Evaluating Skilled Experts:
Optimal Scoring Rules for Surgeons*

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Abstract

We consider settings in which skilled experts have private, heterogeneous types. Contracts that evaluate experts based on outcomes are used to differentiate between types. However, experts can take unobservable actions to manipulate their outcomes, which may harm consumers. For example, surgeons may privately engage in harmful selection behavior to avoid risky patients and hence improve observed performance. In this paper we solve for optimal evaluation contracts that maximize consumer welfare. We find that an optimal contract takes the form of a scoring rule, typically characterized by four regions: (1) high score sensitivity to outcomes, (2) low score sensitivity to outcomes, (3) tenure, and (4) firing or license revocation. When improvement is possible, an optimal contract for the low quality expert is a fixed-length mentorship program. In terms of methods, we draw upon continuous-time techniques, as introduced in Sannikov (2007*b*). Since our problem involves both adverse selection and moral hazard, this paper features novel applications of continuous-time methods in contract design.

1 Introduction

This paper is motivated by the experience with performance reporting for cardiac surgeons. In the past, the only way to distinguish between surgeons of different abilities was through informal word-of-mouth reputations. Those could be biased or inaccurate, and at best incomplete. In recent years, however, as the collection, compilation and dispersion of data have become easier, policy makers have been keen to create "score cards" that evaluate surgeons based on outcomes, with the intention of identifying high-performing and low-performing surgeons. Beginning in the early 1990's, state agencies in New York and Pennsylvania have released hospital- and surgeon-specific data on risk-adjusted cardiac surgery mortality rates (PA in late 1991, NY in late 1992). Other organizations have been following suit (e.g. NJ in 1997, CA in 2001).

There are at least two ways in which score cards can improve patient welfare. First, they can lead to more efficient sorting in the market: better surgeons treat more difficult cases, worse surgeons treat less difficult cases, and very bad surgeons leave the market completely. Secondly, score cards can induce additional investments in quality and higher levels of effort.

Despite the potential benefits, skeptics of score cards argue that they may harm patients because providers are encouraged to distort their behavior. For example, surgeons can inflate their scores through *risk selection*, focusing on patients who are relatively healthy and avoiding patients who are sick. A survey in Pennsylvania reveals that as a consequence of report cards' introduction, 63% of cardiac surgeons report having only accepted healthier candidates for CABG surgery,[1] and 59% of cardiologists report increased difficulty finding surgeons willing to perform needed CABG surgery in severely ill patients (Schneider & Epstein 1996) . A similar survey in New York reports that 62% of surgeons have refused to operate on at least one high-risk CABG patient over the prior year primarily because of public reporting, and that report cards have led to a significantly higher percentage of high-risk CABG patients being treated non-operatively (Burack, Impellizzeri, Homel & Jr. 1999).[2]

In this paper, we present a model with heterogeneous types in which experts (a) know their own private types, (b) generate observable outcomes that can be used to measure performance, and

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[1]CABG refers to "coronary artery bypass graft" surgery, which is the specific cardiac procedure most commonly measured in state score cards.

[2]Dranove, Kessler, McClellan and Satterthwaite (2003) study the empirical effects of introducing cardiac surgery score cards in NY and PA and conclude that the net effects have been to reduce overall welfare, citing increased selection behavior by physicians, higher levels of resource use, and worse health outcomes. Dranove et al. do observe, however, that score cards have led to improved sorting between physicians and patients. See Epstein (2006) for a summary of the empirical and medical literature.

(c) have the ability to take private actions, potentially harmful to consumers, that inflate those outcomes. We take a normative approach and fully characterize an optimal evaluation mechanism that maximizes patient welfare. By doing so, we establish a baseline for the best-case scenario.

The medical literature has emphasized improving risk adjustment techniques as the major solution to performance manipulation issues (Werner, Asch & Polsky 2005). In theory, perfect risk adjustment can eliminate the incentives for risk selection. The fundamental assumption we make, however, is that in practice *a surgeon will always have better information about a patient's risk profile than a scoring mechanism.* So long as risk adjustment is not perfect, the moral hazard for surgeons to distort their behavior is impossible to eliminate, and we can only seek to minimize its harm.[3]

Our model is closely related to the class of "bad reputation" games introduced by Ely and Välimäki (2003). In such games, reputation is labeled "bad" because the introduction of performance histories, i.e. reputation, may adversely affect all players, often leading to complete market breakdown. If represented as a reputation game, our setting with surgeons would also be prone to bad reputation effects. The key tension is that higher outcomes, generally associated with a higher ability type, can be roughly mimicked through risk selection. Hence, making future payoffs depend on those outcomes tempts surgeons to distort their behavior. Ironically, even though performance reporting is introduced to solve the *adverse selection* problem of not knowing whether a surgeon is of high ability or low ability, that introduction creates an inevitable *moral hazard* to manipulate reports. The existing theoretical literature on optimal contract design has not yet shed light on such settings.

The main portion of our paper treats surgeons' types as fixed, disallowing improvement, and focuses on welfare gains from improved sorting. One feature we emphasize is that selection behavior can be welfare-enhancing. In particular, if bad surgeons avoid risky patients, those patients may be better off overall from receiving a substitute procedure or being treated by a different surgeon. This observation has been made in Dranove et al. (2003), where authors cite "the failure of previous studies to consider the entire population at risk for CABG, rather than only those who receive it." Hence, in our model patient welfare improves via two types of sorting: (1) inducing bad surgeons

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[3]Even if risk adjustment becomes perfect, different types of moral hazard may present themselves. For example, when risk factors used for risk adjustment are reported by surgeons or institutions themselves, Epstein (1995) observes that risk adjustment provides incentives for "upcoding." After New York began to report risk-adjusted mortality rates, there was "a dramatic increase in the prevalence reported by hospitals of the co-existing conditions used in the state's risk-adjustment model." In another example, Shahian et al. (2001) note that surgeons can perform additional procedures at the same time as CABG surgery, thereby disqualifying the procedure from being included in reporting (since it is not an isolated CABG).

to avoid difficult cases, and (2) firing bad surgeons. Our primary task then is to characterize an optimal evaluation mechanism that separates good surgeons from bad surgeons while minimizing the incentives for good surgeons to distort behavior and maximizing the incentives for bad surgeons to restrict themselves to good risks.

We find that there exists an optimal evaluation contract that takes the form of a scoring rule, in which a surgeon's past performance is summarized by a single "score." Our key insight for score card design lies in our characterization of rules according to which the score varies with performance, i.e. the score's sensitivity to outcomes. The optimal scoring rule includes the possibility of both firing (e.g. license revocation) and tenure, regardless of the surgeon's type. Interestingly, the scoring rule for the good surgeon includes two additional scoring regimes: one in which the surgeon's score is highly sensitive to performance (a "hot seat"), and another in which the surgeon's score is much less sensitive to performance ("benefit of the doubt"). Furthermore, a simplifying lemma (Lemma 1) shows that in our optimal mechanism the principal does not reveal any information to patients until the surgeon has been fired.

At the end of the paper, we apply our methods to a related but distinct question. We introduce the possibility of improvement through effort and mentored experience. In this case the optimal evaluation mechanism can be implemented as a scoring rule for the good surgeon and a finite-length mentorship program for the bad surgeon. In equilibrium, after completing the mentorship program, bad surgeons elect to improve to good types.

The issue of performance or quality reporting in healthcare is complex and involves many different questions. For example, what defines quality or high performance? What specific measures should be used for evaluation? When should surgeons and institutions be measured jointly and when should they be measured separately? Such questions require deep thought and research by experts in relevant areas before a complete proposal is created. However, most lie outside the realm of economic theory. One question that may lie within the realm of economic theory but that we do not deal with in this paper is whether performance reporting should be made available to the public. Addressing that question requires careful modeling of demand-side behavior. In this paper, we make a brief comment on whether reports should be public, but our focus is largely on the effect of performance evaluation on the behavior of providers.

We emphasize that money does not appear in our model. While that absence limits applications

to other experts, it accurately portrays settings in healthcare. Money can appear in the form of prices that surgeons set or transfers that a contract specifies. In healthcare (as in education), prices rarely play their usual role. Instead, they are typically fixed and constrained from responding to factors such as perceived quality and public reputation. Although we admit that incorporating transfers into the model will be interesting, seeing what we can achieve without transfers is highly relevant. For one thing, as Epstein et al. (2004) note, financial incentives for performance may "threaten the sense of professionalism, autonomy, and job satisfaction among physicians" and "underscore the inadequacy of professionalism as a means of self-regulation and quality assurance." Transfers may be viewed as interference in the art and skill of medicine. Although momentum is building around efforts to link pay with performance, in today's healthcare markets policy makers must rely on mechanisms other than money to positively influence market dynamics.

In this paper, we employ continuous-time techniques. Relative to discrete-time methods, continuous-time methods have proven more successful in answering questions similar to ours. For example, in principal-agent models with pure moral hazard, known methods to solve discrete-time contracts are computationally intensive and their full characterization is difficult (Spear & Srivastava (1987), Phelan & Townsend (1991)). Meanwhile, Sannikov (2007b) has recently shown that contracts in continuous-time have a simple characterization by an ordinary differential equation. In the long-term contracting problem we consider, the additional feature of adverse selection is reminiscent of two-player games with imperfect monitoring, since each type "plays" against a fictional version of the other type. In discrete time, results in repeated games have been limited to identifying achievable payoffs as players become sufficiently patient, i.e. when δ approaches one (e.g. Fudenberg & Levine (1992), Fudenberg, Levine & Maskin (1994)). On the other hand, research using continuous-time methods has succeeded in characterizing the set of payoffs for *fixed discount factors* (Sannikov (2007c), Faingold & Sannikov (2007)). Since computing contracts for a fixed discount factor is essential in our application to surgeons, we choose to use a continuous-time model.

2 Related Literature

Although this paper is related to the literature on credence goods, our main point of departure lies in disregarding the role of prices and focusing on performance measurement as the avenue for

providing incentives.[4] Ely and Välimäki (2003) also study credence goods and disregard prices. Using a stylized model of mechanics, the authors show that the introduction of reputation may lead to complete market breakdown.[5] In our paper, instead of focusing on the specific mechanism of reputation in which experts are evaluated via Bayesian updating on their type, we characterize the optimal evaluation mechanism.

At the end of their paper, Ely and Välimäki (2003) consider a long-term principal-agent setup similar to ours. They show that in the limit as players become sufficiently patient, the bad reputation effect disappears. To prove the result, the authors consider an evaluation mechanism using a "score" that is similar in some respects to the optimal scoring rules we identify. Our use of continuous-time methods, however, enables us to characterize and compute the optimal scoring rule for any discount rate and to verify that a scoring rule is indeed an optimal way to provide incentives.

The origin of continuous-time contracting lies in principal-agent models with pure moral hazard. Sannikov (2007b) shows that an optimal contract with dynamic moral hazard is characterized by a simple ordinary differential equation. Our paper follows his approach. Others have taken different approaches to continuous-time contracting with moral hazard, including Williams (2004), and Westerfield (2006).

Building on the above work, several recent papers have studied continuous-time contracting with both adverse selection and moral hazard, as we study in this paper. Sannikov (2007a) solves for the optimal dynamic financing contract for a cash-constrained entrepreneur with a hidden type. Sung (2005) solves for an optimal continuous-time contract in a setting where the principal can only observe the initial and final values of the underlying signal process. As a result, the optimal contract is linear and the optimal actions of the agent are constant across time. Cvitanic and Zhang (2006) consider a general setup with both moral hazard and adverse selection over an interval of agent types. Rather than continuous payments, they allow for bulk payments only at the end of a finite time period. Because of their generality, Cvitanic and Zhang are not able to fully characterize optimal contracts.

The paper is organized as follows. In Section 3, we describe the model and formally present the

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[4]Dulleck and Kerschbamer (2006) define a credence good as any good or service where an expert knows more about what a consumer needs than the consumer himself. Papers on credence goods include Wolinsky (1993), Taylor (1995), and Dulleck and Kerschbamer (2006).

[5]Ely, Fudenberg and Levine (2002) go on to identify more general conditions under which such "bad reputation effects" occur.

problem. Section 4 introduces the optimal menu of evaluation contracts and solves for the easier contract, the contract for the bad surgeon. Section 5 solves for the harder contract, introducing novel applications of continuous-time techniques in contract design. In Section 6, we complete the characterization of the optimal menu of evaluation contracts. Section 7 presents some numerical examples to provide more concrete insight, and in Section 8 we briefly discuss an extension that incorporates the possibility of improvement. Section 9 concludes.

3   The Model

We consider the example of surgery. There is a principal who designs contracts, and an agent. The agent, whom we call the *surgeon*, lives in all periods $t \in [0, \infty)$. With probability p the surgeon is a good type $\theta = G$, and with probability $1 - p$ he is a bad type $\theta = B$. The surgeon knows his own private type.

At each time t, a mass of patients $D_t \in [0, 1]$ visits the surgeon, and the surgeon chooses a private action $a_t \in A^\theta$ where we let $A^G = A^B = A = \{0, 1\}$.[6] When $D_t = 1$ the surgeon is fully employed, and when $D_t = 0$ he sees no patients. The action a_t can be broadly interpreted as any hidden action that inflates performance. For the purposes of our discussion, however, we interpret a_t concretely as indicating whether the surgeon (privately) engages in selection behavior. In particular, a_t can represent the fraction of risky patients on whom the surgeon chooses to avoid being evaluated by treating with an alternative procedure. Empirical evidence confirms that surgeons are able to shift their practices towards healthier patients. Dranove et al. (2003) find that report cards have caused an increase in the quantity of CABG surgery for healthier patients, explaining that surgeons heavily influence whether a patient receives a CABG or an alternative cardiac procedure such as angioplasty or other revascularization treatments. Additionally, Omoigui et al. (1996) report that in New York the number of patients transferred to the world-renowned Cleveland Clinic has increased by 31% after the release of report cards and that generally these transfer patients have higher risk profiles than patients transferred to the Cleveland Clinic from other states.[7]

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[6]Technically, we can accommodate any finite action space A^θ if both the type-θ surgeon's payoff function and his technology (defined on the next page) are "convex" in a (i.e. increment sizes increase in a). In the limit as $|A|$ goes to ∞, the necessary and sufficient condition approaches linearity of both functions.

[7]One may argue that transferring patients to the Cleveland Clinic is an observable action, but the point is that since the number of transfers is not a reported measure, surgeons so far have been able to use it as a private action to affect performance. Nonetheless, it will be interesting in future work to accommodate additional signals such as a monitoring process of a. That is currently outside the scope of our model.

While the surgeon's action at time t is not observable, if $D_t > 0$ then both the principal and the surgeon observe a noisy signal X_t that summarizes the surgeon's outcomes. The path of this signal depends on both the surgeon's action and his type. We model the signal as a continuous-time diffusion process, which, if $D_t > 0$, evolves according to

$$dX_t = \mu(a_t, \theta)dt + \sigma dZ_t,$$

where $\{Z_t\}_{t \geq 0}$ is a standard Brownian motion and the volatility σ is a constant. We call the drift $\mu(a, \theta)$ the *technology* of the type-θ surgeon. Denote by $\{\mathcal{F}_t\}_{t \geq 0}$ the filtration generated by $\{X_t\}_{t \geq 0}$. If $D_t = 0$, no information is learned.

All else equal, good surgeons generate higher signals than bad surgeons so that $\mu(a, G) > \mu(a, B)$ for all $a \in A$. In other words, for a fixed action, the drift of the good surgeon's signal strictly exceeds the drift of the bad surgeon's signal. However – and here is the critical component of our setup that generates moral hazard – the drift of X_t also increases in a_t so that $\mu(1, \theta) - \mu(0, \theta) = k^\theta > 0$. Engaging in selection behavior improves observed performance. Furthermore, we assume that $k^B > k^G$ so that the marginal improvement in performance is larger for the bad surgeon than for the good surgeon. Hence when both surgeons are selecting risk, there is weaker separation between the two types' performances. This reflects the fact that the good surgeon has a comparative advantage over the bad surgeon in dealing with difficult cases, as suggested by Capps et al. (2001).

The flow payoffs of the type-θ surgeon are $D_t g(a_t, \theta)$. The payoff function $g(a_t, \theta)$ is decreasing in a_t so that the surgeon's payoffs decrease when he engages in selection behavior. In other words, risk selection, i.e. identifying risky patients and then avoiding them or substituting an alternative procedure for them, is costly. Let us normalize $g(0, \theta) = 1$ and let $g(1, \theta) = 1 - \zeta^\theta > 0$. The parameter ζ^θ may be related to the proportion of risky patients in the population, or ζ^θ may reflect the ease or difficulty with which surgeons of different types select risk. We introduce $\lambda^\theta = 1 - \zeta^\theta > 0$ as useful notation. We assume that the surgeon discounts payoffs at some rate $r > 0$, so the type-θ surgeon's total expected average discounted payoff is[8]

$$E\left[r \int_0^\infty e^{-rt} D_t g(a_t, \theta)dt\right].$$

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[8]The factor r normalizes payoffs to the scale of flow payoffs.

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The principal can fully commit at time $t = 0$ to any history-dependent contract. We do not allow renegotation. Her objective is to maximize expected patient welfare. Patient welfare depends on both the surgeon's action and his type. Let $D_t h(a_t, \theta)$ represent the expected flow payoff to patients from a surgeon of type θ who takes action $a_t \in A$. We assume that $h(a_t, G)$ is strictly *decreasing* in its first parameter so that selection behavior by a good surgeon is detrimental to patients. This generates a potentially adverse effect on patient welfare. Meanwhile, we assume that $h(a_t, B)$ is strictly *increasing* in its first parameter. This generates a potentially beneficial effect on patient welfare. It follows that

$$0 = \arg\max_{a' \in A} h(a', G) \qquad \text{and} \qquad 1 = \arg\max_{a' \in A} h(a', B).$$

That curiosity may create an additional tension in the model: although selection behavior by good surgeons is detrimental to patient welfare, selection behavior by bad surgeons improves patient welfare.[9] A simple explanation is that there exists an outside option such that risky patients who would be selected against derive a higher payoff from the outside option than from a surgery performed by a bad surgeon. It is easily the case that a very sick patient will be better off being declined by a bad surgeon for a CABG and then either undergoing an alternative cardiac procedure or finding a different surgeon. Empirically, it has been suggested that sick patients have more to gain from a good surgeon over a bad surgeon. For example, Capps et al. (2001) find that sick patients are more willing to incur travel and financial costs to see a high quality provider.

We assume that patients discount payoffs at the same rate r as surgeons, so aggregate expected average discounted patient welfare, conditional on a surgeon of type θ, is equal to

$$E\left[r \int_0^\infty e^{-rt} D_t h(a_t, \theta) dt\right].$$

One final modeling assumption remains. Recall that we are interested in establishing a baseline for the highest patient welfare achievable. In our analysis, we assume that the good surgeon is not only highly-skilled but also altruistic. Like "good" experts in the bad reputation models, the good surgeon fully aligns his interests with patients and acts to maximize patient welfare. So we have

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[9]This is a feature of the model that has been somewhat neglected in the empirical literature. It is difficult for those studies to take into account patients who do not undergo the specific procedure and instead undergo a substitute procedure.

$g(a, G) \propto h(a, G)$. Meanwhile, the bad surgeon cannot align his interests with patients and instead acts in his own self-interest. In the health literature, economic models of physician behavior often include or discuss altruism (see, for example, De Jaegher and Jegers (2000)).

Adaptation to Other Experts: Mechanics. We emphasize that our model is flexible and can accommodate other skilled experts. We show briefly how an amendment of Ely and Välimäki's mechanics setup (in which both types are strategic) fits into our model (2003).

A mechanic is one of two types: $\theta \in \{G, B\}$. The flow $D_t \in [0, 1]$ reflects car-owners bringing in their cars for repair. Each car is either in need of a tune-up (state t) or in need of an engine change (state e). Only a mechanic can identify whether the state is t or e.

The good mechanic has interests aligned with car-owners. He maximizes his payoff by being honest: performing tune-ups when the state is t and performing engine changes when the state is e. Denote this honest strategy as the action 0^G. The mechanic is also capable of only performing tune-ups regardless of the state, even though doing so gives him a lower payoff. Denote this dishonest strategy as the action 1^G. So the good mechanic chooses an action $a_t^G \in \{0^G, 1^G\}$ where $g(0^G, G) > g(1^G, G)$.

The bad mechanic maximizes his payoff by always performing engine changes, regardless of the state. Denote this engine change strategy by the action 0^B. He is also capable of being honest, although doing so gives him a lower payoff. Denote this honest strategy by 1^B. So the bad mechanic chooses an action $a_t^B \in \{0^B, 1^B\}$ where $g(0^B, B) > g(1^B, B)$.

Let X_t be a noisy summary signal reflecting the number of tune-ups performed by a mechanic. This may be generated by surveying car-owners and informally gathering data about the frequency of tune-ups.[10] Recall that $\mu(a_t, \theta)$ is the drift of the signal X_t. It follows from our setup that $\mu(1^G, G) > \mu(0^G, G)$ and $\mu(1^B, B) > \mu(0^B, B)$ since in each case the action 1^θ involves more tune-ups. We can see that $\mu(1^G, G) > \mu(1^B, B)$ and $\mu(0^G, G) > \mu(0^B, B)$. Finally, we have $\mu(1^G, G) - \mu(0^G, G) > \mu(1^B, B) - \mu(0^B, B)$ so that the performance improvement of a bad mechanic when he switches from only replacing engines to being honest is greater than that of an honest mechanic who switches from behaving honestly to only doing tune-ups.

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[10] (It is essential that the action strategy of the mechanic not be perfectly observed. If the number of tune-ups and the number of engine changes are perfectly observed, we can introduce "noise" car-owners in the mix who in fact do know the true state of their vehicles and thus dictate the repair performed by the mechanic.

3.1 Evaluation Contracts and Scoring Rules

In general, a contract offered by the principal specifies demand D_t (i.e. the mass of patients visiting the surgeon) along with recommended actions at every moment in time contingent on the surgeon's entire signal history. The principal can affect demand either through information she releases, quotas she imposes, or other controls she exercises. We do not need to specify exactly how much power the principal has because, according to Lemma 1, regardless of other powers the principal may have to control demand, there exists an optimal contract in which she will only exercise the power to revoke a surgeon's license. When the principal does not affect demand, we assume that patients arrive at a unit rate $D_t = 1$. Lemma 1 implies that we can restrict attention to contracts in which the principal simply specifies a history-dependent stopping time after which she cuts off demand permanently.

