Putting Your Ballot Where Your Mouth Is - An Analysis of Collective Choice with Communication*

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Abstract

The goal of this paper is to analyze collective decision making with communication. We concentrate on decision panels that are comprised of a collection of agents sharing a common goal, having a joint task, and possessing the ability to communicate at no cost. We first show that communication renders all intermediate threshold voting rules equivalent with respect to the sequential equilibrium outcomes they produce. We then proceed to analyze the mechanism design problem in an environment in which members of the group decide whether to acquire costly information or not, preceding the communication stage. We find that: 1. Groups producing the optimal collective decisions are bounded in size; and 2. The optimal incentive scheme in such an environment balances a tradeoff between inducing players to acquire information and extracting the maximal amount of information from them. In particular, the optimal device may aggregate information suboptimally from a statistical point of view. In addition, we provide some comparative statics results on the optimal extended mechanism for collective choice.

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"What raises us out of nature is the only thing whose nature we can know: language. Through its structure autonomy and responsibility are posited for us. Our first sentence expresses unequivocally the intention of universal and unconstrained consensus."

-Jurgen Habermas.

1 Introduction

The current paper introduces communication between agents in a collective choice setting. We consider a simple setup in which a group of agents chooses one out of two alternatives. Unlike most extant models of collective decision-making (e.g., Austen-Smith and Banks [1996], Feddersen and Pesendorfer [1996, 1997]), we analyze a setting in which agents can communicate before jointly choosing an action.

Our inquiry is motivated by the observation that most group decision processes contain some form of communication phase before collective choices are made. For example, trial jurors converse before casting their votes, hiring committees convene before making their final decisions, and top management teams hold meetings before determining their firm's investment strategies.

The goal of this paper is twofold. First, we demonstrate that communication renders a large class of voting rules equivalent in terms of the sets of sequential equilibrium outcomes they generate. This serves as input to the second part of our investigation in which we analyze a mechanism design problem where homogeneous agents are capable of investing in information acquisition. The designer chooses the size, communication system, and voting rule in order to maximize the (common) expected utility of the collective decision. Our second goal is to characterize the solution of this mechanism design problem.

The characterization of the optimal mechanism yields a few interesting insights: 1. The optimal size of the decision panel is bounded and does not necessarily coincide with the maximal number of agents who can be induced to purchase information in equilibrium; 2.

In order to provide strong incentives for information acquisition, the optimal device does not necessarily utilize all the information that is reported; and 3. The optimal mechanism exhibits many intuitive regularities with respect to the comparative statics of the problem: e.g., the expected social value is monotonic in the cost of information and accuracy of private information.

More formally, in the first part of the paper, we consider the class of threshold voting rules parametrized by $r = 1, ..., n$. Under voting rule r, the first alternative is chosen if and only if at least r agents vote in favor of it. Our first main result illustrates that regardless of the structure of private information, when players can communicate before casting their votes, voting rules $2, \ldots, n-1$ are identical, in the sense that they all yield the same set of sequential equilibrium outcomes.

The reasoning for our equivalence result is as follows. Take an outcome implementable with communication under voting rule $r = 2, ..., n-1$. The revelation principle (see Myerson [1982], Forges [1986]) implies that this equilibrium outcome can be implemented with a communication device in which players truthfully reveal their types to an impartial mediator who disperses recommendations to all players. Each profile of recommended actions corresponds, through r, to one of the two social alternatives. Consider then a modification of this device which prescribes to each profile of private reports an identical recommendation to all players matching the social alternative that would have resulted in the original device. Since $1 < r < n$, any unilateral deviation will not alter the outcome, and so equilibrium incentives are maintained. In particular, the modified device generates an implementable outcome coinciding with the one we started with. Moreover, since all recommendations are unanimous, this remains an equilibrium outcome for any voting rule $r' = 2, ..., n-1$. Our equivalence result then follows.

As it turns out, all equilibrium outcomes can be implemented with only two rounds of unmediated public communication (transmitting information and dispelling recommendations).

We also note that with voting rules 1 and n (unanimity) it is possible to implement only a subset of the outcomes that can be implemented with the "intermediate" voting rules $r = 2, .., n-1$.

The equivalence result is key to the second part of the paper in which we study mechanism design with information acquisition. We consider the standard voting setup (see Feddersen and Pesendorfer [1998] or Persico [2002]). There are two possible states of the world and two alternatives that need to be matched to the states. All agents have the same utility function and a common prior on the state of the world. At the outset of the game, each agent can pay a positive cost and receive, in return, a signal of accuracy $p > \frac{1}{2}$ (i.e., the signal reports the realized state of the world with probability p). Players can then communicate with one another after which they cast their simultaneous votes. The model can serve as a parable to the decision making process of a jury (each juror decides whether to attend the testimonies or not, the jurors then meet to discuss the case, after which they simultaneously cast their votes), an advisory committee (each member invests in information and gives an advice following conversation with the other committee members), etc.

The first part of the paper implies that the designer's choice of voting rule does not affect the collective outcomes and that she can concentrate on communication devices that give only unanimous recommendations.

For any fixed number of agents, the optimal scenario, which we term the first best, entails all agents purchasing information and reporting truthfully. This information is then utilized in a statistically efficient way. That is, for every profile of signals, the optimal device calculates, using Bayes' rule, the probability that each state of the world has been realized and chooses the optimal alternative.

Unfortunately, for large numbers of agents, the marginal contribution of each signal becomes quite low and, since the cost of information is positive, a free rider problem disables the first best scenario from constituting an equilibrium.

The benevolent designer faces two options. The mechanism can simply induce a small number of agents to purchase information and make the best statistical use of it. Alternatively, the designer can alleviate the free rider problem by using an aggregation rule which is not statistically efficient, thus increasing the incentives to acquire information. This approach involves a compromise between achieving more information in the population, but creating intentional distortions in the interpretation of the collective signals.

Our theoretical results indicate that the optimal design employs both approaches. One can achieve the optimal expected social value by using distortionary mechanisms. These are devices that induce more players to acquire information than would be possible if the mechanism were using statistically efficient rules in creating recommendations. Nonetheless, the optimal mechanism does not always exhaust the number of agents that can be induced to purchase information. That is, the optimal size of a panel of decision makers may be smaller than the maximal number of players who can be made to invest in information in equilibrium.

The mechanism design problem described so far depends on essentially three parameters: a preference parameter indicating the relative preference of matching alternative to each state (in the terminology of the jury literature, this is the weight each juror puts on convicting the innocent relative to acquitting the guilty), the accuracy of the private information, and the cost of private information.

There are a few interesting insights regarding the comparative statics of the mechanism design problem. As was already mentioned, the expected (common) utility of the optimal mechanism is decreasing in the cost of information and increasing in the accuracy of available signals. It also appears that as the cost of information increases, the optimal decision panel decreases in size. Finally, the optimal committee size is not monotonic in the signals' accuracy.

The paper is structured as follows. Section 2 overviews some of the related literature. Section 3 describes the general setup of collective choice with communication and provides the comparison between different threshold voting rules. Section 4 analyzes the mechanism design problem. Section 5 concludes. Most technical proofs are relegated to the Appendix.

2 Related Literature

The current paper is linked to a few strands of literature. First, the paper contributes to the literature on mechanism design with endogenous information. While most of this literature deals with auction and public good models (see, e.g., Auriol and Gary-Bobo [1999], Bergemann and Välimäki [2002], and references therein), there are a few exceptions focusing on collective decision-making.

Persico [2002] is possibly the closest paper to ours. He considers jury decisions and allows the jurors to acquire information before voting. In contrast to our model, the jurors are not allowed to communicate. Persico [2002] analyzes the problem of the designer who can choose the size of the jury and the voting rule. While the tension between giving incentives to acquire information and aggregating information efficiently comes through in his framework, the optimal mechanism is very different from ours. In particular, the distinction between different voting rules plays a crucial role in Persico [2002] but becomes irrelevant in our context once we allow for communication.

Li [2001] considers a committee of a fixed size and allows each player to invest in the precision of her private signal. Information is a public good and, thus, there is an insufficient effort to gather information. To alleviate this problem, it is optimal to introduce statistical distortions to the decision rule. In Li [2001] investments as well as signals are publicly observed and thereby verifiable. As we show below, in our setup verifiability assures that a non-distortionary rule is optimal when the committee is large enough. It is in environments in which investment and acquired signals are not transparent (such as in the case of jury decisions, hiring committee decisions, etc.) that distortionary devices end up being optimal.

Mukhopadhaya [1998] restricts attention to majority rule elections and compares committees of different sizes. Players decide whether to acquire information or not before voting (and communication is prohibited). Mukhopadhaya [1998] shows that the quality of the decision may worsen when the size of the committee increases.

Our paper is also connected with a few recent attempts to model strategic voting with communication. Coughlan [2000] adds a straw poll preceding the voting stage. He shows that voters reveal their information truthfully if and only if their preferences are sufficiently close. Doraszelski, Gerardi and Squintani [2002] study a two-player model with communication and voting. Preferences are heterogenous (not necessarily aligned) and private information. They show that some, but not all, information is transmitted in equilibrium, and that communication is beneficial.

In a similar vein, there has been some experimental work on voting with communication. Guarnaschelli, McKelvey, and Palfrey [2000] constructed an experiment replicating Coughlan's [2000] setup. They noted that during deliberations, voters tend to expose their private information but not to the full extent as predicted by Coughlan's [2000] results.

Blinder and Morgan [2000] conducted experiments in which groups were required to solve two problems - a statistical urn problem and a monetary policy puzzle. The groups could converse before casting their votes using either majority rule or unanimity. They found no significant difference in the decision lag when group decisions were made by majority rule relative to when they were made under a unanimity requirement.

The importance of communication in political thought has been acknowledged extensively. Habermas [1976] was one of the first to lay foundations for a universal theory of pragmatism and direct attention to the importance of communication as foundations for social action. His theory served as a trigger for work on political decisions when communication is possible amongst candidates and electors. In fact, the theory of deliberative democracy is a source of an abundance of work in political science on the effects of communication on how institutions function and, consequentially, should be designed (see Elster [1998] for a good review of the state of the art of the field). The research presented here provides an initial formal framework to study some of these issues.

3 Communication and Voting

A group of $n \geqslant 3$ individuals has to select one of two alternatives. We use the terminology of jury models and denote the alternatives by A (acquit) and C (convict). Each player j has a type t_j which is private information. We let T_j denote the set of types of player j, and assume that T_j is finite. $T = \prod_{j=1}^{n} T_j$ denotes the set of profiles of types, and p is the probability distribution over T. A player's utility depends on the profile of types and the chosen alternative. Formally, for each player j there exists a function $u_j : \{A, C\} \times T \rightarrow \mathbb{R}$.

