium. Since the cases go to court with probability 1 in the intermediate region, our result may sound counterintuitive: why is a bad defendant more willing to go to court when the court will find him out almost surely? The reason, again, lies on the conflict of interests between the long-lived and short-lived parties. When the court is very precise, the plaintiff's expected payoff from trial increases and this makes the demand too high for the defendant to tolerate. The defendant is willing to take even a small chance of court error; after all, a single mistake will greatly enhance his reputation when the hard information is very precise. Thus, our paper contributes to the debate over the desirability of court accuracy (see Kaplow [14] for a survey on the issue). Our results suggest that, although greater accuracy will make it more difficult for a guilty defendant to build reputation and get away with their liability cheaply, it will generate an increased burden on the legal system, manifested by greater frequency of trials.

We also examine the effects of shifts in the long-lived player's discount rate and the short-lived players' expert cost. The equilibrium response to an increased plaintiff legal cost is of particular interest since several policy reforms have been enacted precisely in this direction with the aim of curtailing "frivolous" lawsuits, whereby plaintiffs bring up non-meritorious cases solely to extract settlements.[11] Our results throw a caution at such policies: they may actually help a bad defendant build unjust reputation.

[11] A case in point is the Private Securities Litigation Reform Act (PSLRA) of 1995 which made it more difficult for plaintiffs to forward a securities suit.

On the other hand, we offer a fresh perspective on the negative expected value suits (see Bebchuk [3][4] and Katz [15] for some of the existing explanations). It is straightforward to re-formulate our model such that the long-lived player represents a plaintiff and moreover the expected value of each claim is negative. The equilibrium in such a setup implies that frivolous litigations may involve not only the motive of extracting immediate compensation but also willingness to reject settlement offers on the part of the plaintiff to build reputation and reap larger benefits in the future.

Other related literature To our knowledge, our model is the first repeated litigation model with "long horizon".[12] Infinite repetition is not only a realistic feature, but it also enables us to fully analyze the reputation effects in a much clearer way. A number of papers have considered multi-period (but finite) models of litigation but their concerns differ from ours (for instance, see Che and Yi [5], Daughety and Reinganum [8][9], Hua and Spier [13] and Choi [6]). Nonetheless, in Section 5 below, we shall review some of these results in more detail and discuss extensions of our model in the directions adopted by these authors. There are also models that study dynamic settlement bargaining with a single case, where the bargaining protocol matters (Spier [24]). Instead, in this paper we consider repeated settlement bargaining and long-run incentives/behavior. For this reason, we abstract away the issue of bargaining protocol within each case.

1.3 Plan

The rest of the paper is organized as follows. The next section describes a game of repeated settlement bargaining. In Section 3, we construct an equilibrium of the game and, also, conduct some comparative static analysis in order to discuss several policy issues. Section 4 then presents some general characterization results. Finally, we offer some concluding remarks in Section 5. All technical proofs are relegated to Appendix.

[12]See Spier [25] for an excellent survey on the economics of litigation.

2 The Model

2.1 Description

We consider repeated settlement bargaining in discrete time. Periods are indexed by $t = 1, 2, \ldots$. A single long-lived player 1 faces an infinite sequence of short-lived players 2, with a new player 2 entering in every period. Each player 2 brings a claim to player 1. Underwriter/auditor-shareholders disputes in securities class actions, Hospital-patients disputes over medical accidents and landlord-tenant disputes over property damages mentioned in Introduction are some of the applications of the model.

The stake involved in each claim, $H > 0$ is fixed and common knowledge.[13] For example, in a securities class action, it is commonly known that H is the loss in stock value. However, player 1 privately knows his type $\theta \in \{G, B\}$, where G stands for good and B for bad. The bad type is more likely to be liable for each case than the good type. In what follows, we shall assume that type B is liable for each case with probability 1, while type G is not liable with probability 1. As argued in Remark 1 below, this simplifies the analysis. We also note that the assumption is valid in applications, such as the aforementioned Merck/Pfizer cases, where the long-lived player faces repeated disputes over some action that he has already committed privately in the past.

Each player 1-player 2 pair attempt to settle their dispute via voluntary bargaining. Should they fail to reach an agreement, they call upon an external third party, an expert, an arbitrator or a court, to determine whether player 1 is liable or not. Both players are committed to obey the third party's suggestion: player 1, if judged to be liable, should pay H to player 2, and player 2 should receive no compensation otherwise. Seeking a third party incurs a cost $c_i > 0$ to player i, regardless of the verdict. We shall henceforth refer to the third-party as an "expert".

The expert is unbiased but imperfect: independently of the true liability of player 1, he makes an error with probability $1 - q$, where $q \in \left(\frac{1}{2}, 1\right)$ is common knowledge among all players. Specifically, when player 1 is liable (or not liable) for compensation, the expert will incorrectly rule that the player owes nothing (or H) to player 2 with probability $1 - q$. We shall

[13] Our analysis remains unchanged if we instead assume that each stake is drawn from a fixed distribution, which is independent of player 1's private information and commonly known. Then, H would represent the *expected* stake.

henceforth refer to q as the "quality" or "precision" of the expert.

Furthermore, we shall assume that the quality of expert judgement is independent of history and, hence, there is no learning on the part of the expert. For instance, it may be that at any given period there is a pool of experts with average quality q and any past verdict cannot influence the direction of the current case.

The timing of the stage game in period t is as follows. Player 2 makes a take-it-or-leave-it settlement demand $s_t \in \mathbb{R}$, which player 1 can either accept or reject. If s_t is accepted, then player 2 receives s_t from player 1 and leaves the game forever. If, on the other hand, the demand is rejected, an expert is called upon to make a judgement.

Note that if player 1 is of type B his *expected payment* under expert resolution is equal to qH; if he is of type G the corresponding amount is $(1-q)H$. It is assumed throughout that $c_1+c_2 < qH-(1-q)H = (2q-1)H$.

The expert's verdict is publicly observable, and so are the details of an agreed settlement.[14] The first player 2 holds a prior belief, $p_1 \in (0, 1)$, that player 1 is good. Later short-lived players update their beliefs from this prior and the public history that they observe. Let $p_t \in [0, 1]$ denote player 2's posterior belief that player 1 is not liable in period t. Player 1's "reputation" is player 2's current posterior belief on the good type.

Remark 1 *We have assumed that type B (G) is always liable (not liable). Our analysis remains the same by instead assuming the following structure. For each claim, type G is liable with probability $q' < \frac{1}{2}$ and type B is liable with a probability $1 - q'$. Player 1 knows his type, but not his liability for each case due to some randomness (usual in medical malpractice cases, for instance). An expert makes a judgement on player 1's liability for each case with precision q''. It is readily verified that this is isomorphic to the model above with expert quality $q = q'q'' + (1 - q')(1 - q'')$.*

[14] This is the most reasonable description of actual settlement bargaining, and also consistent with the observations of Alexander [1] and Palmrose [22] on securities/auditor litigations. The details of any negotiation process (such as the value of rejected demands) are usually private information known only to the negotiating parties. But, once a deal is struck, the terms of the deal often enter the public domain. It is however possible for player 1 and player 2 to make a confidentiality agreement; we shall discuss this issue later.

2.2 Strategies and equilibrium concept

A (behavioral) strategy of player 1 is a mapping from the set of all possible histories that he can observe at the beginning of each period and the set of all possible settlement demands from player 2 to probability distributions over the set $\{A, R\}$, where A and R denote acceptance and rejection, respectively.

A (behavioral) strategy of player 2 in period t is a mapping from the set of all possible histories that he can observe over preceding $t-1$ periods to probability distributions over all possible settlement demands, $\mathbb{R}$. Note that it is possible *a priori* that player 2 demands more than qH.

We focus on perfect Bayesian equilibria in Markov strategies in which any relevant past history can be summarized by the level of belief that it generates. A Markov strategy for type-θ player 1, r^θ, is

$$r^\theta : [0,1] \times \mathbb{R} \to [0,1]$$

such that $r^\theta(p,s)$ is the probability with which type θ rejects the demand $s \in \mathbb{R}$ at belief $p \in [0,1]$.

On the other hand, the Markovian assumption renders irrelevant the period in which player 2 makes entry and, hence, we shall write a Markov strategy for player 2, d, simply as

$$d : [0,1] \to \triangle(\mathbb{R})$$

such that $d(p) \in \triangle(\mathbb{R})$ for any $p \in [0,1]$.

If (r^B, r^G, d) is a Makrov strategy profile, we write type θ's discounted average expected payment at belief p as $V^\theta(p)$ with discount factor $\delta \in (0,1)$. Note that we have already surpressed the dependence of V^θ on the strategy profile and the discount factor. In what follows, our focus will be on the equilibrium payments of type-B player 1. Thus, when the meaning is clear, we shall refer to $V^B(p)$ simply as $V(p)$. Player 2 maximizes his stage game payoff while player 1 minimizes his (discounted average) expected payment.

A strategy profile (r^B, r^G, d), together with a system of beliefs, forms a Makrov perfect Bayesian equilibrium (MPBE) if the usual condtions are satisfied. See, for instance, Fudenberg and Tirole [11] for a formal definition.

3 Equilibrium

3.1 Behavior of the good type

If a dispute is resolved via the expert, good player 1, in expectation, incurs total payment $(1 - q)H + c_1$. It is then natural that this player 1 should not agree to pay anything above this amount from bargaining with player 2. In this section, we construct an equilibrium of the repeated settlement bargaining game in which, regardless of past history, the good type accepts a settlement demand if and only if it does not exceed $(1-q)H+c_1$; in other words, the good type acts *as if* he commits to this strategy. Note that this behavior emerges as part of an equilibrium rather than as an assumption of the model, often the case in standard reputation models. We shall later discuss other possible equilibrium behavior of the good type in Section 4. Whenever we henceforth refer to player 1, we shall therefore mean the bad type.