Lemma 1. *There exists an optimal contract in which the principal specifies demand $D_t \in \{0, 1\}$. Furthermore, after any moment in time at which $D_t = 0$, it follows that $D_{t'} = 0$ for all $t' > t$.*

Proof. See proof in Appendix A.4. □

So it is optimal for the principal either to fully employ the surgeon or to take away his license, and nothing in between. The intuition is similar to discrete time. Generally, any optimal contract in which $D_t \in (0, 1)$ can be converted to an optimal contract in which $D_t \in \{0, 1\}$ by modifying continuation paths appropriately. Furthermore, it is optimal never to suspend a surgeon's license temporarily but instead to revoke it permanently. The idea is that suspension only serves to "punish" the surgeon while the principal gains no additional information; hence suspension is suboptimal.

Ely and Välimäki (2003) remark that bad reputation effects are generated by short-term interactions and suggest that contracts of a long-term nature improve welfare. Lemma 1 implies the existence of an optimal long-term contract in which the principal does not release any information to patients, who may act myopically; instead, the principal only wants to disclose when the surgeon has been fired.

Since this is a setting with incomplete information where the surgeon's type θ is privately known, the *Revelation Principle* tells us that we can restrict attention to direct revelation mechanisms in which each surgeon announces his type truthfully at time 0. The principal just needs to choose a pair of outcome-dependent stopping rules τ^G and τ^B, which map outcome paths to a stopping time in $[0, \infty)$ (i.e. the point at which licenses are revoked). If the surgeon announces $\theta = G$, he is assigned

the stopping rule τ^G, and if the surgeon announces $\theta = B$, he is assigned the stopping rule τ^B.[11]

We call such contracts *evaluation contracts* and proceed to define them more formally. There are two contracts, one for each type θ. In order to verify that surgeons will reveal their types truthfully and self-select into the appropriate contracts, and also to compute the principal's profits, we conveniently include recommended action strategies into the definition of contracts. Thus, we write an evaluation contract for the good surgeon as

$$C^G = \{\tau^G, \mathcal{A}^G, \hat{\mathcal{A}}^B\}$$

where τ^G is an X_t-measurable stopping time, $\mathcal{A}^G = \{a_t^G\}_{t \in [0, \tau^G]}$ is the recommended (feasible) X_t-measurable action strategy for the good surgeon, and $\hat{\mathcal{A}}^B = \{\hat{a}_t^B\}_{t \in [0, \tau^G]}$ is an optimal (feasible) X_t-measurable strategy for the bad surgeon in this contract. The purpose of introducing $\hat{\mathcal{A}}^B$ is to verify that the bad surgeon does not want to deviate and be assigned to the good surgeon's contract.

Similarly, we write an evaluation contract for the bad surgeon as

$$C^B = \{\tau^B, \mathcal{A}^B, \hat{\mathcal{A}}^G\}$$

where τ^B is an X_t-measurable stopping time, $\mathcal{A}^B = \{a_t^B\}_{t \in [0, \tau^B]}$ is the recommended (feasible) X_t-measurable action strategy for the bad surgeon, and $\hat{\mathcal{A}}^G = \{\hat{a}_t^G\}_{t \in [0, \tau^B]}$ is an optimal (feasible) X_t-measurable strategy for the good surgeon in this contract, introduced to verify that the good surgeon will truthfully reveal his type.

The principal offers the menu of evaluation contracts (C^G, C^B). A menu is *optimal* if and only if it maximizes expected average discounted patient welfare

$$pE\left[r \int_0^{\tau^G} e^{-rt} h(a_t^G, G) dt \mid \theta = G\right] + (1-p)E\left[r \int_0^{\tau^B} e^{-rt} h(a_t^B, B) dt \mid \theta = B\right]$$

over all menus of evaluation contracts (C^G, C^B). A result in this paper is that each evaluation contract in an optimal menu takes the form of a scoring rule. We define a scoring rule as follows:

Definition 1. *An evaluation contract C is a* scoring rule *if it satisfies the following conditions:*

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[11]Note that we can restrict attention to deterministic stopping times since mixing over multiple stopping times is equivalent to a deterministic stopping time in expectation.

1. *there exists a single state variable, which we call the* score, *that completely summarizes the past performance history,*

2. *the score evolves continuously and is measurable with respect to the agent's performance, and*

3. *there exists a threshold such that if the score drops below the threshold, play terminates.*

3.2 What is First-Best?

Suppose the principal can observe the surgeon's type and dictate the surgeon's actions. What is the outcome that maximizes patient welfare? Conditional on the type of surgeon, recall that two variables affect patient welfare: the demand process $\{D_t\}$ and the risk selection policy $\{a_t\}$. We assume that the outside option for patients who do not visit a surgeon yields an expected payoff of 0.

Suppose $\theta = G$. We assume that $h(1, G) > 0$ so that even when a good surgeon is engaging in maximal selection behavior, visiting the good surgeon has positive expected patient payoff. Therefore, when $\theta = G$, the first-best outcome for patient welfare is $D_t = 1$ and $a_t = 0$ for all t.

Now suppose $\theta = B$. The first-best outcome depends on the sign of $h(1, B)$:

- When $h(1, B) > 0$, in expectation it is preferred for patients to visit a bad surgeon who is selecting against risky patients than not to visit him at all. So the first-best outcome is $D_t = 1$ and $a_t = 1$ for all t.

- When $h(1, B) < 0$, in expectation it is preferred for patients to take the outside option (payoff 0) than to see a bad surgeon even when he is selecting risk. So conditional on $\theta = B$, patients never want to visit a bad surgeon. The first-best outcome then is for $D_t = 0$ for all t (making a_t irrelevant for welfare).

3.3 Tradeoffs

To preface the technical analysis that follows, we first provide some intuition about the tradeoffs we are balancing. The Revelation Principle tells us that we can restrict attention to truth-telling pairs of contracts. It will turn out that the contract for the bad type can be computed without concern for the good surgeon's self-selection constraint because the good surgeon will not be tempted to identify

himself as bad. As a result, we are easily able to characterize the contract for the bad surgeon using existing techniques.

The contract for the good type is more difficult to compute because we must be wary of the bad surgeon wanting to deviate and identify himself as good. Thus there are tradeoffs that need to be carefully balanced in the good contract. We can separate welfare effects into current period effects and future period effects. In the current period, we can increase patient welfare by having the good surgeon treat all patients. However, that gain is offset by making it easier for the bad surgeon to imitate the good surgeon and hence more difficult to separate the two types in the future. On the other hand, we can take a loss in patient welfare in the current period by having the good surgeon select against risky patients. That current period loss is offset by making it easier to distinguish good types from bad types in the future.

3.4 The Principal's Problem

The principal wants to specify a menu of evaluation contracts (C^G, C^B) that maximizes patient welfare. Recall that C^G consists of an X_t-measurable stopping time τ^G, a recommended action strategy $\mathcal{A}^G = \{a_t^G\}$ for the good surgeon, and (for convenience) an optimal action strategy $\hat{\mathcal{A}}^B = \{\hat{a}_t^B\}$ for the bad surgeon. C^B is similar. Denote $C^G = \{\tau^G, \mathcal{A}^G, \hat{\mathcal{A}}^B\}$ and $C^B = \{\tau^B, \mathcal{A}^B, \hat{\mathcal{A}}^G\}$. Formally, the principal chooses (C^G, C^B) to maximize discounted expected patient welfare

$$pE\left[r\int_0^{\tau^G} e^{-rt}h(a_t^G, G)dt \mid \theta = G\right] + (1-p)E\left[r\int_0^{\tau^B} e^{-rt}h(a_t^B, B)dt \mid \theta = B\right]$$

subject to the self-selection constraints

$$E\left[r\int_0^{\tau^B} e^{-rt}g(\hat{a}_t^G, G)dt \mid \theta = G\right] \le E\left[r\int_0^{\tau^G} e^{-rt}g(a_t^G, G)dt \mid \theta = G\right] = W^G$$

$$E\left[r\int_0^{\tau^G} e^{-rt}g(\hat{a}_t^B, B)dt \mid \theta = B\right] \le E\left[r\int_0^{\tau^B} e^{-rt}g(a_t^B, B)dt \mid \theta = B\right] = W^B,$$

the incentive compatibility constraints for the recommended action strategies $\mathcal{A}^G$ and $\mathcal{A}^B$

$$\max_{\{a_t'\}} E\left[r\int_0^{\tau^G} e^{-rt}g(a_t', G)dt \mid \theta = G\right] \le W^G$$

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$$\max_{\{a_t'\}} E\left[r\int_0^{\tau^B} e^{-rt}g(a_t', B)dt \mid \theta = B\right] \leq W^B,$$

and the verification constraints[12] for the optimal action strategies $\hat{\mathcal{A}}^G$ and $\hat{\mathcal{A}}^B$

$$\max_{\{a_t'\}} E\left[r\int_0^{\tau^B} e^{-rt}g(a_t', G)dt \mid \theta = G\right] \leq E\left[r\int_0^{\tau^B} e^{-rt}g(\hat{a}_t^G, G)dt \mid \theta = G\right]$$

$$\max_{\{a_t'\}} E\left[r\int_0^{\tau^G} e^{-rt}g(a_t', B)dt \mid \theta = B\right] \leq E\left[r\int_0^{\tau^G} e^{-rt}g(\hat{a}_t^B, B)dt \mid \theta = B\right].$$

Associated with each contract C^θ is an expected payoff W^θ to its intended recipient. Hence (W^G, W^B) denotes the pair of values associated with a menu of evaluation contracts.

Remark: Notice that in the principal's problem there is no participation constraint for either the principal or the surgeon. For the surgeon, he does not have the option of avoiding the principal and rejecting both contracts. The menu of contracts is compulsory. For the principal, since the surgeries will take place regardless, she must at least weakly prefer to offer some menu of contracts.

Remark: Let us summarize the key differences between the good surgeon and the bad surgeon: (1) although both surgeons improve their performance through selection behavior, which is costly, the bad surgeon experiences higher marginal performance gain than the good surgeon; (2) the bad surgeon's payoffs may be affected differently by selection behavior than those of the good surgeon, whose interests are completely aligned with patient welfare; and (3) selection behavior by the bad surgeon improves expected patient welfare, while selection behavior by the good surgeon reduces expected patient welfare.

Remark: We have chosen not to include costs in our objective function. We argue that a patient who needs surgery will inevitably receive some sort of costly treatment, whether it be from a surgeon in our model who accepts the patient's case or from an outside option. If we assume the costs of all such treatments are equal, then maximizing patient welfare is simply a matter of optimal assignment of treatment based on patient welfare, which is the focus of our model. It has been noted that related literature on this topic has largely neglected to assess the impact of report cards on costs, and our model follows in that vein (Dranove, Kessler, McClellan & Satterthwaite 2003).

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[12]These are simply incentive compatibility constraints.

4 The Optimal Menu of Evaluation Contracts

Observe that we can solve for each evaluation contract separately. Since the good surgeon chooses C^G and the bad surgeon chooses C^B, each contract is optimal conditional on its associated type. The good type contract C^G maximizes patient welfare conditional on $\theta = G$:

$$\max_{\tau^G, \{a_t^G\}, \{\hat{a}_t^B\}} E \left[r \int_0^{\tau^G} e^{-rt} h(a_t^G, G) dt \mid \theta = G \right] \quad \text{s.t.}$$

$$E \left[r \int_0^{\tau^G} e^{-rt} g(a_t^G, G) dt \mid \theta = G \right] = W_0^G,$$

$$E \left[r \int_0^{\tau^G} e^{-rt} g(\hat{a}_t^B, B) dt \mid \theta = B \right] \leq \hat{W}^B, \qquad \text{(C-GOOD)}$$

and $\{a_t^G\}$, $\{\hat{a}_t^B\}$ are feasible and incentive compatible, where W_0^G is the good surgeon's expected payoff and $\hat{W}^B$ is the bad surgeon's maximum payoff (the self-selection constraint).

Similarly, the bad type contract C^B maximizes patient welfare conditional on $\theta = B$:

$$\max_{\tau^B, \{a_t^B\}, \{\hat{a}_t^G\}} E \left[r \int_0^{\tau^B} e^{-rt} h(a_t^B, B) dt \mid \theta = B \right] \quad \text{s.t.}$$

$$E \left[r \int_0^{\tau^B} e^{-rt} g(a_t^B, B) dt \mid \theta = B \right] = W_0^B,$$

$$E \left[r \int_0^{\tau^B} e^{-rt} g(\hat{a}_t^G, G) dt \mid \theta = G \right] \leq \hat{W}^G, \qquad \text{(C-BAD)}$$

and $\{a_t^B\}$, $\{\hat{a}_t^G\}$ are feasible and incentive compatible, where W_0^B is the bad surgeon's expected payoff and $\hat{W}^G$ is the good surgeon's maximum payoff (the self-selection constraint).

It turns out that in an optimal evaluation contract, each surgeon's continuation value evolves as a state variable, whose initial value is equal to the surgeon's total expected (average) payoff from the contract at time 0. Notice that each continuation value must fall between 0, the surgeon's continuation value after license revocation, and 1, the (normalized) maximal flow payoff. If the surgeon's continuation value ever reaches 1, this means that the principal can no longer cut off the surgeon. In other words, the surgeon achieves *tenure*. Note that the contract in which the principal completely ignores the surgeon's signal history corresponds to the pair of values $(W^G, W^B) = (1, 1)$.

We characterize each contract separately. We begin by deriving the easier contract, the contract for the bad type, and then continue by tackling the contract for the good type. The solution to the good contract is where we introduce novel techniques. After finding those two evaluation contracts, we take the last step of choosing specific values for the contracts such that total expected patient welfare is maximized and the self-selection constraints hold.

4.1 Optimal Evaluation Contract for the Bad Surgeon

The optimal evaluation contract for the bad surgeon solves the optimization problem (C-BAD). The last constraint reflects the self-selection constraint, and we proceed by ignoring it (claiming it is not binding), solving the maximization problem, and afterwards verifying that the constraint holds in the optimal solution.

Without the self-selection constraint, the optimization problem is very similar to the continuous-time principal-agent problem solved in Sannikov (2007b) and we follow his techniques directly. In this section, we derive the contract informally, leaving the formal proofs to Appendix A.1.3.

Notice that we allow for arbitrarily complex history-dependent contracts. The key simplifying insight from Sannikov (2007b), which also holds in discrete time (Spear & Srivastava 1987), is that the agent's continuation value W_t at a given time t completely summarizes the past history in an optimal contract. Replacing one continuation contract (following some history) with another continuation contract that has the same continuation value W_t does not change the agent's incentives. After any history then, the principal's payoff is maximized if and only if the continuation contract is optimal given the continuation value. Therefore, W_t completely determines the optimal continuation contract. Recalling Definition 1 of a scoring rule, we can think of W_t as the contract's *score.*

Let $\mathcal{A}^B = \{a_t^B\}$ be the action policy for the bad surgeon specified by the contract. Then his continuation value at time t is

$$W_t^B(\mathcal{A}^B) = E^{\mathcal{A}^B}\left[\int_t^{\tau^B} e^{-r(s-t)}g(a_t^B, B)ds \mid \mathcal{F}_t^B\right],$$

where $E^{\mathcal{A}^B}$ denotes the expectation under the probability measure $\mathbb{P}^{\mathcal{A}^B}$ induced by the bad surgeon's strategy $\mathcal{A}^B$. For the remainder of this section, we drop the B notation, unless necessary to avoid confusion. From Proposition 1 of Sannikov (2007b), we know that we can represent $W_t(\mathcal{A}^B)$ as a

diffusion process: there exists a progressively measurable process $\mathcal{Y} = \{Y_t\}$ such that[13]

$$dW_t(\mathcal{A}) = r(W_t(\mathcal{A}) - g(a_t, B))dt + rY_t\sigma dZ_t^{\mathcal{A}} \quad \forall t \geq 0, \tag{4.1}$$

where $\sigma dZ_t^{\mathcal{A}} = dX_t - \mu(a_t, B)dt$. We call Y_t the *sensitivity* of the process to the signals X_t, where Y_t (scaled by $r\sigma$) reflects the volatility of $W_t(\mathcal{A})$. Now, how do we determine whether $\mathcal{A}$ is incentive compatible? For a given strategy $\mathcal{A}$, consider the process $\mathcal{Y} = \{Y_t\}$ such that (4.1) holds. Proposition 2 of Sannikov (2007b) tells us that $\mathcal{A}$ is optimal if and only if $a_t \in \arg\max_{a' \in A} g(a', B) + Y_t\mu(a', B) \ \forall t \geq 0$ holds almost everywhere. In other words, for the specific process $\mathcal{Y}$ associated with a given strategy $\mathcal{A}$, we can summarize incentive-compatible actions a_t as a function of Y_t:

$$a_t(Y_t) = 1 \quad \Longrightarrow \quad Y_t \geq \frac{\zeta^B}{k^B} \quad \text{and} \quad a_t(Y_t) = 0 \quad \Longrightarrow \quad Y_t \leq \frac{\zeta^B}{k^B}. \tag{4.2}$$

This is intuitive: the surgeon optimally engaging in selection behavior corresponds to the score being highly sensitive to signals. Similarly, the surgeon optimally avoiding selection behavior corresponds to the score being less sensitive to signals.

To find the optimal contract that solves (C-BAD), we first use the representation of the surgeon's payoff as a diffusion process to apply dynamic programming principles and write out the Hamilton-Jacobi-Bellman (HJB) equation. We solve the HJB equation and then verify that the solution in fact characterizes an optimal contract.

Let $\Lambda(W)$ denote the maximum payoff to the principal when the bad surgeon gets continuation payoff W. The principal provides incentives by using realized signals to adjust W. Formally, the surgeon's payoff evolves according to:

$$dW_t = r(W_t - g(a_t, B))dt + rY_t\sigma dZ_t \tag{4.3}$$

where $\{a_t\}$ is the surgeon's optimal risk selection strategy and $\{Y_t\}$ is a progressively measurable process (with respect to the signal process $\{X_t\}$).[14] By Ito's Lemma, we can write the law of motion

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[13]To be precise, Proposition 1 of Sannikov (2007b) also tells us that the process $\mathcal{Y}$ is in $\mathcal{L}^*$, i.e. $E^{\mathcal{A}^B}[\int_0^t Y_s^2 ds] < \infty$ for all $t \in [0, \infty)$.

[14]Recall that $dX_t = \mu(a_t, B)dt + \sigma dZ_t$ and $\{Z_t\}$ is a Brownian motion.

for $\Lambda(W)$ as

$$d\Lambda(W) = \left(\Lambda'(W)(r(W - g(a, B))) + \frac{\Lambda''(W)}{2}r^2Y^2\sigma^2\right)dt + \Lambda'(W)rY\sigma dZ.$$

Then, the HJB equation is

$$\Lambda(W) = \max_{a,Y} h(a, B) + \Lambda'(W)(W - g(a, B)) + \frac{\Lambda''(W)}{2}r\sigma^2Y^2.$$

Let $a(W)$ and $Y(W)$ denote the action and sensitivity arguments that serve as maximizers on the right hand side for a given W. We impose the constraint that $a(W) = a(Y(W))$ (from (4.2)). So in fact, we can rewrite the HJB equation in a more convenient form, and with boundary conditions, as

$$\Lambda''(W) = \min_Y \frac{\Lambda(W) - h(a(Y), B) - \Lambda'(W)(W - g(a(Y), B))}{r\sigma^2Y^2/2}$$

$$\text{where} \quad \Lambda(0) = 0 \quad \text{and} \quad \Lambda(1) = h(0, B). \qquad \text{(HJB-BAD)}$$

Notice that if $\Lambda''(W) < 0$, then the minimizing choice of Y in the HJB equation will be the minimum sensitivity needed to enforce action $a(W)$.

Without loss of generality, we normalize $h(a, B)$ so that $h(1, B) = 1$. First, we state a brief lemma, proved in Appendix A.1.1:

Lemma 2. *The solution Λ to (HJB-BAD) is unique and strictly concave.*

Solving (HJB-BAD) gives us the function $\Lambda(W)$, as well as the functions $a(W)$ and $Y(W)$ that serve as minimizers on the RHS of the HJB equation. More formally, we denote the solution to (HJB-BAD) as (Λ, a, Y) where $\Lambda : [0, 1] \to [h(0, B), 1]$, $a : [0, 1] \to A$ and $Y : [0, 1] \to [0, \infty)$. First we will show how to construct a corresponding evaluation contract C^B from that solution, and then we confirm that C^B is indeed optimal.

Denote the evaluation contract corresponding to the solution to (HJB-BAD) by $C^B = \{\tau^B, \mathcal{A}^B\}$, where τ^B is the X_t-measurable stopping time and $\mathcal{A}^B = \{a_t^B\}$ is the recommended incentive-compatible risk selection policy for the bad surgeon. We conjecture that C^B is optimal and construct it as follows. Let the bad surgeon's continuation value at time t be W_t, which starts at an initially

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chosen W_0 and evolves according to

$$dW_t = r(W_t - g(a_t^B, B))dt + rY_t\sigma dZ_t.$$

Set $a_t^B = a(W_t)$ and $Y_t = Y(W_t)$, which we know enforces incentive compatibility of $a(W_t)$. Finally, define τ^B as follows. Let $\bar\tau$ satisfy $\bar\tau = \min\{t \; : \; W_t = 0 \;||\; W_t = 1\}$. Then the stopping time τ^B can be written

$$\tau^B = \left\{ \begin{array}{ll} \bar\tau & \text{if } W_{\bar\tau} = 0 \\ \infty & \text{if } W_{\bar\tau} = 1 \end{array} \right\} \tag{4.4}$$

We show that at any time t, W_t is the expected continuation payoff to the bad surgeon from this contract and $\Lambda(W_t)$ is the expected continuation patient welfare.

Proposition 1 establishes that the above evaluation contract constructed from the solution to (HJB-BAD) is in fact optimal.