The existing models of strategic voting assume that there exists an unknown state of the world (for example, the defendant is either guilty or innocent). Each player receives a signal which is correlated with the state. Conditional on the state of the world, signals are independent across players. Moreover, all players have a preference parameter, which may be either common knowledge or private information. The utility of a player is a function of the state of the world, her preference type, and the chosen alternative. We consider a more general model than the standard voting setup in that we do not impose any restrictions on the set of possible types. In particular, we allow for correlation of the signals across individuals.

The individuals select an alternative by voting. Each player can vote to acquit, a, or to convict, c. We let $V_j = \{a, c\}$ denote the set of actions available to player j, and $V = \{a, c\}^n$ the set of action profiles. Under the voting rule $r = 1, ..., n$, the alternative C is selected if and only if r or more players vote to convict.

Given a voting rule r and a profile of votes v, we let $\psi_r(v)$ denote the group's decision. Formally, $\psi_r : V \rightarrow \{A, C\}$ is defined as follows:

$$\psi_r(v) = \begin{cases} A & \text{if } |\{j : v_j = c\}| < r, \\ C & \text{if } |\{j : v_j = c\}| \geqslant r. \end{cases}$$

The voting rule r defines the following Bayesian game G_r. Nature selects a profile of types in T according to the probability distribution p, then players learn their types, after which they vote simultaneously. If the profiles of types and actions are t and v, respectively, player j obtains $u_j(\psi_r(v), t)$.

We are interested in comparing different voting rules when players are allowed to communicate before casting their votes. We therefore add cheap talk to the game G_r. A cheap talk extension of G_r is an extensive-form game in which the players, after learning their types, exchange messages. At the last stage of the game, the players vote. Payoffs depend on the players' types and votes, but not on their messages. For the moment, we also assume that there exists an impartial and exogenous mediator who helps the players communicate (for a general definition of cheap talk extensions to arbitrary games see Myerson [1991]).

A strategy profile σ of a cheap talk extension of G_r induces an outcome, i.e., a mapping γ_σ from the set of types T into the interval $[0,1]$. $\gamma_\sigma(t)$ denotes the probability that the defendant is convicted when the profile of types is t (and the players adopt the strategy profile σ). We let Γ_r denote the set of outcomes induced by Bayesian Nash equilibria of cheap talk extensions of G_r. The notion of communication equilibrium (Myerson [1982], Forges [1986]) allows us to characterize the set Γ_r. A mapping μ from T into $\Delta(V)$, the set of probability distributions over V, is a communication equilibrium of G_r if and only if the following inequalities hold:[1]

$$\sum_{t_{-j}\in T_{-j}} p(t_{-j}|t_j) \sum_{v\in V} \mu(v|t)\, u_j(\psi_r(v), t) \geqslant \sum_{t_{-j}\in T_{-j}} p(t_{-j}|t_j) \sum_{v\in V} \mu\left(v|t_{-j}, t'_j\right) u_j(\psi_r(v_{-j}, \delta(v_j)), t)$$

$$\forall j = 1, ..., n, \quad \forall \left(t_j, t'_j\right) \in T_j^2, \quad \forall \delta : \{a, c\} \to \{a, c\}.$$

The set Γ_r coincides with the set of outcomes induced by communication equilibria of G_r (Γ_r is therefore a convex polyhedron). Let V_r^C denote the set of profiles of votes that lead to conviction under the voting rule r. Formally, $V_r^C = \{v \in V : \psi_r(v) \geqslant r\}$. Then we have:

[1] As usual, T_{-j} denotes the set of types of players other than j.

$$\Gamma_r = \{\gamma : T \to [0,1] \,|\exists \text{ a communication equilibrium } \mu \text{ of } G_r \\ \text{such that } \gamma(t) = \sum_{v \in V_r^C} \mu(v|t) \text{ for every } t \in T\}.$$

We are now ready to compare the sets Γ_r and $\Gamma_{r'}$ for two different voting rules r and r'. In Proposition 1 we show that, except for the voting rules $r = 1$ and $r = n$,[2] all other "intermediate" rules are equivalent. If players can communicate, every outcome that can be implemented with a voting rule $r \neq 1, n$ can also be implemented with a different voting rule $r' \neq 1, n$. Furthermore, by adopting an extreme voting rule ($r = 1$ or $r = n$), we cannot enlarge the set of equilibrium outcomes.

Proposition 1 $\Gamma_2 = ... = \Gamma_{n-1}$. *Moreover,* $\Gamma_1 \subseteq \Gamma_2$ *and* $\Gamma_n \subseteq \Gamma_2$.

Proof. We first show that for $r = 1, ..., n$, if γ belongs to Γ_r then γ satisfies the following inequality:

$$\sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) [\gamma(t) u_j(C,t) + (1 - \gamma(t)) u_j(A,t)] \geqslant$$

$$\sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) \left[\gamma\left(t_{-j}, t'_j\right) u_j(C,t) + \left(1 - \gamma\left(t_{-j}, t'_j\right)\right) u_j(A,t)\right] \tag{1}$$

$$\forall j = 1, ..., n, \quad \forall \left(t_j, t'_j\right) \in T_j^2.$$

If γ is in Γ_r, there exists a communication equilibrium μ of G_r that induces γ. For every player j and for every pair $\left(t_j, t'_j\right)$ we therefore have:

$$\sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) [\gamma(t) u_j(C,t) + (1 - \gamma(t)) u_j(A,t)] =$$

$$\sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) [(\sum_{v \in V_r^C} \mu(v|t)) u_j(C,t) + (1 - \sum_{v \in V_r^C} \mu(v|t)) u_j(A,t)] =$$

$$\sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) \sum_{v \in V} \mu(v|t) u_j(\psi_r(v), t) \geqslant \sum_{t_{-j} \in T_{-j}} p(t_{-j}|t_j) \sum_{v \in V} \mu\left(v|t_{-j}, t'_j\right) u_j(\psi_r(v), t) =$$

[2] The voting rules $r = 1$ and $r = n$ are the only rules which require a unanimous consensus in order to adopt a certain alternative (A if $r = 1$, C if $r = n$).

$$\sum_{t_{-j}\in T_{-j}} p\left(t_{-j}|t_j\right)[(\sum_{v\in V_r^C} \mu\left(v|t_{-j},t_j'\right))u_j\left(C,t\right)+(1-\sum_{v\in V_r^C}\mu\left(v|t_{-j},t_j'\right))u_j\left(A,t\right)] =$$

$$\sum_{t_{-j}\in T_{-j}} p\left(t_{-j}|t_j\right)\left[\gamma\left(t_{-j},t_j'\right)u_j\left(C,t\right)+\left(1-\gamma\left(t_{-j},t_j'\right)\right)u_j\left(A,t\right)\right],$$

where the inequality follows from the truth-telling constraint of the communication equilibrium μ.

To give an intuition of the above result, consider the communication equilibrium μ which induces γ. Suppose that all players are obedient and that all players different from j are also sincere. Let t_{-j} be the profile of types of j's opponents. By reporting the truth, type t_j of j will induce a lottery between the alternatives A and C, with probabilities $1-\gamma\left(t_{-j},t_j\right)$ and $\gamma\left(t_{-j},t_j\right)$, respectively. If type t_j lies and reports a different message t_j', then the alternative C will be selected with probability $\gamma\left(t_{-j},t_j'\right)$. Inequality (1) simply says that every player j has an incentive to report her type truthfully provided that her opponents are sincere and all players (including j) are obedient.

Consider now a voting rule $r=2,...,n-1$. We now demonstrate that if $\gamma:T\rightarrow[0,1]$ satisfies inequality (1), then γ belongs to Γ_r. Given γ, consider the following mapping $\tilde{\mu}$ from T into $\Delta\left(V\right)$:

$$\tilde{\mu}\left(v|t\right)=\begin{cases}\gamma\left(t\right) & \text{if } v=\left(c,...,c\right),\\ 1-\gamma\left(t\right) & \text{if } v=\left(a,...,a\right),\\ 0 & \text{otherwise.}\end{cases}$$

Obviously, $\tilde{\mu}$ induces γ. It is easy to show that $\tilde{\mu}$ is a communication equilibrium of G_r. First of all, no player has an incentive to disobey the mediator's recommendation. Indeed, when the mediator follows $\tilde{\mu}$, she makes the same recommendation to all players. A player's vote cannot change the final outcome if all her opponents are obedient (notice that we are not considering $r=1$ and $r=n$). Furthermore, the fact that γ satisfies inequality (1) implies that no player has an incentive to lie to the mediator when her opponents are sincere.

We conclude that $\Gamma_2,...,\Gamma_{n-1}$ coincide with the set of the mappings from T into $[0,1]$

which satisfy inequality (1). Moreover, this set contains Γ_1 and Γ_n. ■

The inclusion of Γ_1 and Γ_n in the set Γ_2 may be strict, as the following example illustrates.

Example (strict inclusion of Γ_1 and Γ_2) Suppose that player 1 always prefers acquittal to conviction. That is, let us assume that $u_1(A,t) > u_1(C,t)$ for every t in T. Denote by γ^C the outcome in which the defendant is always convicted, i.e., $\gamma^C(t) = 1$ for every type profile t. Clearly, γ^C belongs to Γ_2. Consider the game G_2 in which the players do not communicate. The symmetric strategy profile in which every type of every player votes c constitutes a Bayesian Nash equilibrium. On the other hand, γ^C cannot belong to the set Γ_n. Consider the voting stage of any cheap talk extension of G_n. Every time player 1 is pivotal, she will vote to acquit.[3] A similar example can be constructed to demonstrate that Γ_1 may be strictly included in Γ_2.

So far we have assumed that each player can communicate privately with a trustworthy mediator. However, in many instances an exogenous mediator is not available and players can only exchange messages with each other. In addition, there are cases, like jury deliberations, in which a player cannot communicate with a subset of players but has to send her message to *all* her opponents (public communication). We would like to investigate how these restrictions affect our results.

To derive Proposition 1 we have used the Bayesian Nash equilibrium concept. However, cheap talk extensions are extensive-form games, and in a Bayesian Nash equilibrium a player could behave irrationally off the equilibrium path. Another way to check the robustness of our result is to consider a stronger solution concept.

