Bargaining collectively*

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Abstract

Many bargaining situations take place between a central agent and a group of individuals acting collectively where (i) the proposals are restricted to treat all the group members equally, (ii) the decisions of the group are reached through a voting process, and (iii) the vote binds all the members of the group. Examples include debt restructuring negotiations between a troubled company and its bondholders; shareholder votes on executive compensation; and collective bargaining between a firm and union members. We study how the equilibrium payoffs of such bargaining situations depend on the decision rule adopted by the group.

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1 Introduction

Many bargaining situations take place between a central agent and a group of individuals acting collectively where (i) the proposals are restricted to treat all the group members equally, (ii) the decisions of the group are reached through a voting process, and (iii) the vote binds all the members of the group. Examples include debt restructuring negotiations between a troubled company and its bondholders; shareholder votes on executive compensation; and collective bargaining between a firm and union members.

The internal decision making rule the group uses has consequences: it determines the likelihood that a proposal will be accepted by the group, and in turn, the offers that are proposed in the first place. As a result, the group can potentially increase its relative bargaining power by an appropriate choice of its internal decision making rule. An immediate question that arises is then what intra-group decision process maximizes the group's total payoff. In this paper, we address this question in the context of a take-it-or-leave-it bargaining game. The basic trade-off is relatively easy to describe. On the one hand, decision processes that are comparatively inimical to agreement harm the group by causing it to reject welfare improving proposals; but on the other hand, this same "toughness" may be helpful in extracting an attractive offer from the opposing party. By explicitly modelling intra-group decision-making we assess the net effect.

Specifically, we consider the following class of bargaining environments. A large group of *ex ante* identical individuals foresees that at some later date it will have to bargain with an opposing party. While still under the veil of ignorance, the group selects a decision rule. The bargaining game, when it arrives, consists of the opposing party making a take-it-or-leave-it offer. We restrict attention to *voting* rules: each member of the group votes on the proposal, and it is accepted if and only if the number of votes in favor exceeds a prespecified threshold. Leading examples are the simple majority rule, the two-thirds supermajority rule, and the unanimity rule.

In general, for any agreement rule the probability of acceptance increases as the offer increases. As a result, the key tradeoff faced by the proposer in choosing his offer is between making a higher offer that is accepted more often, and making a lower offer which increases the proposer's payoff conditional on acceptance, but is accepted less often. This tradeoff determines the equilibrium choice of the offer given the agreement rule used by the group.

To illustrate our results it is useful to consider the following example. An owner-manager seeks to restructure his firm's outstanding debt by offering creditors a share in the firm. If they decline the offer, creditors will liquidate the firm and obtain $100, and the owner-manager will receive nothing. The value of the firm as a going concern is uncertain: it is either $100 or $200, with *ex ante* equal probability. Each agent possesses private and partially informative information about the relative likelihood of the two valuations.

Suppose first that the creditors are using a majority rule. The recent "strategic voting" literature on how agents vote when in possession of private information (Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1997) has established that as the number of voters grows large the aggregate decision asymptotes to the decision that would have been made under full information. In the current paper we extend this literature to the case in which the issue being voted over is itself endogenous. For the case of majority voting rules, we show (Section 4) that the owner-manager's choice boils down to the following: either he can offer creditors 1/2 of the firm, so that they accept whenever the true going concern of the firm is $200; or he can offer creditors all the firm, and gain acceptance all the time. Clearly the former is the more attractive option. In equilibrium, then, the owner-manager offers 1/2 the firm. If the true going concern is $200 the creditors accept, and receive a payoff of $100; while if the true going concern is $100, creditors reject the offer, and obtain $100 in liquidation.

This outcome contrasts sharply with that which arises when the unanimous agreement of creditors is required for acceptance. As previous authors (see Feddersen and Pesendorfer 1998) have shown, information aggregation fails when a unanimity agreement rule is used. That is, creditors will reject some offers that (under full information) deliver more than the liquidation value, but accept others that deliver less. Roughly speaking, whether or not creditors are better off employing unanimity rule instead of majority rule depends on whether the owner-manager internalizes these errors or exploits them.

For the parameter values given above, the former dominates. As we show (see Section 5) the errors made under unanimity are not entirely random. In particular, the creditors will always reject an offer of one half of the firm. That is, for low offers there are only mistaken rejections, and no mistaken acceptances. However, as the offer increases (and therefore becomes more attractive to the creditors), the nature of the errors shifts from mistaken rejections of good offers to mistaken acceptances of bad offers. This gives the owner-manager the incentive to make an offer strictly better than half the firm.

Do the creditors actually appreciate the higher offer they receive when using the unanimity rule? After all, the offer is higher than the one received under a majority rule precisely because they make mistakes. In short, it is not obvious whether higher offers actually translate into higher payoffs. We show that they do. Consequently, it follows that the creditors' equilibrium payoff under the unanimity rule strictly exceeds the amount they obtain under majority vote.

In the above example, the owner-manager makes a relatively low offer (half the firm) when facing a majority rule. Creditors obtain more using the unanimity rule because it engenders mistakes, and these mistakes in turn prevent the owner-manager from making a low offer. However, under only slightly different circumstances the owner-manager makes a high offer against a majority rule. In such circumstances, the mistakes that arise under unanimity rule have a cost, as follows.

Suppose now that the firm is worth either \$150 or \$200 as a going concern. When facing a majority rule the owner-manager must choose between offering 1/2 the firm and gaining acceptance only in the latter case, and offering 2/3 of the firm and gaining acceptance always. It is easily seen that he prefers the latter strategy. In this situation, the owner-manager is able to exploit the errors that arise when creditors use the unanimity rule. As we show (Lemma 8) there exists an offer strictly less than 2/3 that the creditors will always accept when using the unanimity rule. That is, just as all errors in response to an offer of 1/2 take the form of mistaken rejections, all errors in response to the offer 2/3 take the form of mistaken acceptances.

Our main results establish and generalize the claims we have made in the above example. The choice of voting rule only has an effect on equilibrium outcomes if group members are in some way heterogeneous. There are two possibilities. First, they may have different information with respect to the relative desirability of the offer compared to status quo. Second, their intrinsic preferences over outcomes may be diverse even conditional on full information. The model we consider allows for both types of heterogeneity, and contains pure private values and pure common values frameworks as special cases. The main assumption we make is that the proposer and the group members have diametrically opposing preferences: offers which one side prefers are disliked by the other side. This assumption is clearly satisfied for many common bargaining situations.

For this fairly general set of preferences, we are able to evaluate the asymptotic equilibrium payoffs arising when the group uses a majority rule; and to asmptotically bound the equilibrium payoffs for the unananimity rule. Given economic fundamentals, these results are enough to rank alternate agreement rules from the perspective of the two sides. In cases in which group members' heterogeneity derives primarily from their distinct information (that is, we are close to common values) we are able to go further, and give a succint condition for when the group is (and is not) better off using the unananimity rule. As illustrated by our example above, the key determinant is whether, when facing a (hypothetically) fully informed group, the proposer would prefer to make a low offer that is only sometimes accepted, or a high offer that is always accepted.

Inevitably our analysis neglects some important issues. We focus almost exclusively on equilibrium payoffs as the group size grows large. The chief reason for this focus is that it allows us to establish our results with fewer assumptions on preferences and the distributional properties of agents' information. Numerical simulations suggest that the group size needed for our asymptotic results to apply is not large — in many cases the equilibrium with ten agents is very close to the limiting equilibrium.

Related, we ignore the possibility of communication between agents prior to the vote.

To some extent this is consistent with our focus on large groups. Moreover, it is worth pointing out that since agents' preferences include a private values component, the extent to which they are able to communicate their private information to each other may be limited. As a result, there may be some private information in addition to the public signal that is generated through communication. In general, our results would be qualitatively unchanged if in addition to observing their own private signals, agents also have access to a public signal.[1]

Finally, we take as given the information possessed by group members. As noted above, on a technical level our analysis constitutes an extension of a strategic voting game to allow for the endogeneity of the issue being voted over to the voting rule. Other authors have extended this same basic environment to allow for costly information acquisition,[2] as well as pre-vote communication (see the references cited in footnote 1). We leave the integration of these distinct and individually important extensions for future work.

Related literature

There is a considerable literature in recent years on multilateral bargaining, in which more than two agents must agree on the division of a pie: see Baron and Ferejohn (1989), Merlo and Wilson (1989), Eraslan (2002), Eraslan and Merlo (2002), Cai (2000), Kalandrakis (2004), Eraslan and McLennan (2005). However, in many negotiations the proposals must treat all members of some group equally, either for technological reasons (e.g., the building of a bridge), or for institutional/legal reasons (e.g., wage determination, debt restructuring). The literature analysing this important class of bargaining problems is much smaller. Banks and Duggan (2001) establish equilibrium existence and core equivalence, while Cho and Duggan (2003) and Cardona and Ponsati (2005) establish uniqueness. Closest to us are Manzini and Mariotti (2005), who consider a bargaining game between a group and a central agent, and compare the effect of different agreement rules. All the above papers

[1] For analysis of communication prior to decision making, see Coughlan (2000), Austen-Smith and Feddersen (2002), Doraszelski *et al* (2003), and Gerardi and Yariv (2005).

[2] See Persico (2004), Martinelli (2005), and Yariv (2004).

are deterministic complete information models. As such, informational issues do not arise. Moreover, since agreement is always reached, there is no risk of breakdown of agreement from having a "tougher" bargaining stance. In contrast, the possibility of failing to agree to a Pareto improving proposal is central to our analysis and results. Finally, Chae and Moulin (2004) provide a family of solutions to group bargaining from an axiomatic viewpoint. Elbittar *et al* (2004) provide experimental evidence that the choice of voting rule used by a group in bargaining affects outcomes.

Our work is also related to the literature in two-sided asymmetric information bargaining. For the most part, the literature has focused on private values case (see Kennan and Wilson 1993 for a review.) Exceptions are Samuelson (1984), Evans (1989), Vincent (1989), Chatterjee and Samuelson (1987), and Schweizer (1989). These papers are all bilateral bargaining games, and therefore are not equipped to answer key question we after.

On the technical side, our work is closely related with the growing literature on strategic voting — see especially Feddersen and Pesendorfer (1997, 1997, 1998). Our paper contributes to this literature by endogenizing the agenda to be voted upon.

Paper outline

Section 2 describes the model. Section 3 analyzes some general properties of the voting stage. Section 4 characterizes the equilibrium outcomes of the bargaining game when the group uses a majority rule. Section 5 conducts the same exercise when the group adopts unanimity rule. Section 6 compares outcomes from different rules. Section 7 concludes. All proofs are in Appendix A.

2 Model

1. The coalition of responders fixes a decision making process. As discussed, we restrict attention to voting rules: a proposal is accepted and implemented if and only if some fraction α or more of responders vote to accept. That is, the decision rule is completely indexed by the parameter $\alpha \in [0, 1]$. Common examples include the simple majority

rule ($\alpha = \frac{1}{2} + \frac{1}{n}$ if n is even, $\alpha = \frac{1}{2} + \frac{1}{2n}$ if n is odd), the supermajority rule (α chosen so that $n\alpha$ is the smallest integer above $\frac{2n}{3}$), and the unanimity rule, $\alpha = 1$.

2. Each agent $i \in \{0, 1, \ldots, n\}$ privately observes a random variable $\sigma_i \in [\underline{\sigma}, \bar{\sigma}]$. As we detail below, the realization of σ_i affects agent i's preferences and/or information.

3. The proposer selects a proposal $x \in [0, 1]$.

4. Responders $\{1, \ldots, n\}$ simultaneously cast ballots to accept or reject the proposal.

5. If $n\alpha$ or more responders vote to accept the proposal,[3] it is implemented. Otherwise, the status quo prevails.

Preferences

Agent i's relative preferences over the proposal x and the status quo are determined by σ_i, and also by an unobserved state variable $\omega \in \{L, H\}$. We write responder i's utility associated with offer x as $U^\omega(x, \sigma_i, \lambda)$, where $\lambda \in [0, 1]$ is a parameter that determines the relative importance of ω and σ_i. In particular, we assume that $U^\omega(x, \sigma_i, \lambda)$ is independent of σ_i at $\lambda = 0$ and that $U^L(\cdot, \cdot, \lambda) \equiv U^H(\cdot, \cdot, \lambda)$ when $\lambda = 1$. Likewise, we write $\bar{U}^\omega(\sigma_i, \lambda)$ for responder i's utility under the status quo, and make parallel assumptions for $\lambda = 0, 1$. Note that our framework includes pure *common values* ($\lambda = 0$) and pure *private values* ($\lambda = 1$) as special cases. A key object in our analysis is the utility of a responder from the proposal above and beyond the status quo. Accordingly, we define

$$\Delta^\omega(x, \sigma_i, \lambda) \equiv U^\omega(x, \sigma_i, \lambda) - \bar{U}^\omega(\sigma_i, \lambda).$$

Similarly, we write the proposer's utility from having his offer accepted as $V^\omega(x, \sigma_0)$, and his utility under the status quo as $\bar{V}^\omega(\sigma_0)$. Note that we do not require the relative

[3]Throughout, we ignore the issue of whether or not $n\alpha$ were an integer. This issue could easily be handled formally by replacing $n\alpha$ with $[n\alpha]$ everywhere, where $[n\alpha]$ denotes the smallest integer weakly greater than $n\alpha$. Since this formality has no impact on our results, we prefer to avoid the extra notation and instead proceed as if $n\alpha$ is an integer.

weight of ω and σ_0 in determining the proposer's preferences to match the relative weight (given by λ) of ω and σ_i in determining responder i's preferences.

For all preferences $\lambda < 1$, the realization of σ_i provides responder i with useful (albeit noisy) information about the unobserved state variable ω. We assume that the random variables $\{\sigma_i : i = 0, 1, \ldots, n\}$ are independent conditional on ω, and that except for σ_0 (which is observed by the proposer) are identically distributed. Let $F(\cdot|\omega)$ and $F_0(\cdot|\omega)$ denote the distribution functions for the responders and proposer respectively. We assume that both distributions have associated continuous density functions, which we write $f(\cdot|\omega)$ and $f_0(\cdot|\omega)$. We assume that (i) the realization of σ_i is informative about ω, in the sense that the monotone likelihood ratio property (MLRP) holds strictly;[4] but (ii) no realization is perfectly informative, i.e., $\frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L)} > 0$ and $\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)} < \infty$, with similar inequalities for f_0.

Equilibrium

We examine the pure strategy[5] sequential equilibria of the endogenous offer voting game just described. Let $\pi_n^*(\cdot; \lambda, \alpha) : [\underline{\sigma}, \bar{\sigma}] \to [0, 1]$ denote the proposer's offer strategy for the game with n responders using voting rule α and preference parameter λ. (As is standard in the voting literature, we restrict attention to equilibria in which the *ex ante* identical responders behave symmetrically.[6])

Responders are potentially able to infer information about the proposer's observation of σ_0 from his offer, and thus information about the state variable ω. Since only the latter affects responders' preferences, we focus directly on the beliefs about ω after observing an

[4]That is, $\frac{f(\sigma|H)}{f(\sigma|L)}$ and $\frac{f_0(\sigma|H)}{f_0(\sigma|L)}$ are strictly increasing in σ.

[5]In much of the literature concerning voting by differentially informed individuals, voters are assumed to observe binary signals. In such settings non-trivial pure strategy equilibria do not exist. However, in our model voters observe continuous signals. As such, the restriction that voters follow pure strategy equilibria is of no consequence. Moreover, our focus on pure strategy behavior of the proposer is solely for notational convenience: our main results would hold if the proposer were allowed to follow mixed strategies. See Duggan and Martinelli (2001) and Yılmaz (1999).

[6]Duggan and Martinelli (2001) gives conditions under which the symmetric voting equilibrium is the unique equilibrium.

offer x. Let $\beta_n(x;\lambda,\alpha)$ denote the responders' belief that $\omega = H$ after observing offer x in the game with n responders using voting rule α and preference parameter λ.

A symmetric equilibrium of the game is a proposer offer strategy $\pi_n^*(\cdot;\lambda,\alpha)$, a set of responder beliefs $\beta_n(\cdot;\lambda,\alpha)$ and a responder voting strategy $[\underline{\sigma},\bar{\sigma}] \to \{\text{accept,reject}\}$ such that the proposer's offer strategy is a best response to the responders' (identical) strategies; and each responder's strategy maximizes his expected payoff given that all other responders use the same strategy, and his beliefs are $\beta_n(\cdot;\lambda,\alpha)$; and the beliefs themselves are consistent.

At a minimum, belief consistency requires that having received an offer x, responders are not more (respectively, less) confident that the state is H than the proposer himself is after he sees the most (respectively, least) pro-H signal $\sigma_0 = \bar{\sigma}$ (respectively, $\sigma_0 = \underline{\sigma}$). That is, for all offers x,

$$\frac{\beta_n(x;\lambda,\alpha)}{1-\beta_n(x;\lambda,\alpha)} \in \left[\frac{f_0(\underline{\sigma}|H)}{f_0(\underline{\sigma}|L)}\frac{\Pr(H)}{\Pr(L)}, \frac{f_0(\bar{\sigma}|H)}{f_0(\bar{\sigma}|L)}\frac{\Pr(H)}{\Pr(L)}\right]. \tag{1}$$

Interpretations

Possible interpretations of the game include the following:

1. A debtor offers an equity stake in his firm in exchange for the retirement of debt held by n creditors. He offers a total share x of the firm. If the creditors reject the offer the firm is liquidated. Let $\frac{1}{n}U^\omega(x,\sigma_i,\lambda)$ be the value of an x/n share to each creditor, $\frac{1}{n}\bar{U}^\omega(\sigma_i,\lambda)$ be the value of receiving $1/n$ of the liquidation value,[7] $V^\omega(x,\sigma_0)$ be the debtor's valuation of the remaining $1-x$ share if his offer is accepted, and $\bar{V}^\omega(\sigma_0)$ his payoff in liquidation.

2. An employer is in wage negotiations with workers. He offers a wage x, which worker i values at $U^\omega(x,\sigma_i,\lambda)$. If the offer is rejected, workers strike: $\bar{U}^\omega(\sigma_i,\lambda)$ is worker

[7]Note that these preferences are isomorphic under any monotone transformation, and so in particular to multiplication by n.

i's expected payoff from the strike. The firm's total profits if the offer is accepted are $nV^{\omega}(x, \sigma_0)$, and its expected total profits if a strike ensues are $n\bar{V}^{\omega}(\sigma_0)$.

3. Legislators are either left- or rightwing. Likewise, policies are either left- or rightwing. A coalition of all rightwing legislators forms and proposes a policy x. All members of the coalition are in favor; the policy is only adopted if a sufficient fraction of the leftwing legislators also vote in favor. Leftwing legislators prefer more leftwing policies, which are indexed by higher x; rightwing legislators have the opposite preferences.

Assumptions

We make the following assumptions:

Assumption 1 Δ^{ω}, *V and $\bar{V}$ are continuously differentiable in their arguments.*

Assumption 2 $\Delta^H \geq \Delta^L$ *and* Δ^{ω} *is increasing in* σ_i*; both relations are strict for* $x > 0$.