Proposition 1. *The contract constructed from the unique solution (Λ, a, Y) to (HJB-BAD) characterizes an optimal evaluation contract for the bad surgeon C^B that solves (C-BAD). This evaluation contract takes the form of a scoring rule in which the score is the bad surgeon's continuation value W_t. The initial score W_0 corresponds to the bad surgeon's expected payoff from the entire contract, and $\Lambda(W_0)$ corresponds to the expected patient welfare from the entire contract.*

When the score is W_t, the expected continuation patient welfare is $\Lambda(W_t)$, the sensitivity of the score to signals is $Y(W_t)$ and the optimal selection policy is $a(W_t)$. Also, $Y(W_t) > 0$ for all $W_t \in (0, 1)$ and hence the score hits 0 or 1 in finite time. If the score hits 0 before it hits 1, the surgeon's license is revoked; if the score hits 1 before it hits 0, the surgeon is awarded tenure.

Proof. See Appendix A.1.3. □

Broadly, the optimal evaluation contract is a scoring rule in which the evolution of the score is governed by the sensitivity rule $Y(W)$. If the surgeon performs too poorly and his score drops below a threshold, his license is revoked and he is fired. On the other hand, if the surgeon performs very well and his score rises above a threshold, he is awarded tenure. Proposition 2 below tells us that in an optimal contract the bad surgeon is always incentivized to engage in selection behavior, which is welfare-improving, unless he has achieved tenure.
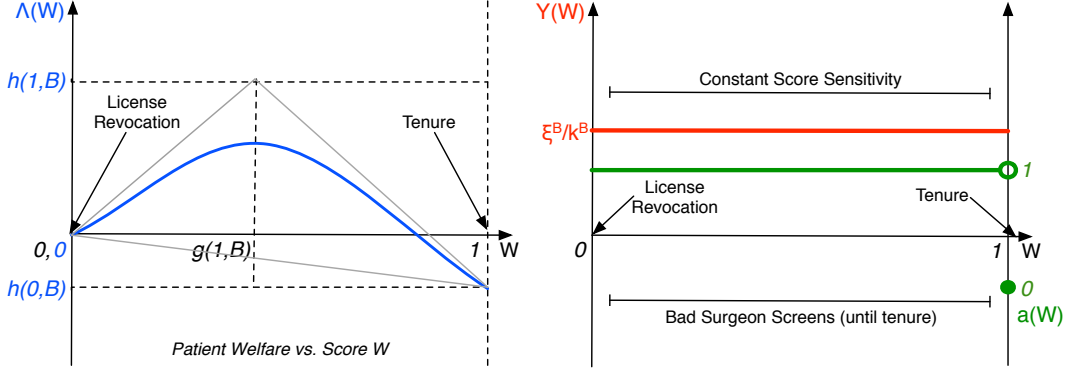
Figure 1: For C^B – Patient Welfare $\Lambda(W)$, Score Sensitivity $Y(W)$, Selection Policy $a(W)$

Proposition 2. *In the solution to (HJB-BAD), $a(W) = 1$ for all $W \in [0, 1)$ and therefore optimally $Y(W) = \frac{\zeta^B}{k^B}$ for all $W \in [0, 1)$. In other words, until tenure, the bad surgeon always engages in risk selection and the contract's sensitivity to outcomes is constant.*

Proof. See Appendix A.1.2. □

In an optimal bad contract, the result that an untenured bad surgeon always engages in selection behavior is intuitive. Given that we ignore the good surgeon's self-selection constraint as non-binding, making an untenured bad surgeon engage in selection behavior by increasing the score sensitivity improves patient welfare under the bad contract without causing value to be destroyed under the good contract.

The main features of the scoring rule are depicted in Figure 1. On the left panel is patient welfare as a function of the score W. Notice that patient welfare is bounded by the triangle formed by the feasible set of bad surgeon-patient welfare pairs $(0, 0)$, $(g(1, B), h(1, B))$, and $(g(0, B), h(0, B))$. Also plotted as a function of the score, the right panel presents the score sensitivity rule and the bad surgeon's optimal selection policy, both of which we have shown to be constant.

Comparative Statics. How does the optimal contract for the bad type change as we vary the underlying parameters? The shape of the optimal contract depends on the technology parameter k^B, the cost of risk selection ζ^B, the interest rate r, the signal volatility σ, and the patient welfare function $h(a, B)$. As k^B, the difference between the drift of the bad surgeon's signal when he selects risk and the drift of his signal when he does not select risk, increases, the more discerning the
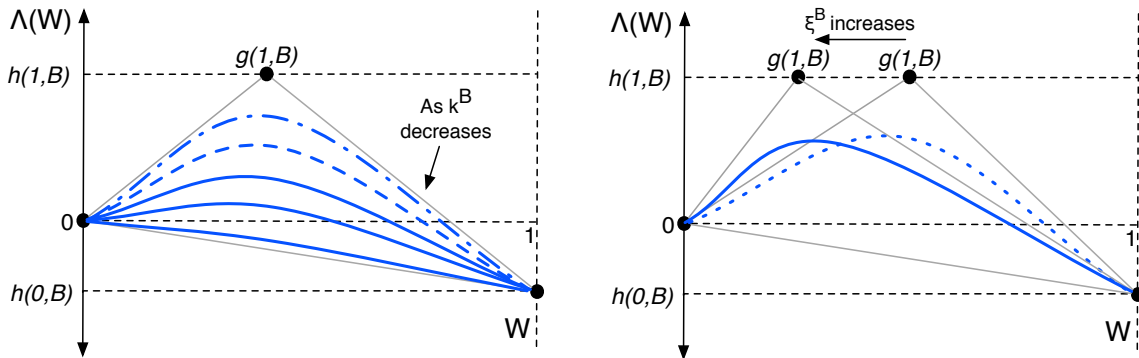
Figure 2: For C^B – Changes in $\Lambda(W)$ as k^B, ζ^B vary (for fixed r, σ, g, h)

principal can be and the higher his maximal payoff for a given W. In other words, Λ becomes more concave (i.e. $\Lambda'(0) \to \frac{h(1,B)}{g(1,B)}$). On the other hand, as k^B decreases, Λ becomes less concave (i.e. $\Lambda'(0) \to -h(0,B)$) and the principal's payoff decreases. See the left panel of Figure 2. Note that holding k^B fixed and decreasing $r\sigma^2$ has the same effect as holding $r\sigma^2$ fixed and increasing k^B.

Now consider if we increase ζ^B, the cost of risk selection. There are two effects. First, the minimum sensitivity $\frac{\zeta^B}{k^B}$ needed to enforce action $a_t = 1$ increases, which decreases the concavity of Λ. Secondly, the bad surgeon's payoff when he engages in selection behavior, $g(1, B)$, decreases. Hence, for the region to the right of the maximum Λ, patient welfare decreases. See the right panel of Figure 2.

Finally, consider what happens as the values of $h(a, B)$ change. This alters the size and position of the triangle that bounds the optimal contract Λ. As expected, as either $h(1, B)$ or $h(0, B)$ increases, patient welfare in the optimal contract Λ for a given W also increases.

We still need to verify that the good surgeon's self-selection constraint is in fact not binding. The verification is simpler after we derive the optimal contract for the good surgeon, which we do in the following section. At the end of that section we confirm that the good surgeon's self-selection constraint holds.

The techniques we have used to solve for the bad surgeon's optimal contract are directly analogous to those developed in Sannikov (2007b). In the following section, we solve for the good surgeon's optimal contract. Because we cannot ignore the self-selection constraint, the problem becomes much more complex. To solve it, we adopt novel applications of continuous-time techniques.

5 Optimal Evaluation Contract for the Good Surgeon

The optimal evaluation contract for the good surgeon solves the optimization problem (C-GOOD). Unlike solving for the bad contract, we cannot ignore the self-selection constraint in this problem because it will be binding. Therefore, we must keep track of two state variables – both the continuation payoff to the good surgeon and the continuation payoff to the bad surgeon. Furthermore, the principal's payoff (i.e. patient welfare) will be a function of both continuation values. In this section, we provide an informal derivation of the optimal contract, and we perform most formal verifications in Appendix A.2.

Let $\mathcal{A}^G = \{a_t^G\}$ be the good surgeon's optimal risk selection policy, and $\mathcal{A}^B = \{a_t^B\}$ be the bad surgeon's optimal risk selection policy.[15] Their respective continuation values in the contract at time t are

$$W_t^G(\mathcal{A}^G) = E^{\mathcal{A}^G}\left[\int_t^{\tau^G} e^{-r(s-t)}g(a_t^G, G)ds \mid \mathcal{F}_t^G\right] \quad \text{and}$$

$$W_t^B(\mathcal{A}^B) = E^{\mathcal{A}^B}\left[\int_t^{\tau^G} e^{-r(s-t)}g(a_t^B, B)ds \mid \mathcal{F}_t^B\right],$$

where $E^{\mathcal{A}^\theta}$ denotes the expectation under the probability measure $\mathbb{P}^{\mathcal{A}^\theta}$ induced by the type-θ surgeon's strategy $\mathcal{A}^\theta$. Again, by Proposition 1 of Sannikov (2007b) we can represent $W_t^G(\mathcal{A}^G)$ and $W_t^B(\mathcal{A}^B)$ as diffusion processes so that there exist progressively measurable processes $\mathcal{Y}^G = \{Y_t^G\}$ and $\mathcal{Y}^B = \{Y_t^B\}$ such that[16]

$$dW_t^G(\mathcal{A}^G) = r(W_t^G(\mathcal{A}^G) - g(a_t^G, G))dt + rY_t^G\sigma dZ_t^{\mathcal{A}^G} \quad \forall t \geq 0, \tag{5.1}$$

$$dW_t^B(\mathcal{A}^B) = r(W_t^B(\mathcal{A}^B) - g(a_t^B, B))dt + rY_t^B\sigma dZ_t^{\mathcal{A}^B} \quad \forall t \geq 0. \tag{5.2}$$

As before, we call Y_t^θ the *sensitivity* of the process to the signals X_t, from the perspective of the type-θ surgeon. Lemma 10, proven in Appendix A.2.1, gives the incentive compatibility conditions for $\mathcal{A}^G$ and $\mathcal{A}^B$ (and the corresponding sensitivity processes $\mathcal{Y}^G$ and $\mathcal{Y}^B$):

$$a_t^G(Y_t^G) = 1 \quad \Longrightarrow \quad Y_t^G \geq \frac{\zeta^G}{k^G} \quad \text{and} \quad a_t^G(Y_t^G) = 0 \quad \Longrightarrow \quad Y_t^G \leq \frac{\zeta^G}{k^G}$$

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[15]We have dropped the hat over $\hat{\mathcal{A}}^B = \{\hat{a}_t^B\}$ and will continue to do so in this section.

[16]Again, to be precise, Proposition 1 of Sannikov (2007b) also tells us that the processes $\mathcal{Y}^\theta$ are in $\mathcal{L}^*$, i.e. $E^{\mathcal{A}^\theta}[\int_0^t (Y_s^\theta)^2 ds] < \infty$ for all $t \in [0, \infty)$.

$$a_t^B(Y_t^B) = 1 \implies Y_t^B \geq \frac{\zeta^B}{k^B} \text{ and } a_t^B(Y_t^B) = 0 \implies Y_t^B \leq \frac{\zeta^B}{k^B}. \tag{5.3}$$

So for each type of surgeon, there exists a threshold sensitivity above which he engages in selection behavior and below which he does not.

To find the optimal contract that solves (C-GOOD), we attempt a similar approach as before. First we use the representation of the surgeons' payoffs as diffusion processes so that we can apply dynamic programming principles to find the relevant Hamilton-Jacobi-Bellman (HJB) equation. We solve the HJB equation and then show that the solution in fact characterizes an optimal contract.

Let $\Pi(W_t^G, W_t^B)$ denote the maximum expected payoff to the principal (i.e. patient welfare) when the good surgeon gets continuation payoff W_t^G and the bad surgeon gets continuation payoff W_t^B. Formally, the principal adjusts the surgeons' payoffs according to

$$dW_t^G = r(W_t^G - g(a_t^G, G))dt + rY_t^G \sigma dZ_t^G$$

$$dW_t^B = r(W_t^B - g(a_t^B, B))dt + rY_t^B \sigma dZ_t^B$$

where $\sigma dZ_t^G = dX_t - \mu(a^G, G)dt$ and $\sigma dZ_t^B = dX_t - \mu(a^B, B)dt$, so that $\sigma dZ_t^B = \sigma dZ_t^G + (\mu(a^G, G) - \mu(a^B, B))dt$. From the point of view of the principal, who is expecting the good surgeon, we have

$$dW_t^B = r(W_t^B - g(a_t^B, B) + Y_t^B(\mu(a_t^G, G) - \mu(a_t^B, B)))dt + rY_t^B \sigma dZ_t^G.$$

So the drift of the bad surgeon's continuation value from the principal's perspective involves a correction term, which includes the bad surgeon's perceived sensitivity. We apply Ito's Lemma to arrive at the law of motion for $\Pi(W_t^G, W_t^B)$. It follows that the HJB equation is

$$\Pi(W_t^G, W_t^B) = \max_{a^G, a^B, Y^G, Y^B} h(a^G, G) + \Pi_1(W_t^G, W_t^B)(W_t^G - g(a^G, G))$$
$$+ \Pi_2(W_t^G, W_t^B)(W_t^B - g(a^B, B) + Y^B(\mu(a^G, G) - \mu(a^B, B)))$$
$$+ \frac{\Pi_{11}(W_t^G, W_t^B)}{2}r\sigma^2(Y^G)^2 + \frac{\Pi_{22}(W_t^G, W_t^B)}{2}r\sigma^2(Y^B)^2 + \Pi_{12}(W_t^G, W_t^B)r\sigma^2 Y^G Y^B.$$

Analyzing that equation is unwieldy. Fortunately, recall that we have assumed that good surgeons are fully altruistic. So, $h(a, G) \propto g(a, G)$ and the good surgeon makes risk selection decisions completely in the interests of patients. A contract that maximizes the good surgeon's payoff also maximizes patient welfare. So we can re-write the optimization problem (C-GOOD) with the good surgeon's

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payoff as the objective function:

$$\max_{\tau^G, \{a_t^G\}, \{a_t^B\}} E\left[r \int_0^{\tau^G} e^{-rt} g(a_t^G, G) dt \mid \theta = G\right] \quad \text{s.t.} \tag{5.4}$$

$$E\left[r \int_0^{\tau^G} e^{-rt} g(a_t^B, B) dt \mid \theta = B\right] \leq \hat{W}^B, \tag{5.5}$$

and $\{a_t^G\}$, $\{a_t^B\}$ are feasible and incentive compatible, where $\hat{W}^B$ is the maximum allowable payoff to the bad surgeon.

It follows that $\Pi(W_t^G, W_t^B) \propto W_t^G$, so we can write patient welfare as a function of W_t^B only, i.e. $\Pi(W_t^B)$. Without loss of generality, we normalize that constant of proportionality to be 1, so that $\Pi(W_t^B) = W_t^G$. From Ito's Lemma, the drift of $\Pi(W_t^B)$ is

$$r\Pi'(W_t^B)(W_t^B - g(a_t^B, B) + Y^B(\mu(a_t^G, G) - \mu(a_t^B, B)))) + \frac{\Pi''(W_t^B)}{2} r^2 \sigma^2 (Y_t^B)^2$$

and the volatility is $r\sigma\Pi'(W_t^B)Y_t^B$. We can match up terms with the law of motion for W_t^G to obtain useful formulas characterizing the relationship between W_t^G and W_t^B. In particular, matching up volatilities implies

$$Y_t^G = \Pi'(W_t^B)Y_t^B,$$

which describes the relationship between the score sensitivity from the good surgeon's perspective Y_t^G and the score sensitivity from the bad surgeon's perspective Y_t^B. When $\Pi'(W_t^B)$ is large (i.e. the good surgeon has a lot to gain from a score increase), the score sensitivity from the good surgeon's perspective is higher than the score sensitivity from the bad surgeon's perspective. The opposite holds when $\Pi'(W_t^B)$ is small.

The HJB equation that selects a^G, a^B, and Y (we drop the B superscript) to maximize the good surgeon's payoff $\Pi(W^B)$ is as follows:

$$\Pi(W^B) = \max_{a^G, a^B, Y} g(a^G, G) + \Pi'(W^B)(W^B - g(a^B, B) + Y(\mu(a^G, G) - \mu(a^B, B))) + \frac{\Pi''(W^B)}{2} r\sigma^2 Y^2.$$

Let $a^G(W^B)$, $a^B(W^B)$ and $Y(W^B)$ denote the action and sensitivity policies that serve as maximizers on the right hand side of the HJB equation for a given W^B. We impose the constraint that $a^G(W^B)$ and $a^B(W^B)$ have to be optimal actions given $Y(W^B)$, i.e. $a^G(W^B) = a^G(\Pi'(W)Y(W^B))$ and

$a^B(W^B) = a^B(Y(W^B))$ (from 5.3). Thus, there is a mapping from Y to incentive-compatible pairs (a^G, a^B), and in fact we are maximizing over Y in the HJB equation. More conveniently, the HJB equation can be written

$$\Pi''(W^B) = \min_Y \frac{\Pi(W^B) - g(a^G(Y), G) - \Pi'(W^B)(W^B - g(a^B(Y), B) + Y(\mu(a^G(Y), G) - \mu(a^B(Y), B)))}{r\sigma^2 Y^2/2}$$

$$\text{where } \Pi(0) = 0 \text{ and } \Pi(1) = 1. \tag{HJB-GOOD}$$

Additionally, we require $a^G(Y) = 1$ only if $Y \geq \frac{\zeta^G}{\Pi'(W^B)k^G}$ and $a^G(Y) = 0$ only if $Y \leq \frac{\zeta^G}{\Pi'(W^B)k^G}$, while $a^B(Y) = 1$ only if $Y \geq \frac{\zeta^B}{k^B}$ and $a^B(Y) = 0$ only if $Y \leq \frac{\zeta^B}{k^B}$. The following lemma is proved in Appendix A.2.2.

Lemma 3. *The solution Π to (HJB-GOOD) is unique, strictly concave and strictly increasing.*

At this point it is instructive to highlight one major difference between this contracting problem for the good surgeon and the contracting problem solved in Sannikov (2007b). In the latter problem, the sensitivity Y appears in the HJB equation only once in the denominator. As a result, the sensitivity that minimizes the right hand side of the HJB equation is always the minimum sensitivity needed to enforce a given action. In our problem, however, the sensitivity Y appears both in the numerator as well as the denominator. As a result, the Y that minimizes the right hand side of the HJB equation for a given action is not necessarily the minimum sensitivity that enforces that action.

Suppose we solve the HJB equation (HJB-GOOD) and generate the solutions $\Pi(W^B)$, $a^G(W^B)$, $a^B(W^B)$ and $Y(W^B)$. More formally, we denote the solution to (HJB-GOOD) as (Π, a^G, a^B, Y) where $\Pi : [0, 1] \to [0, 1]$, $a^G : [0, 1] \to A$, $a^B : [0, 1] \to A$ and $Y : [0, 1] \to [0, \infty)$.

We construct a corresponding evaluation contract as follows, conjecturing that it solves (C-GOOD). Denote the contract by $C^G = \{\tau^G, \mathcal{A}^G, \mathcal{A}^B\}$, where τ^G is an X_t-measurable stopping time, $\mathcal{A}^G = \{a_t^G\}$ is the recommended incentive-compatible risk selection policy for the good surgeon, and $\mathcal{A}^B = \{a_t^B\}$ is the optimal risk selection policy for the bad surgeon (if he were to deviate and take this contract). Let the deviating bad surgeon's continuation value be W_t^B, which starts at an initially chosen W_0^B and evolves according to

$$dW_t^B = r(W_t^B - g(a_t^B, B))dt + rY_t\sigma dZ_t^{\mathcal{A}^B},$$

which from the principal's perspective is

$$dW_t^B = r(W_t^B - g(a_t^B, B) + Y_t(\mu(a_t^G, G) - \mu(a_t^B, B)))dt + rY_t\sigma dZ_t^{A^G}.$$

Set $a_t^B = a^B(W_t^B)$ and $a_t^G = a^G(W_t^B)$. We also set $Y_t = Y(W_t)$, which we know enforces incentive compatibility of $a^B(W_t^B)$ and $a^G(W_t^B)$. Finally, define the stopping time τ^G as follows. Define $\bar{\tau} = \min\{t \ : \ \Pi(W_t) = 0 \ || \ \Pi(W_t) = 1\}$. Then the stopping time τ^G can be written

$$\tau^G = \left\{ \begin{array}{ll} \bar{\tau} & \text{if } \Pi(W_{\bar{\tau}}) = 0 \\ \infty & \text{if } \Pi(W_{\bar{\tau}}) = 1 \end{array} \right\} \tag{5.6}$$

We show that at any time t, W_t^B is the expected continuation payoff to the deviating bad surgeon from this contract and $\Pi(W_t)$ is the expected continuation patient welfare (which is also equal to the good surgeon's continuation value).

Proposition 3 establishes that the above evaluation contract constructed from the solution to (HJB-GOOD) is in fact optimal.

Proposition 3. *The contract constructed from the unique solution (Π, a^G, a^B, Y) to (HJB-GOOD) characterizes an optimal evaluation contract for the good surgeon C^G that solves (C-GOOD). This evaluation contract takes the form of a scoring rule in which the score is the bad surgeon's continuation value W_t. The initial score W_0 corresponds to the deviating bad surgeon's expected payoff from the entire contract if he chooses it, and $\Pi(W_0)$ corresponds to the expected patient welfare (and hence the good surgeon's expected payoff) from the entire contract.*

Suppose the score is W_t. Then the expected continuation patient welfare is $\Lambda(W_t)$. From the good surgeon's perspective, the sensitivity of the score to signals is $\Pi'(W_t)Y(W_t)$ and the optimal risk selection policy is $a^G(W_t)$. From the bad surgeon's perspective, if he has deviated and accepted C^G, the sensitivity of the score to signals is $Y(W_t)$ and the optimal risk selection policy is $a^B(W_t)$. Furthermore, $Y(W_t) > 0$ for all $W_t \in (0, 1)$ and hence the score W_t hits 0 or 1 in finite time. If the score hits 0 before it hits 1, we know $\Pi(W_t) = 0$ and hence the surgeon's license is revoked. If the score hits 1 before it hits 0, we know $\Pi(W_t) = 1$ and hence the surgeon is awarded tenure.

Proof. See Appendix A.2.3 for proof. □

We now highlight the second major difficulty that arises when solving this contracting problem for

the good surgeon but does not arise when solving the contracting problem for the bad surgeon (i.e. the problem solved in Sannikov (2007b)). Recall that in the case of the bad surgeon, we appeal to a simplifying insight that we can replace one continuation contract with another continuation contract without changing the agent's incentives, so long as both contracts have the same continuation value to the agent. Therefore, we know that any continuation contract in the bad surgeon's optimal contract is itself optimal and hence on the curve Λ.

