Preliminary Draft

# Mortgage Origination and the Rise of Securitization: An Incomplete-Contracts Model

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**Abstract:** Recent empirical studies find that securitised mortgages yield higher default rates than those which are originated and held by the same party, raising a number of theoretical puzzles which we address with a model based on incomplete contracts. Our model that combines the features of diversion (along the lines of Hart and Moore 1989, 1998), with asymmetric information regarding borrowers default risk (as in Stiglitz and Weiss 1981) and soft-information screening by lenders. We show that securitisation weakens the incentive to screen compared with bank originated-and-held-loans, and that securitisation is more prevalent (inter-alia) with rising house prices, lower interest rates, reduced liquidation costs and higher bank regulation costs. Extending our basic model to the case of stochastic prices allows us to analyse strategic default: Enforcement of recourse loans or policies to encourage renegotiation reduces repayments and default rates in our model. In a final extension, we assume a rise in the volume of mortgage-default properties raises the depreciation rate of such properties in the second-hand market. This systemic factor introduces multiple equilibria, i.e. either securitised or bank-held mortgages (not both at the same time) exist within a region. Securitised mortgages can generate significantly lower welfare if depreciation is sufficiently high so that mortgage markets in this region are fragile if external factors can shift the equilibrium from banking (with its low default rate) to securitisation (with its higher rate).

**Journal of Economic Literature codes:** G21, D86, D43, D82, L14, G38

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# 1. Introduction

Recent empirical studies of the period prior to the financial crisis have found that securitized mortgages had significantly higher default rates than loans originated and held by the same institution.[1] This presents a number of puzzles for economists. Why should it matter which party holds foreclosure rights? Why could securitizable incentive contracts not be written to address moral hazard or adverse selection? Even without such contracts, why were low-quality securities not recognised as lemons and rejected by the market?

A popular answer to these questions is that originators sold low quality loans for the purpose of securitization, and investors bought them because they underestimated their riskiness, perhaps due to biased ratings. While it has some merit, this explanation also has a number of difficulties. The argument that investors exhibited systemic errors of perception can be challenged on the grounds that most mortgage-backed securities (by value) were held by sophisticated financial institutions which stood to lose a significant amount of money in buying lemons. Taking this further, although systematic errors of perception are an important future line of research, economic theory does not currently have a convincing way of modelling boundedly-rational agents. It therefore makes sense to first seek answers using extant theory.

Our paper provides an alternative explanation, based on the 'new' incomplete-contracts theory of debt begun by Hart and Moore (1989, 1998) and Aghion and Bolton (1992). As in Aghion and Bolton, a wealth-constrained borrower—a mortgagee—enjoys a non-pecuniary payoff, in our case from living in the home. As in Hart and Moore, diversion of funds, but not physical assets, is central to our theory: Parties in our two-period model can divert funds held from one period to the next on a one-for-one basis, but they cannot divert the physical property—the home—over which the mortgage is written. The only exception to the ability to divert funds are regulated intermediaries we refer to throughout the paper as 'banks' (for ease of expression). Prudential regulation ensures that depositors receive their principal plus interest in expectation.[2] Unregulated intermediaries, which we call 'brokers', are unable to commit *not* to divert funds.

We follow the traditional credit-rationing literature (e.g. Stiglitz and Weiss 1981), in that bor-

[1] See, for example, Berndt and Gupta (2009), Keys et al. (2010), Krainer and Laderman (2009), Mian and Sufi (2009) and Nadauld and Sherlund (2009).

[2] An equivalent assumption is that there exists a costly technology, such as an accounting system, which allows private intermediaries to commit to repay should they incur the cost of using it.

rowers have private information on their type. Similarly to the moral-hazard literature on borrowing (e.g. Gorton and Pennacchi 1995), borrower type is soft information which can be uncovered with costly screening effort. Intermediaries in our model can choose to function as banks or brokers, and compete through loan offers to a borrower. A loan package consists of two strategic variables, a repayment and a decision whether or not to screen the borrower using soft-information. Screening reveals the borrower's type, which, in our framework, is whether or not a loan to the customer generates a loss.

We fully characterize the equilibrium for the ensuing game. For one parameter set, the unique equilibrium is an unscreened loan originated by a broker, which is immediately securitized and transferred to investors. The securitized mortgage may be either a high quality loan or a lemon. For the complementary set of parameters, a bank originates the loan and funds it through deposits, holding it through the second period. Being the holder of the right to foreclose as well as being the originator, the bank finds it worthwhile to screen, so that any mortgage it makes consists only of a high quality loan. The trade-off between banking and broking is therefore one of high quality loans with regulatory and screening costs in the case of banking, versus no regulatory or screening costs but mortgage-backed securities consisting of both good loans and lemons, in the case of broking. Investors are willing to purchase mortgage-backed securities from brokers because the securities offer a non-negative expected return. They make a loss if the borrower is a bad type and an offsetting gain if the borrower is a good type.

Contractual incompleteness due to diversion underlies our explanation for the observed higher default rates of securitized mortgages. As in Aghion and Bolton (1992) and Hart and Moore (1998, 1989), contractual incompleteness explains why it matters which party holds the right to foreclose.

Our theory yields sharp comparative-statics conclusions on securitization. We predict that there will be more broker-originated mortgages and securitization in housing markets with steeply rising prices, lower foreclosure/liquidation costs, and lower interest rates. Securitization also increases with the introduction of cheaper underwriting (perhaps due to computer automation), higher banking regulatory costs, and higher expected incomes. The results accord well with recent empirical research on mortgage lending.[3] We also use the benchmark model to analyze the comparative

[3]Using zip-code level data on mortgage lending, Mian and Sufi (2009) find that rising house prices, lower interest rates, lower origination costs, and lender moral hazard are each associated with increased securitization. Nadauld and Sherlund (2009) similarly find an association between rising house prices and securitization.

statics of default rates. An exogenous switch from banking to broking increases the frequency of foreclosure.[4] We study the impact of a measure of borrower quality on default, analogous to a FICO score. For very low or very high levels, an increase in quality results in a gradual decline in defaults on the bank-held mortgages. A striking result provides a theoretical basis for the finding by Keys et al (2010) of a discrete rise in the rate of default at some threshold level of quality.[5] At an intermediate level of borrower quality, our model predicts a discrete jump up in both defaults and loan volume. At this threshold, the mortgage originator switches from bank to broker and the jump occurs because banks screen and brokers do not, thus granting loans to both good and bad borrower types.

We extend the benchmark case by introducing stochastic future house prices to the model, in order to analyze the impact of the collapse of house prices experienced before the financial crisis, and possible policy measures to address such events. Under the US practice of no-recourse loans, whereby borrowers are not liable in default beyond losing their home, this allows borrowers to strategically default when the repayment exceeds the price of a new home (i.e. 'underwater loans'). We find that strategic default by high income customers occurs in equilibrium, which leads to increased repayments, lower expected welfare and possibly lower loan volumes. Full-recourse loans, whereby borrower's future assets can be seized in lieu of repayment, recovers the outcomes of the non-stochastic benchmark case. Stochastic house prices allows us to study the impact of renegotiation. Following Hart and Zingales (2008), we assume that renegotiation is harder under securitization than banking, due to the diffusion of invesors in the former case. We find that the introduction of renegotiation expands the set of parameters where banking is an equilibrium, reduces repayments and increases welfare.

Our most important extension is to a introduce a key systemic factor into the benchmark model, and a continuum of borrowers, in order to capture important general-equilibrium type effects that are absent in the benchmark case. We assume that the greater is the rate of default, the higher

[4]Identifying the causal effect of switching from banking to broking on default rates is difficult for reasons we discuss in the paper. Attempting to control for other factors, Berndt and Gupta (2009) find that loans that were originated to be distributed had higher default rates than those that were originated to be held. Similar results using different identification strategies are found in Krainer and Laderman (2009), Mian and Sufi (2009), Nadauld and Sherlund (2009), and Keys et al. (2010).

[5]They find that 620 is a threshold FICO credit score. Below 620 there is discretely more bank lending and above it more broking and securitization. Default rates fall with the FICO score everywhere except at the threshold 620, where there is the discrete rise, congruent with our theory.

are the 'frictional' or depreciation losses from liquidation. In practice, outside of our model, this arises because increased default leads to a larger stock of empty homes in a quantity-clearing housing market.[6] Securitization now imposes a negative-externality, altering the equilibrium, because in a banking equilibrium a broker which deviates by entering ends up serves all borrowers including lemons, leading to increased market-wide liquidation costs. Accordingly, the systemic factor allows us to succinctly capture the social trade-off between securitization and traditional banking: The welfare benefit of securitization is that loans are extended to parties who would not otherwise get them. The welfare cost is due a larger measure of defaults, and the larger liquidation costs due greater depreciation of stock which this entails.

Perhaps the most important welfare feature we study is market stability. The introduction of a market liquidation cost which increases with the rate of default, makes the mortgage market unstable in the following sense: Rising liquidation costs introduces a region of multiple equilibria in which either all loans are securitized by a broker, or all are originated and held by a bank. This allows us to compare welfare across actual equilibria rather than between one equilibrium and some counter-factual equilibrium. A parameter shift such as a fall in the interest rate may now lead to a shift between banking and broking equilibria. The consequence would be a discrete jump in default rates and hence a discrete jump in liquidation costs and a discrete fall in welfare: The mortgage market is unstable due to the possibility of securitization. We apply the idea of 'evolutionarily stable equilibria' to argue that introducing securitizing brokers to the banking equilibrium should tend to drive banks out, but introducing banks to a broking equlibrium has no such effect.

## 2. Literature

The classical literature on mortgage lending, begun by Stiglitz and Weiss (1981), and developed by Bester (1985, 1987), Besanko and Thakor (1987) and many others, is focused on why credit may be rationed in equilibrium. To study current events, however, the focus needs to be on the apparently excessive extension of credit. Thus, our theory needs to explain how it may be that credit is not rationed in equilibrium. The core assumptions of the credit-rationing literature are

[6] Depreciation due to idleness is a feature of the housing market. As Karl Case (2008) argues "Home prices are subject to inertia and are sticky downward. Housing markets have traditionally been quantity clearing markets, with excess inventories absorbed only as new households are formed."

(a) that borrowers have private information on their credit-worthiness, and (b) that some form of single crossing condition holds, which allows for contractual screening through menus consisting of a repayment along with some combination of collateral and (probabilistic) ex-ante rationing. While we retain the assumption of private information, we assume limited income (to focus on the lower end of the mortgage market) and rule out ex-post but not ex-ante public randomization.[7] These assumptions obviate contractual screening in our model and (endogenously) yield simple debt, without ex-ante random rationing, as the optimal contractual form.

A more recent strand of literature focused on moral hazard associated with loan sales and is closer in spirit to the topic of this paper—securitization—than the credit-rationing literature. The basic tenant, as pointed out earlier in Diamond (1984),[8] is that diminished incentives to screen means that marketed loans will be tend to be worthless. To explain the growing trend toward loan sales, Gorton and Pennacchi (1995) assumed that fractions of loans with limited implicit guarantees could be sold, but that full transfer is prevented by regulation.[9] However, this restriction has gradually eroded over time, and with the Gramm–Leach–Bliley Act of 1999, such restrictions on banks were eased. This did not appear to result in improved screening, as would be predicted by theory if loans were sold which incorporated optimal incentive schemes for bank monitoring.

The two literatures are subject to one of the central puzzles our theory seeks to address, i.e. that they do not explain why securitized loans based on comprehensive contracts fail to yield identical outcomes to bank-held loans. As mentioned, Aghion and Bolton (1992), and Hart and Moore (1989, 1998) provide the key, by emphasising the incompleteness of contracts. However, these papers are not directly applicable to the problem of mortgage lending and securitization for several reasons. First, they are based on symmetric information. Asymmetric information, assumed in our model, is a vital ingredient of such lending. Second, the contracts written by creditors in these papers could be sold without altering outcomes. In other words, these theories provide insights but are not directly adaptable to the issue of securitization. Our model combines features of the credit-rationing, moral hazard and incomplete contracts literature on debt into a carefully developed

[7]Ex-post randomization requires that a mortgage is able to be be liquidated at random regardless of whether or not the borrower repays. Some households would repay and be liquidated, others would not repay and remain in their properties. We rule out this possibility. See Appendix ,

[8]See the discussion on the necessary illiquidity of intermediary assets on p410 of this article.

[9]The Glass–Steagall Act of 1933 limited banks activities in the securities markets, effectively preventing the full transfer or securitization of loans.

theory of mortgage securitization. Borrowers have asymmetric information. Intermediaries can can uncover this information at a cost. Parties, except regulated banks, can divert funds held between periods. Simple debt is optimal. As well as addressing important theoretical puzzles, we have been careful develop a theory with empirical content, to uncover a key welfare trade-off, and to examine some of the policy solutions suggested in the literature.

One additional paper, Greenbaum and Thakor (1987), hereafter GT, requires closer attention because it is one of the few examples of a paper in the contractual-screening literature with implications for securitization. Deposit funding in GT involves depositors' active participation in the loan. They pay a screening cost, becoming fully informed about the borrower's type, allowing them to sign the efficient, full-information loan contract, which uses all the risk-neutral intermediary's capital to insure risk-averse depositors in case the borrower defaults. With securitization, investors' involvement is more arm's-length. Instead of screening, investors infer the borrower's type from the loan's terms, in particular from the percentage of the loan the intermediary "backstops" for investors in case of default.

Our approach has several advantages relative to GT. First, our analysis does not involve the considerable complication of signaling contracts and so is simpler at the same time it allows us to be more rigorous. Rather than making behavioral assumptions, we provide foundations based on the theory of incomplete contracts from which we derive optimal behavior for all agents. For example, rather than assuming that the intermediary always screens as in GT, this is a central question to be studied in our analysis. GT's implication that depositors pay more attention to the quality of mortgages issued by their banks than investors in mortgage-backed securities seems counterfactual. Even if one argues that investors in mortgage-backed securities should have been more vigilant about quality of the mortgages involved in the run-up to the crisis, it is hard to argue that they paid less attention than the typical depositor, whose deposits are government-insured and who does not veto individual loans of the thousands originated by his or her bank. In our model, neither depositors nor investors screen.

# 3. Model

We model the mortgage market as a game of incomplete information involving two periods, 1 and 2, and three groups of risk-neutral players: consumers, who require financing for a house; investors, who supply financial capital, and intermediaries, who link demanders and suppliers of financial capital by originating mortgages. We first describe the contractual setting, then characterize the players and the market in which they operate, and conclude with discussion of the timing diagram, which will serve to recapitulate the model's main elements.

## 3.1. Contractual Environment

We begin with a discussion of the contractual environment, highlighting two key features: diversion and moral hazard. These features dictate the form of mortgage contracts, leading to the simple-debt form used in practice, and will provide an endogenous difference between banks and brokers, the two types of intermediary we will study.

Diversion is a key feature of the environment leading to a high degree of contractual incompleteness. Specifically, all players are assumed to be able to divert any funds which they hold from one period to the next for their own consumption. Allowing for such complete diversion is an extreme assumption imposed to streamline the analysis: we think of it as a metaphor for perhaps more moderate financial market imperfections.

Two exceptions constrain diversion. First, an intermediary can be prevented from diverting deposited funds by subjecting itself to costly prudential regulation. We will later identify an intermediary's regulated operations as banking and its unregulated operations as broking. Banks can raise deposits from investors because regulation commits them to repaying the principal and interest on the deposits. Brokers cannot raise deposits because they cannot commit not to divert the funds.[10] The second constraining factor follows from Hart and Moore's (1998) idea that physical assets cannot be diverted and are therefore subject to a threat of seizure. The relevant physical asset here is the house: the threat that the house will otherwise be taken incentivizes the borrower to make mortgage payments.

[10] An alternative interpretation to regulation is that an intermediary can commit itself to a costly accounting system which credibly prevents it from diverting funds. The key point is that there is a cost wedge between the activities of banking and broking.

Diversion helps narrow down the set of feasible contracts considerably. Theorem 3, stated and proved in Appendix C, implies that the combination of diversion with the ability of lenders to seize the physical home leads optimal mortgage contracts to have the form of simple debt. That is, the mortgage specifies a repayment in period 2 for the amount borrowed in period 1. The amount borrowed exactly equals the amount needed to finance the house. If the borrower fails to make the repayment, the lender can seize the house from the borrower. If the borrower makes the repayment, it can continue to enjoy the property through period 2. We will take for granted that mortgages have this contractual form in the remainder of the text of the paper, streamlining the discussion.

The second key feature of the contractual environment is a moral-hazard problem between investors (as principal) and the intermediary (as agent), which is exacerbated by the high degree of contractual incompleteness. At a cost, the intermediary can screen out consumer types who are unlikely to be able to make their repayments. However, investors have limited contractual means to induce the intermediary to screen because any promised payments between them can be diverted. Here is where the ability to raise deposits provides banks with an advantage: deposits provide a bank with a source of funds that it can use to hold the mortgage lien as an asset on its own balance sheet until period 2. As the mortgage lien holder, the bank becomes the residual claimant of the repayment stream, incentivizing it to undertake efficient actions such as screening borrowers. By contrast, brokers cannot use deposits to hold mortgage liens in this way. Instead, mortgage liens originated by brokers must immediately be transferred to investors. Investors become the residual claimants of the repayment stream—not the broker—dulling the broker's incentives to screen the borrower. We identify the broker's method of originating mortgages and transferring them directly to investors as "securitization."

### 3.2. Consumer

We next characterize the players in the model. A single consumer has the opportunity of purchasing a new house in period 1, providing utility $u_1$ from the consumption of housing services in period 1 and $u_2$ if he remains in the house through period 2. His utility function is additively separable in housing services and income. He has no initial wealth, so needs to obtain a mortgage to purchase a home.[11] The consumer makes mortgage repayments using his period-2 income, which depends on his type, $\theta$.

The "good" consumer type has probability $h \in (0,1)$ of earning high income and complementary probability $(1-h)$ of earning low income in period 2. Let $y > 0$ be the high income level and normalize low income to 0. The "bad" type earns no income in period 2 with certainty. Nature draws the good type with probability $\gamma \in (0,1)$ and the bad type with complementary probability $1-\gamma$. The specific realization of the consumer's type is his private information, but the distribution of types (i.e., $\gamma$) and all other parameters of the model are public information. Let $\theta$ be an indicator for consumer type, i.e., $\theta = 1$ for the good type and $\theta = 0$ for the bad type.

The consumer should be envisaged as being part of a segment of the market sharing attributes $h, y, \gamma, u_1$ and $u_2$ containing both good and bad types. As explained further below, the consumer's market segment represents the "hard" information about him and type is thought of as "soft" information. Other segments might simultaneously exist having different parameters, but for simplicity we will consider just this one.

## 3.3. Investors

Investors are the only source of financial capital in the model. They provide a perfectly elastic supply of credit. Given that they are risk neutral, this means they are willing to sign any contract providing an expected return greater than or equal to principal plus the risk-free interest rate, $r \geq 0$.

