

An Equilibrium Model of Federal Mandates*

Jacques Crémer

University of Toulouse, CNRS, GREMAQ, IDEI

Thomas R. Palfrey

California Institute of Technology

November 22, 2002

Abstract

This paper proposes a framework for studying policy making in a federal system in the presence of spillover externalities. Local jurisdictions choose policies by majority rule subject to constraints that are set by majority rule, but at the federation level. We characterize the induced preferences of voters for federal policies, prove existence of local majority rule equilibrium, provide an example of nonexistence of global majority rule equilibrium, and explore the welfare properties of federal standards with spillovers.

1 Introduction

A common justification for the role of a federal government is to solve problems of externalities between the members of the federation. These externalities can take many forms, indeed it is difficult to imagine public policies that are actually immune to interjurisdictional externalities, especially in a highly mobile society. Health and education policies are obvious examples, as are environmental, industrial, and agricultural regulation. Even policies that are nominally local, such as zoning laws and criminal statutes, have significant implications for the welfare of adjoining jurisdictions, and have even

*This is a preliminary draft. The paper has benefited from helpful discussions with and comments from Jenna Bednar, Norman Schofield, Barry Weingast, and seminar participants at Caltech, Harvard, LSE, Malaga, Ohio State, Princeton, Toulouse, and the 2002 meetings of the Public Choice Society. We gratefully acknowledge the financial support of LEESP, CNRS, and NSF.

broader impacts through their effects on the location choices of consumers and firms. If each district makes decisions independently, there will be a failure to equate marginal social cost with marginal social benefit, due to the gap between private cost and benefits and social costs and benefits. The potential of a significant welfare-enhancing role of centralized policy making, by a federal government, is obvious. In fact if one simply applies the same basic principles from which Coase argued for the merger of two firms, when there are production externalities, then the logic is compelling. But are these the right principles to apply? In this paper, we demonstrate that that this line of argument can be misleading.

On the one hand, the merger metaphor seems to capture the correct logic, since we are considering public goods, which will not be provided efficiently by markets due to free rider problems. While they can be locally provided, their effects spill over to other districts. Therefore, this puts us in a second best situation, and so traditional economic theory suggests that either mergers or rationally chosen taxes and subsidies should work.

In fact, this traditional approach has spawned many papers in the fiscal federalism literature. A typical model of this genre follows roughly the following scenario. Local governmental units apply taxes and subsidies to finance the production of public goods, as a second best method for (partially) correcting for externality-induced inefficiencies. These externalities may be either direct, or indirect, for example due to congestion effects resulting from relocation of residents in response to taxes.¹ These taxes and subsidies are arrived at by maximizing a utilitarian social welfare function, usually assumed to be the same in all districts, subject to technological constraints, and market conditions of demand and supply. A noncooperative game ensues between the local districts, with each district taking as given the economic and fiscal behavior of the other districts. At a Nash equilibrium, each separately chooses their own taxes and subsidies to maximize the same social welfare function, but applied only to their population. This equilibrium is then compared to a "cooperative" solution in which taxes and subsidies are decided centrally, in a manner that can rationally correct for the external effects across districts (subject, of course, to second-best considerations).²

However, at all levels of government, these decisions – taxes, subsidies, regulations, etc. – are always made within the constraints of political, not

¹See, for example, Gordon (1983) and the references cited there. A number of other papers look at issues related to mobility and "voting with your feet", in the tradition of Tiebout. See for example, Epple and Romer (1991).

²Some of the same issues arise in problems of production externalities, multiproduct (or multiplant) production, where cartels or mergers are assumed to perform functions similar to those of a central government.

economic, institutions. This means that the natural mechanisms for aggregating preferences and deciding policy involve legislatures, elections, and voting, instead of firms, markets, buying, and selling. This key difference – the political dimension – suggests serious limitations with the standard economic approach. These limitations can be viewed as falling into two categories. The first is normative: the efficiency problem is compounded by a preference aggregation problem. While, with some stretch of the imagination, one can treat firms as unitary actors, such an assumption is quite dubious in the case of voters, politicians, and governments. Voters will typically have idiosyncratic preferences over policy choices, and these may differ systematically across jurisdictions. It is this heterogeneity that creates the preference aggregation problem, so federal and local policies are chosen by voting schemes which require a different approach from the standard normative analysis of externalities. In fact, the welfare function to maximize is itself determined endogenously, by the political process. Consequently, different districts may implicitly optimize much different welfare functions, and the aggregation of these welfare functions into a "federation" welfare function in some cases may not even be well-defined.

Second, the mechanisms available in the political arena are not as rich as the mechanisms available in an economic setting. In particular, a feature that is virtually universal to political processes is that direct sidepayments are limited or, in some cases, altogether absent.³ This changes the nature of equilibrium in the models and the nature of second best solutions. In particular, with voting mechanisms instead of sidepayments, equilibrium in the resulting game is driven by marginal actors who are pivotal in a voting game. In contrast, equilibrium in market games are determined by marginal utilities and costs, which is the driving force behind standard efficiency concepts of either the first or second best variety. Unfortunately, there is no guarantee that the preferred policies of the pivotal voters, say the median voter, lead to outcomes closely resembling classical economic efficiency.

In the context of locally provided public goods and multiple jurisdictions, there is even a third difficulty, in that the political decisions at the local and federal levels will be dependent on each other. On the one hand, local jurisdictions are constrained in their policy choices by decisions at the federal level. Of course, there are effects in the other direction as well, since federal policies are made by legislative policies that are composed of representatives of the various jurisdictions who anticipate the effect of federal policies on

³This paper makes no attempt to provide an explanation for this. There are, in fact, some possibilities for transfers, under the guise of campaign finance, vote trading, cross jurisdictional block grants, and other products of distributive politics. However, to a first approximation transfers can be viewed as very limited.

their jurisdictions.

This paper considers a very simple version of the problem of externalities with two levels of government, one level, which we call the local level, and another level, which we call the federal level.⁴ A nonexcludable public good is provided (and financed) by each jurisdiction at the local level and there are positive spillovers across local jurisdictions.⁵ Voters have single peaked preferences along a single issue dimension. Local jurisdictions make decisions independently by majority rule, taking into account the equilibrium policies of other jurisdictions. We assume the political process is open and competitive, so equilibrium outcomes are determined by the preference of the median voter in each district. In the autarky equilibrium, without any federal policy, the public good is underprovided, relative to the optimal level, using a utilitarian welfare function that is constructed from the median voter utilities in each district. The role of the federal government is limited to simple constraints on local policies, which we call *federal mandates*. A federal mandate places a lower bound on the amount of public good that each district must provide. We assume that federal mandates are also made in a competitive majority rule institution, so we use majority rule equilibrium as the solution.

The role of a federal government creates a two stage game, where the federal mandate is decided first (by majority rule), followed by a noncooperative game between the (median voters of) local jurisdictions. Because the federal stage is followed by a local policy making stage, the voter induced preferences over federal mandates is quite complicated. We characterize these preferences and show that they are generally multipeaked, which can lead to a nonexistence problem. However, by extending a result of Kramer and Klevorick (1975) we establish existence of a local majority rule equilibrium, and characterize the range of equilibrium outcomes.

We then illustrate the welfare effects of federal mandates by comparing a regime with federal mandates to a regime without. When the spillover effects are small, then federal mandates lead to *worse* outcomes than the autarky solution. The direction of distortion is that equilibrium federal mandates will be set *too high* relative to the optimum. That is, while the autarky solution results in underprovision of the public goods, federal mandates overcompensates if the spillover effects are small enough. We show unambiguously that the regime with federal mandates leads to higher production of the public good. Hence, to a first approximation, the relevant consideration is therefore whether or not the spillover effect is sufficiently large enough to warrant

⁴Of course there are many tiers of a federal system, ranging from cities, towns, townships, counties, and so forth. Hopefully a model with two levels captures the most salient features of a federal system.

⁵The model can be easily translated into a model with negative spillovers.

federal intervention. However, in general it is not possible to unambiguously sign the welfare effects, since there are systematic redistributive features to federal mandates. In particular, low demanders of the public good from districts who are also relatively low demand districts are made worse off. Not surprisingly, it is the voters from high demand districts who are made better off. The reason for this is that federal mandates create a constraint that is only binding on the lowest demand districts, and can actually lead to a reduction in production by the high demand districts.