In this case of the good surgeon, however, we are managing the incentives of two agents. If the continuation contract for a given continuation value W_t achieves a lower patient welfare than the optimal contract for that value W_t, then we cannot simply replace the first contract by the second one. The reason is that doing so will alter the incentives of the good surgeon. In order to show that at any point in time the continuation contract in the optimal contract achieves a continuation patient welfare that lies on the curve Π, we must appeal to a more complex argument. We do so in Lemma 16 of the Appendix.

We have now fully characterized an optimal evaluation contract for the good surgeon by an HJB equation. In the next section, we analyze properties of the contract.

5.1 Properties of the Good Surgeon's Evaluation Contract

We characterize both the optimal risk selection policy for the good surgeon $a^G(W)$ and the optimal risk selection policy for the deviating bad surgeon $a^B(W)$. Clearly, the good surgeon's policy is of greater interest than the bad surgeon's, since under the optimal menu of contracts only the good surgeon is assigned to the good contract. Nonetheless, characterizing both policies gives a more complete picture that is helpful in understanding the incentives the good surgeon faces.

Consider the good surgeon. We find that his optimal risk selection policy takes the shape of a weakly decreasing step function. We also find that the sensitivity of the score to signals from the good surgeon's perspective is strictly decreasing in W. Thus, when the score is low, the contract is highly sensitive to signals and the good surgeon optimally engages in selection behavior; meanwhile, when the score is high, the contract is less sensitive to signals and the good surgeon optimally stops selecting risk. Notice that the good surgeon selects risk even in our case when he is fully altruistic and aligns his interests directly with patients. We discuss this in further detail below.

The score at which the good surgeon switches from selecting risk to not selecting risk corresponds

generically to a discontinuous downward jump in the sensitivity. Thus, there are two distinct regions: a "hot seat" region of high sensitivity in which the good surgeon's score is low, the scoring rule is stricter, and his performance is scrutinized more carefully; and a "benefit of the doubt" region of low sensitivity in which the good surgeon's score is higher, the scoring rule is more lenient, and his performance is scrutinized less carefully. Interestingly, the optimal risk selection policy for the deviating bad surgeon turns out also to take the form of a weakly decreasing step function.

We summarize the results in the following proposition.

Proposition 4. *In an optimal evaluation contract for the good surgeon characterized by Proposition 3, both the recommended risk selection policy for the good surgeon and the optimal risk selection policy for the deviating bad surgeon are weakly decreasing step functions: as the score increases, both types of surgeon (weakly) reduce their selection behavior. Furthermore, in this contract the score sensitivity from the good surgeon's perspective $\Pi'(W)Y(W)$ is strictly decreasing. At the score W' where the good surgeon switches from engaging in selection behavior to avoiding selection behavior, $Y(W')$ (and hence $\Pi'(W')Y(W')$) generically features a discontinuous jump, separating the low-score region with high score sensitivity (the "hot seat") from the high-score region with low score sensitivity ("benefit of the doubt").*

Proof. See Appendix A.3.1 for proof. □

Although weakly decreasing functions admit the possibility of the optimal policies being constant before tenure, it turns out that neither surgeon optimally selects risk over the entire range of scores.

Proposition 5. *In an optimal evaluation contract for the good surgeon characterized by Proposition 3, there always exists an $\epsilon > 0$ such that for high enough scores $W \in (1 - \epsilon, 1]$, both the good surgeon and the deviating bad surgeon optimally avoid selection behavior.*

Proof. See Appendix A.3.1 for proof. □

To summarize, the good surgeon's contract takes the form of a scoring rule where the score is the state variable W (i.e. the deviating bad surgeon's continuation value). The score evolves in accordance with the rule $Y^B(W) = Y(W)$, which reflects the sensitivity of the score to realizations of the surgeons' signals. From the perspective of the good surgeon, the sensitivity of the score is $Y^G(W) = \Pi'(W)Y(W)$. A high sensitivity indicates the scoring rule is "strict" and hence surgeons
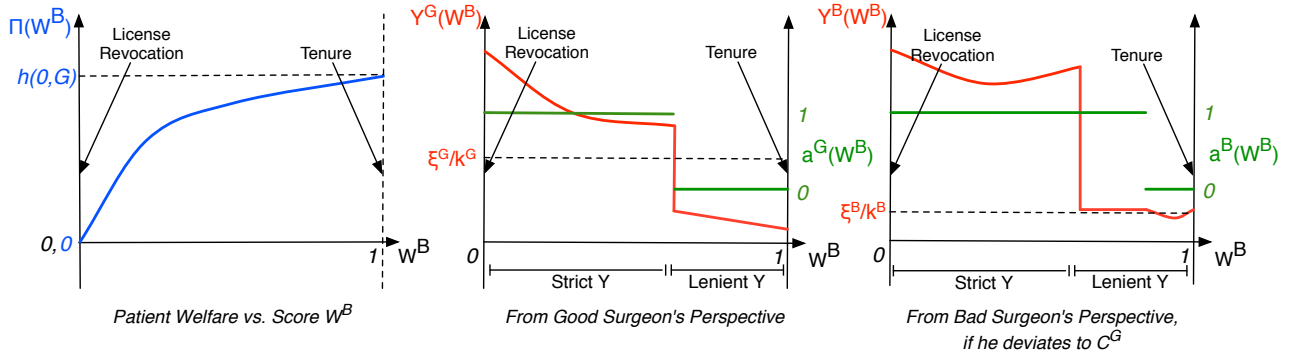
Figure 3: For C^G – Patient Welfare Π; Sensitivity and Selection Policies Y^G, a^G; Sensitivity and Selection Policies Y^B, a^B

are likely to engage in risk selection. A low sensitivity indicates the scoring rule is "lenient" and hence surgeons are likely to avoid selection. Implementation of the good surgeon's scoring rule then typically involves four regions: a "hot seat" region in which the good surgeon's performance is judged strictly and thus he selects risk; a "benefit of the doubt" region in which his performance is judged leniently and hence he avoids selection; a license revocation region when his score falls too low and play terminates; and finally a point of tenure at which his payoffs are fixed at the maximum regardless of performance. Movement between regions depends on the good surgeon's signals, with a well-performing surgeon typically moving away from the strict region towards the lenient region and finally to the point of tenure.

In Figure 3, we depict the main features of the good surgeon's optimal scoring rule C^G. In the left panel, patient welfare is plotted as a strictly increasing, strictly concave function of the score. In the middle panel is the scoring rule from the perspective of the good surgeon, who is expected to select this scoring rule. Notice that the score sensitivity from the good surgeon's perspective is weakly decreasing, and that there is a jump downward separating the region of high sensitivity from the region of low sensitivity. In the right panel we also include the scoring rule from the perspective of the bad surgeon if he were to deviate and select this scoring rule. Note that the score sensitivity is not necessarily monotonic from the bad surgeon's perspective.

We can look at the shape of the contract to consider the tradeoffs between current period gains and future period gains. When the score is low, the good surgeon engages in risk selection because the deviating bad surgeon is also selecting risk, and it is optimal to suffer the current period loss in
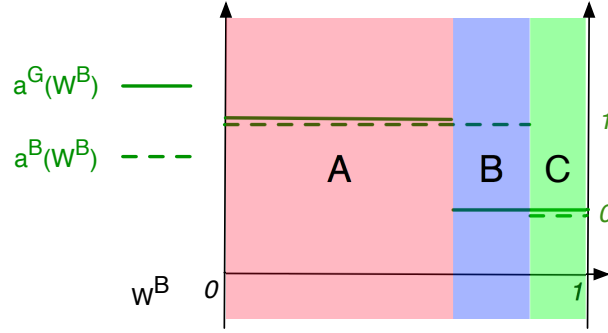
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Figure 4: Illustration of Current Period versus Future Period Tradeoffs

order to more easily separate from the bad surgeon in the future. This corresponds to region A in Figure 4. In region B, even though the bad surgeon is still selecting risk to inflate his performance, the good surgeon stops selecting risk, choosing to take the current period gain rather than work to separate himself from the deviating bad surgeon. Finally, in region C, the bad surgeon is no longer willing to engage in costly selection behavior, and the good surgeon can avoid selection altogether, capturing current period gains while still being able to separate himself.

Comparative Statics. We briefly give some intuition behind what happens as we change the values of the underlying parameters in the optimal good contract. In particular, we are interested in what happens (a) as ζ^G, the good surgeon's cost of selection, changes, (b) as k^G, the effectiveness of screening for the good surgeon changes, (c) as $\mu(0, G) - \mu(0, B)$, the difference between the two surgeons' technologies, changes, and (d) as r, the interest rate, or σ^2, the square of the signal volatility, changes.[17]

As ζ^G increases, the effect moves in two ways. First, patient welfare increases because the good surgeon engages in less selection behavior; however, this allows the bad surgeon to derive more value from deviating to the good contract and hence we must give him more value in the bad contract, decreasing patient welfare. Furthermore, patients suffer more harm when the good surgeon selects risk.

Given an increase in k^G, patient welfare increases because the good surgeon engaging in selection

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[17]Note that holding all parameters for the good type constant and then adjusting parameters for the bad type also changes the optimal contract, but we choose not to focus on those here as such changes would also change the shape of the optimal bad contract.

becomes more effective in separating himself. Similarly, as $\mu(0, G) - \mu(0, B)$ increases, again patients benefit because the good surgeon can identify himself more easily. Finally, as r or σ increases, the signal effectively becomes less informative and therefore patient welfare decreases.

5.2 Verifying Self-Selection Constraint

We still need to verify that the good surgeon's self-selection constraint is not binding. Suppose it were. Then C^B would give the bad surgeon a payoff of W^B and the good surgeon a payoff of W^G where $W^G > \Pi(W^B)$. This would then violate our claim that C^G is the optimal contract that maximizes the good surgeon's payoff for a given payoff of the bad surgeon. Therefore, we conclude that the good surgeon's self-selection constraint must not be binding.

6 Finding the Optimal Menu of Evaluation Contracts

Now that we have characterized the optimal contract for the good surgeon and the optimal contract for the bad surgeon, the last step is for the principal to choose appropriate initial conditions for the pair of contracts so as to maximize total discounted expected patient welfare.

Recall that for an optimal bad contract C^B, expected patient welfare is $\Lambda(\tilde{W}_0^B)$ when the expected payoff to the bad surgeon is $\tilde{W}_0^B$. On the other hand, for an optimal good contract C^G, expected patient welfare is $\Pi(\check{W}_0^B)$ when the expected payoff to the bad surgeon is $\check{W}_0^B$. In order for the bad surgeon's self-selection constraint to hold, the initial conditions of the contracts must be chosen such that $\tilde{W}_0^B \geq \check{W}_0^B$.

So the principal wants to choose initial scores $\check{W}_0^B, \tilde{W}_0^B$ for the optimal good and optimal bad contracts respectively, such that the principal solves $\max_{\check{W}_0^B, \tilde{W}_0^B} p\Pi(\check{W}_0^B) + (1-p)\Lambda(\tilde{W}_0^B)$ subject to $\tilde{W}_0^B \geq \check{W}_0^B$. Since $\Pi(\check{W}_0^B)$ is weakly increasing in its argument, the solution always involves $\tilde{W}_0^B = \check{W}_0^B$, and the maximization problem becomes

$$\max_{W_0^B} p\Pi(W_0^B) + (1-p)\Lambda(W_0^B),$$

which is maximized when

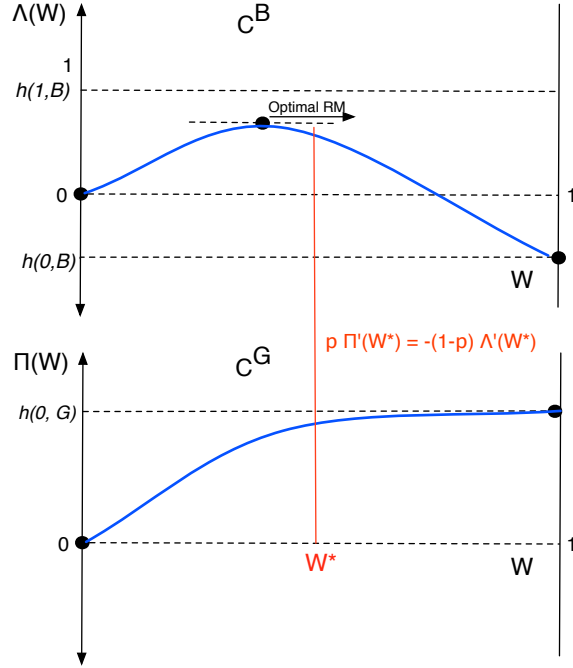$$p\Pi'(W^B) = -(1-p)\Lambda'(W^B). \tag{6.1}$$

Figure 5: Optimal initial score W^* for optimal menu of scoring rules (C^G, C^B)

See Figure 5. Notice that the optimal W^* always occurs to the right of the score for which $\Lambda(W)$ is maximized. Therefore, if $\Lambda'(0) > 0$, then it is never optimal to fire both surgeons immediately. On the other hand, if the proportion of good surgeons p in the population is large enough, or if the expected patient welfare from a bad surgeon taking all cases is not too low, then it may be optimal to avoid evaluation contracts altogether (i.e. tenure all surgeons immediately).

Notice that when W^* is in the region of high sensitivity, this suggests that any surgeon in the good contract starts out in a "hot seat." Immediately, his performance is scrutinized carefully and his outcomes carry a heavy weight. Only when the surgeon has proven himself and pushed his score past the threshold into the region of low sensitivity does he get a break. On the other hand, when W^* is in the region of low sensitivity, all surgeons start out in the "benefit of the doubt" region of the good contract. It is only when a surgeon has experienced some poor performance and his score drops into the high sensitivity region that his performance is scrutinized carefully.

Remark: As Ely and Välimäki mention in their analysis (2003), there are two issues with separating contracts. First, conditional on the good type, the contract is not renegotiation-proof. The good surgeon and the principal would like to renegotiate and tenure the surgeon immediately once he has

been identified as good. We assume that there is no renegotiation. Secondly, conditional on the bad type, it may not be sequentially rational for the principal to follow through with the contract. It is never sequentially rational if $h(1, B) < 0$. For now we assume that the principal can commit to the bad contract, but in Section 8 we revisit the issue.

Comparative Statics. How does the choice of W^* change as we adjust the values of the underlying parameters?

First, let's look at p, the *ex ante* probability that the surgeon is of the good type. As p increases, the ratio at the optimal W^* of $\frac{-\Lambda'(W^*}{\Pi'(W^*)}$ increases as well. Hence the optimal W^* increases in p, as does expected patient welfare. Another way to think about this is that as the proportion of good surgeons p increases, the good surgeon is more likely to begin with the benefit of the doubt (i.e. in the lenient scoring region) than to begin in the hot seat (i.e. in the strict scoring region).

To look at comparative statics of other underlying parameters, we have to turn attention to how such changes affect the optimal evaluation contracts C^G and C^B themselves. Generally, all else equal, the more concave the function $\Pi(W)$ in C^G, the smaller W^* is and the higher the patient welfare achieved. Similarly, all else equal, the more concave the function $\Lambda(W)$ in C^B, again the smaller W^* is and the higher the patient welfare achieved. Since any change in parameters involves several moving parts, we best illustrate comparative statics through numerical examples presented in the following section.

7 Numerical Examples

To illustrate some comparative statics, we look at several numerical examples and compute optimal contracts.

In Example 1, we consider the base case of equal costs of selection and overlap in surgeons' technologies. Example 2 adjusts the bad surgeon's technology so that there is no longer overlap. In Example 3, we return to the base case of Example 1 but make the cost of selection for the bad surgeon high relative to the good surgeon. Finally, in Example 4, we amend the base case so that the effectiveness of selection for the good surgeon, k^G, is close to that of the bad surgeon, k^B.

Example 1. Suppose that both the bad surgeon and the good surgeon experience the same cost of selection, i.e. $\zeta = \zeta^G = \zeta^B$, and that the bad surgeon's marginal performance gain from selection is twice the good surgeon's marginal performance gain, so $k^B = 2k^G$. Furthermore, there is overlap between the two surgeon's technologies, so that $\mu(0, G) < \mu(1, B)$. Figure 7 summarizes the optimal menu of contracts with such a setup. In the top panel, we have an analog of Figure 5, in which patient welfare is plotted against the score in both C^G and C^B. In the bottom panels, we have analogs of the middle and right panels of Figure 3 for the good contract: from each surgeon's perspective, the score sensitivity and the optimal selection policy are plotted against the score.

As expected, expected patient welfare $\Pi(W)$ is strictly increasing and concave in the score. Furthermore, the optimal selection policies of both surgeons take the form of weakly decreasing step functions. The bottom row of plots is particularly interesting: these two plots represent the score sensitivity to signals from each surgeon's perspective. Recall that $Y^G(W)$, the score sensitivity for the good surgeon, is equal to $\Pi'(W)Y^B(W)$. There are four regions in the evaluation contract:

1. $W = 0$, which reflects license revocation,

2. $W \in [0, \approx 0.2)$, which reflects the region of high score sensitivity,

3. $W \in [\approx 0.2, 1)$, which reflects the region of low score sensitivity, and

4. $W = 1$, which reflects tenure.

When $p = 0.5$, the optimal W^* at which scores are initialized is around 0.64, so that good surgeons begin in the region of low score sensitivity. Define the *inefficiency* of an outcome as a measure of the distance from first-best welfare. Relative to having no contracts, the optimal contracts in this base case example eliminates 42% of the inefficiency.

One might imagine that overlap in the two surgeons' technologies is essential in generating regions where the good surgeon engages in selection. However, even if the technologies do not overlap, the optimal evaluation contract for the good surgeon looks quite similar, although patient welfare increases. Consider Example 2.

Example 2. Suppose that the surgeons continue to experience the same common cost of selection and the same difference in marginal performance gains from selection. However, there is no longer

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overlap between the two technologies, so $\mu(0, G) > \mu(1, B)$. Figure 8 summarizes the optimal menu of evaluation contracts in such a case.

Notice that the downward jump in score sensitivities gets smaller. The intuition is that it is easier to differentiate the surgeons, and so the drop in sensitivities need not be as drastic. The optimal W^* drops to 0.51 and eliminated efficiency relative to no contracts is 64%, which is high relative to the base case.

The optimal selection policy for the deviating bad surgeon need not always switch from selection to no selection at a later score than the good surgeon's policy. In fact, if the cost of selection for the bad surgeon is high enough, he may stop selecting risk before the good surgeon. Consider the following example.

Example 3. Suppose that the bad surgeon's marginal performance gain is still twice the good surgeon's marginal performance gain from selection, and there is again overlap between the surgeons' technologies. But now the bad surgeon's cost of selection is high (while the good surgeon's cost of selection is low). Then Figure 9 summarizes the optimal good evaluation contract. The optimal W^* is 0.58 and eliminated efficiency relative to no contracts is a low 21.8%, so introducing contracts is relatively less helpful than the base case.

If we now remove the overlap from Example 3 and leave the cost of selection for the bad surgeon high enough relative to the good surgeon, it turns out that the bad surgeon's optimal selection policy may actually become flat. At all scores W, the bad surgeon chooses not to engage in selection behavior. See Example 4.

Example 4. We return to the base case of Example 1 but increase k^G so that now $k^B = 1.1 k^G$. Figure 10 summarizes the optimal menu of evaluation contracts. Since the effectiveness of selection for the good surgeon is higher, effectively he is able to separate himself more easily. The optimal W^* is 0.61 and eliminated inefficiency is 45% relative to no contracts.

8   Possibility of Improvement through Mentorship

Before concluding, we show how our methods can be used to consider a related question. Given that we have applied the Revelation Principle, our optimal mechanism takes the form of two separating

contracts, one of which will be chosen by the good surgeon in equilibrium and the other of which will be chosen by the bad surgeon in equilibrium. In our setup, because we not allow for improvement, once a surgeon has identified himself as bad, the optimal contract is constructed in order to minimize his harm.

In practice, however, it is likely that improvement is possible. In particular, if a bad surgeon performs some volume of surgeries in a mentored setting, i.e. with additional guidance and oversight, it is conceivable that he can improve to a good type. We observe that it is common for physicians to undergo a mentored training program (e.g. a fellowship) after which their quality is believed to improve. Suppose this is the case.

Let's consider the existence of a mentorship program with the following characteristics:

- If the bad surgeon is under mentorship, a patient who visits the bad surgeon receives a higher payoff if the bad surgeon does not engage in selection behavior than if he does engage in selection behavior, i.e. $h(0, B) > h(1, B)$.

- A surgeon can only go through the mentorship program once.

- A surgeon must complete the entire mentorship program in order to be eligible to practice surgery in the future. After completing the mentorship program, the surgeon is assigned to the good contract.

- If the mentorship program is of length at least $T^* > 0$, so the bad surgeon has a minimum level of mentored experience, then the bad surgeon improves to a good surgeon so long as he invests effort, which is unobservable. Let K be the cumulative cost of effort to the surgeon.[18]

We show that such a program is optimal. More specifically, an optimal contract for the bad surgeon is a fixed-length mentorship program in which (a) the surgeon's outcomes are ignored and (b) the bad surgeon in equilibrium chooses to exert costly effort to become a good surgeon.

Lemma 4. *Suppose $h(0, B) > h(1, B)$. Then the optimal contract C^B is as follows: for all W, $\Lambda(W) = h(0, B)W$, $a(W) = 0$ and $Y(W) = 0$.*

Proof. This follows directly from looking at the optimization problem (C-BAD) when $h(a, B)$ is decreasing in a and we ignore the good surgeon's self-selection constraint. To maximize patient

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[18]We assume that the surgeon either invests full effort and expends the whole cost K, or invests no effort and expends 0.
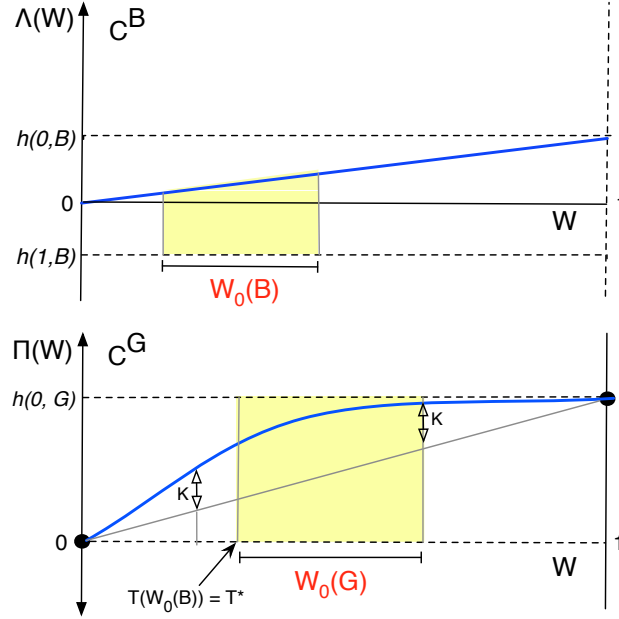
Figure 6: Optimal initial scores when improvement is possible through a training program

welfare for a given payoff W_0^B to the bad surgeon, we want him never to engage in selection behavior, which we can achieve for free (without introducing value-destroying volatility to provide incentives).