3.2 First intuition

In the repeated settlement bargaining game, player 2 may learn about the type of the long-lived player from two sources: player 1's equilibrium actions and expert verdicts, if any. We call the tacit information inferred from the equilibrium behavior "soft" information and the expert's explicit verdict "hard" information. The important aspect of our equilibrium construction captures the interplay between these two channels of information.

Let us first think of possible equilibrium dynamics intuitively. On the one hand, if player 2's posterior belief (on the good type) is high, his expected payoff from resolving the dispute through the expert is low and, moreover, he has to pay a cost to obtain a verdict. Thus, intuitively, when the posterior belief is sufficiently high, player 2 should make a low settlement demand that is going to be accepted by both types of player 1 and the dispute is resolved without expert intervention. In this case, there is no learning from either soft or hard information for future player 2.

On the other hand, if the posterior belief is low, player 2 expects to win a large compensation if the case goes to the expert. Therefore, player 2 should make a large demand that the good type will not tolerate. How should the bad type respond?

If the bad type accepts this demand, he reveals his type and consequently his future compensation expenditure will be high. He cannot therefore accept it with probability 1; otherwise, the equilibrium posterior (on the good type) following rejection must be 1, and the bad type would mimic the good type by rejecting the demand.

This brings us to the critical question of whether the bad type has incentives to fully mimic the good type in equilibrium and reject the high demand with probability 1. In the game, rejecting a demand invokes an expert signal, which is imperfect but informative ($q > \frac{1}{2}$). Thus, mimicking the good type will reduce the bad type's reputation at the next period *in expectation*. This suggests that, when reputation is very low, fully mimicking the good type is costly in terms of both current period and continuatin payoffs. Player 1 should then play a mixed strategy: he rejects the high demand with an interior probability. The role of randomization here is to mitigate the effect of a non-favorable expert verdict (that is, a verdict that player 1 is liable). Since the good type rejects the demand for sure and the bad type rejects it only occasionally, the act of rejection will itself enhance player 1's reputation and may even overturn the effect of a non-favorable verdict. Player 2 learns through both soft and hard information.

However, when reputation is sufficiently close to the point beyond which player 2 finds optimal to make a low settlement demand, the bad type may still wish to fully mimic the good type, reject the high demand with probability 1 and count on the chance that expert verdict favors him. If he is lucky, his reputation will enter the region in which player 2 makes only a small settlement demand, accepted by both types, and learning stops altogether.

The above arguments therefore suggest that an equilibrium can be characterized by two cutoff beliefs that quantify the "low " and "high " reputation regions. We next show that this is indeed the case.

3.3 Formal description

We now elaborate on the above intuition and formally construct an equilibrium characterized by two threshold beliefs, $0 < p^* < p^{**} < 1$.[15] The equilibrium displays the following dynamics around three corresponding "regions":

The low reputation region, $(0, p^*)$. This is a region of learning through

[15] In the next section, we characterize the key properties of all Markov perfect Bayesian equilibria.

both soft and hard information. Player 2 finds it optimal to make a high settlement demand, equal to $qH - c_2$, which the good type will reject for sure; the bad type responds to such a demand by randomization. Therefore, the act of rejection itself leads to reputation building, and the subsequent expert signal will also lead to learning from player 2. A favorable verdict enhances reputation further, while a non-favorable verdict brings reputation back down.

The intermediate reputation region, (p^*, p^{**}). This is a region of learning through hard information alone. Here, player 2 also makes a high demand equal to $qH - c_2$ and both types reject it with probability 1. Player 2 does not learn about player 1's private information via player 1's act of rejection *per se*; rather, the learning takes place only through the realization of the expert's verdict.

The high reputation region, $(p^{**}, 1)$. This is a region of no learning. The full benefit of reputation is obtained. Player 2 makes a low settlement demand, equal to $(1-q)H + c_1$, that both types accept for sure.

Finally, the behavior at p^* and p^{**} are chosen so that the equilibrium conditions hold. It is worth noting that p^* and p^{**} are generic points in the following sense. At any belief $p \in (0, p^*)$, the posterior will reach p^* after a rejection and, moreover, a finite number of successive favorable expert verdicts will take the belief from p^* to p^{**}. The equilibrium behavior at these two thresholds are critical in the equilibrium construction. The next proposition states the equilibrium formally.

Proposition 1 *There exists $\bar{\delta} \in (0,1)$ such that for any $\delta > \bar{\delta}$ the repeated settlement bargaining game admits a Markov perfect Bayesian equilibrium with the following properties. There exist two thresholds, $0 < p^* < p^{**} < 1$, such that:*

- *If $p = 0$, player 2 demands $qH + c_1$ with probability 1; player 1 (the bad type) accepts it with probability 1.*
- *If $p \in (0, p^*]$, player 2 demands $qH - c_2$ with probability 1; player 1 rejects it with probability $r(p)$, where*

$$r(p) = \frac{p}{p^*} \frac{1-p^*}{1-p} \leq 1.$$

- *If $p \in (p^*, p^{**})$, player 2 demands $qH - c_2$ with probability 1; player 1 rejects it with probability 1.*

- *If $p = p^{**}$, player 2 demands $(1-q)H + c_1$ with probability x and $qH - c_2$ with probability $1-x$ for some $x \in [0,1)$; player 1 accepts the low demand with probability 1, while rejecting the other demand with probability 1.*

- *If $p \in (p^{**}, 1]$, player 2 demands $(1-q)H + c_1$ with probability 1; player 1 accepts this demands with probability 1.*

- *Beliefs: after observing player 1's acceptance of any demand above $(1-q)H + c_1$, player 2 assigns zero probability to the good type; in all other circumstances, beliefs are updated according to Bayes' rule whenever possible.*

The equilibrium displays some interesting features beyond the threshold dynamics. First, starting from any interior prior, the posterior reaches the high reputation region $(p^{**}, 1)$ and then stay there forever with an interior probability. *Reputation can be built.* Second, player 1 will also fail to build reputation with a positive probability; this happens in the low reputation region $(0, p^*)$ where player 1 randomizes and reveals his type occasionally. *Reputation is a valuable asset.* Third, if the prior falls in the low reputation region, the equilibrium payment converges to $V(0)$ (the payment once the type has been revealed) as the discount factor goes to 1. *The gain from reputation building is small asymptotically.* Finally, in the low reputation region where both soft and hard information are present, *the soft information can overturn the hard information* when their forces pull in opposite directions. In particular, when p is low enough relative to p^*, the overall effect of reputation will be positive: even after a non-favorable expert verdict the subsequent posterior at the beginning of the next period will remain higher than the current period's initial level. Our next proposition summarizes these findings formally. Their proofs are straightforward.

Proposition 2 *Consider the MPBE in Proposition 1.*

- *"Reputation can be built."*

  Suppose that $p_1 \in (0, p^{**}]$. Then, the probability with which the equilibrium posterior reaches the region $(p^{**}, 1)$ is positive.

- *"Reputation is a valuable asset."*

  Suppose that $p_1 \in (0, p^{**}]$. Then, the probability with which the equilibrium posterior falls to 0 is positive.

- *"The gain from reputation building is small asymptotically."*

  Suppose that $p_1 \in (0, p^*)$*. Then,* $V(p_1)$ *converges to* $V(0)$ *as* δ *goes to 1.*

- *"Soft information can overturn hard information."*

  Suppose that $p_t \in \left(0, \frac{p^*(1-q)}{p^*(1-q)+(1-p^*)q}\right)$*. Suppose also that, in this period* t*, player 1 rejects player 2's demand and the subsequent expert verdict is non-favorable. Then, we have*

$$p_{t+1} = \frac{p^*(1-q)}{p^*(1-q)+(1-p^*)q} > p_t.$$

We next discuss how the equilibrium responds to shifts in some key parameters. Of particular interest is how the thresholds change in response to increased patience and expert quality.[16]

Proposition 3 *Consider the MPBE in Proposition 1.*

- *As* δ *goes to 1,* p^* *goes to* 0*;* p^{**} *is independent of* δ*.*
- *As* q *goes to 1,* p^* *goes to* 0*;* p^{**} *goes to* $\frac{H-c_1-c_2}{H}$*.*

The impact of increased patience falls only on the lower threshold, p^*, which decreases. Thus, it expands the region in which player 1 fully mimics the good type and rejects the equilibrium demands for sure, thereby relying solely on expert signals. Although expert resolution, on average, worsens reputation, a more patient long-lived player is willing to try his luck earlier, in an effort to move into the no-learning region above the upper threshold, p^{**}, where he pays only a small amount of compensation.

As the expert quality increases, the intermediate reputation region also expands. But here, this effect is achieved by a reduced lower threshold *and* an increased upper threshold, p^{**} (whose corresponding limit is less than 1). This first implies that the no-learning region shrinks, and we may interpret this as suggesting that reputation is indeed more difficult to build when the expert is more accurate.

[16]Note that, due to the construction seen in the next subsection, p^* responds *discretely* to changes in the parameters. We therefore report its limits.

While this last observation is intuitive, the fact that the intermediate region expands as q goes to 1 is somewhat surprising. This says that, when the expert is very accurate, the parties will almost always resort to external intervention, rather than making voluntary settlements and saving on expert costs. Why is this? The reason is that when q is very large a single piece of good luck is all that is needed for player 1 to jump into the no-learning region and reap the full benefits of reputation. Given this, what player 2 asks for at low levels of reputation is too much for player 1 to accept.

3.4 Details of construction

We now demonstrate the technique behind the equilibrium construction which we believe to be innovative and interesting in its own right. Some readers may, however, wish to skip this part and move directly to the next sub-section which contains some graphical illustrations of the equilibrium.