Assumption 3 *For all* λ, $\Delta^H(0, \bar{\sigma}, \lambda) < 0$ *and* $\Delta^H(1, \bar{\sigma}, \lambda) > 0$.

Assumption 4 *For all* x, $V^{\omega}(x, \sigma_0) - \bar{V}^{\omega}(\sigma_0) \geq 0$ *for* $\omega = L, H$ *and all* σ_0.

Assumption 5 Δ^{ω} *is strictly increasing in* x*, with the derivative bounded uniformly away from* 0*. Likewise,* V *is strictly decreasing in* x*, with the derivative bounded uniformly away from* 0.

Assumption 1 is entirely standard. For future reference, observe that $|\Delta^{\omega}|$ is bounded above since Δ^{ω} is continuous in all three of its arguments and has compact domain. Assumption 2 says responders are more pro-acceptance when $\omega = H$ than $\omega = L$, and when the responder-specific variable σ_i is higher. Since higher values of σ_i are more likely when $\omega = H$ (this is MLRP, see above), the content of Assumption 2 (beyond being a simple normalization) is that the "private" and "common" components of responder utility act in the same direction.

Assumption 3 says that the responders regard the worst offer ($x = 0$) as worthless, i.e., they prefer the status quo. On the other hand, there are some offers which the responders view as worthwhile under some conditions — in particular, responder i prefers the best offer ($x = 1$) to the status quo when $\omega = H$ and $\sigma_i = \bar{\sigma}$.

Assumption 4 says that the proposer strongly dislikes the status quo relative to the range of possible alternatives: regardless of the state, he would prefer to have any proposal $x \in [0, 1]$ implemented.[8]

Finally, Assumption 5 says that the proposer and responders have diametrically opposing preferences: higher x makes the responders more pro-agreement, but reduces the proposer's payoff if his proposal is accepted. Moreover, changes in the proposal x always have a non-negligible effect on preferences.

3 The voting stage

Fix a preference parameter λ and a number of responders n, and consider the situation faced by responders after the proposer offers x. Having observed the offer, each responder i attaches a subjective probability $b = \beta_n (x; \lambda, \alpha)$ to the state variable ω being H; and has privately observed σ_i. Taking the strategies of other responders as given, let PIV denote the event that his vote is pivotal, and ACC and REJ respectively the events in which the offer is accepted and rejected regardless of agent i's vote. His payoff from voting against the proposal is

$$E_b \left[\bar{U}^\omega (x, \sigma_i, \lambda) | PIV, \sigma_i\right] \Pr_b (PIV|\sigma_i) + E_b \left[\bar{U}^\omega (x, \sigma_i, \lambda) | REJ, \sigma_i\right] \Pr_b (REJ|\sigma_i)$$
$$+ E_b \left[U^\omega (x, \sigma_i, \lambda) | ACC, \sigma_i\right] \Pr_b (ACC|\sigma_i)$$

[8] In general, one can clearly think of a broader range of proposals $[0, \infty)$, but with the proposer prefering the status quo to offers $x \in (0, \infty)$. The content of Assumption 4 is that $x = 1$ is the highest proposal the proposer is prepared to make for *any* pair (ω, σ_0). For instance, in our debt renegotation example, a debtor (the proposer) would prefer being left with any fraction $1 - x$ of the firm to liquidation, if (as is typical) in the latter case he is left with nothing.

where $\Pr_b$ and E_b denote the subjective probability and expectation given b. Likewise, agent i's payoff from voting for the proposal is

$$E_b\left[U^\omega\left(x,\sigma_i,\lambda\right)|PIV,\sigma_i\right]\Pr_b\left(PIV|\sigma_i\right)+E_b\left[\bar{U}^\omega\left(x,\sigma_i,\lambda\right)|REJ,\sigma_i\right]\Pr_b\left(REJ|\sigma_i\right)$$
$$+E_b\left[U^\omega\left(x,\sigma_i,\lambda\right)|ACC,\sigma_i\right]\Pr_b\left(ACC|\sigma_i\right)$$

Thus agent i votes to accept proposal x if and only if

$$E_b\left[U^\omega\left(x,\sigma_i,\lambda\right)|PIV,\sigma_i\right]\geq E_b\left[\bar{U}^\omega\left(x,\sigma_i,\lambda\right)|PIV,\sigma_i\right]. \tag{2}$$

Condition (2) says that in choosing whether to accept or reject an offer, a responder i focuses only on the states in which he is pivotal. Even though he does not observe σ_j $(i\neq j)$, and does not know whether or not he is actually pivotal, in casting his vote he considers only the payoffs in events in which he is pivotal, and takes into account any information thus revealed.

Since the random variables σ_i are independent conditional on ω,

$$\begin{aligned}\Pr_b\left(\omega|PIV,\sigma_i\right) &= \frac{\Pr_b\left(\omega,PIV,\sigma_i\right)}{\Pr_b\left(PIV,\sigma_i\right)}=\frac{\Pr\left(PIV,\sigma_i|\omega\right)\Pr_b\left(\omega\right)}{\Pr_b\left(PIV,\sigma_i\right)}\\ &= \frac{\Pr\left(PIV|\omega\right)\Pr\left(\sigma_i|\omega\right)\Pr_b\left(\omega\right)}{\Pr_b\left(PIV,\sigma_i\right)}.\end{aligned} \tag{3}$$

Substituting (3) into inequality (2), and noting that $\Pr_b(H)=b=1-\Pr_b(L)$, agent i votes to accept proposal x after observing signal σ_i if and only if

$$\Delta^H\left(x,\sigma_i,\lambda\right)\Pr\left(PIV|H\right)f\left(\sigma_i|H\right)b+\Delta^L\left(x,\sigma_i,\lambda\right)\Pr\left(PIV|L\right)f\left(\sigma_i|L\right)(1-b)\geq 0. \tag{4}$$

By MLRP, it is immediate from (4) that each responder follows a cutoff strategy, in the sense of voting to accept whenever his/her signal exceeds some critical level:

Lemma 1 ***(Cutoff rules)***

In any equilibrium, for each responder i there must exist a cutoff signal $\sigma_i^*(x)\in[\underline{\sigma},\bar{\sigma}]$ such that agent i votes to accept proposal x if his signal is more positive than $\sigma_i^*(x)$, i.e., $\sigma_i>\sigma_i^*(x)$; and votes to reject the proposal if his signal is more negative, i.e., $\sigma_i<\sigma_i^*(x)$.

As noted, throughout we focus on symmetric equilibria in which the *ex ante* identical responders follow the same voting strategy. In light of Lemma 1, let $\sigma_n^*(x,b,\lambda,\alpha) \in [\underline{\sigma},\bar{\sigma}]$ denote the common cutoff signal when responders attach a probability b to $\omega = H$, and the preferences parameter and voting rule are λ and α respectively.[9] For clarity of exposition, we will suppress the arguments n, x, b, λ and α unless needed.

Evaluating explicitly, the probability that a responder is pivotal is given by

$$\Pr(PIV|\omega) = \binom{n-1}{n\alpha-1}(1-F(\sigma^*(x)|\omega))^{n\alpha-1}F(\sigma^*(x)|\omega)^{n-n\alpha}. \tag{5}$$

The acceptance condition (4) then rewrites to:

$$\begin{aligned}&\Delta^H(x,\sigma_i,\lambda)(1-F(\sigma^*(x)|H))^{n\alpha-1}F(\sigma^*(x)|H)^{n-n\alpha}f(\sigma_i|H)\,b\\&+\Delta^L(x,\sigma_i,\lambda)(1-F(\sigma^*(x)|L))^{n\alpha-1}F(\sigma^*(x)|L)^{n-n\alpha}f(\sigma_i|L)(1-b)\geq 0\end{aligned} \tag{6}$$

If there exists a $\sigma^* \in [\underline{\sigma},\bar{\sigma}]$ such that responder i is indifferent between accepting and rejecting the offer x exactly when he observes the signal $\sigma_i = \sigma^*$, then the equilibrium can be said to be a *responsive equilibrium*. That is, a responsive equilibrium exists whenever the equation

$$-\frac{\Delta^H(x,\sigma_i,\lambda)}{\Delta^L(x,\sigma_i,\lambda)}\frac{b}{1-b}\frac{f(\sigma^*|H)}{f(\sigma^*|L)}\frac{1-F(\sigma^*|L)}{1-F(\sigma^*|H)} = \left(\frac{(1-F(\sigma^*|L))^{\alpha}F(\sigma^*|L)^{1-\alpha}}{(1-F(\sigma^*|H))^{\alpha}F(\sigma^*|H)^{1-\alpha}}\right)^n \tag{7}$$

has a solution $\sigma^* \in [\underline{\sigma},\bar{\sigma}]$. Notationally, we represent a responsive equilibrium by its corresponding cutoff value $\sigma^* \in [\underline{\sigma},\bar{\sigma}]$.

We turn now to existence and uniqueness of responsive equilibria. For a given subjective probability b that $\omega = H$, it is useful to define the function

$$Z(x,\sigma;n,\alpha,\lambda,b) \equiv \Delta^H(x,\sigma)\frac{b}{1-b}\frac{f(\sigma|H)}{f(\sigma|L)}\left(\frac{F(\sigma|H)}{F(\sigma|L)}\right)^{n-n\alpha}\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n\alpha-1}+\Delta^L(x,\sigma)$$

When $Z(x,\sigma)$ is strictly positive (negative), then if all but one of the responders are using a cutoff strategy σ, and the remaining responder i observes $\sigma_i = \sigma$, then that responder is strictly better off voting to accept (reject) the proposal x. Similarly, if $Z(x,\sigma) = 0$ then there is a responsive equilibrium in which all responders use the cutoff strategy σ.

[9] As we show below, there exists a unique cutoff signal.

By the Theorem of the Maximum, $\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} Z(x,\sigma)$ and $\min_{\sigma\in[\underline{\sigma},\bar{\sigma}]} Z(x,\sigma)$ are both continuous in x. So we can define

$$\underline{x}_n(\alpha,\lambda,b) = \begin{cases} \min\{x|\max_\sigma Z(x,\sigma)\geq 0\} & \text{if } \{x|\max_\sigma Z(x,\sigma)\geq 0\}\neq\varnothing \\ 1 & \text{otherwise} \end{cases} \quad (8)$$

$$\bar{x}_n(\alpha,\lambda,b) = \begin{cases} \max\{x|\min_\sigma Z(x,\sigma)\leq 0\} & \text{if } \{x|\min_\sigma Z(x,\sigma)\leq 0\}\neq\varnothing \\ 0 & \text{otherwise} \end{cases} . \quad (9)$$

Observe that $\underline{x}_n(\alpha,\lambda,b) > 0$ since by Assumptions 2 and 3 $Z(0,\sigma) < 0$ for all σ.

Lemma 2 ***(Existence and uniqueness)***

Fix a set of beliefs $b\in(0,1)$, a voting rule α and preferences λ. Then:

1. *For any n, a responsive equilibrium $\sigma^*(x)\in[\underline{\sigma},\bar{\sigma}]$ exists if and only if $x\in[\underline{x}_n,\bar{x}_n]$. When a responsive equilibrium exists it is the unique symmetric responsive equilibrium.*

2. *The equilibrium cutoff $\sigma^*(x)$ is decreasing and continuously differentiable over $(\underline{x}_n,\bar{x}_n)$, and $\sigma^*(x)\to\bar{\sigma}$ as $x\to\underline{x}_n$ and $\sigma^*(x)\to\underline{\sigma}$ as $x\to\bar{x}_n$.*

3. *For $\alpha<1$, if $\Delta^H(x,\bar{\sigma})>0>\Delta^L(x,\underline{\sigma})$, then there exists an N such that $x\in(\underline{x}_n,\bar{x}_n)$ for $n\geq N$.*

4. *For $\alpha=1$, if $\Delta^H(x,\bar{\sigma})>0\geq\Delta^H(x,\underline{\sigma})$, then there exists an N such that $x\in(\underline{x}_n,\bar{x}_n)$ for $n\geq N$.*

In addition to responsive equilibria, non-responsive equilibria also exist. Specifically, for any $\alpha>0$ there is an equilibrium in which each agent votes to reject regardless of his signal, i.e., $\sigma^*=\bar{\sigma}$. Likewise, for any $\alpha<1$ there is an equilibrium in which each responder votes to accept regardless of his signal, i.e., $\sigma^*=\underline{\sigma}$.

Lemma 3 ***(Rejection equilibrium)***

Fix beliefs b, a voting rule $\alpha>\frac{1}{2}+\frac{1}{2n}$ and preferences λ. Let $(\underline{x}_n,\bar{x}_n)$ be the interval defined in Lemma 2. Then if $x\leq\underline{x}_n$ the only trembling-hand perfect equilibrium is the non-responsive equilibrium in which each responder always rejects.

Given Lemmas 2 and 3, we adopt the following equilibrium selection rule. Take $(\underline{x}_n, \bar{x}_n)$ the interval defined by Lemma 2. When $x \leq \underline{x}_n$ the only trembling-hand perfect equilibrium is the (non-responsive) rejection equilibrium. When $x \in (\underline{x}_n, \bar{x}_n)$ a responsive equilibrium exists, and we follow the literature in assuming it is played. Finally, when $x \geq \bar{x}_n$ we assume that (non-responsive) acceptance equilibrium is played. Doing so ensures continuity of the cutoff signal σ^* in the offer x, and the acceptance equilibrium is certainly trembling-hand perfect. Moreover, although the rejection equilibrium is also potentially trembling-hand perfect, in this latter case the trembles required do not satisfy the cutoff rule property of Lemma 1.[10]

How does the equilibrium respond to changes in responders' beliefs? The following is a straightforward corollary of Lemma 2:

Corollary 1 ***(Change in beliefs)***
Fix n, λ, α, and suppose that a responsive equilibrium exists for an offer x and beliefs b. Then for any beliefs $b' > b$, either a responsive equilibrium exists for offer x, or else the equilibrium is the acceptance equilibrium.

The heart of our analysis concerns the effect of the voting rule on the proposer's offer x, and in turn the effect on responder and proposer payoffs. Notationally, we write $\Pi_n^P(x, \lambda, \alpha, \sigma_0, b)$ for the proposer's expected payoff from offer x under voting rule α, responder preferences λ, proposer signal σ_0, and responder beliefs b; and $\Pi_n^V(x, \lambda, \alpha, b)$ for the responder's expected payoff from offer x under voting rule α, responder preferences λ, and responder beliefs b. Before turning to the details, we note a second straightforward corollary of Lemma 2:

Corollary 2 ***(Continuity and differentiability of payoffs)***
Fix a set of responder beliefs b. Then $\Pi_n^V(x, \lambda, \alpha, b)$ and $\Pi_n^P(x, \lambda, \alpha, \sigma_0, b)$ are continuous

[10]Slightly more formally, if tremble strategies were required to satisfy the mild monotonicity restriction that voting to accept is weakly more likely after a higher signal, then the acceptance equilibrium would be the *only* trembling-hand perfect equilibrium when $x \geq \bar{x}_n$.

functions of the offer x, and are differentiable except at the boundaries of the responsive equilibrium range, $\underline{x}_n(\alpha, \lambda, b)$ and $\bar{x}_n(\alpha, \lambda, b)$.

4 Majority voting

We first characterize the equilibrium payoffs for any non-unanimity voting rule $\alpha < 1$. We will need a couple pieces notation. First, for $\omega = L, H$, define $\sigma_\omega(\alpha)$ implicitly by

$$1 - F(\sigma_\omega(\alpha) | \omega) = \alpha.$$

Then define $x_\omega(\lambda, \alpha)$ implicitly by

$$\Delta^\omega(x_\omega(\lambda, \alpha), \sigma_\omega(\alpha), \lambda) = 0.$$

By assumption, $\Delta^\omega(x, \sigma_\omega, \lambda)$ is strictly negative at $x = 0$, and is strictly increasing in x. Consequently $x_\omega(\lambda, \alpha)$ is well-defined unless $\Delta^\omega(x, \sigma_\omega, \lambda) < 0$ at $x = 1$. For this case, we write $x_\omega(\lambda, \alpha) = \infty$.

Note that $\sigma_\omega(\alpha)$ is strictly decreasing. Likewise, $x_\omega(\lambda, \alpha)$ is strictly increasing in α, except for at the common values extreme $\lambda = 0$, in which case it is constant in α. Moreover, by Assumption 3, $x_H(\lambda) \neq \infty$ for all λ sufficiently small.

Our first result extends Feddersen and Pesendorfer's (1997) finding that under majority rule, the aggregate decision of responders matches that which would be obtained if the responders had full information. The differences relative to their analysis are (i) signals are continuous in our model,[11] and (ii) the proposal being voted over varies as the number of responders grows large. Because there is no reason to require the proposal to have a well-defined limit, we state our result in terms of the limits infimum and supremum.

Formally, let $A_n(x, b, \lambda, \alpha)$ denote the event in which the offer x:

Lemma 4 ***(Acceptance probabilities under a majority voting rule)***
Suppose a majority voting rule $\alpha < 1$ is in effect. Take any $\lambda \in [0, 1]$, and consider a

[11] Again Duggan and Martinelli (2001) and Yılmaz (1999).

sequence of offers x_n. *If* $\liminf x_n > x_\omega(\lambda,\alpha)$ *then* $\Pr(A_n(x_n)|\omega) \to 1$ *and if* $\limsup x_n < x_\omega(\lambda,\alpha)$ *then* $\Pr(A_n(x_n)|\omega) \to 0$.

Our next result characterizes how the proposer responds to the voting behavior described in Lemma 4. Since when n is large the aggregate behavior of responders matches that in which the state variable ω is directly observed, the basic choice of the proposer boils down to the following. Either he can offer x_H and attain acceptance when $\omega = H$ but not when $\omega = L$; or he can offer $x_L > x_H$ and attain acceptance both when $\omega = L$ and H. For any given realization of σ_0, the proposer's choice thus clearly depends on the sign of

$$W(\sigma_0;\lambda,\alpha) \equiv \left(\Pr(H|\sigma_0)\,V^H(x_H,\sigma_0) + \Pr(L|\sigma_0)\,\bar{V}^L(\sigma_0)\right) - E\left[V^\omega(x_L,\sigma_0)\,|\sigma_0\right]. \quad (10)$$

Note that the condition $W(\sigma_0) > 0$ is equivalent to

$$\Pr(H|\sigma_0)\left(V^H(x_H,\sigma_0) - V^H(x_L,\sigma_0)\right) < \Pr(L|\sigma_0)\left(V^L(x_L,\sigma_0) - \bar{V}^L(\sigma_0)\right). \quad (11)$$

When $W(\sigma_0) > 0$ then the proposer's payoff from having $x_H(\lambda)$ accepted with probability $\Pr(H|\sigma_0)$ is above his payoff from having $x_L(\lambda)$ accepted all the time. As such, his equilibrium offer must converge to $x_H(\lambda)$ as $n \to \infty$. Likewise, if $W(\sigma_0) < 0$, then his equilibrium offer converges to $x_L(\lambda)$. In the case that $x_H(\lambda)$ but not $x_L(\lambda,\alpha)$ is feasible (i.e., $x_L(\lambda,\alpha) = \infty$), again the proposer's offer converges to $x_H(\lambda)$. Finally, if neither $x_H(\lambda)$ nor $x_L(\lambda)$ is feasible, then for a large number of responders the proposer's offer is irrelevant, since it is always rejected.