$\square$

For any given starting value W_0, this contract has a straightforward description. In each period, the surgeon's score W_t declines by a deterministic amount, independent of his signal:

$$dW_t = r(W_t - g(a_t, B))dt.$$

Therefore, at the end of a fixed period of time – which we denote as $T(W_0)$ and we observe is increasing in W_0 – the score hits 0 and the contract ends. So an optimal contract with starting value W_0 can be interpreted as a mentorship program of length $T(W_0)$. For $W_0 \in (0, 1)$, $T(0) = 0 < T(W_0) < T(1) = \infty$.

Because the only change has been to the patient welfare function when the surgeon is bad, construction of the optimal contract for the good type remains the same. To determine the optimal menu of contracts, the principal chooses an initial score $W_0(G)$ for the good contract and a length $T(W_0(B))$ for the bad contract to maximize expected unconditional patient welfare. Formally, his

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optimization problem becomes

$$\max_{W_0(G),W_0(B)} p\Pi(W_0(G)) + (1-p)(\Lambda(W_0(B)) + e^{-rT(W_0(B))}\Pi(W_0(G)))$$

$$\text{s.t.} \quad W_0(B) + e^{-rT(W_0(B))}(\Pi(W_0(G)) - K) \geq W_0(G),$$

$$\Pi(W_0(G)) - K \geq W_0(G) \quad \text{and} \quad T(W_0(B)) \geq T^*$$

where the first constraint is the bad surgeon's self-selection constraint, the second constraint verifies that it is rational for the bad surgeon to expend K to become a good surgeon, and the third constraint ensures that the mentorship program is sufficiently long to be effective. Notice that we are still bounded away from full efficiency under the good contract. Furthermore, since $T(1) = \infty$, optimally the mentorship program is of finite length.

The self-selection constraint will be binding (else, we could just increase $W_0(G)$ and achieve higher patient welfare). So we essentially maximize over only $W_0(G)$, which will imply $W_0(B)$ through the self-selection constraint. Meanwhile, the second and third constraints will restrict $W_0(G)$ to an interior subinterval of $[0,1]$.[19] On the one hand, we want $\tilde{W}_0$ to be small so that the bad surgeon becomes good quickly. On the other hand, we want $\tilde{W}_0$ to be big so that the good contract can offer a high payoff to the good surgeon (without attracting the bad surgeon). See Figure 6 for an illustration.

Notice that if $h(0, B) > 0$, then it is sequentially rational for the principal to follow through with the bad contract even when he knows the surgeon is bad.

9 Discussion

In Dranove et al. (2003), the authors hint at the possibility of improved report card design, claiming "report cards could be constructive if designed in a way to minimize the incentives and opportunities for provider selection." This paper takes a step in that direction. Our motivating belief is that performance reporting in healthcare will always be subject to information asymmetries that enable gaming. Hence, in this paper we characterize optimal contracts for evaluating experts whose types are heterogeneous but who can take private actions to manipulate performance. Such contracts

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[19]We assume that K and T^* are not too high so that this subinterval has positive measure.

maximize patient welfare while minimizing welfare-harming gaming incentives.

We apply the Revelation Principle and focus on menus of separating contracts. An optimal contract takes the form of a scoring rule. The contract for the bad type is simple, implementing a fixed action so long as the surgeon has not achieved tenure by performing exceptionally well or had his license revoked by performing poorly. The contract for the good type is the more interesting one, since its design must take into consideration the incentives of both the good surgeon and the bad surgeon who deviates. The form of this contract makes intuitive sense. If the surgeon's performance has been exceptionally high, he achieves tenure and is no longer subject to scoring. If his performance has been exceptionally low, his license is revoked. In between, the score varies with performance. In particular, the score sensitivity decreases in past performance: when a surgeon's past performance has been better, the score sensitivity he faces is lower. Furthermore, there are two distinct scoring regions: first, a region of high score sensitivity in which the good surgeon optimally engages in selection behavior, and secondly, a region of low score sensitivity in which the good surgeon optimally avoids selection behavior.

Our analysis reveals two features of provider selection that have not been highlighted in the literature. First, a certain amount of gaming by providers may be inevitable. Even when the good surgeon makes decisions completely in the interests of patients, in our optimal contract he typically engages in selection behavior. Secondly, risk selection may in fact be welfare-enhancing. For one thing, risk selection enables good surgeons to separate themselves from bad surgeons who are trying to look like good types, thereby benefiting future patients. Moreover, patients may be better off when bad surgeons avoid working on difficult cases.

Finally, we note that this issue of performance reporting in healthcare is closely related to the issue of pay-for-performance, which considers tying payment to performance, i.e. contracting upon outcome-contingent transfers. Such contracts may be a fruitful extension of this paper's techniques.

To conclude, we stress that the use of performance measures to identify the quality of experts is essential. Difficult to avoid, however, is the information asymmetry that enables experts to "game" the system, particularly in healthcare and in education where market-determined prices are absent. Such gaming is not always harmful, however, and the real question is a matter of balancing the appropriate tradeoffs.
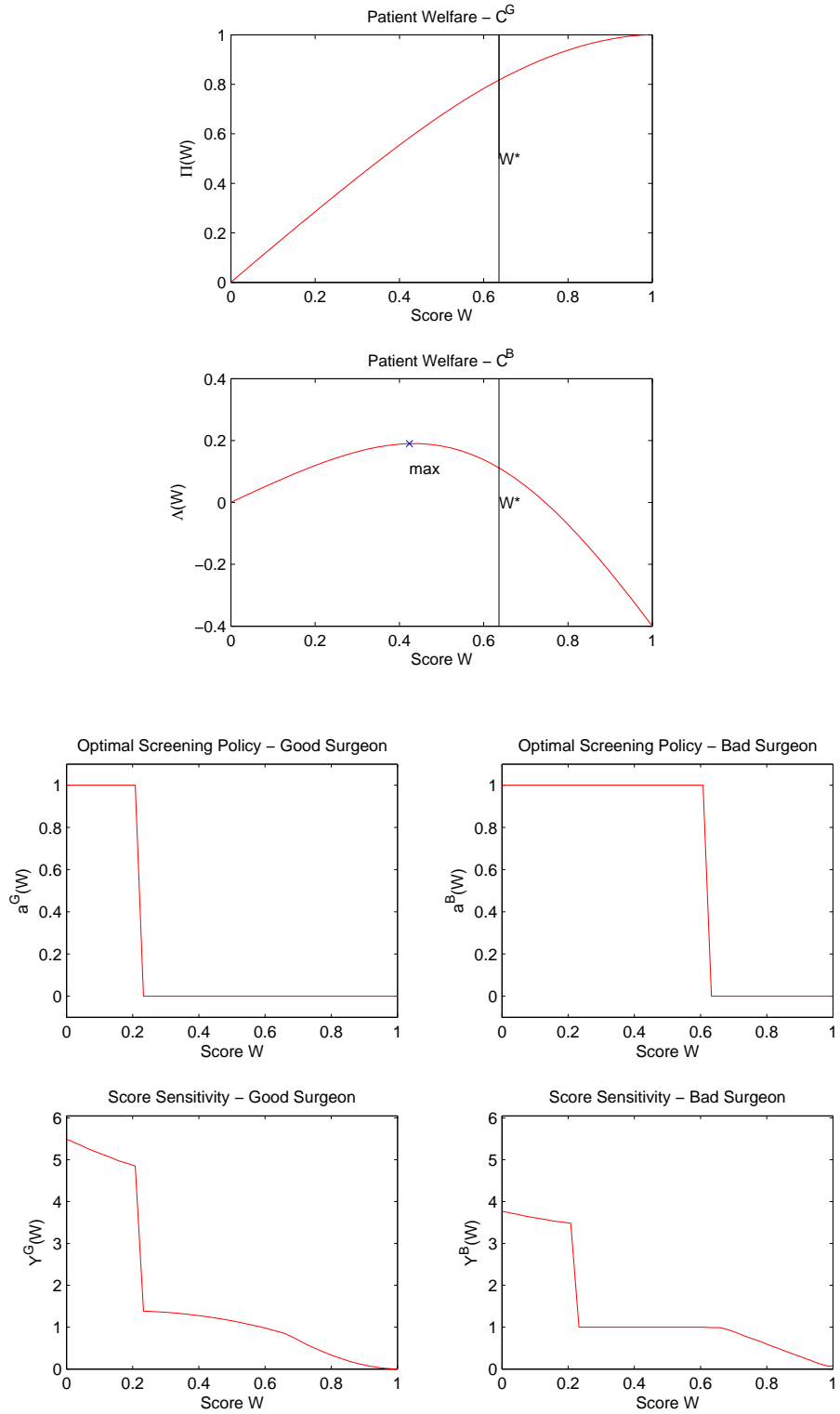
Figure 7: Example 1 ($r\sigma^2 = 0.5$, $\zeta = 0.4$, $\mu^G \in \{0.8, 1.0\}$, $\mu^B \in \{0.5, 0.9\}$, $h(\cdot, B) \in \{-0.4, 0.4\}$)
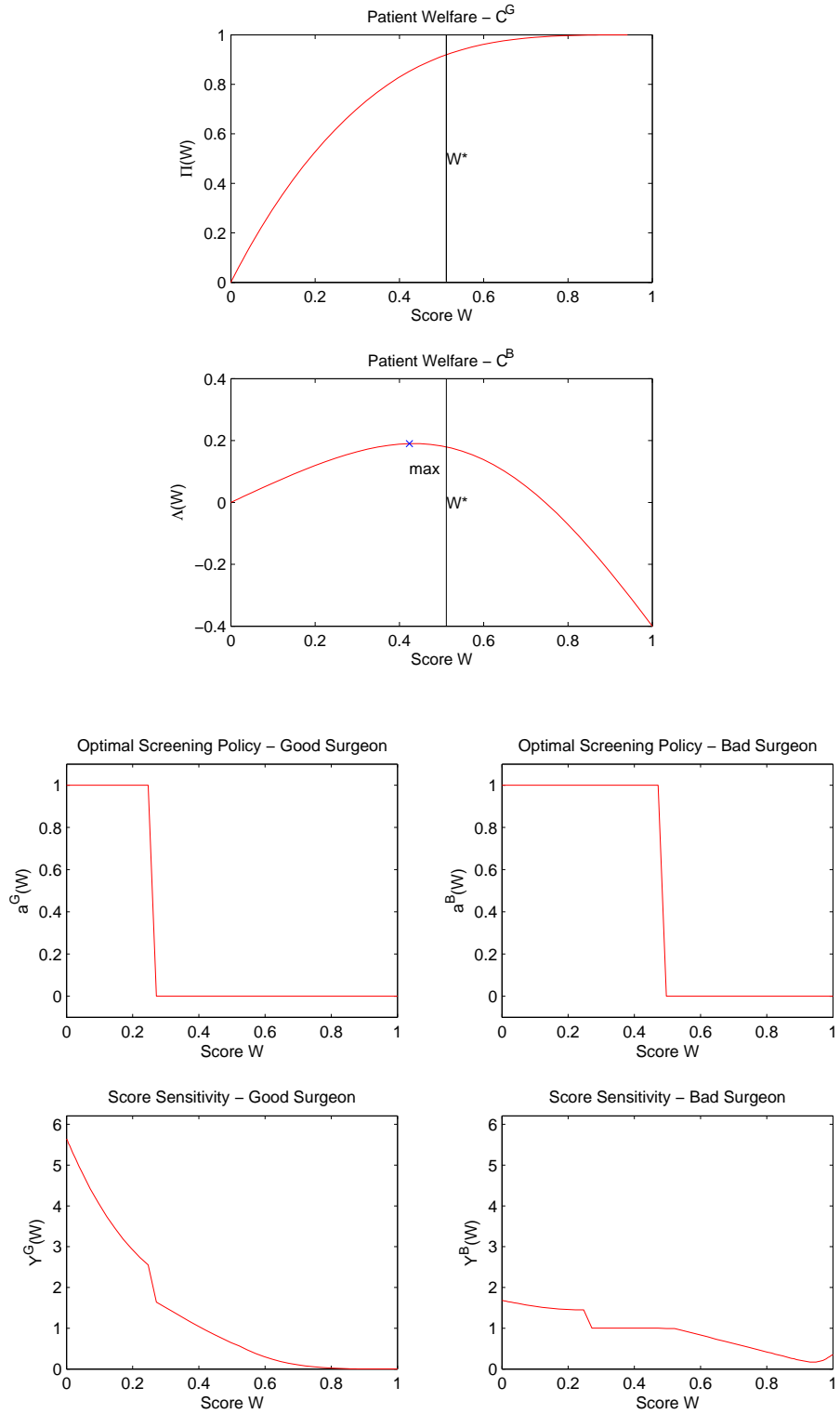
Figure 8: Example 2 ($r\sigma^2 = 0.5$, $\zeta = 0.4$, $\mu^G \in \{0.8, 1.0\}$, $\mu^B \in \{0.1, 0.5\}$, $h(\cdot, B) \in \{-0.4, 0.4\}$)
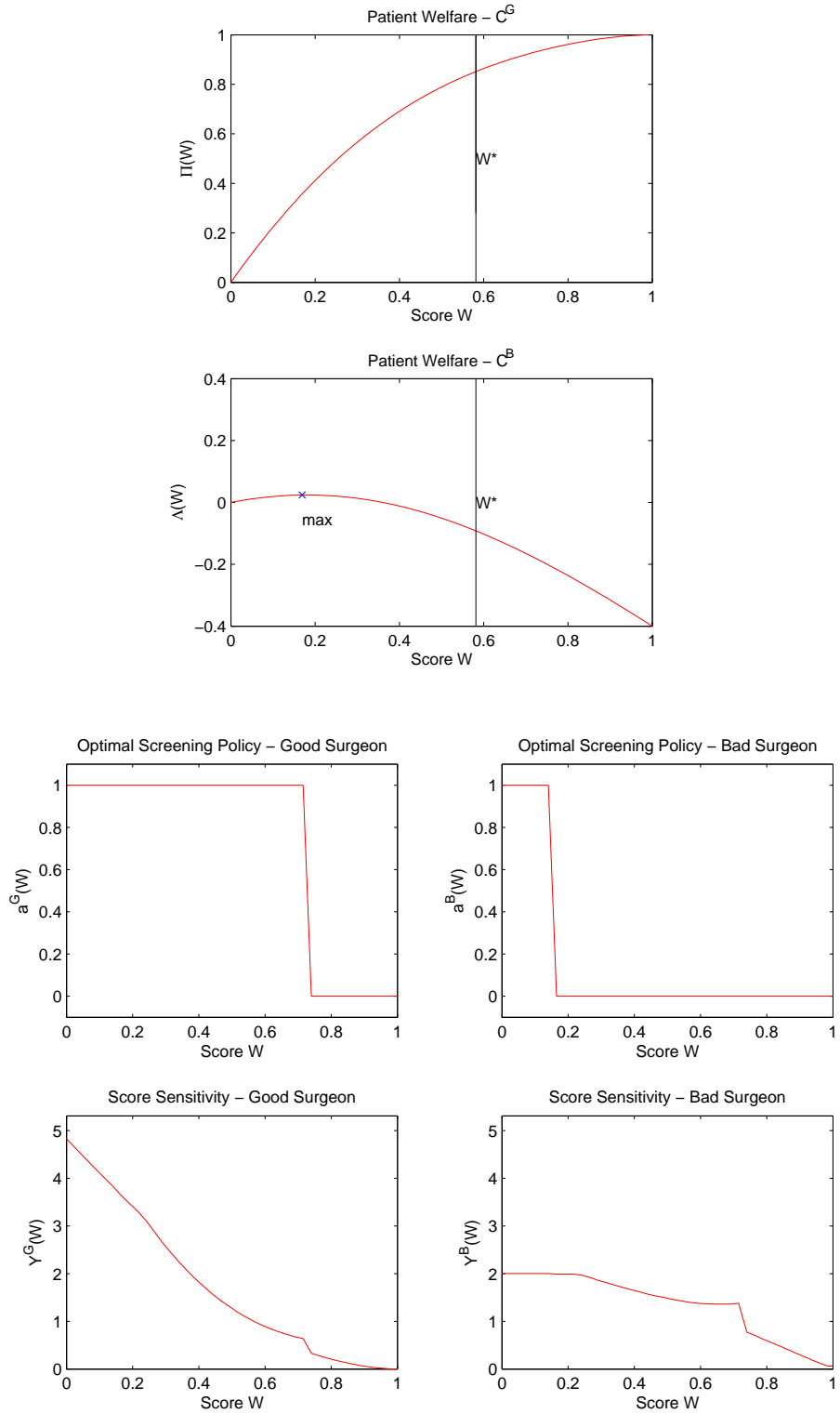
Figure 9: Example 3 ($r\sigma^2 = 0.5$, $\zeta^G = 0.1$, $\zeta^B = 0.8$, $\mu^G \in \{0.8, 1.0\}$, $\mu^B \in \{0.5, 0.9\}$, $h(\cdot, B) \in \{-0.4, 0.4\}$)
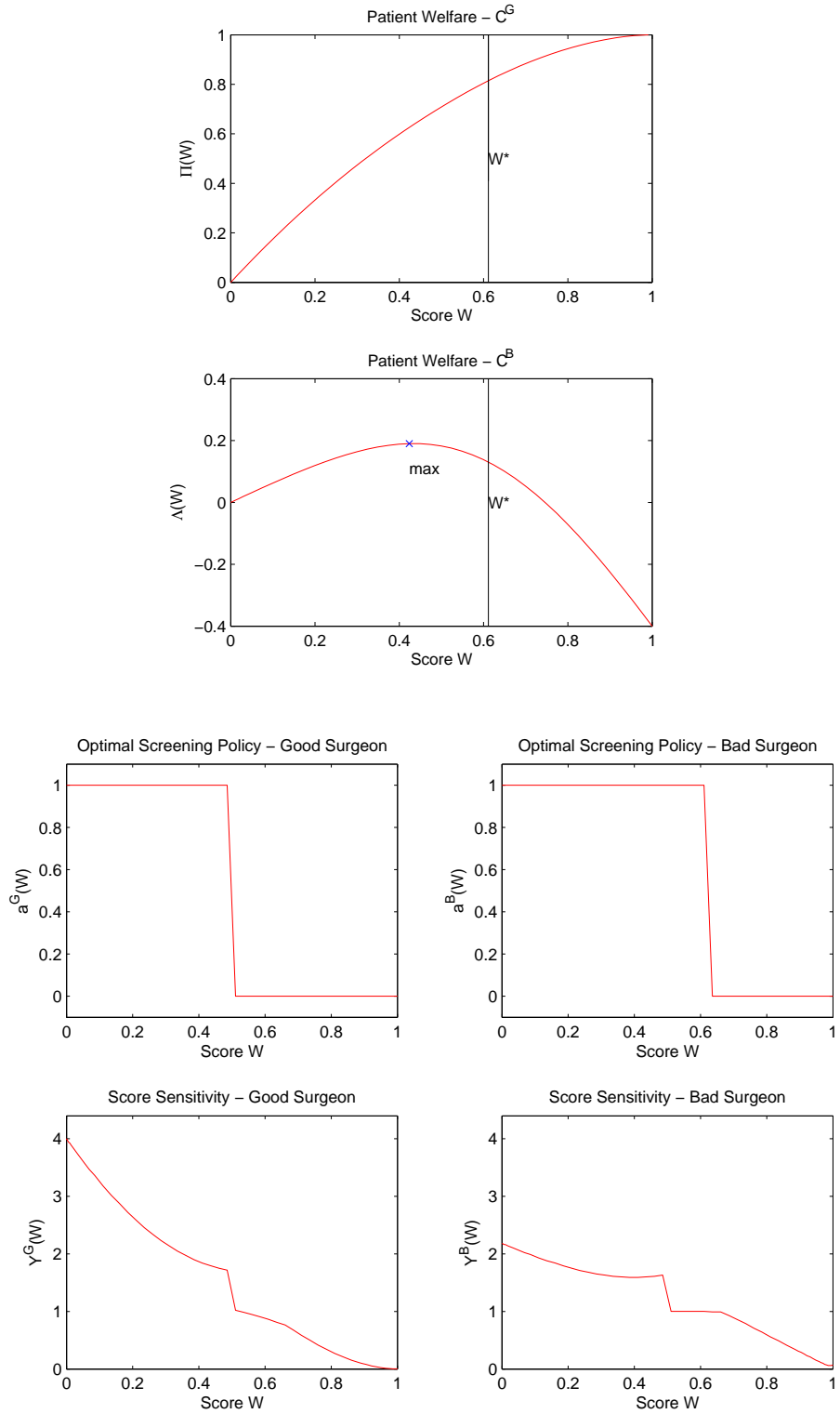
Figure 10: Example 4 ($r\sigma^2 = 0.5$, $\zeta^G = 0.4$, $\zeta^B = 0.4$, $\mu^G \in \{0.8, 1.15\}$, $\mu^B \in \{0.5, 0.9\}$, $h(\cdot, B) \in \{-0.4, 0.4\}$)

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A Appendix

A.1 Optimal Contract for the Bad Surgeon

In this section we simplify notation and let $g(a) = g(a, B)$, $\zeta = \zeta^B$, and $\lambda = \lambda^B$.

A.1.1 Proof of Lemma 2

We prove the two results in Lemma 2 separately. Let $\Lambda(W)$ be a solution to the HJB equation and its boundary conditions (HJB-BAD).

Lemma 5. $\Lambda(W)$ *is strictly concave for all* $W \in [0, 1]$.