Given a voting rule r, we define a cheap talk extension with public communication of G_r as follows. After learning their types, the players undergo a finite number of rounds of

[3] Of course, γ^C will not be an equilibrium outcome of the voting rule $r = 2, ..., n-1$ if we rule out weakly dominated strategies. This raises the question of what is the set of outcomes that can be implemented with a solution concept stronger than Bayesian Nash equilibrium. We will discuss this issue below.

communication. In each round one or more individuals send a message to all players. In the last stage, the players cast their vote simultaneously and the defendant is convicted if r or more players vote to convict. We let Γ_r^P denote the set of outcomes that are induced by sequential equilibria of cheap talk extensions with public communication of G_r.

Notice that for every $r = 1, ..., n$, the set Γ_r^P is included in Γ_r. In fact, any sequential equilibrium of a cheap talk extension is obviously a Bayesian Nash equilibrium, and any outcome that can be implemented without a mediator can be also implemented with a mediator. We now demonstrate that the opposite inclusion holds for $r = 2, ..., n-1$.

Proposition 2 *For every $r = 2, ..., n-1$, $\Gamma_r^P = \Gamma_r$.*

Proof. We need to show that every outcome γ in Γ_r is induced by a sequential equilibrium of a cheap talk extension with public communication of G_r. Consider the following game. In stage 1, all players announce their types publicly (the players make their announcements simultaneously). In stage 2, players 1 and 2 announce two numbers in the unit interval to all players (again, these announcements are simultaneous). Finally, in stage 3 the players cast their votes.

Consider the following strategy profile. In stage 1, all players reveal their types truthfully. In stage 2, both player 1 and player 2 randomly select a number in the unit interval, according to the uniform distribution.

Finally, let us describe how the players vote in stage 3. Suppose that the vector of reports in stage 1 is t. Let ω_j, $j = 1, 2$, denote the number announced by player j in stage 2. Let $\chi : [0,1]^2 \to [0,1]$ denote the following function of ω_1 and ω_2 :

$$\chi(\omega_1, \omega_2) = \begin{cases} \omega_1 + \omega_2 & \text{if } \omega_1 + \omega_2 \leqslant 1 \\ \omega_1 + \omega_2 - 1 & \text{if } \omega_1 + \omega_2 > 1 \end{cases}.$$

If $\chi(\omega_1, \omega_2) \leqslant \gamma(t)$ all players vote to convict. If $\chi(\omega_1, \omega_2) > \gamma(t)$ all players vote to acquit.

Of course, this strategy profile induces the outcome γ. It is also easy to check that our strategy profile is a sequential equilibrium (consistent beliefs can be derived from any sequence of completely mixed strategy profiles converging to the equilibrium profile). Of course, a player does not have a profitable deviation in stage 3 since her vote does not affect the final outcome. In stage 2, players 1 and 2 perform a jointly controlled lottery which determines how the players will vote. Since ω_1 is independent of ω_2 and uniformly distributed, $\chi(\omega_1, \omega_2)$ is also independent of ω_2 and uniformly distributed. Thus in stage 2, player 2 is indifferent between all announcements (clearly, the same argument can be applied to player 1). Finally, the players' reports in stage 1 determine which lottery will be used in the subsequent steps of the game. Inequality (1) guarantees that each player has an incentive to be sincere provided that all her opponents behave likewise. ■

The proof of Proposition 2 also demonstrates that all the equilibrium outcomes of an intermediate voting rule can be implemented with a single game. Two rounds of public communication followed by the voting stage are what we need to implement any equilibrium outcome. Suppose that an external designer can decide how the players can communicate, i.e., she can choose the cheap talk extension that will be played. The designer may be interested in implementing a specific outcome. To accomplish this, the designer should do two things. First, she should find a cheap talk extension with an equilibrium that induces the desired outcome. Second, the designer should induce the players to play that equilibrium. Our analysis shows that, without loss of generality, the designer can restrict attention to the simple game described in the proof of Proposition 2. The only problem is to induce the players to play the appropriate equilibrium.

Our analysis is a first attempt to compare different voting rules when players can communicate. However, we believe that there are still some important open questions.

It follows from Proposition 2 that our result on the equivalence of the intermediate rules continues to hold even if we assume that a reliable mediator is not available and require the players to be sequentially rational. On the other hand, sequential rationality does not allow us to rule out weakly dominated strategies. We feel that this is the main objection to our results. Consider the equilibrium described in the proof of Proposition 2. After stage 1, all players know the profile of types t. Suppose that the messages sent by players 1 and 2 are such that in stage 3 all players are supposed to vote to acquit. Obviously, there could be a player who, given the available information t, prefers to convict. This player does not have any incentive to vote c because her vote will not affect the final outcome. However, if either the player uses only undominated strategies or the players tremble at the voting stage and cast the wrong vote with a small but positive probability, conviction would be a plausible action for the player. Thus, it would be worthwhile to characterize the outcomes of a voting rule that can be implemented with a solution concept stronger than sequential equilibrium.

Furthermore, Proposition 2 relies on the fact that players can send their messages simultaneously. This assumption may be reasonable if players can exchange written messages. Most of the time, however, members of a committee talk. It would therefore be interesting to see what happens if we restrict attention to sequential communication, and consider cheap talk extensions with only one sender in each round of communication.

4 Mechanism Design with Information Acquisition

In this section we assume that information is costly and study the problem of a designer who can choose the procedure according to which collective decisions are made.

We concentrate on the case replicating the standard committee voting problem (see, e.g., Feddersen and Pesendorfer [1998] or Persico [2002]). There are two states of the world, I (innocent) and G (guilty), with prior distribution $(P(I), P(G))$. As in Section 3, the alternatives (or decisions) are acquittal, A, and conviction, C. There is an infinite number of identical agents. All the agents as well as the mechanism designer share the same preferences which depend on the state of the world and the final decision. Let q be a number in $(0,1)$. The common utility is given by:

$$u(d,\omega) = \begin{cases} -q & \text{if } d = C \text{ and } \omega = I \\ -(1-q) & \text{if } d = A \text{ and } \omega = G \\ 0 & \text{otherwise.} \end{cases}$$

where d and ω denote the collective decision and the state of the world, respectively.

To intuit this utility specification, consider a jury decision scenario. Jurors prefer to make the correct decision, i.e., acquitting the innocent and convicting the guilty (in this case we normalize the utility to zero). The ratio $\frac{q}{1-q}$ can be thought of as the jurors' perceived cost of convicting the innocent relative to that of acquitting the guilty.

Each agent can purchase a signal of accuracy $p > \frac{1}{2}$. That is, upon paying the cost $c > 0$,[4] the agent receives a signal $s \in \{i, g\}$ satisfying $\Pr(s = i|I) = \Pr(s = g|G) = p$. If more than one agent purchases information, we assume their signals are conditionally independent. Moreover, we only attend to the case in which an agent can buy at most one signal. While these assumptions may not always be completely realistic, they serve as a first approximation and make our benchmark model tractable.

In this environment there are infinitely many ways to make a collective decision. First, we can have committees of different sizes. Second, for a committee of a given size we can choose different voting rules. Finally, we can select different procedures according to which the members of a committee communicate before casting their votes. Of course, these variables will affect the agents' decisions (whether they acquire information or not, as well as how they communicate and vote) and, therefore, the quality of the final decision. We now analyze the problem of designing the optimal mechanism. To accomplish this, we study the following game.

Stage 1 The designer chooses an *extended mechanism*, i.e., the size of the committee n, the voting rule $r = 1, ..., n$, and a cheap talk extension (i.e., how the players can exchange messages before voting).

[4] To help the reader, we decide to follow the convention of using the variable c to denote the cost. Although we already used c in Section 3 to denote the vote to convict, no confusion arises.

Stage 2 All agents observe the designer's mechanism. Each agent $j = 1, ..., n$ decides whether to purchase a signal. These choices are made simultaneously.

Stage 3 Each member of the committee $j = 1, ..., n$ does not observe whether other members have acquired information in Stage 2.[5] All the agents in the committee exchange messages (as specified by the cheap talk extension) and vote.

Note that the chosen extended mechanism is comprised partly by the size of the decision panel. This is where we use the assumption that there exists an infinite pool of identical players free to be selected by the designer as participants in the collective decision making procedure.

Stages 2 and 3 constitute an extensive-form game played by the agents $1, ..., n$. We restrict attention to sequential equilibria in which the players use pure (behavioral) strategies in Stage 2, and are allowed to randomize in Stage 3. A strategy profile of this game determines an outcome (i.e., the probabilities that the correct decision is made in state I and in state G) and therefore, the expected common utility of the decision. The designer chooses the mechanism to maximize her utility (from the decision). In particular, the designer does not take into account the cost incurred by the informed agents. There are different situations in which this assumption is appropriate. The designer may be a principal who delegates the final decision to a committee. Alternatively, the decision may affect the welfare of every individual in a large society and the designer can be a benevolent planner (Persico [2002]). In this case, any increase in the utility from the decision can compensate for the information costs paid by the agents.

In the terminology of Section 3, each agent j's type in Stage 3 is captured by $t_j \in \{\phi, i, g\}$, where ϕ stands for an agent who does not purchase information, and i or g

[5] Our analysis would, in fact, be tremendously simplified if investments were overt (see Footnote 10). However, in many situations in which agents engage in information acquisition, investment in information is indeed covert and signals are non-verifiable. For example, jurors would have a hard time proving they had attended testimonies, committee members do not check their colleagues have gone over the relevant background information before convening, etc.

stand for an agent who purchases information and receives i or g as the realized signals, respectively.

The assumption that players can invest in information, thereby endogenizing their types, is a deviation from the basic model described in the previous section. We find this model particularly appealing both on realistic and theoretical grounds. Indeed, when thinking about real-life decision panels, players need to decide whether (and sometimes to what extent) to invest in information acquisition: jurors choose whether to attend or not the testimonies presented to them, hiring committee members decide whether to carefully go over the candidate's portfolio or not, etc. Theoretically, this framework allows us to study mechanism design in situations where there are two forces at play. On the one hand, the mechanism should use the information available as efficiently as possible. On the other hand, the mechanism needs to provide agents with incentives to invest in information.

The analysis presented in Section 3 simplifies enormously the designer's problem. Indeed, once players decide whether to acquire information or not, and the appropriate signals are realized, we are in the setup of the previous section and all intermediate voting rules result in identical sets of equilibrium outcomes. This implies that when the size of the committee is $n > 2$, we can ignore the choice of the voting rule. Moreover, without loss of generality, we can restrict attention to communication devices that give unanimous recommendations to all the players. That is, for every profile of types, the device chooses (possibly probabilistically), an alternative in $\{A, C\}$, which is recommended to all players. In other words, when the designer chooses a committee of size $n > 2$, she needs only to select an incentive compatible communication device $\gamma : T_1 \times ... \times T_n \to [0, 1]$, where $T_j = \{\phi, i, g\}$. Each player $j = 1, ..., n$ reports her type to the device, and $\gamma(t)$ denotes the probability that the defendant is convicted when the vector of reports is t. Of course, incentive compatibility requires that each player has an incentive to report her type truthfully provided that all her opponents do the same.