Figure 1: Equilibrium demands

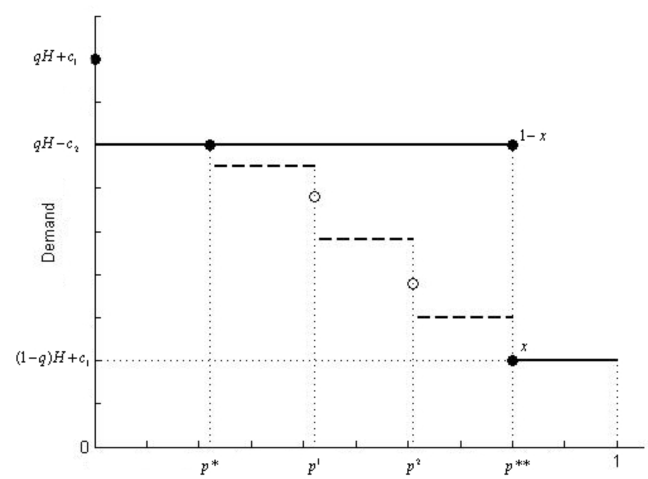


Figure 1 describes player 2's equilibrium demand as a function of the posterior belief p. Here, the critical aspect of our equilibrium construction is that the demand at low beliefs ($p \leq p^*$) must be exactly $qH - c_2$. The reasoning is as follows.[17]

[17]This is in fact a general property of an equilibrium. A formal statement and its proof

Note first that player 1 must use a cutoff strategy; that is, in equilibrium, if he accepts a (high) demand, he must accept any demand that is smaller. Since rejected demands are not observable, rejecting any demand generates the same posterior belief and, hence, continuation payment. This implies that, if player 1 (weakly) prefers to accept than to reject a demand, he must strictly prefer to accept any lower demand.

Now, suppose that the equilibrium demand were higher than $qH - c_2$ and, moreover, player 1 would accept it. Then, the acceptance must occur for sure. This is because, otherwise, player 2 could profitably deviate by demanding slightly less than the equilibrium amount, and the deviation would be met with sure acceptance (via the cutoff strategy argument).

But then, since only the good type would reject the demand, the posterior following rejection on such an equilibrium path would actually be 1 and, therefore, the bad type would himself have an incentive for deviation; by rejecting the equilibrium demand, he could obtain a low settlement demand in every period thereafter.[18]

On the other hand, player 2 clearly has no incentive to demand anything less than $qH - c_2$, as this is precisely the amount that he expects to achieve from the bad type under expert resolution. These arguments lead to the property that, when p is low, the only demand that can be accepted in equilibrium must equal $qH - c_2$.

Note that, if $p \in (p^*, p^{**})$, player 2's equilibrium demand is rejected for sure by player 1. We can construct other equilibria in which, in this intermediate range of beliefs, player 2 makes demands other than $qH - c_2$, as long as it is sufficiently large that player 1 finds it optimal to reject it. Similarly, at p^{**}, player 2 can randomize between $(1 - q)H + c_1$ and any high demand that is rejected. In Figure 1 above, the dotted lines and empty dots drawn in the (p^*, p^{**}) region represent player 1's *reservation* demands. These values are formally characterized in Appendix.

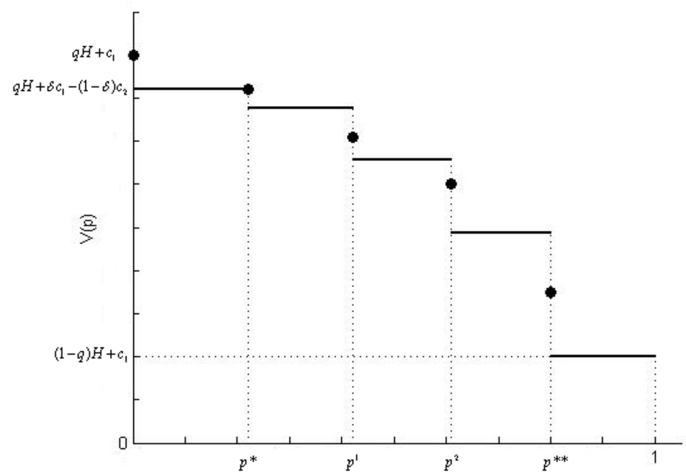
Figure 2 describes player 1's equilibrium (discounted average) expected payment. This figure illustrates another key element behind the construction of the equilibrium.

If $p \in (0, p^*]$, player 1 is indifferent between accepting and rejecting the demand $qH - c_2$. His expected payment is then given by what he obtains

appears in Appendix (Lemma 2).

[18]For the deviation to be profitable, of course, the discount factor must be sufficiently large.

Figure 2: Equilibrium payments



from accepting and revealing his type. If player 1's type is revealed, the demand will be $qH + c_1$ in every period thereafter (and he is going to accept it) and, therefore, we can see that

$$V(p) = (1 - \delta)(qH - c_2) + \delta(qH + c_1) = qH + \delta c_1 - (1 - \delta)c_2. \qquad (1)$$

If $p > p^{**}$, on the other hand, player 2 demands $(1 - q)H + c_1$ which player 1 accepts and, therefore, $V(p) = (1-q)H+c_1$. The payment in the intermediate region, (p^*, p^{**}), follows a decreasing step function, while the payment when p is exactly at p^{**} is determined by player 2's randomization.

We now provide a full sketch of the equilibrium construction with the aid of Figure 2. All omitted technical details, as well as a more formal description of the proposed equilibrium strategies and beliefs appear in Appendix.

Characterizing p^{**} First, let us determine the upper belief threshold. At p^{**}, player 2 should be indifferent between demanding $(1-q)H+c_1$, which is accepted for sure, and a high demand, which is rejected for sure. If his demand is rejected, the expected payoff is $p^{**}(1 - q)H + (1 - p^{**})qH - c_2$. Therefore, from the indifference condition $p^{**}(1 - q)H + (1 - p^{**})qH - c_2 = (1 - q)H + c_1$, we derive

$$p^{**} = \frac{(2q-1)H - c_1 - c_2}{(2q-1)H} \in (0,1). \tag{2}$$

It is important to note that p^{**} is *independent of the discount factor*. No matter how patient or impatient player 1 is, he must pass this threshold in order to entertain the full benefits of reputation.

Characterizing p^* Note that, in the equilibrium, $V(p) = V(p^*)$ (as in (1) above) if $p \in (0, p^*)$, and $V(p) = (1-q)H + c_1$ if $p \in (p^{**}, 1]$. Let p^1 be the belief that is updated from p^* after an expert verdict favorable to player 1, given that rejection prior to the verdict itself does not affect the belief. That is,

$$p^1 = \frac{p^* q}{p^* q + (1-p^*)(1-q)}.$$

We define p^2 to be the belief obtained from p^1 after another favorable verdict, and $p^3, p^4, \ldots$ are defined similarly. We write $p^0 = p^*$.

Importantly, the assumption that the quality of expert judgement, q, is symmetric across player 1 types implies that the posterior updated from p^n following a non-favorable verdict is exactly p^{n-1}. It is straightforward to verify that, for any integer n, if

$$p^n = \frac{p^{n-1} q}{p^{n-1} q + (1-p^{n-1})(1-q)},$$

then

$$p^{n-1} = \frac{p^n (1-q)}{p^n (1-q) + (1-p^n) q}.$$

Consider any positive integer n such that $p^n \in (p^*, p^{**})$. Player 1's equilibrium expected payment at p^n, $V(p^n)$, contains three parts: (i) since he rejects player 2's demand, he expects to pay $qH + c_1$ in the current period, via expert resolution; (ii) if the verdict is non-favorable, which occurs with probability q, his continuation payment at the next period will be $V(p^{n-1})$; and, if the verdict is favorable, which occurs with probability $1-q$, his continuation payment at the next period will be $V(p^{n+1})$. Thus, we obtain the following recursive equation to characterize $V(p^n)$:

$$V(p^n) = (1-\delta)(qH + c_1) + \delta q V(p^{n-1}) + \delta(1-q) V(p^{n+1}). \tag{3}$$

We solve this second-order difference equation (for integer values on n) with the initial conditions:

$$V(p^0) = qH + \delta c_1 - (1-\delta)c_2, \quad (4)$$
$$V(p^0) = (1-\delta)(qH + c_1) + \delta q V(p^0) + \delta(1-q)V(p^1). \quad (5)$$

These initial conditions arise because we set $p^0 = p^*$ and, at any $p < p^*$, player 1 randomizes such that $V(p) = V(p^*)$, in equilibrium.

The solution, $V(p^n)$, can easily be shown to be strictly decreasing and also divergent. This implies that there must exist some finite integer N such that $V(p^N) > (1-q)H + c_1$ and $V(p^{N+1}) \leq (1-q)H + c_1$. We then define the lower threshold belief, $p^* = p^0$, such that $p^{**} = p^N$. In other words, p^* is the posterior belief that is reached from p^{**}, which is already fixed, after N successive non-favorable expert decisions. Although p^{**} does not depend on δ, the other threshold turns out to be tied to the discount factor; specifically, an increase in δ (weakly) raises the number of steps between the two thresholds, implying a lower value for p^*.

Equilibrium payments The solution to the recursive equation (3) above, together with the initial conditions (4) and (5), gives equilibrium payments at posterior beliefs that are located between and reachable from the two thresholds, corresponding to the solid dots in Figure 2 ($V(p^0) = V(p^*), \ldots, V(p^N) = V(p^{**})$). However, recall that $(1-q)H + c_1$ is the lowest possible equilibrium payment. Then, while $V(p^N)$ characterizes the equilibrium payment at p^{**}, the values of the solution to (3) beyond this point cannot be equilibrium payments since they, by definition, fall below $(1-q)H + c_1$.