The formal result is as follows. We take care to establish uniform convergence:

Lemma 5 ***(The proposer's offer under a majority voting rule)***
Suppose a majority voting rule $\alpha < 1$ *is in effect. Then:*

1. If $x_L(\lambda,\alpha) \neq \infty \neq x_H(\lambda,\alpha)$, then for any $\varepsilon, \delta > 0$ there exists an $N(\varepsilon,\delta)$ such that

   (a) $|\pi_n^*(\sigma_0;\lambda,\alpha) - x_H(\lambda,\alpha)| < \delta$ and $|\Pr(A_n|\sigma_0,H) - 1| < \delta$ whenever $W(\sigma_0) > \varepsilon$ and $n \geq N(\varepsilon,\delta)$.

(b) $|\pi_n^* (\sigma_0; \lambda, \alpha) - x_L (\lambda, \alpha)| < \delta$ and $|\Pr (A_n|\sigma_0) - 1| < \delta$ whenever $W (\sigma_0) < -\varepsilon$ and $n \geq N (\varepsilon, \delta)$.

2. If $x_H (\lambda, \alpha) \neq \infty$ and $x_L (\lambda, \alpha) = \infty$, for any $\delta > 0$ there exists an $N (\delta)$ such that $|\pi_n^* (\sigma_0; \lambda, \alpha) - x_H (\lambda, \alpha)| < \delta$ and $|\Pr (A_n|\sigma_0, H) - 1| < \delta$ for all σ_0.

3. If $x_L (\lambda, \alpha) = x_H (\lambda, \alpha) = \infty$, then for any $\delta > 0$ there exists an $N (\delta)$ such that $\Pr (A_n|\sigma_0, \omega) < \delta$ for all $\sigma_0, \omega = L, H$.

As stated, Lemma 5 does not cover equilibrium behavior when $W (\sigma_0) = 0$. In general, this knife-edge condition will hold only for finitely many realizations of σ_0. In particular, from condition (11) it is clear that $W (\sigma_0) = 0$ at most one value of σ_0 if the proposer's payoffs $V^\omega (x, \sigma_0)$ and $\bar{V}^\omega (\sigma_0)$ are independent of σ_0 — or more generally, if the private values component of proposer payoffs is sufficiently small, i.e., $\left|\frac{\partial}{\partial \sigma_0} V^\omega (x, \sigma_0)\right|$ and $\left|\frac{\partial}{\partial \sigma_0} \bar{V}^\omega (\sigma_0)\right|$ are sufficiently small for all x and σ_0. For the remainder of the paper we make the following very mild assumption:

Assumption 6 *$W (\sigma_0; \lambda, \alpha) = 0$ for at most finitely many values of σ_0 when $x_L (\lambda, \alpha) \neq \infty \neq x_H (\lambda, \alpha)$.*

When $x_H (\lambda, \alpha) \neq \infty$ and $x_L (\lambda, \alpha) = \infty$, we set $W (\sigma_0; \lambda, \alpha) = \infty$. When $x_L (\lambda, \alpha) = x_H (\lambda, \alpha) = \infty$ we set $W (\sigma_0; \lambda, \alpha) = 0$.

From Lemma 5 it is straightforward to establish the limiting payoffs of the proposer and the responders under any majority voting rule. Notationally, we write $\Pi_n^{*P} (\lambda, \alpha)$ and $\Pi_n^{*V} (\lambda, \alpha)$ for the proposer's and responders' expected equilibrium payoffs.

Proposition 1 ***(The proposer's payoff under a majority voting rule)***
Suppose a majority voting rule $\alpha < 1$ is in effect.

$$\begin{aligned} \Pi_n^{*V} (\lambda, \alpha) \quad \rightarrow \quad & E_{\sigma_i, \omega} \left[\bar{U}^\omega (\sigma_i)\right] + \int_{\sigma_0 \ s.t. \ W(\sigma_0)<0} E_{\sigma_i, \omega} \left[\Delta^\omega (x_L, \sigma_i, \lambda) |\sigma_0\right] dF_0 (\sigma_0) \\ & + \int_{\sigma_0 \ s.t. \ W(\sigma_0)>0} \Pr (H|\sigma_0) E_{\sigma_i} \left[\Delta^H (x_H, \sigma_i, \lambda) |H\right] dF_0 (\sigma_0) \end{aligned}$$

$$\begin{aligned}\Pi_n^{*P}(\lambda,\alpha) &\rightarrow \int_{\sigma_0 \ s.t.\ W(\sigma_0)<0} E_\omega\left[V^\omega(x_L,\sigma_0)|\sigma_0\right]dF_0(\sigma_0) \\ &+\int_{\sigma_0 \ s.t.\ W(\sigma_0)>0}\left(\Pr(H|\sigma_0)V^H(x_H,\sigma_0)+\Pr(L|\sigma_0)\bar{V}^L(\sigma_0)\right)dF_0(\sigma_0).\end{aligned}$$

5 The unanimity rule

As we saw in the previous section, when facing a majority vote the proposer will offer something close to $x_H(\lambda,\alpha)$ (respectively, $x_L(\lambda,\alpha) > x_H(\lambda,\alpha)$) after observing a σ_0 such that $W(\sigma_0)$ is positive (negative). In this section, we turn to the proposer's offer when he faces responders who employ a unanimity rule (i.e., $\alpha = 1$).

We will first show that when responders' preferences are sufficiently close to the common values extreme (i.e., λ small enough), then compared to their behavior under a majority voting rule, they accept "low" offers less often but "high" offers more often. Specifically, whereas under a majority voting rule an offer slightly above $x_H(\lambda,\alpha)$ is accepted with probability approaching one conditional on $\omega = H$, under the unanimity rule such an offer is accepted with vanishingly small probability. Conversely, under a majority voting rule an offer slightly below $x_L(\lambda,\alpha)$ is rejected with probability approaching one conditional on $\omega = L$; but under the unanimity rule, it is certain to be accepted.

Based on these observations, intuition suggests that after a realization of σ_0 such that $W(\sigma_0)$ is positive, the proposer will make a higher offer when responders use a unanimity rule: against a majority rule his favorite offer is close to $x_H(\lambda,\alpha)$, but against a unanimity rule, this — and all lower offers — is rejected almost for sure. Conversely, the proposer will actually make a lower offer against a unanimity rule after any σ_0 for which $W(\sigma_0)$ is negative: against a majority rule his favorite offer is close to $x_L(\lambda,\alpha)$, while against a unanimity rule he can assure himself of acceptance with an offer less than $x_L(\lambda,\alpha)$.

Loosely speaking, it then follows that if $W(\sigma_0)$ is positive with high probability, responders are better off in *ex ante* terms by adopting a unanimity agreement rule. Likewise, if

$W(\sigma_0)$ is negative with high probability, responders are *ex ante* better off by adopting a majority agreement rule.

These arguments apply only when preferences are sufficiently close to the common values extreme. Consequently, establishing them formally requires us to take what is essentially a double limit: we must allow the preference parameter λ to approach 0 at the same time as the number of responders grows large. A further complication is that, as under majority rules, there is no reason to suppose *a priori* that either $x_n(\lambda)$ or the corresponding acceptance probability converges in either λ or n.

Formally, we handle these difficulties by stating our results about offers and acceptance probabilities in terms of

$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} x_n(\lambda) - x_\omega(\lambda)$$
$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr\left(A_n\left(x_n\left(\lambda\right), \lambda\right) | \omega\right).$$

These expressions serve to give a lower bound on the offer and acceptance probability as n grows large and λ approaches 0 at the same time. To see this, observe that the lower bound

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr\left(A_n\left(x_n\left(\lambda\right), \lambda\right) | \omega\right)$$

increases as N grows and Λ shrinks, and so by taking the supremum of this expression over Λ and N we characterize the limiting behavior of the lower bound of $\Pr(A_n|\omega)$. Likewise, the expression

$$\inf_{\Lambda \geq \lambda_0, N} \sup_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr\left(A_n\left(x_n\left(\lambda\right), \lambda\right) | \omega\right)$$

captures the limiting behavior of the upper bound of $\Pr(A_n|\omega)$.

Finally, for the case where we set λ_0 to be strictly positive,

$$\begin{aligned}
\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr\left(A_n\left(x_n\left(\lambda\right), \lambda\right) | \omega\right) &= \lim_{N \to \infty} \inf_{n \geq N} \Pr\left(A_n\left(x_n\left(\lambda_0\right), \lambda_0\right) | \omega\right) \\
&= \liminf \Pr\left(A_n\left(x_n\left(\lambda_0\right), \lambda_0\right) | \omega\right)
\end{aligned}$$

and we obtain simply the single limit as $n \to \infty$ for preference parameter λ_0.

Voting under the unanimity rule

Lemma 4 above characterized responders' aggregate response to an arbitrary offer under any majority rule. Lemmas 6 and 7 do the same for the unanimity rule. First, Lemma 6 says that if the proposer's offers stay bounded away from $x_H(\lambda)$ as n grows large, then the probabilities that these offers are accepted likewise stay bounded away from 0.

Lemma 6 ***(Intermediate offers accepted under unanimity)***
Suppose the unanimity rule $\alpha = 1$ is in effect. Take $x_n(\lambda)$ a set of offers. Then for any $\lambda_0 \geq 0$, if

$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} x_n(\lambda) - x_H(\lambda) > 0 \tag{12}$$

then

$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n | H) > 0 \tag{13}$$

At first glance Lemma 6 is surprising: one might have conjectured that agreement is impossible to obtain when the unanimous consent of a large number of responders is required. The reason why agreement is in fact possible is that each individual votes to accept with a probability that approaches one as the number of responders grows large. Such a strategy is individually rational because, given that a large number of other individuals are voting to accept, each responder can conclude that conditional on being pivotal there is strong evidence that the offer is worth accepting.

Informally, Lemma 6 is established by contradiction. Here, we give the argument for a fixed preference parameter λ close to 0. Suppose that, contrary to the claim, the acceptance probability converges to 0. Algebraically, this means that

$$(1 - F(\sigma_n^* | H))^n \to 0.$$

This implies that the equilibrium σ_n^* is either a responsive equilibrium or a rejection equilibrium (since if instead $\sigma_n^* = \underline{\sigma}$, $(1 - F(\sigma_n^* | H))^n = 1$ for any n). Moreover, if σ_n^* approaches $\underline{\sigma}$, it does so at a "slower" rate than n grows large. But these comparative speeds of convergence in turn imply (via l'Hôpital's rule) that as $n \to \infty$,

$$\frac{\left(1 - F\left(\sigma_n^*|H\right)\right)^n}{\left(1 - F\left(\sigma_n^*|L\right)\right)^n} \to \infty.$$

This says that, conditional on being pivotal, each individual can infer that the true realization of ω is almost certainly H. Since by hypothesis the proposer's offer is bounded away from $x_H(\lambda)$, the benefits of accepting the offer when $\omega = H$ are likewise bounded away from 0. As such, each individual i should vote to accept independent of his own observation σ_i. But this contradicts the observation above that σ_n^* is either a responsive or rejection equilibrium.

Lemma 6 complements Lemma 7, and says that if instead the proposer's offer converges to $x_H(\lambda)$ as $n \to \infty$, then likewise the probability that it is accepted converges to zero.

Lemma 7 ***(Low offers rejected under unanimity)***

Suppose the unanimity rule $\alpha = 1$ is in effect. Take $x_n(\lambda)$ a set of offers. Then for any $\lambda_0 \geq 0$:

1. *If*

$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} x_n(\lambda) - x_H(\lambda) \leq 0 \tag{14}$$

then

$$\sup_{\Lambda \geq \lambda_0, N} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n) = 0 \tag{15}$$

2. *If for all $\varepsilon > 0$ there exists an N such that*[12]

$$\inf_{\Lambda \geq \lambda_0} \sup_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} x_n(\lambda) - x_H(\lambda) \leq \varepsilon \tag{16}$$

then

$$\inf_{\Lambda \geq \lambda_0, N} \sup_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n) = 0. \tag{17}$$

[12]The hypothesis (16) of Part 2 of Lemma 7 says, loosely, that as a function of λ the expression $x_n(\lambda) - x_H(\lambda)$ converges uniformly to 0, or else is negative.

As for Lemma 6, we offer a rough proof by contradiction for a fixed preference parameter λ. Suppose that contrary to the claim the acceptance probability stays bounded away from 0. Algebraically this means that the expression

$$\left(1 - F\left(\sigma_n^* | H\right)\right)^n$$

stays bounded away from 0, which in turn implies that either $\sigma_n^* = \underline{\sigma}$, or else that σ_n^* approaches $\underline{\sigma}$ "faster" than n approaches ∞. These comparative speeds of convergence in turn imply (via l'Hôpital's rule) that as $n \to \infty$,

$$\left(1 - F\left(\sigma_n^* | L\right)\right)^n$$

stays bounded away from 0 also. Consequently, even after conditioning on being pivotal each individual i attaches a strictly positive probability to both $\omega = L$ and $\omega = H$. By hypothesis, the proposer's offer approaches $x_H(\lambda)$ as n grows large, and so conditional on seeing $\sigma_i = \sigma_H = \underline{\sigma}$, the benefits from accepting the offer approach zero when $\omega = H$, and are strictly negative when $\omega = L$. But then each individual i should vote to reject the offer over at least some neighborhood of observations of σ_i around $\underline{\sigma}$. But this contradicts the observation above that σ_n^* must approach $\underline{\sigma}$.

Thus far we have concentrated on the aggregate response of responders to offers that are close to $x_H(\lambda)$. We now turn to the opposite extreme of offers close to $x_L(\lambda)$. Recall that under majority voting, the proposer makes an offer that is close to $x_L(\lambda)$ whenever $W(\sigma_0) < 0$, but never makes an offer that is significantly greater.

Our next result shows that when the proposer faces a unanimity rule, the maximum offer he finds it worthwhile to make is lower than $x_L(\lambda)$. Specifically, we show that there is an offer strictly less than $x_L(\lambda)$ that responders accept with certainty if n is sufficiently large.

Lemma 8 ***(An upper bound on offers under unanimity)***
Suppose the unanimity rule $\alpha = 1$ is in effect, and $x_L(\lambda) \neq \infty$ for all λ small enough. Then there exists a $\kappa_x > 0$ and a $\Lambda > 0$ such that the offer $x_L(\lambda) - \kappa_x$ is accepted with certainty for all $\lambda \leq \Lambda$ and all n. As such, $\pi_n^*(\sigma_0; \lambda) \leq x_L(\lambda) - \kappa_x$.

Proposer's offer under unanimity

From Lemmas 6 and 7, offers that converge to $x_H(\lambda)$ are rejected, while offers that remain bounded away are accepted with a probability that stays bounded away from 0. Since the proposer prefers acceptance to rejection, his optimal offers stay bounded away from $x_H(\lambda)$:

Lemma 9 ***(The proposer's offer under a unanimity voting rule)*** *Suppose the unanimity rule $\alpha = 1$ is in effect. Then there exists a $\kappa_x > 0$ such for all $\varepsilon > 0$, there exists an $N(\varepsilon)$ such that for all σ_0,*

$$\sup_{\Lambda \geq \lambda_0} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N(\varepsilon)} \pi_n^* (\sigma_0; \lambda) - x_H(\lambda) \geq \kappa_x - \varepsilon.$$

That is, the proposer's offer is bounded away from $x_H(\lambda)$ when n is sufficiently large and λ is sufficiently small.

Responder payoffs

When σ_0 is such that $W(\sigma_0) > 0$, for any majority rule α responders would receive a higher offer if instead they employ the unanimity rule (provided n is large enough). Although at first glance a higher offer might be thought to trivially improve responder welfare, this is not the case. When responders use a unanimity rule, their aggregate decision is frequently suboptimal *ex post*, in the sense that they reject offers they should accept, and accept offers they should reject. It is conceivable that a higher offer would serve to increase the error rate to such an extent that it actually lowers their welfare.

Our next result explicitly characterizes the effect of a change in the proposer's offer on the responders' payoff when the unanimity rule is in place. It shows that in spite of the above caveat, the net effect is indeed positive.

Lemma 10 *Suppose the unanimity rule $\alpha = 1$ is in effect. Then holding responder beliefs fixed, for $x \neq \underline{x}_n(\alpha, \lambda, b), \bar{x}_n(\alpha, \lambda, b)$,*

$$\begin{aligned} \frac{\partial \Pi_n^V}{\partial x} &\geq b \Pr(A_n(x) | H) E\left[\frac{\partial}{\partial x} \Delta^H(x, \sigma, \lambda) | H, \sigma \geq \sigma^*\right] \\ &\quad + (1-b) \Pr(A_n(x) | L) E\left[\frac{\partial}{\partial x} \Delta^L(x, \sigma, \lambda) | L, \sigma \geq \sigma^*\right]. \end{aligned}$$

We are now ready to state our main conclusion in this section. As we have seen, the proposer's equilibrium offers stay bounded away from $x_H(\lambda)$ when he faces a unanimity rule (Lemma 9). Moreover, when responders employ the unanimity rule, then higher offers improve their welfare — even though they also affect the voting equilibrium (Lemma 10). From these observations, it follows that the responders' equilibrium payoff under the unanimity rule is bounded away from their status quo payoff, $E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right]$.

Proposition 2 ***(A lower bound for the responders' payoff under the unanimity rule)***
There exists a $\gamma > 0$ such that

$$\sup_{N,\Lambda\geq\lambda_0}\inf_{\lambda_0\leq\lambda\leq\Lambda,\lambda>0,n\geq N}\Pi_n^{*V}(\lambda,\alpha=1)-E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i,\lambda)\right]\geq\gamma.$$

Lemma 8 gives an upper bound for the proposer's offer against a unanimity rule. An immediate consequence is the following upper bound for the responders' payoff:

Proposition 3 ***(An upper bound for the responders' payoff under the unanimity rule)***
Suppose the unanimity rule $\alpha = 1$ is in effect, and $x_L(\lambda,\alpha) \neq \infty$ for all λ small enough. Then there exists a γ and $\Lambda > 0$ such that for all $\lambda \leq \Lambda$

$$\Pi_n^{*V}(\lambda,\alpha=1)\leq E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i,\lambda)\right]+E_{\sigma_i,\omega}\left[\Delta^{\omega}(x_L(\lambda),\sigma_i,\lambda)\right]-\gamma.$$

6 Comparing majority and unanimity voting rules

Propositions 1, 2 and 3 characterize the responders' payoff from majority and unanimity voting rules when the number of responders is large. In this section we build on these results to compare the payoffs of responders and the proposer under different voting rules, when responders' payoffs are close to common values ($\lambda \approx 0$).