To summarize, when $n > 2$ our design problem can be described by the following simplified timeline:

Stage 1* The designer chooses the size of the committee n and an incentive compatible device γ (that is, a device that induces truthful revelation).

Stage 2* All agents observe the designer's mechanism. Each agent $j = 1, ..., n$ (simultaneously) decides whether to purchase a signal.

Stage 3* All players report (truthfully) their types to the device simultaneously.

Although this problem is much simpler to analyze than the original one, in order to characterize the optimal mechanism we still have to consider all possible incentive compatible devices (for any committee size) and all equilibria. We now present a number of steps that further simplify the designer's problem.

Given the committee size n and an incentive compatible device γ, consider the game in which the agents $1, ..., n$ decide whether to become informed or not. In general, this game admits multiple equilibria. In particular, there may be an equilibrium in which all players acquire information, and equilibria in which one or more players do not pay the cost c to observe a signal.[6] However, without loss of generality, we can restrict attention to equilibria in which every member of the committee buys the signal. Consider an equilibrium in which only players $1, ..., n'$ acquire the signal, where $n' < n$. It is easy to show that the same equilibrium outcome can be implemented with a committee of size n' and a communication device γ' such that all the agents become informed. For every vector of reports $t_1, ..., t_{n'}$, let $\gamma'(t_1, ..., t_{n'}) = \gamma(t_1, ..., t_{n'}, \phi, ..., \phi)$. In the original equilibrium, the first n' players know that players $n' + 1, ..., n$ do not purchase the signal and report message ϕ to the device γ (remember that γ induces truthful revelation). If players $1, ..., n$ decide to acquire information and be sincere under γ then they have an incentive to do the same under γ'. Therefore, in the remainder of the section we focus on incentive compatible devices that

[6] As already mentioned, we do not allow for mixed strategies in this stage of the game.

admit equilibria in which all agents acquire information. We call these devices *admissible.* It is important to note that admissible devices are characterized by two classes of incentive compatibility constraints. The first is the already introduced truthful revelation constraint. The second guarantees that each player best responds by acquiring information.

The next step of our analysis is to show that we can ignore what a communication device specifies when one or more players report the message ϕ. Consider an admissible device $\gamma : \{\phi, i, g\}^n \to [0,1]$. Let $U_j\left(t_j, t_j'\right)$ denote the expected utility (from the decision) of player j when her type is t_j, she reports message t_j' and all her opponents acquire information and are sincere. *Truthful revelation* holds if and only if:

$$U_j\left(t_j, t_j\right) \geqslant U_j\left(t_j, t_j'\right) \quad \forall j = 1, ..., n, \quad \forall \left(t_j, t_j'\right) \in \{\phi, i, g\}^2. \tag{2}$$

The expected utility of player j when she does not acquire information can be expressed as:

$$U_j\left(\phi, \phi\right) = \Pr\left(i\right) U_j\left(i, \phi\right) + \Pr\left(g\right) U_j\left(g, \phi\right),$$

where $\Pr\left(s\right)$ denotes the probability that agent j will observe signal $s = i, g$ if she acquires information. It follows that agent j will purchase the signal if and only if the following, *information acquisition* constraint, is satisfied:

$$\Pr\left(i\right) U_j\left(i, i\right) + \Pr\left(g\right) U_j\left(g, g\right) - c \geqslant \Pr\left(i\right) U_j\left(i, \phi\right) + \Pr\left(g\right) U_j\left(g, \phi\right). \tag{3}$$

Constraints (2) and (3) imply that a necessary condition for a communication device to be admissible is that for every player $j = 1, ..., n$:

$$\Pr\left(g\right) \left(U_j\left(g, g\right) - U_j\left(g, i\right)\right) \geqslant c, \tag{4}$$

$$\Pr\left(i\right) \left(U_j\left(i, i\right) - U_j\left(i, g\right)\right) \geqslant c. \tag{5}$$

These inequalities guarantee that agent j prefers to buy the signal and be sincere rather than not buy the signal and always report one of $s = i, g$ (i in the first inequality, g in the second one).

We now explain in which sense inequalities (4) and (5) are also a sufficient condition for a device to be admissible. Consider a device $\gamma : \{\phi, i, g\}^n \to [0,1]$, and suppose inequalities (4) and (5) hold. This device may not be incentive compatible. In particular, a player may have an incentive to lie when her type is ϕ. Consider, however, the outcome induced by γ when all players acquire information and are sincere. This outcome can be implemented with the following admissible device γ'. Given γ, consider player $j = 1, ..., n$ and assume that all her opponents acquire information and are sincere. Suppose that player j does not observe a signal and has to choose between message i and message g. Denote by s_j the message that agent j prefers to send.[7] Given the device γ, we construct γ' as follows:

$$\gamma'(t) = \begin{cases} \gamma(t) & \text{if } t \in \{i, g\}^n \\ \gamma(s_j, t_{-j}) & \text{if } t = (\phi, t_{-j}) \text{ and } t_{-j} \in \{i, g\}^{n-1} \end{cases},$$

and we assign an arbitrary value to $\gamma'(t)$ when two or more players report message ϕ. Intuitively, when each player different from j sends either i or g, the device γ' interprets message ϕ of player j as message s_j.

Notice that the expressions in inequalities (4) and (5) do not depend on what the device specifies when some players report ϕ. We can, therefore, think of an admissible device as a mapping $\gamma : \{i, g\}^n \to [0,1]$ which satisfies conditions (4) and (5).

An admissible device γ is *symmetric* if for every vector $(t_1, ..., t_n)$ in $\{i, g\}^n$ and every permutation φ on $\{1, ..., n\}$, $\gamma(t_1, ..., t_n) = \gamma\left(t_{\varphi(1)}, ..., t_{\varphi(n)}\right)$. In a symmetric device, the probability that the defendant is convicted depends only on the number of messages g (or i) but not on the identity of the players who send g. We now argue that there is no loss of generality in considering only symmetric devices. In fact, suppose that γ is an admissible device and consider a permutation φ on $\{1, ..., n\}$ (let Λ_n denote the set of such permutations). Consider the device γ_φ, where $\gamma_\varphi(t_1, ..., t_n) = \gamma\left(t_{\varphi(1)}, ..., t_{\varphi(n)}\right)$ for every $\{t_1, ..., t_n\}$ in $\{i, g\}^n$. Since all players are identical and γ is admissible, the device γ_φ is also admis-

[7] That is, $s_j \in \{i, g\}$ and is such that $\Pr(i) U_j(i, s_j) + \Pr(g) U_j(g, s_j) \geqslant \Pr(i) U_j\left(i, s_j'\right) + \Pr(g) U_j\left(g, s_j'\right)$, where $s_j' = i, g$.

sible and outcome equivalent to γ. Of course, convex combinations of admissible devices are admissible. It follows that the symmetric device $\tilde{\gamma} = \left(\frac{1}{|\Lambda_n|}\right) \sum_{\varphi \in \Lambda_n} \gamma_\varphi = \frac{1}{n!} \sum_{\varphi \in \Lambda_n} \gamma_\varphi$ is admissible and outcome equivalent to the original device γ.

A symmetric device can be represented as a mapping $\gamma : \{0, 1, ..., n\} \to [0, 1]$, where $\gamma(k)$ denotes the probability that the defendant is convicted (alternative C is chosen) when k players report the guilty signal g (each player can report either i or g). For a symmetric device $\gamma : \{0, 1, ..., n\} \to [0, 1]$ conditions (4) and (5) can be expressed as:

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k+1; n) (\gamma(k+1) - \gamma(k)) \geqslant c, \tag{ICi}$$

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k; n) (\gamma(k) - \gamma(k+1)) \geqslant c, \tag{ICg}$$

where $f(\cdot; n) : \mathbb{R} \to \mathbb{R}$ is defined by:

$$f(x; n) = -qP(I)(1-p)^x p^{n-x} + (1-q) P(G) p^x (1-p)^{n-x}.$$

For each $n > 2$, we look for the optimal admissible device, i.e., the admissible device that maximizes the expected utility of the decision. This amounts to solving the following linear programming problem P_n :

$$\max_{\gamma:\{0,\ldots,n\}\to[0,1]} -(1-q) P(G) + \sum_{k=0}^{n} \binom{n}{k} f(k; n) \gamma(k)$$

$$\text{s.t. } (ICi), (ICg).$$

We denote by $\bar{\gamma}_n$ the solution to problem P_n (if it exists), and by $V(n)$ the expected utility of the optimal device. If problem P_n does not have any feasible solution, we set $V(n) = -1$.

To complete the description of all mechanisms, we need to consider committees with less than three players. We let $V(0) = \max\{-qP(I), -(1-q)P(G)\}$ denote the expected utility of the optimal decision when no information is available.

Suppose $n = 1$, i.e., the designer delegates the final decision to a single agent. Let $\hat{V}(1)$ be the expected utility of the optimal decision of the agent when she acquires information.[8] If the benefit of buying the signal, $\hat{V}(1) - V(0)$, is greater than or equal to the cost c, the agent will acquire information, and we set $V(1) = \hat{V}(1)$. Otherwise, we set $V(1) = -1$.

Finally, suppose that the designer chooses a committee with two agents. We define problem P_2 in the same way as P_n for $n > 2$. We let $V(2)$ denote the value of the objective function at the solution (if it exists). Notice, however, that problem P_2 does not guarantee that $V(2)$ can be achieved by the extended mechanism designer. In fact, when $n = 2$, a player can be pivotal at the voting stage and we need to take into account her incentives to follow the mediator's recommendation (constrains (ICi) and (ICg) do not capture these incentives). In other words, $V(2)$ represents an upper bound of the expected utility that can be achieved with two players. If the problem P_2 does not admit any feasible solution, we set $V(2) = -1$.

The optimal mechanism consists of the optimal size of the committee n^*, and the optimal admissible device $\bar{\gamma}_{n^*}$. n^* is such that $V(n^*) \geqslant V(n)$, for every nonnegative integer n.[9] In what follows, we will demonstrate that the optimal size of the committee is always finite. Furthermore, when the cost of acquiring information is sufficiently low, the optimal size n^* is greater than 2 (see below) and, therefore, the expected utility $V(n^*)$ can be achieved.