Proof. First, we prove that either $\Lambda(W)$ is concave for all $W \in [0, 1]$, convex for all $W \in [0, 1]$, or a straight line for all $W \in [0, 1]$. We do so by showing that if $\Lambda''(W) = 0$ at any point W, then $\Lambda(W)$ must be a straight line. Suppose $\Lambda''(W) = 0$ at $\hat{W}$. Then $\Lambda(W) = \Lambda(\hat{W}) + \Lambda'(\hat{W})(W - \hat{W})$ for all W since

$$\Lambda''(W) = \min_Y \frac{\Lambda(\hat{W}) + \Lambda'(\hat{W})(W - \hat{W}) - h(a(Y), B) - \Lambda'(\hat{W})(W - g(a(Y)))}{r\sigma^2 Y^2/2}$$

is constant for all W.

At $W = 0$, it is either the case that $a(0) = 1$ or $a(0) = 0$. Suppose $a(0) = 1$. Then

$$\Lambda''(0) = \frac{-1 + \Lambda'(0)\lambda}{r\sigma^2(Y(0))^2/2}$$

Since the denominator is always positive, the sign of $\Lambda''(0)$ is determined by the numerator. In order for $\Lambda''(0) > 0$, it must be that $\Lambda'(0) > \frac{1}{\lambda}$. However, this implies that $\Lambda(W)$ will reach a point (x, y) such that $x > \lambda$ and $y > 1$, which is not feasible. For $\Lambda''(0) = 0$, then $\Lambda'(W) = \frac{1}{\lambda}$ for all W, and the boundary condition $\Lambda(1) = h(0, B)$ does not hold generically. So if $a(0) = 1$, then Λ must be strictly concave for all W.

Now suppose $a(0) = 0$. Then

$$\Lambda''(0) = \frac{-h(0, B) + \Lambda'(0)}{r\sigma^2(Y(0))^2/2}$$

For $\Lambda''(0) > 0$, it must be that $\Lambda'(0) > h(0, B)$. However, this will violate the boundary condition $\Lambda(1) = h(0, B)$. For $\Lambda''(0) = 0$, then $\Lambda'(0) = h(0, B)$, which is consistent with the boundary conditions, but given that $\Lambda''(0) < 0$ for $a(0) = 1$, it follows that $a(0) = 0$ is not a minimizer on the right hand side of the HJB equation. Therefore $a(0) = 1$ and Λ is strictly concave for all W. $\square$

Lemma 6. $\Lambda(W)$ *is unique.*

Proof. Consider equation (HJB-BAD) with initial conditions $\Lambda(0) = 0$ and $\Lambda'(0) = \Lambda_0$. Since the RHS of the equation is continuous and differentiable in $W^B, \Lambda(W^B)$ and $\Lambda'(W^B)$ for $W^B \in [0, 1]$, it follows that solutions exist, are unique, and are continuous in Λ_0. Recall from Lemma 5 that for $\Lambda_0 > \frac{1}{\lambda}$, the solution to the HJB equation is convex and the boundary condition $\Lambda(1) = h(0, B)$ does not hold because the solution will hit the boundary at $W = 1$ at too high of a point. It must be that $\Lambda_0 < \frac{1}{\lambda}$. Since Λ is concave, if $\Lambda_0 < h(0, B)$, then $\Lambda(1) = h(0, B)$ also cannot hold because the solution will hit the boundary at $W = 1$ at too low of a point. Since the solution is continuous in Λ_0, there must exist some $\Lambda_0 \in (h(0, B), \frac{1}{\lambda})$ such that $\Lambda(1) = h(0, B)$. $\square$

A.1.2 Proof of Proposition 2

Proof. (Proposition 2) Suppose $a(W) = 0$ for some $W \in [0, 1)$. Then Lemma 5 tells us that $\Lambda''(W) < 0$ so the tangent line at W passes above the point $(1, h(0, B))$. If the tangent line passes above the point $(1, h(0, B))$, then the HJB equation for $a(W) = 0$, i.e.

$$\Lambda''(W) = \frac{\Lambda(W) - h(0, B) - \Lambda'(W)(W - 1)}{r\sigma^2(Y(W))^2/2}$$

implies that $\Lambda''(W) > 0$ (since the numerator is positive), which is a contradiction. To maximize patient welfare, we want to choose the minimum sensitivity that enforces the optimal action. Hence $Y(W) = \frac{\zeta^B}{k^B}$ for all $W \in [0, 1)$. $\qquad\square$

A.1.3 Proof of Proposition 1

Lemma 5 tells us that the solution to the HJB equation is concave. Proposition 2 tells us that we can restrict attention to the following HJB equation from (HJB-BAD):

$$\Lambda''(W) = \frac{\Lambda(W) - 1 - \Lambda'(W)(W - \lambda)}{r\sigma^2(\frac{\zeta}{k^B})^2/2}$$

with boundary conditions $\Lambda(0) = 0$ and $\Lambda(1) = h(0, B)$.

First, we show how to construct contracts from a given solution to the HJB equation. We follow the proof of Proposition 3 from Sannikov (2007b) almost identically. We continue to let $g(a) = g(a, B)$, $\zeta = \zeta^B$, and $\lambda = \lambda^B$.

Lemma 7. *Consider a solution Λ of equation (HJB-BAD). Let $a : [0, 1] \to A$ and $Y : [0, 1] \to [0, \infty)$ be the minimizers in (HJB-BAD). For any initial condition $W_0 \in (0, 1)$ there is a unique weak (in the sense of probability law weak) solution to the equation*

$$dW_t = r(W_t - g(a(W_t)))dt + rY(W_t)(dX_t - \mu(a(W_t), B)dt) \tag{A.1}$$

that specifies $\{W_t\}$. The contract $C^B = (\tau^B, \mathcal{A})$ with $\mathcal{A} = \{a_t^B\}$ gives the bad surgeon continuation value W_t at any time t, and is defined by $a_t^B = a(W_t)$ and τ^B being the first time that W_t hits either 0 or 1. C^B is incentive-compatible and implies an expected payoff of W_0 to the bad surgeon and an expected patient welfare (conditional on $\theta = B$) of $\Lambda(W_0)$.

Proof. From Proposition 2, we know that $a(W_t) = 1$ for all $W_t \in [0, 1)$. Theorem 5.5.15 from Karatzas and Shreve (1991) tells us that there is a unique, weak (in the sense of probability law weak) solution to (A.1) because the drift and volatility of W_t are bounded on $[0, 1]$, and the volatility is $r\sigma\frac{\zeta}{k^B} > 0$ (notice that $dW_t = r(W_t - \lambda)dt + r\sigma\frac{\zeta}{k^B}dZ_t^{\mathcal{A}}$).

We now show that $W_t = W_t(\mathcal{A})$, where $W_t(\mathcal{A})$ is the surgeon's true continuation value from the contract $(\tau^B, \mathcal{A})$. From the representation (4.1), we can write

$$d(W_t(\mathcal{A}) - W_t) = r(W_t(\mathcal{A}) - W_t)dt + r(Y_t - Y(W_t))\sigma dZ_t^{\mathcal{A}} \implies$$

$$E_t[W_{t+s}(\mathcal{A}) - W_{t+s}] = e^{rs}(W_t(\mathcal{A}) - W_t).$$

Since $E_t[W_{t+s}(\mathcal{A}) - W_{t+s}]$ must remain bounded because both W_t and $W_t(\mathcal{A})$ are bounded, we

conclude that $W_t = W_t(\mathcal{A})$ for all $t \geq 0$. In particular, the surgeon gets $W_0 = W_0(\mathcal{A})$ from the entire contract. Note that the contract is incentive compatible because we require $a(W_t) = a(Y(W_t))$.

To see that patient welfare is $\Lambda(W_0)$, consider

$$K_t = r \int_0^t e^{-rs} h(1, B) ds + e^{-rt} \Lambda(W_t).$$

By Ito's lemma, the drift of K_t is

$$re^{-rt} \left((h(1, B) - \Lambda(W_t)) + \Lambda'(W_t)(W_t - \lambda) + r\sigma^2 (\frac{\zeta}{kB})^2 \frac{\Lambda''(W_t)}{2} \right).$$

By the HJB equation, the value of this expression is always 0. Therefore, K_t is a bounded martingale, so $E[K_t] = K_0$ for all $t \geq 0$. Since $K_0 = \Lambda(W_0)$, we conclude that expected patient welfare is $\Lambda(W_0)$. $\qquad \square$

Now we show that any incentive compatible contract that gives the bad surgeon a payoff of W achieves patient welfare at most $\Lambda(W)$. So $\Lambda(W)$ is the upper bound on patient welfare.

Lemma 8. *Consider a solution Λ of equation (HJB-BAD). Any incentive-compatible contract $C = \{\tau, \mathcal{A}\}$ achieves patient welfare at most $\Lambda(W_0(\mathcal{A}))$.*

Proof. Given an incentive-compatible contract $C = \{\tau, \mathcal{A}\}$, denote the surgeon's continuation value by $W_t = W_t(\mathcal{A})$ (as in (4.1)). Define

$$K_t = r \int_0^t e^{-rs} h(a_t, B) ds + e^{-rt} \Lambda(W_t)$$

where $\{a_t\} = \mathcal{A}$. By Ito's Lemma, the drift of K_t is

$$re^{-rt} \left((h(a_t, B) - \Lambda(W_t)) + \Lambda'(W_t)(W_t - g(a_t)) + r\sigma^2 (Y_t)^2 \frac{\Lambda''(W_t)}{2} \right).$$

We just need to show that the drift is non-positive. Consider $a_t = 1$ and $a_t = 0$.

Suppose $a_t = 1$. Then $Y_t \geq \frac{\zeta}{kB}$. Since $\Lambda''(W) \leq 0$, it follows from (HJB-BAD) that the drift of K_t is non-positive. Now suppose $a_t = 0$. Then, the drift of K_t is

$$re^{-rt} \left(1 - \Lambda(W_t) + \Lambda'(W_t)(W_t - \Lambda) + r\sigma^2 (Y_t)^2 \frac{\Lambda''(W_t)}{2} \right)$$

which is strictly negative. $\qquad \square$

Lemma 9. *For any t, when the continuation value to the bad surgeon is W_t, the continuation expected patient welfare is $\Lambda(W_t)$. In other words, the optimal contract stays on the curve Λ.*

Proof. A short argument verifies that the continuation value for patient welfare always stays on the curve Λ. The key idea, as we have argued in Section 4.1, is that in an optimal contract the agent's continuation value completely summarizes the past history. Therefore, after any history, the principal's payoff is maximized if and only if the continuation contract is optimal given the continuation value.

Consider the optimal contract. Suppose that for some continuation value W_t for the bad surgeon, the continuation value for patient welfare is $\hat{\Lambda} \neq \Lambda(W_t)$. If $\hat{\Lambda} > \Lambda(W_t)$, then Lemma 8 is violated because we have a contract, namely the continuation contract at W_t, which achieves a higher patient welfare than $\Lambda(W_t)$. Now suppose $\hat{\Lambda} < \Lambda(W_t)$. Then we can simply replace the continuation contract at W_t with the optimal contract that achieves $\Lambda(W_t)$ and strictly improve patient welfare, thus violating the optimality of the original contract. Incentive compatibility is maintained because we have substituted a continuation contract with the same continuation value to the bad surgeon. $\square$

A.2 Optimal Contract for the Good Surgeon

Although the proofs in this section are inspired by the techniques developed in Sannikov (2007b), they require novel applications of continuous-time techniques due to the presence of adverse selection. Because incentives of two types of agents, i.e. an agent and a deviating agent, are being considered, the sensitivity Y appears not only in the denominator but in the numerator as well. As a result, the sensitivity that minimizes the HJB equation for a given action is not always the minimum sensitivity that enforces that action.

A.2.1 Incentive Compatibility

Lemma 10. *For a given pair of strategies $\mathcal{A}^G$ and $\mathcal{A}^B$ for the bad and good type surgeon, let $\{Y_t^G\}$ and $\{Y_t^B\}$ be associated processes in the representation $W_t^G(\mathcal{A}^G)$ and $W_t^B(\mathcal{A}^B)$. A pair of strategies (a^G, a^B) is optimal if and only if*

$$a^G \in \arg\max_a g(a, G) + Y^G \mu(a, G) \ \text{ and} \tag{A.2}$$

$$a^B \in \arg\max_a g(a, B) + Y^B \mu(a, B) \tag{A.3}$$

Proof. The proofs for both the good surgeon's optimal strategy $\mathcal{A}^G$ and the bad surgeon's optimal strategy $\mathcal{A}^B$ are essentially identical to the proof of Proposition 2 in Sannikov (2007b), and we do not repeat the analysis here.

$\square$

A.2.2 Proof of Lemma 3

Lemma 11. *The RHS of the HJB equation (HJB-GOOD) is continuous and differentiable in Y.*

Proof. First, consider the case that $\Pi'(W^B) > \frac{\zeta^G k^B}{\zeta^B k^G}$. For $0 < Y < \frac{\zeta^G}{\Pi'(W^B)k^G}$, we have $a^G(Y) = 0$ and $a^B(Y) = 0$, and the RHS is continuous. When $Y = \frac{\zeta^G}{\Pi'(W^B)k^G}$, the good type is indifferent between $a^G(Y) = 1$ and $a^G(Y) = 0$ while the bad type chooses $a^B(Y) = 0$. Since the denominator of the RHS doesn't depend on $a^G(Y)$, we only need to compare the numerators when $a^G(Y) = 0$ and $a^G(Y) = 1$, respectively:

$$\Pi(W^B) - g(0, G) - \Pi'(W^B)(W^B - g(0, B)) + Y(\mu(0, G) - \mu(1, B)) \ \text{ and}$$
$$\Pi(W^B) - g(1, G) - \Pi'(W^B)(W^B - g(0, B)) + Y(\mu(1, G) - \mu(1, B)).$$

Subtracting the second from the first yields $-\zeta^G + \frac{\zeta^G}{k^G}k^G = 0$. For $\frac{\zeta^G}{\Pi'(W^B)k^G} < Y < \frac{\zeta^B}{k^B}$, we have $a^G(Y) = 1$ and $a^B(Y) = 0$. At $Y = \frac{\zeta^B}{k^B}$, the bad type is indifferent between $a^B(Y) = 0$ and $a^B(Y) = 1$. Again, comparing numerators we have

$$\Pi(W^B) - g(1, G) - \Pi'(W^B)(W^B - g(0, B)) + Y(\mu(0, G) - \mu(0, B)) \text{ and}$$
$$\Pi(W^B) - g(1, G) - \Pi'(W^B)(W^B - g(1, B)) + Y(\mu(0, G) - \mu(1, B)).$$

Subtracting the second from the first yields $\Pi'(W^B)\zeta^B - \Pi'(W^B)\frac{\zeta^B}{k^B}k^B = 0$. So we have our result.
$\square$

Lemma 12. *Given initial conditions $\Pi(0) = 0$ and $\Pi'(0) = \Pi_0$, there exists a solution to (HJB-GOOD) that is unique and continuous in Π_0. Moreover, initial conditions with $\Pi''(0) < 0$ result in a concave solution.*

Proof. Consider the HJB equation (HJB-GOOD) with initial conditions $\Pi(0) = 0$ and $\Pi'(0) = \Pi_0$. Since the RHS is continuous and differentiable in all its arguments over all Y, it is Lipschitz continuous. It also satisfies linear growth conditions. Therefore, solutions exist, are unique and are continuous in Π_0.

Suppose $\Pi(W)$ is concave at some point. Then $\Pi''(W) < 0$ for all $W \in [0, 1]$. The reason is that if $\Pi''(W) = 0$ at any point, say $\hat{W}$, it follows that $\Pi(W)$ must be a straight line, i.e. $\Pi(W) = \Pi(\hat{W}) + \Pi'(\hat{W})(W - \hat{W})$ for all W since

$$\min_Y \frac{\Pi(\hat{W}) + \Pi'(\hat{W})(W - \hat{W}) - g(a^G(Y), G) - \Pi'(\hat{W})(W - g(a^B(Y), B)) + Y(\mu(a^G(Y), G) - \mu(a^B(Y), B)))}{r\sigma^2 Y^2/2}$$

takes the same value for all W. A corollary is that if $\Pi''(W) > 0$ at any point, then $\Pi''(W) > 0$ for all W.
$\square$

Lemma 13. *There exists a unique solution $\Pi(W)$ to the HJB function that satisfies the boundary conditions $\Pi(0) = 0$ and $\Pi(1) = 1$. Moreover, $\Pi(W)$ is strictly increasing and strictly concave for all $W \geq 0$.*

Proof. Suppose a solution exists. First we show it is unique. Consider two solutions Π and $\bar{\Pi}$ of the HJB equation where Π satisfies $\Pi(0) = 0$ and $\Pi'(0) = \Pi_0 \geq 1$ and $\bar{\Pi}$ satisfies $\bar{\Pi}(0) = 0$ and $\bar{\Pi}'(0) = \Pi_1 > \Pi_0$. We claim that $\bar{\Pi}'(W) > \Pi'(W)$ for all $W > 0$, which implies $\bar{\Pi}(W) > \Pi(W)$ for all $W > 0$ (and thus the solution that satisfies both boundary conditions must be unique).

Suppose it is not true that $\bar{\Pi}'(W) > \Pi'(W)$ for all $W > 0$. Let $\hat{W}$ be the smallest W such that $\bar{\Pi}'(W) = \Pi'(W)$. Then $\bar{\Pi}'(W) > \Pi'(W)$ for all $0 < W < \hat{W}$ and it must be that $\bar{\Pi}(\hat{W}) > \Pi(\hat{W})$. Look at the HJB equation at $\hat{W}$ for both $\bar{\Pi}$ and Π. Let Y be the minimizer in the HJB equation for $\bar{\Pi}$ at that point. Since the derivatives are the same for both functions at this point, the optimal $a^G(Y)$ and $a^B(Y)$ are the same, and the only differences in the two equations is $\Pi(\hat{W})$ versus $\bar{\Pi}(\hat{W})$. It follows then that $\hat{\Pi}''(\hat{W}) > \Pi''(\hat{W})$, which is inconsistent. So the claim must be true.

Next, we show that the solution exists. From Lemma 12, which tells us that solutions are unique and continuous in Π_0, there exists a $\Pi_0 > 1$ such that the unique solution to the HJB equation with initial conditions $\Pi(0) = 0$ and $\Pi'(0) = \Pi_0$ satisfies $\Pi(1) = 1$. The reason is that as we increase Π_0 from 1, the corresponding solutions strictly dominate each other in the sense that $\Pi^A(W) > \Pi^B(W)$ for all $W > 0$ if $\Pi^A(W)$ is the solution for the HJB equation with a higher initial condition Π_0

than that for $\Pi^B(W)$. So there must exist a unique Π_0 such that the corresponding solution exactly satisfies the boundary conditions.

Now we show the solution Π is strictly concave for all $W > 0$. Let $\hat{\Pi}$ be the solution to the HJB equation with initial conditions $\Pi(0) = 0$ and $\Pi'(0) = 1$. $\hat{\Pi}$ is strictly concave. Looking at the HJB equation (HJB-GOOD), observe that for $Y = \min\{\frac{\zeta^G}{k^G}, \frac{\zeta^B}{k^B}\}$, we have $a^G(Y) = 0$ and $a^B(Y) = 0$, which imply the RHS is at most $\frac{-(\mu(0,G)-\mu(0,B))}{r\sigma^2 Y^2/2} < 0$. Therefore $\hat{\Pi}''(0) < 0$ and by Lemma 12 $\Pi(W)$ is strictly concave for all $W > 0$. Now suppose the solution Π is not concave. Then it must be convex (it cannot be a straight line). But a convex solution must pass below $\hat{\Pi}$ and this contradicts our first claim that $\bar{\Pi}'(W) > \Pi'(W)$ for all $W > 0$. Therefore, Π must be strictly concave.

Finally, since $\Pi(W)$ is strictly concave, $\Pi(0) = 0$, $\Pi(1) = 1$ and $\Pi(W) \leq 1$ for all W, it follows that Π must always be strictly increasing. $\qquad\square$

A.2.3   Proof of Proposition 3

Lemma 14. *Consider a solution Π of equation (HJB-GOOD). Let $a^G : [0,1] \to A$, $a^B : [0,1] \to A$ and $Y : [0,1] \to [0,\infty)$ be the minimizers in (HJB-GOOD). For any initial condition $W_0 \in (0,1)$, there exists a unique weak (in the sense of probability law weak) solution to the equation*

$$dW_t = r(W_t - g(a^B(W_t), B) + Y(W_t)(\mu(a^G(W_t), G) - \mu(a^B(W_t), B)))dt + rY(W_t)\sigma dZ_t^G \quad \text{(A.4)}$$

that specifies $\{W_t\}$. The contract $C^G = \{\tau^G, \mathcal{A}^G, \mathcal{A}^B\}$ with $\mathcal{A}^G = \{a_t^G\}$ and $\mathcal{A}^B = \{a_t^B\}$ gives the deviating bad surgeon continuation value W_t at any time t, and is defined by $a_t^G = a^G(W_t)$, $a_t^B = a^B(W_t)$, and τ^G being the first time that W_t hits either 0 or 1. C^G is incentive-compatible and implies an expected payoff of W_0 to the bad surgeon and an expected payoff of $\Pi(W_0)$ to the good surgeon.

Proof. Theorem 5.5.15. of Karatzas and Shreve (1991) tells us that there is a unique weak solution of (A.4) (in the sense of probability law) if the drift and volatility of W_t are bounded, and the volatility is strictly positive. We just need to show that $Y(W_t) > 0$ and is finite. Suppose $Y(W_t) = 0$. Then $a^G(W_t) = 0$ and $a^B(W_t) = 0$, and the right hand side of the HJB equation (HJB-GOOD) would have $\Pi(W) - 1 - \Pi'(W)(W - 1)$ as the numerator. But notice that $\Pi'(W) > \frac{1-\Pi(W)}{1-W}$ for all W since Π is strictly concave. Therefore the numerator is positive, implying $\Pi''(W) > 0$, which is a contradiction. So $Y(W_t) > 0$. Next we show that $Y(W_t)$ is bounded from above. Suppose $Y(W_t)$ is large enough so that $a^G(W_t) = 1$ and $a^B(W_t) = 1$, and the numerator on the RHS of (HJB-GOOD) becomes $\Pi(W_t) - \lambda^G - \Pi'(W_t)(W_t - \lambda^B) - \Pi'(W_t)Y(W_t)(\mu(1, G) - \mu(1, B))$. Is it ever the case that the RHS is strictly decreasing in Y_t so that the minimizing choice $Y(W_t)$ is unbounded? This is only the case if $\Pi''(W) > 0$, which is again a contradiction.