This paper is by no means the first to model the political dimension of federalism issues. The closest papers are Cremer and Palfrey (2000, 2002), which investigated political equilibrium models of federal mandates in the absence of externalities. Bednar (2001) models the federation stability problem as a repeated game in which local jurisdictions can "cheat" on public policy agreements. A similar motivation lies behind the analysis of De Figueiredo and Weingast (2001) and the empirical work of Alesina and Spolaore (2002). Cremer and Palfrey (1996, 1999) characterize voter preferences over different rules of representation and degrees of centralization, as derived from both individual and jurisdictional characteristics, and study the theoretical implications of these induced preferences for constitutional design. In a different vein, a number of papers are concerned with the issue of interjurisdictional redistribution and the efficiency implications of different federal structures.⁶ Finally, there is a large literature in the Tiebout (1956) tradition that considers mobility across jurisdictions.⁷ We do not consider the mobility issue here.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 analyzes the autarky case, without a federal government. Section 4 analyzes the two stage game with a federal stage followed by a local stage. We show existence of a (strict) local majority rule equilibrium, and find a general characterization of the range of possible equilibria. Section 5 works out a detailed example illustrating the induced preferences, as well as illustrating why a global majority rule equilibrium may fail to exist. Section 6 addresses welfare comparisons between the equilibrium under an autarky regime and a regime with federal mandates.

2 The Basic Model

We consider a confederation composed of D districts, where D is an odd integer greater than or equal to 3. Each district has an odd number of

⁶See for example Persson and Tabellini (1996).

⁷See for example Epple and Romer (1991) or Nechyba (1997).

voters. Each district is to decide on a level of a local non-excludable public good, such as air pollution reduction. We denote by $x_d \in \mathfrak{R}$ the level chosen in district d . The locally provided public good is subject to externalities in the form of spillover effects. That is, the utility of a voter i in district d (denoted voter (i, d)) depends not only on x_d , but also on the levels in the other districts, and on the cost of producing x_d , which we assume is linear in x_d . We consider the case where the spillover effects are positive.

Specifically, we will assume two different forms for the utility function of agent (i, d) . In the logarithmic form, denoting $X_{-d} \equiv \sum_{d' \neq d} x_{d'}$ we have

$$\begin{aligned} u_{id}(x) &= t_{id} \ln(x_d + \beta X_{-d}) - x_d \text{ if } x_d + \beta X_{-d} > 0 \\ &= -\infty \text{ otherwise} \end{aligned}$$

where $\beta > 0$, and $t_{id} > 0$ is referred to as voter i 's *type*.⁸ The voter type, t_{id} , is exactly the ideal point of voter (i, d) if $x_{d'} = 0$ for all $d' \neq d$. Higher types refer a higher level of the public good in their own districts than do lower types, if other districts produce nothing. Let m_d be the median type in district d . For convenience we assume that each district has a different median type, and that districts are labeled in order of their median type, so

$$d < d' \iff m_d < m_{d'},$$

and within district we assume that the index of voters is ordered by type, so

$$i < i' \iff t_{id} < t_{i'd}.$$

There is substitutability between production in one's own district and production in the other districts, and the coefficient β measure the degree of substitutability.⁹

In the separable form, we have

$$u_{id}(x) = v_{id}(x_d) + \beta w_{id}(x_{-d}),$$

where v_{id} is single peaked and w_{id} is a strictly increasing function of the increasing in its arguments, and where β is a non negative real number:

⁸Implicitly, we are assuming that all districts are the same size, so that their externality effects are symmetric. This could be generalized, indeed if we think of x_d as expenditures on the public good per head, it does not make sense that districts of different sizes yield the same externalities towards the other districts. Also, note that if $t_{id} < 0$ then voter (i, d) considers x a public bad.

⁹One can also think of β as measuring the strength of the spillover effects. The special case of $\beta = 0$ corresponds to no externality. This was studied in Cremer and Palfrey (2000, 2002).

agent (i, d) bears the cost of the creation of the externality in his district, so that there is a finite amount of x_d which he finds optimal; on the other hand, there is no limits to the amount of the positive externality that the would like the other districts to produce. The type of agent (i, d) will be

$$t_{id} = \max_{x_d} v_{id}(x_d)$$

and the median type in the district will satisfy

$$m_d = \text{med}_i t_{id}.$$

2.1 Externality-induced preferences

Due to the spillover effects, a voter's preferences over local public good provision in their own district actually depend on the amount of the public good being provided by the other districts. In the logarithmic case, given any profile of public good production by the other districts, $x_{-d} = (x_1, \dots, x_{d-1}, x_{d+1}, \dots, x_D)$, the *conditional ideal point* of voter (i, d) , which we denote by $\hat{x}_{id}(x_{-d})$, is obtained by differentiating (1), to get the first order condition:

$$\hat{x}_{id}(x_{-d}) = t_{id} - \beta X_{-d}$$

The second order conditions for a maximum hold, so this characterizes i 's ideal policy, given the policies in the other districts. There are several interesting features of these *externality induced preferences*. First, every voter's ideal point is (weakly) decreasing in the public good levels of all other districts. This represents the substitution effect of the spillovers. The greater the spillover effect, the greater the substitution (free riding) effect. It is also easy to see that all voters are better off as other districts produce more, since the externality is positive. It is this free rider problem that leads to the intuition that federal mandates may increase efficiency.¹⁰ However, this free riding problem also leads to complex strategic interactions between the districts, since the (conditional) ideal point of the median voter in a district will depend on the policies adopted by the other districts. A second feature of the induced preferences is that the identity of the median voter in a district is independent of the policies of the other districts, since each ideal point is

¹⁰Bednar(1999) explores the repeated game equilibria in a related model of free riding across districts, but where $t_{id} = 0$ for all voters and $x_{id}^*(x_{-d}) = 0$ for all voters, all districts, and all values of x_{-d} . In her model, x corresponds to the degree of compliance to some regulation or standard.

simply shifted downward by the constant, $\beta \sum_{d' \neq d} x_{d'}$. Because they are all shifted down, the order of the conditional ideal points is preserved.

In the separable case, voter (i, d) conditional ideal point is independent of the production in the other districts.

3 Equilibrium without a federal policy

We first consider the case where there is no federal policy. That is, the districts are unconstrained. Each district is free to choose any policy level in their own district. As in Crémer and Palfrey (2000, 2002), the game within a district is modeled as a competitive outcome, driven by the preferences of the median voter. This can be rationalized as the equilibrium of a game across jurisdictions, where the equilibrium outcome in each district corresponds to the most preferred policy of the median voter, given the outcomes in the other district. Formally this is modeled as a proposal game, whereby each voter in each district simultaneously makes a proposal and the outcome for their district is the median proposal. A Nash equilibrium of this game will result in a profile of district policies, $x^* = (x_1^*, \dots, x_D^*)$ such that, for all d , x_d^* is the conditional ideal point of voter (m_d, d) given x_{-d}^* . In the logarithmic case, since the identity of the median voter in d does not depend on x_{-i} , the first order condition for district d , is:

$$x_d^* = t_{m_d} - \beta X_{-d}^*$$

or

$$(1 - \beta)x_d^* = t_{m_d} - \beta X^*$$

where $X_{-d}^* = \sum_{d' \neq d} x_{d'}^*$ and $X^* = \sum_d x_d^*$ is total public good production. The maximization problem is concave, so this is indeed a maximum.¹¹ This condition implies a result that is simple, but has important consequences for the equilibrium with federal mandates. That result is the *substitution principle*. Increased public good levels in one district will result in lower public good provision in all other districts. This reflects the downward sloping reaction functions, which occur because utility is nonseparable in the production

¹¹Not only is this a Nash equilibrium outcome of the announcement game, but it has a stronger property as well. Fixing the outcomes in all districts except d , each voter in d has an optimal strategy that is independent of the announcements of the other members of district d . Thus voters have what might be called conditionally dominant strategies. They are not, strictly speaking, dominant strategies since best responses will depend on the median announcements in the other districts.

of different districts, and spillovers are positive. Combining the first order conditions for all districts and solving gives:

$$\begin{aligned} X^* &= \frac{\sum t_{m_d}}{1 + (n-1)\beta} \\ x_d^* &= \frac{t_{m_d} - \beta \frac{\sum t_{m_d}}{1 + (n-1)\beta}}{1 - \beta} \end{aligned} \quad (1)$$