## 3.4. Intermediaries

The consumer cannot access access investors' financial capital directly but must go through an intermediary. Intermediaries are indexed by $i = 1, \ldots, N$, where $N \geq 2$. They compete for the consumer's business in the mortgage market by posting contract terms $(R_i, S_i)$ associated with a loan of $p_1$. $R_i$ is the amount that the borrower is required to repay at the start of period 2 to avoid foreclosure. $S_i$ is an indicator for whether $i$ screens on the basis of soft information, with $S_i = 1$ indicating screening and $S_i = 0$ no screening. The household observes the terms $(R_i, S_i)$ offered by

[11] The assumption of zero wealth is made to simplify the analysis. In Appendix C, we show that key features of equilibrium contracts are unchanged if a consumer has positive wealth, as long as such wealth is not too large.

all active intermediaries and then chooses whether or not to accept one of their contracts.[12] If the intermediary screens the consumer, it incurs a cost $k_I > 0$ and imposes cost $k_C > 0$ on the borrower (non-pecuniary because he has no income at that point). To simplify the exposition, we will take $k_C$ to be negligible in the remainder of the text. The results are stated and proved in the appendix for arbitrary $k_C > 0$.[13] After screening, the intermediary decides whether or not to originate a mortgage to the consumer.

We make the following assumptions about the information available to intermediaries and other players. All the parameters of the model (aside from the consumer's type, $\theta$) are public information. We equate knowledge of these parameters with the sort of "hard" information embodied in FICO and other commercially available credit scores. The model makes this information costless to acquire for simplicity. We equate the borrower's type $\theta$ with additional "soft" information on the household's income prospects which can only be uncovered through the costly screening process. The act of screening and its soft-information outcome are assumed to be unobservable to outside parties: this is the source of the potential moral-hazard problem between the intermediary and investors, who might like the intermediary to screen bad loan risks.

An intermediary can function in one of two modes, as a bank or as a broker. Operating as a bank is synonymous with consenting to regulation. As discussed above, regulation commits the bank not to divert deposits, enabling it to finance the mortgage by raising deposits from investors. Regulation has a downside, involving an extra amount $d \in (0,1)$ per dollar of deposits taken, em-

[12]The restriction to deterministic screening can be justified by analyzing an extended model with a dynamic "pre-application" stage in which commitment to stochastic screening is allowed. The timing of the pre-application stage is as follows. First, active intermediaries post contracts $(R_i, \sigma_i)$, where $\sigma_i \in [0,1]$ is the probability of screening (i.e., the probability that $S_i = 1$). Next, the household observes $(R_i, \sigma_i)$ and pre-applies to all active intermediaries. Next, the outcome of the screening randomizations $S_i$ are realized. Finally, the household observes $S_i$ and chooses an intermediary. In the equilibrium of this game, the intermediary chooses a corner solution, either $\sigma_i = 0$ or $\sigma_i = 1$, equivalent to a deterministic screening policy in the benchmark model without the pre-application stage.

The pre-application model rests on two ideas. The first is that in practice households can apply to (or at least gather information on) multiple intermediaries before they select one. The other idea is that a household can withdraw its application and avoid being screened. Unlike a complusory tax audit, for example, screening for a mortgage involves the borrower's and intermediary's voluntary participation, thus making it reasonable to assume the household can observe the screening commitment and withdraw if it does not wish to be screened.

[13]The formal apparatus behind "taking $k_C$ to be negligible" involves first solving for equilibrium for arbitary $k_C > 0$ and then examining the limit as $k_C \downarrow 0$. The analysis in the text is fully rigorous for this special (limiting) case. The reason for introducing a positive value of $k_C$, even if negligible, is to rule out an expanded set of broking equilibria manifested at the $k_C = 0$ boundary. These equilibria arise because, off the equilibrium path, a specified mass of bad types apply to a screening bank even though they would not receive a loan. These bad types would lower the profitability of banking, which can shift the equilibrium to broking.

bodying the cost of maintaining a rigorous accounting system in addition to any reserves required by the regulator. For example, to fund a loan of size $p_1$ (as we will see, the period-1 price of a house), the bank would have to raise $p_1/(1-d)$ in deposits. Accounting for the interest rate $r$ required by investors on their deposits, the bank's opportunity cost of funds for this loan is

$$\frac{(1+r)p_1}{1-d}. \tag{1}$$

Although involving an additional cost, as discussed above, deposits have the advantage of providing a source of funds which allows the bank to hold the mortgage lien on its balance sheet, incentivizing it to undertake efficient actions such as borrower screening.

The alternative mode of operation for an intermediary is to remain unregulated, operating as a broker. Because they do not face the regulatory overhead cost, brokers can originate a loan with a lower opportunity cost of funds. In the above example involving a loan of size $p_1$, the broker's opportunity cost of funds is

$$(1+r)p_1. \tag{2}$$

Brokers cannot commit not to divert deposits, so are unable to finance the mortgage by raising deposits from investors. Brokers' only available option is to originate a mortgage and immediately securitize it, i.e., directly transferring the mortgage lien to investors.

Figure 1 provides a schematic diagram of the two modes of operation for an intermediary. While we allow intermediaries to choose their mode of operation, the analysis would be identical if the mode were exogenous, provided that the market has at least two of each exogenous type (at least two banks and two brokers).

## 3.5. Housing Market

New houses are available in perfectly elastic supply at a price of $p_1$ in period 1 and $p_2$ in period 2. A house which was occupied in period 1 can be resold on the market in period 2. After subtracting the transactions costs involved, the seller obtains only a fraction of the sale price, $\lambda p_2$, where $\lambda \in (0,1)$ measures the liquidation value of previously occupied houses.

We will not model general equilibrium of the housing market, instead taking $p_1$, $p_2$, and $\lambda$ to be exogenous. We will impose one weak condition on these otherwise free parameters, which would follow from a simple general-equilibrium model:

$$(1+r)p_1 > \lambda p_2. \tag{3}$$

If (3) were violated, investors could profit from buying houses at $p_1$ in period 1 and reselling for a return of $\lambda p_2$ in period 2, even if the houses were left vacant in the interim. This behavior would drive up $p_1$ until (3) was reestablished. Condition (3) is not crucial for the analysis, but it does eliminate trivial cases in which screening is worthless because lenders profit from serving the bad type, who is guaranteed to default.

### 3.6. Summary of Timing

Figure 2 summarizes the timing of the game. Events related to the consumer appear above the timeline and those related to intermediaries appear below. At the start of period 1, nature draws the consumer's type $\theta \in \{0,1\}$. This is private information for the consumer. Competing intermediaries $i = 1,\ldots,N$ simultaneously post mortgage contracts $(R_i, S_i)$. The consumer observes the contracts and chooses one, $i$, or none of them. Intermediary $i$ screens (if $S_i = 1$) or not ($S_i = 0$) as specified in the contract, expending $k_I$ if it screens. Intermediary $i$ then obtains funds from investors. If it is a bank, $i$ raises deposits and if it is a broker, $i$ securitizes the loan to investors. The consumer uses the loan to purchase the house at price $p_1$ and derives utility $u_1$ from its services.

In period 2, the consumer's income is realized, either $y$ or 0. The consumer next decides whether or not to repay $R_i$. If the consumer defaults, the lien holder—investors if the mortgage was securitized, the bank if not—forecloses on the house, obtaining liquidation value $\lambda p_2$. If the consumer repays, he stays in the house and obtains utility $u_2$ from its services.

## 4. Equilibrium

### 4.1. Existence and Characterization

In this section we solve for the perfect Bayesian equilibrium of this sequential game of incomplete information. Our focus will be on finding the terms of the equilibrium mortgage contract, denoted $(R^*, S^*)$, emerging from competition among the intermediaries. Several insights help pin down their strategies. First, the profit from serving the bad consumer type is negative. To see this, note that from expressions (1) and (2), the opportunity cost of the funds for the mortgage is at least $(1+r)p_1$. The bad type can never repay in period 2 because it earns no income, so the return on the mortgage comes from the liquidation proceeds $\lambda p_2$. By (3), however, the liquidation proceeds cannot cover the cost of funds.

Thus, competition among intermediaries is directed toward maximizing the good type's payoff. Because intermediaries would never deny a loan to a good type, and the direct screening cost imposed on the consumer $k_C$ is negligible, the good type's payoff from a mortgage contract does not depend (directly at least) on the screening term $S_i$ but only on the repayment term $R_i$. The good type obviously prefers lower values of $R_i$. Hence competition among intermediaries is of the familiar Bertrand form, generating the lowest repayment $R_i$ subject to the intermediary's breaking even.

Before solving for equilibrium value of the continuous variable $R^*$, we will analyze intermediaries' discrete decisions. Intermediaries have two discrete decisions, which are both binary: choosing whether to function as a bank or a broker and then choosing whether to specify screening in the contract, $S_i \in \{0,1\}$. We will argue that, of the four combinations of the two binary decisions, only two are relevant in equilibrium. Screening brokers and non-screening banks can be ruled out, leaving screening banks and non-screening brokers as the only possibilities.

To rule out screening brokers, note that they are unregulated, so cannot raise deposits. A broker's only option is to securitize the mortgage, directly transferring the lien to investors after origination. Because brokers and investors can divert funds, any promised transfers conditional on outcomes—such as successful repayment by the consumer—are not credible. The broker obtains at best a fixed commission for origination, transferring the residual claim on the borrower's repayment stream to the investor. Because it is not a residual claimant on the repayment stream, the broker has no incentive to screen the consumer. Screening provides no benefits for the broker, only costs. Besides the expense $k_I$, screening reduces the probability of a successful origination (and the resulting commission) if the bad type is rejected. Thus brokers do not screen in equilibrium.

To rule out non-screening banks, note that raising deposits involves a higher opportunity cost of funds than securitizing, (1) rather than (2), due to the additional regulatory expense. A non-screening bank will always be undercut by a broker in Bertrand competition. The only way a bank could be observed in equilibrium is if it used deposits to pursue a strategy not open to brokers, that is, to give itself an incentive to screen the consumer. Hence non-screening banks are ruled out in the benchmark model.[14] Furthermore, there is no reason for the bank to pay the regulatory and screening costs if it does not reject the bad type. Thus, if a bank is observed in equilibrium, it must screen and reject the bad type.

Bertrand competition ends up driving the equilibrium repayment down to the intermediary's zero-profit level. Let $R^{SBK}$ be the zero-profit repayment for a screening bank and $R^{NBR}$ be the zero-profit repayment for a non-screening broker. We will solve for $R^{SBK}$ and $R^{NBR}$ and compare them to determine which form of intermediary "wins" in Bertrand competition.

First, consider the outcome with screening banks. As argued above, the bank rejects the bad type in equilibrium. Given that he would certainly be rejected, the bad type does not apply to a screening bank for a mortgage because he thus avoids even a negligible personal screening cost $k_C$.[15] Assume for the moment that the good type accepts the mortgage contract. Further, assume that he is willing and able to repay the screening bank's posted repayment, $R_i$, when his income is high, i.e. $y$. (We will investigate the conditions under which these assumptions hold shortly.) The profit for a representative screening bank then is

$$\gamma \left[ hR_i + (1-h)\lambda p_2 - \frac{(1+r)p_1}{1-d} - k_I \right]. \tag{4}$$

With probability $\gamma$, the consumer is a good type and applies to the bank for a mortgage. Conditional on applying, with probability $h$, the good type earns high income and repays the bank $R_i$. With probability $1-h$, the consumer cannot repay; the bank forecloses and earns proceeds $\lambda p_2$. The bank's costs include the opportunity cost of funds for a regulated intermediary from (1) and the screening cost $k_I$. Setting (4) to zero and solving yields the zero-profit repayment for screening banks:

$$R^{SBK} = \lambda p_2 + \frac{1}{h} \left[ \frac{(1+r)p_1}{1-d} + k_I - \lambda p_2 \right]. \tag{5}$$

Next, consider non-screening brokers. We argue in a series of steps that if a broker is active

[14] In the extension with renegotiation in Section 6, the ability to renegotiate is an additional advantage of banks over brokers. If this advantage is sufficiently great, banks may be observed in equilibrium even if they do not screen.

[15] This is where the introduction of a small but positive $k_C$ is used in the analysis. See footnote 13 for further discussion.

in equilibrium, this broker must be the only active intermediary and must serve all consumers of all types. Note first that if brokers are active, all bad types end up being served by a broker. The bad type strictly benefits from obtaining a mortgage because he gains at least the utility $u_1$ from period-1 housing services. Because brokers do not screen, they have no information to exclude the bad type. Let $i$ index an active broker serving a bad type with some probability. Note second that because bad types generate losses $i$ must have some positive probability of serving a good type as well. Moreover, it can be shown that this probability must be 1 in equilibrium. If this probability were less than 1, the good type must be indifferent between $i$ and some other intermediary, implying that this other intermediary must be offering the same repayment $R_i$ as does $i$. But $i$ then has a strictly profitable deviation: $i$ can obtain a discrete jump in the probability of attracting the good type to 1 with an infinitesimal reduction in $R_i$. Hence any intermediary having any probability of serving a bad type must serve all good types, so there cannot be any other active intermediaries, because there are only loss-making bad types left to serve. Note finally that since $i$ is the only active intermediary, by our previous argument all bad types will apply and be served by $i$. In sum, if a broker is active in equilibrium, it alone must serve all consumers of all types.

Because brokers immediately securitize mortgages, broker $i$'s profit must come in the form of a fixed origination fee, denoted $F_i$. This fee must be low enough to ensure that investors' ex ante expected return net of the fee is sufficient to cover their opportunity cost of funds:

$$\gamma h R_i + (1-\gamma h)\lambda p_2 - (1+r)p_1 - F_i \geq 0. \tag{6}$$

Because $i$ serves all consumers of all types, the probability that the borrower makes the repayment $R_i$ is $\gamma h$, the unconditional probability of being a good type times the probability that the good type earns high income. (Again we are making the implicit assumption that the high-income consumer is willing and able to make the repayment, an assumption we will investigate shortly.) With probability $1-\gamma h$, the consumer earns no income and thus defaults. Investors then earn foreclosure proceeds $\lambda p_2$. Subtracting the investors' opportunity cost of funds in the unregulated case given by (2) and $i$'s origination fee leaves (6). No screening costs need to be subtracted because neither $i$ nor the investors screen. Bertrand competition among intermediaries results in the zero-profit origination fee $F_i = 0$ and the zero-profit repayment, which can be found by substituting $F_i = 0$ into (6), treated as an equality, and solving:

$$R^{NBR} = \lambda p_2 + \frac{1}{\gamma h}\left[(1+r)p_1 - \lambda p_2\right]. \tag{7}$$

We argued above that Bertrand competition among intermediaries selects the mode of lending that provides the good type with a higher expected payoff at the repayment levels calculated above, which by construction are feasible for the lenders to offer. The good type's payoff from an arbitrary mortgage contract $(R_i, S_i)$ is

$$u_1 + h(u_2 + y - R_i), \tag{8}$$

consisting of the utility in period 1 from housing consumption and, if he earns positive income, which happens with probability $h$, the utility from period-2 housing consumption and the income $y - R_i$ left over after the mortgage repayment. Notice that, because the consumer's personal screening costs are negligible, the only contractual term showing up in the good type's payoff is $R_i$. Because it enters with a minus sign, the good type chooses the intermediary with the lower repayment. Equilibrium involves a screening bank if $R^{SBK} < R^{NBR}$ and a non-screening broker if $R^{NBR} < R^{SBK}$

The preceding analysis took for granted that the consumer would accept either contract and would repay if he earned positive income. We need to tie up this loose end by deriving conditions under which the consumer behaves this way. In particular, given an arbitrary mortgage contract $(R_i, S_i)$, we will derive a constraint ensuring that the good consumer type accepts the contract (participation constraint) and a constraint ensuring that the high-income consumer repays (repayment constraint).

Begin with the repayment constraint. Clearly a consumer with no period-2 income will always default regardless of type. It remains to see when a consumer with positive income would repay or default. As can be seen with the help of the timeline in Figure 2, in the continuation game following realization of high income, incomplete information plays no material role. The last decision in the continuation game is the lien holder's choice of whether or not to foreclose. Foreclosing is a dominant strategy if the consumer defaults (regardless of any private consumer information) because the payoff is $\lambda p_2$ from doing so rather than nothing. Anticipating this, the high-income consumer will repay (regardless of any private information) if his utility from staying in the house exceeds the payoff from the alternatives. Repayment yields the consumer $u_2 + y - R_i$, default and repurchase yields $u_2 + y - p_2$, and default without repurchase yields $y$. Comparing these payoffs, the consumer weakly prefers repayment when

$$R_i \leq \min\{y, p_2, u_2\} \equiv m, \tag{9}$$

and defaults otherwise.

The consumer's repayment decision constrains the set of feasible values of the zero-profit repayments. For example, if $R^{SBK} > m$, screening banks are ruled out in equilibrium because the consumer never repays the loan to a screening bank: the intermediary earns just the foreclosure proceeds $\lambda p_2$, which by (3), cannot cover its cost of funds. Similar reasoning implies that non-screening brokers are ruled out in equilibrium if $R^{NBR} > m$. Repayment of $R^{SBK}$ or $R^{NBR}$ by the consumer is therefore feasible whenever (9) holds at these values.

Next consider the consumer's participation constraint. The good type weakly prefers to accept an offer from an intermediary if its payoff in (8) is non-negative. Rearranging provides a bound on the repayment:

$$R_i \leq y + u_2 + \frac{u_1}{h}. \tag{10}$$

The right-hand side of (10) obviously exceeds $m$. Thus we can ignore the participation constraint because it is automatically satisfied if the repayment constraint (9) holds.

Summarizing the preceding analysis, the equilibrium involves lending by the intermediary with the lower of $R^{NBR}$ and $R^{SBK}$ unless both exceed $m$, in which case no repayment can simultaneously satisfy the repayment constraint and allow intermediaries to break even. The following proposition states these results formally for reference.

**Proposition 1.** *Assume* (3) *holds. The equilibrium falls into one of the following three cases.*

*(i) If $R^{NBR} < \min\{R^{SBK}, m\}$, then brokers originate all mortgages in equilibrium, securitizing them immediately. Equilibrium mortgage terms are $R^* = R^{NBR}$ and $S^* = 0$. Both good and bad consumer types are served.*

*(ii) If $R^{SBK} < \min\{R^{NBR}, m\}$ then banks originate all mortgages by raising deposits from investors and hold the mortgages for both periods. Equilibrium mortgage terms are $R^* = R^{SBK}$ and $S^* = 1$. Only good consumer types receive mortgages. Bad types are screened and rejected if they apply and so do not apply in equilibrium.*

*(iii) If $m < \min\{R^{NBR}, R^{SBK}\}$, then there is no mortgage lending in equilibrium.*

*For each of the cases (i)–(iii), the stated outcome is also an equilibrium if any of the strict inequalities involved holds as an equality. There are no other equilibria.*

Appendix B provides a fully rigorous proof for general values of $k_C > 0$, verifying the existence of the posited equilibria and proving uniqueness by ruling out an exhaustive set of alternatives.