This raises a critical issue in our equilibrium construction. Note that $V(p^N)$ is computed from

$$V(p^N) = (1-\delta)(qH + c_1) + \delta q V(p^{N-1}) + \delta(1-q)V(p^{N+1}),$$

where $V(p^{N+1}) \leq (1-q)H + c_1$. How then do we support $V(p^N)$ as an equilibrium payment? The answer is found in player 2's randomization at p^{**}. Here, player 2 is indifferent between having the demand $(1-q)H + c_1$ accepted and having another, higher demand rejected for sure by both types. We can derive a unique mixed strategy by player 2 at p^{**} such that the equilibrium holds.[19]

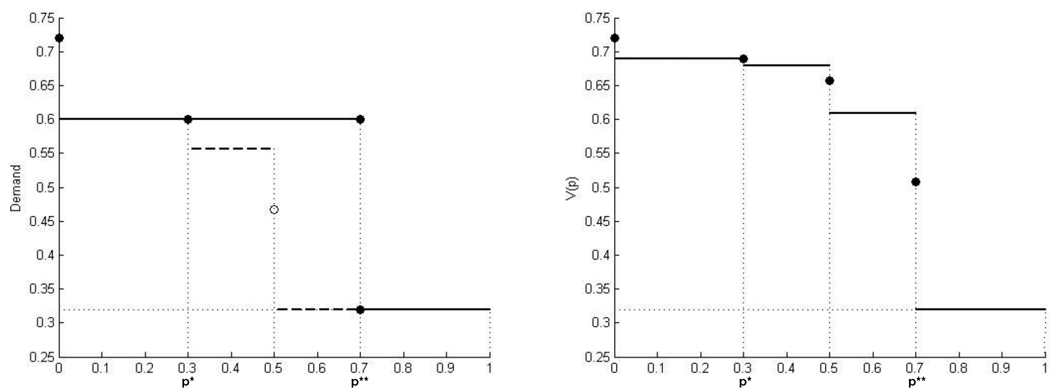
[19] In the degenerate case where the solution to the recursive equation yields a value exactly $(1-q)H + c_1$, at p^{**}, player 2 demands a high amount that is rejected with probability 1.

Finally, we can tie down the equilibrium payments at other, remaining beliefs, along the "steps" between p^* and p^{**}, by solving the same recursive equation (3) with different initial conditions, specifically, that the first value is $qH + \delta c_1 - (1-\delta)c_2$ and the $(N+1)$-th value $(1-q)H + c_1$.

3.5 Further illustrations

We next provide some graphical illustrations of the above equilibrium with specific parameter values. Figure 3 shows equilibrium demands by player 2 (together with the bad type's reservation demands) and equilibrium payments of the bad type when $H = 1$, $\delta = 0.75$, $q = 0.7$, $c_1 = 0.02$ and $c_2 = 0.1$.

Figure 3: Example



In addition, Figure 4 illustrates the probability of rejection, that is, the likelihood of expert intervention in the corresponding equilibrium. This further clarifies the relationship between reputation, or the "merit" of a dispute, and settlement. As shown in the previous figure, the *amount* of settlement, if agreed, is fixed in the interval $(0, p^*)$; however, the *rate* of settlement (or expert intervention) is falling (or increasing) in the merit over this region before reaching zero (or 1) in the intermediate region. Note also that the settlement rate rises to some interior level at p^{**} (due to player 2's randomization) and then all the way to 1 in the high reputation region.

Figure 4: Rejection rate

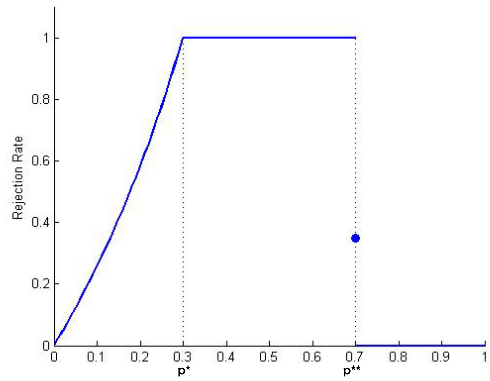


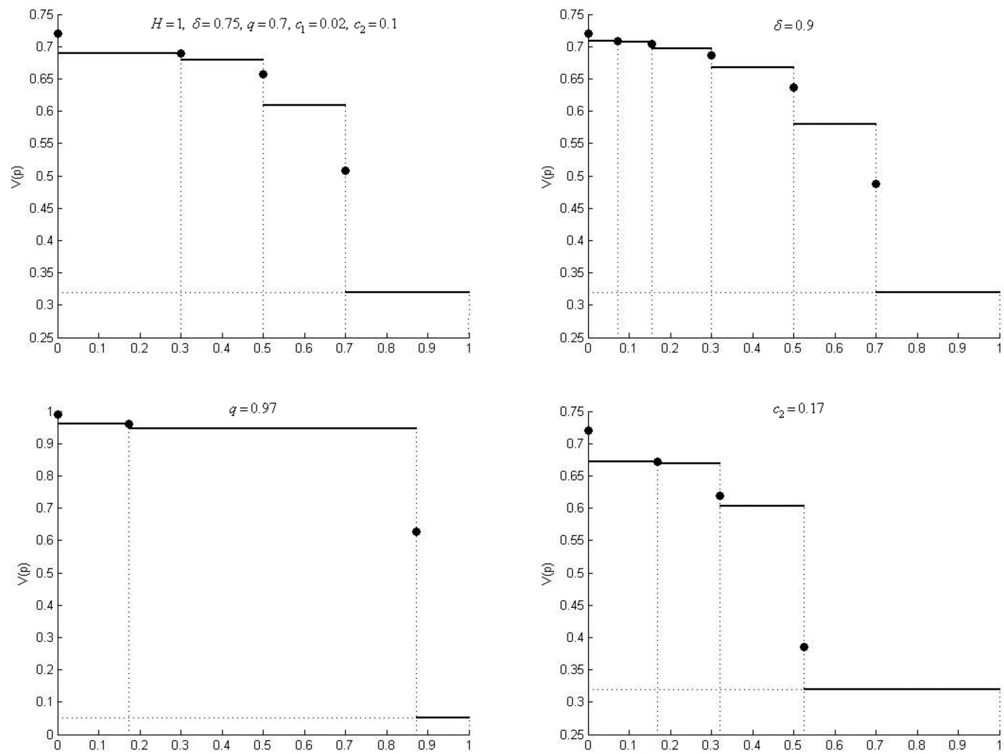
Figure 5 illustrates the comparative static results in terms of payments. The first graph here reproduces the equilibrium payments of the example above, while the other three show how the payments change after an increase in δ, q and, also, c_2, respectively.

An increase in δ from 0.75 to 0.9 indeed expands the intermediate region by inducing more "steps"; the high reputation region and the corresponding payments remain the same but the lower threshold falls and the payment at the low reputation region is pushed up.

Raising the expert precision from 0.7 to 0.97 shows an even more drastic change. The intermediate region is vastly expanded but it involves only one step. Both thresholds move, in opposite directions. It is more difficult to build reputation and reach the high reputation region; moreover, the payments during the reputation building process are also higher than the benchmark. However, should player 1 succeed in reaching beyond the (increased) upper threshold, the benefits will actually be greater (lower payments). As mentioned in Introduction, this again illustrates the possibility that, somewhat paradoxically, greater accuracy of the court will impose a greater burden on itself.

The final graph illustrates the effect of an increase in c_2 (from 0.1 to 0.17), the expert cost incurred by player 2. Here, at any p, player 1's compensation expenditure is lower, or the same, compared to the benchmark. Thus, making the expert more costly to player 2 may improve the benefits of player 1's reputation building. This observation throws a caution at some policies

Figure 5: Comparative statics



targeted at reducing the volume of “frivolous” lawsuits (e.g. PSLRA of 1995 in securities litigations). Although it may deter some non-meritorious suits, higher litigation costs for plaintiffs could perversely aid the cause of a bad defendant who wants to get away with his responsibility cheaply.

4 Some general properties of an equilibrium

The equilibrium constructed in the previous section exhibits a particular behavioral pattern. We now turn to the question of whether any aspects of the equilibrium apply more generally to an equilibrium.

Our next Proposition characterizes some general properties of an MPBE, while maintaining the following assumptions:

A1. The good type, regardless of past history, accepts a demand if and only if it does not exceed $(1-q)H+c_1$.

A2. Acceptance of a demand strictly greater than $(1-q)H+c_1$ reveals that player 1 is bad, both on and off the equilibrium path.

Proposition 4 *Assume A1 and A2, and let* $p^{**} = \frac{(2q-1)H-c_1-c_2}{(2q-1)H}$. *Then, there exists* $\bar{\delta} \in (0,1)$ *such that, for any* $\delta > \bar{\delta}$, *any MPBE of the repeated settlement bargaining game satisfies the following properties:*

- $V(0) = qH + c_1$.
- *For any* $p \in (0,1]$, $V(p) \in [(1-q)H + c_1, qH + \delta c_1 - (1-\delta)c_2]$.
- *For any* $p \in (p^{**}, 1]$, $V(p) = (1-q)H + c_1$.
- *There exists* $p^* \in (0, p^{**})$ *such that, for any* $p \in (0, p^*)$, $V(p) = qH + \delta c_1 - (1-\delta)c_2$.

Thus, we are able to obtain payment bounds for any equilibrium and, moreover, establish that the lower bound must be achieved when reputation is sufficiently high while the upper bound is met when reputation is sufficiently low. These bounds correspond precisely to the payments of the equilibrium constructed in the previous section for sufficiently high/low reputation levels.

In principle, player 2's mixed strategy can involve any distribution over the real line. This potentially makes the problem of characterizing an equilibrium very complicated in the present repeated game setup. But, we show that there are only two demand levels that can be accepted with a positive probability by player 1 on the equilibrium path: either $(1-q)H + c_1$ or $qH - c_2$. All other demands are rejected for sure. (An intuition for this has already been given in the previous section.) This will greatly simplify our analysis and enable us to reach Proposition 4.