Of course, in the separable case we have

$$x_d^* = m_d \text{ for all } d.$$

3.1 Socially optimal production

We next show that (due to the positive spillovers), in the equilibrium of the voting game, total production is less than the socially optimal level, which we denote X^{**} , provided all voters consider x a public good ($t_{id} > 0$). To show this, define a socially optimal profile of outputs as a vector $x^{**} = (x_1^*, \dots, x_D^*)$ that maximizes the sum of the utilities of the median voters of all districts. Under this assumption, the social welfare function can be written:

$$W(x_1, \dots, x_D) = \sum_{d=1}^D \{t_{m_d} \ln(x_d + \beta X_{-d}) - x_d\}.$$

The first order conditions for a maximum with respect to x_d is:

$$\frac{t_{m_d}}{x_d + \beta X_{-d}} + \sum_{\delta \neq d} \frac{\beta t_{m_\delta}}{x_\delta + \beta X_{-d}} = 1$$

Hence, since $t_{m_d} > 0$ for all d , we must have $x_d + \beta X_{-d} > 0$ for all d , so:

$$\begin{aligned} \frac{t_{m_d}}{x_d^{**} + \beta X_{-d}^{**}} &< 1 \\ \implies \\ t_{m_d} &< x_d^{**} + \beta X_{-d}^{**} \end{aligned}$$

for all d . Summing these inequalities gives:

$$\sum_{d=1}^D t_{m_d} < \sum_{d=1}^D \{x_d^{**} + \beta X_{-d}^{**}\}$$

implying

$$X^{**} > \frac{\sum t_{m_d}}{1 + (n-1)\beta}$$

The same result trivially holds true in the separable case.

4 Equilibrium with a federal policy

As shown above, the free riding problem results in underprovision of the public good when each district decides independently. Thus there is a potential role for federal policy to remedy this problem. In this section, we consider the effect of a simple federal policy called a *standard*. The federal standard, denoted F , imposes a minimum level of the public good that must be produced by each district. There are many examples of standards of this sort for local public goods with spillovers across jurisdictions. For example, in environmental policy, the federal government mandates water quality standards, air quality standards, and emissions standards for automobiles. States and local jurisdictions in some cases may augment these standards, for example in the case of California emissions standards for new automobiles.

As in Cremer and Palfrey (2000,2002), we model the federal standard setting process as the first stage of a two stage game. The second stage of the game is analyzed the same as in the previous section, but F distorts the induced preference of the voters, so the local standard setting equilibrium changes as a function of F . This feeds back and changes the voter's induced preferences over F in the first stage. The equilibrium in the first stage has voters making simultaneous proposals for F , with the median proposal winning.

4.1 Equilibrium in the Second Stage

4.1.1 Logarithmic case

This section characterizes the equilibrium in the second stage, modeling it in the same way as above, except that the induced ideal points are different if $F \neq 0$. Because of the nature of the free-riding problem, the imposition of a federal mandate this has subtle consequences for the inter-district equilibrium choices of x_d . In particular, x_d is not monotonic in F . That is, induced preferences can be either increasing or decreasing in F . However, we can show that X^* is nondecreasing in F .

For the analysis below, recall that the Nash equilibrium of the local standard setting game without federal standards is characterized by D equations of the form:

$$x_d^* = t_{m_d} - \beta X_{-d}^*, \quad d = 1, 2, \dots, D$$

With a federal standard, F , these D conditions become:

$$x_d^*(F) = \max\{F, t_{m_d} - \beta X_{-d}^*(F)\}, \quad d = 1, 2, \dots, D$$

The solution to this set of equations is unique for each F , and has several properties that are summarized below. First, for any district d , there is some value of F , at which the constraint that $x_d \geq F$ will become binding, given X_{-d}^* . We call this the *critical federal standard for district d* .

Definition 1 *The critical federal standard for district d , F_d , is the minimum value of F for which $x_d^*(F) = F$.*

Note that these critical levels are endogenous, since the critical level of district d will depend on F and the public good production of the other districts, which depend on F , and so on. However, it is easy to show that the constraint is first binding on district 1, then district 2, and so forth, so that these levels exist. Furthermore, for values of $F < F_d$, district d 's output is either constant or *even shrinking*. The reason an unconstrained district's output may be decreasing in F is that the districts for which F is binding will be producing more, and d 's reaction function is downward sloping. Therefore, while F is forcing low demand districts to produce more, it produces an equilibrium effect of actually *decreasing* the production by the high demand districts, creating a secondary problem of inefficiency.

4.1.2 Separable case

In the separable case, the policy adopted in a district is independent of the policies adopted in the other districts. This implies that $F_d = m_d$ for all d . Therefore, the equilibrium policies in the second stage, given F , are simply:

$$x_d^*(F) = \max[m_d, F]. \quad (2)$$

4.2 Equilibrium in the First Stage

The analysis in the previous section simply looks at how the final stage interdistrict equilibrium will respond to different levels of F . The majority rule equilibrium in the first stage assumes sophisticated voting, that is, voters vote over levels of the federal standard, F , correctly anticipating the effect of F on (second-stage) equilibrium policies in each district. Therefore, In order to characterize the majority rule equilibrium in the first stage, we first need to derive the indirect preferences over the federal mandate, F , for each voter t_{id} . Note that these indirect preferences are in fact *endogenously determined*, in the sense that the induced preferences of voter (i, d) depends on all the other voters in the system (including those outside his own district), and also depends on the interdistrict equilibrium outcomes.

4.2.1 Induced voter preferences for F

We use the terminology U_{id} in order to denote voter t_{id} 's indirect utility over F . The following lemmas summarize the main properties of U . Lemma 4 describes the indirect preferences of relatively low demand voters (relative to their district median) and lemma 5 describes the indirect preferences of the relatively high demand voters. The main difference is that low demand voters will have single peaked indirect utility, while high demand voters will not. The reason for this is twofold. First, low demand voters can only be constrained by F , so they have an ideal point equal to F_d , and hence are never better off by higher mandates. Second, in contrast, high demand voters may benefit from higher F once it constrains higher demand districts. This may happen, for example, if voter t'_{id} 's unconstrained ideal point is greater than F_{d+1} .

It is fairly easy to see that, since x_d^* is not monotonic in F , induced preferences are not single peaked, but may have multiple peaks. This leads to complications similar those resulting from the double-peaked induced preferences of some voters in Crémer and Palfrey (2002). In that paper, we were able to show the existence of a majority rule equilibrium in the first stage. That is, there existed a value for F such that there was no alternative federal mandate that was preferred by a majority to F . Because preferences are not single peaked, this existence result is not generally guaranteed. However, a weaker majority rule equilibrium, called *local majority rule equilibrium* (LMRE) can be shown to exist. A local majority rule equilibrium is any policy F with the property that there is no policy in the neighborhood of F that is majority preferred to F . This result is originally due to Kramer and Klevorick, who proved existence (Kramer and Klevorick 1974) and demonstrated a useful application of the result (Klevorick and Kramer 1973).

In this paper, we apply a stronger definition of LMRE and show its existence, using a different argument. Existence of the standard LMRE in our model could be established in either of two ways. One way is to show that Kramer and Klevorick's (1974) sufficient conditions are satisfied in our model; a second, direct, way is simply to note that any low value of F , such that $F < F_1$ is a LMRE, since neither F nor any standard in the neighborhood of F would be binding on any district. Hence, for $F < F_1$ induced preferences for a federal standard are completely flat for all voters. We refer to such equilibria as *weak* equilibria and do not consider them in this paper. That is we limit attention to (*strict*) LMRE standards that create a binding constraint on at least one district. Proving existence of strict LMRE requires a somewhat different proof. Our proof is constructive and implies tight upper and lower bounds of the equilibrium federal standards. This allows us

to characterize the minimum and maximum federal standards that are local majority rule equilibria.

4.2.2 Local Majority Rule Equilibria

This section provides a *definition, constructive proof of existence, and characterization* of Local Majority Rule Equilibrium (*LMRE*) in our model. Recall that the policy space be the real line, \mathfrak{R} . For one-dimensional voting problems over the real line, a general definition of LMRE is as follows.

Definition 2 *A policy $x \in \mathfrak{R}$ is a (strict) local majority rule equilibrium (LMRE) if there exist $\epsilon > 0$ such that for (i) for all $x' \in (x, x + \epsilon)$, $\exists i(x')$ such that $U_{i(x')}(x') < U_{i(x')}(x)$ and (ii) for all $x' \in (x - \epsilon, x + \epsilon)$, the number of voters, j , such that $U_j(x') > U_j(x)$ is strictly less than $(N + 1)/2$.*

For the existence proof below, we only use the following properties of $U_{id}(F)$, which hold in our model, for both the logarithmic specification and the separable specification.¹²

Property 1 *For all i , the function U_i has a finite number of local extrema in the region $[F_1, \infty)$.*

Property 2 *There exist a uniform bound $M > 0$, such that for all i the function U_i is decreasing for $x \geq M$.*

The next assumption ensures that the utility functions are never locally constant.