Figure 3 depicts the equilibrium. Curve $AG$ is the good type's indifference curve, along which $R^{SBK} = R^{NBR}$. Ignoring the repayment constraint for the moment, this curve by itself delineates when there is broking versus banking in equilibrium. In the dark-shaded region above the curve—i.e., for high values of $\gamma$ and $k_I$—non-screening brokers undercut screening banks. The reverse is true in the light-shaded region below the curve. To see why these regions are positioned as they are, it is obvious that non-screening brokers are more efficient at supplying mortgages to the good type for sufficiently high $\gamma$. The only advantage of banks is in their holding of mortgages, which incentivizes them to screen out loss-making bad types. If the share of good types is sufficiently high (implying that the share of bad types is sufficiently low), the advantage of banks disappears. What remains is the brokers' advantage in economizing on the costs of regulation and screening. Obviously non-screening brokers are more efficient than screening banks if the screening cost $k_I$ is sufficiently high.

The repayment constraint (9) for broking, which $R^{NBR}$ must satisfy, is represented in Figure 3 by the vertical line $BF$. It is vertical because $R^{NBR}$ is independent of screening costs. To the left of this line, the share of good types $\gamma$ is too low for broking to be feasible; but to the right, $\gamma$ is sufficiently high. The repayment constraint for banking, which $R^{SBK}$ must satisfy, is represented by the horizontal line $CE$. It is horizontal because $R^{SBK}$ is independent of $\gamma$, which in turn is true because a bank only serves good types, so its profit margin is independent of the share of good types. Above this line screening banks are not feasible because screening cost $k_I$ is too high. Below the line, this mode is feasible. Both modes of lending are infeasible in the unshaded, rectangular region, corresponding to case (iii) in Proposition 1. No mortgages are signed in this region. The share of bad types is too high for non-screening brokers to be viable, and the cost of screening is too high for screening banks to be viable. A whole section below (Section 4.2) is devoted to a more detailed discussion of comparative-statics results with respect to these parameters as well as for parameters not plotted on the axes, changes in which can be represented by shifting the curves in the figure.

Proposition 1 implies that only one form of intermediary is active in the market.[16] This prediction may seem unrealistic at first glance. However, the model can easily be viewed as applying to one among several market segments, each with different observable characteristics (reflected in parameters $\gamma$, $h$, $y$, $p_1$, etc.), each of which can be served by the intermediary specified by the proposition. This view allows different forms of intermediary to be active at the same time.[17]

## 4.2. Comparative Statics

The model yields sharp comparative-statics results, which as discussed in the introduction are consistent with key aspects of the history of securitized mortgage lending in the United States over the last several decades. We divide the comparative-statics analysis into three parts. First, we consider how parameter changes affect the prevalence of mortgage lending of any form. Second, we examine the impact of parameter changes on the prevalence of securitized versus bank-held mortgages. Finally, we examine the impact of parameter changes on mortgage prices, quantities, and default rates. The analysis of default rates will shed some light on the stability of the mortgage market even in our partial-equilibrium setting.

The next proposition provides comparative-statics results for overall mortgage lending, whether by screening banks or securitizing brokers.

**Proposition 2.** *There is (weakly) more mortgage contracting the greater are $y$, $u_2$, $p_2$, $h$, $\lambda$ and $\gamma$ and the lower are $p_1$, $r$, $d$ and $k_I$.*

These results are intuitive: the parameter changes all serve to relax the repayment constraint (9) for both lending modes.

The proposition can be easily proved using Figure 3. Whether or not there is mortgage lending depends only on the size of the unshaded "no contract" region, not on the indifference curve $AG$, which delineates the lending mode rather than the existence of a mortgage. Consider an increase in $y$. As can be seen from the formulae (5), (7), and (9), an increase in $y$ has no effect on $R^{SBK}$ or

[16] On the boundary $DG$, there is an equilibrium with screening banks and another with non-screening brokers. There are no equilibria in which both forms of intermediary are active in the market together. This follows from the argument in the text leading up to Proposition 1 that, if a broker is active, it must be the sole active intermediary.

[17] Both forms of intermediary can be active in an alternative version of the model in which good types always have positive income and bad types have positive income with probability $h$ (i.e., we switch from the case in which bad types are perfectly bad to that in which good types are perfectly good). In this alternative, lending to bad types is no longer guaranteed to be unprofitable. There exists an intermediate region of parameters in which banks and brokers simultaneously offer mortgages, banks serving good types and brokers serving bad types.

$R^{NBR}$ but increases $m$. A consumer with higher income can afford a greater range of repayments. This relaxes the repayment constraints for both banking and broking, represented by an upward shift in $CE$ and a leftward shift in $BF$, shrinking the size of the unshaded region. Thus there is lending for a larger set of parameters besides $y$. An increase in $u_2$ has a similar effect, relaxing the repayment constraints in this case by increasing the consumer's attachment to his house, making him less likely to strategically default.

A rise in $p_2$ also makes repayment more likely, but through two distinct channels. One channel is through a reduction in strategic defaults. Defaulting and purchasing an equivalent home becomes more expensive with $p_2$, represented mathematically by an increase in $m$ on the right-hand side of (9). The other channel is through an increase in foreclosure proceeds, which allows intermediaries to reduce the break-even repayment levels $R^{SBK}$ and $R^{NBR}$. One can see this mathematically by differentiating (5) and (7): $\partial R^{SBK}/\partial p_2 = -\lambda(1-h)/h < 0$ and $\partial R^{NBR}/\partial p_2 = -\lambda(1-\gamma h)/\gamma h < 0$. An increase in $\lambda$ has this same effect on foreclosure proceeds, thus also reducing the zero-profit repayments, relaxing the repayment constraints. A rise in $h$ also reduces the zero-profit repayments, in this case by increasing the probability that the consumer can afford the repayment.

Reductions in $r$ and $p_1$ reduce the loan amount, therefore also relaxing the repayment constraints by reducing $R^{SBK}$ and $R^{NBR}$, shifting $CE$ up and $BF$ right, shrinking the unshaded region. A reduction in $d$ reduces the bank's operating costs, reducing $R^{SBK}$ ($R^{NBR}$ is left unchanged because brokers do not pay regulatory costs), thus shrinking the unshaded region.

The effect of a rise in $\gamma$ or a fall in $k_I$ are easy to see in Figure 3 because rather than shifting the curves, these variables appear directly on the axes. Because the unshaded region is in the upper left of the graph, either parameter change may cause a point in the unshaded region to move outside but would never move a point outside back in. Turning from the graph to the underlying economic intuition, when the share of good types increases, $R^{NBR}$ falls because brokers serve both types. $R^{SBK}$ is unchanged because banks only serve good types, so their profit *margin* is unaffected by the share of good types. When the screening cost $k_I$ falls, $R^{NBR}$ is left unchanged but $R^{SBK}$ falls.

The next proposition examines the impact of parameter changes on the prevalence of broking/securitization, represented by the area of the dark-shaded region in Figure 3. This area depends on the position of the vertical line $BF$, which we have already analyzed, representing the repayment constraint for broking, as well as the position of the good type's indifference curve $AG$. To understand how the parameters affect the position of $AG$, substitute the expressions for the zero-profit repayments, (5) and (7), into the equation $R^{SBK} = R^{NBR}$ and rearrange, giving the following equation for $AG$:

$$\frac{d}{1-d}(1+r)p_1 + k_I = \frac{1-\gamma}{\gamma}[(1+r)p_1 - \lambda p_2]. \tag{11}$$

The left-hand side captures the inefficiencies associated with a banking contract, including the regulatory cost $d$ per dollar of the total amount due depositors, as well as the screening cost $k_I$. The right-hand side captures the inefficiencies associated with the broking contract from the good type's perspective. Because lenders earn zero profit from mortgages in equilibrium, the good type is the residual claimant of any profit or loss generated by the mortgage. The right-hand side of () consists of the expected loss $(1+r)p_1 - \lambda p_2$ from serving a bad type scaled by the odds $(1-\gamma)/\gamma$ of serving a bad type. This loss gets passed on to the good type through a higher repayment in a broking contract. (Banks avoid this loss by deterring bad types through a commitment to screen.)

**Proposition 3.** *There is more broking/securitization the greater is $y$, $u_2$, $p_2$, $h$, $\lambda$, $\gamma$, $d$ and $k_I$ and the lower is $p_1$ and $r$.*

Before we prove these results, it is worth emphasizing that they are consistent with the changes in economic conditions that accompanied the rise in securitization over the last several decades. The housing boom prior to the crisis with its sharply rising prices can be captured by an increase in $p_2$ holding $p_1$ constant. Proposition 3 implies that this parameter change would lead to more securitization. The proposition also predicts that a low interest rate spurs securitization. Some argue that Federal Reserve Board policy kept interest rates artificially low prior to the crisis. Securitization is also spurred by general boom in economic conditions, as captured by various parameters in the model including a higher income $y$, a higher probability of high income $h$, or higher probability that a consumer is a good type $\gamma$. An increase in the cost of traditional banking—as captured by $d$ and $k_I$—also spurs securitization.

Turning to a proof of Proposition 3, by (), increases in $y$, $u_2$ and $h$ do not affect the position of $AG$ in Figure 3. However, by Proposition 2, increases in these variables relax the repayment constraint for broking, shifting $BF$ left, thereby increasing securitization. An increase in $p_2$ also relaxes the repayment constraint for broking, shifting $BF$ left. It also shifts $AG$ left by inducing a larger reduction in $R^{NBR}$ than $R^{SBK}$. This is because foreclosure proceeds, which depend on $p_2$

matter more with broking than banking. Brokers serve bad as well as good types, and so their mortgages have a higher associated default rate *ceteris paribus*. A rise in $\lambda$ or $\gamma$ also benefit broking contracts relatively more than banking contracts, shifting $AG$ left, while also relaxing the repayment constraints for broker mortgages. Finally, higher $d$ or $k_I$ increases $R^{SBK}$ but leaves $R^{NBR}$ constant, shifting $AG$ left, expanding the set of cases in which brokers can undercut bankers.

The last two comparative-statics results in the proposition involve a fall in $p_1$ or $r$, which together comprise the opportunity cost of the loan to investors, $(1+r)p_1$. This reduction in the opportunity cost reduces $R^{NBR}$, shifting $BF$ left, expanding the set of parameters for which broking is feasible. This reduction also reduces $R^{SBK}$, so it is not at first obvious which way $AG$ shifts. We will argue that $AG$ shifts left as well, so that the expansion of the broking region is unambiguous. The easiest way to see this point is to treat $(1+r)p_1$ as the numeraire by which we divide all the dollar values in (). Then it is clear that a reduction in $(1+r)p_1$ has two real effects. One is a rise in the real screening cost $k_I/(1+r)p_1$, decreasing the attractiveness of banking relative to broking. The other is a rise in the real foreclosure proceeds $\lambda p_2/(1+r)p_1$, increasing the attractiveness of broking relative to banking. Bother effects contribute to a leftward shift in $AG$, expanding the broking region.

So far, the comparative-statics results have touched on the size of the no-contract and broking regions in Figure 3. For completeness, it is worth summarizing the effect of parameter changes on the remaining—banking—region. For most of the parameters, the effect on banking is ambiguous. For example, consider $p_1$ and $r$. As just mentioned, a decrease in these expands the broking region. But because repayments $R^{NBR}$ and $R^{SBK}$ fall, the no-contract region shrinks. The net effect on the size of the banking region is ambiguous. For most of the other parameters, the same is true: whatever expands the broking region contracts the no-contract region, and whatever contracts the broking region expands the no-contract region, generating an ambiguous effect on the residual, banking, region. As the next proposition states, there are three parameters for which we can be more concrete.

**Proposition 4.** *There is more banking with screening the lower are $\gamma$, $d$ and $k_I$. Other parameters have an ambiguous effect.*

A fall in $\gamma$, which is equivalent to an increase in the share of bad types in the market, has no impact on $R^{SBK}$ because screening banks do not serve any bad types. Thus banking feasibility is unaffected, implying that line $CE$ in Figure 3 does not shift. However, an increase in the share of bad types increases $R^{NBR}$ because the good type must finance not only the state of nature where it has low income, but also the state where the consumer is a bad type. The right-hand side of the indifference condition () rises, resulting in a rightward shift in $AG$. Hence the banking region expands unambiguously. A fall in $d$ or $k_I$ reduces $R^{SBK}$ and leaves $R^{NBR}$ unchanged, shifting $CD$ upwards and $AG$ outwards, thus expanding the banking region.

Propositions 2–4 effectively consider each region of Figure 3 in isolation. However, there is a relationship among them. Banking can form a buffer between the no-contract region, in which the environment is inhospitable to lending, and the broking region, in which conditions are ripe for efficient lending. In the banking region, extra costs can be expended to make lending feasible when it would not be otherwise.

**Proposition 5.** *There exist individual parameters for which the banking interval is intermediate between a broking interval at one extreme (either low or high) and a no-contract interval at the other extreme. There is never a broking interval between no-contract and banking intervals.*

Proposition 5 is a corollary of a much richer comparative-statics result, Theorem 2, which characterizes how equilibrium outcomes change with increases in the value of every parameter in the model over its entire range. Theorem 2 is stated and proved in Appendix B.

Additional comparative-statics results provide a rigorous theoretical explanation for recent empirical work by Keyes *et al.* (2010). The authors assert that in the U.S. mortgage market, before the crisis, a threshold FICO score of 620 emerged as an industry standard or "rule of thumb": it was relatively easy for originators to securitize mortgages for borrowers rated above this threshold and difficult for them to securitize loans with scores below. Mortgages below the threshold thus tended to be held by the originator as they are by screening banks in our model. Keyes *et al.* also found a discontinuous increase in default risk as the FICO score moved from just below the threshold to just above. As we will show below, this predicted by our theory: banks use additional soft information for (retained) mortgages just below some threshold, screening out the bad consumer type and moderating the default risk otherwise embodied in such loans. Consumers above the threshold are not screened, implying that mortgage liens consist of both the good and bad consumer type. The consequence is a discrete upwards jump in default risk.

Three parameters in the model, $h$, $\gamma$, and $y$, are related to a consumer's FICO score. Figure 4 illustrates how these parameters affect equilibrium mortgage quantity and quality. The graphs are drawn for one of the parameters, $h$, but the graphs for $\gamma$ and $y$ are similar. The figure is drawn for the interesting case in which each of the three possibilities—no contract, banking, broking—arises for certain values of $h$. In particular, there is no contract below $h'$, banking between $h'$ and $h''$, and broking above $h''$.[18]

As the first panel in Figure 4 shows, equilibrium mortgage quantity (captured in this representative consumer model by the probability that the consumer obtains a mortgage) is weakly increasing in $h$. In the initial no-contract interval, quantity equals 0. In the next banking interval, good types receive mortgages. At the threshold $h''$ between the banking and broking intervals quantity jumps as now all consumers receive mortgages. As shown in the lower panel, default risk also jumps at the margin between banking and broking. Technically, default risk jumps from $1-h''$ to $1-\gamma h''$ at $h = h''$. We expect this jump to be substantial in practice since it captures the bank's use of soft information to screen borrowers who have FICO scores around the margin where securitization becomes feasible. The model thus provides a rigorous explanation of the discontinuity in default risk for securitized and non-securitized mortgages at a FICO of 620 along with other comparative statics in Keyes' et al. (2010).

For reference, the next proposition formalizes the most important of the comparative-statics results for $h$ and $\gamma$.

**Proposition 6.** *Consider the consumer's credit worthiness parameters $h$, $\gamma$, or $y$. An increase in any of these parameters weakly increases mortgage volume (captured by the probability that the consumer receives a mortgage). An increase any of these parameters weakly decreases the default probability over the parameter's whole range except for an upward jump at the margin between banking and broking (whenever this margin exists).*

Proposition 6 is also a corollary of the much more general comparative-statics result, Theorem 2, stated and proved in Appendix B.

## 4.3. Welfare

Propositions 1–6 make positive predictions regarding equilibrium. Here, we answer normative questions, comparing social welfare in the equilibrium under broking to that under banking.

[18]Other cases are possible, including one of the three intervals being empty. Proposition 5 rules out the possibility that broking is observed between no-contract and banking intervals. See Proposition **??** in Appendix B for details.

Because intermediaries and investors earn zero expected profit in equilibrium, social welfare is identical to consumers' payoff. In the continuation equilibrium with banking, the consumers' payoff is given by the good type's payoff alone because the bad type is screened and does not receive a mortgage. Substituting $R^{SBK}$ from (5) into the expression for the good type's payoff, (8), yields expected social welfare with screening banks

$$W^{SBK} = \gamma[u_1 + h(u_2 + y - R^{SBK})]. \tag{12}$$

The expression is pre-multiplied by the probability of a good type, $\gamma$. In the continuation equilibrium with broking, both good and bad types obtain a positive payoff. The expected payoff across these types is

$$W^{NBR} = \gamma[u_1 + h(u_2 + y - R^{NBR})] + (1-\gamma)u_1. \tag{13}$$

With probability $\gamma$, the consumer is the good type, obtaining a payoff found by substituting $R^{NBR}$ into expression (8). With probability $(1-\gamma)$, the consumer is the bad type, and obtains $u_1$: after consuming housing services in period 1, he is always forced out of the house because he never has the income to repay.

Subtracting (12) from (13) yields

$$W^{NBR} - W^{SBK} = \gamma h(R^{SBK} - R^{NBR}) + (1-\gamma)u_1. \tag{14}$$

The right-hand side has two terms. The first captures the good type's payoff from broking relative to banking. This is the same as the condition determining the form of intermediary emerging in equilibrium. In particular, it equals 0 when $R^{SBK} = R^{NBR}$ so that the good type is indifferent between broking and banking. The second term, $(1-\gamma)u_1$, reflects the bad type's payoff from a mortgage. It represents a wedge between the equilibrium and the efficient form of intermediary. The bad type's payoff contributes to social welfare, but is not valued by lenders in equilibrium as they are unable to finance loss-making bad types. This positive externality associated with broking leads to insufficient broking with securitization and excessive banking with screening in equilibrium. Formally, we have the following proposition.

**Proposition 7.** *If equilibrium entails securitizing brokers, this is the socially optimal form of in-*

*termediary. There is a set of parameters for which equilibrium entails screening banks but the continuation equilibrium with a securitizing broker would be socially more efficient.*

The formal proof is almost immediate from (14). If equilibrium involves broking, then $R^{NBR} \leq R^{SBK}$ by of Proposition 1, implying that the first term of (14) is non-negative. Given the second term is positive, $W^{NBR} > W^{SBK}$. One can easily generate cases in which equilibrium involves inefficient banking. Take any parameters for which both forms of intermediary are feasible but banking emerges in equilibrium. In the limit as $u_1 \to \infty$, we have $W^{NBR} > W^{SBK}$.