The important feature of an equilibrium is that, when p is sufficiently low (but strictly positive), the equilibrium payment of player 1 is equal to $qH+\delta c_1-(1-\delta)c_2$. Notice that, as δ goes to 1, this upper bound approaches $qH+c_1$, the payment that the long-lived player expects to incur in each period if the short-lived players *know* that he is bad. Thus, when δ is sufficiently close to 1, there exists a level of belief low enough at which the value of reputation building is arbitrarily small.

It must also be noted that the above equilibrium properties are not tied to the specific behavior assumed from the good type (assumption A1). In particular, we may think of another class of equilibria in which the good type takes a *tougher* stance and accepts a demand if and only if it is less than or equal to some level below $(1-q)H + c_1$. It is not difficult to see that, appropriately modifying off-the-equilibrium beliefs (assumption A2), we can reach essentially the same conclusions as in Propositions 1-4. The upper threshold belief, p^{**}, and the corresponding lower bound on payments will change, but the overall reputation dynamics as well as the upper payment bound at low levels of reputation will remain as before.

5 Concluding Discussion

In this paper we have maintained several simplifying assumptions to faciliate the analysis. We now offer some concluding discussion by addressing these assumptions. In doing so, we shall also discuss some related work in litigation research. Our model provides a benchmark for future research in a number of directions.

5.1 Robustness to (un-)observability of settlement demands

We have assumed, as is usually the case in securities class actions and medical malpractice litigations, that the accepted settlement demands are publicly observable while the rejected settlement demands are not. Our equilibrium in Section 3 is robust to the (un-)observability of the settlement details.

It is straightforward to see that the equilibrium continues to be valid when the rejected demands are also publicly observable. Even though we have assumed that short-lived players do not observe previously rejected demands, it is common knowledge in equilibrium that the rejected demands must always be $qH - c_2$. Thus, it does not depend on whether this amount is observable or not.

Daughety and Reinganum [9] consider endogenous settlement in a two-period model where the parties can choose whether their settlement agreement will be open or confidential and, furthermore, the arrival rate of a second plaintiff is lower with confidential settlement. They also assume that the court's decision is perfect. This last assumption implies that the long-lived defendant's type is perfectly revealed if the first case goes to court, independently of the verdict, and that the second period game is degenerate after a trial in the first period. They show that there can be a first-plaintiff benefit via confidential settlements.[20]

In contrast, we assume that the court is imperfect. It is possible to incorporate into our model endogenous determination of the settlement mode. However, our equilibrium is robust under the following natural specification of belief upon observing a confidential settlement: player 2 assigns probability 1 to the bad type. After all, it is natural that the good type who is innocent has nothing to hide. This eliminates any benefit of confidentiality. Indeed, we believe that this reasoning explains why settlements once reached are observable in securities class actions or medical malpractice cases where the arrival of litigations are public and independent of the previous settlements (shareholders simply react to a loss in value, while patients to their own experience of accident).

To justify benefits of confidential settlements with an imperfect court, it seems that we need to allow the arrival of future short-lived players to itself depend on the mode of settlement. We leave this to future research.[21]

5.2 Other assumptions

In our model, the bargaining within each period takes a simple format: the uninformed player makes a take-it-or-leave-it offer. Such simplicity allows us to concentrate on the long-lived player's dynamic incentives, as done also in Schmidt [23], Daughety and Reinganum [8][9] and others. The one-sided offer by the uninformed player however rules out complex signaling effects. Spier [24] considers settlement bargaining between a single pair of defendant and plaintiff under more complex bargaining procotols.

The stake (or the distribution thereof) in each dispute is assumed to be common knowledge. This seems to be a reasonable description of securities

[20]Daughety and Reinganum [8] consider a similar model in which the defendant's type is independent across the two periods and, therefore, the issue of learning does not arise.

[21]Choi [6] studies a multi-period model in which an incumbent patentee has an option of launching an infringement suit when faced with entry of an imitator. There is, however, no asymmetric information; rather, the author addresses the incumbent's optimal timing of a lawsuit. The issue of timing is in general an interesting aspect of a dynamic problem but it is not clear how this can be incorporated into a model of reputation.

class actions where the stakes can be traced to the loss in share value. Introducing private information over the magnitude of the stake, in addition to private information on liability, will significantly complicate the analysis beyond the scope of the present paper.

Finally, we also assume that the a third party verdict is based on the merit of the current case alone, regarding whether player 1 is liable or not. Che and Yi [5] consider a two-period model with correlated decisions. Here, the probability with which the second plaintiff would win in court depends on the outcome of the first period litigation. This offers another interesting extension for future research. However, as argued before, time-independent expert precision is plausible.

6 Appendix

6.1 Proof of Proposition 1

Our proof of this result is based on the following construction. We first need some notation. Let

$$\begin{aligned}\Phi^1(p) &= \frac{pq}{pq+(1-p)(1-q)} \\ \Phi^{-1}(p) &= \frac{p(1-q)}{p(1-q)+(1-p)q}.\end{aligned}$$

That is, when the belief is p, if both types of player 1 go to expert and the verdict is not liable (or liable), then the increased (or decreased) updated belief is equal to $\Phi^1(p)$ (or $\Phi^{-1}(p)$). Notice that $\Phi^{-1}\left(\Phi^1(p)\right) = p$ for any p.

Furthermore, for any positive integer k, define $\Phi^k(p)$ recursively such that $\Phi^2(p) = \Phi^1\left(\Phi^1(p)\right)$, $\Phi^3(p) = \Phi^1\left(\Phi^2(p)\right)$ and, hence, $\Phi^k(p) = \Phi^1\left(\Phi^{k-1}(p)\right)$. In other words, when the initial belief is p, if both types of player 1 go to expert k consecutive times and the verdict favors player 1 on each occasion, then the posterior belief updated from p is $\Phi^k(p)$, Similarly, we define $\Phi^{-k}(p)$ as the posterior reached from p after k successive non-favorable expert decisions for player 1. Also, let $\Phi^0(p) \equiv p$.

Next, let $\bar{\delta}$ solve the following:

$$[1-\bar{\delta}q][qH+\bar{\delta}c_1-(1-\bar{\delta})c_2] = (1-\bar{\delta})(qH+c_1)+\bar{\delta}(1-q)[(1-q)H+c_1]. \quad (6)$$

It is straightforward to observe that such $\bar{\delta}$ must belong to $(0, 1)$.

Fix any $\delta > \bar{\delta}$, and consider the profile (r^B, r^G, d) below, where $p^*, p^{**} \in (0, 1)$, $x \in [0, 1)$ and $\xi(p)$ are to be defined later.

First, player 2's strategy, d, is such that:

- At $p = 0$, it demands $qH + c_1$ with probability 1;
- For any $p \in (0, p^{**})$, it demands $qH - c_2$ with probability 1;
- For any $p \in (p^{**}, 1]$, it demands $(1 - q)H + c_1$ with probability 1;
- For $p = p^{**}$, it demands $(1 - q)H + c_1$ with probability x and $qH - c_2$ with probability $1 - x$.

Second, type-G player 1's strategy, r^G, is such that, for any p, it accepts a demand s if and only if $s \leq (1 - q)H + c_1$.

Third, type-B player 1's strategy, r^B, is such that:

- At $p = 0$, it accepts a demand s if and only if $s \leq qH + c_1$;
- For any $p \in (0, p^*]$,
  - it rejects any $s > qH - c_2$ with probability 1;
  - it accepts any $s < qH - c_2$ with probability 1;
  - it rejects $s = qH - c_2$ with probability $r(p)$, where $r(p)$ satisfies

    $$p^* = \frac{p}{p + (1 - p)r(p)},$$

    and therefore,

    $$r(p) = \frac{p}{p^*} \frac{1 - p^*}{1 - p} \leq 1.$$

    (Notice that $r(p^*) = 1$.)
- For any $p \in (p^*, p^{**}]$,
  - it rejects any $s > \max\{(1 - q)H + c_1, \xi(p)\}$ with probability 1;
  - it accepts any $s \leq \max\{(1 - q)H + c_1, \xi(p)\}$ with probability 1.

- For any $p \in (p^{**}, 1]$,
  - it rejects any $s > (1-q)H + c_1$ with probability 1;
  - it accepts any $s \leq (1-q)H + c_1$ with probability 1.

Finally, the belief is updated by Baye's rule and the equilibrium strategies whenever possible. We also assume that the posterior belief assigns probability 1 to type B after an acceptance of a demand higher than $(1-q)H+c_1$.

We now define p^*, p^{**}, x and $\xi(p)$. Along the way, the equilibrium payment of type B, $V(p)$, will also be obtained.

Defining p^{**} At the upper threshold level of belief, p^{**}, player 2 must be indifferent between demanding $(1-q)H + c_1$, which is accepted with probability 1, and demanding $qH - c_2$, which is rejected with probability 1. Thus, it is computed from the equation

$$(1-q)H + c_1 = p^{**}((1-q)H - c_2) + (1-p^{**})(qH - c_2),$$

which yields

$$p^{**} \equiv \frac{(2q-1)H - c_1 - c_2}{(2q-1)H} \in (0, 1).$$

Defining p^* At p^*, type B is indifferent between accepting and rejecting $qH - c_2$. Let $V_0 \equiv V(p^*)$ and $V_n \equiv V\left(\Phi^n(p^*)\right)$. Then, since acceptance of the equilibrium demand leads to revelation, we first have

$$V_0 = (1-\delta)(qH - c_2) + \delta(qH + c_1) = qH + \delta c_1 - (1-\delta)c_2. \tag{7}$$

Rejection, on the other hand, yields the following:

$$V_0 = (1-\delta)(qH + c_1) + \delta q V_0 + \delta(1-q)V_1, \tag{8}$$

where the current period expected payment equals $qH + c_1$, the next period continuation expected payment following a favorable verdict (which takes place with probability $1-q$) is V_1 and the corresponding payment following a non-favorable verdict is also V_0 (since type B randomizes at any $p < p^*$).