Property 3 *For any $x \in (F_1, \infty)$ there exists $\eta > 0$ such that, for all i , U_i is strictly monotone on each of the intervals $(x - \eta, x)$ and $(x, x + \eta)$.*

We state the following theorem.

Theorem 3 *Under properties 1 to 3, there exists at least one strict LMRE.*

The proof yields a characterization of the greatest and least strict LMRE. We will present this characterization below. We first establish two preliminary results. The first preliminary result establishes a necessary condition for F to be a strict LMRE. deals with the issue of "strictness."

Lemma 4 *If a policy F is a strict LMRE then $F \geq F_1$.*

¹²For proofs, see the appendix.

Proof. Suppose $F < F_1$, then the constraint is not binding on any district, so U_i is locally flat for all voters, which contradicts (i) in the definition of strict LRME. Hence $F \geq F_1$. ■

We next establish that any LMRE will be a local maximum of the utility function of at least one agent, and these local maxima are therefore natural “candidate equilibria”.

Lemma 5 *If a policy F is a LMRE then there exists an i such that F is a local maximum of U_i .*

Proof. If $F = F_1$, then F is a local maximum for the median voter in district 1. Therefore, let $F > F_1$ and suppose, for all i , that F is not a local maximum. We will show that this leads to a contradiction. By lemma ??, there exists η such that for all i the function U_i is strictly monotone on $(x - \eta, x + \eta)$. Hence, it is either strictly increasing or strictly decreasing for at least $(N + 1)/2$ voters. Assume it is strictly increasing. Then $U_i(x') > U_i(x)$ for at least $(N + 1)/2$ voters and for all $x' \in (x, x + \eta)$, which by definition 2 implies it is not an LMRE. ■

Accordingly, we define the set of candidates as the set of local extrema of the utility functions:

Definition 6 *A candidate, F , is any policy which is a local extremum of at least one U_i .*

By lemma 4 we need only consider candidates $F \geq F_1$. For any candidate, the voters fall into one of four categories, depending on their induced preferences, locally around F . For at least one voter, F is a local maximum. For the remaining voters, U_i is either increasing in a neighborhood of F , decreasing in a neighborhood of F , or has a local *minimum* at F . The following definition formally defines these four categories of voters.

Definition 7 *For any candidate $F \geq F_1$, we will say that*

- *i surely votes to the right of F if there is an open interval (x, y) with $x < F < y$ such that U_i is nondecreasing on that interval — let us call $\mathcal{R}(F)$ the set of voters that surely vote to the right of F ;*
- *i surely votes to the left of F if there is an open interval (x, y) with $x < F < y$ such that U_i is strictly decreasing on that interval — let us call $\mathcal{L}(F)$ the set of voters that surely vote to the left of F ;*
- *i votes exactly for F if F is a (weak) local maximum of U_i — let us call $\mathcal{E}(F)$ the set of voters that vote exactly for F ;*

- i votes either to the left or to the right of F if F is a strict local minimum of U_i — let us call $\mathcal{LR}(F)$ the set of voters that vote either to the left or to the right of F .

Lemma 8 For any candidate F , $\{\mathcal{L}(F), \mathcal{R}(F), \mathcal{E}(F), \mathcal{LR}(F)\}$ is a partition of the set of voters.

A proof of this lemma left to the reader. Note that $\mathcal{E}(F)$ is defined in terms of weak local maxima, while $\mathcal{LR}(F)$ is defined with respect to strict local minima. For $F > F_1$, this distinction never matters, since U_i is not locally constant in that region. For expositional reasons, in the proof we will sometimes treat the case of $F = F_1$ separately. The reason $F = F_1$ is slightly different is that *all* voters are indifferent between F_1 and any point below F_1 . Therefore, $\mathcal{L}(F_1)$ and $\mathcal{LR}(F_1)$ are both empty. For $F < F_1$, all voters are in $\mathcal{E}(F)$ since preferences are locally constant in that region. However, since preferences are locally constant when $F < F_1$, there cannot be a strict LMRE in that region.

Using this partition of voters, we obtain the following characterization of strict LMRE.

Lemma 9 A candidate F is a strict LMRE if and only if

$$|\mathcal{L}(F)| + |\mathcal{LR}(F)| \leq \frac{N-1}{2} \quad (3)$$

and

$$|\mathcal{R}(F)| + |\mathcal{LR}(F)| \leq \frac{N-1}{2}. \quad (4)$$

and

$$F \geq F_1. \quad (5)$$

Proof.

i. Necessity:

First, by lemma 4, $F \geq F_1$ is a necessary condition for a strict LMRE. To establish necessity of the other conditions, we suppose $F \geq F_1$ is a strict LMRE and show that this implies inequalities (3) and (4). If $F > F_1$, let $\epsilon(F)$ satisfy the conditions in the definition of LMRE, and Property 3. Since F is an LMRE, we have $U_i(x) \leq U_i(F)$ for at least $(N+1)/2$ voters for all $x \in (F, F + \epsilon(F))$. Since the functions U_i are strictly monotone on

$(F, F + \epsilon(F))$, we get $|\mathcal{E}(F)| + |\mathcal{L}(F)| \geq \frac{N+1}{2}$, which, by 8, implies (4). The fact that inequality (3) holds is proved in a similar fashion.

Suppose $F = F_1$. Since F_1 is a LMRE, we must have $U_i(x) \leq U_i(F_1)$ for at least $(N+1)/2$ voters for any $x \in (F_1, F_1 + \epsilon(F_1))$, where $\epsilon(F_1)$ satisfies the conditions in the definition of LMRE. Therefore $|\mathcal{E}(F)| \geq \frac{N+1}{2}$, implying $|\mathcal{LR}(F_1)| + |\mathcal{R}(F_1)| \leq \frac{N+1}{2}$. To show (3), simply observe that $\mathcal{L}(F_1)$ and $\mathcal{LR}(F_1)$ are both empty.

ii. Sufficiency:

Assume now that inequalities (3) and (4) hold, and $F > F_1$. Take any $x' \in (F - \eta, F)$, where η satisfies the condition in Property 3. By Property 3, U_i is not locally constant on this open interval for any i , and we have $U_i(x') > U_i(F)$ if and only if $i \in \mathcal{L}(F) \cup \mathcal{LR}(F)$, and therefore, by (3) there are fewer than $(N+1)/2$ voters who strictly prefer x' to F . Consider $x' \in (F, F + \eta)$. In this case, we have $U_i(x') > U_i(F)$ if and only if $i \in \mathcal{R}(F) \cup \mathcal{LR}(F)$, and therefore, by inequality (4) there are fewer than $(N+1)/2$ voters who strictly prefer x' to F . Hence F is an LMRE. Strictness follows from local nonconstancy and lemma 5. Next suppose $F = F_1$ and inequalities (3) and (4) hold. A similar argument to the case of $F > F_1$ shows that, for all $x' \in (F_1 - \epsilon, F_1 + \epsilon)$, the number of voters, i , such that $U_i(x') > U_i(F_1)$ is strictly less than $(N+1)/2$. Furthermore, if ϵ is chosen small enough, then for all $x' \in (F_1, F_1 + \epsilon)$, $U_i(x') < U_i(F_1)$ for the median voter of district 1, so F_1 satisfies the strictness condition in the definition of LMRE. ■

Let F^{\max} be the greatest candidate.¹³ For any candidate $F > F_1$, let us call F^- the greatest candidate strictly less than F , and for any $F < F^{\max}$, let F^+ be the least candidate strictly greater than F .

Lemma 10 *For any candidate $F > F_1$:*

$$i \in \mathcal{R}(F^-) \cup \mathcal{LR}(F^-) \Rightarrow i \in \mathcal{R}(F) \cup \mathcal{E}(F)$$

Proof. Consider $i \in \mathcal{R}(F^-) \cup \mathcal{LR}(F^-)$. Because U_i is strictly increasing on $(F^-, F^- + \eta)$, and all local extrema of U_i are candidates, the least local maximum of U_i greater than F^- is greater than or equal to F . If it is equal, then $i \in \mathcal{E}(F)$; if it is smaller, then $i \in \mathcal{R}(F)$. Therefore $i \in \mathcal{R}(F) \cup \mathcal{E}(F)$. ■

Proof of Theorem 3: As the final step in showing existence of a strict LMRE, let \tilde{F} be the least candidate $F \geq F_1$ for which inequality (4) holds. We will show that \tilde{F} is a strict LMRE. Strictness follows immediately, so we only need to verify part (ii) of the definition.