Responder payoffs

Recall that when responders use a majority rule $\alpha < 1$, the proposer's choice of offer effectively reduces to deciding between offering $x_H(\lambda, \alpha)$ and having it accepted with probability $\Pr(H|\sigma_0)$, and offering $x_L(\lambda, \alpha) > x_H(\lambda, \alpha)$ and securing acceptance all the time. The function $W(\sigma_0; \lambda, \alpha)$ is defined to be the gain (if positive) from pursuing the former strategy over the latter.

Whether the responders *ex ante* prefer to use a majority or unanimity rule depends on the sign of $W(\sigma_0; \lambda, \alpha)$. Observe that at the common values extreme ($\lambda = 0$), $x_\omega(\lambda, \alpha)$ is independent of the voting rule α, and hence $W(\sigma_0; \lambda, \alpha)$ is also.

Directly from Proposition 1, if $W(\sigma_0; \lambda, \alpha) > 0$ for all σ_0, then as the number of responders grows large

$$\Pi_n^{*V}(\lambda, \alpha) \to E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] + \Pr(H) E_{\sigma_i}\left[\Delta^H(x_H(\lambda, \alpha), \sigma_i, \lambda)|H\right].$$

The first term represents the responders' payoff under the status quo. In general, the second term can be either positive or negative. However, by definition, $\Delta^H(x_H(\lambda, \alpha), \sigma_H, \lambda) = 0$, and Δ^H is independent of σ_i when $\lambda = 0$. Consequently $E_{\sigma_i}\left[\Delta^H(x_H(\lambda), \sigma_i, \lambda)|H\right]$ approaches 0 as $\lambda \to 0$, and so the responders' payoff approaches their status quo payoff. Put differently, against a majority rule the proposer is able to reduce the responders' payoff all the way to their outside option.

In contrast, when responders use the unanimity rule, from Proposition 2 we know their payoff is bounded away from their status quo payoff as n grows large and preferences approach the common values extreme. Consequently:

Corollary 3 *Fix a majority voting rule $\alpha < 1$. If $W(\sigma_0; \lambda = 0, \alpha) > 0$ for all σ_0, then there exists a $\gamma_\Pi > 0$ and a $\bar{\lambda} > 0$ such that whenever $\lambda \leq \bar{\lambda}$, for all n large enough the responders are better off under the unanimity rule,*

$$\Pi_n^{*V}(\lambda, 1) \geq \Pi_n^{*V}(\lambda, \alpha) + \gamma_\Pi. \quad (18)$$

Next, we consider the case in which $W(\sigma_0; \lambda, \alpha) < 0$ for all σ_0. From Proposition 1, as the number of responders grows large

$$\Pi_n^{*V}(\lambda, \alpha) \to E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i, \lambda)\right] + E_{\sigma_i,\omega}\left[\Delta^{\omega}(x_L(\lambda, \alpha), \sigma_i)\right].$$

In contrast, when responders use the unanimity rule there exists an offer strictly less than $x_L(\lambda, \alpha = 1)$ that the responders will always accept (see Lemma 8). As such, the responders' payoff in this case is bounded away from

$$E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i, \lambda)\right] + E_{\sigma_i,\omega}\left[\Delta^{\omega}(x_L(\lambda, \alpha = 1), \sigma_i, \lambda)\right]$$

from above (Proposition 3). It follows that when $W(\sigma_0; \lambda, \alpha) < 0$ for all σ_0, the responders actually receive a higher payoff from using the majority rule. Loosely speaking, the proposer is able to exploit the faliure of the unanimity rule to aggregate responders' information.

Corollary 4 *Fix a majority voting rule $\alpha < 1$. If $W(\sigma_0; \lambda = 0, \alpha) < 0$ for all σ_0, then there exists a $\bar{\lambda} > 0$ such that whenever $\lambda \leq \bar{\lambda}$, for all n large enough the responders are worse off under the unanimity rule,*

$$\Pi_n^{*V}(\lambda, 1) < \Pi_n^{*V}(\lambda, \alpha).$$

To summarize, for low offers agreement is harder to reach under the unanimity rule. In the common values setting, it is not impossible, however. The unanimity rule engenders a better offer from the proposer, making the responders better off. For higher offers, however, the so-called swing responder's curse has the implication that agreement is *more* likely under the unanimity rule. In this case, the unanimity rule is not the tougher rule — it is the softer one. Using the unanimity rule then lowers the responders' payoff. Which of the two cases applies depends on the offer the proposer would make against fully informed responders.

The proposer's payoff

We now turn to the effect of the voting rule on the proposer's payoff. Despite the conflicting preferences of the proposer and the responders, there is no reason to expect that an increase in the responders' payoff will immediately translate into a lower payoff for the proposer. For example, if the responders are less risk averse than the proposer, there might be welfare gains from adopting unanimity rule due to risk sharing it provides. With the additional assumption that both the responders and the proposer are risk neutral, such risk sharing benefits disappear. We exploit the resultant welfare loss due to failure of information aggregation under unanimity rule, and show that the proposer is better off when negotiating with responders using a majority rule if he is not willing or able to make very high offers.

Formally, there are several distinct cases to consider. We continue to focus on preferences close to the common values extreme ($\lambda \approx 0$). Note that at the common values extreme $x_H(\lambda = 0) \neq \infty$ by Assumption 3. As such, when $\omega = H$ the status quo is Pareto inefficient: the proposer prefers acceptance of any offer to the status quo, and there are at least some offers that the responders prefer.

We first consider the case in which, at the common values extreme, $x_L(\lambda = 0) = \infty$. This means that conditional on $\omega = L$, the status quo is Pareto efficient. Since $x_H(\lambda = 0) \neq \infty$ and $x_L(\lambda = 0) = \infty$ then against any majority rule the proposer offers x_H when the number of responders is large, and the responders accept with a probability converging to 1 (respectively, 0) when $\omega = H$ (respectively, $\omega = L$). That is, conditional on the realization of ω a majority voting rule results in an outcome that is (asymptotically) Pareto efficient.

From Corollary 3 above, we also know that in this case responders are better off using the unanimity rule, since doing so induces the proposer to make a more attractive offer. When preferences are linear, it follows that since the outcome under majority rule is Pareto efficient, and responders receive more under the unanimity rule, the proposer's payoff under the unanimity rule must be lower.

Specifically, consider the following additional assumption on preferences:

Assumption 7 *(i)* $V^\omega(x = 1, \sigma_0)$ *and* $\bar{V}(\sigma_0)$ *are independent of* σ_0*; (ii)* $V^\omega(x = 1, \sigma_0) = \bar{V}(\sigma_0)$*; (iii)* $U^\omega(x, \sigma_i, \lambda = 0)$ *and* $V^\omega(x = 1, \sigma_0)$ *are linear in* x*, with* $U^\omega_x(x, \sigma_i, \lambda = 0) / V^\omega_x(x, \sigma_0)$ *independent of* ω*.*

Assumption 7 says (i) the proposer's payoffs have no private value component, (ii) the proposer is indifferent between the status quo and having his most generous offer $x = 1$ accepted, and (iii) the relative value of changes in the offer x for the responders and the proposer is always the same. These conditions are satisfied in standard "split-the-dollar" type bargaining games.

Proposition 4 ***(The proposer's payoff in a split-the-dollar environment)*** *Fix a majority voting rule $\alpha < 1$. If Assumption 7 holds, $x_L(\lambda, \alpha) = \infty$ and $x_H(\lambda, \alpha) \neq \infty$ for λ small enough, then there exists a $\bar{\lambda} > 0$ such that whenever $\lambda \leq \bar{\lambda}$, for all n large enough the proposer is worse off under the unanimity rule,*

$$\Pi_n^{*P}(\lambda, 1) < \Pi_n^{*P}(\lambda, \alpha).$$

Arguably the most restrictive aspect of Assumption 7 is that the relative value of changes in the offer x for the responders and the proposer is always the same. This implies that no welfare gains can be achieved by increasing the acceptance probability when $\omega = L$ while decreasing it when $\omega = H$. In general, offers slightly above x_H are accepted with higher probability after $\omega = L$ under the unanimity rule, but lower probability after $\omega = H$. Consequently, if the proposer's relative valuation of x were much higher when $\omega = L$ than $\omega = H$, it would be quite possible for *both* the responders and the proposer to be better off under the unanimity rule.

Thus far we have focused on the case in which $x_H(\lambda = 0) \neq \infty$ and $x_L(\lambda = 0) = \infty$. At the opposite extreme, if $x_L(\lambda = 0) \neq \infty$ and $W(\sigma_0) < 0$ for all σ_0, then against a majority rule the proposer offers $x_L(\lambda = 0)$ and the acceptance probability after both $\omega = L, H$ converges to 1. But from Lemma 8, there is an offer strictly lower than $x_L(\lambda = 0)$ that responders would always accept under a unanimity rule. We then easily obtain:

Corollary 5 *Fix a majority voting rule $\alpha < 1$. If $W(\sigma_0; \lambda = 0, \alpha) < 0$ for all σ_0, then there exists a $\bar{\lambda} > 0$ such that whenever $\lambda \leq \bar{\lambda}$, for all n large enough the proposer is better off against the unanimity rule,*

$$\Pi_n^{*P}(\lambda, 1) > \Pi_n^{*P}(\lambda, \alpha).$$

For the case in which $x_L(\lambda = 0) \neq \infty$ and $W(\sigma_0) > 0$, Corollary 3 above established that responders are better off using the unanimity rule. In contrast to the case above in which $x_L(\lambda = 0) = \infty$, however, the equilibrium outcome when $\omega = L$ is *not* Pareto efficient. Specifically, the proposer makes an offer close to $x_H(\lambda = 0)$, which the responders reject when $\omega = L$ — even though the status quo outcome that then ensues is Pareto dominated by the acceptance of *some* offer. On the other hand, when facing a unanimity rule the proposer may raise his offer and increase the acceptance probability in state L.

The proposer's payoff is consequently subject to two offsetting effects: against the unanimity rule he makes a higher offer, which lowers his payoff, but which also moves the outcome after $\omega = L$ closer to Pareto efficiency. It is easy to produce examples in which either one of these effects dominates. In particular, even when Assumption 7 holds there are instances in which the equilibrium outcome under a majority voting rule is Pareto dominated by the equilibrium outcome under the unanimity rule.

Private values

Conventional wisdom identifies two opposing effects of adopting a unanimous voting rule. On the one hand, unanimity makes agreement harder to obtain. On the other hand, this "toughness" may be useful in negotiation. Corollary 3 identifies a fairly general set of circumstances under which the latter effect dominates: whenever responders are close enough to common values, the increase in the proposer's offer relative to that obtained under majority voting more than compensates for the increased probability of mistakenly rejecting the offer.

One way to think about this result is that when responders vote strategically, the requirement of unanimity is not as inimical to agreement as it might at first seem. Recall that each responder conditions his or her vote only on the circumstances under which it is actually pivotal. Given a unanimity agreement rule, this means that a responder considers the impact of voting to accept an offer conditional on all other responders accepting — in other words, conditional on all other responders viewing the offer as attractive. Such a responder will vote to accept unless his own signal is very pro status quo.

In contrast, as we move to a situation in which responders are further away from the common values benchmark, we reach a situation in which agreement is indeed extremely difficult to obtain under unanimity. This is most easily seen at the extreme of fully private values preferences: each responder will vote to accept only if his own valuation of the offer on the table beats his payoff under the status quo. Formally, we introduce the following (mild) assumption:

Assumption 8 $\Delta^H(x = 1, \underline{\sigma}, \lambda = 1) < 0$.

Assumption 8 simply says that at the private values extreme, a responder who received the most pro status quo realization of σ_i prefers the status quo to even the most generous offer. When it holds, we obtain:

Lemma 11 ***(Private values responder payoffs under unanimity)***
Suppose the unanimity rule is in effect ($\alpha = 1$), and that responders have preferences sufficiently close to private values such that $\Delta^H(x = 1, \underline{\sigma}, \lambda) < 0$. Then under any sequence of offers $x_n \leq 1$, the acceptance probability converges to 0 and the responder payoff converges to $E\left[\bar{U}^\omega(\sigma_i, \lambda)\right]$.

Lemma 11 says that when Assumption 8 holds, agreement is impossible to obtain when responders use the unanimity rule. In contrast, given Assumption 3, there is *some* majority rule $\hat{\alpha}$ under which the responders will accept at least some offers. Clearly the proposer is better off if responders adopt such a rule. However, because of the private values component of responder preferences, it is quite possible that $E[\Delta^\omega(x, \sigma_i, \lambda) | A_n]$ is negative: that is, even though an offer is only accepted if it makes the marginal responder better off than the status quo, on average the coalition of responders may be worse off.

A pair of conditions that are sufficient to guarantee that there exists a majority voting rule that makes the responder as well as the proposer better off than the unanimity rule are:

$$\Delta^H(1, E[\sigma_i | H], \lambda = 1) \quad > \quad 0 \tag{19}$$

$$\frac{\partial^2}{\partial \sigma_i^2} \Delta^H (x, \sigma_i, \lambda) \quad \geq \quad 0 \text{ for all } \lambda. \tag{20}$$

Of these, (20) is close to a normalization: for any given λ, we can always monotonically transform preferences so that it holds. Condition (19) is stronger, and says that if the proposer makes the best offer possible, $x = 1$, and $\omega = H$, then a responder with the *average* private valuation $E[\sigma_i|H]$ prefers the offer to the status quo. When both conditions are satisfied we obtain:

Proposition 5 ***(Unanimity worse for private values creditors)***
Suppose that conditions (19) and (20) hold. Then there exists a $\Lambda < 1$ such that whenever $\lambda \geq \Lambda$, i.e., responders' preferences are sufficiently close to private values, then there exists a majority voting rule $\hat{\alpha} < 1$ such that the equilibrium outcome under $\hat{\alpha}$ Pareto dominates the equilibrium outcome under the unanimity rule.

7 Discussion

There are many instances in which a group of individuals is engaged in collective bargaining. In such instances, it is often tempting to model the group as a single individual. One way to view our paper is as an exploration of the extent to which this approach is justified. When the group uses a majority rule, and the main source of intra-group heterogeneity is different information, it is indeed the case that the response of a large group to an offer made by the opposing party matches that of a single individual endowed with the same information. However, by adopting unanimity rule the group will cause its joint behavior to diverge from that of an individual. Our results suggest that under some circumstances such a purposeful deviation is benificial to the group.

A somewhat related issue is the extent to which our results would change if instead of analysing a take-it-or-leave game we instead allowed for alternating offers. However, to do so would introduce the non-trivial complication that the group would have to decide on a procedure for how to select a counter-offer. In particular, what is the optimal way for a group to decide which member(s) should make the offer?[13] We leave this undoubtedly important issue for future research.

[13]See Haller and Holden (1997) for an analysis of a related issue: should an agreement reached by a group representative by subject to ratification by the rest of the group?

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A Appendix

We repeatedly use the following minor observation:

Lemma 12 *$F(\sigma|H)/F(\sigma|L)$ is increasing in σ, and is bounded above by 1. As a consequence, $F(\sigma|H) \le F(\sigma|L)$, and is strict if strict MLRP holds. Finally, $(1-F(\sigma|L))/(1-F(\sigma|H))$ is decreasing in σ, and bounded below by $f(\bar{\sigma}|L)/f(\bar{\sigma}|H) \in (0,1)$.*

Proof: Rewriting, we must show that

$$\frac{\int^{\sigma} f(\tilde{\sigma}|L)\frac{f(\tilde{\sigma}|H)}{f(\tilde{\sigma}|L)}d\tilde{\sigma}}{\int^{\sigma} f(\tilde{\sigma}|L)\,d\tilde{\sigma}}$$

is increasing in σ. Differentiating, we must show

$$f(\sigma|L)\frac{f(\sigma|H)}{f(\sigma|L)}\int^{\sigma} f(\tilde{\sigma}|L)\,d\tilde{\sigma} > f(\sigma|L)\int^{\sigma} f(\tilde{\sigma}|L)\frac{f(\tilde{\sigma}|H)}{f(\tilde{\sigma}|L)}d\tilde{\sigma},$$

which is immediate from MLRP. The proof that $(1-F(\sigma|L))/(1-F(\sigma|H))$ is decreasing is exactly parallel; its lower bound follows from l'Hôpital's rule. ∎

Proof of Lemma 1

Since $\Delta^H(x,\sigma_i,\lambda) \ge \Delta^L(x,\sigma_i,\lambda)$, (4) is only satisfied for σ_i if $\Delta^H(x,\sigma_i,\lambda) \ge 0$. It then follows from MLRP that if (4) is satisfied for some value of σ_i, it is strictly satisfied for all higher values. ∎

Proof of Lemma 2

1. By definition, if $x < \underline{x}$ then $Z(x,\cdot) < 0$, while if $x > \bar{x}$ then $Z(x,\cdot) > 0$. For $x \in [\underline{x},\bar{x}]$ we claim that $Z(x,\sigma) = 0$ for some unique σ, which we write as $\sigma^*(x)$. Existence is immediate, since $\max_\sigma Z(x,\sigma) \ge 0 \ge \min_\sigma Z(x,\sigma)$, and $Z(x,\sigma)$ is continuous in σ.

   For uniqueness, suppose that $Z(x,\sigma^{**}) = Z(x,\sigma^*) = 0$ for some $\sigma^{**} > \sigma^*$. By Assumption 2, $\Delta^H(x,\sigma^*) > \Delta^L(x,\sigma^*)$, and so $\Delta^H(x,\sigma^*) > 0 > \Delta^L(x,\sigma^*)$. Since Δ^H and Δ^L are both increasing in σ, it follows that $Z(x,\sigma)$ is strictly increasing for $\sigma \ge \sigma^*$, and so $Z(x,\sigma^{**}) = 0$ is contradicted.

2. To see that $\sigma^*(x)$ is decreasing, consider x and $x' > x$ in $(\underline{x}_n, \bar{x}_n)$. Since $Z(x, \sigma^*(x)) = 0$, then $Z(x', \sigma^*(x)) > 0$. Exactly as above, $Z(x', \sigma)$ is increasing in σ for $\sigma \geq \sigma^*(x)$. It follows that $\sigma^*(x') < \sigma^*(x)$.

   The Implicit Function Theorem implies that $\sigma(x)$ is continuously differentiable over $(\underline{x}, \bar{x})$.

   We claim that $\sigma^*(\underline{x}) = \bar{\sigma}$ and $\sigma^*(\bar{x}) = \underline{\sigma}$. For $\sigma^*(\underline{x}) = \bar{\sigma}$, suppose to the contrary that $\sigma^*(\underline{x}) < \bar{\sigma}$. So $Z(\underline{x}, \sigma^*(\underline{x})) = 0$ and so $\Delta^H(\underline{x}, \sigma^*(\underline{x})) > 0 > \Delta^L(\underline{x}, \sigma^*(\underline{x}))$. Since $\Delta^H(\underline{x}, \sigma^*(\underline{x})) > 0$ it follows that $Z(\underline{x}, \bar{\sigma}) > 0$; but then by continuity there exists an $x < \underline{x}$ such that $Z(x, \bar{\sigma}) > 0$ also, contradicting the definition of $\underline{x}$. Likewise, for $\sigma^*(\bar{x}) = \underline{\sigma}$ suppose to the contrary that $\sigma^*(\bar{x}) > \underline{\sigma}$. So $Z(\bar{x}, \sigma^*(\bar{x})) = 0$ and so $\Delta^H(\bar{x}, \sigma^*(\bar{x})) > 0 > \Delta^L(\bar{x}, \sigma^*(\bar{x}))$. Since $\Delta^H(\bar{x}, \sigma^*(\bar{x})) > 0$ it follows that $Z(x, \sigma)$ is locally strictly increasing in σ at $(\bar{x}, \sigma^*(\bar{x}))$. Given this, by continuity there exists a (x, σ) such that $x > \bar{x}$ and $\sigma < \sigma^*(\bar{x})$ for which $Z(x, \sigma) < 0$, contradicting the definition of $\bar{x}$.