Note here that, since we assume $(2q-1)H > c_1 + c_2$, $V_0 > (1-q)H + c_1$ and that, since $\delta > \bar{\delta}$, $V_1 > (1-q)H + c_1$ (see (6) above for the definition of $\bar{\delta}$).

Next, consider the equilibrium payment V_n (at $p = \Phi^n(p^*)$) for any integer $n \geq 1$. Here, since the equilibrium demand is rejected for sure, the continuation payment must satisfy the following recursive structure:

$$V_n = (1-\delta)(qH + c_1) + \delta q V_{n-1} + \delta(1-q)V_{n+1}. \qquad (9)$$

Define $N = \sup\{n \in \mathbb{Z} : V_n > (1-q)H + c_1\}$, where $\mathbb{Z}$ denotes the set of integers; i.e. N is the largest integer n such that $V_n > (1-q)H + c_1$.

Then, given Claim 1 below, define $p^* = \Phi^{-N}(p^{**}) \in (0,1)$. Since $V_1 > (1-q)H + c_1$, N must be positive and, hence, $p^* < p^{**}$ as required by the equilibrium.

Claim 1 (1) *V_n is strictly decreasing in n.*
(2) *N is finte.*

Proof. (1) Notice that $V_0 < qH$ and V_0 is a convex combination of $qH + c_1$ and V_1. Then $V_1 < V_0$. Suppose $V_n < V_{n-1} < \cdots < V_0 < qH$. From (9), V_n is a convex combination of $qH + c_1$, V_{n-1}, and V_{n+1}, and hence $V_{n+1} < V_n$. The monotonicity of V_n follows by induction.

(2) Suppose to the contrary that N is infinite. That is, $V_n > (1-q)H + c_1$ for all n. Then, since V_n is strictly decreasing, V_n converges to V_∞ such that $(1-q)H + c_1 \leq V_\infty < qH + c_1$. But, from (9), it follows that $V_\infty = qH + c_1$. This is a contradiction. ■

Defining x At p^{**}, player 2 demands $(1-q)H + c_1$ with probability x and $qH - c_2$ with probability $1-x$; both types of player 1 accept the first demand with probability 1 and reject the second demand with probability 1. This implies that the equilibrium posterior at the next period must be such that:

- if $(1-q)H + c_1$ is accepted then the posterior remains at p^{**};
- if a demand is rejected, followed by a favorable verdict to player 1, then the posterior moves up to $\Phi^1(p^{**})$; and
- if a demand is rejected, followed by a non-favorable verdict to player 1, then the posterior moves down to $\Phi^{-1}(p^{**})$.

Thus, we have

$$V(p^{**}) \equiv V_N = x\left[(1-\delta)((1-q)H + c_1) + \delta V_N\right] + (1-x)X, \qquad (10)$$

where V_N is given by the second-order difference equation (9) with the two initial conditions V_0 and V_1 as in (7) and (8) above, and

$$X \equiv (1-\delta)(qH + c_1) + \delta q V_{N-1} + \delta(1-q)((1-q)H + c_1). \qquad (11)$$

Claim 2 *There exists a unique $x \in [0, 1)$ that satisfies (10).*

Proof. Simple computation shows that

$$x = \frac{X - V_N}{X - (1-\delta)((1-q)H + c_1) - \delta V_N}.$$

Note first that $V_N \leq X$. This follows from comparing (11) above to the recursive equation

$$V_N = (1-\delta)(qH + c_1) + \delta q V_{N-1} + \delta(1-q)V_{N+1},$$

where, by assumption, $V_{N+1} \leq (1-q)H + c_1$. Also, we have $V_N > (1-\delta)((1-q)H + c_1) + \delta V_N$ because, again by assumption, $V_N > (1-q)H + c_1$. Thus, $x \in [0, 1)$. ■

Equilibrium payments At this juncture, we characterize the equilibrium expected payments of type B. The following is clear:

- For any $p \leq p^*$, $V(p) = V_0$.
- For any $p = \Phi^n(p^*)$ with an integer $1 \leq n \leq N$, $V(p) = V_n$; in particular, $V(p^{**}) = V_N$.
- For any $p > p^{**}$, $V(p) = (1-q)H + c_1$.

We now pin down payments when $p \in (p^*, p^{**})$ but $p \neq \Phi^n(p^*)$ for any integer $1 \leq n \leq N$.

Claim 3 *Fix any integer $n \in [1, N]$ and any $p, p' \in (\Phi^{n-1}(p^*), \Phi^n(p^*))$. Then, we have*

$$V(p) = V(p') < V_0.$$

Proof. Consider the following recursive structure: for any integer k,

$$W_k = (1-\delta)(qH + c_1) + \delta q W_{k-1} + \delta(1-q)W_{k+1}$$

such that $W_0 = V_0$ and $W_{N+1} = (1-q)H + c_1$, where N is defined as above.

Note that we have

$$\Phi^{-n}(p) = \Phi^{-n}(p') < p^* \text{ and } \Phi^{-n+1}(p) = \Phi^{-n+1}(p') > p^*;$$
$$\Phi^{N-n+1}(p) = \Phi^{N-n+1}(p') > p^{**} \text{ and } \Phi^{N-n}(p) = \Phi^{N-n}(p') < p^{**}.$$

Thus, it is straightforward to see that

$$W_n = V(p) = V(p').$$

Also, from the same arguments for Claim 1 above, we can show that W_k is strictly decreasing. ■

Defining $\xi(p)$ For any $p \in (p^*, p^{**})$, $\xi(p)$ satisfies

$$(1-\delta)\xi(p) + \delta(qH + c_1) = V(p).$$

It remains to be shown that the profile (r^B, r^G, d) defined above, together with the stated beliefs, indeed constitutes an MPBE.

First, given r^B and r^G, and the definition of p^{**}, it is straightforward to establish optimality of player 2 strategy, d. In particular, note that it is never optimal for player 2 to make a demand $s \in ((1-q)H + c_1, qH - c_2)$.

Second, we check optimality of r^G, the strategy of type G. This is clear since player 2 never makes a demand less than $(1-q)H+c_1$, which is precisely the amount that this type expects to pay in total in case the dispute goes to the expert in any period.

Finally, we check optimality of r^B.

- It is straightforward to check optimality of r^B at $p = 0$.
- Fix any $p \in (0, p^*]$. Suppose first that the demand, s, is less than $qH - c_2$. If type B accepts this demand, the continuation payment amounts to

$$(1-\delta)s + \delta(qH + c_1) < V_0,$$

while, since rejected demands are not observable, the continuation payment from rejecting continues to be V_0. Thus, accepting any $s < qH - c_2$ for sure is optimal. A symmetric argument establishes that rejecting any $s > qH - c_2$ for sure is optimal. The rejection probability $r(p)$, supports the indifference conditions captured by (7) and (8) above.

- Fix any $p \in (p^*, p^{**})$. Here, by Claims 1 and 3 above, we have $V(p) < V_0$, and accepting the demand $qH - c_2$ yields precisely $V_0 = (1-\delta)(qH - c_2) + \delta(qH + c_1)$ due to revelation. Thus, rejecting the equilibrium demand, $qH - c_2$, is optimal.[22]

- Consider $p = p^{**}$. If type B accepts the equilibrium demand $qH - c_2$, he reveals his type and, hence, obtains a continuation payment V_0. If he rejects this demand, on the other hand, he obtains

  $$(1-\delta)(qH + c_1) + \delta q V_{N-1} + \delta(1-q)((1-q)H + c_1) \equiv X < V_0,$$

  where the last inequality can be obtained from the proof of Claim 3 above. Thus, it is optimal to reject $qH - c_2$.

  Next, consider the demand $(1-q)H + c_1$. Rejection, again, yields a continuation payment X, while acceptance leads to a payment $(1-\delta)((1-q)H + c_1) + \delta V_N$. Since $V_N < X$ and $(1-q)H + c_1 < X$, acceptance is optimal.

- Fix any $p \in (p^{**}, 1]$. Since player 2 plays a pure strategy here, and by A1, accepting the equilibrium demand $(1-q)H + c_1$ cannot reduce the equilibrium posterior. Thus, accepting yields a continuation payment $(1-q)H + c_1$. On the other hand, rejection yields, at best, a continuation payment

  $$(1-\delta)(qH + c_1) + \delta\left((1-q)H + c_1\right),$$

  implying optimality of acceptance.

6.2 Proof of Proposition 3

1. We have already established that p^{**} is independent of δ. By definition, p^* is the posterior probability after N consecutive non-favorable expert decisions

[22] Note that type B's equilibrium cutoff demand here is given by $\max\{(1-q)H + c_1, \xi(p)\}$ (see the specification of r^B). It is easily seen that $\xi(p) < qH - c_2$.

starting from p^{**}. Therefore, to show p^* goes to 0 as δ goes to 1, it suffices to establish that $N(\delta)$ goes to ∞ as δ goes to 1.

We first note that that $V(p^n) - V(p^0) \to 0$ as δ goes to 1 for any fixed n. This follows directly from equations (3) and (4) in the main text. Since $V(p^0) > (1-q)H + c_1$ even when $\delta \to 1$, $N(\delta)$ goes to ∞ by definition.

2. It is immediate from the definition of $p**$ that $p^{**} \to \frac{H-c_1-c_2}{H}$ as $q \to 1$. By equations (4) and (5), $V(p^1) \to -\infty$ as $q \to 1$. Therefore, $N \to 1$ as $q \to 1$ and, hence, p^* becomes the posterior probability obtained after a single non-favorable expert decision starting from p^{**}, that is, $p^* = \frac{p^{**}(1-q)}{p^{**}(1-q)+(1-p^{**})q}$. Given the limit of p^{**}, it follows immediately that $p^* \to 0$ as $q \to 1$.