¹³We know a greatest candidate exists since U_i is eventually decreasing for all i .

First, suppose $\tilde{F} > F_1$. Because \tilde{F} satisfies (4), by lemma 9, if it is not a LMRE it cannot satisfy (3), and we must have

$$\left| \mathcal{L}(\tilde{F}) \right| + \left| \mathcal{LR}(\tilde{F}) \right| \geq \frac{N+1}{2}.$$

By lemma 8, this implies

$$\left| \mathcal{E}(\tilde{F}) \right| + \left| \mathcal{R}(\tilde{F}) \right| \leq \frac{N-1}{2},$$

and by (10)

$$\left| \mathcal{R}(\tilde{F}^-) \right| + \left| \mathcal{LR}(\tilde{F}^-) \right| \leq \frac{N-1}{2},$$

which contradicts the definition of \tilde{F} . Hence \tilde{F} is a LMRE.

Suppose instead that $\tilde{F} = F_1$. As shown earlier, $\mathcal{L}(F_1)$ and $\mathcal{LR}(F_1)$ are empty. This implies $\left| \mathcal{E}(\tilde{F}) \right| \geq (N+1)/2$, so it is a (strict) LMRE. ■

Theorem 11 *The least candidate F such that inequality (4) and (5) hold, and the greatest candidate such that (3) and (5) hold are, respectively, the least and the greatest strict LMRE.*

Proof. The proof follows immediately. ■

4.2.3 A bound on LMREs in the separable case

In the separable case, equation (2) implies that for all d , X_d is increasing in F , and strictly increasing if $F \geq m_d$. We then have

$$U_{id}(F) = v_{id}(\max[m_d, F]) + \beta w_{id}(X_{-d}(F)).$$

This implies

Lemma 12 *The function $U_{id}(F)$ is strictly increasing on $(F_1, \max[m_d, t_{id}])$.*

Proof. For $F < m_d$ we have $U_{id}(F) = v_{id}(m_d) + \beta w_{id}(X_{-d}(F))$. Since $F \in (F_1, \max[m_d, t_{id}])$ this implies that X_{-d} is increasing in F , so the second term is increasing since w_{id} is an increasing function. The first term is constant, since m_d is independent of F . Next suppose $t_{id} > m_d$ and $F \in (m_d, t_{id})$. Then $U_{id}(F) = v_{id}(F) + \beta w_{id}(X_{-d}(F))$, and both terms of this sum are increasing in F . ■

We first state the following lemma.

Lemma 13 *Let X be such that U_{id} is strictly increasing on (F_1, X) for $(N + 1)/2$ voters, and let F^* be a LMRE. Then $F^* \geq X$.*

Proof. Suppose $F \in (F_1, X)$ and U_{id} is increasing on (F_1, X) for $(N + 1)/2$ voters. All of these voters are in $\mathcal{R}(F)$ which implies that $|\mathcal{R}(F')| + |\mathcal{LR}(F')| > (N - 1)/2$. Therefore F is not a LMRE. ■

Let us define

$$\tilde{F} = \text{med}_{(i,d)}(\max[m_d, t_{id}]).$$

The previous lemma immediately implies the following corollary.

Corollary 14 *If F^* is a LMRE then $F^* \geq \tilde{F}$.*

Proof. Suppose to the contrary that $F < \tilde{F}$. Without loss of generality let $F \geq F_1$. Then U_{id} is strictly increasing for all voters for whom $t_{id} \geq \tilde{F}$. But $\tilde{F} = \text{med}_{(i,d)}(\max[m_d, t_{id}])$, so there are at least $(N + 1)/2$ such voters, so the lemma applies and F is not a LMRE. ■

Note that this result holds true even if the spillover effect is very small. Hence, for the case of small externalities, we obtain the same results as in Crémer and Palfrey (2000), which showed that there was overproduction when federal production of local public goods is more efficient than local production.

5 An example with three districts

This section gives an example illustrating how induced preferences for federal standards vary across districts and across voters in a district, and shows a robust example where a local equilibrium exists but there is no (global) majority rule equilibrium federal standard.

We assume that there are three districts, and that agents have logarithmic preferences. At this point, we do not need to specify number of voters in each district, as the equilibrium profile of production across districts is a function only of the median voter types.

5.1 District equilibrium, conditional on F

With a federal mandate, the constraint $x_d \geq F$ may be binding on one or more districts. To compute the district equilibrium, we partition the values of F according to regions with differ by the number of districts for which the constraint is binding.

5.1.1 F not binding on any district

>From (1), the solution is :

$$\begin{aligned} x_3^* &= \frac{\beta t_1 + \beta t_2 - \beta t_3 - t_3}{-\beta - 1 + 2\beta^2} \\ x_2^* &= \frac{\beta t_1 + \beta t_3 - \beta t_2 - t_2}{(\beta - 1)(2\beta + 1)} . \\ x_1^* &= \frac{\beta t_2 + \beta t_3 - \beta t_1 - t_1}{(\beta - 1)(2\beta + 1)} \end{aligned}$$

Since $t_1 < t_2 < t_3$ it follows that $x_d \geq F$ is not binding for any d for values of F satisfying:

$$F < \frac{\beta t_3 + \beta t_2 - \beta t_1 - t_1}{2\beta^2 - \beta - 1}.$$

5.1.2 F binding on district 1

If F is binding on district 1, and only on district 1, the x_d 's must solve

$$\begin{aligned} F &= x_1 \\ t_2 &= x_2 + \beta(x_1 + x_3) \\ t_3 &= x_3 + \beta(x_2 + x_1) \end{aligned}$$

The solution is :

$$\begin{aligned} x_1^* &= F \\ x_3^* &= \frac{\beta F + \beta t_2 - \beta^2 F - t_3}{\beta^2 - 1} . \\ x_2^* &= \frac{\beta F + \beta t_3 - \beta^2 F - t_2}{\beta^2 - 1} \end{aligned}$$

From the analysis of the case where F is not binding on any district, we know that the constraint $x_1 \geq F$ becomes binding on district 1 precisely when $F = \frac{\beta t_3 + t_2 \beta - \beta t_1 - t_1}{2\beta^2 - \beta - 1} \equiv F_1$. Similarly, the constraint later becomes binding on district 2 as soon as $\frac{\beta F + \beta t_3 - \beta^2 F - t_2}{\beta^2 - 1} = F$, or, $F_2 = \frac{\beta t_3 - t_2}{(2\beta + 1)(\beta - 1)}$. Observe that $F_1 < F_2$ since $t_1 < t_2$. Hence the second region, where the federal mandate binds only on district 1, is defined by

$$\frac{\beta t_3 + \beta t_2 - \beta t_1 - t_1}{2\beta^2 - \beta - 1} \leq F \leq \frac{\beta t_3 - t_2}{2\beta^2 - \beta - 1}.$$

5.1.3 F binding on district 1 and 2

To compute the equilibrium when F is binding on districts 1 and 2, but not on district 3, we solve:

$$\begin{aligned}
F &= x_1 \\
F &= x_2 \\
t_3 &= x_3 + \beta(x_2 + x_1)
\end{aligned}$$

and obtain:

$$\begin{aligned}
x_1^* &= F \\
x_2^* &= F \\
x_3^* &= t_3 - 2\beta F
\end{aligned}
.$$

Notice that x_3 is decreasing in F , which was proved in an earlier lemma. The constraint $x_1 \geq F$ becomes binding on district 2 when $t_3 - 2\beta F = F$, implying that $F_3 = \frac{t_3}{2\beta+1} > F_2 > F_3$. Hence the third region, where the federal mandate binds on districts 1 and 2, but not 3, is defined by

$$\frac{-t_2 + \beta t_3}{2\beta^2 - \beta - 1} \leq F \leq \frac{t_3}{2\beta + 1}.$$