4.1 Features of The Optimal Extended Mechanism

The extended mechanism the designer chooses is comprised of the size of the decision panel as well as the incentive scheme it will operate under. In Subsection 4.1.1 we tackle the first aspect of the this design problem. Namely, we illustrate that the optimal committee is of bounded size. In Subsection 4.1.2 we illustrate some traits of the optimal device the

[8] Formally, $\hat{V}(1) = \max_{\gamma:\{0,1\}\to[0,1]} -(1-q)P(G) + \sum_{k=0}^{1} f(k;1)\gamma(k)$.

[9] Notice that $V(0) > -1$, and, thus, $n > 1$ can be the optimal size only if problem P_n admits a feasible solution. Similarly, $n = 1$ can be optimal only if the agent who has to make the final decision acquires information.

designer would choose. In particular, we show that imperfectly aggregating the available information may induce more players to acquire information, thereby yielding a higher overall expected utility level.

4.1.1 The Scope of The Committee

Our first result, Proposition 3, shows that the solution to our design problem always exists. In fact, we show that when the size of the committee is very large it is impossible to give the incentive to all the members to acquire information. That is, committees with too many members do not have any admissible device. Intuitively, when there are many agents in the committee, the marginal contribution of an additional signal is relatively small. Therefore, each agent has an incentive to save the cost c and benefit from the information acquired by the other participants. In other words, in large committees there is a severe free rider problem.

Proposition 3 *Fix $P(I)$, q, p and c. There exists $\bar{n}$ such that for every $n \geqslant \bar{n}$, problem P_n does not have any feasible solution.*

Proof. See Appendix.

Clearly, it follows from Proposition 3 that the optimal size of the committee n^* is finite and smaller than $\bar{n}$. This, in turn, implies that when information is costly, the probability of making the wrong decision is bounded away from zero. This observation stands in contrast to the underlying message of the information aggregation literature (see, e.g., Feddersen and Pesendorfer [1996, 1997]) in which a large pool of agents yields complete aggregation of all of the available information.

4.1.2 Optimal Distortionary Devices

The next question is how the optimal device uses the information of the agents. To analyze this problem, let us first consider the case in which the designer makes the final decision after observing n free signals. This will give us an upper bound on what the designer can achieve when she chooses a committee of size n and information is costly.

To find the optimal decision rule, we simply need to maximize the objective function of problem P_n (without the constraints). We let γ_n^B denote the solution to this maximization problem. γ_n^B, which we call a *Bayesian* device, is of the form:

$$\gamma_n^B(k) = \begin{cases} 0 & \text{if } f(k;n) \leqslant 0 \\ 1 & \text{if } f(k;n) > 0 \end{cases},$$

(in fact, when $f(k;n) = 0$, $\gamma_n^B(k)$ can be any number in the unit interval).

To interpret this result, notice that $f(k;n)$ is positive (negative) if and only if the cost of convicting the innocent q is smaller (greater) than the probability that the defendant is guilty given that k of n signals are g.

The function $f(\cdot;n)$ is increasing and we let $z(n)$ be defined by $f(z(n);n) = 0$. We have:

$$z(n) = \frac{1}{2}\left(n + \frac{\ln\left(\frac{qP(I)}{(1-q)P(G)}\right)}{\ln\left(\frac{p}{1-p}\right)}\right).$$

Another way to express the Bayesian device γ_n^B is:

$$\gamma_n^B(k) = \begin{cases} 0 & \text{if } k \leqslant z(n) \\ 1 & \text{if } k > z(n) \end{cases}.$$

For small values of n, $z(n)$ can be negative or greater than n. In the first case, the optimal decision is always to convict the defendant. In the latter case, the optimal decision is always to acquit. These cases arise when the designer is very concerned with a particular mistake (acquitting the guilty or convicting the innocent), and the signal is not very accurate, i.e., p is close to $1/2$. In both cases the n signals are of no value. For large values of n, however, $z(n)$ is positive and smaller than n ($z(n)/n$ converges to $1/2$ as n goes to infinity), and the defendant will be convicted if and only if the designer observes sufficiently many guilty signals.

We let $\hat{V}(n)$ denote the expected utility of the Bayesian device:

$$\hat{V}(n) = -(1-q)\, P(G) + \sum_{k\in\{0,..,n\},\ k>z(n)} \binom{n}{k} f(k;n),$$

The utility $\hat{V}(n)$ is nondecreasing in the number of signals n. Moreover, $\hat{V}(n)$ is strictly greater than $V(0)$, the expected utility of the optimal uninformed decision, if and only if $z(n)$ belongs to $(0,n)$. If $z(n)$ is not in $(0,n)$, then $V(0) = \hat{V}(1) = ... = \hat{V}(n)$.

When n becomes unboundedly large, the Bayesian device uses an infinitely increasing number of i.i.d. signals. The law of large numbers ensures that all uncertainty vanishes asymptotically. In particular, $\hat{V}(n)$ converges to zero, the no uncertainty value, when n goes to infinity.[10]

We now return to the original design problem. Clearly, the expected utility of the optimal admissible device $V(n)$ cannot be greater than $\hat{V}(n)$, since the Bayesian device γ_n^B is the solution to the unconstrained problem. On the other hand, when the Bayesian device γ_n^B is admissible, we have $V(n) = \hat{V}(n)$. In this case the designer is able to give the incentive to the n agents to acquire the signal and, at the same time, to make the best use of the available information. Proposition 4 shows that this can happen if and only if the contribution of the last signal to the utility of a single decision maker is greater than or equal to its cost.

Proposition 4 *For every $n \geqslant 2$, $V(n) = \hat{V}(n)$ if and only if $\hat{V}(n) - \hat{V}(n-1) \geqslant c$.*

Proof. See Appendix.

[10] Note that if information acquisition is overt and $c < 1$, then $\hat{V}(n)$ is implementable (in Nash equilibrium) for sufficiently large n. Indeed, consider the following scenario. The designer selects the Bayesian device γ_n^B as long as everyone purchases information, and a device γ that makes a choice contrary to the Bayesian prescription if any agent does not purchase information (i.e., for all k, $\gamma(k) = 1 - \gamma_n^B(k)$). The strategy profile under which all players acquire information and are always sincere constitutes a Nash equilibrium. Under this profile, the expected utility of the decision approaches 0. If one player deviates and does not acquire information, she drives the common utility to a level that approaches -1. Finally, no agent has an incentive to lie upon acquiring information.

The following example provides an intuition for Proposition 4. The Bayesian device γ_9^B of a committee of size 9 selects conviction if at least 5 players report a guilty signal. Consider the Bayesian device γ_8^B for the committee of size 8. There are two cases, depending on p, q and $P(I)$: (a) the device selects conviction when there are at least 4 guilty signals; (b) it selects conviction when there are at least 5 guilty signals. Consider the committee with 9 members and suppose that each opponent of player 1 acquires information and is sincere. Player 1's expected utility if she also acquires information and is sincere is equal to $\hat{V}(9) - c$. However, if player 1 does not purchase the signal and reports message g in case (a) and message i in case (b), she gets $\hat{V}(8)$. The proof of Proposition 4 formalizes this argument.

Let n^B denote the greatest integer for which the Bayesian device is admissible (we assume that there exists at least one such integer). That is, $\hat{V}\left(n^B\right) - \hat{V}\left(n^B - 1\right) \geqslant c$, and $\hat{V}(n) - \hat{V}(n-1) < c$ for every $n > n^B$. The existence of n^B is guaranteed by the fact that the sequence $\left\{\hat{V}(1), ..., \hat{V}(n), ...\right\}$ converges (to zero). The designer can induce more than n^B players to acquire information only if she selects a device that aggregates the available information suboptimally. On the other hand, more information will be available in larger committees. How should the designer solve this trade-off? Is the optimal size of the committee equal to or larger than n^B? Before answering these questions, let us explain why we believe they are important.

Suppose n is such that the Bayesian device is admissible. We now show that there is a very simple mechanism that does not require communication and allows the designer to obtain utility $\hat{V}(n)$. Let k_n be the smallest integer greater than $z(n)$. Notice that k_n belongs to $\{1, ..., n\}$ since $\hat{V}(n)$ can be greater than $\hat{V}(n-1)$ only if $z(n)$ is in $(0, n)$. Consider the following game. Each agent decides whether to buy a signal or not. Then the players vote, and the defendant is convicted if and only if at least k_n agents vote to convict. This game admits an equilibrium in which each player acquires the signal and votes sincerely (i.e., she votes to convict if and only if she observes signal g). The expected utility of the decision when the agents play this equilibrium is, of course, $\hat{V}(n)$.

Consider our design problem. If the optimal size of the committee is n^B, then communication is unnecessary and the designer simply needs to select the voting rule k_{n^B} (this is the optimal mechanism in Persico [2002] where communication is not allowed). On the other hand, if the optimal size is larger than n^B, then communication may play a very important role in the solution to the designer's problem. Proposition 5 shows that this is indeed the case (at least when the cost is sufficiently low).

Before formally stating the result, we need to introduce one technical assumption. We say the environment is *regular* if $\frac{\ln\left(\frac{qP(I)}{(1-q)P(G)}\right)}{\ln\left(\frac{p}{1-p}\right)}$ is not an integer. This implies that for all n, $z(n)$, the Bayesian threshold value, is not an integer. In a regular environment, if n is such that $\hat{V}(n) > V(0)$, then for all $n' \geqslant n$, $\hat{V}(n'+1) > \hat{V}(n')$. Note that assuming the environment is regular is not restrictive since this is, generically, the case.

Proposition 5 *Fix $P(I)$, q and p and assume the environment is regular. Let $n^*(c)$ denote the optimal size of the committee when the cost of acquiring information is c. There exists $\bar{c} > 0$ such that for every $c < \bar{c}$, $V(n^*(c)) < \hat{V}(n^*(c))$.*

Proof. See Appendix.

In the proof of Proposition 5, we show that if n^B is sufficiently large (or equivalently, if c is sufficiently low), then there exists a non Bayesian device for a committee of size n^B+1 that is better than the Bayesian device with n^B players, i.e., $V\left(n^B+1\right) > \hat{V}\left(n^B\right)$. Proposition 5 provides only sufficient conditions for the optimal size to be greater than n^B. Of course, if the optimal size happens to be $n^* = 1$, then $V(n^*) = \hat{V}(n^*)$. However, the authors have not found any example in which the optimal size n^* is greater than one and coincides with n^B.