# 5. Equilibrium Strategic Default

The benchmark model allowed for the possibility of strategic default. In particular, if the contractual repayment satisfied $R_i > p_2$, then the consumer would prefer to default on the mortgage and purchase a different home at price $p_2$, thus saving $R_i - p_2$. Still, strategic default never occurred in equilibrium. If $R_i > p_2$, the consumer always defaults, and the bank only obtains the foreclosure proceeds, which by (3) are insufficient to cover the principal and interest on the initial loan: For any case where there would be strategic default, there simply would be no mortgage lending.

This section extends the model to the case of stochastic period-2 house prices, $\tilde{p}_2$. This extension allows for the possibility of strategic default in equilibrium—for low realizations of $\tilde{p}_2$—yet for lending to still be feasible—as repayments can be made for higher realizations of $\tilde{p}_2$. Besides allowing the possibility of equilibrium strategic default, the model with stochastic house prices is interesting in its own right because it allows for an analysis of the economic impact of volatile house prices, an important recent issue in the mortgage market. A further use of the stochastic-price model is that it will allow us to analyze renegotiation between the consumer and bank, which may happen if a buyer can threaten to purchase a new home at a lower price than the contractual repayment. We analyze renegotiation in Section 6.

## 5.1. Model with Stochastic Prices

Let the second-period house price be a Bernoulli variable $\tilde{p}_2$ taking on high value $\bar{p}_2$ with probability $\phi$ and low value $\underline{p}_2$ with probability $1 - \phi$. For ease of comparison, assume the mean of $\tilde{p}_2$

is the same as in the benchmark model:

$$p_2 = \phi \bar{p}_2 + (1-\phi)\underline{p}_2. \tag{15}$$

Several additional parameter restrictions will help streamline the analysis. First, we will consider positive values of $\underline{p}_2$ in a neighborhood of 0, indicated with the limit notation $\underline{p}_2 \to 0$. By making $\underline{p}_2$ as low as possible, this restriction represents extreme volatility, thus making the contrast with the benchmark model as stark as possible. It also pins down the consumers' behavior in the low-price state, ensuring he strategically defaults if he does not default for other reasons (such as if he has no income). Second, we restrict attention to sufficiently high values of $y$ and $u_2$—indicated with the limit notation $y, u_2 \to \infty$—so that these parameters do not constrain feasibility. This eliminates all the channels of consumer default except for the focus of this section: the consumer defaulting when the second-period house price is so low that it is cheaper to buy a different house than to repay the existing mortgage. After taking limits, repayment constraints $R^{NBR}, R^{SBK} \leq m = \min\{y, p_2, u_2\}$ in the benchmark model would reduce to $R^{NBR}, R^{SBK} \leq p_2$. Finally, we restrict attention to values of $p_2$ sufficiently high that both forms of lending would be feasible in the benchmark model: $R^{NBR}, R^{SBK} < p_2$. As we will show, under this restriction, both modes of lending will also be feasible under stochastic prices and the equilibrium form of lending will remain the same when moving from deterministic to stochastic prices. This will enable us to derive comparative-static results for the effect of a move from deterministic to stochastic prices on the equilibrium mode of lending.

## 5.2. Equilibrium with Stochastic Prices

As in the deterministic-price model, the consumer defaults if he has low income. The new outcome with stochastic prices is that the consumer now also defaults in the high-income, low-price state. We know the consumer strategically defaults in this state because the limit value $\underline{p}_2 \to 0$ of the low price is less than any finite repayment, so the consumer always finds it cheaper abandon his current house, saving the mortgage repayment, and buying a different house. Thus the only state in which the consumer repays is when he earns positive income and the house price is high. Bertrand competition drives this repayment to the zero-profit level, denoted $\hat{R}^{NBR}$ for a mortgage originated by a broker and $\hat{R}^{SBK}$ for a mortgage originated by a screening bank.

Repayment $\hat{R}^{SBK}$ satisfies the screening bank's zero profit condition

$$\gamma\left[h\phi\hat{R}^{SBK}+(1-\phi)\lambda\underline{p}_2+(1-h)\phi\lambda\bar{p}_2-\frac{(1+r)p_1}{1-d}-k_I\right]=0. \tag{16}$$

The bank only makes the loan to the good type, which nature selects with probability $\gamma$. Conditional on this, it receives repayment $\hat{R}^{SBK}$ only in the high-income, high-price state, which occurs with probability $h\phi$. The bank obtains foreclosure proceeds $\lambda\underline{p}_2$ in the low-price state regardless of the borrower's income ( i.e. with probability $1-\phi$) and $\lambda\bar{p}_2$ in the low-income, high-price state with probability $(1-h)\phi$. The last two terms on the left-hand side of (16) capture the bank's cost of making the loan.

Repayment $\hat{R}^{NBR}$ yields zero profits for investors in the securitized mortgage given zero originating fee for the broker:

$$\gamma h\phi\hat{R}^{NBR}+(1-\phi)\lambda\underline{p}_2+(1-\gamma h)\phi\lambda\bar{p}_2-(1+r)p_1=0. \tag{17}$$

Investors only receive the repayment when the consumer is the good type, has high income, and the house price is high. When the price is low, the consumer—regardless of its type—defaults, and the broker obtains $\lambda\underline{p}_2$. Whenever price is high and the consumer is the good type but without high income—which happens with probability $(1-\gamma h)\phi$—the broker receives the high liquidation value $\lambda\bar{p}_2$. The last term on the left-hand side of (17) captures the investors' opportunity cost of funds for the loan amount.

Using the formulas for $R^{SBK}$ and $R^{NBR}$ from (5) and (7) and rearranging, (16) and (17) give

$$R^{SBK} = \phi\hat{R}^{SBK}+(1-\phi)\lambda\underline{p}_2 \tag{18}$$

$$R^{NBR} = \phi\hat{R}^{NBR}+(1-\phi)\lambda\underline{p}_2. \tag{19}$$

These equations are intuitive. Because the lender's costs are the same regardless of whether house prices are deterministic or stochastic, the lender's expected revenue with stochastic prices must be the same as the expected revenue generating zero profits with deterministic prices (although the repayments, of course, will be different). Further, the components of expected revenue only differ between the stochastic and deterministic cases in the high-consumer-income state; in the other states of nature the consumer always defaults, leaving the intermediary with the liquidation revenue $\lambda p_2$. When house price is deterministic, revenue in the high-consumer-income state is simply the repayment given by the left-hand sides of (18) and (19). When house price is stochastic, the high-income consumer only repays when house price is high, which happens with probability $\phi$. With probability $1-\phi$ price is low, the consumer strategically defaults and the intermediary receives only the liquidation value $\lambda p_2$.

Using the formula for $R^{SBK}$ together with condition (3) (ensuring that foreclosure proceeds do not cover the loan cost) we obtain $R^{SBK} > \lambda \underline{p}_2$. But then (18) implies $\hat{R}^{SBK} > R^{SBK}$ and similarly (19) yields $\hat{R}^{NBR} > R^{NBR}$. Thus mortgage repayments must increase when prices are stochastic. Intuitively, mortgage repayments must cover the additional contingency of strategic default in the stochastic case, along with default caused by low income which happens in both cases.

We have seen that Bertrand competition in the deterministic-price model leads to the form of lending with the lower of the two repayments $R^{NBR}$, $R^{SBK}$. Similar logic implies that Bertrand competition in the stochastic-price model leads to the form of lending with the lower of the two repayments $\hat{R}^{NBR}$, $\hat{R}^{SBK}$. But (18) and (19) imply that $\hat{R}^{NBR}$ and $\hat{R}^{SBK}$ preserve whatever inequality exists between $R^{NBR}$ and $R^{SBK}$ because the same constant is added in both equations. One can check that if we substitute the parameter restrictions we have adopted in this section ($y, u_2 \to \infty$, and $R^{NBR}, R^{SBK} < p_2$) back into Proposition 1, there is always lending in equilibrium in the deterministic model. The repayment constraints $R_i \leq m$ do not bind. To prove that the equilibrium form of lending is the same in the deterministic- and stochastic-price model requires one more step. We need to demonstrate that $\hat{R}^{NBR}$ and $\hat{R}^{SBK}$ satisfy their respective repayment constraints. Because we have taken $y, u_2 \to \infty$, the last step reduces to showing that $\hat{R}^{SBK}$ and $\hat{R}^{NBR}$ are not so high as to induce strategic default by the high-income consumer in the high-price state: i.e., $\hat{R}^{SBK}, \hat{R}^{NBR} < \bar{p}_2$. This fact is established in the proof of Proposition 8 in Appendix B. We have the following.

**Proposition 8.** *Impose the parameter restrictions $y, u_2 \to \infty$, $\underline{p}_2 \to 0$, $R^{NBR}, R^{SBK} < p_2$, and equation* (15)*.*

*(i) In the benchmark, deterministic-price model, a mortgage contract is always signed in equilibrium. Let $R^*$ denote the equilibrium repayment. If $R^{NBR} < R^{SBK}$, the contract is of-*

*fered by a securitizing broker, specifying repayment $R^* = R^{NBR}$. If $R^{SBK} < R^{NBR}$, the contract is offered by a screening bank, specifying repayment $R^* = R^{SBK}$.*

*(ii) In the stochastic-price model, a mortgage contract is also always signed in equilibrium, offered by the same form of lender as in the benchmark model. The good type of consumer strategically defaults in the high-income, low-price state. The repayment $\hat{R}^*$ is higher than in benchmark model, satisfying $R^* = \phi\hat{R}^* + (1-\phi)\lambda \underline{p}_2$. Expected welfare is lower than in the benchmark model by $\gamma h(1-\phi)(1-\lambda)\underline{p}_2$.*

The expression for the welfare loss in moving from deterministic to stochastic prices can be derived by direct calculation. Intuitively, the welfare loss stems from the new possibility of strategic default: the borrower strategically defaults in the high-income, low-price state, which arises with probability $\gamma h(1-\phi)$ (from an ex-ante perspective, before the consumer's type is drawn). The welfare loss is the friction $(1-\lambda)\underline{p}_2$ associated with foreclosure in the low-price state.

## 5.3. Non-Recourse Mortgages

The model of stochastic prices can be used to derive policy implications for the relative efficiency of recourse and non-recourse mortgages. A recourse mortgage allows the lien holder to seize assets beyond the original house to recover mortgage debts. A non-recourse loan prohibits recovery beyond the original house. The analysis so far has implicitly focused on non-recourse mortgages because they best embody our stark assumptions on the borrower's ability to divert all assets except for the original house. However, a slight modification of the model allows some scope for recourse mortgages. While maintaining the assumption that the lender cannot seize the borrower's income directly in our model, we will now assume that a recourse loan allows the lender to seize a new property.

There is no difference between recourse and non-recourse mortgages in the low-income state because the borrower has no assets to seize. Nor is there a difference when the borrower makes the required repayment. The only possible difference arises when the borrower is tempted to strategically default on the first house and buy a second one for a price less than the repayment. As we have seen, this sort of strategic default arises in equilibrium with non-recourse mortgages. The threat of seizure of the new property provided by a recourse loan eliminates this sort of strategic default.

We saw in Proposition 8 that with non-recourse mortgages, the possibility of strategic default arising in equilibrium in the stochastic-price model (case (ii)) causes repayments to rise and welfare to fall relative to the equilibrium in the deterministic-price model (case (i)). With recourse mortgages, this is no longer true. Recourse mortgages eliminate strategic default, returning the equilibrium to case (i) of the proposition. The policy implication of the analysis is that if recourse loans were to replace the typical non-recourse style in the United States, mortgage repayments would fall and welfare would rise.[19]

# 6. Renegotiation

In this section, we analyze another possible benefit to banks from holding mortgages in addition to screening: a bank is in a good position to renegotiate mortgages that would go into default in the face of an adverse housing market. As argued by Hart and Zingales (2008), diffuse investors who hold securitized mortgages originated by brokers would have much greater difficulty renegotiating with borrowers threatening to default.

Renegotiation avoids cases in which the borrower strategically defaults if forced to make the contractual repayment but would be willing to stay in the current house at a reduced repayment. This benefits the lender if the reduced repayment still exceeds the foreclosure proceeds. Such cases arise in our model with stochastic prices. In particular, in the state with high income but low price $\underline{p}_2$, the consumer strategically defaults. The lender obtains $\lambda \underline{p}_2$ from foreclosure, but the borrower would be willing to pay as much as $\underline{p}_2$ to stay in the home.

We will thus adopt the previous section's stochastic-price model for our analysis of renegotiation. To focus on the cases of most interest, we maintain the parameter restrictions from the previous section. We will make the stark assumption that banks can costlessly renegotiate the loans they hold, but securitized loans cannot be renegotiated. The bargaining process is also simplified by giving the bank all the bargaining power vis-á-vis the borrower in the event of renegotiation.[20]

[19] As a matter of law, only six of the 50 U.S. states explicitly prohibit recourse mortgages. In practice, however, lenders generally regard the costs of judicial foreclosure to be too high to make such recovery worthwhile, so that non-recourse loans are the standard in the United States. See Ellis (2008).

[20] The model abstracts from some obvious drawbacks of renegotiation. If the borrower had private information about $\tilde{p}_2$, there would be scope for him to claim that the price is $\underline{p}_2$ rather than $\bar{p}_2$ to receive a reduced repayment. A lender might gain from committing not to renegotiate to avoid giving away information rent to the borrower. This benefit from committing not to renegotiate is absent from our model because information about the state of prices is symmetric. The assumption of symmetric information is realistic in the post-crisis economy, in which it is widely known that house prices had fallen substantially. It may be less realistic in periods in which local conditions are

The continuation equilibria for each form of lending are similar to those found in the stochastic-price model. Indeed, the outcome with a broker is identical because securitized mortgages cannot be renegotiated when second-period price is low: consumers continue to strategically default. With banking, the only difference is that instead of allowing strategic default in the high-income, low-price state, the bank renegotiates the mortgage. Since the bank has all the bargaining power, the renegotiated repayment is set at the highest amount that the borrower would be willing to pay instead of moving, i.e., the price of a different house, $\underline{p}_2$. Note that this is more than the liquidation value $\lambda \underline{p}_2$ the bank would obtain if it simply foreclosed. The revenue increase which renegotiation allows alters the zero-profit repayment under renegotiation, which we denote as $\hat{\hat{R}}^{SBK}$:

$$\gamma \left[ h\phi \hat{\hat{R}}^{SBK} + h(1-\phi)\underline{p}_2 + (1-h)\lambda \underline{p}_2 - \frac{(1+r)p_1}{1-d} - k_I \right] = 0. \tag{20}$$

Using the formula for $R^{SBK}$ from (5) and rearranging, (20) gives

$$R^{SBK} = \phi \hat{\hat{R}}^{SBK} + (1-\phi)\underline{p}_2. \tag{21}$$

A comparison of (18) with (21) shows that $\hat{\hat{R}}^{SBK} < \hat{R}^{SBK}$, implying that renegotiation reduces the contractual repayment relative to the case with stochastic prices but no renegotiation. Renegotiation thus gives an additional advantage to banks relative to brokers in addition to screening. A natural question is whether the renegotiation effect could be substantial enough that banks emerge as the equilibrium lending form even if screening is prohibitively expensive. That is, is it possible to observe non-screening banks? In general, the answer is yes. If the cost of screening and the benefit of renegotiation are sufficiently high, non-screening banks may be more efficient than either screening banks or brokers.[21] Under the maintained parameter restriction $\underline{p}_2 \to 0$, however, the

responsible for most of the fluctuations in house prices.

[21]The full analysis for larger values of $\underline{p}_2$ in which non-screening banks can emerge in equilibrium is available from the authors on request. One can show that two conditions are sufficient for equilibrium to involve non-screening banking:

$$p_1(1+r)\left(\frac{d}{1-d}\right) < \gamma h(1-\phi)(1-\lambda)\underline{p}_2$$

$$(1-\gamma)\left[\frac{p_1(1+r)}{1-d} - \lambda \underline{p}_2\right] < \gamma k_I.$$

The first condition ensures the good type prefers non-screening banking to broking. The second condition ensures the good type prefers non-screening banking to screening banking.

renegotiation benefit is vanishingly small, so non-screening banks do not emerge in equilibrium.

**Proposition 9.** *In the stochastic-price model, maintain the parameter restrictions $y, u_2 \to \infty$, $\underline{p}_2 \to 0$, $R^{NBR}, R^{SBK} < p_2$, and equation (15). Renegotiation enlarges the set of parameters for which banking is the equilibrium form of lending, emerging in equilibrium when*

$$R^{SBK} < R^{NBR} + (1-\phi)(1-\lambda)\underline{p}_2. \tag{22}$$

*Renegotiation reduces the equilibrium contractual repayment offered by a bank: $\hat{\hat{R}}^{SBK} < \hat{R}^{SBK}$. There is weakly more banking than is socially efficient even with renegotiation.*

The proof in the appendix derives condition (22). One might think that the social benefit of renegotiation—avoiding foreclosure waste $(1-\lambda)\underline{p}_2$—might tip the balance of social welfare toward banking. While this is a social benefit, it is also a private benefit and so does not change the wedge between private and social preference toward banking. The positive externality associated with lending to bad types is still present with broking, and so equilibrium involves socially too little broking whether or not there is renegotiation.

# 7. Systemic Factors

To this point, we have conducted partial-equilibrium analysis. The representative-consumer model we used was simple to analyze but left no role for correlation in shocks in income, house prices, and foreclosure losses across consumers. Many commentators point to these systemic factors as causing a cascade of problems that precipitated the global financial crisis. This section shows how to extend the basic model to allow the analysis of such systemic factors.

## 7.1. Model with a Systemic Factor

For concreteness, we will focus on one simple systemic factor, allowing foreclosure frictions to increase in the number of borrowers in foreclosure. Because securitized mortgages serve bad types and thus have higher default rates than bank mortgages, securitization can exert a negative externality on the market. This negative externality raises the possibility of excessive securitization in equilibrium, contrasting our finding in the absence of systemic factors that there could only be socially too little securitization (see Proposition 7).

Return to the model in Section 3 with deterministic price $p_2$. Rather than a representative consumer, assume now that the market has a unit mass of ex ante identical consumers. Let $\gamma$ be the measure of good types and $1-\gamma$ the measure of bad types. Instead of taking the fraction of period-2 house value that can be recovered in foreclosure to be a constant $\lambda$, we will now assume that it is a decreasing function $\lambda(\delta)$ of the number of period-2 defaults in the market, $\delta \in [0,1]$. We will sometimes refer to the original model as the exogenous-$\lambda$ model and the current extension the endogenous-$\lambda(\delta)$ model.