5.1.4 F binding on all three districts

Finally, if $F > F_3 = \frac{t_3}{2\beta+1}$ then the federal mandate constraint is binding everywhere, and the trivial solution is:

$$\begin{aligned}
x_1^* &= F \\
x_2^* &= F \\
x_3^* &= F
\end{aligned}$$

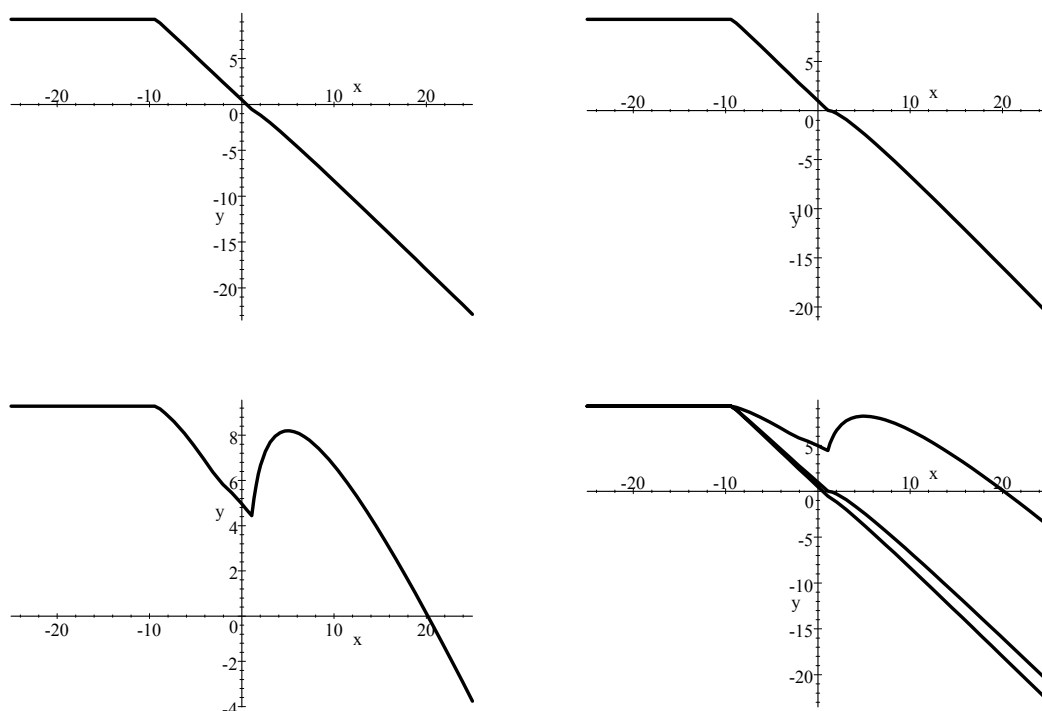
5.2 Computing LMREs

In order to compute the equilibrium federal mandate in this example, we need to look at the induced preferences for *all* voters in *all* districts. All of the analysis so far in this example did not depend on the preferences of any voters *except* for the median voter of each district. However, to compute an equilibrium federal mandate, we will need to know the induced preferences of *all* voters. To make things concrete, we assume that there are 3 voters in each district and let $\beta = 0.9$. The median voters for the three districts are given by $t_1 = 1$, $t_2 = 2$, and $t_3 = 3$. Then in every district we will assume the left most voter has $t_d^L = 0.5$ and the right most voter has $t_d^R = 5.0$. Simple algebra shows that $F_1 = -9.3$, $F_2 = -2.5$, and $F_3 = 1.1$. The induced preferences for the voters are graphed below.¹⁴

¹⁴All computations were done using Maple.

5.2.1 Induced preferences of district 1 voters

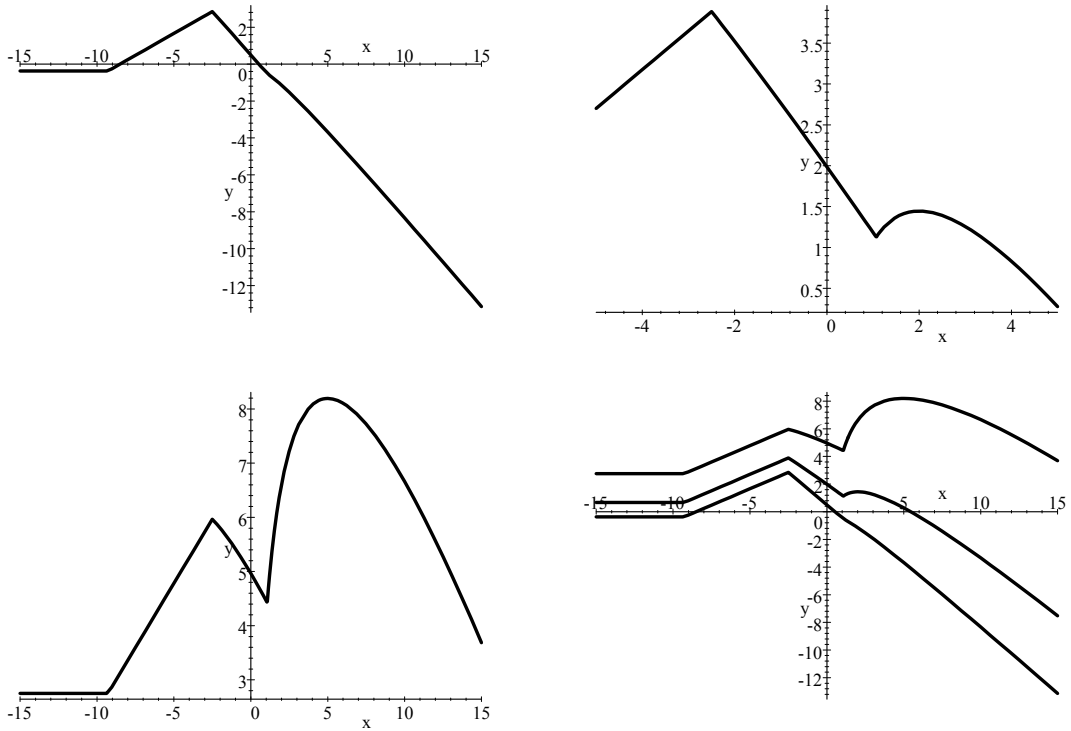
The induced utility functions of the three voters in district 1 are given below. Observe that for each voter, it is decreasing in the region $[F_1, F_2]$, and, of course constant below F_1 . For the rightmost voter in district 1, the induced utility is double-peaked, since it is increasing between F_3 and that voter's ideal point, $t = 5$. The fourth figure graphs all three voters' induced utilities on a single graph.



5.2.2 Induced preferences of district 2 voters

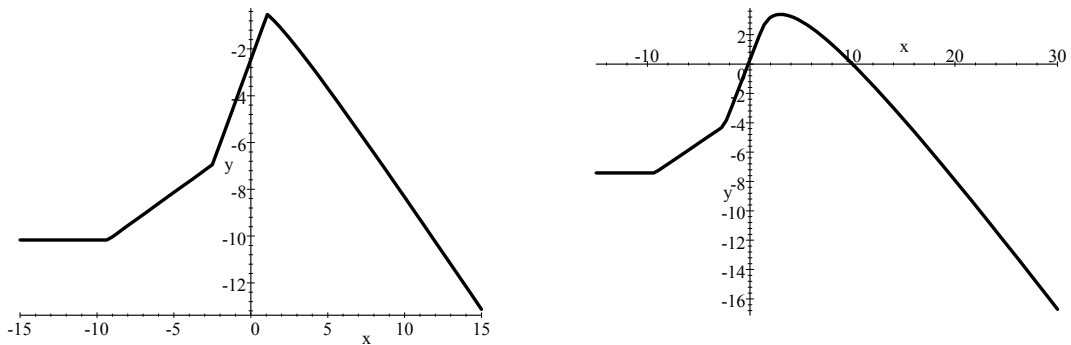
These graphs are produced in a similar way to the graphs of the district 1 voters. For all these voters, utility is *increasing* in the region $[F_1, F_2]$, and, of course constant below F_1 . The reason it increases in that region is due to the substitution effect. District 1 is constrained, and district 2 is not constrained in this region, so district 1 is increasing output one-for-one as F increases, while district 2 is cutting back at a slower rate. In effect, district 2 voters are better off, because district 1 is forced to produce spillovers that are valuable to district 2 voters. Notice that for the rightmost *and leftmost* voters in district 2, the induced utility is double-peaked. For all voters, induced utility is decreasing in the region $[F_2, F_3]$. The fourth figure graphs

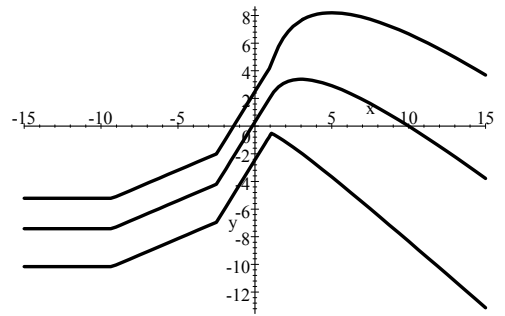
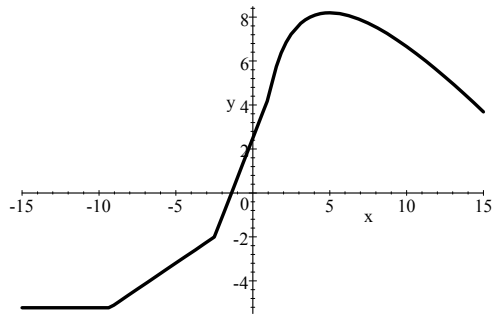
all three district 2 voters' induced utilities on a single graph.