3. Immediate from the observations that as $n \to \infty$, the expression

$$\frac{f(\sigma|H)}{f(\sigma|L)} \left(\frac{F(\sigma|H)}{F(\sigma|L)}\right)^{n-n\alpha} \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n\alpha-1}$$

   tends to 0 when $\sigma = \underline{\sigma}$ and tends to ∞ when $\sigma = \bar{\sigma}$.

4. Similar.

∎

Proof of Lemma 3

Let Z be as defined in the proof of Lemma **2**. Since $x \leq \underline{x}_n$, from the proof of Lemma **2**

$$\Delta^H(x, \sigma) \frac{b}{1-b} \frac{f(\sigma|H)}{f(\sigma|L)} \left(\frac{F(\sigma|H)}{F(\sigma|L)}\right)^{n-n\alpha} \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n\alpha-1} + \Delta^L(x, \sigma) \leq 0$$

for all σ. It follows that

$$\Delta^H(x,\bar{\sigma})\frac{b}{1-b}\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)}\left(\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)}\right)^{n\alpha-1}+\Delta^L(x,\bar{\sigma})\leq 0.$$

It then follows that for any σ_i,

$$\Delta^H(x,\sigma_i)\frac{b}{1-b}\frac{f(\sigma_i|H)}{f(\sigma_i|L)}\left(\frac{f(\bar{\Sigma}|H)}{f(\bar{\sigma}|L)}\right)^{n\alpha-1}+\Delta^L(x,\sigma_i)\leq 0$$

(for this expression could only be strictly positive if $\Delta^H(x,\sigma_i)$ were strictly positive; but then by MLRP it would be strictly positive at $\sigma_i=\bar{\sigma}$). Finally, since $\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)}>1$, for any $m<n\alpha-1$,

$$\Delta^H(x,\sigma_i)\frac{b}{1-b}\frac{f(\sigma_i|H)}{f(\sigma_i|L)}\left(\frac{f(\bar{\sigma}|H)}{f(\bar{\sigma}|L)}\right)^{m}+\Delta^L(x,\sigma_i)<0.$$

In words, this last inequality says that responder i, having observed his own signal σ_i, will reject the offer x even if he conditions on the event that $m<n\alpha-1$ other responders observe the most pro-acceptance signal $\bar{\sigma}$. This has two implications.

First, the equilibrium in which all responders reject always is a trembling-hand perfect equilibrium: for if all responders tremble and accept with probability ε independent of their own signal, it remains a best response to reject the offer. This follows since it is a best response to reject the offer given the information that $m<n\alpha-1$ responders have observed $\bar{\sigma}$, it is certainly a best response to reject given no information.

Second, we claim that the equilibrium in which all responders accept is not trembling-hand perfect. Recall that responder i's vote only matters if exactly $n\alpha-1$ other responders vote to accept. This event only arises if at least $n-n\alpha$ of the $n-1$ other responders tremble. As the probability of trembles converges to zero, responder i's best response is determined entirely by the event in which exactly $n-n\alpha$ other responders tremble. But by assumption $n-n\alpha<n\alpha-1$, and so even if responder i infers from $n-n\alpha$ trembles that $n-n\alpha$ responders have observed $\bar{\sigma}$, his best response is to reject. As such, the equilibrium in which all creditors accept always cannot be trembling-hand perfect. ■

Proof of Lemma 4

The proof largely parallels those in the existing literature, and is omitted for reasons of space. It is available from the authors' webpages.

Proof of Lemma 5

We focus on Part 1a. The main idea is straightforward: for any σ_0 such that $W(\sigma_0) > 0$, the proposer prefers offering $x_H(\lambda, \alpha)$ and gaining acceptance if and only if $\omega = H$ to offering $x_L(\lambda, \alpha)$ and gaining acceptance all the time. Given the limiting behavior of responders established in Lemma 4, intuitively it follows that the proposer's offer converges to $x_H(\lambda, \alpha)$ as the number of responders grows large. The main difficulty encountered in the formal proof is establishing uniform convergence: for any $\varepsilon, \delta > 0$, there is some $N(\varepsilon, \delta)$ such that when $n \geq N(\varepsilon, \delta)$, the proposer's offer lies within δ of $x_H(\lambda, \alpha)$ for all σ_0 such that $W(\sigma_0) > \varepsilon$.

Part 1b and 2 are proved by similar arguments, which for conciseness we omit. Part 3 is immediate from Lemma 4.

Take any $\varepsilon, \delta > 0$. Throughout the proof, we omit all λ and α arguments for readability.

Preliminaries: The first part of the proof consists of defining bounds which we will use to establish uniform convergence below. We urge all except masochistic readers to proceed directly to Part A.

Choose a $\delta_1 \in (0, \delta]$ sufficiently small such that $x_H + \delta_1 < x_H - \delta_1$ and for any σ_0, the proposer prefers $x_H$accepted always to a slightly smaller offer accepted slightly less often,

$$V^H(x_H, \sigma_0) > (1-\delta) V^H(x_H - \delta_1, \sigma_0) + \delta \bar{V}^H(\sigma_0); \tag{21}$$

and $x_L - \delta_1$ and $x_L$are close enough such that the expected proposer payoffs are similar

$$E[V^\omega(x_L - \delta_1, \sigma_0)|\sigma_0] - E[V^\omega(x_L, \sigma_0)|\sigma_0] \leq \frac{\varepsilon}{4}. \tag{22}$$

Given δ_1, choose $\hat{\varepsilon}$,$\tilde{\varepsilon}$,$\breve{\varepsilon} > 0$ and $\kappa \in \left(0, \frac{\delta_1}{2}\right)$ sufficiently small such that $x_H + \kappa < \min\{1, x_L\}$ and for all σ_0,

$$V^H\left(x^H, \sigma_0\right) - V^H\left(x^H + \kappa, \sigma_0\right) \leq \frac{\varepsilon}{4}, \tag{23}$$

(i.e., κ small) and

$$\left|\hat{\varepsilon}\left(V^H\left(x^H+\kappa,\sigma_0\right)-\bar{V}_H\left(\sigma_0\right)\right)+\hat{\varepsilon}\left(\bar{V}_L\left(\sigma_0\right)-V^L\left(x^H+\kappa,\sigma_0\right)\right)\right|\le\frac{\varepsilon}{4} \tag{24}$$

(i.e., $\hat{\varepsilon}$ small) and

$$\begin{aligned}&\tilde{\varepsilon}\Pr\left(H|\sigma_0\right)\left(V^H\left(0,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)+\tilde{\varepsilon}\Pr\left(L|\sigma_0\right)\left(V^L\left(0,\sigma_0\right)-\bar{V}^L\left(\sigma_0\right)\right)\\ <\;&\left(1-\hat{\varepsilon}\right)\Pr\left(H|\sigma_0\right)\left(V^H\left(x_H+\kappa,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\end{aligned} \tag{25}$$

(i.e., $\tilde{\varepsilon}$ small) and

$$\begin{aligned}&\Pr\left(H|\sigma_0\right)\left(\left(1-\hat{\varepsilon}\right)\left(V^H\left(x^H+\kappa,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)-\left(V^H\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\right)\\ >\;&\Pr\left(L|\sigma_0\right)\check{\varepsilon}\left(V^L\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^L\left(\sigma_0\right)\right).\end{aligned} \tag{26}$$

(i.e., $\hat{\varepsilon}$ and $\check{\varepsilon}$ small: (26) can be satisfied since $\kappa<\delta_1$ and so $V^H\left(x_H+\delta_1,\sigma_0\right)<V^H\left(x^H+\kappa,\sigma_0\right)$) and

$$\begin{aligned}&\Pr\left(H|\sigma_0\right)\left(\left(1-\hat{\varepsilon}\right)V^H\left(x^H+\kappa,\sigma_0\right)+\hat{\varepsilon}\bar{V}^H\left(\sigma_0\right)\right)+\Pr\left(L|\sigma_0\right)\bar{V}^L\left(\sigma_0\right)\\ >\;&\Pr\left(H|\sigma_0\right)\left(\left(1-\delta\right)V^H\left(x_H-\delta_1,\sigma_0\right)+\delta\bar{V}^H\left(\sigma_0\right)\right)\\ &+\Pr\left(L|\sigma_0\right)\left(\check{\varepsilon}V^L\left(x_H-\delta_1,\sigma_0\right)+\left(1-\check{\varepsilon}\right)\bar{V}^L\left(\sigma_0\right)\right)\end{aligned} \tag{27}$$

(i.e., $\hat{\varepsilon}$ and $\check{\varepsilon}$ small: (27) can be satisfied since (21) holds).

Let $\underline{b}$ and $\bar{b}$ respectively denote the most pro-L and pro-H beliefs possible, i.e.,

$$\frac{\underline{b}}{1-\underline{b}}=\frac{f_0\left(\underline{\sigma}|H\right)}{f_0\left(\underline{\sigma}|L\right)}\frac{\Pr\left(H\right)}{\Pr\left(L\right)}\text{ and }\frac{\bar{b}}{1-\bar{b}}=\frac{f_0\left(\bar{\sigma}|H\right)}{f_0\left(\bar{\sigma}|L\right)}\frac{\Pr\left(H\right)}{\Pr\left(L\right)}.$$

Define the following offer sequences, which we use throughout the proof:

$$\begin{aligned}\hat{x}_n&\equiv x_H+\kappa\\ \tilde{x}_n&\equiv x_H-\delta_1\\ \check{x}_n&\equiv x_L-\delta_1\end{aligned}$$

By Lemma 4, $\Pr\left(A_n\left(\hat{x}_n\right)|H\right)\to 1$, $\Pr\left(A_n\left(\tilde{x}_n,\bar{b}\right)|\omega\right)\to 0$ for $\omega=L,H$, and $\Pr\left(A_n\left(\check{x}_n,\bar{b}\right)|L\right)\to 0$. Thus for any $\hat{\varepsilon},\tilde{\varepsilon},\check{\varepsilon}>0$ there exist $N\left(\hat{\varepsilon}\right),N\left(\tilde{\varepsilon}\right),N\left(\check{\varepsilon}\right)$ such that when $n\ge N\left(\hat{\varepsilon}\right)$,

$\Pr(A_n(\hat{x}_n)|\omega) \geq 1-\hat{\varepsilon}$; when $n \geq N(\tilde{\varepsilon})$, $\Pr\left(A_n\left(\tilde{x}_n, \bar{b}\right)|\omega\right) \leq \tilde{\varepsilon}$ for $\omega = L, H$; and when $n \geq N(\check{\varepsilon})$, $\Pr\left(A_n\left(\check{x}_n, \bar{b}\right)|\omega\right) \leq \check{\varepsilon}$. Let $N(\varepsilon, \delta) = \max\{N(\hat{\varepsilon}), N(\tilde{\varepsilon}), N(\check{\varepsilon})\}$. Note that by construction $N(\varepsilon, \delta)$ depends only on ε and δ, and not σ_0.

Part A: If $W(\sigma_0) \geq \varepsilon$ then $|\pi_n^*(\sigma_0) - x_H| \leq \delta_1$ when $n \geq N(\varepsilon, \delta)$.

Proof: Fix a realization of σ_0 such that the hypothesis holds, and let $x_n = \pi_n^*(\sigma_0)$.

First, we claim that $x_n \geq x_H - \delta_1 = \tilde{x}_n$ whenever $n \geq N(\varepsilon, \delta)$. If this were not the case, there must exist some $m \geq N(\varepsilon, \delta)$ such that $x_m < \tilde{x}_m$. The acceptance probability of x_m given ω is consequently less than that of $\tilde{x}_m$ under the most pro-acceptance beliefs $\bar{b}$, which is in turn less than $\tilde{\varepsilon}$. The acceptance probability of $\hat{x}_m$ given H is at least $1-\hat{\varepsilon}$. It follows from (25) that the proposer's payoff is higher under $\hat{x}_m$ than under x_m. But this contradicts the optimality of the proposer's offer x_m.

Second, and for use below, we claim that $x_n < x_L - \delta_1$ when $n \geq N(\varepsilon, \delta)$. If this were not the case, there must exist some $m \geq N(\varepsilon, \delta)$ such that $x_m \geq x_L - \delta_1 = \check{x}_m$. By Assumption 4 the proposer is always happier when his offer is accepted; and so if $x_m \geq \check{x}_m$ the proposer's expected payoff is bounded above by $E[V^\omega(\check{x}_m, \sigma_0)|\sigma_0]$. In contrast, under the sequence of offers $\hat{x}_n$ defined above the proposer's payoff converges to

$$\Pr(H|\sigma_0)V^H(\hat{x}_n, \sigma_0) + \Pr(L|\sigma_0)\bar{V}^L(\sigma_0).$$

By (24), whenever $n \geq N(\varepsilon, \delta)$ the proposer's payoff is within $\frac{\varepsilon}{4}$ of this expression. It follows that the proposer's payoff under $\hat{x}_m$ is larger than under x_m by at least $\frac{\varepsilon}{4}$. But this contradicts the optimality of the proposer's offers x_n.

Third, we claim that when $n \geq N(\varepsilon, \delta)$ then $x_n \leq x_H + \delta_1$. If this were not the case, there must exist some $m \geq N(\varepsilon, \delta)$ such that $x_m \geq x_H + \delta_1$. Moreover, from above we know that $x_m \leq x_L - \delta_1 = \check{x}_n$. The difference in the proposer's expected payoff under the two offers x_m and $\hat{x}_m$ is

$$\begin{aligned}
&\Pr(H|\sigma_0)\Pr(A_m(\hat{x}_m)|H)\left(V^H(\hat{x}_m, \sigma_0) - \bar{V}^H(\sigma_0)\right) \\
&- \Pr(H|\sigma_0)\Pr(A_m(x_m)|H)\left(V^H(x_m, \sigma_0) - \bar{V}^H(\sigma_0)\right) \\
&+ \Pr(L|\sigma_0)\Pr(A_m(\hat{x}_m)|L)\left(V^L(\hat{x}_m, \sigma_0) - \bar{V}^L(\sigma_0)\right)
\end{aligned}$$

$$-\Pr\left(L|\sigma_0\right)\Pr\left(A_m\left(x_m\right)|L\right)\left(V^L\left(x_m,\sigma_0\right)-\bar{V}^L\left(\sigma_0\right)\right),$$

which is bounded below by

$$\begin{aligned}&\Pr\left(H|\sigma_0\right)\Pr\left(A_m\left(\hat{x}_m\right)|H\right)\left(V^H\left(\hat{x}_m,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\\&-\Pr\left(H|\sigma_0\right)\left(V^H\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\\&-\Pr\left(L|\sigma_0\right)\Pr\left(A_m\left(x_m\right)|L\right)\left(V^L\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^L\left(\sigma_0\right)\right).\end{aligned}$$

Since $x_m \leq \check{x}_m$, then $\Pr\left(A_m\left(x_m\right)|L\right) \leq \Pr\left(A_m\left(\check{x},\bar{b}\right)|L\right)$, and so since $m \geq N\left(\varepsilon,\delta\right)$ the above expression is in turn greater than

$$\begin{aligned}&\Pr\left(H|\sigma_0\right)\left(1-\hat{\varepsilon}\right)\left(V^H\left(\hat{x}_m,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\\&-\Pr\left(H|\sigma_0\right)\left(V^H\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^H\left(\sigma_0\right)\right)\\&-\Pr\left(L|\sigma_0\right)\check{\varepsilon}\left(V^L\left(x_H+\delta_1,\sigma_0\right)-\bar{V}^L\left(\sigma_0\right)\right).\end{aligned}$$

By (26) this is strictly positive. But this contradicts the optimality of the proposer's offers x_n.

Part B: If $W\left(\sigma_0\right) \geq \varepsilon$, then $1-\Pr\left(A_n\left(x_n\right)|H\right) \leq \delta$ when $n \geq N\left(\varepsilon,\delta\right)$.

Proof: Again, fix a realization of σ_0 such that the hypothesis holds, and let $x_n = \pi_n^*\left(\sigma_0\right)$. From Part A, if $n \geq N\left(\varepsilon,\delta\right)$ then $|x_n - x_H| \leq \delta_1$.

Suppose that contrary to the claim, there exists an $m \geq N\left(\varepsilon,\delta\right)$ such that $\Pr\left(A_m\left(x_m\right)|H\right) < 1-\delta$. Note that $x_m \geq \tilde{x}_m$. Moreover, since $x_m \leq x_H+\delta_1 < \check{x}_m$, then $\Pr\left(A_n\left(\check{x}_m,\bar{b}\right)|L\right) \leq \check{\varepsilon}$ when $n \geq N\left(\varepsilon,\delta\right)$. So the proposer's payoff is bounded above by

$$\Pr\left(H|\sigma_0\right)\left(\left(1-\delta\right)V^H\left(\tilde{x}_m,\sigma_0\right)+\delta\bar{V}^H\left(\sigma_0\right)\right)+\Pr\left(L|\sigma_0\right)\left(\check{\varepsilon}V^L\left(\tilde{x}_m,\sigma_0\right)+\left(1-\check{\varepsilon}\right)\bar{V}^L\left(\sigma_0\right)\right).$$

Under the offers $\hat{x}_m$, the proposer's payoff is bounded below by

$$\Pr\left(H|\sigma_0\right)\left(\left(1-\hat{\varepsilon}\right)V^H\left(\hat{x}_m,\sigma_0\right)+\hat{\varepsilon}\bar{V}^H\left(\sigma_0\right)\right)+\Pr\left(L|\sigma_0\right)\bar{V}^L\left(\sigma_0\right).$$

By (27) the latter is strictly greater, contradicting the optimality of the proposer's offers x_n. ∎

Proof of Lemma 6

Fix λ_0. The basic idea of the proof is as follows. In Claim 1, we choose a Λ and an N such that when $0 < \lambda \in [\lambda_0, \Lambda]$ and $n \geq N$ the voting equilibrium is "well-behaved." In Claim 2, we show that if $\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n | H) = 0$, then this would allow us to form a sequence in which the number of responders grows large "faster" than the probability that each responder votes to accept approaches 1. In Claim 3, we then show just the opposite: given the "well-behaved" properties we established in Claim 1, then the number of responders must grow large "slower" than the probability that each responder votes to accept approaches 1.