## 7.2. Equilibrium with a Systemic Factor

With some additional notation, we can characterize equilibrium in a way quite similar to what we have seen in Proposition 1. Consider an equilibrium in which screening banks are active. Previous arguments can be used to show that no brokers are active, that all good types are served, that only good types are served, and that the borrower repays if and only if his income is high. Thus the measure of defaults in a banking equilibrium is $\gamma(1-h)$. Let $\lambda^{SBK} = \lambda(\gamma(1-h))$ be the fraction of period-2 house value recovered in this equilibrium. Consider an equilibrium in which brokers are active. Previous arguments can be used to show that no banks are active, that all consumers are served, but that only the high-income, good type repays. Thus the measure of defaults is $1-\gamma h$ in this equilibrium. Let $\lambda^{NBR} = \lambda(1-\gamma h)$ be the fraction of period-2 house value recovered in this equilibrium. Notice $\lambda^{SBK} > \lambda^{NBR}$ because $\gamma(1-h) < 1-\gamma h$ and $\lambda(\delta)$ is an decreasing function. A further piece of notation, it will be useful in the analysis to emphasize the dependence of the zero-profit repayments on $\lambda$ alone by writing the left-hand side of (5) as $R^{SBK}(\lambda)$ and the left-hand side of (7) as $R^{NBR}(\lambda)$, recognizing that in fact they are potentially functions of all of the underlying parameters. With this notation in hand, notice the similarity between the following characterization of equilibrium and Proposition 1.

**Proposition 10.** *Assume* (3) *holds. Equilibrium of the endogenous-$\lambda(\delta)$ model falls into one of the following three cases.*

*(i) If $R^{NBR}(\lambda^{NBR}) < \min\{R^{SBK}(\lambda^{NBR}), m\}$, then brokers originate all mortgages in equilibrium, securitizing them immediately. Equilibrium mortgage terms are $R^* = R^{NBR}(\lambda^{NBR})$ and $S^* = 0$. Both good and bad consumer types are served.*

*(ii) If $R^{SBK}(\lambda^{SBK}) < \min\{R^{NBR}(\lambda^{NBR}), m\}$ then banks originate all mortgages by raising deposits from investors and hold the mortgages for both periods. Equilibrium mortgage*

*terms are* $R^* = R^{SBK}(\lambda^{SBK})$ *and* $S^* = 1$. *Only good consumer types receive mortgages. Bad types are screened and rejected if they apply and so do not apply in equilibrium.*

*(iii) If* $m < \min\{R^{NBR}(\lambda^{NBR}), R^{SBK}(\lambda^{SBK})\}$*, then there is no mortgage lending in equilibrium.*

*For each of the cases (i)–(iii), the stated outcome is also an equilibrium if any of the strict inequalities involved holds as an equality. There are no other equilibria.*

The only difference with Proposition 1 is that the relevant default rates at which the zero-profit repayments are evaluated differ across terms. In case (i), the broking equilibrium is stable unless a deviating screening bank enters. If it enters, existing brokers are serving all consumers, so all consumers will continue to be served after the deviation, resulting in $1-\gamma h$ defaults. Thus $\lambda^{NBR}$ is the relevant value of $\lambda$ at which to evaluate both the equilibrium repayment $R^{NBR}(\lambda^{NBR})$ and the deviating repayment $R^{SBK}(\lambda^{NBR})$. In case (ii), deviating entry by a broker changes the lending environment from one in which only good types are served, so that $\lambda^{SBK}$ would be the relevant value of $\lambda$, to one in which all types are served, so that $\lambda^{NBR}$ becomes the relevant value of $\lambda$. Thus there is an asymmetry between cases (i) and (ii). In case (i), the deviating bank's entry does not change the overall quality of loans offered on the market, which are extended to good and bad types in any event. In case (ii), the deviating broker's entry reduces the overall quality of loans offered, thus making deviation less profitable than in the case in which $\lambda$ was fixed.

While the conditions for broking or banking were mutually exclusive when $\lambda$ was exogenous (except on a boundary), with endogenous $\lambda(\delta)$ there can be a region of overlap where there are multiple equilibria. This can be seen in Figure 5. Curve $DG$ delineates indifference between banking and broking when the market default rate is $\lambda^{NBR}$, i.e., delineates the set of parameters for which $R^{NBR}(\lambda^{NBR}) = R^{SBK}(\lambda^{NBR})$. This is the same as the curve $DG$ in Figure 3 if we fix the exogenous $\lambda$ behind Figure 3 so that it equals $\lambda^{NBR}$. Curve $D'G'$ delineates the set of parameters for which $R^{SBK}(\lambda^{SBK}) = R^{NBR}(\lambda^{NBR})$. In the region between $DG$ and $D'G'$, the parameters can support both a banking equilibrium and a broking equilibrium.[22]

[22] The fact that this region is non-empty is an implication of Proposition 11. The proposition provides a tight bound on the welfare difference between banking and broking equilibria, implying that a sequence of parameter vectors exist approaching the bound. But then a banking and a broking equilibrium exists for each of the parameter vectors in the sequence.

### 7.3. Welfare with a Systemic Factor

The addition of systemic factors allows us to enrich the welfare analysis from Section 4.3 in several dimensions. First, by restricting attention to parameters for which either lending mode can arise in an equilibrium, we can compare welfare across actual equilibria rather than comparing equilibrium welfare to welfare in a counterfactual outcome. More importantly, securitization only had a positive externality in the previous analysis, leading to the stark conclusion that securitization would never be socially excessive. Introducing systemic factors introduces the possibility of negative externalities associated with securitization. Whether there is too much or two little securitization involves an interesting theoretical tradeoff that is also of practical interest, embodying concerns raised by commentators about the recent wave of securitization.

We will focus on parameters for which there are a banking and a broking equilibrium, i.e., the region between curves $D'G'$ and $DG$ in Figure 5. To be strictly in this region, the conditions behind both case (i) and case (ii) of Proposition 10 must hold. Combining these conditions,

$$R^{SBK}(\lambda^{SBK}) < R^{NBR}(\lambda^{NBR}) < \min\{R^{SBK}(\lambda^{NBR}), m\}. \tag{23}$$

Following the logic of Section 4.3, we can derive the difference between welfare in the broking and banking equilibria as

$$W^{NBR} - W^{SBK} = \gamma h[R^{SBK}(\lambda^{SBK}) - R^{NBR}(\lambda^{NBR})] + (1-\gamma)u_1. \tag{24}$$

This is the same as equation (14) except the equilibrium repayments, $R^{SBK}(\lambda^{SBK})$ and $R^{NBR}(\lambda^{NBR})$, now reflect endogenous foreclosure frictions. The first term on the right-hand side of (24) is negative in the multiple-equilibrium region by (23). This is the negative externality associated with securitization, worsening market foreclosure frictions. The second term on the right-hand side of (24) is the positive externality associated with securitization, increasing homeownership among bad types, familiar from the analysis with fixed $\lambda$.

We have already seen cases in which there is socially too little securitization. Proposition 7 found this result in the absence of systemic factors. By continuity, the result continues to hold in the presence of systemic factors if they are not too important; i.e., if $\lambda(\delta)$ is fairly inelastic. The next proposition fleshes out the opposite possibility, examining whether there are cases in which there can be too socially much securitization and, if so, how much welfare can possibly be lost moving from a banking to a broking equilibrium. The answer turns out to be yes, there can be socially too much securitization; the proposition provides a formula for the maximum welfare loss from securitization.

**Proposition 11.** *Consider any fixed values $\gamma \in (0,1)$, $h \in (0,1)$, $u_1 > 0$, and $p_2 > 0$. The welfare loss in moving from a banking to a broking equilibrium, $W^{SBK} - W^{NBR}$, can be no greater than*

$$\gamma^2 h(1-h)p_2 - (1-\gamma)u_1 \tag{25}$$

*for any values of the other parameters and for any decreasing $\lambda(\delta)$ on $[0,1]$ that support both a banking and a broking equilibrium. Bound* (25) *is tight in that there exist values of the other parameters and $\lambda(\delta)$ in the multiple-equilibrium set for which the welfare loss from securitization can be made arbitrarily close to* (25).

The proof, which amounts to solving a constrained-optimization problem with many variables and constraints via the Kuhn-Tucker method, is provided in Appendix B.

Proposition 11 has a number of relevant implications. It implies that in the multiple-equilibrium region, the broking equilibrium can be socially less efficient than then banking one. This can be seen by substituting a high value of $p_2$ and low value of $u_1$ into (25). It also implies that the loss from securitization can be arbitrarily high, as can be seen by substituting increasingly high values of $p_2$ into (25).

In the presence of systemic factors, renegotiation and non-recourse loans have additional social benefits. We already saw that renegotiation and non-recourse loans reduce strategic default (see Sections 5.3 and 6), a social benefit that is fully internalized by the contracting parties. With systemic factors, this reduction in strategic default reduces the overall default rate, which has a positive external benefit of reducing market-wide foreclosure frictions.

Securitization can be seen to increase the fragility of the financial system along several dimensions in the model. One dimension can be understood by looking more carefully at the proof of Proposition 11. The proof shows that the bound in (25) is approached as the parameters in the multiple-equilibrium region approach the boundary with the solely-banking region, curve *DG* in Figure 5. But this implies that a small increase in what one might otherwise consider a beneficial parameter such as $\gamma$ can cause a discontinuous fall in social welfare by introducing the possibility of socially inferior broking equilibria. The fact that a slight improvement in the lending environment, by shifting the market toward securitization, can cause a large fall in welfare can be interpreted as a fragility in the financial system. This sort of fragility did not arise in the absence of systemic factors because expression for the difference in social welfare (14) was everywhere continuous in the parameters, even at the boundary between banking and broking regions. It should be emphasized that a parameter change shifting the market from the banking to the multiple-equilibrium region does not automatically shift the equilibrium to broking—we have not yet provided a theory of equilibrium selection in the presence of multiple equilibria—but admits broking as a new possibility.

One approach to equilibrium selection follows the spirit of evolutionary stability (see, e.g., Axelrod 1984). A equilibrium strategy is evolutionarily stable if it is robust to "mutations", i.e., to the introduction of a small numbers of agents playing a non-equilibrium strategy. In our region of multiple equilibria, the broking equilibrium is robust to the introduction of a bank offering the equilibrium bank contract. The bank does not worsen the default rate, so the brokers continue to earn their equilibrium (zero) profit. The banking equilibrium is not robust to the introduction of a broker offering the equilibrium securitized mortgage. All bad types would apply, worsening the default rate, reducing the bank's profits below the equilibrium (zero) level, eventually leading to their exit. In this sense, securitization has a parasitic effect on bank lending, increasing the fragility of the system. The reverse is not true: banks do not impair securitized mortgages.

# 8. Conclusion

Recent history has seen a rapid expansion in the securitization of mortgages and a substantially higher rate of default when compared with bank-held loans. The natural question that emerges is why should there be a difference in the quality of loans, depending on whether investors or banks hold the right to foreclose? We have argued that incomplete contracts must be part of the explanation, developing and analyzing a model which features significant contractual incompleteness due to the ability of parties to divert funds. Banks can commit to repay depositors because of regulation, whereas brokers cannot, financing loans through immediate sale of mortgage liens to investors—i.e. securitization. This breaks the link between brokers' screening decision and the right to foreclose, reducing the incentive to undertake soft-information screening of consumers, leading to increased default rates for securitized mortgages.

Our results are consistent with recent empirical findings. As well as the basic link between increased default and securitization, we predict that rising house prices, lower liquidation costs, lower interest rates, higher regulatory costs and cheaper underwriting lead to increased securitization and default. We provide a theoretical basis for Keys et al's (2010) finding of a discrete jump in default rates around the FICO threshold of 620.

While welfare analysis demonstrates that there is insufficient securitization, extending our model to the case of stochastic prices yields insights regarding the stability of mortgage markets under different funding modes. When house prices are low, high income consumers will be tempted to strategically default on a non-recourse loan, which are the standard type of mortgage in the United States. Enforcement of recourse loans would reduce repayments as well as eliminate such strategic default and improve market stability. We analyze the impact of renegotiation, assuming that banks are able to do so and diffuse investors are not. Renegotiation also reduces repayments, reduces strategic default, and increases the incentive for bank origination and screening, thus improving mortgage market and hence macroeconomic stability. Policies to encourage renegotiation thus would provide some of the benefits of recourse mortgages loans, if such are not feasible.

## Appendix A: Lemmas

To streamline the proofs of the main propositions, we split out some technical details in a series of lemmas, stated and proved in this appendix. Many of the lemmas are concerned with the properties of the function

$$\pi(\theta, R, D) = \theta h R + (1-\theta h)\lambda p_2 - \frac{(1+r)p_1}{1-D}. \tag{A1}$$

Equation (A1) is the expected profit of a lender facing regulatory cost $D \in \{0, d\}$ that has offered a mortgage to a consumer who is known to be of type $\theta \in \{0,1\}$ and who makes repayment $R$ when he has positive income. This is a continuation profit, which does not take into account sunk screening costs.

**Lemma 1.** *$R^{NBR}$ satisfies*

$$\gamma\pi(1, R^{NBR}, 0) + (1-\gamma)\pi(0, R^{NBR}, 0) = 0$$

*and $R^{SBK}$ satisfies*

$$\pi(1, R^{SBK}, D) = k_I.$$

*Proof.* The equations are true by construction of $R^{NBR}$ and $R^{SBK}$. They can be verified by direct substitution of (7) and (5) into (A1). *Q.E.D.*

**Lemma 2.** *$\pi(\theta, R, D)$ is non-decreasing in $R$ for all $\theta \in \{0,1\}$. In particular, $\pi(\theta, R, D)$ is independent of $R$ for $\theta = 0$ and strictly increasing in $R$ for $\theta = 1$.*

*Proof.* Differentiating (A1) yields $\partial\pi/\partial R = \theta h$, which equals 0 for $\theta = 0$ and is strictly positive for $\theta = 1$. *Q.E.D.*

**Lemma 3.** *$\pi(\theta, R, D)$ is strictly decreasing in $D$.*

*Proof.* Differentiating (A1) yields

$$\frac{\partial\pi}{\partial D} = -\frac{(1+r)p_1}{(1-D)^2} < 0.$$

*Q.E.D.*

**Lemma 4.** *$\pi(0, R, D) < 0$ for all $R$ and for all $D \in [0,1]$.*

*Proof.* We have

$$\begin{aligned}\pi(0, R, D) &= \lambda p_2 - \frac{(1+r)p_1}{1-D} \\ &\leq \lambda p_2 - (1+r)p_1.\end{aligned}$$

The first line follows from substituting $\theta = 0$ into (A1) and the second line from straightforward algebra. The last expression is negative by condition (3). *Q.E.D.*

**Lemma 5.** *$\pi(1, R, d) \leq k_I$ for all $R \leq R^{SBK}$ with strict inequality for $R < R^{SBK}$.*

*Proof.* By construction, the zero-profit repayment $R^{SBK}$ satisfies $\pi(1, R^{SBK}, d) - k_I = 0$; this can also be verified by direct substitution of (5) into (A1). Therefore $k_I = \pi(1, R^{SBK}, d) \geq \pi(1, R, d)$ for $R \leq R^{SBK}$ by Lemma 2. For $R < R^{SBK}$, we have $k_I > \pi(1, R, d)$ by Lemma 2. *Q.E.D.*

**Lemma 6.** $\min\{R^{NBR}, R^{SBK}\} > 0$.

*Proof.* We have

$$R^{NBR} = \frac{1}{\gamma h}\left[(1+r)p_1 - (1-\gamma h)\lambda p_2\right] \tag{A2}$$

$$\geq \frac{1}{\gamma h}\left[(1+r)p_1 - \lambda p_2\right] \tag{A3}$$

and

$$R^{SBK} = \frac{1}{h}\left[\frac{(1+r)p_1}{1-d} + k_I - (1-h)\lambda p_2\right] \tag{A4}$$

$$\geq \frac{1}{h}[(1+r)p_1 - \lambda p_2], \tag{A5}$$

where (A2) follows from (7), (A4) follows from (5), and (A3) and (A5) follow from sliminating positive terms. By (3), (A3) and (A5) are positive. *Q.E.D.*

# Appendix B: Proofs of Propositions

## Proof of Proposition 1

Proposition 1 is stated for the limiting case $k_C \downarrow 0$. Our strategy for proving Proposition 1 will be to state and prove a more general result, Theorem 1, which applies to the case of any $k_C > 0$. Proposition 1 will then follow as a limiting special case.

**Theorem 1.** *Assume* (3) *holds. Consider any* $k_C > 0$. *Equilibria fall into one of the following three cases.*

*(i) Assume*

$$R^{NBR} < \min\{R^{SBK} + k_C/h, m\}. \tag{B1}$$

*Then brokers originate all mortgages in equilibrium, securitizing them immediately. Equilibrium mortgage terms are* $R^* = R^{NBR}$ *and* $S^* = 0$. *Both good and bad consumer types are served.*

*(ii) Assume*

$$R^{NBR} > \min\{R^{SBK} + k_C/h, m\} \tag{B2}$$

$$R^{SBK} < \min\{y + u_2 + (u_1 - k_C)/h, m\}. \tag{B3}$$

*Then banks originate all mortgages by raising deposits from investors and hold the mortgages for both periods. Equilibrium mortgage terms are* $R^* = R^{SBK}$ *and* $S^* = 1$. *Only good consumer types receive mortgages. Bad types are screened and rejected if they apply and so do not apply in equilibrium.*

*(iii) Assume*

$$R^{NBR} > \min\{R^{SBK} + k_C/h, m\} \tag{B4}$$

$$R^{SBK} > \min\{y + u_2 + (u_1 - k_C)/h, m\}. \tag{B5}$$

*Then there is no mortgage lending in equilibrium.*

*For each of the cases (i)–(iii), the outcome is also an equilibrium if any of the strict inequalities involved holds as an equality. There are no other equilibria.*

## Proof of Theorem 1

Theorem 1 and its corollary, Proposition 1, are central to the paper. The proof is fairly involved and so is divided into a number of subsections. The first subsection updates the relevant analysis in the text, generalizing the analysis from Section 4.1 to allow for arbitrary $k_C > 0$. The next subsections establish existence of equilibrium in cases (i)–(iii) by direct construction. The last subsections are the most intricate, establishing uniqueness by ruling out an exhaustive set of alternatives.

**Generalizing Section 4.1.** Equations 4–(7) and the accompanying analysis continue to hold without modification. The good type's payoff from a loan in (8) must be updated to reflect the personal screening cost:

$$u_1 + h(u_2 + y) - T_i, \tag{B6}$$

where

$$T_i = hR_i + S_i k_C \tag{B7}$$

is the total expected cost of the loan facing the good type. The good type's payoff depends on the terms of the mortgage contract only through $T_i$. By (B6), he chooses the contract offering the lowest $T_i$ provided (B6) is non-negative at this $T_i$. If (B6) is negative, anticipating a negative payoff from any mortgage offered, he instead exits the market. As in Section 4, $R_i$ must satisfy the repayment constraint (9) or else the good type would always default and the resulting foreclosure proceeds would not be sufficient to allow the lender to break even by (3). However, it is no longer the case that the repayment constraint (9) implies that the participation constraint is satisfied: (B6) can be negative when (9) is satisfied because of the addition of screening cost to $T_i$ in (B7). Therefore, we need to consider both repayment and participation constraints in the subsequent analysis.

The bad type's strategy is easy to characterize. We argued in the text that the lender only screens if it uses the information to exclude bad types. Thus a bad type never accepts a contract with $S_i = 1$ because he faces screening cost $k_C$ for no benefit. The bad type is indifferent among contracts with $S_i = 0$ because he never has income to repay, so the level of $R_i$ is irrelevant to him. The bad type obtains $u_1 > 0$ from accepting a contract with $S_i = 0$, so always accepts one of them.