5.2.3 Induced preferences of district 3 voters

For district 3 voters, utility is *increasing* in the region $[F_1, F_3]$, and, of course constant below F_1 . Notice that all these voters' preferences are single peaked. This will always be the case for voters from the district with the highest median. For all voters, induced utility is decreasing in the region $[F_2, F_3]$. The fourth figure graphs all three district 2 voters' induced utilities on a single graph.



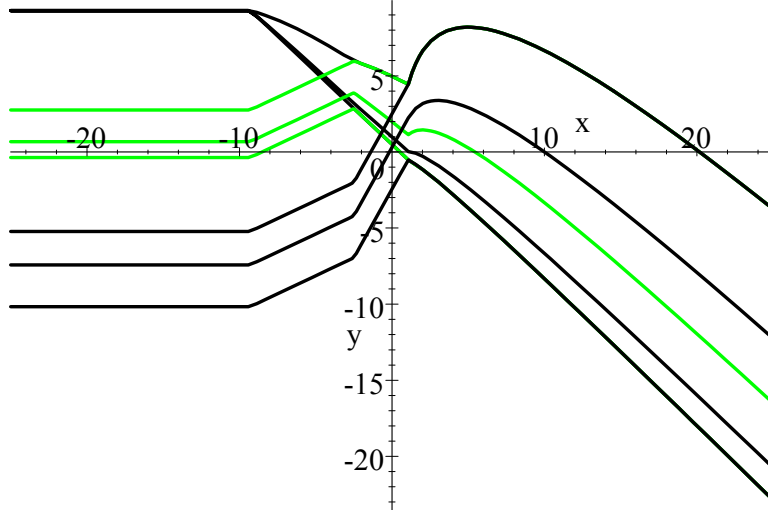


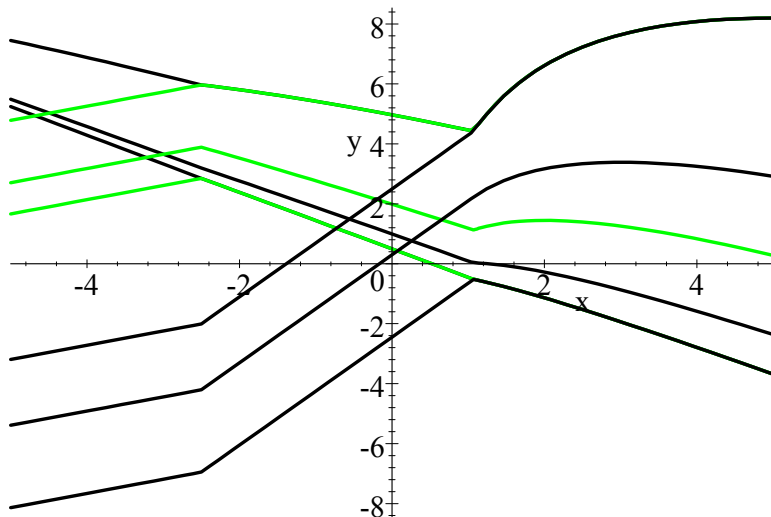
5.3 Equilibrium

To find the local majority rule equilibrium, we can use the algorithm implied by the characterization earlier in the paper. Doing so, we find there are two LMRE. The leftmost LMRE occurs at $F_L^* = F_2$ and the rightmost one occurs at $F_R^* = 2 (= t_2)$. So, for this example, the characteristics of the middle district are reflected in both local equilibria.

To find global equilibria, we only have to check the two local equilibria, since all global majority rule equilibria must also be local majority rule equilibria. First consider $F_L^* = F_2$. This is not a global equilibrium because it is “too low.” A majority of voters would prefer a much *higher* federal standard (even above F_H^*), for example $F = 3$. To see this, simply notice that all district 3 voters are better off at $F = 3$ and so are high demand ($t = 5$) voters in the other two districts. Next, consider $F_R^* = 2$. It is easy to check that it is also not a global equilibrium, this time because because it is “too high.” A majority of voters would prefer a much *lower* federal standard (even below F_L^*), for example $F = -5$. It is easily checked that all district 1 voters prefer -5 to 2, as does the lowest demand voter and the median voter in district 2.

Therefore, in this example, there are two LMRE and no global LMRE.





6 Welfare effects of federal mandates with externalities

In a previous paper (Cremer and Palfrey 2000), we argue that federal mandates can have a negative impact on welfare in the absence of externalities. In particular, we showed in that paper that more voters will be made worse off by mandates than will be made better off. The logic was simply that all voters will have an incentive to push for mandates up to their ideal point, not taking into account the negative effect this may have on low demand voters. The resulting equilibrium federal mandate is therefore equal to the overall median ideal point. Districts whose medians are above this point will be unaffected, while more than half the voters in a district whose median is below the overall median will be made worse off.

With externalities, we will still evaluate welfare on the basis of the preferences of the median voters of each district, but the comparison between regimes is much more complex. First, there is the problem of multiple equilibria (and possible nonexistence of global equilibria), due to the externalities. Second, public good provision is already too low relative to the optimum, so intuitively, this would suggest that the median voter in every district can be made better off *if* total public good provision is increased in a particular way. But in order to make the median voter in every district better off, the increase in output may have to be divided across the districts in a special way,

and this may not be consistent with LMRE in some environments. Indeed, this is one of the effects of federal standards with spillovers that enter in a nonseparable way. There is two perverse (but intuitive) effects that lead to an inefficient distribution of the increase in total production, with low demand districts bearing the brunt of the increase. First, standards bind first on the low demand districts, and for districts with sufficiently high demand, the constraint will not be binding. Second, the nonseparability leads to a substitution effect, in which the high demand districts actually decrease their production at the same time the low demand districts are forced to increase their production.

Due to these added complications, our analysis in this section is divided into parts. The first part studies the effect of federal mandates on the equilibrium total production of the public good, and the distribution of production across districts, and how these two things affect welfare of the median voters of each district.

6.1 Welfare effects in the logarithmic model

The main effect is to increase total production, as proved earlier in Lemma 3. In the case of autarky (no federal mandates), there is an inefficiently low level of total production, so in this respect federal mandates are welfare improving. A corollary to Lemma 3 is that there exists a federal mandate (not necessarily an equilibrium), such that the total production of public good equals X^{**} . This follows because $X^*(0) = X^*$, $X^*(F)$ is continuously increasing above F_1 , and $X^*(F) = DF$ for $F > F_D$. Hence there exists some point at which $X^*(F) = X^{**}$. Therefore, in principle for any β , there is some federal standard with the property that the resulting total production be efficient. However, not all allocations of public productions are efficient. This is clear from inspection of the first order conditions for the efficient solution (see section 3). One can show that the optimal district productions are ordered by the preferences t_d .¹⁵

The claims above are true for any level of F , so they will be true in equilibrium. To illustrate the distributional effects and the differential impact of mandates across districts, we return to the example of the previous section and show that the equilibrium federal mandates do not necessarily make all

¹⁵For example, with two districts, the conditions imply:

$$\frac{t_1}{(1-\beta)x_1^{**} + \beta X^{**}} = \frac{t_2}{(1-\beta)x_2^{**} + \beta X^{**}}$$

so $x_1^{**} > x_2^{**}$ if and only if $t_1 > t_2$.

district medians better off, in both of the LMRE of that example. The reason for this failure is due to distributional effects. While total production is increasing (which is good), it affects different districts in different ways. In particular, it is increasing most in the low demand districts, rather than the high demand districts, since the mandates become binding first for the lowest demand districts. When the mandate becomes binding for a district, the utility of the median voter of that district is decreasing, at least for some range (lemma X). Nonseparability of the utility functions creates an additional negative impact on constrained districts, since high demand districts will decrease production, according to the substitution principle. In fact, high demand districts may produce even less under the equilibrium federal mandate than they did in the autarky solution.