Clearly C^G is incentive compatible since we require it in the solution to the HJB equation. To show that $W_t = W_t(\mathcal{A}^B)$ and that C^G implies an expected payoff of W_0 to the bad surgeon and an expected payoff of $\Pi(W_0)$ to the good surgeon, we follow identical steps to the second half of the proof of Lemma 7 and do not repeat the analysis here. $\qquad\square$

Lemma 15. *Consider a solution Π of equation (HJB-GOOD). Any incentive-compatible contract $C = \{\tau, \mathcal{A}^G, \mathcal{A}^B\}$ achieves expected discounted patient welfare at most $\Pi(W_0(\mathcal{A}^B))$.*

Proof. Given an incentive-compatible contract $C = \{\tau, \mathcal{A}^G, \mathcal{A}^B\}$, denote the bad surgeon's continu-

ation value by $W_t = W_t(\mathcal{A}^B)$ (as in (5.2)). Let $\mathcal{A}^B = \{a_t^B\}$ and $\mathcal{A}^G = \{a_t^G\}$. Define

$$K_t = r \int_0^t e^{-rs} g(a_t^G, G) ds + e^{-rt} \Pi(W_t).$$

By Ito's Lemma, the drift of K_t is

$$re^{-rt} \left(g(a_t^G, G) - \Pi(W_t) + \Pi'(W_t)(W_t - g(a_t^B, B) + Y_t(\mu(a_t^G, G) - \mu(a_t^B, B))) + \frac{\Pi''(W_t)}{2} r\sigma^2 Y_t^2 \right).$$

We just need to show that the drift is non-positive. By definition of the HJB equation, however, the drift must be non-positive because

$$\Pi(W_t) \geq g(a_t^G, G) + \Pi'(W_t)(W_t - g(a_t^B, B) + Y_t(\mu(a_t^G, G) - \mu(a_t^B, B))) + \frac{\Pi''(W_t)}{2} r\sigma^2 Y_t^2$$

for all Y_t and corresponding incentive-compatible pairs (a_t^G, a_t^B).

It follows that K_t is a bounded supermartingale until the stopping time, and hence the expected patient welfare a time 0 is less than or equal to $E^{\mathcal{A}^G}[K_t] \leq K_0 = \Pi(W_0)$. $\qquad \square$

The final step is to show that the continuation value of the good surgeon (i.e. expected patient welfare) stays on the curve $\Pi(W_t)$ for any time t almost everywhere when the bad surgeon's continuation value is W_t. To do so, we use an "escape argument" along the lines of those made in Sannikov (2007c) and Faingold and Sannikov (2007).

Lemma 16. *Consider a solution Π of equation (HJB-GOOD). Consider the corresponding contract constructed according to Lemma 14 with initial condition W_0^B. At any time t, the continuation value of the good surgeon is equal to $\Pi(W_t^B)$ (i.e. stays on the curve Π).*

Proof. Suppose the claim is not true and that in the optimal contract the continuation value of the good surgeon follows a process Π_t such that $\Pi_t > \Pi(W_t^B)$ for some t. From the Martingale Representation Theorem we know there exists a process $\{\gamma_t\}$ such that Π_t satisfies

$$d\Pi_t = r(\Pi_t - g(a_t^G, G))dt + r\sigma\gamma_t dZ_t^{\mathcal{A}^G}.$$

Let $L_t = \Pi_t - \Pi(W_t)$. The drift of L_t can be written as $rL_t + d(a_t^G, a_t^B, W_t)$ where $d(a_t^G, a_t^B, W_t)$ is equal to

$$r(\Pi(W_t) - g(a_t^G, G)) - r\Pi'(W_t)(W_t - g(a_t^B, B) + Y(W_t)(\mu(a_t^G, G) - \mu(a_t^B, B))) - \frac{\Pi''(W_t)}{2} r^2\sigma^2(Y(W_t))^2$$

and the volatility $v(\gamma_t, W_t)$ is equal to

$$r\sigma(\gamma_t - \Pi'(W_t)Y(W_t)).$$

Incentive compatibility requires that

$$a_t^G \in \arg\max_a g(a, G) + \gamma_t \mu(a, G) \tag{A.5}$$

We want to show that if $L_{\hat{t}} > 0$ for some $\hat{t}$, then L_t grows arbitrarily large with positive probability, which then leads to a contradiction since both Π_t and $\Pi(W_t^B)$ are bounded processes. To do so,

proving the following claim is sufficient: For every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $t \geq 0$ either (a) the drift of L_t is greater than $rL_t - \epsilon$ or (b) the absolute value of the volatility of L_t is greater than δ.

Suppose $E_t > 0$ for some t. There exists constants $c_1, c_2 > 0$ such that $|r\sigma\gamma_t - r\sigma\Pi'(W_t)Y_t| > c_2$ for all $|\gamma_t| > c_1$ since $\Pi'(W)Y(W_t)$ is bounded ($\Pi'(W)$ is bounded by initial condition $\Pi'(0) = \Pi_0$ and we have argued in the proof of Lemma 14 that $Y(W_t)$ is bounded).

Fix some $\epsilon > 0$. Consider the set Φ of tuples $(a^G, a^B, W, \gamma) \in A \times A \times [0,1] \times \mathbb{R}$ with $|\gamma| \leq c_1$ such that a^G, a^B are incentive compatible and $d(a^G, a^B, W) \leq -\epsilon$. Since d is a continuous function and Φ is a closed subset of the compact set $\{(a^G, a^B, W, \gamma) \in A \times A \times [0,1] \times \mathbb{R} \ : \ |\gamma| \leq c_1\}$, it follows that Φ is compact.

Notice that $|v(\gamma_t, W_t)|$ is also continuous. Hence it achieves its minimum η on Φ. We claim $\eta > 0$. Suppose $\eta = 0$. Then we argue that $d(a^G, a^B, W_t) = 0$, which would contradict our definition of the set Φ. When the volatility of L_t is exactly 0, then $\gamma_t = \Pi'(W_t)Y(W_t)$ and therefore

$$a_t^G \in \arg\max_a g(a, G) + \Pi'(W_t)Y(W_t)\mu(a, G)$$

i.e. $a_t^G = a^G(W_t)$ and therefore $d(a^G, a^B, W_t) = 0$. So $\eta > 0$ and $|v(\gamma_t, W_t)| \geq \eta$ on Π. Then it follows that for all $(a^G, a^B, W, \gamma) \in A \times A \times [0,1] \times \mathbb{R}$ that satisfy incentive compatibility constraints (A.5), either $d(a^G, a^B, W_t) > -\epsilon$ or $|v(\gamma_t, W_t)| \geq \min\{c, \eta\} > 0$. So we have proven the claim and we have our contradiction.

An identical argument leading to a contradiction holds if $\Pi_t < \Pi(W_t)$ for some t. $\qquad\square$

A.3 Properties of the Good Surgeon's Contract

A.3.1 Proof of Proposition 4

To prove the first part of the proposition, for the good surgeon's optimal action policy, proving the following lemma is sufficient.

Lemma 17. If $a^G(W) = 0$ for some W, then $a^G(W') = 0$ for all $W' > W$.

Proof. Consider some point W at which $a^G(W) = 0$. Let $Y = Y(W)$ be the sensitivity in the optimal solution at point W, which enforces $a^G(Y) = 0$ and $a^B(Y) = a \in A = \{0,1\}$. It follows from the HJB equation that for all Y' such that Y' enforces $a^G(Y') = 1$ (and hence $Y' > Y$), we have

$$\frac{\Pi(W) - 1 - \Pi'(W)(W - g(a,B) + Y(\mu(0,G) - \mu(a,B))}{r\sigma^2 Y^2/2}$$

$$\leq \frac{\Pi(W) - \lambda^G - \Pi'(W)(W - g(a^B(Y'), B) + Y'(\mu(1,G) - \mu(a^B(Y'), B))}{r\sigma^2 (Y')^2/2}. \tag{A.6}$$

We want to show that for any $W' > W$, it is strictly better to use Y (which must also enforce $a^G(Y) = 0$ at W' since $\Pi'(W') < \Pi'(W)$) than any Y' that enforces $a^G(Y') = 1$. Note that if Y' enforces $a^G(Y') = 1$ at W', then it also enforces $a^G(Y') = 1$ at W (again since $\Pi'(W') < \Pi'(W)$). Also Y and Y' enforce the same a^B at each point. So we just need to show that

$$\frac{\Pi(W') - 1 - \Pi'(W')(W' - g(a,B) + Y(\mu(0,G) - \mu(a,B))}{r\sigma^2 Y^2/2}$$

$$\leq \frac{\Pi(W') - \lambda^G - \Pi'(W')(W' - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))}{r\sigma^2(Y')^2/2}. \tag{A.7}$$

Notice that if the right hand side of (A.7) is positive, the result follows trivially. Therefore we only need to consider $\Pi(W) - \lambda^G < \Pi'(W)(W - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))$ (because otherwise it implies $\Pi(W') - \lambda^G > \Pi'(W')(W' - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))$, i.e. the RHS of (A.7) is positive). So we can rewrite (A.6), which we know to be true, as

$$\frac{\Pi(W) - 1 - \Pi'(W)(W - g(a, B) + Y(\mu(0, G) - \mu(a, B))}{\Pi(W) - \lambda^G - \Pi'(W)(W - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))} \geq \frac{Y^2}{(Y')^2}. \tag{A.8}$$

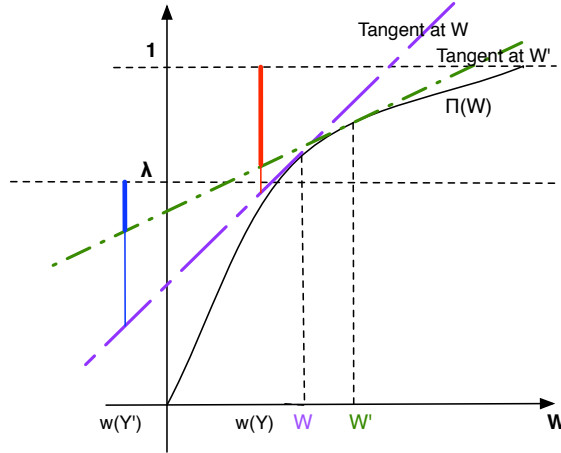(A.7) follows if this inequality holds:

$$\frac{\Pi(W') - 1 - \Pi'(W')(W' - g(a, B) + Y(\mu(0, G) - \mu(a, B))}{\Pi(W') - \lambda^G - \Pi'(W')(W' - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))}$$
$$\geq \frac{\Pi(W) - 1 - \Pi'(W)(W - g(a, B) + Y(\mu(0, G) - \mu(a, B))}{\Pi(W) - \lambda^G - \Pi'(W)(W - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))}$$

which we can re-write with positive numerator and denominator:

$$\frac{1 - \Pi(W') + \Pi'(W')(W' - g(a, B) + Y(\mu(0, G) - \mu(a, B))}{\lambda^G - \Pi(W') + \Pi'(W')(W' - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))}$$
$$\geq \frac{1 - \Pi(W) + \Pi'(W)(W - g(a, B) + Y(\mu(0, G) - \mu(a, B))}{\lambda^G - \Pi(W) + \Pi'(W)(W - g(a^B(Y'), B) + Y'(\mu(1, G) - \mu(a^B(Y'), B))}$$

Let $w(Y) = g(a, B) - Y(\mu(0, G) - \mu(a, B))$ and $w(Y') = g(a^B(Y'), B) - Y'(\mu(1, G) - \mu(a^B(Y'), B))$. We can interpret that equation in the following figure:

The fraction at W' is the ratio between the dark red line and the dark blue line. Meanwhile, the fraction at W is the ratio between the light red line and the light blue line. Since the slope of the tangent line at W' is smaller than the slope of the tangent line at W, the ratio is smaller at W'. So we are done. $\square$

To prove the remaining claims in Proposition 4, we introduce some notation. Define $Y^*(a^B, a^G)$ as the unconstrained value of Y for which the RHS of the HJB equation reaches its extremum given an action pair $(a^B, a^G) \in \{0,1\} \times \{0,1\}$. Writing out the FOC and setting it equal to 0, we get that the extremum is reached at

$$Y^*(a^B, a^G) = 2 \frac{\Pi(W) - g(a^G, G) - \Pi'(W)(W - g(a^B, B))}{\Pi'(W)(\mu(a^G, G) - \mu(a^B, B))}.$$

$Y = 0$ is also a possibility but there the RHS of the HJB equation is undefined (i.e. it blows up). If $\mu(a^G, G) > \mu(a^B, B)$, the RHS of the HJB equation is increasing in Y so long as

$$Y > 0 \quad \text{and} \quad Y > Y^*(a^B, a^G), \quad \text{or} \quad Y < 0 \quad \text{and} \quad Y < Y^*(a^B, a^G)$$

Thus, when $Y^*(a^B, a^G) > 0$, the minimum of the RHS of the HJB equation is reached at $Y^*(a^B, a^G)$. Otherwise, the function asymptotes to $-\infty$ as Y approaches 0 (so the minimizing Y is the smallest Y).

When $\mu(a^G, G) < \mu(a^B, B)$ (which is only possible at $(1,0)$ when there is overlap, i.e. $\Delta\mu_0 = \mu(0, G) - \mu(0, B) < k^B$), the RHS of the HJB equation is increasing in Y when

$$0 < Y < Y^*(a^B, a^G), \quad \text{or} \quad 0 > Y > Y^*(a^B, a^G)$$

Thus, when $Y^*(a^B, a^G) > 0$, the function asymptotes to $-\infty$ as Y approaches 0 (so the minimizing Y is the smallest Y). When $Y^*(a^B, a^G) < 0$, the minimum of the RHS is reached at $Y^*(a^B, a^G)$ and the minimizing Y is the biggest feasible Y.

We can rewrite $Y^*(a^B, a^G)$ as follows:

$$Y^*(a^B, a^G) = 2 \frac{\frac{\Pi(W) - 1 - \Pi'(W)(W-1)}{\Pi'(W)} + a^G \frac{\zeta^G}{\Pi'(W)} - a^B \zeta^B}{(\Delta\mu_0 - a^B k^B + a^G k^G)}.$$

Notice that as $W \to 1$, the term on top $\frac{\Pi(W) - 1 - \Pi'(W)(W-1)}{\Pi'(W)} \to 0$. Furthermore, as $W \to 1$, $\Pi'(W) < 1$.

The following table summarizes the minimizing values of Y for each action pair (a^B, a^G). First, suppose there is no overlap, so $\Delta\mu_0 > k^B$:

(a^B, a^G)	Feasible, so $Y^*(a^B, a^G)$	Infeasible, but $Y^*(a^B, a^G) > 0$	$Y^*(a^B, a^G) < 0$
$(0,0)$	$0 \leq Y^*(0,0) \leq \frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G}$	$\min\{\frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G}\}$	N/A
$(1,0)$	$\frac{\zeta^B}{k^B} \leq Y^*(1,0) \leq \frac{\zeta^G}{\Pi'(W)k^G}$	$\frac{\zeta^B}{k^B}$ or $\frac{\zeta^G}{\Pi'(W)k^G}$	$\frac{\zeta^B}{k^B}$
$(0,1)$	$\frac{\zeta^G}{\Pi'(W)k^G} \leq Y^*(0,1) \leq \frac{\zeta^B}{k^B}$	$\frac{\zeta^B}{k^B}$ or $\frac{\zeta^G}{\Pi'(W)k^G}$	N/A
$(1,1)$	$\frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G} \leq Y^*(1,1)$	$\max\{\frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G}\}$	$\max\{\frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G}\}$

Now suppose there is overlap, so $\Delta\mu_0 < k^B$:

(a^B, a^G)	$If Y^*(a^B, a^G) > 0$	$If Y^*(a^B, a^G) < 0$
$(1,0)$	$\frac{\zeta^B}{k^B}$	$\frac{\zeta^G}{\Pi'(W)k^G}$

Lemma 18. *There exists an $\epsilon \in (0,1]$ such that for all $W \in [1 - \epsilon, 1]$, an optimal action strategy is*

$a^G(W) = 0$ *and* $a^B(W) = 0$, *i.e. the good surgeon and the deviating bad surgeon both do not screen patients.*

Proof. Suppose W is close to 1. Let's compare the feasible minimum achieved by the RHS of the HJB equation for the action pair $(0,0)$ and the feasible minimum achieved by the RHS of the HJB equation for the action pair $(1,1)$. We know that $Y^*(0,0) > 0$ and $Y^*(0,0)$ is feasible for all W close to 1. Suppose $Y^*(1,1) > 0$ so that $Y^*(1,1)$ defines a minimum. Let's compare $(0,0)$ and $(1,1)$ at $Y = Y^*(1,1)$. The RHS of the HJB equation for $(1,1)$ is

$$\frac{\Pi(W) - 1 - \Pi'(W)(W-1) + \zeta^G - \Pi'(W)\zeta^B - \Pi'(W)Y(\Delta\mu_0 - k^B + k^G)}{r\sigma^2 Y^2/2}$$

and the RHS of the HJB equation for $(0,0)$ is

$$\frac{\Pi(W) - 1 - \Pi'(W)(W-1) - \Pi'(W)Y\Delta\mu_0}{r\sigma^2 Y^2/2}$$

So long as $\zeta^G - \Pi'(W)\zeta^B + \Pi'(W)Y(k^B - k^G) > 0$, then the term for $(0,0)$ is smaller. That condition is equivalent to

$$Y > \frac{\zeta^B - \frac{\zeta^G}{\Pi'(W)}}{k^B - k^G}$$

But notice that $Y^*(1,1) \to 2\frac{-\zeta^B + \frac{\zeta^G}{\Pi'(W)}}{\Delta\mu_0 - k^B + k^G}$ as $W \to 1$ so that $Y^*(1,1) > 0$ implies that $\frac{\zeta^G}{\Pi'(W)} > \zeta^B$. So the condition holds, and $(0,0)$ achieves a lower value.

Now suppose $Y^*(1,1) < 0$ so that the minimizing Y is $\max\{\frac{\zeta^B}{k^B}, \frac{\zeta^G}{\Pi'(W)k^G}\}$. If $\frac{\zeta^G}{\Pi'(W)k^G} > \frac{\zeta^B}{k^B}$, then $Y = \frac{\zeta^G}{\Pi'(W)k^G} > \frac{\zeta^B - \frac{\zeta^G}{\Pi'(W)}}{k^B - k^G}$ and again $(0,0)$ achieves a lower value than $(1,1)$. However, if $\frac{\zeta^G}{\Pi'(W)k^G} < \frac{\zeta^B}{k^B}$, then the minimum is achieved for $(1,1)$ at $\frac{\zeta^B}{k^B}$. Notice that in this case, $(1,1)$ at $Y = \frac{\zeta^B}{k^B}$ achieves the same value as $(0,1)$, so the minimum value of the RHS of the HJB equation for $(0,1)$ is at least as small as the RHS of the HJB equation for $(1,1)$. Therefore, we need only compare $(0,1)$ and $(0,0)$ when $\frac{\zeta^G}{\Pi'(W)k^G} < \frac{\zeta^B}{k^B}$.

Suppose $\frac{\zeta^G}{\Pi'(W)k^G} < \frac{\zeta^B}{k^B}$ for W close to 1. We want to show that the feasible minimum achieved by the RHS of the HJB equation for the action pair $(0,1)$ is always higher than the feasible minimum achieved by the action pair $(0,0)$. Notice that for W close to 1, it must be that $Y^*(0,0)$ is feasible. What about $Y^*(0,1)$? We know that $0 < Y^*(0,1) < \frac{\zeta^G}{\Pi'(W)k^G}$ for W close to 1. We show that the RHS of the HJB equation reaches a lower value for $Y^*(0,0)$ at $(0,0)$ than at $Y^*(0,1)$ at $(0,1)$, and therefore it follows that $(0,0)$ is preferred. Directly plugging in terms, we get

$$\frac{-(\Pi'(W)\Delta\mu_0)^2}{2r\sigma^2(\Pi(W) - 1 - \Pi'(W)(W-1))} < 0$$

Doing the same for $Y^*(0,1)$, the RHS of the HJB equation for $(0,1)$ becomes

$$\frac{-(\Pi'(W)(\Delta\mu_0 + k^G))^2}{2r\sigma^2(\Pi(W) - 1 - \Pi'(W)(W-1) + \zeta^G)}$$

Comparing the two, we find the following is a necessary and sufficient condition for $(0,0)$ being smaller:

$$\frac{-(\Pi'(W)\Delta\mu_0)^2}{2r\sigma^2(\Pi(W)-1-\Pi'(W)(W-1))} < \frac{-(\Pi'(W)(\Delta\mu_0+k^G))^2}{2r\sigma^2(\Pi(W)-1-\Pi'(W)(W-1)+\zeta^G)}$$

$$\frac{-(\Delta\mu_0)^2}{(\Pi(W)-1-\Pi'(W)(W-1))} < \frac{-(\Delta\mu_0+k^G)^2}{(\Pi(W)-1-\Pi'(W)(W-1)+\zeta^G)}$$

$$\frac{-(\Delta\mu_0)^2}{\epsilon} < \frac{-(\Delta\mu_0+k^G)^2}{(\epsilon+\zeta^G)}$$

$$\zeta^G(\Delta\mu_0)^2 > \epsilon k^G(2\Delta\mu_0+k^G)$$

where $\epsilon = \Pi(W)-1-\Pi'(W)(W-1) \to 0$ as $W \to 1$. Thus for values of W close to 1, $(0,0)$ attains a lower value for the HJB equation than $(0,1)$.

Finally we show that $(0,0)$ is preferred to $(1,0)$. Notice that as W approaches 1, $Y^*(1,0)$ approaches $\frac{\zeta^B}{k^B-\Delta\mu_0}$. If there is overlap, so that $\Delta\mu_0 < k^B$, then $Y^*(1,0) > 0$ and the minimizing Y is $\frac{\zeta^B}{k^B}$. On the other hand, if there is no overlap, then $Y^*(1,0) < 0$ and the minimizing Y is again $\frac{\zeta^B}{k^B}$. Since the RHS of the HJB equation takes the same value for $(1,0)$ and $(0,0)$ when $Y = \frac{\zeta^B}{k^B}$, it follows that the feasible minimum for $(0,0)$ is weakly lower than that for $(1,0)$.