4.2 Comparative Statics

In this section we analyze how the optimal extended mechanism and the quality of the decision depend on the primitives of the model. We first look at the impact that changes in the information cost c and in the accuracy of the signal p have on the expected utility of the designer and on the optimal size of the committee. We then perform the comparative statics for the agents' preferences. Finally, we focus on the optimal admissible device.

The Cost of Information

The first, obvious, result is that the expected utility of the optimal mechanism is decreasing in the cost of information acquisition. This follows from the fact that for any given size of the committee, the utility of the optimal devices increases (weakly) when the cost decreases. In fact, if a device is admissible when the cost is c, then the device is also admissible when the cost is lower than c.

We now consider how the optimal size is affected by a change in the information cost. Clearly, given any cost c with optimal size $n^*(c)$, we can always find another cost c', sufficiently lower than c, such that $n^*(c')$ is greater than $n^*(c)$. In fact, it is enough to choose c' such that the Bayesian device γ_n^B is admissible for some n greater than $n^*(c)$. Unfortunately, it is less straightforward to perform the comparative statics for small changes of the information cost. In all the examples we have constructed, the optimal size decreases when the information cost increases. However, we have not been able to prove that this is a general result. To illustrate what constitutes a problem, we consider two committees of size n and $n+1$. For any cost c, let us consider the difference between the utility of the optimal device at $n+1$ and the utility of the optimal device at n. It is possible to construct examples such that this difference is positive for low and high values of the cost, but is negative for intermediate values (in a sense, the utility does not exhibit a single crossing property).[11] In other words, suppose we start with a level of the cost such that size $n+1$ is better than size n. In general, we cannot conclude that this relation holds when the cost becomes smaller. Of course, this discussion does not show that the optimal size may increase with the cost. It only explains why it could be difficult to obtain analytical results. But it remains an open question whether the optimal size is indeed always decreasing in

[11] A possible example is the following: $P(I) = \frac{1}{2}$, $q = 0.82$, $p = 0.55$ and $n = 7$.

the cost or not.

The Signals' Accuracy

As one would expect, the utility of optimal extended mechanism increases when the signal becomes more accurate, i.e. when p increases. In fact, a stronger result holds. For a committee of a given size n, the utility of the optimal device increases when the quality of the signal improves. Intuitively, when the signal is more accurate, the device can ignore some information and replicate an environment with less information. Formally, let γ be an admissible device when the accuracy of the signal is p. Suppose now the accuracy is $\hat{p} > p$ and consider the following device $\hat{\gamma}$. The simplest way to describe $\hat{\gamma}$ is to imagine that each player j reports her signal $s_j = i, g$ to the mediator. The mediator then generates a new variable $s'_j = i, g$ at random according to the distribution:

$$\Pr\left(s'_j = i | s_j = i\right) = \Pr\left(s'_j = g | s_j = g\right) = \frac{p + \hat{p} - 1}{2\hat{p} - 1}.$$

The variables $s'_1, ..., s'_n$ are independent of each other. Finally, the mediator applies the original device γ to the vector $(s'_1, ..., s'_n)$.[12] Notice that the mediator's distribution is appropriately chosen so that $\Pr\left(s'_j = i | I\right) = \Pr\left(s'_j = g | G\right) = p$. Thus, the expected utility of the device $\hat{\gamma}$ when the accuracy is $\hat{p}$ coincides with the utility of γ when the accuracy is p. It is also easy to show that $\hat{\gamma}$ is admissible (when the accuracy is $\hat{p}$). This implies that for any committee size, the utility of the optimal device is increasing in p.

While in our model the designer always benefits from a more informative signal, this is not necessarily the case when communication is not possible. For example, in Persico [2002] the utility of the optimal mechanism can decrease when p increases. When players

[12] That is, for every vector $(s_1, ..., s_n)$ in $\{i, g\}^n$,

$$\hat{\gamma}(s_1, ..., s_n) = \sum_{(s'_1, ..., s'_n) \in \{i,g\}^n} \Pr\left((s'_1, ..., s'_n) \,|\, (s_1, ..., s_n)\right) \gamma(s'_1, ..., s'_n),$$

where $\Pr\left((s'_1, ..., s'_n) \,|\, (s_1, ..., s_n)\right)$ denotes the probability that the mediator generates the vector $(s'_1, ..., s'_n)$ when the vector of reports is $(s_1, ..., s_n)$.

are not allowed to communicate, the designer can induce n agents to acquire information if and only if $\hat{V}(n) - \hat{V}(n-1)$ is greater than the information cost. When the signal is very informative, $\hat{V}(n)$ is close to zero for n relatively small, and therefore it is impossible to induce many players to acquire information. In contrast, when the signal is less accurate, the difference $\hat{V}(n) - \hat{V}(n-1)$ can be larger than the cost for large values of n. It is possible for the designer to prefer having many uninformative signals over a few very accurate ones.[13]

As far as the optimal size of the committee is concerned, simple examples indicate that it is not monotonic in p. Consider, for example, the case $P(I) = \frac{1}{2}$, $q = 0.62$, and $c = 0.004$. The optimal size is 13 for $p = 0.55$, 24 for $p = 0.65$, and 15 for $p = 0.75$.

The Preference Parameter

The agents and the designer's preferences are characterized by the parameter q, the cost of convicting the innocent. We do not have a formal result for the relation between q and the optimal size of the committee. However, all the examples that we have constructed suggest that it depends on the comparison between q and $P(G)$, the probability that the defendant is guilty. If q is greater than $P(G)$, the agents (and the designer) are more concerned with the error of convicting the innocent, and an uninformed agent would acquit the defendant. In this case, the optimal size of the committee decreases when q increases. Conversely, if q is smaller than $P(G)$, the optimal uninformed decision is to convict the defendant. In this case, the optimal size is increasing in q. Our examples also show that the utility of the optimal mechanism is not monotonic in q (even if we restrict attention to values of q above or below $P(G)$). Finally, notice that similar results hold for the prior distribution, since q and $P(I)$ play an interchangeable role throughout all of our analysis.

The Optimal Device

As was illustrated in Proposition 5, for certain parameters, the optimal device does not

[13] To give a concrete example, let us assume that $P(I) = \frac{1}{2}$, $q = 0.82$, and $c = 0.0013$. When $p = 0.85$, the optimal size is 10 and the utility is -0.001455 (the voting rule is $r = 6$). However, when $p = 0.95$, the optimal size is 4 and the utility is -0.001459 (the voting rule is $r = 3$).

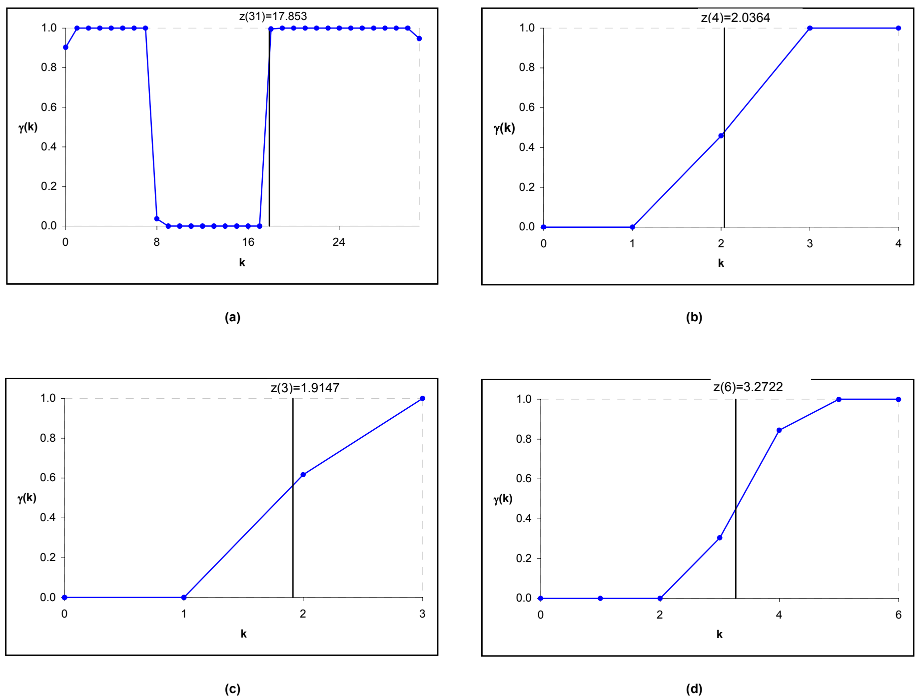

Figure 1: The Optimal Devices

coincide with the corresponding Bayesian device. It is then natural to compare $\bar{\gamma}_{n^*}$ with the Bayesian $\gamma^B_{n^*}$. There are three main observations that may prove significant for future theoretical investigations of the optimal extended mechanism.

First, unlike its Bayesian counterpart, $\bar{\gamma}_{n^*}$ may be non-monotonic in the number of guilty signals k. For the sake of illustration, consider the example in which $P(I) = \frac{1}{2}$, $c = 0.002$, $p = 0.55$, and $q = 0.72$. In this case, the optimal size of the team is $n^* = 31$. Figure 1a plots $\bar{\gamma}_{n^*}$ and the Bayesian threshold $z(n^*)$ and illustrates that $\bar{\gamma}_{n^*}$ may indeed be non-monotonic.

Second, when $\bar{\gamma}_{n^*}$ is monotonic, there is no global regularity in its relation to the Bayesian incentive scheme. Specifically, it can be above $\gamma^B_{n^*}$ (thereby inducing a higher or equal probability of conviction for any profile of signals), as illustrated in Figure 1b[14] for

[14]Note that the Bayesian device $\gamma^B_{n^*}$ is in fact a step function achieving the value of 0 for any $k \leqslant z(n^*)$

the case of $P(I) = \frac{1}{2}$, $c = 0.035, p = 0.75$, and $q = 0.52$. It can be below $\gamma^B_{n^*}$ (thereby inducing a lower or equal probability of conviction), as illustrated in Figure 1c for the case of $P(I) = \frac{1}{2}$, $c = 0.014, p = 0.95$, and $q = 0.92$. It can also be neither below nor above $\gamma^B_{n^*}$, as illustrated in Figure 1d for the case of $P(I) = \frac{1}{2}$, $c = 0.014, p = 0.85$, and $q = 0.72$.

Third, in cases as described above, an agent is pivotal for more than one value of guilty signals. Intuitively, increasing an agent's probability of being pivotal (relative to the Bayesian device) increases her incentives to purchase information. In such situations, this effect is stronger than the statistical efficiency loss.

5 Conclusions

A group can be identified as a collection of agents satisfying one or more of the following three elements: sharing a common goal, having a joint task, or possessing the ability to communicate and exchange information at no costs. The current paper considers groups satisfying all three conditions and introduces a model of group decision making under uncertainty.