**Existence in Case (i).** Assume (B1). Posit the following equilibrium outcome. Two brokers are the only active intermediaries, offering mortgage contracts $(R^*, S^*) = (R^{NBR}, 0)$ and charging no origination fee. All consumers are served. Pin down the bad type's strategy when he is faced with several non-screening contracts, among which he is indifferent, by assuming that he accepts the one with the lowest repayment. We will show that no player strictly gains from deviating.

Consider deviations by a consumer. Given the consumer strategies outlined in the previous subsection, the only deviation remaining to be checked is whether the consumer would gain by rejecting the contract. The good type earns payoff

$$u_1 + h(y + u_2) - hR^{NBR}.$$

This is positive because $R^{NBR} < m \leq y$, where the first inequality holds by (B1) and the second from the definition $m \equiv \min\{y, p_2, u_2\}$. Thus the good type prefers not to reject. The previous subsection argued that the bad type always accepts some mortgage with no screening if offered.

Consider deviations by an intermediary. Inactive intermediaries earn zero profit. Active brokers earn zero profit because they charge no origination fee. Investors in the securitized mortgages originated by active brokers earn zero profit from contract $(R^{NBR}, 0)$ by Lemma 1. It remains to show that there is no deviation by an intermediary that will generate positive profit.

If an inactive intermediary enters and matches the active brokers' offer $(R^{NBR}, 0)$, it (and its investors if any) earn the same (zero) profit as the original active lenders.

Suppose an intermediary (active or not) deviates to $(R_i, 0)$ for some $R_i > R^{NBR}$. This deviation will not attract good types and so will not generate positive profit. Similarly, deviating to $(R^{NBR}, 1)$ would not attract good types because it involves additional screening cost $k_C$.

A deviation by a broker to $(R_i, 0)$ for some $R_i < R^{NBR}$ will attract all consumers, generating expected profit

$$\gamma\pi(1, R_i, 0) + (1-\gamma)\pi(0, R_i, 0) < \gamma\pi(1, R^{NBR}, 0) + (1-\gamma)\pi(0, R^{NBR}, 0).$$

The inequality follows from Lemma 2. The right-hand side equals 0 by Lemma 1.

A deviation by a bank to $(R_i, 1)$ for some $R_i < R^{NBR}$ will attract either no consumers or only good consumers. If the deviation attracts all good types, the expected profit is

$$\gamma[\pi(1, R_i, d) - k_I] \quad < \quad \gamma[\pi(1, R^{SBK} + k_C/h, d) - k_I] \tag{B8}$$

$$= \quad -\gamma k_C. \tag{B9}$$

To see (B8), note $R_i < R^{NBR} < R^{SBK} + k_C/h$, where the second inequality follows from (B1). Then apply Lemma 2. Condition (B9) follows from substituting from (5) into the expression $R^{SBK} + k_C/h$, and then substituting this expression into (A1).

Note that no intermediary can gain by randomizing over its contract, since the consumer observes all intermediaries' entry decisions and contracts before making its choice. Strict randomization only serves to put positive probability on pure strategies that yield no greater profit than the pure strategy equilibrium.

The preceding arguments also hold if the inequality in (B1) is weak.

**Existence in Case (ii).** Assume (B2) and (B3). Posit the following equilibrium outcome. Two banks are the only active intermediaries, offering mortgage contracts $(R^*, S^*) = (R^{SBK}, 1)$. Only good types apply for mortgages, and they are served.

Consider deviations by a consumer. Given the consumer strategies derived in the previous subsection, the only deviations remaining to be checked are consumer types' decisions to apply for a mortgage. The good type earns payoff

$$u_1 + h(y + u_2) - hR^{SBK} - k_C,$$

which is positive by (B3). So the good type prefers not to reject. As argued in the previous subsection, the bad type never applies for a screening contract.

Consider deviations by an intermediary. As argued above for case (i), we need to show that there is no deviation by an intermediary which generates positive profit. If an inactive intermediary enters and matches active banks' offer $(R^{SBK}, 1)$, it earns the same (zero) profit as they.

Suppose an intermediary (active or not) deviates to $(R_i, 1)$ for some $R_i > R^{SBK}$. This deviation attracts no consumers and so generates zero profit. Suppose an intermediary deviates to $(R_i, 0)$ for some $R_i > R^{SBK}$. This deviation attracts only bad types, generating profit $\pi(0, R_i, 0) = \lambda p_2 - (1 + r)p_1 < 0$ by (3).

Suppose $i$ deviates to $(R_i, 0)$ for some $R_i \leq R^{SBK}$. Its expected profit is

$$\gamma\pi(1, R_i, 0) + (1 - \gamma)\pi(0, R_i, 0) < \gamma\pi(1, R^{NBR}, 0) + (1 - \gamma)\pi(0, R^{NBR}, 0).$$

The inequality can be established as follows. We first argue that (B2) and (B3) imply $R^{SBK} < R^{NBR}$. To see this, note (B2) implies either $R^{NBR} > R^{SBK} + k_C/h$ or $R^{NBR} > m$. In the former case, clearly $R^{NBR} > R^{SBK}$. In the latter case, $R^{NBR} > m > R^{SBK}$ by (B3). Hence $R^{SBK} < R^{NBR}$ in either case. But then $R_i \leq R^{SBK} < R^{NBR}$ together with Lemma 2 establishes the inequality. The right-hand side of the inequality equals 0 by Lemma 1. Thus the proposed deviation is not strictly profitable.

Suppose $i$ deviates to $(R_i, 1)$ for some $R_i < R^{SBK}$. Its expected profit is

$$\gamma[\pi(1, R_i, d) - k_I] < \gamma[\pi(1, R^{SBK}, d) - k_I] = 0,$$

where the inequality follows from Lemma 2 and the equality from Lemma 1.

The preceding arguments also hold if one or both of the inequalities in (B2) and (B3) is weak.

**Existence in Case (iii).** Assume (B4) and B5. Posit the equilibrium outcome that no lenders offer mortgages. Given that all lenders are inactive, they earn zero profit. To establish existence, we need to check that there are no lending opportunities providing positive profits.

As a preliminary step, we will show that (B4) and (B5) imply $R^{NBR} > m$. We have

$$\begin{aligned} R^{NBR} &> \min\left(R^{SBK} + \frac{k_C}{h}, m\right) \\ &\geq \min\left(\min\left(y + u_2 + \frac{u_1}{h}, m + \frac{k_C}{h}\right), m\right) \\ &\geq m. \end{aligned}$$

The first inequality holds by (B4). The second inequality holds by substituting the bound on $R^{SBK}$ from (B5). The third inequality holds because $h + u_2 + u_1/h > m$ by definition of $m$ and because $m + k_C/h > m$ for $k_C, h > 0$.

Suppose an inactive broker deviates by offering contract $(R',0)$. If $R' > m$, this contract will certainly generate negative profit because the good type would always default given the repayment constraint is violated. So assume $R' \leq m$. All types would sign the deviating contract. The combined profit of the broker and its investors is

$$\begin{aligned}\gamma\pi(1,R',0)+(1-\gamma)\pi(0,R',0) &\leq \pi(1,m,0)+(1-\gamma)\pi(0,m,0)\\ &< \pi(1,R^{NBR},0)+(1-\gamma)\pi(0,R^{NBR},0).\end{aligned}$$

The first inequality holds by Lemma 2 and $R' \leq m$. The second inequality holds by Lemma 2 and $R^{NBR} > m$. The last expression equals 0 by Lemma 1. There is thus no strictly profitable deviation. (Indeed, the deviation is strictly unprofitable, a fact we will return to below.)

Suppose an inactive bank deviates by offering contract $(R',1)$. As above, this contract will certainly generate negative profit unless $R' \leq m$. This contract would violate the good type's participation constraint unless the total expected cost $T'$ it imposes on the good type, which by (B7) is $T' = hR' + k_C$, is not so high that (B6) is negative. Substituting and rearranging, the participation constraint is $R' \leq y + u_2 + (u_1 - k_C)/h$. Putting the conditions on $R'$ together,

$$R' \leq \min\left(y+u_2+\frac{u_1-k_C}{h}, m\right),$$

implying $R' < R^{SBK}$ by (B5) The bank's expected profit from this contract is

$$\gamma[\pi(1,R',d)-k_I] < \gamma[\pi(1,R^{SBK},d)-k_I].$$

The inequality follows from $R' < R^{SBK}$ and Lemma 2. The right-hand side of the inequality equals 0 by Lemma 1. Again, we see the proposed deviation is not strictly profitable. (Indeed, it is strictly unprofitable, a fact we will return to below.)

**Introduction to Uniqueness Proof.** We will derive a series of restrictions on equilibria, which together constrain the set of equilibria not to fall outside the characterization in Theorem 1.

**Restriction 1.** *All active intermediaries $i$ (i.e., all $i$ who serve a consumer with positive probability) impose the same total expected costs on the good type: $T_i = T^E$, where $T^E$ is the total expected cost for the good type in equilibrium.*

*Proof.* Consider an outcome in which such intermediaries total costs are ordered by size as $T_{(1)} \leq T_{(2)} \leq \cdots \leq T_{(N)}$. Suppose one of the inequalities is strict, say $T_{(i)} < T_{(i+1)}$. Then intermediaries $i+1$ and above serve no good types. Since they serve the consumer with positive probability, they must serve the bad type with positive probability. This leads to negative profits and they would strictly gain from a deviation to be inactive. Thus, the outcome cannot be an equilibrium. *Q.E.D.*

**Restriction 2.** *There is no equilibrium in which non-screening banks are active.*

*Proof.* Suppose at least one non-screening bank is active in a putative equilibrium, offering contract $(R',0)$. By Restriction 1, all active intermediaries impose the same total expected cost on good types. Thus all active non-screening intermediaries must offer contract $(R',0)$. Let $\gamma'$ and $\beta'$ be the measure of good and bad types, respectively, served by all active non-screening banks. Because at least one of them is active, either $\gamma' > 0$ or $\beta' > 0$. In fact we must have $\gamma' > 0$ or else some non-screening bank would be strictly unprofitable and would benefit from the deviation of exiting the market. Let $\gamma''$ and $\beta''$ be the measure of good and bad types, respectively, served by all active non-screening brokers. We must have

$$\gamma' + \gamma \leq \gamma. \tag{B10}$$

Further,

$$\beta' + \beta'' = 1 - \gamma \tag{B11}$$

because, as argued in the text, the whole $1-\gamma$ measure of bad types is served if any non-screening intermediary is active because they obtain positive utility from a mortgage.

For the outcome to be an equilibrium, total profit for the combination of non-screening intermediaries and their investors cannot be negative or at least one of them would deviate to exiting the market. Thus

$$0 \le \gamma' \pi(1,R',d) + \beta' \pi(0,R',d) + \gamma'' \pi(1,R',0) + \beta'' \pi(0,R',0) \quad \text{(B12)}$$
$$< (\gamma' + \gamma'')\pi(1,R',0) + (\beta' + \beta'')\pi(0,R',0) \quad \text{(B13)}$$
$$\le \gamma\pi(1,R',0) + (1-\gamma)\pi(0,R',0). \quad \text{(B14)}$$

The right-hand side of (B12) adds up the profits of non-screening intermediaries and their investors. Condition (B13) holds by Lemma 4 and $\gamma' > 0$. Condition (B14) uses (B11) and $\pi(1,R',0) > 0$. To see this last inequality, note $\pi(0,R',0) < 0$ by Lemma 4. But then for (B13) to be positive, $\pi(1,R',0) > 0$.

Evaluated at repayment $R'$, as we see, the profit (B14) is positive. If we instead substitute $R^{NBR}$ for the repayment in (B14), the expression would be 0 by Lemma 1. Hence by Lemma 2, $R' > R^{NBR}$.

An inactive broker could profit by offering contract $(R'',0)$ for any $R'' \in (R^{NBR}, R')$. It would attract all good types because it imposes lower total expected cost on the good type than the contract $(R',0)$, which we know some good type accepted since $\gamma' > 0$. At worst the contract $(R'',0)$ also attracts all bad types. The deviator's profit is at least

$$\gamma\pi(1,R'',0) + (1-\gamma)\pi(0,R'',0) > \gamma\pi(1,R^{NBR},0) + (1-\gamma)\pi(0,R^{NBR},0).$$

The inequality holds by Lemma 2 and $R'' > R^{NBR}$. The right-hand side of the inequality equals 0 by Lemma 1 *Q.E.D.*

**Restriction 3.** *Non-screening brokers and screening banks cannot both be active in equilibrium.*

*Proof.* Since screening banks do not serve bad types, to be active they must serve some measure of good types. At most the remaining measure, call it $\gamma'$, is served by active brokers, where $\gamma' < \gamma$. Brokers together serve all $1-\gamma$ bad types. Letting $R'$ be the repayment specified by a broker mortgage, the brokers and their investors earn combined profit

$$\gamma' \pi(1,R',0) + (1-\gamma)\pi(0,R',0). \quad \text{(B15)}$$

If (B15) is negative, there is at least one broker or investor who would benefit by deviating to exiting the market.

So assume (B15) is non-negative. Now $\pi(0,R',0) < 0$ by Lemma 4. But then the non-negativity of (B15) implies $\pi(1,R',0) > 0$. Hence,

$$\gamma\pi(1,R^{NBR},0) + (1-\gamma)\pi(0,R^{NBR},0) = 0 \quad \text{(B16)}$$
$$\le \gamma' \pi(1,R',0) + (1-\gamma)\pi(0,R',0) \quad \text{(B17)}$$
$$< \gamma\pi(1,R',0) + (1-\gamma)\pi(0,R',0), \quad \text{(B18)}$$

where (B16) follows from Lemma 1, (B17) follows from the maintained assumption that (B15) is non-negative, and (B18) follows from $\gamma' < \gamma$ and $\pi(1,R',0) > 0$. Lemma 2 implies that the second terms of (**??**) and (B18) are independent of $R$ and thus equal. The fact that (**??**) is strictly less than (B18) thus implies $\pi(1,R^{NBR},0) < \pi(1,R',0)$, implying $R^{NBR} < R'$ by Lemma 2. But then the total cost imposed by the broker contract on good types exceeds $T^{NBR}$. An inactive broker could gain from deviating to some repayment $R'' \in (R^{NBR}, R')$. This would attract all good types and in the worst case all bad types. See the proof of Restriction 2 for the rest of the argument that this deviation is strictly profitable. The profitable deviation in this remaining case rules out any equilibrium in which both brokers and banks are active together. *Q.E.D.*

**Restriction 4.** *If $T^{SBK} > T^{NBR}$, there is no equilibrium with active screening banks. If $T^{NBR} > T^{SBK}$, there is no equilibrium with active non-screening brokers.*

*Proof.* Assume $T^{SBK} > T^{NBR}$. Suppose there is an active screening bank charging repayment $R' < R^{SBK}$. By Lemma 5, this bank must earn negative profit inclusive of screening cost. This outcome cannot be an equilibrium because the bank could profitably deviate to being inactive. Suppose that the active screening bank charges repayment $R' \ge R^{SBK}$. Then the total cost it imposes on a good type exceeds $T^{SBK}$. An inactive broker can profitably enter with contract $(R'',0)$ for $R'' \in (R^{NBR}, T^{SBK}/h)$. The total cost this contract imposes on a good type is $hR'' < T^{SBK}$, so the contract would attract the good type. In the worst case, it also attracts the bad type with certainty. See the proof of Restriction 2 for the rest of the argument that this deviation is strictly profitable. Thus we have established that some intermediary has a profitable deviation in any outcome with an active screening bank.

A similar argument establishes that there is no equilibrium with active non-screening brokers if $T^{NBR} > T^{SBK}$. *Q.E.D.*

**Uniqueness in Case (i).** Restrictions 2 and 3 rule out all alternative equilibrium configurations except for an outcome with only screening banks or a no-contract outcome. We will rule out each alternative in turn.

Condition B1 implies $R^{NBR} < R^{SBK} + k_C/h$, in turn implying $T^{NBR} = hR^{NBR} < hR^{SBK} + k_C = T^{SBK}$. By Restriction 4, there can be no active screening banks in equilibrium.

Consider an outcome with no accepted contract. A broker can deviate by offering $(R', 0)$ for any $R' \in (R^{NBR}, m)$. This contract would be accepted by all types. Because $R' < m$, the high-income consumer would make the repayment. See the proof of Restriction 2 for the rest of the argument that this deviation is strictly profitable. This strictly profitable deviation rules out no-contract equilibria.

**Uniqueness in Case (ii).** The argument for case (ii) parallels the previous one for case (i), so is omitted.

**Uniqueness in Case (iii).** Restrictions 2 and 3 rule out all alternative equilibrium configurations except for an outcome with only screening banks or only non-screening brokers. The proof of existence in case (iii) above showed that a contract offered by either lending mode would either be rejected by all borrowers or would generate strictly negative profit. Thus no alternative configuration can be an equilibrium. This completes the proof of Theorem 1. *Q.E.D.*

## Completing Proof of Proposition 1

To complete the proof of Proposition 1, the reader can check that inserting the limit $k_C \downarrow 0$ in conditions (B1)–(B5) give the conditions stated in Proposition 1. *Q.E.D.*

## Proof of Propositions 5 and 6

These propositions are corollaries of a much richer result, Theorem 2. We will state and prove Theorem 2 before returning to prove its corollaries.

**Theorem 2.** *Enumerated are comparative-static exercises changing a single parameter, holding all other parameters constant. The thresholds bounding the intervals are functions of the other parameters, but these arguments are suppressed for brevity. All intervals can be empty unless otherwise noted.*

*(i) Hold constant all parameters except y. There exists thresholds $y'$ and $y''$ with $y' \leq y''$ such that the equilibrium involves no contract for $y < y'$, banking for $y \in (y', y'')$, and broking for $y > y''$. The preceding statements also hold for parameters $p_2$, $u_2$, and $\lambda$ in place of y. For all of these parameters but $\lambda$, the interval in which there is no contract—respectively, $[0, y')$, $[0, p_2')$, and $[0, u_2')$—is guaranteed to be nonempty.*

*(ii) Hold constant all parameters except r. There exists thresholds $r'$ and $r'' \geq r'$ such that the equilibrium involves broking for $r < r'$, banking for $r \in (r', r'')$, and no contract for $r > r''$. The preceding statements also hold for parameter $p_1$ in place of r. For each parameter, the interval in which there cannot be banking—respectively, $(r'', \infty)$ and $(p_1'', \infty)$—is guaranteed to be nonempty.*

*(iii) Hold constant all parameters except h. There exists threshold $h'$ such that the equilibrium involves banking for all $h < h'$ or no contract for all $h < h'$. For $h > h'$ equilibrium involves broking. The preceding statements also hold for parameters $\gamma$ and $\lambda$ in place of h.*

*(iv) Hold constant all parameters except d. There exists threshold $d'$ such that the equilibrium involves banking for $d < d'$ and either broking for all $d > d'$ or no contract for all $d > d'$. The preceding statements also hold for parameters $k_C$ and $k_I$ in place of d. For each parameter, the interval in which there cannot be banking—respectively, $[0, d')$, $[0, k_C')$, and $[0, k_I')$—is guaranteed to be nonempty.*

*(v) Hold constant all parameters except $u_1$. There exists threshold $u_1'$ such that the equilibrium involves broking for all $u_1 < u_1'$ or no contract for all $u_1 < u_1'$. For $u_1 > u_1'$ equilibrium involves banking.*

## Proof of Theorem 2

We prove the proposition for each of cases (i)–(v) in turn. For each case, we provide a proof for one representative parameter. The proof for the other parameters is similar and omitted.