Claim 1: There exists a pair (Λ, N) and a $\delta > 0$ such that for $0 < \lambda \in [\lambda_0, \Lambda]$ and $n \geq N$,

$$\left(\frac{1 - F\left(\sigma_n^*\left(x_n\left(\lambda\right), \lambda\right)|L\right)}{1 - F\left(\sigma_n^*\left(x_n\left(\lambda\right), \lambda\right)|H\right)}\right)^n \geq \delta \text{ if } \sigma_n^*\left(x_n\left(\lambda\right), \lambda\right) > \underline{\sigma} \tag{28}$$

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0} \sigma_n^*\left(x_n\left(\lambda\right), \lambda\right) \leq \left(\underline{\sigma} + \bar{\sigma}\right)/2. \tag{29}$$

Proof of Claim 1:

By condition (12), for all $\varepsilon > 0$ sufficiently small there exists a $\Lambda \geq \lambda_0$ and N_0 such that $x_n(\lambda) - x_H(\lambda) \geq \varepsilon$ whenever $0 < \lambda \in [\lambda_0, \Lambda]$ and $n \geq N_0$.

Note that $\sigma_H = \underline{\sigma}$, so $\Delta^H\left(x_H\left(\lambda\right), \underline{\sigma}\right) = 0 < \Delta_H\left(x_H\left(\lambda\right), \bar{\sigma}\right)$. Consider the most negative beliefs possible, i.e., $\inf\left\{\frac{\beta(x_n(\lambda))}{1-\beta(x_n(\lambda))} : n, \lambda\right\}$. From Lemma 2, there exists an $N_1 \geq N_0$ such that if $n \geq N_1$ and $0 < \lambda \in [\lambda_0, \Lambda]$, then there is a responsive voting equilibrium when the offer is $x_n\left(\lambda\right) \geq x_H\left(\lambda\right) + \varepsilon$ and the beliefs are $\inf\left\{\frac{\beta(x_n(\lambda))}{1-\beta(x_n(\lambda))} : n, \lambda\right\}$. Consequently, by Corollary 1 it follows that if $0 < \lambda \in [\lambda_0, \Lambda]$ and $n \geq N_1$, then given the actual beliefs $\frac{\beta(x_n(\lambda))}{1-\beta(x_n(\lambda))}$ associated with offer $x_n\left(\lambda\right)$, either a responsive equilibrium exists, or else the equilibrium is the acceptance equilibrium.

If the equilibrium for some $x_n\left(\lambda\right)$ and λ is the acceptance equilibrium, then $\sigma_n^*\left(x_n\left(\lambda\right), \lambda\right) = \underline{\sigma}$. On the other hand, if the equilibrium is a responsive equilibrium then since $\sigma_n^*\left(x_n\left(\lambda\right), \lambda\right) \geq \underline{\sigma} = \sigma_H$, and by definition $\Delta^H\left(x_H\left(\lambda\right), \sigma_H, \lambda\right) = 0$, it follows that there exists an $\hat{\varepsilon} > 0$ such that $\Delta^H\left(x_n\left(\lambda\right), \sigma_n^*, \lambda\right) > \hat{\varepsilon}$ under these same conditions. As in the proof of Lemma 4, the remaining terms on the lefthand side of the equilibrium condition (7) are also bounded away from 0. Consequently, there exists a $\delta > 0$ and and $N_2 \geq N_1$ such that when $0 < \lambda \in [\lambda_0, \Lambda]$ and $n \geq N_2$, inequality (28) holds.

We claim that as a consequence,

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0} \sigma_n^* (x_n (\lambda), \lambda) \to \underline{\sigma} \text{ as } n \to \infty. \tag{30}$$

The proof of this claim is as follows. Suppose to the contrary that for all $0 < \lambda \in [\lambda_0, \Lambda]$ there exists a subsequence of $\sigma_n^* (x_n (\lambda), \lambda)$ which stays bounded away from $\underline{\sigma}$. Clearly along this subsequence the equilibria are responsive. From Lemma 12, $\frac{1-F(\sigma|L)}{1-F(\sigma|H)} < 1$ for any $\sigma \in (\underline{\sigma}, \bar{\sigma})$, and converges to $f(\bar{\sigma}|L) / f(\bar{\sigma}|H) < 1$ as $\sigma \to \bar{\sigma}$. This gives a contradiction to (28).

Given this, it is possible to choose $N \geq N_2$ such that (29) holds. This completes the proof of Claim 1.

Claim 2: Suppose that

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr (A_n|H) = 0, \tag{31}$$

where N and Λ are as defined in Claim 1. Then there exists a sequence $(\lambda_n, m(n))$ for which $\lambda_0 \leq \lambda_n \leq \Lambda, \lambda_n > 0$ and $m(n) \geq N$ and for which

$$\left(1 - F\left(\sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) |H\right)\right)^{m(n)} \to 0 \text{ as } n \to \infty \tag{32}$$

$$m(n) \to \infty \text{ as } n \to \infty \tag{33}$$

$$\sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) > \underline{\sigma} \text{ and } \sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) \to \underline{\sigma} \text{ as } n \to \infty \tag{34}$$

$$\left(\frac{1 - F\left(\sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) |L\right)}{1 - F\left(\sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) |H\right)}\right)^{m(n)} \to \hat{\delta} > 0 \text{ as } n \to \infty \tag{35}$$

Proof of Claim 2:

Equality (31) can only hold if there exists some sequence $(\lambda_n, m(n))$ for which $\lambda_0 \leq \lambda_n \leq \Lambda, \lambda_n > 0$ and $m(n) \geq N$ and (32) holds.

If $m(n)$ were bounded above, then (32) could hold only if $\sigma_{m(n)}^* \left(x_{m(n)} (\lambda_n), \lambda_n\right) \to \bar{\sigma}$. Since this would contradict (29), it follows that $m(n) \to \infty$.

If $\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right) = \underline{\sigma}$ then $F\left(\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right)|H\right) = 0$; so to avoid violating (32), we need $\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right) > \underline{\sigma}$ for all n large enough. Without loss, choose the sequence $(\lambda_n, m(n))$ so that it has this property directly. Given that $\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right) > \underline{\sigma}$ and $m(n)\to\infty$, it is then immediate from (28) that $\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right) \to \underline{\sigma}$.

The sequence defined by (35) is bounded, and so by Bolzano-Weierstrass has a convergent subsequence. Again, without loss, assume that the sequence $(\lambda_n, m(n))$ was chosen so that it has this property directly. By (28), the limit of expression (35) is above δ. This completes the proof of Claim 2.

Claim 3: Suppose that $(\lambda_n, m(n))$ is a sequence such properties (33) - (35) holds. Then

$$\left(1 - F\left(\sigma^*_{m(n)}\left(x_{m(n)}(\lambda_n),\lambda_n\right)|H\right)\right)^{m(n)}$$

has a non-zero limit.

Proof of Claim 3:

For readability, for the remainder of the proof we suppress the arguments $\left(x_{m(n)}(\lambda_n),\lambda_n\right)$ in writing $\sigma^*_{m(n)}$. We evaluate the limit of $\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)^{m(n)}$. Since $\sigma^*_{m(n)}$ is not differentiable with respect to $m(n)$, we use the discrete version of l'Hôpital's rule, which gives[14]

$$\lim \ln\left(1-F\left(\sigma^*_{n_m}|H\right)\right)^{n_m} = \lim \frac{n_{m+1}-n_m}{\frac{1}{\ln 1-F\left(\sigma^*_{n_{m+1}}|H\right)} - \frac{1}{\ln 1-F\left(\sigma^*_{n_m}|H\right)}}$$

Expanding, the right hand side is equal to

$$\begin{aligned}
& \lim\left(m(n+1)-m(n)\right)\frac{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)} \\
= \; & \lim\left(m(n+1)-m(n)\right)\frac{\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}}{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}-\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}
\end{aligned}$$

[14] The discrete version of l'Hôpital's rule holds provided that $\frac{1}{\ln 1-F\left(\sigma^*_{m(n)}(\lambda_n)|H\right)}\to\infty$, which is satisfied since $\sigma^*_{m(n)}(\lambda_n)\to\underline{\sigma}$.

$$\times\frac{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}-\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}\frac{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}{\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}\frac{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}}$$

Since $\sigma^*_{m(n)}\to\underline{\sigma}$,[15]

$$\frac{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}-\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)} \to \frac{f\left(\underline{\sigma}|L\right)}{f\left(\underline{\sigma}|H\right)}-1 \tag{36}$$

$$\frac{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}} \to \left(\frac{f\left(\underline{\sigma}|L\right)}{f\left(\underline{\sigma}|H\right)}-1\right)^{-1} \tag{37}$$

By strict MLRP, $f\left(\underline{\sigma}|L\right)/f\left(\underline{\sigma}|H\right)>1$. Thus provided

$$\frac{\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}}{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}-\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}$$

exists and is finite, the right hand side limit exists and is finite. But again by the discrete version of l'Hôpital's rule,

$$\lim\ln\left(\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}\right)^{m(n)}=\lim\left(m\left(n+1\right)-m\left(n\right)\right)\times\lim\frac{\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}}{\ln\frac{1-F\left(\sigma^*_{m(n)}|L\right)}{1-F\left(\sigma^*_{m(n)}|H\right)}-\ln\frac{1-F\left(\sigma^*_{m(n+1)}|L\right)}{1-F\left(\sigma^*_{m(n+1)}|H\right)}}$$

which is finite by assumption. Thus $\lim\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)^{m(n)}$ exists and is finite. This completes the proof of Claim 3.

[15] The next two limits follow since

$$\begin{aligned}\ln\left(1-F\left(\sigma|R\right)\right) &= -F\left(\sigma|R\right)+o\left(F\left(\sigma|R\right)^2\right)\\ F\left(\sigma|R\right) &= \left(\sigma-\underline{\sigma}\right)f\left(\underline{\sigma}|R\right)+o\left(\left(\sigma-\underline{\sigma}\right)^2\right)\end{aligned}$$

and so

$$\ln\left(1-F\left(\sigma|R\right)\right)=-\left(\sigma-\underline{\sigma}\right)f\left(\underline{\sigma}|R\right)+o\left(\left(\sigma-\underline{\sigma}\right)^2\right).$$

Proof of Main Result:

Take Λ and N as defined in Claim 1. By Claim 2, if

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n|H) = 0,$$

then there exists a sequence $(\lambda_n, m(n))$ such that properties (32) - (35) hold. But then by Claim 3, property (32) cannot hold, a contradiction. Consequently

$$\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n|H) > 0,$$

which suffices to establish the result. ∎

PROOF OF LEMMA 7

Part 1: Fix λ_0. Suppose to the contrary that (15) does not hold. So there exists a Λ and N such that $\inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n|H) > 0$. From (14), we can construct a sequence (λ_m, n_m) for which $\lambda_0 \leq \lambda_m \leq \Lambda, \lambda_m > 0$ and $n_m \geq N$, with $n_m \to \infty$ and

$$\inf_m x_{n_m}(\lambda_n) - x_H(\lambda_m) \leq 0.$$

By Bolzano-Weierstrass, $x_{n_m}(\lambda_m) - x_H(\lambda_m)$ must have a convergent subsequence. Likewise, $\Pr(A_{n_m}(x_{n_m}(\lambda_m), \lambda_m))$ must have a convergent subsequence. Thus without loss, assume that (λ_m, n_m) is chosen so that

$$\lim x_{n_m}(\lambda_m) - x_H(\lambda_m) \leq 0 \quad (38)$$

$$\lim \Pr(A_{n_m}(x_{n_m}(\lambda_m), \lambda_m)|H) > 0 \quad (39)$$

Similar to in the proof of Lemma 6, from (39), $\sigma^*_{n_m}(x_{n_m}(\lambda_m), \lambda_m) \to \underline{\sigma}$ since $n_m \to \infty$. Moreover, by definition $\Delta^H(x_H(\lambda), \sigma_H, \lambda) = 0$ and so $\Delta^L(x_H(\lambda), \sigma_H, \lambda) < 0$. Since $\sigma_H = \underline{\sigma}$, and since beliefs are bounded, it follows that for for n_m large enough a responder would strictly prefer to reject an offer of $x_{n_m}(\lambda_m)$. Consequently the equilibrium does not feature unconditional acceptance, and is instead responsive: $\sigma^*_{n_m}(x_{n_m}(\lambda_m), \lambda_m) > \underline{\sigma}$ for n_m large.

As such, the equilibrium condition (7) must hold. Again, since $\underline{\sigma} = \sigma_H$, and by definition $\Delta^H(x_H(\lambda), \sigma_H, \lambda) = 0$, from (38), $\lim \Delta^H\left(x_{n_m}(\lambda_m), \sigma^*_{n_m}, \lambda_{n_m}\right) \leq 0$. If the inequality is strict we have an immediate contradiction to the equilibrium condition. Otherwise, since $\frac{\beta(x_n)}{1-\beta(x_n)}$ is bounded away from infinity, the equilibrium condition (7) implies that

$$\left(\frac{1 - F\left(\sigma^*_{n_m}(x_{n_m}(\lambda_m), \lambda_m)|L\right)}{1 - F\left(\sigma^*_{n_m}(x_{n_m}(\lambda_m), \lambda_m)|H\right)}\right)^{n_m} \to 0 \text{ as } m \to \infty \tag{40}$$

Suppressing the arguments $(x_{n_m}(\lambda_m), \lambda_m)$ for clarity, by the discrete version of l'Hôpital's rule,

$$\lim \ln\left(\frac{1 - F\left(\sigma^*_{n_m}|L\right)}{1 - F\left(\sigma^*_{n_m}|H\right)}\right)^{n_m} = \lim \frac{n_{m+1} - n_m}{\frac{1}{\ln \frac{1-F\left(\sigma^*_{n_{m+1}}|L\right)}{1-F\left(\sigma^*_{n_{m+1}}|H\right)}} - \frac{1}{\ln \frac{1-F\left(\sigma^*_{n_m}|L\right)}{1-F\left(\sigma^*_{n_m}|H\right)}}}$$

provided the right hand side exists. Expanding in the same manner as in Lemma 6, the right hand side is equal to

$$\lim (n_{m+1} - n_m) \frac{\ln\left(1 - F\left(\sigma^*_{n_{m+1}}|H\right)\right) \ln\left(1 - F\left(\sigma^*_{n_m}|H\right)\right)}{\ln\left(1 - F\left(\sigma^*_{n_m}|H\right)\right) - \ln\left(1 - F\left(\sigma^*_{n_{m+1}}|H\right)\right)}$$

$$\times \frac{\ln\left(1 - F\left(\sigma^*_{n_m}|H\right)\right) - \ln\left(1 - F\left(\sigma^*_{n_{m+1}}|H\right)\right)}{\ln \frac{1-F\left(\sigma^*_{n_m}|L\right)}{1-F\left(\sigma^*_{n_m}|H\right)} - \ln \frac{1-F\left(\sigma^*_{n_{m+1}}|L\right)}{1-F\left(\sigma^*_{n_{m+1}}|H\right)}} \frac{\ln \frac{1-F\left(\sigma^*_{n_{m+1}}|L\right)}{1-F\left(\sigma^*_{n_{m+1}}|H\right)}}{\ln\left(1 - F\left(\sigma^*_{n_{m+1}}|H\right)\right)} \frac{\ln \frac{1-F\left(\sigma^*_{n_m}|L\right)}{1-F\left(\sigma^*_{n_m}|H\right)}}{\ln\left(1 - F\left(\sigma^*_{n_m}|H\right)\right)}$$

From (39), $\lim\left(1 - F\left(\sigma^*_{n_m}|H\right)\right)^{n_m}$ exists and is strictly positive. By the discrete version of l'Hôpital's rule,

$$\lim \frac{n_{m+1} - n_m}{\frac{1}{\ln 1-F\left(\sigma^*_{n_{m+1}}|H\right)} - \frac{1}{\ln 1-F\left(\sigma^*_{n_m}|H\right)}} = \lim \ln\left(1 - F\left(\sigma^*_{n_m}|H\right)\right)^{n_m} > -\infty.$$

From limits (36) and (37) (see the proof of Lemma 6), it follows that $\lim \ln\left(\frac{1-F\left(\sigma^*_{n_m}|L\right)}{1-F\left(\sigma^*_{n_m}|H\right)}\right)^{n_m}$ exists and is finite. But this contradicts (40), completing the proof.

Part 2: Suppose to the contrary that (17) does not hold. So there exists a $\delta > 0$ such that for any Λ and N $\sup_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N} \Pr(A_n) \geq \delta$. So we can construct a sequence (λ_m, n_m) for which

$$\Pr\left(A_{n_m}(x_{n_m}(\lambda_m), \lambda_m)|H\right) \geq \delta$$

and $n_m \to \infty$ and either $\lambda_m = \lambda_0$ if $\lambda_0 > 0$, or $\lambda_m \to 0$ if $\lambda_m = 0$. By (16),

$$\limsup_m x_{n_m}(\lambda_m) - x_H(\lambda_m) \leq 0.$$

As in Part 1, by Bolzano-Weierstrass we can without loss assume that the subsequence (λ_m, n_m) is chosen so that equations (38) and (39) hold. The remainder of the proof exactly parallels that of Part 1. ∎

PROOF OF LEMMA 8

By definition, $\Delta^L(x_L(\lambda = 0, \alpha = 1), \sigma_L, \lambda = 0) = 0$, and by Assumption 5 Δ^L is strictly increasing in x. By Assumption 3, $\Delta^L(x = 0, \sigma_L, \lambda = 0) < 0$, and so certainly $x_L(0, 1) > 0$. Moreover, $\sigma_L(\alpha = 1) = \underline{\sigma}$. Consequently by choosing x sufficiently close to $x_L(0, 1)$, the term $\Delta^L(x, \underline{\sigma})$ can be made negative and arbitrarily close to 0. As such, there exists a $\kappa_x > 0$ such that

$$-\frac{\Delta^H(x_L(0,1) - \kappa_x, \underline{\sigma}, \lambda = 0)}{\Delta^L(x_L(0,1) - \kappa_x, \underline{\sigma}, \lambda = 0)} \frac{f(\underline{\sigma}|H)\Pr(H)}{f(\underline{\sigma}|L)\Pr(L)} \frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L)} > 1.$$

By continuity, it follows that there exists a $\Lambda > 0$ such that for all $\lambda \leq \Lambda$,

$$-\frac{\Delta^H(x_L(\lambda,1) - \kappa_x, \underline{\sigma}, \lambda)}{\Delta^L(x_L(\lambda,1) - \kappa_x, \underline{\sigma}, \lambda)} \frac{f(\underline{\sigma}|H)\Pr(H)}{f(\underline{\sigma}|L)\Pr(L)} \frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L)} > 1.$$

By MLRP $\frac{1-F(\sigma|H)}{1-F(\sigma|L)} \geq 1$ for any σ (see Lemma 12). Consequently, the expression

$$\Delta^H(x_L(\lambda,1) - \kappa_x, \underline{\sigma}, \lambda) \frac{f(\underline{\sigma}|H)\Pr(H)}{f(\underline{\sigma}|L)\Pr(L)} \frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L)} \left(\frac{1-F(\underline{\sigma}|H)}{1-F(\underline{\sigma}|L)}\right)^{n-1}$$
$$+\Delta^L(x_L(\lambda,1) - \kappa_x, \underline{\sigma}, \lambda)$$

is strictly positive. Since for any consistent beliefs $\frac{b}{1-b} \geq \frac{f(\underline{\sigma}|H)\Pr(H)}{f(\underline{\sigma}|L)\Pr(L)}$, and both $\frac{f(\sigma|H)}{f(\sigma|L)}$ and $\frac{1-F(\sigma|H)}{1-F(\sigma|L)}$ are increasing in σ, it follows that $Z(x_L(\lambda,1) - \kappa_x, \sigma; n, \alpha = 1, \lambda, b) > 0$ for all σ. Thus $x_L(\lambda,1) - \kappa_x > \bar{x}_n(\alpha = 1, \lambda, b)$. That is, the offer $x_L(\lambda,1) - \kappa_x$ is always accepted. ∎