Therefore, it is always the case that for W close enough to 1, it is optimal to have $a^G(W) = 0$ and $a^B(W) = 0$. $\qquad\square$

Let $\mathcal{W}^H$ denote the interval such that $a^G(W) = 0$ if $W \in \mathcal{W}^H$. Let $\mathcal{W}^L$ denote the interval such that $a^G(W) = 1$ if $W \in \mathcal{W}^L$. Note that $\mathcal{W}^H = [0,1]\backslash\mathcal{W}^L$.

Lemma 19. *If $a^B(W) = 1$ for some $W \in \mathcal{W}^L$, then $a^B(W') = 1$ for all $W' < W$.*

Proof. Suppose $a^B(W) = 1$ for some $W \in \mathcal{W}^L$. We show that $a^B(W') = 1$ for all $W' < W$. For $a^B(W) = 1$, there must exist a $Y' \geq \frac{\zeta^B}{k^B}$ such that

$$\frac{\Pi(W) - \lambda^G - \Pi'(W)(W - \lambda^B + Y'\Delta\mu_1)}{\Pi(W) - \lambda^G - \Pi'(W)(W - 1 + Y(\Delta\mu_1 + k^B))} \geq \left(\frac{Y'}{Y}\right)^2. \tag{A.9}$$

for all $Y < \frac{\zeta^B}{k^B}$ where $\Delta\mu_1 = \mu(1,G) - \mu(1,B)$.[20] Notice that Y' still enforces $a^G(W') = 1$ for $W' < W$. Also notice that for all $Y < \frac{\zeta^B}{k^B}$, $1 - Y(\Delta\mu_1 + k^B) > \lambda^B - Y'\Delta\mu_1$. Suppose $W - \lambda^B + Y'\Delta\mu_1 < 0$. Then, in order for $\Pi''(W) < 0$ it is necessary that $\Pi(W) < \lambda^G$. We can see from the following picture that the ratio on the LHS of (A.9) increases for any $W' < W$. Let $w(Y) = 1 - Y(\Delta\mu_1 + k^B)$ and $w(Y') = \lambda^B - Y'\Delta\mu_1$.

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[20] As in the proof of Lemma 17, if the denominator is positive, we are done.
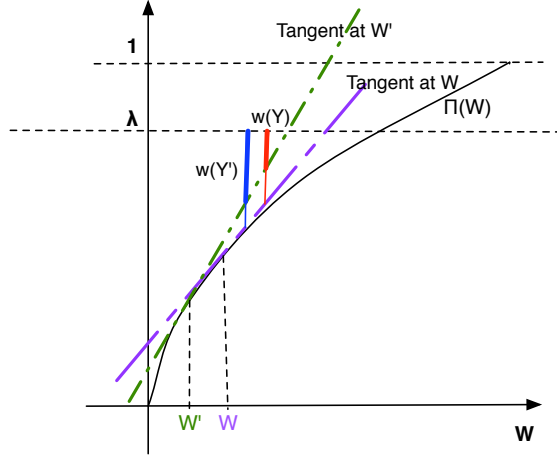
The thin blue line represents the numerator at W and the thin red line represents the denominator at W. The thick blue line represents the numerator at W', and the thick red line represents the denominator at W'. Notice that concavity implies that the ratio between the blue line and the red line increases as W moves to W'.

Now suppose $W - \lambda^B + Y'\Delta\mu_1 > 0$. Then the numerator in the fraction (which is negative) decreases by $(\Pi'(W') - \Pi'(W))(W - \lambda^B + Y'\Delta\mu_1) > 0$, which is greater than the amount by which the denominator (which is also negative) decreases, i.e. $(\Pi'(W') - \Pi'(W))(W - 1 + Y(\Delta\mu_1 + k^B))$.[21] Therefore the ratio must increase, so $a^B(W') = 1$. $\qquad\square$

Lemma 20. $a^B(W)$ *is a weakly decreasing step function in* $W \in [0, 1]$.

Proof. Consider $W \in \mathcal{W}^H$. We claim that if $Y^*(0, 0)$ is feasible, then $a^B(W) = 0$ is optimal. Suppose $Y^*(0, 0)$ is feasible. First consider when there is overlap so $\Delta\mu_0 < k^B$. In this case, if $Y^*(1, 0) > 0$, then the minimum is achieved at $\frac{\zeta^B}{k^B}$ and so $(0, 0)$ is weakly preferred. If $Y^*(1, 0) < 0$, then the minimum is achieved at $\frac{\zeta^G}{\Pi'(W)k^G}$, where $(1, 0)$ takes the same value as $(1, 1)$. Since $(1, 1)$ is not optimal, $(1, 0)$ cannot be.

Now consider when there is no overlap so $\Delta\mu_0 > k^B$. In this case if $Y^*(1, 0) < 0$, then the minimum for $(1, 0)$ is reached at $\frac{\zeta^B}{k^B}$, at which point the same value is implemented as $(0, 0)$ at that value of Y. Hence, $(0, 0)$ is weakly preferred. If $Y^*(1, 0) > 0$ on the other hand, then $Y^*(1, 0)$ implements a global minimum. Let's compare $(1, 0)$ and $(0, 0)$ both at $Y^*(1, 0)$. The term for $(0, 0)$ is smaller if and only if $0 < \Pi'(W)(Yk^B - \zeta^B)$, i.e. $Y > \frac{\zeta^B}{k^B}$, which is true whenever $Y^*(1, 0)$ is feasible. If $Y^*(1, 0)$ is too small, then the minimizing Y is $\frac{\zeta^B}{k^B}$, which implies that $(0, 0)$ leads to a lower value. So we have proven our claim.

Notice that $Y^*(0, 0)$ is decreasing in W so that if $Y^*(0, 0)$ is feasible at some W, then it is feasible for all $W' > W$. Therefore, if $(0, 0)$ holds for some $W \in \mathcal{W}^H$, then $(0, 0)$ is optimal for all $W' > W$.

So we know $Y(W)$ is a step function over $\mathcal{W}^H$ and over $\mathcal{W}^L$. We just need to make sure now that $a^B(W)$ doesn't jump from 0 to 1 right when $a^G(W)$ jumps from 1 to 0. In order for that to happen, the jump must occur exactly when $\frac{\zeta^G}{\Pi'(W)k^G} = \frac{\zeta^B}{k^B}$, i.e. $\Pi'(W) = \frac{\zeta^G}{\zeta^B}\frac{k^B}{k^G} > \frac{\zeta^G}{\zeta^B}$. So as W increases towards this jump from the left (i.e. $\Pi'(W)$ decreases towards $\frac{\zeta^G}{\zeta^B}\frac{k^B}{k^G}$), $(0, 1)$ must be optimal. And

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[21]Note that if the denominator is ever positive, the claim holds trivially.

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as W decreases towards this jump from the right (i.e. $\Pi'(W)$ increases towards $\frac{\zeta^G}{\zeta^B}\frac{k^B}{k^G}$), $(1,0)$ must be optimal.

Suppose that is the case. For $(0,1)$ to be optimal, $Y^*(0,1)$ must be feasible because otherwise we would just replace $(0,1)$ with $(0,0)$ or $(1,0)$ and get the same value. But notice that $Y^*(0,1)$ is continuous in W. As $\Pi'(W)$ decreases towards $\frac{\zeta^G}{\zeta^B}\frac{k^B}{k^G}$, it must be that $Y^*(0,1)$ is converging towards $\frac{\zeta^B}{k^B}$, which is not true. Hence, we must not have that discontinuous jump. $\qquad\square$

Lemma 21. *The function $Y^G(W) = \Pi'(W)Y(W)$ is a strictly decreasing function.*

Proof. As W goes from 0 to 1, there are two possibilities for the path of action pairs (a^B, a^G) that we have characterized. First, the path may go from $(1,1)$ to $(1,0)$ to $(0,0)$, or just from $(1,0)$ to $(0,0)$. If the path begins at $(1,1)$, then $Y^*(1,1)$ is the minimizing choice of Y, so that $\Pi'(W)Y(W) = \frac{\Pi(W)-1-\Pi'(W)(W-1)+\zeta^G-\Pi'(W)\zeta^B}{\Delta\mu_0 - k^B + k^G}$ and hence $\Pi'(W)Y(W) > \frac{\zeta^G}{k^G}$ and $\Pi'(W)Y(W) > \Pi'(W)\frac{\zeta^B}{k^B}$. As W increases, $(1,0)$ becomes optimal and for $Y^*(1,0)$ to be the minimizing Y, then $\frac{\zeta^G}{k^G} > \Pi'(W)Y(W) > \Pi'(W)\frac{\zeta^B}{k^B}$. Finally, as W approaches 1 and $(0,0)$ is optimal, then $\frac{\zeta^G}{k^G} > \Pi'(W)\frac{\zeta^B}{k^B} > \Pi'(W)Y(W)$. Hence it follows that $Y(W)$ is decreasing.

The other possible path of action pairs is from $(1,1)$ to $(0,1)$ to $(0,0)$, or just from $(0,1)$ to $(0,0)$. If the path begins at $(1,1)$, then $Y^*(1,1)$ is the minimizing choice of Y, so that $\Pi'(W)Y(W) > \Pi'(W)\frac{\zeta^B}{k^B} > \frac{\zeta^G}{k^G}$. As W increases, $(0,1)$ becomes optimal and thus $\Pi'(W)\frac{\zeta^B}{k^B} > \Pi'(W)Y(W) > \frac{\zeta^G}{k^G}$. Finally, as W approaches 1 and $(0,0)$ is optimal, then $\Pi'(W)Y(W) < \frac{\zeta^G}{k^G}$ and $\Pi'(W)Y(W) < \Pi'(W)\frac{\zeta^B}{k^B}$. Hence it follows again that $Y(W)$ is decreasing. $\qquad\square$

Lemma 22. *There is generically a discontinuous jump downward in $Y(W)$ when $a^G(W)$ goes from 1 to 0.*

Proof. The jump occurs either as the optimal pair of actions goes from $(1,1)$ to $(1,0)$ or from $(0,1)$ to $(0,0)$. Let's consider the former first. In order for $(1,0)$ to be possible, it must be that $\frac{\zeta^G}{\Pi'(W)k^G} > \frac{\zeta^B}{k^B}$. Furthermore, it must be that when $(1,0)$, $Y^*(1,0) < \frac{\zeta^G}{\Pi'(W)k^G}$ or else $(1,1)$ would achieve a weakly lower value. However, the minimizing Y for $(1,1)$ must be at least as large as $\frac{\zeta^G}{\Pi'(W)k^G}$. So this implies that for there to be no jump $Y^*(1,0) = Y^*(1,1) = \frac{\zeta^G}{\Pi'(W)k^G}$, which does not hold generically.

Now consider $(0,1)$ to $(0,0)$. In order for $(0,0)$ to be the case, $Y^*(0,0)$ must be feasible. For $(0,1)$, we need $\frac{\zeta^B}{k^B} > \frac{\zeta^G}{\Pi'(W)k^G}$. So $Y^*(0,0) \leq \frac{\zeta^G}{\Pi'(W)k^G}$. In order for there not to be a jump, we need $Y^*(0,0) = Y^*(0,1) = \frac{\zeta^G}{\Pi'(W)k^G}$, which does not hold generically. $\qquad\square$

A.4   Proof of Lemma 1

To prove Lemma 1, we need to consider more general contracts of the form $C = \{\mathcal{D}, \mathcal{A}^G, \mathcal{A}^B\}$ where $\mathcal{D} = \{D_t\}_{t\geq 0}$, $\mathcal{A}^G = \{a_t^G\}_{t\geq 0}$ and $\mathcal{A}^B = \{a_t^B\}_{t\geq 0}$ (of course, we require feasibility so $D_t \in [0,1]$, $a_t^G, a_t^B \in A$ for all $t \geq 0$). So rather than a stopping time τ such that $D_t = 1$ for all $t < \tau$ and $D_t = 0$ for all $t \geq \tau$, contracts can now specify D_t anywhere between 0 and 1 at all times.

First, consider the contract for the bad type. The principal's amended optimization problem is the following: given some $\hat{W}^B \in [0,1]$,

$$\max_{D_t, \{a_t^B\}} E\left[r \int_0^\infty e^{-rt} D_t h(a_t^B, B) dt \mid \theta = B\right] \quad \text{s.t.} \tag{A.10}$$

$$E\left[r \int_0^\infty e^{-rt} D_t g(a_t^B, B) dt \mid \theta = B\right] \geq \hat{W}^B \tag{A.11}$$

where $\{a_t^B\}$ is incentive compatible, $a_t^B \in A$, and $D_t \in [0,1]$.

Next, consider the contract for the good type. The principal's amended optimization problem is the following: given some $\hat{W}^B \in [0,1]$,

$$\max_{D_t, \{a_t^G\}, \{a_t^B\}} E\left[r \int_0^\infty e^{-rt} D_t g(a_t^G, G) dt \mid \theta = G\right] \quad \text{s.t.} \tag{A.12}$$

$$E\left[r \int_0^\infty e^{-rt} D_t g(a_t^B, B) dt \mid \theta = B\right] \leq \hat{W}^B \tag{A.13}$$

where $\{a_t^G\}, \{a_t^B\}$ are incentive compatible, $a_t^G, a_t^B \in A$ and $D_t \in [0,1]$.

Just as before, we conjecture that an optimal contract for each type is characterized by the solution to the relevant HJB equation. By following the methods of this paper directly, we can verify our conjecture and prove uniqueness and concavity of the solution (for the sake of space, we do not repeat the analysis here).

For the bad type, we denote patient welfare by $F(W)$ and find that the HJB equation is as follows:

$$F''(W) = \min_{D,Y} \frac{F(W) - Dh(a(D,Y),B) - F'(W)(W - Dg(a(D,Y),B))}{r\sigma^2 \mathbb{I}_{D_t>0} Y^2/2}$$

$$\text{s.t.} \quad F(0) = 0 \text{ and } F(1) = h(0,B), \tag{A.14}$$

where $a(D,Y) = 0$ only if $\frac{Y}{D} \leq \frac{\zeta^B}{k^B}$ and $a(D,Y) = 1$ only if $\frac{Y}{D} \geq \frac{\zeta^B}{k^B}$. Let $D(W)$, $Y(W)$ and $a(W) = a(D(W), Y(W))$ be the minimizers on the right hand side of the HJB equation. This unique and concave solution to the HJB equation corresponds to the following contract, which we can verify is optimal. The contract uses W_t as a state variable and sets $D_t = D(W_t)$, $Y_t = Y(W_t)$ and $a_t = a(W_t)$ for all W_t. W_t is initialized at some W_0 and solves

$$dW_t = r(W_t - D_t g(a_t, B))dt + rY_t \mathbb{I}_{D_t>0}\sigma dZ_t.$$

For the good type, we denote patient welfare by $G(W^B)$ and can write the HJB equation as

$$G''(W^B) = \min_{D,Y} \frac{G(W^B) - Dg(a^G(D,Y),G) - G'(W^B)(W^B - Dg(a^B(D,Y),B)) + Y(\mu(a^G(D,Y),G) - \mu(a^B(D,Y),B)))}{r\sigma^2 \mathbb{I}_{D_t>0} Y^2/2}$$

$$\text{s.t.} \quad G(0) = 0 \text{ and } G(1) = 1, \tag{A.15}$$

where $a^G(D, Y) = 0$ only if $\frac{Y}{D} \leq \frac{\zeta^G}{G'(W^B)k^G}$ and $a^G(D, Y) = 1$ only if $\frac{Y}{D} \geq \frac{\zeta^G}{G'(W^B)k^G}$, while $a^B(D, Y) = 0$ only if $\frac{Y}{D} \leq \frac{\zeta^B}{k^B}$ and $a^B(D, Y) = 1$ only if $\frac{Y}{D} \geq \frac{\zeta^B}{k^B}$. Let $D(W^B)$, $Y(W^B)$, $a^G(W^B) = a^G(D(W^B), Y(W^B))$, and $a^B(W^B) = a^B(D(W^B), Y(W^B))$ be the minimizers in the HJB equation. This unique and concave solution to the HJB equation corresponds to the following contract, which we can verify is optimal. The contract uses W_t^B as a state variable and sets $D_t = D(W_t^B)$, $Y_t = Y(W_t^B)$, $a_t^G = a^G(W_t^B)$, and $a_t^B = a^B(W_t^B)$. W_t^B is initialized at some initial value W_0^B and solves

$$dW_t^B = r(W_t^B - D_t g(a_t^B, B) + \mathbb{I}_{D_t > 0} Y_t^B (\mu(a_t^G, G) - \mu(a_t^B, B))) dt + rY_t^B \mathbb{I}_{D_t > 0} \sigma dZ_t^{\mathcal{A}^G}.$$

Lemma 23. *In an optimal contract, a principal chooses $D_t \in \{0, 1\}$.*

Proof. We can prove the result for the bad contract from first principles without using the HJB representation. Suppose C^B is an optimal contract for the bad type that specifies demand $D_t \in (0, 1)$. Formally, the contract specifies an X_t-measurable demand process $\mathcal{D} = \{D_t\}$ and an X_t-measurable incentive-compatible advice of actions $\mathcal{A} = \{a_t\}$ for the bad surgeon. Following Proposition 1 of Sannikov (2007b), we know that we can represent patient welfare $F_t(\mathcal{D}, \mathcal{A})$ in this contract as a diffusion process: there exists an X_t-measurable process $\mathcal{Y} = \{Y_t\}$ such that

$$dF_t(\mathcal{D}, \mathcal{A}) = r(F_t(\mathcal{D}, \mathcal{A}) - D_t h(a_t, B)) dt + rY_t \sigma dZ_t^{\mathcal{A}}.$$

Recognizing that we can similarly represent the bad surgeon's value as a diffusion process, we apply Proposition 2 of Sannikov (2007b) to observe that, for an action a_t to be incentive compatible, it must satisfy:

$$a_t \in \arg\max_{a' \in A} D_t g(a', B) + Y_t \mu(a', B).$$

So $a_t = 0$ only if $\frac{Y_t}{D_t} \leq \frac{\zeta^B}{k^B}$, and $a_t = 1$ otherwise.

Now, we define a feasible and incentive compatible contract $\hat{C}^B$ such that $\hat{a}_t = a_t$ for all t and $\hat{D}_t = 1$ whenever $D_t > 0$ and $\hat{D}_t = 0$ whenever $D_t = 0$. We guarantee incentive compatibility by defining the sensitivity process so that a_t remains optimal: whenever $D_t > 0$, $\hat{Y}_t = \frac{Y_t}{D_t}$, and whenever $D_t = 0$, $\hat{Y}_t = Y_t$. It now suffices to show that in order for C^B to be optimal, it must be that $D_t = 1$ whenever $D_t > 0$.

Define the time-t expectation of aggregate patient welfare by

$$\hat{V}_t = r \int_0^t e^{-rs} \hat{D}_s h(a_s, B) ds + e^{-rt} F_t(\mathcal{D}, \mathcal{A})$$

if the principal has followed contract $\hat{C}^B$ until time t and then plans to follow contract C^B after time t. The drift of the process under the probability measure $\mathbb{P}^{\mathcal{A}}$ is

$$d\hat{V}_t = re^{-rt} \hat{D}_t h(a_t, B) dt - re^{-rt} D_t h(a_t, B) dt + re^{-rt} \sigma Y_t dZ_t^{\mathcal{A}}$$

$$\implies d\hat{V}_t = re^{-rt} (\hat{D}_t - D_t) h(a_t, B) dt + re^{-rt} \sigma Y_t dZ_t^{\mathcal{A}}$$

If $\hat{D}_t > D_t$ on a set of positive measure, then the drift of $\hat{V}$ is positive on a set of positive measure.

Thus, there exists a time $t > 0$ such that

$$E^{\mathcal{A}}[\hat{V}_t] > \hat{V}_0 = F_0(\mathcal{D}, \mathcal{A})$$

which contradicts the contract C^B being optimal. Therefore, if C^B is optimal, $\hat{D}_t = D_t$ for all t and thus $D_t \in \{0, 1\}$.

Now consider an optimal contract C^G for the good type. Recall that $D_t = D(W_t)$ where $D(W_t)$ is the minimizer on the right hand side of the HJB equation. The HJB equation is linear in D though, so clearly the minimizing value is either $D = 0$ or $D = 1$. Therefore $D_t \in \{0, 1\}$. □

Lemma 24. *In an optimal contract, if $D_t = 0$ for some $t \geq 0$, then $D_{t'} = 0$ for all $t' \geq t$, i.e. there is only license revocation, not license suspension.*

Proof. First, consider the contract for the bad type. The HJB equation (A.14) can be written

$$F(W) = \max_{a,D,Y} Dh(a, B) + F'(W)(W - Dg(a, B)) + \frac{F''(W)}{2} r\sigma^2 \mathbb{I}_{D>0} Y^2.$$

If $D = 0$ for any $W > 0$, then $F(W) = \max_{a,Y} F'(W)W = F'(W)W$ and $F(W)$ is a straight line. On the other hand, If $D = 1$ for that $W > 0$, then $F(W) = \max_{a,Y} h(a, B) + F'(W)(W - g(a, B)) + \frac{F''(W)}{2} r\sigma^2 Y^2$. We know that there exists a concave solution F that satisfies the relevant boundary conditions. This is strictly preferred to the straight line solution which arises if $D = 0$ for any $W > 0$. Thus, it cannot be that $D_t = 0$ for any t unless $W_t = 0$ (i.e. the license has been revoked).

A similar argument holds for the good type contract. The HJB equation A.15 can be written

$$G(W^B) = \max_{a^G, a^B, D, Y} Dg(a^G, G) + G'(W^B)(W^B - Dg(a^B, B) + \mathbb{I}_{D>0} Y(\mu(a^G, G) - \mu(a^B, B)))$$

$$+ \frac{G''(W^B)}{2} r\sigma^2 \mathbb{I}_{D>0} Y^2.$$

Again, notice that if $D = 0$ for any $W > 0$, then $G(W) = G'(W)W$ and G is a straight line. Since we know there exists a concave solution that satisfies the boundary conditions, it cannot be that a straight line is optimal. So it cannot be that $D_t = 0$ for any t unless $W_t = 0$ (i.e. the license has already been revoked). □

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