6.3 Proof of Proposition 4

First of all, given assumption A1 and the Markov restriction, it is straightforward to observe that, at $p = 0$, $qH + c_1$ will be demanded and accepted for sure (thus, $V(0) = qH + c_1$) and, at $p = 1$, $(1-q)H + c_1$ will be demanded and accepted for sure (thus, $V(1) = (1-q)H + c_1$).

We now proceed by establishing the following Lemmas.

Lemma 1 ***(Cutoff of Acceptance)*** *Fix any δ and any MPBE. Also, fix any posterior p, and consider a demand $s > (1-q)H + c_1$. The following is true on or off the equilibrium path:*

(1) *If type B accepts s with a positive probability, then he must accept any $s' < s$ with probability 1.*

(2) *If type B rejects s with a positive probability, then it must reject any $s' > s$ with probability 1.*

Proof. (1) If s is accepted, the continuation (discounted average expected) payment from accepting s must be at least as good as that from rejecting it.

Since rejected demands are not observable, rejecting any demand results in the same continuation payment. Also, by assumption A2, accepting any demand strictly above $(1-q)H+c_1$ leads to the same continuation payment at the next period (equal to $qH+c_1$). Then, accepting any $s' \in ((1-q)H+c_1, s)$ must be strictly better than rejecting it since it yields a lower immediate payment.

On the other hand, accepting a demand $s' \le (1-q)H+c_1$ needs not lead to revelation but the continuation payment at the next period must still be bounded above by $qH + c_1$ and, hence, the same arguments imply that such a demand must also be accepted for sure.

(2) If s is rejected, the continuation payment from rejecting s must be at least as good as that from accepting it. Rejecting s or s' results in identical expected payments, both in the current period and each forthcoming period; on the other hand, while accepting s' and s yield the same continuation payment as of the next period, accepting $s' > s$ involves a strictly higher stage expected payment than accepting s. Thus, any $s' > s$ must be rejected for sure. ■

Lemma 2 *Fix any $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$ and any MPBE. Also, fix any posterior $p \in (0,1)$. Suppose that, in equilibrium, player 2 makes a demand s which player 1 accepts with a positive probability. Then, s is either $(1-q)H + c_1$ or $qH - c_2$.*

Proof. The proof is by contradiction. We consider the following cases.

Case 0. $s < (1-q)H + c_1$ or $s > qH + c_1$.

Any demand $s < (1-q)H + c_1$ is dominated by $(1-q)H + c_1$ since type G accepts $(1-q)H + c_1$ and player 2's stage payoff from type B is $qH - c_2 > (1-q)H + c_1$ should he reject $(1-q)H + c_1$. Therefore, in equilibrium, player 2 will not demand $s < (1-q)H + c_1$. This contradicts the assumption that s is demanded in equilibrium.

If type B accepts a demand $s > qH + c_1$, by assumption A2, he will reveal his type and the subsequent payment is $qH + c_1$ each period. If he rejects s, his current period expected payment is $qH + c_1$ while future expected payments are bounded above by $qH + c_1$. Therefore, $s > qH + c_1$, if demanded, will be rejected by type B for sure. This contradicts the assumption that s is accepted.

Case 1. $s \in ((1-q)H + c_1, qH - c_2)$.

But then, player 2 can profitably deviate by not demanding s and, instead, demanding any $s' > qH + c_1$. By assumption A1, type G rejects both s and s' for sure; from Case 0 above, we know that type B must also reject s' for sure. But player 2 expects to earn $qH - c_2 > s$ from type B by seeking an expert and, therefore, would strictly prefer to have s' rejected than to have s accepted. This is a contradiction.

Case 2. $s \in (qH - c_2, qH + c_1]$ and type B rejects s with probability $r \in (0,1)$.

But then, consider player 2 deviating by demanding $s - \epsilon > qH - c_2$ instead of s for some small $\epsilon > 0$. By Lemma 1, such a demand must be accepted by type B for sure; by A1, type G rejects $s - \epsilon$. The deviation payoff then amounts to

$$p((1-q)H - c_2) + (1-p)(s-\epsilon),$$

while the corresponding equilibrium payoff is

$$p((1-q)H - c_2) + (1-r)(1-p)s + r(1-p)(qH - c_2).$$

Thus, such a deviation is profitable if $\epsilon < r(s - qH + c_2)$. This is a contradiction.

Case 3. $s \in (qH - c_2, qH + c_1]$ and type B accepts s with probability 1.

Let r^B be the given equilibrium strategy of type B, and let $s^* > qH - c_2$ denote the supremum of demands that it accepts with probability 1 at p; that is, $s^* = \sup\{s : r^B(p, s) = 0\}$.

Then, by Lemma 1, $r^B(p, s') = 0$ for any $s' \in (qH - c_2, s^*)$, and $r(p, s'') = 1$ for any $s'' \in (s^*, \infty)$. Therefore, player 2's payoff is s' by demanding s' and $qH - c_2 < s^*$ by demanding s''. However, both s' and s'' are dominated by $s^* - \frac{s^* - s'}{2}$ which is accepted for sure, yielding a payoff of $s^* - \frac{s^* - s'}{2} > qH - c_2$. Therefore, given our arguments against Cases 0 and 1 above, player 2 will not make a demand other than $(1-q)H + c_1$ or s^* in equilibrium.

Suppose now that player 2 demands s^* with a positive probability. We shall show that this is impossible.

On the one hand, if player 2's equilibrium strategy demands s^* with a positive probability, type B must accept it with probability 1 by the same argument as in Case 2; otherwise, player 2 could profitably deviate by demanding $s^* - \epsilon$ instead of s^* for some small enough $\epsilon > 0$.

On the other hand, type B has an incentive to deviate by rejecting s^* if $\delta > \frac{c_1 + c_2}{(2q-1)H + c_1 + c_2}$. As we have already established, in equilibrium, the demand can only be either $(1-q)H + c_1$ or s^*, where the former demand is accepted for sure by both types and the latter is accepted for sure by type B while rejected for sure by type G. It then follows that the equilibrium posterior at the next period after observing rejection in the current period must be 1.

Thus, the deviation results in each subsequent player 2 demanding $(1-q)H + c_1$ and, hence, the continuation payment

$$(1-\delta)(qH + c_1) + \delta((1-q)H + c_1). \tag{12}$$

But, in equilibrium, acceptance of s^* results in revelation (assumption A2) and, hence, the continuation payment

$$(1-\delta)s^* + \delta(qH + c_1). \tag{13}$$

Since $s^* > qH - c_2$ and $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$, (13) exceeds (12) and, therefore, the deviation is profitable. This is a contradiction. ■

We now proceed to prove each claim of Proposition 4 in turn. Fix any $\delta > \frac{c_1+c_2}{(2q-1)H+c_1+c_2}$, as required by Lemma 2 above, and any Markov perfect Bayesian equilibrium. Also, for ease of exposition, let $\overline{V} = qH + \delta c_1 - (1-\delta)c_2$.

Lemma 3 *For any* $p \in (0,1)$, $V(p) \in \left[(1-q)H + c_1, \overline{V}\right]$.

Proof. First of all, the lower bound is immediate since, with assumption A1, any demand less than $(1-q)H + c_1$ is strictly dominated for player 2 and thus will never occur in equilibrium.

Next, we establish the upper bound. Let us consider two cases in turn.

First, suppose that every *equilibrium* demand of player 2 is accepted by type B. Then, player 2 must play pure strategy (given the assumption that each equilibrium demand is accepted, player 2 cannot randomize between a low demand and a high demand).

Then, by Lemma 2, the equilibrium demand is either $qH - c_2$ or $(1-q)H + c_1$. If the demand is $(1-q)H + c_1$, by assumption A1, no belief updating occurs and, therefore, $V(p) = (1-q)H + c_1 < \overline{V}$. If the demand is $qH - c_2$, type B reveals himself and hence by the Markov property

$$V(p) = (1-\delta)(qH - c_2) + \delta(qH + c_1) = \overline{V}.$$

Second, suppose that, at p, some equilibrium demand is rejected with a positive probability. Let s_* be the infimum of these demands that are rejected by type B at p. By Lemma 1, all demands below s_* will be accepted and all demands above s_* will be rejected by this type.

Note that type B's equilibrium payment, $V(p)$, is bounded above by rejecting all demands. In particular, given the definition of s_*, the upper bound equals the continuation payment from rejecting an *equilibrium* demand $s_* + \epsilon$, for some $\epsilon \geq 0$.

But, at the same time, since $s_* + \epsilon$ occurs and is rejected in equilibrium, type H's equilibrium payment at p is bounded above by the continuation payment from accepting $s_* + \epsilon$. Therefore, it must be that

$$V(p) \leq (1-\delta)(s_* + \epsilon) + \delta(qH + c_1),$$

where $qH + c_1$ is the maximum possible continuation payment.

Now, by the definition of s_*, we can take $\epsilon \to 0$ and, hence, obtain

$$V(p) \leq (1-\delta)s_* + \delta(qH + c_1). \tag{14}$$

From (14), we are done if $s_* \leq qH - c_2$. We simply note that it is impossible that $s_* > qH - c_2$. The reasoning is as follows. Suppose not. By the definiton of s_*, there exists an equilibrium demand $s \geq s_*$ such that s is rejected and player 2 obtains a payoff of $qH - c_2$. But, by the definition of s_*, any $s_* - \epsilon > qH - c_2$ will be accepted by type B which gives player 2 a payoff of $s_* - \epsilon > qH - c_2$. Therefore, s cannot be demanded in equilibrium. This is a contradiction. ■

Lemma 4 *Let* $p^{**} = \frac{(2q-1)H - c_1 - c_2}{(2q-1)H}$. *For any* $p \in (p^{**}, 1)$, $(1-q)H + c_1$ *is demanded and accepted for sure.*

Proof. By demanding $(1-q)H + c_1$, player 2 obtains a payoff of at least

$$(1-q)H + c_1 \tag{15}$$

since the good type accepts it and he can obtain $qH - c_2 > (1-q)H + c_1$ if the bad type ever rejects the demand. Note that all lower demands are strictly dominated by $(1-q)H + c_1$.