Proof of Lemma 9

The proof is by contradiction. Suppose that contrary to the claim, for all $\kappa_x > 0$ there exists an $\varepsilon > 0$ such that for all N there is a σ_0 such that

$$\sup_{\Lambda} \inf_{\lambda < \Lambda, n \geq N} \pi_n^* (\sigma_0; \lambda, \alpha = 1) - x_H(\lambda) < \kappa_x - \varepsilon. \tag{41}$$

We claim this implies that $\sup_{\Lambda,N} \inf_{0<\lambda\leq\Lambda, n\geq N} \pi_n^* (\sigma_0; \lambda, \alpha = 1) - x_H(\lambda)$ is weakly negative. For suppose to the contrary that this expression equals a strictly positive constant C. Then for any $\varepsilon > 0$, there is some N such that

$$\sup_{\Lambda} \inf_{0<\lambda\leq\Lambda, n\geq N} \pi_n^* (\sigma_0; \lambda, \alpha = 1) - x_H(\lambda) \geq C - \varepsilon.$$

But this contradicts (41) above. For the remainder of the proof we write $x_n(\lambda) = \pi_n^*(\sigma_0; \lambda, \alpha = 1)$. From Lemma 7, for $\omega = L, H$,

$$\sup_{\Lambda,N} \inf_{0<\lambda\leq\Lambda, n\geq N} \Pr(A_n(x_n(\lambda))|\omega) = 0.$$

The proposer's expected payoff under offer $x_n(\lambda)$ is

$$\begin{aligned}
\Pi_n^P(x_n(\lambda), \lambda, \alpha, \sigma_0, \beta) &= E\left[\bar{V}(\omega, \sigma_0)|\sigma_0\right] \\
&\quad + \sum_{\omega} \Pr(A_n(x_n(\lambda))|\omega) \Pr(\omega|\sigma_0) \left(V(x_n(\lambda), \omega, \sigma_0) - \bar{V}(\omega, \sigma_0)\right) \\
&\leq E\left[\bar{V}(\omega, \sigma_0)|\sigma_0\right] \\
&\quad + \Pr(A_n(x_n(\lambda))|H) \sum_{\omega} \Pr(\omega|\sigma_0) \left(V(x_n(\lambda), \omega, \sigma_0) - \bar{V}(\omega, \sigma_0)\right)
\end{aligned}$$

by Assumption 4 and since $\Pr(A_n(x_n(\lambda))|H) \geq \Pr(A_n(x_n(\lambda))|L)$. From this, it follows that

$$\sup_{\Lambda,N} \inf_{0<\lambda\leq\Lambda, n\geq N} \Pi_n^P(x_n(\lambda), \lambda, \alpha, \sigma_0, \beta) = E\left[\bar{V}(\omega, \sigma_0)|\sigma_0\right].$$

Now, consider an alternative set of offers defined by $\tilde{x}_n(\lambda) = x_H(\lambda) + \varepsilon$. By Lemma 6,

$$\sup_{\Lambda,N} \inf_{0<\lambda\leq\Lambda, n\geq N} \Pr(A_n(\tilde{x}_n(\lambda))|H) > 0.$$

By Assumption 4, the proposer's expected payoff is certainly greater than

$$E\left[\bar{V}(\omega, \sigma_0)|\sigma_0\right] + \Pr(A_n(\tilde{x}_n(\lambda))|H) \Pr(H|\sigma_0) \left(V(\tilde{x}_n(\lambda), H, \sigma_0) - \bar{V}(H, \sigma_0)\right).$$

Consequently, for ε chosen small enough,

$$\sup_{\Lambda,N} \inf_{0<\lambda\leq\Lambda,n\geq N} \Pi_n^P \left(\tilde{x}_n\left(\lambda\right),\lambda,\alpha,\sigma_0,\beta\right) > E\left[\bar{V}\left(\omega,\sigma_0\right)|\sigma_0\right].$$

But then the proposer's original set of offers $x_n\left(\lambda\right)$ cannot have been an equilibrium, giving a contradiction. ∎

Proof of Lemma 10

When x is such that the equilibrium is either a non-responsive rejection equilibrium, or a non-responsive acceptance equilibrium, the result follows trivially. Below, we focus on the case in which there is a responsive voting equilibrium given x and b.

Fix a set of beliefs b and an offer x. For the remainder of the proof, we write $b\left(H\right)$ for b and $b\left(L\right)$ for $1-b$. The average responder's expected payoff can be decomposed as

$$\begin{aligned}\Pi_n^V\left(x,\lambda,\alpha,b\right) &= \sum_\omega b\left(\omega\right)\left(E\left[\left(1-\lambda\right)\bar{v}\left(\omega\right)+\lambda\bar{u}\left(\sigma_i\right)|\omega\right]\right) \\ &\quad +\sum_\omega b\left(\omega\right)\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\frac{1}{n}\sum_i\Delta^\omega\left(x,\sigma_i,\lambda\right)|A_n\left(x,b\right),\omega\right].\end{aligned}$$

Now,

$$\begin{aligned}&\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\frac{1}{n}\sum_i\Delta^\omega\left(x,\sigma_i,\lambda\right)|A_n\left(x,b\right),\omega\right] \\ =\; &\frac{\partial}{\partial x}\left(\int_{\sigma^*}\cdots\int_{\sigma^*}\frac{1}{n}\sum_i\Delta^\omega\left(x,\sigma_i,\lambda\right)dF\left(\sigma_1|\omega\right)\ldots dF\left(\sigma_n|\omega\right)\right) \\ =\; &\frac{\partial}{\partial x}\left(\left(1-F\left(\sigma^*|\omega\right)\right)^{n-1}\int_{\sigma^*}\Delta^\omega\left(x,\sigma,\lambda\right)dF\left(\sigma|\omega\right)\right) \\ =\; &-\frac{\partial\sigma^*}{\partial x}\left(1-F\left(\sigma^*|\omega\right)\right)^{n-1}\Delta^\omega\left(x,\sigma^*,\lambda\right)f\left(\sigma^*|\omega\right) \\ &-\frac{\partial\sigma^*}{\partial x}f\left(\sigma^*|\omega\right)\left(n-1\right)\left(1-F\left(\sigma^*|\omega\right)\right)^{n-2}\int_{\sigma^*}\Delta^\omega\left(x,\sigma,\lambda\right)dF\left(\sigma|\omega\right) \\ &+\left(1-F\left(\sigma^*|\omega\right)\right)^{n-1}\int_{\sigma^*}\Delta_x^\omega\left(x,\sigma,\lambda\right)dF\left(\sigma|\omega\right). \qquad (42)\end{aligned}$$

Since

$$\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)=\frac{\partial}{\partial x}(1-F\left(\sigma^*|\omega\right))^n=-n\frac{\partial\sigma^*}{\partial x}f\left(\sigma^*|\omega\right)\left(1-F\left(\sigma^*|\omega\right)\right)^{n-1},$$

and

$$(1-F\left(\sigma^*|\omega\right))^{n-1}\int_{\sigma^*}\Delta_x^\omega\left(x,\sigma,\lambda\right)dF\left(\sigma|\omega\right)=\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)\left(x,\sigma\right)|\omega,\sigma\geq\sigma^*\right]$$

then expression (42) rewrites as

$$\begin{aligned}
&\frac{1}{n}\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)\left(\Delta^\omega\left(x,\sigma^*,\lambda\right)+\frac{n-1}{1-F\left(\sigma^*|\omega\right)}\int_{\sigma^*}\Delta^\omega\left(x,\sigma,\lambda\right)dF\left(\sigma|\omega\right)\right)\\
&+\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)|\omega,\sigma\geq\sigma^*\right]\\
=\ &\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)\left(\frac{1}{n}\Delta^\omega\left(x,\sigma^*,\lambda\right)+\frac{n-1}{n}E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)\left(x,\sigma\right)|\omega,\sigma\geq\sigma^*\right]\right)\\
&+\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)|\omega,\sigma\geq\sigma^*\right]
\end{aligned}$$

Consequently

$$\begin{aligned}
\frac{\partial\Pi_n^V}{\partial x}\ =\ &\frac{1}{n}\sum_\omega b\left(\omega\right)\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)\Delta^\omega\left(x,\sigma^*,\lambda\right)\\
&+\frac{n-1}{n}\sum_\omega b\left(\omega\right)\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)\left(x,\sigma\right)|\omega,\sigma\geq\sigma^*\right]\\
&+\sum_\omega b\left(\omega\right)\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)|\omega,\sigma\geq\sigma^*\right].
\end{aligned}$$

Now, from the equilibrium condition (7):

$$\frac{\frac{\partial\Pr(A_n(x,b)|H)}{\partial x}}{\frac{\partial\Pr(A_n(x,b)|L)}{\partial x}}=\frac{f\left(\sigma^*|H\right)\left(1-F\left(\sigma^*|H\right)\right)^{n-1}}{f\left(\sigma^*|L\right)\left(1-F\left(\sigma^*|L\right)\right)^{n-1}}=-\frac{\Delta^L\left(x,\sigma^*,\lambda\right)}{\Delta^H\left(x,\sigma^*,\lambda\right)}\frac{b\left(L\right)}{b\left(H\right)}$$

and so

$$\sum_\omega b\left(\omega\right)\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)\Delta^\omega\left(x,\sigma^*,\lambda\right)=0.$$

Since $\Delta^\omega\left(x,\sigma,\lambda\right)$ is increasing in σ (Assumption 2), then it follows that

$$\sum_\omega b\left(\omega\right)\frac{\partial}{\partial x}\Pr\left(A_n\left(x,b\right)|\omega\right)E\left[\Delta_x^\omega\left(x,\sigma,\lambda\right)\left(x,\sigma\right)|\omega,\sigma\geq\sigma^*\right]\geq0$$

also. The result then follows. ∎

Proof of Proposition **2**

We start with some preliminary bounds:

From Lemma 9, we know that there exists a $\kappa_x > 0$ and an N_x such that for all σ_0,

$$\sup_{\Lambda \geq \lambda_0} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N_x} \pi_n^* (\sigma_0; \lambda, \alpha = 1) - x_H(\lambda) \geq \frac{\kappa_x}{2}.$$

For any λ, define a sequence of offers by $\hat{x}_n(\lambda) = x_H(\lambda) + \kappa_x/4$. Let $\underline{b}$ and $\bar{b}$ respectively denote the most pro-L and pro-H beliefs possible, i.e., $\frac{\underline{b}}{1-\underline{b}} = \frac{f_0(\underline{\sigma}|H)}{f_0(\underline{\sigma}|L)} \frac{\Pr(H)}{\Pr(L)}$ and $\frac{\bar{b}}{1-\bar{b}} = \frac{f_0(\bar{\sigma}|H)}{f_0(\bar{\sigma}|L)} \frac{\Pr(H)}{\Pr(L)}$. From Lemma 6, there exists a $\kappa_A > 0$ and an N_A such that

$$\sup_{\Lambda} \inf_{\lambda \in (0,\Lambda], n \geq N_A} \Pr(A_n(\hat{x}_n(\lambda), \underline{b}) | H) \geq \kappa_A.$$

Since $\frac{\partial}{\partial x}\Delta^H$ is bounded uniformly away from 0, there exists a $\gamma > 0$ such that $E\left[\frac{\partial}{\partial x}\Delta^H(x, \sigma_i, \lambda) | H, \sigma_i \geq \sigma^*\right] > \gamma$ for all x and λ.

By Lemma 7, choose an N_L such that

$$\inf_{\Lambda \geq \lambda_0, N} \sup_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq N_L} \Pr\left(A_n\left(x_H(\lambda), \bar{b}\right) | L\right) \max\left|\Delta^L\right| \leq \underline{b}\kappa_A \gamma \frac{\kappa_x}{8}. \tag{43}$$

We are now ready to establish the result.

For any λ and n, consider an equilibrium $(\pi_n^*, \sigma_n^*, \beta_n)$. Define $\hat{N} = \max\{N_x, N_A, N_L\}$. We show below that for any σ_0,

$$\sup_{\Lambda \geq \lambda_0} \inf_{\lambda_0 \leq \lambda \leq \Lambda, \lambda > 0, n \geq \hat{N}} \Pi_n^V(\pi_n^*(\sigma_0), \lambda, \alpha, \beta_n(\pi_n^*(\sigma_0))) - E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] \geq \underline{b}\kappa_A \gamma \frac{\kappa_x}{4}. \tag{44}$$

Inequality (44) is enough to establish the Proposition, as follows. We first claim that from

(44) it follows that there exists a $\bar{\Lambda}$ such that

$$\inf_{\lambda \in (0,\bar{\Lambda}], n \geq \hat{N}} \Pi_n^V(\pi_n^*(\sigma_0), \lambda, \alpha, \beta_n(\pi_n^*(\sigma_0))) - E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] \geq \underline{b}\kappa_A \gamma \frac{\kappa_x}{8}. \tag{45}$$

We prove this claim by establishing that there exists a $\bar{\Lambda}$ such that

$$\inf_{n \geq \hat{N}} \Pi_n^V(\pi_n^*(\sigma_0), \lambda, \alpha, \beta_n(\pi_n^*(\sigma_0))) - E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] > \underline{b}\kappa_A \gamma \frac{\kappa_x}{8} \tag{46}$$

whenever $\lambda \in (0, \bar{\Lambda}]$. Suppose to the contrary that (46) does not hold. Then for any Λ there exists a $\tilde{\lambda} \leq \Lambda$ such that

$$\inf_{n \geq \hat{N}} \Pi_n^V \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \leq \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}.$$

But then

$$\inf_{\lambda \in (0, \Lambda], n \geq \hat{N}} \Pi \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \leq \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}$$

and

$$\sup_{\Lambda > 0} \inf_{\lambda \in (0, \Lambda], n \geq \hat{N}} \Pi_n^V \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \leq \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}$$

also, contradicting (44).

From (45), we then know that

$$\begin{aligned}
& \sup_{\Lambda > 0} \inf_{\lambda \in (0, \Lambda], n \geq \hat{N}} \int \Pi_n^V \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) dF\left(\sigma_0\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \\
\geq \quad & \sup_{\Lambda \in (0, \bar{\Lambda}]} \inf_{\lambda \in (0, \Lambda], n \geq \hat{N}} \int \Pi_n^V \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) dF\left(\sigma_0\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \\
\geq \quad & \sup_{\Lambda \in (0, \bar{\Lambda}]} \int \inf_{\lambda \in (0, \Lambda], n \geq \hat{N}} \Pi_n^V \left(\pi_n^* \left(\sigma_0\right), \lambda, \alpha, \beta_n \left(\pi_n^* \left(\sigma_0\right)\right)\right) dF\left(\sigma_0\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \\
\geq \quad & \sup_{\Lambda \in (0, \bar{\Lambda}]} \int \left(E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] + \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}\right) dF\left(\sigma_0\right) - E_{\sigma_i, \omega} \left[\bar{U}^\omega \left(\sigma_i, \lambda\right)\right] \\
= \quad & \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}.
\end{aligned}$$

Setting $\gamma = \underline{b} \kappa_A \gamma \frac{\kappa_x}{8}$ then gives the result.

Proof of inequality (44):

Consider a particular realization of σ_0. For the remainder of the proof, we write $x_n^* (\lambda)$ for $\pi_n^* (\sigma_0, \lambda)$, and drop λ where doing so will create no confusion. Likewise, let $b_n (\lambda)$ denote the responders' equilibrium beliefs at the offer $x_n^* (\lambda)$, i.e., $b_n (\lambda) = \beta_n (x_n^* (\lambda), \lambda)$ for $\omega = L, H$.

Arithmetically, the responders' expected payoff from the equilibrium offer x_n^* given the equilibrium beliefs b_n can be decomposed as

$$\Pi_n^V \left(x_n^*, \lambda, \alpha, b_n\right) \quad = \quad \Pi_n^V \left(x_H (\lambda), \lambda, \alpha, b_n\right)$$

$$+\Pi_n^V\left(\hat{x}_n\left(\lambda\right),\lambda,\alpha,b_n\right)-\Pi_n^V\left(x_H\left(\lambda\right),\lambda,\alpha,b_n\right)$$
$$+\Pi_n^V\left(x_n^*,\lambda,\alpha,b_n\right)-\Pi_n^V\left(\hat{x}_n\left(\lambda\right),\lambda,\alpha,b_n\right). \tag{47}$$

From Lemma 10, the derivative of Π_n^V between $x_H\left(\lambda\right)$ and $\hat{x}_n\left(\lambda\right)$ is positive, and so

$$\Pi_n^V\left(x_n^*,\lambda,\alpha,b_n\right)-\Pi_n^V\left(\hat{x}_n\left(\lambda\right),\lambda,\alpha,b_n\right)\geq 0$$

By the construction of $\hat{x}_n\left(\lambda\right)$, the acceptance probability given offer $\hat{x}_n\left(\lambda\right)$ and beliefs b_n satisfies

$$\sup_{\Lambda\geq\lambda_0}\inf_{\lambda_0\leq\lambda\leq\Lambda,\lambda>0,n\geq N_A}\Pr\left(A_n\left(\hat{x}_n\left(\lambda\right),b_n\right)|H\right)>\kappa_A,$$

since under beliefs b_n acceptance is (weakly) more likely than under $\underline{b}$ (see Corollary 1). Consequently, since $x_n^*-\hat{x}_n\left(\lambda\right)\geq\frac{\kappa_x}{4}$, then by Lemma 10,

$$\sup_{\Lambda\geq\lambda_0}\inf_{\lambda_0\leq\lambda\leq\Lambda,\lambda>0,n\geq\max\{N_x,N_R,N_A\}}\Pi_n^V\left(x_n^*,\lambda,\alpha,b_n\right)-\Pi_n^V\left(\hat{x}_n\left(\lambda\right),\lambda,\alpha,b_n\right)\geq\underline{b}\kappa_A\gamma\frac{\kappa_x}{4}.$$

Finally,

$$\begin{aligned}&\Pi_n^V\left(x_H\left(\lambda\right),\lambda,\alpha,b_n\right)E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i,\lambda\right)\right]\\ =\;&\Pr\left(A_n\left(x_H\left(\lambda\right),b_n\right)|L\right)E\left[\Delta^L\left(x_H\left(\lambda\right),\sigma_i,\lambda\right)|A_n\left(x_H\left(\lambda\right),b_n\right),L\right].\end{aligned}$$