**Case (i).** We will prove the statement for $y$. Hold all the other parameters constant. We first show that there exists income $y' > 0$ such that there is no lending in equilibrium. By (7), $R^{NBR}$ is independent of $y$. By Lemma 6, $R^{NBR} > 0$. Thus there exists $y' > 0$ such that $y' < R^{NBR}$. Now $R^{NBR} > y' \geq \min\{y', p_2, u_2\}$ implies that the condition for a broking equilibrium in case (i) of Proposition 1 is violated even if treated as a weak inequality. Hence there is no broking in equilibrium. Similar analysis applies to banking. Thus there is no equilibrium with lending for income $y'$.

Next, suppose there is no lending in equilibrium for income $y'$. We will show there is no lending in equilibrium for all $y \in [0, y']$. Given there is no lending for income $y'$, there is no broking for $y'$. Because $R^{NBR}$ is independent of $y$ and $m = \min\{y, p_2, u_2\}$ is weakly increasing in $y$, a reduction in $y$ only tightens the conditions for broking in case (i) of Proposition 1. Therefore, there is no broking for all $y \in [0, y']$. Similar analysis applies to banking. Thus there is no equilibrium with lending for income $y \in [0, y']$.

Next, suppose there is an equilibrium with broking for income $y''$. We will show that there is an equilibrium with broking for all $y \geq y''$. By case (i) of Proposition 1, $R^{NBR} \leq \min\{y'', p_2, u_2\} \leq \min\{y, p_2, u_2\}$. Thus the proposition implies there is broking for income $y \geq y''$.

**Case (ii).** We will prove the statement for $p_1$. Hold all the other parameters constant. Write the zero-profit broking repayment as $R^{NBR}(p_1)$ to emphasize its dependence on $p_1$. By (7), $dR^{NBR}/dp_1 = (1+r)/\gamma h$. Thus $\lim_{p_1 \to \infty} R^{NBR} = \infty$. Hence there exists $p_1''$ such that $R^{NBR}(p_1'') > \min\{y, p_2, u_2\}$. For this value of the first-period house price, the condition for a broking equilibrium in case (i) of Proposition 1 is violated even if treated as a weak inequality. Hence there is no broking in equilibrium. Similar analysis applies to banking. Thus there is no equilibrium with lending if the first-period house price is $p_1''$.

Next, suppose there is no lending in equilibrium for first-period house price $p_1''$. We will show there is no lending in equilibrium for all $p_1 \geq p_1''$. We have $R^{NBR}(p_1) \geq R^{NBR}(p_1'') > m$. The first inequality follows from $dR^{NBR}/dp_1 > 0$. The second inequality follows because there is no lending in equilibrium, so no broking, so the condition in case (i) of Proposition 1 must be violated even as a weak inequality. But then Proposition 1 implies there is no broking for first-period house price $p_1$. Similar analysis applies to banking. Thus there is no equilibrium with lending for all $p_1 \geq p_1''$.

Next, suppose there is an equilibrium with broking for first-period house price $p_1'$. We will show that there is an equilibrium with broking for all $p_1 \leq p_1'$. We have $m \geq R^{NBR}(p_1') \geq R^{NBR}(p_1)$. The first inequality follows from case (i) of Proposition 1 and the fact that there is a broking equilibrium for $p_1'$. The second inequality follows from $dR^{NBR}/dp_1 > 0$. By case (i) of Proposition 1, there exists a broking equilibrium for first-period house price $p_1$.

**Case (iii).** Hold all the other parameters but $\gamma$ constant. Assume there is a broking equilibrium for some $\gamma'$. We will show there is a also a broking equilibrium for all $\gamma \geq \gamma'$. Rewrite the zero-profit broking repayment as $R^{NBR}(\gamma)$ to emphasize its dependence on $\gamma$. We have $m \geq R^{NBR}(\gamma') \geq R^{NBR}(\gamma)$. The first inequality follows from case (i) of Proposition 1 and the fact that there is a broking equilibrium for $\gamma'$. The second inequality follows because $dR^{NBR}/d\gamma < 0$ as can be seen from (7). Proposition 1 implies there is a broking equilibrium for all $\gamma \geq \gamma'$.

Next, assume there is no broking equilibrium for $\gamma'$. Then for all $\gamma \leq \gamma'$, $R^{NBR}(\gamma) > R^{NBR}(\gamma') > m$. The first inequality follows from $dR^{NBR}/d\gamma < 0$. The second inequality follows because there is no broking for $\gamma'$, so the condition in case (i) of Proposition 1 must be violated even as a weak inequality. Therefore there is no broking equilibrium for all $\gamma \leq \gamma'$. Whether the equilibrium involves no contract or banking for $\gamma \leq \gamma'$ depends on the relative values of $R^{SBK}$ and $\min\{y + u_2 + (u_1 - k_C)/h, m\}$. As can be seen from (5) or direct inspection, neither of these expressions depend on $\gamma$.

**Case (iv).** We will prove the statement for $k_I$. Hold all the other parameters constant. Write the zero-profit banking repayment as $R^{SBK}(k_I)$ to emphasize its dependence on $k_I$. By (5), $\lim_{k_I \to \infty} R^{SBK} = \infty$. Hence there exists $k_I'$ such that $R^{SBK}(k_I') > \min\{y + u_2 + (u_1 - k_C)/h, m\}$. For this value of the screening cost, the condition for a banking equilibrium in case (ii) of Proposition 1 is violated even if treated as a weak inequality. Hence there is no banking in equilibrium for $k_I'$. By (5), $dR^{SBK}/dk_I = 1/h > 0$. Hence, for all $k_I \geq k_I'$, $R^{SBK}(k_I) \geq R^{SBK}(k_I')$. Hence there is no banking in equilibrium for all $k_I \geq k_I'$.

Suppose instead that a banking equilibrium exists for some $k_I'$. Both conditions in case (ii) of Proposition 1 must be satisfied at least as weak inequalities. In particular, the one depending on $k_I$ must hold:

$$R^{SBK}(k_I') \leq \min\{y + u_2 + (u_1 - k_C)/h, m\}.$$

But for all $k_I \in [0, k_I']$, $R^{SBK}(k_I) \le R^{SBK}(k_I')$. Therefore, both conditions in case (ii) of Proposition 1 are satisfied for all $k_I \in [0, k_I']$, implying there exists a banking equilibrium for these values of the screening cost.

If equilibrium does not involve banking, whether it involves broking or no contract depends on the relative values of $R^{NBR}$ and $m$, which are both independent of $k_I$.

**Case (v).** Hold all the other parameters but $u_1$ constant. Assume there exists a banking equilibrium for some $u_1'$. We will show there exists a banking equilibrium for all $u_1 \ge u_1'$. By (5), $R^{SBK}$ is independent of $u_1$. Since $u_1'$ satisfies the conditions in case (ii) of Proposition 1 at least as weak inequalities,

$$\begin{aligned} R^{SBK} &\le \min\left\{y + u_2 + \frac{u_1' - k_C}{h}, m\right\} \\ &\le \min\left\{y + u_2 + \frac{u_1 - k_C}{h}, m\right\}. \end{aligned}$$

Thus the conditions in case (ii) are also satisfied by all $u_1 \ge u_1'$, implying there exists a banking equilibrium for all $u_1 \ge u_1'$.

If equilibrium does not involve banking, whether it involves broking or no contract depends on the relative values of $R^{NBR}$ and $m$, which are both independent of $u_1$. *Q.E.D.*

## Completing Proof of Proposition 5

We have thus proved Theorem 2. To complete the proof of Proposition 5, the reader can check parameter by parameter that if the broking interval borders a no-contract interval, then there is no banking in equilibrium for any value of the parameter. It remains to demonstrate a case in which banking appears in an intermediate interval between no-contract and broking intervals. We will do this by fixing all the parameters except for $p_1$ and consider how the mode of lending changes as $p_1$ is increased starting from a lower bound of $\lambda p_2/(1+r)$; below this bound, (3) is violated.

Assume $p_2 < \min\{y, u_2\}$, implying $m = p_2$. Further assume $d + \gamma < 1$. Define

$$\begin{aligned} p_1' &= \frac{\gamma(1-d)}{(1-d-\gamma)(1+r)}\left[k_I + \left(\frac{1-\gamma}{\gamma}\right)\lambda p_2\right] \\ p_1'' &= \left(\frac{1-d}{1+r}\right)\{[1+(1-\lambda)h]p_2 - k_I. \end{aligned}$$

Direct calculation verifies that $p_1'$ satisfies equation . Hence $R^{NBR} = R^{SBK}$ at $p_1'$. Substituting $p_1''$ into (5) shows that $R^{SBK}$, when evaluated at $p_1''$, equals $p_2$.

We have our case. For $p_1 < p_1'$, we have $R^{NBR} < R^{SBK}$ and $R^{NBR} < p_2 = m$. By case (i) of Proposition 1, the equilibrium involves broking. For $p_1 \in (p_1', p_1'')$, we have $R^{SBK} < R^{NBR}$ and $R^{SBK} < m$. By case (ii) of Proposition 1, the equilibrium involves banking. For $p_1 > p_1''$, $m < \min\{R^{NBR}, R^{SBK}\}$. By case (iii) of Proposition 1, there is no lending in equilibrium. *Q.E.D.*

## Completing Proof of Proposition 6

We will analyze the comparative-statics effects of the credit-worthiness parameters $h$, $\gamma$, and $y$ on mortgage volume and default probability in turn.

**Mortgage Volume.** Consider an increase in $y$ holding all other parameters constant. Case (i) of Theorem 2 states that this parameter change moves the equilibrium from a no-contract to a banking to a broking interval. One or more of these intervals can be empty. The probability that the consumer receives a mortgage equals 0 in the no-contract interval, $\gamma$ in the banking interval (because all good types and only good types are served), and 1 in the broking interval (because all consumers are served). Thus mortgage volume is non-decreasing in $y$.

The analysis is similar for parameters $h$ and $\gamma$, except case (iii) of Theorem 2 is the relevant result.

**Default Probability.** Consider an increase in $y$ holding all other parameters constant. As discussed in the previous subsection, this parameter change moves the equilibrium from a no-contract to a banking to a broking interval. The probability of default conditional on a mortgage being signed is undefined in the no-contract interval. The conditional default probability is $1-h$ in the banking interval, which is the probability a good type earns low income. This is constant, and so non-increasing, over the banking interval. The conditional probability the consumer does not default equals $\gamma h$, the probability the consumer is the good type and earns high income. Thus the conditional probability of default is $1-\gamma h$. This is a constant, and so non-increasing, over the broking interval. The conditional default probability jumps from $1-h$ to $1-\gamma h$ at the boundary between banking and broking.

The analysis is similar for parameters $h$ and $\gamma$, except case (iii) of Theorem 2 is the relevant result. *Q.E.D.*

## Proof of Proposition 8

Maintain the parameter restrictions stated in the proposition: $y, u_2 \to \infty$, $\underline{p}_2 \to 0$, $R^{NBR}, R^{SBK} < p_2$, and equation (15). We will first show that bank lending is feasible with stochastic prices. As argued in the text, bank lending is feasible as long as the repayment does not induce strategic default in the high-income, high-price state: $\hat{R}^{SBK} \leq \bar{p}_2$. We have

$$\begin{aligned}\phi \hat{R}^{SBK} + (1-\phi)\lambda \underline{p}_2 &= R^{SBK} \\ &< p_2 \\ &= \phi \bar{p}_2 + (1-\phi)\underline{p}_2,\end{aligned}$$

where the first line follows from (18), the second line follows from the maintained assumption $R^{SBK} < p_2$, and the last line follows from (15). Rearranging,

$$\hat{R}^{SBK} < \bar{p}_2 + \left(\frac{1-\phi}{\phi}\right)(1-\lambda)\underline{p}_2.$$

But by maintained assumption, $\underline{p}_2 \to 0$, implying $\hat{R}^{SBK} < \bar{p}_2$.

Similar arguments show $\hat{R}^{NBR} < \bar{p}_2$, showing that broker mortgages are also feasible in the stochastic-price model under the maintained assumptions.

The last step in the proof is to compute the expected welfare loss in moving from deterministic to stochastic prices. Let $\hat{W}^{SBK}$ be expected welfare with a bank loan and $\hat{W}^{NBR}$ with a broker loan in the stochastic-price model. Because lenders and investors break even, welfare is given by the consumer's utility. With a bank mortgage, $\hat{W}^{SBK}$ equals

$$\begin{aligned}&\gamma\{u_1 + h[\phi(u_2 + y - \hat{R}^{SBK}) + (1-\phi)(u_2 + y - \underline{p}_2)] - k_C\} \\ =\ & W^{SBK} - \gamma h(1-\phi)(1-\lambda)\underline{p}_2.\end{aligned}$$

The equality follows from substituting for $\hat{R}^{SBK}$ from (18) and using the expression for $W^{SBK}$ from (12). Thus

$$W^{SBK} - \hat{W}^{SBK} = \gamma h(1-\phi)(1-\lambda)\underline{p}_2.$$

Similarly, one can show

$$W^{NBR} - \hat{W}^{NBR} = \gamma h(1-\phi)(1-\lambda)\underline{p}_2.$$

*Q.E.D.*

## Proof of Proposition 9

Maintain the parameter restrictions stated in the proposition: $y, u_2 \to \infty$, $\underline{p}_2 \to 0$, $R^{NBR}, R^{SBK} < p_2$, and equation (15). We will show that the good type obtains higher expected utility from the zero-profit banking than broking contract if (22) holds. The good type's expected utility from a zero-profit banking contract is

$$u_1 + h\left[u_2 + y - \phi\hat{R}^{NBR} - (1-\phi)\underline{p}_2\right] \tag{B19}$$

and from the zero-profit broking contract is

$$u_1 + h\left[u_2 + y - \phi\hat{\hat{R}}^{SBK} - (1-\phi)\underline{p}_2\right]. \tag{B20}$$

Expression (B19) exceeds (B20) if $\hat{\hat{R}}^{SBK} < \hat{R}^{NBR}$. Using (21) to substitute for $\hat{\hat{R}}^{SBK}$, using (19) to substitute for $\hat{R}^{NBR}$, and rearranging yields (22). *Q.E.D.*

## Proof of Proposition 11

Fix any values of $\gamma \in (0,1)$, $h \in (0,1)$, $u_1 > 0$, and $p_2 > 0$. We will look for values of the other parameters and a function $\lambda(\delta)$ that together maximize $W^{SBK} - W^{NBR}$ subject to the constraint that both banking and broking equilibria exist in the case and $\lambda(\delta)$ is an admissible foreclosure friction function: i.e., that $\lambda(\delta)$ is a decreasing function on $[0,1]$. One simplification is that the shape of $\lambda(\delta)$ over its range is immaterial except for the values it takes on at the equilibrium default rates, $\lambda^{NBR}$ and $\lambda^{SBK}$. Thus we will treat those as two additional variables in the maximization problem. For $\lambda(\delta)$ to be an admissible foreclosure function, $0 < \lambda^{SBK} < \lambda^{NBR} < 1$. We will treat these strict inequalities as equalities because we are looking for a supremum on the welfare difference and do not require that it will be reached by some parameter vector.

Using (24), the problem is to choose non-negative values of $\lambda^{NBR}$, $\lambda^{SBK}$, $d$, $y$, $r$, $u_2$, $p_1$, and $k_I$ to maximize

$$\gamma h[R^{NBR}(\lambda^{NBR}) - R^{SBK}(\lambda^{SBK})] - (1-\gamma)u_1 \tag{B21}$$

subject to

$$\lambda^{NBR} \leq \lambda^{SBK} \tag{B22}$$

$$\lambda^{SBK} \leq 1 \tag{B23}$$

$$d \leq 1 \tag{B24}$$

$$\max\{\lambda^{SBK}, \lambda^{NBR}\} p_2 \leq (1+r)p_1 \tag{B25}$$

$$R^{SBK}(\lambda^{SBK}) \leq R^{NBR}(\lambda^{NBR}) \tag{B26}$$

$$R^{NBR}(\lambda^{NBR}) \leq R^{SBK}(\lambda^{NBR}) \tag{B27}$$

$$\max\{R^{SBK}(\lambda^{SBK}), R^{NBR}(\lambda^{NBR})\} \leq m. \tag{B28}$$

Constraint (B22) and (B23) were mentioned in the previous paragraph as ensuring the foreclosure function is admissible. Constraint (B24) ensures $d$ is a fraction as required. Constraint (B25) maintains the house-price assumption (3) for both $\lambda^{SBK}$ and $\lambda^{NBR}$. The final three constraints ensure that the conditions behind case (i) and (ii) of Proposition 10 are met so that the parameters are in the multiple-equilibrium region.

Several steps can help simplify the maximization problem. Given (B22) holds, (B25) reduces to $\lambda^{SBK} p_2 \leq (1+r)p_1$. Given (B26) holds, (B28) reduces to $R^{NBR}(\lambda^{NBR}) \leq m$. Constraints (B22), (B23), and (B26) can be ignored; we will show later that the solution satisfies them. The optimal values of $y$ and $u_2$ are easy to characterize. The variables only appear in (B28) through the dependence of $m$ on them. Increasing $y$ and $u_2$ weakly relaxes this constraint. Any values $y, u_2 \geq p_2$ are thus optimal. Thus $m = p_2$ at an optimum. Finally, because certain parameters occur in the problem in certain configurations, several changes of variable simplify the problem. Let

$$\begin{aligned} \hat{p}_1 &= (1+r)p_1 \\ \hat{k}_I &= k_I + \left(\frac{d}{1-d}\right)(1+r)p_1. \end{aligned}$$

Substituting $\lambda^{SBK}$ into (5) to find $R^{SBK}(\lambda^{SBK})$, substituting $\lambda^{NBR}$ into (7) to find $R^{NBR}(\lambda^{NBR})$, substituting the resulting expressions for $R^{SBK}(\lambda^{SBK})$ and $R^{NBR}(\lambda^{NBR})$ into the previous maximization problem and making the other changes indicated in the previous paragraph, the problem reduces to one of choosing $\lambda^{NBR}$, $\lambda^{SBK}$, $\hat{p}_1$, and $\hat{k}_I$ to maximize

$$\gamma h \left[\lambda^{NBR} p_2 - \frac{1}{\gamma h}(\hat{p}_1 - \lambda^{NBR} p_2) - \lambda^{SBK} p_2 - \frac{1}{h}(\hat{p}_1 + \hat{k}_I - \lambda^{SBK} p_2)\right] - (1-\gamma)u_1 \tag{B29}$$

subject to

$$\lambda^{SBK} p_2 \leq \hat{p}_1 \tag{B30}$$

$$\lambda^{NBR} p_2 + \frac{1}{\gamma h}(\hat{p}_1 - \lambda^{NBR} p_2) \leq \lambda^{NBR} p_2 + \frac{1}{h}(\hat{p}_1 + \hat{k}_I - \lambda^{NBR} p_2) \tag{B31}$$

$$\lambda^{NBR} p_2 + \frac{1}{\gamma h}(\hat{p}_1 - \lambda^{NBR} p_2) \leq p_2. \tag{B32}$$

The objective function (B29) is decreasing in $\hat{k}_I$. The only other place this variable appears is in constraint (B31), which obviously binds at an optimum. Treating it as an equality and rearranging,

$$\hat{k}_I = \left(\frac{1}{\gamma} - 1\right)(\hat{p}_1 - \lambda^{NBR} p_2). \tag{B33}$$

The objective function (B29) is increasing in $\lambda^{SBK}$. The only other place this variable appears is in constraint (B30), which obviously binds at an optimum, implying $\lambda^{SBK} = \hat{p}_1/p_2$.