Our analysis yielded four key insights. First, communication between group members in collective choice scenarios is consequential to the resulting equilibrium outcomes. Specifically, communication renders all intermediate threshold voting rules equivalent with respect to the sequential equilibria outcomes they generate. Second, when members of the group decide whether to acquire costly information or not preceding the communication stage, groups producing the optimal collective decisions are bounded in size. Third, the optimal incentive scheme in such an environment balances a trade-off between inducing players to acquire information and extracting the maximal amount of information from them. In particular, the optimal device may aggregate information suboptimally from a statistical point of view. Fourth, the comparative statics of extended mechanism design for collective choice exhibit some regularities.

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and the value of 1 for any $k > z(n^*)$.

In what follows we outline several avenues in which the current framework can be extended.

First, since our result concerning the equivalence of intermediate threshold rules could potentially be extremely important to mechanism design pertaining to collective choice, as well as provide some formal foundations for deliberative democracy theories in political science, there are a few directions in which its robustness should be put to test. The authors are interested in a comparison between the equilibria outcomes predominantly under three modifications: allowing the players to tremble in their action choices (e.g., by looking at the sets of trembling hand equilibria instead of Nash or sequential equilibria), considering sequential communication protocols in which players do not send messages simultaneously but rather take turns, and regarding players as having a bounded ability to calculate best responses.

Second, it would be interesting to investigate additional extended mechanisms. For instance, one could consider a setup in which the designer is able to subsidize agents' information. One example would be a department chair investing in the creation of a centralized web-site containing all information pertaining to job candidates. Such an investment would potentially reduce the cost of studying each candidate's portfolio for all of the hiring committee members. Formally, in the extended mechanism the designer has to choose the level of informational subsidies in addition to the size of the group and the communication protocol. Selecting a high level of subsidies creates a trade-off between inducing more agents to acquire information, and internalizing some of this information cost by the designer.

A third interesting extension concerns the homogeneity of the players. So far we have considered homogenous decision panels, in the sense that all players, including the mechanism designer, share the same preferences. Concretely, in our model, both the designer and all of the players share the same utility parameter q. However, in many situations it is conceivable that agents have heterogenous preferences. One could then study the extended mechanism design problem in which, at stage 1, the designer chooses the distribution of preference parameters of the decision panel members, in addition to choosing the panel's size and the device. An analysis of such a scenario would entail defining carefully the goal of the designer (maximizing her own preferences, as characterized by one given q, or implementing a point in the Pareto frontier of the equilibria set). We are especially interested in the optimal composition of the decision panel. In particular, would the designer choose a committee comprised of agents with preferences coinciding with her own or would she choose agents with diverging tastes (as observed, e.g., in the optimal choice of central banker - see, for example, Alesina and Gatti [1995] and references therein)?

Appendix

Proof of Proposition 3

We prove Proposition 3 by demonstrating that when the size of the committee is sufficiently large there is no device that satisfies constraint (ICi). The choice of the constraint is arbitrary since we could prove the same result for constraint (ICg).[15]

Given a device $\gamma : \{0, ..., n\} \to [0, 1]$, let $H_n(\gamma)$ denote the left hand side of constraint (ICi):

$$H_n(\gamma) = \sum_{k=0}^{n-1} \binom{n-1}{k} f(k+1; n)\left(\gamma(k+1) - \gamma(k)\right).$$

For ease of presentation, we will drop the argument n in f throughout this proof. $H_n(\gamma)$ can be expressed as:

$$H_n(\gamma) = -f(1)\gamma(0) + \sum_{k=1}^{n-1} \binom{n-1}{k-1} (1-p)^k p^{n-k-1} \frac{1}{k} h(k)\gamma(k) + f(n)\gamma(n),$$

where

$$h(k) = qP(I)(n(1-p) - k) + (1-q)P(G)\left(\frac{1-p}{p}\right)^{n-2k-1}(k - np).$$

[15] In this proof, we show how to construct an upper bound on the size of the committees that have admissible devices. The fact that we do not consider both constraints at the same time implies that our bound is not tight. Admissible devices may not exist even when the size of the committee is smaller than our bound.

For n sufficiently large, $-f(1) > 0$ and $f(n) > 0$. Moreover, $h(k) < 0$ for k in $[n(1-p), np]$.

Let $\lfloor n(1-p) \rfloor$ denote the greatest integer smaller than $n(1-p)$. For every $k = 1, ..., \lfloor n(1-p) \rfloor - 1$, we have:

$$h(k) \geqslant qP(I) - (1-q)P(G)\left(\frac{1-p}{p}\right)^{n(2p-1)+1}(np-1).$$

Notice that the right hand side of the above inequality is positive when n is sufficiently large. Similarly, let $\lceil np \rceil$ be the smallest integer greater than np. For $k = \lceil np \rceil + 1, ..., n-1$,

$$h(k) \geqslant -qP(I)(np-1) + (1-q)P(G)\left(\frac{p}{1-p}\right)^{n(2p-1)+1},$$

and the right hand side is positive for n sufficiently large.

Consider the following maximization problem:

$$\max_{\gamma:\{0,...,n\}\to[0,1]} H_n(\gamma),$$

and let γ^+ denote the solution. Define $\bar{H}_n = H_n(\gamma^+)$. It follows from the analysis above that when n is sufficiently large, the device γ^+ is of the form:

$$\gamma^+(k) = \begin{cases} 1 & \text{if } k = 0, ..., k', k'', ..., n \\ 0 & \text{if } k = k'+1, ..., k''-1 \end{cases}$$

where k' is either $\lfloor n(1-p) \rfloor - 1$ or $\lfloor n(1-p) \rfloor$, and k'' is either $\lceil np \rceil$ or $\lceil np \rceil + 1$. This, in turn, implies:

$$\bar{H}_n = -\binom{n-1}{k'} f(k'+1) + \binom{n-1}{k''-1} f(k'').$$

When n is sufficiently large, both $-f(k'+1)$ and $f(k'')$ are positive. Furthermore,

$$-f(k'+1) \leqslant qP(I)(1-p)^{n(1-p)-1} p^{np+1} - (1-q)P(G)p^{n(1-p)-1}(1-p)^{np+1}$$

since k' belongs to $[n(1-p)-2, n(1-p))$, and

$$f(k'') \leqslant -qP(I)(1-p)^{np+2} p^{n(1-p)-2} + (1-q)P(G)p^{np+2}(1-p)^{n(1-p)-2}$$

since k'' belongs to $(np, np+2]$.

To bound the binomial coefficients in the expression of $\bar{H}_n$, we now introduce Stirling's approximation (see Feller [1968]):

$$l_1(n) = \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n} < n! < \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n+\frac{1}{12n}} = l_2(n).$$

For every n, the function $\frac{l_2(n)}{l_1(x) l_1(n-x)}$ is increasing for $x < \frac{n}{2}$, and decreasing for $x > \frac{n}{2}$. This and the range of k' and k'' imply:

$$\binom{n-1}{k'} = \frac{n-k'}{n} \frac{n!}{(k')!\,(n-k')!} \leqslant \frac{np+2}{n} \frac{l_2(n)}{l_1(k')\, l_1(n-k')} \leqslant$$

$$\frac{np+2}{n} \frac{l_2(n)}{l_1(n(1-p))\, l_1(np)} = \frac{np+2}{n} \frac{e^{\frac{1}{12n}}}{\sqrt{2\pi n}\,(1-p)^{n(1-p)+\frac{1}{2}} p^{np+\frac{1}{2}}}.$$

Similarly,

$$\binom{n-1}{k''-1} = \frac{k''}{n} \frac{n!}{(k'')!\,(n-k'')!} \leqslant \frac{np+2}{n} \frac{l_2(n)}{l_1(k'')\, l_1(n-k'')} \leqslant \frac{np+2}{n} \frac{l_2(n)}{l_1(np)\, l_1(n(1-p))}.$$

After substituting the above inequalities in the expression of $\bar{H}_n$ and performing some algebraic manipulations, we get:

$$\bar{H}_n \leqslant \frac{np+2}{n} \frac{e^{\frac{1}{12n}}}{\sqrt{2\pi n}} \left(qP(I)(1-p) + (1-q) P(G) p\right) \frac{p^{\frac{1}{2}}}{(1-p)^{\frac{5}{2}}}.$$

The right hand side of the above inequality decreases in n and converges to zero as n goes to infinity. Thus, the claim of Proposition 3 follows. ■

Proof of Proposition 4

First, suppose that $\hat{V}(n) = V(0) = \hat{V}(n-1)$. It follows that either $\gamma_n^B(0) = ... = \gamma_n^B(n)$, or $\gamma_n^B(0) = 0$, $f(0;n) = 0$ and $\gamma_n^B(1) = ... = \gamma_n^B(n) = 1$. In both cases, the left hand side of constraint (ICg) is zero.

Thus, we now assume that $\hat{V}(n) > V(0)$. This implies that $z(n)$ is in $(0, n)$ and k_n, the smallest integer greater than $z(n)$, belongs to $\{1, ..., n\}$. Depending on the distance between k_n and $z(n)$, there are two cases to consider.

Case (i): $0 < k_n - z(n) \leqslant \frac{1}{2}$. In this case, it is easy to check that:

$$\hat{V}(n) - \hat{V}(n-1) = \binom{n-1}{k_n - 1} f(k_n; n),$$

and

$$-f(k_n - 1; n) \geqslant f(k_n; n).$$

Case (ii): $\frac{1}{2} < k_n - z(n) \leqslant 1$. Then we have:

$$\hat{V}(n) - \hat{V}(n-1) = -\binom{n-1}{k_n - 1} f(k_n - 1; n),$$

and

$$-f(k_n - 1; n) < f(k_n; n).$$

The Bayesian device is admissible if it satisfies the following constraints:

$$\binom{n-1}{k_n - 1} f(k_n; n) \geqslant c, \tag{6}$$

$$-\binom{n-1}{k_n - 1} f(k_n - 1; n) \geqslant c. \tag{7}$$

Clearly, the two inequalities above hold if and only if $\hat{V}(n) - \hat{V}(n-1) \geqslant c$. ■

Proof of Proposition 5

For every c, let $n^B(c)$ denote the largest integer for which the Bayesian device is admissible. We show that if $n^B(c)$ is sufficiently large then $V\left(n^B(c) + 1\right) > \hat{V}\left(n^B(c)\right)$. This will complete the proof of Proposition 5 since $n^B(c)$ is decreasing in c.