In that example, there are two LMRE, which we denote $F_{low}^* = F_2 = -2.5$ and $F_{hi}^* = t_2 = 2$. Neither is a global MRE, since F_{low}^* is defeated by a high standard (e.g., $F = 3$) and F_{hi}^* is defeated by a low standard (e.g., $F = -5$). However, the question we ask here is whether either of these LMRE are better than having no federal standard at all. For the first equilibrium, F_{low}^* , it is easy to show that the median voter in the lowest demand district is strictly worse off compared to the situation with no federal standards. However, both median voters of the other districts are better off. This is also true for F_{hi}^* . So, in this example a majority of the medians are better off. In fact, a majority of the voters overall are better off. Therefore, either of these local equilibria will win, if they are voted against a status quo of no standard at all, under a closed rule. In this sense (admittedly weak), both are more efficient. One can also show that both are more efficient than no standard using various other criteria, such as the utilitarian rule.

6.2 Welfare effects with separable preferences

As noted earlier in the paper, the separable preferences case provides a relatively easy model to explore the properties of the equilibria. We showed several results above. The most relevant for welfare comparisons is the observation that every LMRE is greater than or equal to $\tilde{F} = \text{med}_{(i,d)}(\max[m_d, t_{id}])$. That is, the set of LMRE is bounded below by the standard that arises as a global majority rule equilibrium when there are no externalities. In particular this implies continuity of some results without externalities, in Cremer and Palfrey (2000). That is, for small values of $\beta > 0$, there will be small changes in the equilibria and hence small changes in the utilities of each of the voters. Therefore, the negative effects based on utilitarian criteria for welfare (summing utilities) will still hold if the spillover effects are small. However, the results that more voters are worse off than are better off may no longer

hold, since $\beta > 0$ implies that many of the high valuation voters who are indifferent between regimes when $\beta = 0$, now are strictly better off with federal standards, due to the spillover effects. Thus, there is a discontinuous effect with respect to the number of voters that are made better off. That is, with $\beta = 0$ a majority would oppose a regime with federal standards, but for small values of $\beta > 0$ a majority would prefer a regime with federal standards to a regime with no federal standards.

7 Concluding remarks

The existence of externalities in the form of positive spillovers lead to significant effects on equilibrium federal standards. These effects are manifested in a number of systematic ways. Naive intuition suggests that federal standards may be a valuable way to overcome the free riding problem among districts in a federation. However, this intuition is complicated due to non single peaked preferences and the equilibrium effects of federal standards on the subgame between local districts.

The first result is that majority rule equilibria may no longer exist. Preferences are not single peaked, since low demand voters are worse off when the mandate binds for their district, but then better off when the mandate binds for other districts. This can lead to majority rule cycles, as demonstrated in the example.

Second, in spite of the potential cycling problem, local majority rule equilibria are guaranteed to exist. Of particular interest are the strict local majority rule equilibria which create binding constraints on some districts, with these constraints creating secondary effects through the equilibrium in the district subgame. We identified the properties of local majority rule equilibria and characterized the range of these equilibria. The range can be quite large, as demonstrated in the example.

Third, the welfare effects are much more complicated than in the original model of Cremer and Palfrey (2000), where there were no spillover effects. The sets of voters who benefit or are made worse off follows an interesting pattern. The value of having federal standards is that it increases the total level of spending on public goods above an inefficiently low level. However, this increase in federal standards is achieved in an inefficient way, because the standards bind first on low demand districts, and last on the highest demand districts, while precisely the opposite pattern would be optimal. With nonseparable preferences, this problem is further exacerbated by a substitution effect, whereby high demand districts actually *reduce* production at that same time low demand districts are being forced to produce more. Thus,

low demand voters from low demand districts are made worse off by federal standards, unless the spillover effects are large, but low demand voters in high demand districts are big winners. Their district produces less, but they benefit from the spillovers generated by increased production in low demand districts.

Because of the confounding effects of higher total production, but perverse distributive effects across districts, we obtained few unambiguous results about the welfare effects in this model. However, in the case of separable preferences, we are able to obtain some conclusions since the subgame between the districts is very simple. In that case, we obtain lower bounds on the LMRE which indicate that if the spillover effects are sufficiently small, federal standards will be set too high, as in Cremer and Palfrey (2000). However, in contrast to that earlier paper, a majority of voters may be made better off even with small spillovers.

While the approach taken here sheds some light on the effectiveness (or ineffectiveness) of federal standards to overcome free riding between districts, it begs the question of what alternatives may be possible, and how well these alternative institutions perform. Hence we see a mechanism design approach as a natural next step in the research agenda. The idea would be to model, institutions, in a general way, as game forms that provide the right incentives for more efficient district decisions for public good production. The use of federal standards, whereby a federation-wide minimum is established is perhaps the simplest class of such mechanisms. More complex mechanisms would allow for the possibility of different standards for different districts, in the form of granting exceptions or exclusions, or possibly employ the use of non-majoritarian methods for voting over mechanisms. Such arrangements could possibly overcome some of the perverse distributive effects of simple federal mandates, and would also be consistent with features of some existing federal policies.

REFERENCES

- Alesina, A. and E. Spolaore, "On the Number and Size of Nations." *Quarterly Journal of Economics* 112 (1997): 1027-56.
- Bednar, J. "Shirking and Stability in Federal Systems." June 2001.
- Bednar, J. "Formal Theories of Federalism" *Newsletter of APSA section on Comparative Politics*, Winter 2000.
- Crémer, J. and T. Palfrey. "In or Out? Centralization by Majority Vote." *European Econ. Rev.* 40 (January 1996):43-60.
- Crémer, J. and T. Palfrey. "Political Confederation." *Am. Pol. Sci. Rev.* 93 (March 1999): 69-83.
- Crémer, J. and T. Palfrey. "Federal Mandates with Local Agenda-setters," *Review of Economic Design*, forthcoming 2002.
- Crémer, J. and T. Palfrey. "Federal Mandates by Popular Demand," *Journal of Political Economy* 108 (October 2000): 905-27.
- De Figueiredo, R. and B. Weingast. "Self-enforcing Federalism." 2001.
- Epple, D. and Romer, T., "Mobillity and Redistribution." *J. P. E.* 99 (1991): 828-58.
- Gordon, R. "An Optimal Taxation Approach to Fiscal Federalism," *Quarterly Journal of Economics* 98 (November 1983): 567-87.
- Klevorick, Alvin K. and Gerald H. Kramer. "Social Choice on Pollution Management: The Genossenschaften" *Journal of Public Economics* 2 (1973): 101-46.
- Kramer, Gerald H. and Alvin K. Klevorick. "Existence of a Local Cooperative Equilibrium in a Class of Voting Games" *Review of Economic Studies* 41 (October 1974): 539-47.
- La Malène, C. "L'Application du Principe de Subsidiarité", Délégation du Sénat pour l'Union Européenne. Rapport 46. 1996/7.
http://cubitus.senat.fr/rap/r96-46/r96-46_mono.html
- Moulin, H. "On Strategyproofness and Single Peakedness." *Public Choice* 35, no. 4 (1980):437-55.

- Nechyba, T. "Existence of Equilibrium and Stratification in Local and Hierarchical Tiebout Economies with Property Taxes and Voting." *Economic Theory* 10 (August 1997):277-304.
- Persson, T. and G. Tabellini, "Fiscal Federal Constitutions: Risk Sharing and Moral Hazard." *Econometrica* 64, no. 3 (1996): 623-46.
- Peterson, P. *The Price of Federalism*. New York:The Twentieth Century Fund, 1995.
- Smith, W. and B. Shin. "Regulating Infrastructure: Perspectives on Decentralization." in *Decentralizing Infrastructure: Advantages and Limitations*. A. Estache, ed. Washington, D.C.:The World Bank, 1995.
- Tiebout, C., "A Pure Theory of Local Public Expenditures." *J. P. E.* 64 (1956): 416-24.