Substituting this value of $\lambda^{SBK}$ and the value of $\hat{k}_I$ from (B33) into (B29) and rearranging, the problem reduces to one of choosing $\lambda^{NBR}$ and $\hat{p}_1$ to maximize

$$\gamma(1-h)(\hat{p}_1 - \lambda^{NBR} p_2) - (1-\gamma)u_1 \tag{B34}$$

subject to

$$\lambda^{NBR} p_2 + \frac{1}{\gamma h}(\hat{p}_1 - \lambda^{NBR} p_2) \leq p_2. \tag{B35}$$

The objective function is increasing in $\hat{p}_1$. An increase in $\hat{p}_1$ tightens (B35), implying that this constraint binds at an optimum.

Solving (B35) as an equality for $\hat{p}_1$ and substituting this value of $\hat{p}_1$ into (B34) leaves the unconstrained problem of choosing $\lambda^{NBR} \geq 0$ to maximize

$$\gamma^2 h(1-h) p_2 (1 - \lambda^{NBR}) - (1-\gamma)u_1. \tag{B36}$$

The solution is $\lambda^{NBR} = 0$, which upon substituting into (B36), gives the bound in (25).

We have solved for the maximum welfare loss from securitization with weak inequality constraints. If any of the constraints are strict inequalities, the bound is a supremum that can be approached but perhaps not attained. *Q.E.D.*

# Appendix C: Foundations of Simple Debt and Pooling Contracts

The appendix provides two theorems that restrict the form of the mortgage contract in our model. Our aim is to show that the contractual form taken for granted in the analysis in the text is in fact the optimal form, and indeed is optimal in a more general setting assumed in our model is essentially endogenous, and independent of some of the simplifying assumptions in the model, in particular the assumption of zero income. The first result demonstrates that contracts have the simple debt form. The theorem provides two sets of sufficient conditions for pooling, rather than sorting contracts to obtain in equilibrium. Notation and structure follow the basic model, with the following modifications:

- $h_\theta$ is the probability that type $\theta \in \{g, b\}$ (where $g$ represents the good type, and $b$ represents the bad type) has high period-2 income, $1 > h_g > h_b \geq 0$ ($h_b = 0$ is assumed in basic model).
- $\hat{\theta} \in \{g, b\}$ is an announcement of type.
- Period-2 income is $y_L$ or $y_H$, $y_H > y_L$ ($y_L = 0$ in basic model). Following the paper, we assume $y_L + w < \lambda p_2$, i.e. liquidation yields higher revenue to the lender than the highest repayment the bad consumer can afford. The bad consumer is thus loss-making for lenders.
- $\hat{I} \in \{H, L\}$ is an announcement of period-2 income, $I \in \{H, L\}$ is the true type.
- $w \geq 0$ is period-1 wealth measured in period-2 dollars (assumed zero in the basic model).

**Theorem 3.** *If there is no public randomizing device in period 2, period-2 income is low, and period-1 wealth is insufficient to fund borrowing ex-ante, then the only ex-post incentive-compatible contract has a simple-debt form.*

*Proof.* Let $(\alpha_{\hat{I}}, R_{\hat{I}}, w_I)$ be a general deterministic mechanism, where $\alpha_{\hat{I}} \in \{0, 1\}$ is an indicator function and $R_{\hat{I}}$ is a payment from the consumer to the lender, and $w_I \leq w$ is a payment from the consumer's initial wealth.The mechanism depends on the period-1 announcement $\hat{\theta}$, but we suppress this for notational simplicity. The consumer's payoff under the mechanism is $y_I + w + \alpha_{\hat{I}} u_2 - R_{\hat{I}}$ if $R_{\hat{I}} \leq m_I$ and $y_I + w$ if $R_{\hat{I}} > m_I$, where $m_I \equiv \min\{y_I + w_I, p_2, u_2\}$ is the consumer's ex-post participation constraint. The IC constraint for the high income consumer is

$$\alpha_H u_2 - R_H \geq \alpha_L u_2 - R_L \tag{C1}$$

Period-1 feasibility for the lender implies that $R_H > y_L + w$, i.e. payment of the consumer's wealth plus the low level of income in both the high and low income states is not sufficient to fund borrowing. Any mechanism with $R_H > y_L + w$ is always incentive compatible for the low-income consumer, who faces the IR constraint

$$\alpha_L u_2 - R_L \geq 0. \tag{C2}$$

Note that since $R_H > y_L + w \geq R_L$, satisfaction of (C1) and (C2) is only possible if $\alpha_H = 1$, $\alpha_L = 0$, $R_L = 0$ and $R_H \leq m_H$. This is the simple debt form. *Q.E.D.*

**Theorem 4.** *Ex-ante incentive compatible contracts are pooling with both types accepting a loan in equilibrium if period-1 wealth w is sufficiently limited.*

*Proof.* First note that it cannot be part of a Bertrand equilibrium with a separating contract with at least two lenders, for a loan to be given to a loss-making type: If both types are loss-making, then no contracts will be offered. If one type is loss making and the other is not, a lender will deviate from any putative equilibrium in which loans are given to both types, and only offer a contract that the profitable type will prefer, leaving others to offer the contract to the loss-making type. The only separating contract is some $(x, R_H, R_L)$, where $x \leq w$ is an up-front payment, which is accepted by the good type and rejected by the bad type. Noting by the last result that if low income is realized in period 0, the consumer is foreclosed, the good type will prefer this contract to no contract whenever

$$u_1 + w - x + h_g(y_H + u_2 - R_H) + (1 - h_g)y_L \geq w + h_g y_H + (1 - h_g)y_L.$$

The bad type will prefer no contract whenever

$$u_1 + w - x + h_b(y_H + u_2 - R_H) + (1 - h_b)y_L \leq w + h_b y_H + (1 - h_b)y_L.$$

These reduce, respectively, to

$$u_1 + h_g(u_2 - R_H) \geq x$$

and

$$u_1 + h_b(u_2 - R_H) \leq x.$$

If wealth is limited such that $u_1 + h_b(u_2 - R_H) > w$, then a sorting contract is infeasible. *Q.E.D.*

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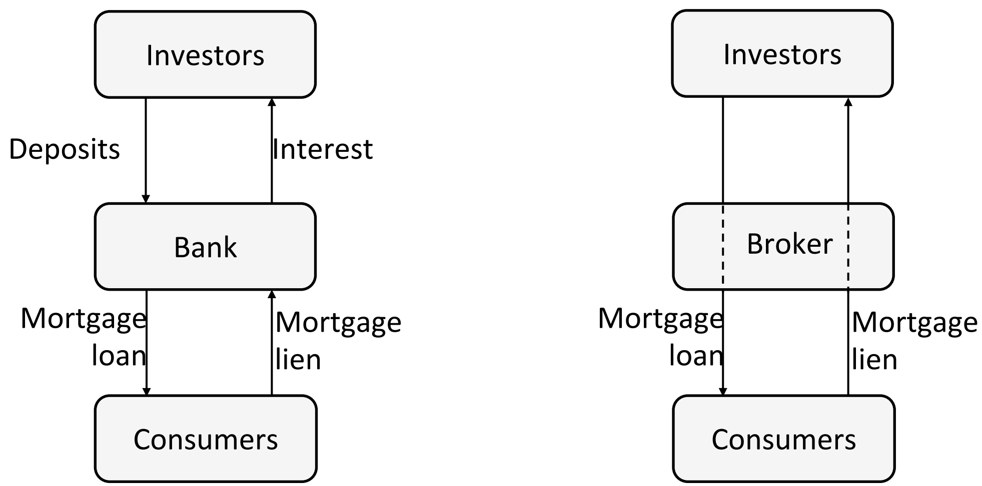


**Figure 1: Modes of Operation for an Intermediary**

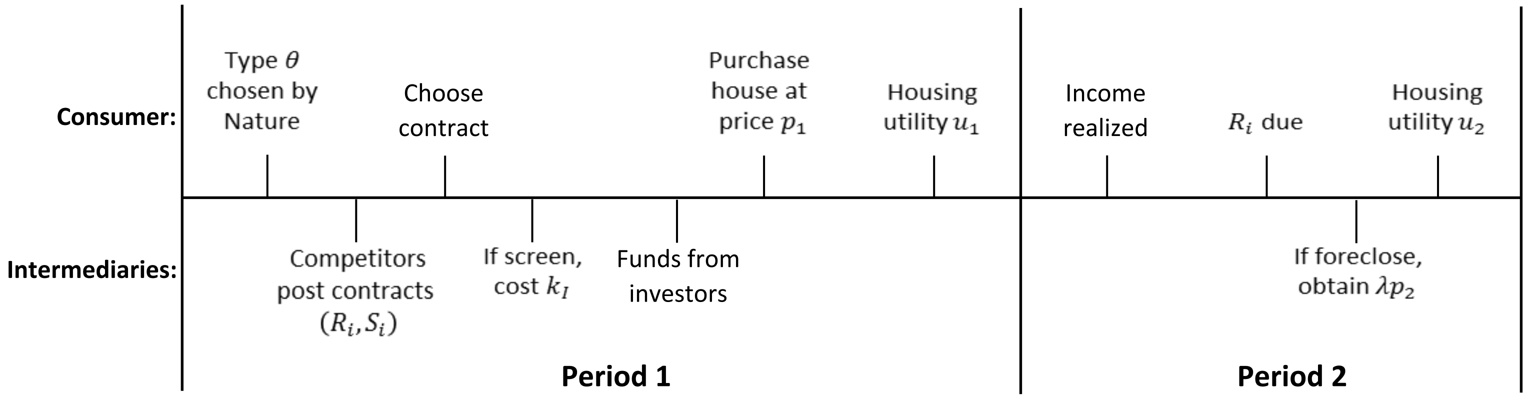


**Figure 2: Timing of Model**

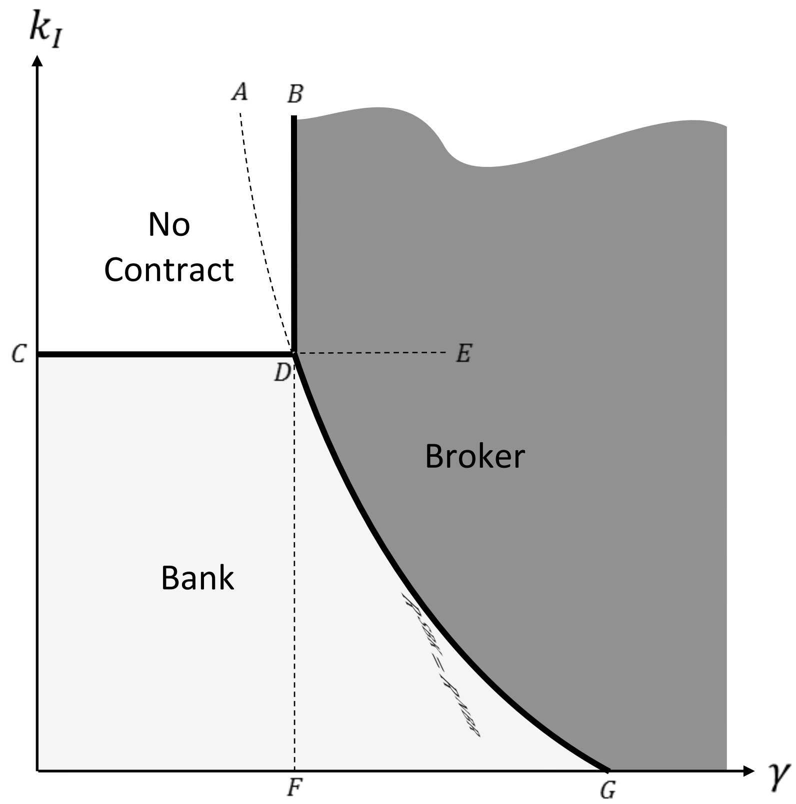


**Figure 3: Equilibrium Characterization**

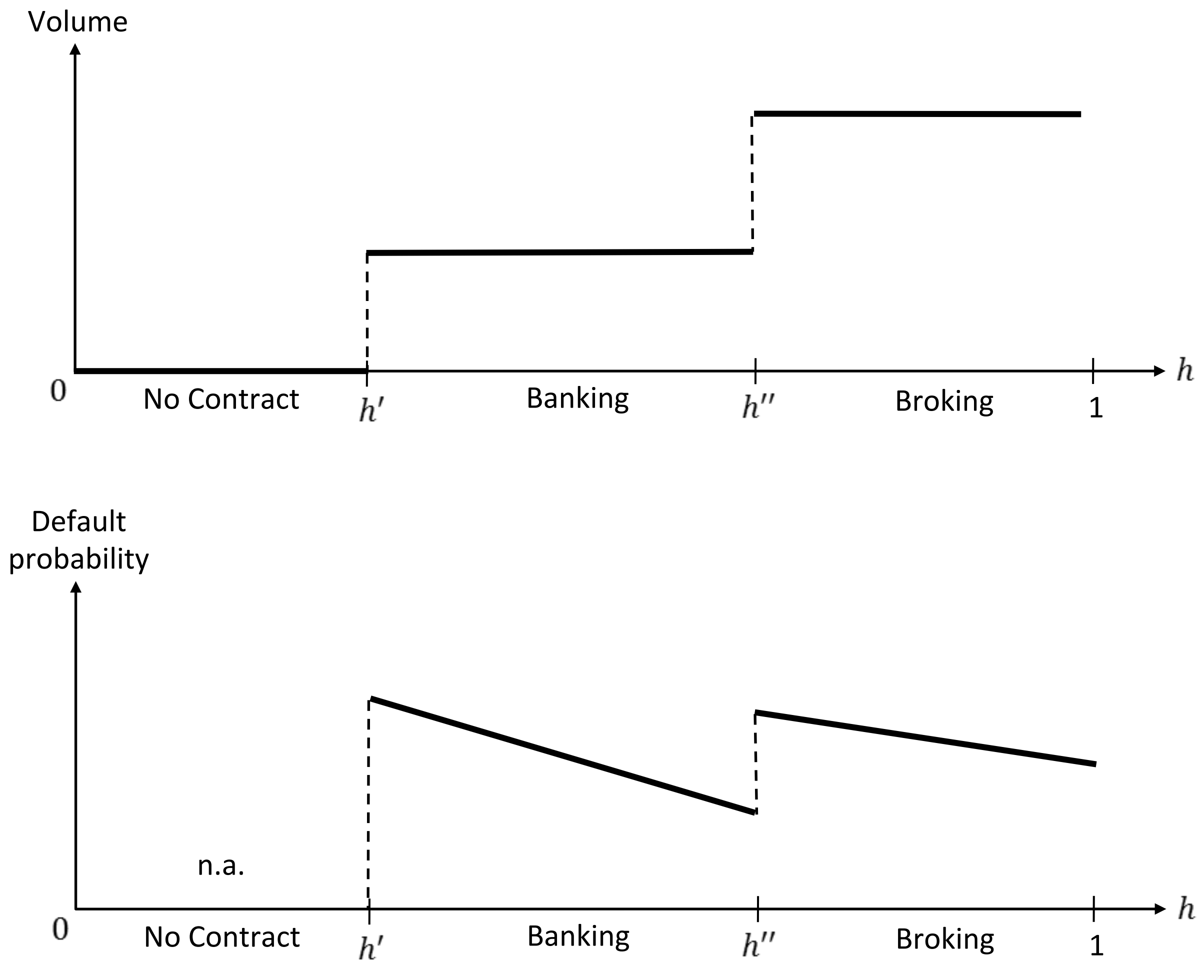


**Figure 4: Effect of $h$ on Mortgage Quantity and Quality**

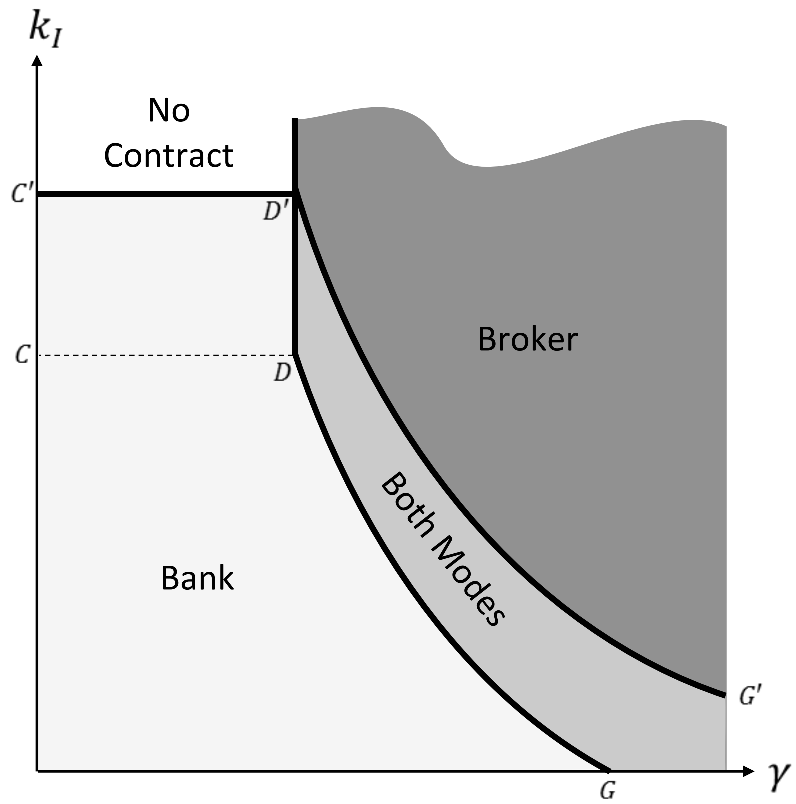


**Figure 5: Multiple Equilibria with Systemic Factors**