We now fix c and write n for $n^B(c)$. We assume $0 < k_n - z(n) < \frac{1}{2}$ (the proof for the case $\frac{1}{2} < k_n - z(n) < 1$ is similar and is therefore omitted).[16] The Bayesian device γ_n^B is admissible, and so inequalities (6) and (7) hold.

[16]Since the environment is regular, $k_n - z(n) \neq \frac{1}{2}, 1$.

Consider now a committee of size $n+1$. For every α in the unit interval, let the device $\gamma_\alpha : \{0, ..., n+1\}$ be defined by:

$$\gamma_\alpha (k) = \begin{cases} 0 & \text{if } k = 0, ..., k_n - 1 \\ \alpha & \text{if } k = k_n \\ 1 & \text{if } k = k_n + 1, ..., n+1 \end{cases}.$$

The constraints that the device γ_α has to satisfy to be admissible can be expressed as:

$$F(\alpha) = \binom{n}{k_n} f(k_n + 1; n+1) + \alpha \left[\binom{n}{k_n - 1} f(k_n; n+1) - \binom{n}{k_n} f(k_n + 1; n+1) \right] \geqslant c, \tag{8}$$

$$L(\alpha) = -\binom{n}{k_n} f(k_n; n+1) + \alpha \left[\binom{n}{k_n} f(k_n; n+1) - \binom{n}{k_n - 1} f(k_n - 1; n+1) \right] \geqslant c. \tag{9}$$

The function F is decreasing in α. We now assume that n is sufficiently large, so that $k_n - 1 \geqslant n(1-p)$ and $k_n \leqslant np$. This implies:

$$F(0) = \binom{n}{k_n} f(k_n + 1; n+1) > \binom{n-1}{k_n - 1} f(k_n; n) \geqslant c,$$

and that L is an increasing function that satisfies:

$$L(1) = -\binom{n}{k_n - 1} f(k_n - 1; n+1) > -\binom{n-1}{k_n - 1} f(k_n - 1; n) \geqslant c.$$

We let $\hat{\alpha}_1$ denote the greatest value of α for which the device γ_α satisfies constraint (8). Similarly, we let $\hat{\alpha}_2$ denote the smallest value of α for which the device γ_α satisfies constraint (9). Notice that $-f(k_n - 1; n) \geqslant f(k_n; n)$ since we are assuming that $k_n - z(n)$ is in $\left(0, \frac{1}{2}\right)$ (see the proof of Proposition 4). Thus, the cost c can be at most $\binom{n-1}{k_n-1} f(k_n; n)$ since the Bayesian device γ_n^B is admissible. It follows that:

$$\hat{\alpha}_1 \geqslant \frac{\binom{n}{k_n} f(k_n + 1; n+1) - \binom{n-1}{k_n-1} f(k_n; n)}{\binom{n}{k_n} f(k_n + 1; n+1) - \binom{n}{k_n-1} f(k_n; n+1)} \equiv \alpha_1,$$

$$\hat{\alpha}_2 \leqslant \frac{\binom{n-1}{k_n-1} f(k_n; n) + \binom{n}{k_n} f(k_n; n+1)}{\binom{n}{k_n} f(k_n; n+1) - \binom{n}{k_n-1} f(k_n - 1; n+1)} \equiv \alpha_2.$$

With a slight abuse of notation we let $V(\alpha)$ denote the expected utility of the device γ_α :

$$V(\alpha) = -(1-q) P(G) + \alpha \binom{n+1}{k_n} f(k_n; n+1) + \sum_{k=k_n+1}^{n+1} \binom{n+1}{k} f(k; n+1).$$

The difference between $V(\alpha)$ and $\hat{V}(n)$ is equal to:

$$V(\alpha) - \hat{V}(n) = \binom{n}{k_n} f(k_n; n+1) \left(\frac{n+1}{n-k_n+1} \alpha - 1 \right).$$

Let $\alpha^* = \frac{n-k_n+1}{n+1}$. Then $V(\alpha)$ is greater than $\hat{V}(n)$ if and only if $\alpha < \alpha^*$.

It remains to be shown that $\alpha_2 < \alpha^*$ and $\alpha_2 < \alpha_1$ for sufficiently large values of n. Let us start with the first inequality. We need to show:

$$(n-k_n+1) \left[\tbinom{n}{k_n} f(k_n; n+1) - \tbinom{n}{k_n-1} f(k_n-1; n+1) \right] >$$

$$(n+1) \left[\tbinom{n-1}{k_n-1} f(k_n; n) + \tbinom{n}{k_n} f(k_n; n+1) \right]$$

which can be rewritten as:

$$-\binom{n-1}{k_n-1} \left(nf(k_n; n+1) + nf(k_n-1; n+1) + (n+1) f(k_n; n) \right) > 0.$$

We divide the above the expression by $\tbinom{n-1}{k_n-1}$, and notice that

$$f(k_n; n+1) + f(k_n-1; n+1) = f(k_n-1; n).$$

We obtain:

$$-nf(k_n-1; n) - (n+1) f(k_n; n) > 0.$$

After dividing both sides by $qP(I)(1-p)^{z(n)} p^{n-z(n)}$ and rearranging terms we have:

$$\left(\frac{p}{1-p} \right)^{1-2\lambda} > \frac{n+p}{n+1-p},$$

where $\lambda = k_n - z(n)$. The left hand side is greater than 1 since λ belongs to $\left(0, \frac{1}{2}\right)$, while the right hand side is decreasing in n, and converges to 1 as n goes to infinity.

We now compare α_1 and α_2. We divide both the numerator and the denominator of α_1 by $\binom{n-1}{k_n-1}qP(I)(1-p)^{z(n)}p^{n-z(n)}$, rearrange terms and obtain:

$$\alpha_1 = \frac{\left((1-p)^{\lambda}p^{-\lambda} - p^{\lambda}(1-p)^{-\lambda}\right) + \left(\frac{n}{k_n}\right)\left(-(1-p)^{1+\lambda}p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}\right)}{\left(\frac{n}{k_n}\right)\left(-(1-p)^{1+\lambda}p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}\right) + \left(\frac{n}{n-k_n+1}\right)\left((1-p)^{\lambda}p^{1-\lambda} - p^{\lambda}(1-p)^{1-\lambda}\right)}.$$

We now take the limit of α_1 as n goes to infinity. Both $\left(\frac{n}{k_n}\right)$ and $\left(\frac{n}{n-k_n+1}\right)$ converge to 2 as n grows large. Thus, we have:

$$\bar{\alpha}_1 = \lim_{n\to\infty}\alpha_1 = \frac{\frac{1}{2}\left((1-p)^{\lambda}p^{-\lambda} - p^{\lambda}(1-p)^{-\lambda}\right) - (1-p)^{1+\lambda}p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}}{-(1-p)^{1+\lambda}p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda} + (1-p)^{\lambda}p^{1-\lambda} - p^{\lambda}(1-p)^{1-\lambda}}.$$

In a similar way we derive:

$$\bar{\alpha}_2 = \lim_{n\to\infty}\alpha_2 = \frac{\frac{1}{2}\left(-(1-p)^{\lambda}p^{-\lambda} + p^{\lambda}(1-p)^{-\lambda}\right) - (1-p)^{\lambda}p^{1-\lambda} + p^{\lambda}(1-p)^{1-\lambda}}{-(1-p)^{\lambda}p^{1-\lambda} + p^{\lambda}(1-p)^{1-\lambda} + (1-p)^{\lambda-1}p^{2-\lambda} - p^{\lambda-1}(1-p)^{2-\lambda}}.$$

It is tedious but simple to show that $\bar{\alpha}_2 < \bar{\alpha}_1$ for every p in $\left(\frac{1}{2}, 1\right)$ and every λ in $\left(0, \frac{1}{2}\right)$.

■

References

[1] Alesina, A. and Gatti, R. [1995], "Independent Central Banks: Low Inflation at No Costs?", *American Economic Review*, Papers and Proceedings, Volume 85, pages 196-200.

[2] Auriol, E. and Gary-Bobo, R. J. [1999], "On The Optimal Number of Representatives," CMS-EMS Discussion Paper 1286.

[3] Austen-Smith, D. and Banks, J. [1996], "Information Aggregation, Rationality, and The Condorcet Jury Theorem," *American Political Science Review*, Volume 90(1), pages 34-45.

[4] Bergemann, D. and Välimäki, J. [2002], "Information Acquisition and Efficient Mechanism Design," *Econometrica*, Volume 70(3), pages 1007-1033.

[5] Blinder, A. S., Morgan, J. [2000], "Are Two Heads Better than One?: An Experimental Analysis of Group vs. Individual Decisionmaking," *National Bureau of Economic Research* Working Paper 7909.

[6] Coughlan, P. [2000], "In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting," *American Political Science Review*, Volume 94(2), pages 375-393.

[7] Doraszelski, U., Gerardi, D., and Squintani, F. [2002], "Communication and Voting with Double-Sided Information," mimeo.

[8] Elster, J. [1998], *Deliberative Democracy*, Cambridge University Press.

[9] Feddersen, T. J. and Pesendorfer, W. [1996], "The Swing Voter's Curse," *American Economic Review*, Volume 86(3), pages 408-424.

[10] Feddersen, T. J. and Pesendorfer, W. [1997], "Voting Behavior and Information Aggregation in Elections with Private Information," *Econometrica*, Volume 65(5), pages 1029-58.

[11] Feddersen, T. J. and Pesendorfer, W. [1998], "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting," *American Political Science Review*, Volume 92(1), pages 23-35.

[12] Feller, W. [1968], *An Introduction to Probability Theory and Its Applications,* Volume I, Third Edition, John Whiley & Sons.

[13] Forges, F. [1986], "An Approach to Communication Equilibria," *Econometrica*, Volume 54(6), pages 1375-85.

[14] Guarnaschelli, S., McKelvey, R. C., and Palfrey, T. R. [2000], "An Experimental Study of Jury Decision Rules," *American Political Science Review*, Volume 94(2), pages 407-423.

[15] Habermas, J. [1976], *Sprachpragmatic und Philosophie and Zur Rekonstruktion des Historischen Materialismus*, Suhrkamp Verlag, Frankfurt am Main.

[16] Li, H. [2001], "Theory of Conservatism," *Journal of Political Economy*, Volume 109(3), pages 617-636.

[17] Mukhopadhaya, K. [1998], "Jury Size and The Free Rider Problem," mimeo.

[18] Myerson, R. B. [1982], "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, Volume 10(1), pages 67-81.

[19] Myerson, R. B. [1991], *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press.

[20] Persico, N. [2002], "Committee Design with Endogenous Information," forthcoming in *The Review of Economic Studies.*