By demanding $qH - c_2$, player 2 obtains at most

$$p((1-q)H - c_2) + (1-p)(qH - c_2) \tag{16}$$

since type G will reject it, leading to expected payoff of $(1-q)H - c_2$ for player 2, and $qH - c_2$ is player 2's expected payoff regardless of type B's response. Note that all demands in $((1-q)H + c_1, qH - c_2)$ are weakly dominated by $qH - c_2$, because type G rejects the demand and player 2's payoff is lower than $qH - c_2$ if type B ever accepts it.

Now, by Lemmas 1-2, any demand greater than $qH - c_2$ is rejected by both types for sure, which gives player 2 a payoff of $p((1-q)H - c_2) +$

$(1-p)(qH-c_2)$. Therefore, we only need to compare (15) with (16). Since $p > p^{**}$, the former is larger, implying that $(1-q)H + c_1$ must be demanded for sure.

Then, since player 2 plays a pure strategy here, and by A1, accepting the equilibrium demand $(1-q)H + c_1$ cannot reduce the equilibrium posterior. Thus, accepting yields a continuation payment $(1-q)H + c_1$ to type B. On the other hand, rejection yields, at best, a continuation payment

$$(1-\delta)(qH + c_1) + \delta\left((1-q)H + c_1\right),$$

implying that $(1-q)H + c_1$ is accepted for sure. ■

In order to pin down our final claim, we first need the following Lemma.

Lemma 5 *Consider the state space $P \subset [0,1]$ such that $P = P_1 \cup P_2 \cup P_3$. Let $V(p)$ be the discounted average expected payment at p (with discount factor $0 < \delta < 1$).*

At any $p \in P_3$, with probability $1-q$ the immediate payment is 0 and the new state becomes $p' = \Phi^1(p)$; with probability q, the payment is H and the new state becomes $p'' = \Phi^{-1}(p)$, where $\Phi^1(\cdot)$ and $\Phi^{-1}(\cdot)$ are as defined in the proof of Proposition 1 above. If $p \in P_1$, $V(p) = v_1 > 0$; If $p \in P_2$, $V(p) = v_2 > 0$.

We then have the following: If $qH \geq \min\{v_1, v_2\}$, then $V(p) \geq \min\{v_1, v_2\}$ for any $p \in P_3$.

Proof. Suppose not. Let $v_3 = \inf_{p \in P_3} V(p)$. Then, by assumption, $v_3 < \min\{v_1, v_2\}$. For any small $\varepsilon > 0$, there exists $p^\varepsilon \in P_3$ such that $V(p^\varepsilon) < v_3 + \varepsilon$. We know that

$$\begin{aligned} V(p^\varepsilon) &= (1-\delta)qH + \delta((1-q)V(p') + qV(p'')) \\ &\geq (1-\delta)qH + \delta \min\{V(p'), V(p'')\}. \end{aligned}$$

Therefore,

$$\begin{aligned} \min\{V(p'), V(p'')\} &\leq \delta^{-1}(V(p^\varepsilon) - (1-\delta)qH) \\ &\leq \delta^{-1}[v_3 + \varepsilon - (1-\delta)v_3 + (1-\delta)v_3 - (1-\delta)qH] \\ &< v_3 + \delta^{-1}[\varepsilon + (1-\delta)(v_3 - qH)]. \end{aligned}$$

Taking ε to 0, we have $\min\{V(p'), V(p'')\} < v_3 + \delta^{-1}(1-\delta)(v_3 - qH)$. However, we know that, by assumption, $v_3 < \min\{v_1, v_2\} \leq qH$. It then follows that $\min\{V(p'), V(p'')\} < v_3$. This contradicts the definition of v_3. ■

We are now ready to complete the proof of Proposition 4 with the following Lemma.

Lemma 6 *There exists* $p^* \in (0, p^{**})$ *such that, for any* $p \in (0, p^*)$, $V(p) = \overline{V}$.

Proof. We shall follow a series of steps.

Step 1. Fix any $p < p^{**}$, and suppose that player 2 demands $(1-q)H + c_1$ in equilibrium. Then, type B must reject this demand with a positive probability, and hence the equilibrium posterior belief after a rejection but before the expert verdict does not exceed p.

Proof of Step 1. Suppose to the contrary that player 1 accepts the demand for sure. Player 2's payoff will be $(1-q)H + c_1$. We shall argue that $(1-q)H + c_1$ is strictly dominated and cannot be an equilibrium demand.

Consider another demand $qH - c_2$. If player 1 is type G, then he will reject it and player 2's payoff will be $(1-q)H - c_2$; if player 1 is type B, then whether or not he rejects $qH - c_2$, player 2 will earn $qH - c_2$ in expectation. Therefore, player 2's expected payoff is $p(1-q)H + (1-p)qH - c_2$. Since $p < p^{**}$, this amount is greater than $(1-q)H + c_1$. That is, $qH - c_2$ dominates $(1-q)H + c_1$.

Since $(1-q)H+c_1$ is rejected with positive probability, all higher demands are rejected for sure by Lemma 1. It follows that in this case rejection reduces the posterior belief.

Step 2. Fix any $p < p^{**}$. One of the following holds:

(a) $V(p) = \overline{V}$; or

(b) player 1 weakly prefers to reject any equilibrium demand and the equilibrium posterior immediately after the rejection (before the expert verdict) does not exceed p.

Proof of Step 2. There are two cases to consider.

Case 1: $(1-q)H + c_1$ is demanded with a positive probability in equilibrium.

Then, by Step 1, (b) holds.

Case 2: $(1-q)H + c_1$ is demanded with probability 0 in equilibrium.

In this case only $qH - c_2$ can be possibly accepted by Lemma 2.

- If type B's equilibrium strategy prescribes that $qH - c_2$ be rejected for sure, then the belief will not change after rejection; hence, (b) holds.

- If it prescribes that $qH - c_2$ be accepted with a positive probability, then all demands greater than $(1-q)H + c_1$ but less than $qH - c_2$ is going to be accepted for sure, and they are dominated by $qH - c_2$ for player 2 (because only type B accepts these demands).

Now, there are two possibilities here.

First, if $qH - c_2$ is not demanded in equilibrium by player 2, then all equilibrium demands are rejected and, therefore, belief never changes; hence, (b) holds.

Second, if $qH - c_2$ is demanded in equilibrium with a positive probability by player 2, then type B's continuation payment from rejecting any demand is higher than or equal to that from accepting $qH - c_2$. The latter amounts to

$$(1-\delta)(qH - c_2) + \delta(qH + c_1) = \overline{V}.$$

But, since $V(p) \leq \overline{V}$ by Lemma 3, it must be that $V(p) = \overline{V}$; hence, (a) holds.

At this point, for any positive integer k, let $p_k = \Phi^{-k}(p^{**})$, as defined in the proof of Proposition 1 above.

<u>Step 3</u>. Fix any $p \in [p_{k+1}, p_k)$, and suppose that

$$\overline{V} \geq (1-\delta^k)qH + \delta^k(1-q)H + c_1.$$

Then, we have

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}.$$

Proof of Step 3. We employ induction. First, consider any $p \in [p_1, p^{**})$. By Step 2, we have either $V(p) = \overline{V}$ or an equilibrium demand is rejected and so $V(p)$ is given by the continuation payment from the rejection.

In the latter case, clearly, $V(p) \geq (1-\delta)(qH + c_1) + \delta((1-q)H + c_1)$. Thus,

$$V(p) \geq \min\{(1-\delta)qH + \delta(1-q)H + c_1, \overline{V}\}.$$

Next, assume that, for any $p \in [p_k, p_{k-1})$,

$$V(p) \geq \min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}.$$

We want to show that, for any $p \in [p_{k+1}, p_k)$,

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}.$$

Again, given Step 2 above, consider the continuation payment when any equilibrium demand here is rejected such that the posterior immediately after rejection does not go above p.

Rejection results in the current period expected payment of $qH + c_1$. If the subsequent expert verdict is favorable, the next period's posterior belongs to $[p_k, p_{k-1})$ and, hence, the corresponding continuation payoff must be at least $\min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}$, by assumption.

If the expert verdict is not favorable then the next period's posterior must belong to $[p_{k+2}, p_{k+1})$. By Lemma 5 (taking $P_3 = [p_{k+2}, p_{k+1})$, $P_1 = [p_k, p_{k-1})$, $P_2 = \{p : V(p) = \overline{V}\} \backslash (P_1 \cup P_3)$), the corresponding continuation payment must also be bounded below by $\min\{(1-\delta^k)qH + \delta^k(1-q)H + c_1, \overline{V}\}$.

Thus, we have

$$\begin{aligned} V(p) &\geq \min\{(1-\delta)(qH + c_1) + \delta\left[(1-\delta^k)qH + \delta^k(1-q)H + c_1\right], \overline{V}\} \\ &= \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\}, \end{aligned}$$

and induction closes the proof of Step 3.

Now, let K be the largest integer such that $\overline{V} \geq (1-\delta^K)qH + \delta^K(1-q)H + c_1$. Then, Step 3 immediately implies that, for any $p \in [p_{k+1}, p_k)$, $k \geq K$, we must have

$$V(p) \geq \min\{(1-\delta^{k+1})qH + \delta^{k+1}(1-q)H + c_1, \overline{V}\} = \overline{V}.$$

Since, by Lemma 3 we already know that $V(p) \leq \overline{V}$ for any $p \in (0,1)$, it follows that $V(p) = \overline{V}$ for any $p < p_K$. ■

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