From (43), regardless of whether the expression $E\left[\Delta^L\left(x_H\left(\lambda\right),\sigma_i,\lambda\right)|A_n\left(x_H\left(\lambda\right),b_n\right),L\right]$ is positive or negative,

$$\sup_{\Lambda\geq\lambda_0}\inf_{\lambda_0\leq\lambda\leq\Lambda,\lambda>0,n\geq N_L}\Pi_n^V\left(x_H\left(\lambda\right),\lambda,\alpha,b_n\right)-E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i,\lambda\right)\right]\geq-\underline{b}\kappa_A\gamma\frac{\kappa_x}{8}.$$

From (47), it follows that

$$\sup_{\Lambda\geq\lambda_0}\inf_{\lambda_0\leq\lambda\leq\Lambda,\lambda>0,n\geq\max\{N_x,N_R,N_A,N_L\}}\Pi_n^V\left(x_n^*,\lambda,\alpha,b_n\right)-E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i,\lambda\right)\right]\geq\underline{b}\kappa_A\gamma\frac{\kappa_x}{4}.$$

The result then follows. ∎

Proof of Corollary 3:

From Proposition 2, there exists a $\gamma > 0$, an N_1 and $\Lambda_1 > 0$ such that

$$\inf_{0<\lambda\leq\Lambda_1, n\geq N_1} \left(\Pi_n^{*V}(\lambda,1) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right]\right) \geq \frac{\gamma}{2}.$$

Choose Λ_α such that

$$\Pr(H) E_{\sigma_i}\left[\Delta^H(x_H(\lambda,\alpha),\sigma_i,\lambda)|H\right] \leq \frac{\gamma}{8},$$

for all $\lambda \in (0,\Lambda_\alpha]$. Choose $\Lambda_W > 0$ such that $\min_{\sigma_0\in[\underline{\sigma},\bar{\sigma}]} W(\sigma_0;\lambda,\alpha) > 0$ whenever $\lambda \leq \Lambda_W$. Then from Proposition 1, for any $\lambda \leq \min\{\Lambda_\alpha,\Lambda_W\}$ there exists an $N_\alpha(\lambda)$ such that

$$\Pi_n^{*V}(\lambda,\alpha) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right] \leq \frac{\gamma}{4} \text{ whenever } n \geq N_\alpha(\lambda).$$

Consequently, whenever $\lambda \leq \min\{\Lambda_\alpha,\Lambda_W,\Lambda_1\}$ and $n \geq \max\{N_\alpha(\lambda),N_1\}$,

$$\Pi_n^{*V}(\lambda,1) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right] \geq \frac{\gamma}{2} > \frac{\gamma}{4} \geq \Pi_n^{*V}(\lambda,\alpha) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right].$$

Setting $\gamma_\Pi = \gamma/4$ and $\bar{\lambda} = \min\{\Lambda_\alpha,\Lambda_W,\Lambda_1\}$ completes the proof. ∎

Proof of Corollary 4:

From Proposition 3, there exists a $\gamma > 0$ and a $\Lambda > 0$ such that whenever $\lambda \leq \Lambda$,

$$\Pi_n^{*V}(\lambda,1) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right] \leq E_{\sigma_i,\omega}\left[\Delta^\omega(x_L(\lambda,1),\sigma_i,\lambda)\right] - \gamma.$$

As such, there exists a $\Lambda_1 \leq \Lambda$ such that whenever $\lambda \leq \Lambda_1$,

$$\Pi_n^{*V}(\lambda,1) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right] \leq E_{\sigma_i,\omega}\left[\Delta^\omega(x_L(0,1),\sigma_i,0)\right] - \frac{\gamma}{2}.$$

Moreover, there exists a Λ_α such that when $\lambda \leq \Lambda_\alpha$

$$\left|E_{\sigma_i,\omega}\left[\Delta^\omega(x_L(0,\alpha),\sigma_i,0)\right] - E_{\sigma_i,\omega}\left[\Delta^\omega(x_L(\lambda,\alpha),\sigma_i,\lambda)\right]\right| < \frac{\gamma}{8}.$$

As such, when $\lambda \leq \min\{\Lambda_\alpha, \Lambda_1\}$ from Proposition 1 there exists an N_α such that when $n \geq N_\alpha$,

$$\begin{aligned} \Pi_n^{*V}(\lambda,\alpha) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right] &\geq E_{\sigma_i,\omega}\left[\Delta^\omega\left(x_L(\lambda,\alpha),\sigma_i,\lambda\right)\right] - \frac{\gamma}{4} \\ &> \Pi_n^{*V}(\lambda,1) - E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i,\lambda)\right]. \end{aligned}$$

■

Proof of Proposition 3:

Immediate from Lemma 8 and Lemma 10. ■

Proof of Proposition 4

For each n and λ and the majority voting rule α, fix an equilibrium π_n^* and β_n, and define $\hat{x}_n(\sigma_0;\lambda) = \pi_n^*(\sigma_0;\lambda,\alpha)$ and $\hat{b}_n(\sigma_0;\lambda) = \beta_n(\pi^*(\sigma_0;\lambda,\alpha),\lambda,\alpha)$ for the equilibrium offers and beliefs given voting rule α, and

$$\hat{A}_n(\sigma_0,\lambda) = A_n\left(\hat{x}_n(\sigma_0;\lambda), \hat{b}_n(\sigma_0;\lambda), \lambda, \alpha\right)$$

for the associated acceptance event. Define $x_n(\sigma_0;\lambda)$, $b_n(\sigma_0;\lambda)$ and $A_n(\sigma_0,\lambda)$ similarly for the unanimity voting rule.

By Assumptions 3 and 4,

$$U^H(x=1,\sigma_i,\lambda=0) + V^H(x=1,\sigma_0) > \bar{U}^H(\sigma_i,\lambda=0) + \bar{V}^H(\sigma_0);$$

and Assumption 7 combined with the hypothesis that $x_L(\lambda,\alpha) = \infty$ imply

$$U^L(x=1,\sigma_i,\lambda=0) + V^L(x=1,\sigma_0) < \bar{U}^L(\sigma_i,\lambda=0) + \bar{V}^L(\sigma_0).$$

Under Assumption 7, there exists a linear transformation of the payoff functions such that (after the transformation)

$$\kappa^\omega \equiv U^\omega(x,\sigma_i,\lambda=0) + V^\omega(x,\sigma_0) - \left(\bar{U}^\omega(\sigma_i,\lambda=0) + \bar{V}^\omega(\sigma_0)\right)$$

is independent of the offer x. From above, $\kappa^H > 0 > \kappa^L$.

From Corollary 3, there exists a $\bar{\lambda} > 0$ and a γ_Π such that whenever $\lambda \leq \bar{\lambda}$, (18) holds for n greater than some $N_\Pi(\lambda)$.

By continuity, for any $\varepsilon > 0$ there exists a $\Lambda(\varepsilon) > 0$ such that for any σ_0, σ_i and $\lambda \in [0, \Lambda(\varepsilon)]$,

$$\left| U^\omega(\hat{x}_n(\sigma_0;\lambda), \sigma_i, \lambda) + V^\omega(\hat{x}_n(\sigma_0;\lambda), \sigma_0) - \bar{U}^\omega(\sigma_i, \lambda) - \bar{V}^\omega(\sigma_0) \right| \geq |\kappa^\omega| - \varepsilon.$$

By Lemma 5, when λ is close enough to 0 the acceptance probabilities under the equilibrium offer given the the majority rule α converge to 0 and 1 in $\omega = L, H$ respectively, i.e., $\Pr\left(\hat{A}_n(\sigma_0, \lambda) | L\right) \to 0$ and $\Pr\left(\hat{A}_n(\sigma_0, \lambda) | H\right) \to 1$ as $n \to \infty$. As such, there exists a $\Lambda > 0$ and an $N(\lambda)$ for each $\lambda \leq \Lambda$ such that if $n \geq N(\lambda)$, the expression

$$\int_{\sigma_0} \Pr(A_n(\sigma_0, \lambda) | \omega) \left(E_{\sigma_i}\left[U^\omega(x_n(\sigma_0;\lambda), \sigma_i, \lambda) | \omega, A_n(\sigma_0, \lambda)\right] + V^\omega(x_n(\sigma_0;\lambda), \sigma_0)\right) dF(\sigma_0)$$
$$- \int_{\sigma_0} \Pr\left(\hat{A}_n(\sigma_0, \lambda) | \omega\right) \left(E_{\sigma_i}\left[U^\omega(\hat{x}_n(\sigma_0;\lambda), \sigma_i, \lambda) | \omega, \hat{A}_n(\sigma_0, \lambda)\right] + V^\omega(\hat{x}_n(\sigma_0;\lambda), \sigma_0)\right) dF(\sigma_0)$$

is bounded above by $\gamma_\Pi/2$ for both $\omega = L, H$. That is, since (as noted in the main text) the outcome under the majority voting rule is approximately Pareto efficient, and payoffs are linear, then under the unanimity rule the sum of the payoffs is generally lower, and is certainly not significantly higher.

From this, it follows immediately that when $\lambda \leq \Lambda$ and $n \geq N(\lambda)$,

$$\Pi_n^{*V}(\lambda, 1) + \Pi_n^{*P}(\lambda, 1) \leq \Pi_n^{*V}(\lambda, \alpha) + \Pi_n^{*P}(\lambda, \alpha) + \frac{\gamma_\Pi}{2}.$$

Consequently, when $\lambda \leq \min\left\{\bar{\lambda}, \Lambda\right\}$ and $n \geq \max\{N_\Pi(\lambda), N(\lambda)\}$, then $\Pi_n^{*P}(\lambda, 1) \leq \Pi_n^{*P}(\lambda, \alpha) - \gamma_\Pi/2$. ■

Proof of Lemma 11

Let b_n be the corresponding sequence of beliefs. If the equilibrium is responsive, the equilibrium cutoff value is a solution to $Z(x_n, \sigma) = 0$. But by the assumption $\Delta^H(x = 1, \underline{\sigma}, \lambda)$, it is immediate that there exists a $\varepsilon > 0$ independent of n such that $Z(x_n, \cdot)$ is strictly negative over $[\underline{\sigma}, \underline{\sigma} + \varepsilon]$. As such, $\sigma_n^* > \underline{\sigma} + \varepsilon$. On the other hand, if the equilibrium is unresponsive then since $\bar{x}_n(\alpha, \lambda, b_n) = 1$ it is the rejection equilibrium, and $\sigma_n^* = \bar{\sigma}$. In either case, the probability that each responder votes to accept is bounded away from 1, and so the probability that the offer x_n is accepted converges to 0.. ∎

Proof of Proposition 5

Fix a preference parameter λ sufficiently small such that $\Delta^H(x = 1, \underline{\sigma}, \lambda) < 0$ (this is possible by Assumption 8), and choose $\varepsilon > 0$. By Assumption 3, $\Delta^H(x = 1, \bar{\sigma}, \lambda) > 0$. Recall that $\sigma_H(\alpha)$ is decreasing α. Provided λ and ε are small enough, there exists a majority voting rule $\hat{\alpha} < 1$ such

$$\Delta^H(1, \sigma_H(\hat{\alpha}), \lambda) > 0 > \Delta^L(1, \sigma_L(\hat{\alpha}), \lambda) \tag{48}$$

and

$$\sigma_H(\hat{\alpha}) + \varepsilon < E[\sigma_i | H]. \tag{49}$$

From (48), $x_H(\lambda, \hat{\alpha}) \neq \infty = x_L(\lambda, \hat{\alpha})$, and so by Proposition 1 the responders' payoff has the following limit as the number of responders grows large:

$$\Pi_n^{*V}(\lambda, \alpha) \to E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i)\right] + \Pr(H) E\left[\Delta^H(x_H(\lambda, \hat{\alpha}), \sigma_i, \lambda) | H\right].$$

By Jensen's inequality, the above limiting expression is weakly greater than

$$\begin{aligned} & E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i)\right] + \Pr(H) \Delta^H(x_H(\lambda, \hat{\alpha}), E[\sigma_i | H], \lambda) \\ \geq \; & E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i)\right] + \Pr(H) \Delta^H(x_H(\lambda, \hat{\alpha}), \sigma_H(\hat{\alpha}) + \varepsilon, \lambda) \\ \geq \; & E_{\sigma_i, \omega}\left[\bar{U}^\omega(\sigma_i)\right] + \Pr(H) \varepsilon \min_{\sigma \in [\sigma_H(\hat{\alpha}), \sigma_H(\hat{\alpha}) + \varepsilon]} \frac{\partial}{\partial \sigma} \Delta^H(x_H(\lambda, \hat{\alpha}), \sigma, \lambda), \end{aligned}$$

where the first inequality follows from (49). In contrast, from Lemma 11, under the unanimity rule the responders' payoff converges to $E\left[\bar{U}^\omega(\sigma_i, \lambda)\right]$. Thus when the number of responders n is sufficiently large, the responders' payoff is higher under the majority rule

$\hat{\alpha}$. For the proposer, simply observe that the acceptance probability under $\hat{\alpha}$ converges to $\Pr(H)$ while the acceptance probability under the unanimity rule converges to 0. From (48), $x_H(\lambda, \hat{\alpha}) < 1$, and so the proposer is strictly better off when his offer is accepted. This completes the proof. ∎

B Technical appendix (omitted from paper)

PROOF OF LEMMA 4

We prove the lemma in four steps.

Claim 1 *If* $\limsup x_n < x_H(\lambda)$ *then* $\liminf \sigma_n^* > \sigma_H$.

Proof: By hypothesis, there exists ε such that $x_n \leq x_H(\lambda) - \varepsilon$ for all n large enough. Suppose now to the contrary that $\liminf \sigma_n^* \leq \sigma_H$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_H + \delta$.

By definition $\Delta^H(x_H(\lambda), \sigma_H, \lambda) = 0$; so for δ small enough, there exists a $\hat{\varepsilon}$ such that $\Delta^H(x_n, \sigma_n^*, \lambda) < -\hat{\varepsilon}$. Moreover, $\Delta^L(x_n, \sigma_n^*, \lambda) \leq \Delta^H(x_n, \sigma_n^*, \lambda)$. Consequently $Z(x_n, \sigma_n^*) < 0$. As such, σ_n^* is not a responsive equilibrium; and since $x_n \leq \bar{x}_n$ then σ_n^* is not an acceptance equilibrium either. The only remaining possibility is that σ_n^* is a rejection equilibrium — but then $\sigma_n^* = \bar{\sigma}$, which gives a contradiction when δ is chosen small enough. ∎

Claim 2 *If* $\limsup x_n < x_L(\lambda)$ *then* $\liminf \sigma_n^* > \sigma_L$.

Proof: By hypothesis, there exists ε such that $x_n \leq x_L(\lambda) - \varepsilon$ for all n large enough. Suppose now to the contrary that $\liminf \sigma_n^* \leq \sigma_L$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_L + \delta$.

By definition $\Delta^L(x_L(\lambda), \sigma_L, \lambda) = 0$; so for δ small enough, there exists a $\hat{\varepsilon}$ such that $\Delta^L(x_n, \sigma_n^*, \lambda) < -\hat{\varepsilon}$.

Next, define

$$\phi = \max_{\sigma \in [\underline{\sigma}, \sigma_L + \delta]} \frac{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}}{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}$$

Note that the function $(1-q)^\alpha q^{1-\alpha}$ is increasing for $q \in (0, 1-\alpha)$ and decreasing for $q \in (1-\alpha, 1)$. Recall that by definition $F(\sigma_L|L) = 1-\alpha$, and by Lemma 12 $F(\sigma|H) < F(\sigma|L)$

for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi < 1$ for δ chosen small enough, and so

$$\left(\frac{(1 - F(\sigma^* | H))^\alpha F(\sigma^* | H)^{1-\alpha}}{(1 - F(\sigma^* | L))^\alpha F(\sigma^* | L)^{1-\alpha}} \right)^n \leq \phi^n \to 0.$$

Since σ_n^* is bounded away from $\bar{\sigma}$, then $1 - F(\sigma_n^* | H)$ is bounded away from 0. By belief consistency, $\frac{\beta_n(x_n)}{1-\beta_n(x_n)}$ is bounded away from infinity. Consequently $Z(x_n, \sigma_n^*) < 0$ for n sufficiently large. A contradiction then follows as in Claim 1. ∎

Claim 3 *If* $\liminf x_n > x_L(\lambda)$ *then* $\limsup \sigma_n^* < \sigma_L$.

Proof: By hypothesis, there exists ε such that $x_n \geq x_L(\lambda) + \varepsilon$ for all n large enough. Suppose that contrary to the claim $\limsup \sigma_n^* \geq \sigma_L$. So for any δ, there exists a subsequence such that $\sigma_n^* \geq \sigma_L - \delta$. By definition, $\Delta^L(x_L(\lambda), \sigma_L, \lambda) = 0$; so for δ small enough, there exists a $\hat{\varepsilon}$ such that $\Delta^L(x_n, \sigma_n^*, \lambda) > \hat{\varepsilon}$. Moreover, $\Delta^H(x_n, \sigma_n^*, \lambda) \geq \Delta^L(x_n, \sigma_n^*, \lambda)$. Consequently $Z(x_n, \sigma_n^*) > 0$ for n sufficiently large. So σ_n^* cannot be a responsive equilibrium; and since $x_n \geq \underline{x}_n$ it is not a rejection equilibrium either. The only remaining possibility is that σ_n^* is an acceptance equilibrium — but then $\sigma_n^* = \underline{\sigma}$, which gives a contradiction when δ is chosen small enough. ∎

Claim 4 *If* $\liminf x_n > x_H(\lambda)$ *then* $\limsup \sigma_n^* < \sigma_H$.

Proof: By hypothesis, there exists ε such that $x_n \geq x_H(\lambda) + \varepsilon$ for all n large enough. Suppose now to the contrary that $\limsup \sigma_n^* \geq \sigma_H$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \geq \sigma_H - \delta$.

By definition $\Delta^H(x_H(\lambda), \sigma_H, \lambda) = 0$; so for δ small enough, there exists a $\hat{\varepsilon}$ such that $\Delta^H(x_n, \sigma_n^*, \lambda) > \hat{\varepsilon}$.

Next, define

$$\phi = \min_{\sigma \in [\sigma_H - \delta, \bar{\sigma}]} \frac{(1 - F(\sigma | H))^\alpha F(\sigma | H)^{1-\alpha}}{(1 - F(\sigma | L))^\alpha F(\sigma | L)^{1-\alpha}}$$

Note that the function $(1 - q)^\alpha q^{1-\alpha}$ is increasing for $q \in (0, 1 - \alpha)$ and decreasing for $q \in (1 - \alpha, 1)$. Recall that by definition $F(\sigma_H | H) = 1 - \alpha$, and by Lemma 12 $F(\sigma | H) <$

$F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi > 1$ for δ chosen small enough, and so

$$\left(\frac{(1 - F(\sigma^*|H))^\alpha F(\sigma^*|H)^{1-\alpha}}{(1 - F(\sigma^*|L))^\alpha F(\sigma^*|L)^{1-\alpha}}\right)^n \geq \phi^n \to \infty.$$

From Lemma 12, the term $\frac{1-F(\sigma|L)}{1-F(\sigma|H)}$ lies above $f(\bar{\sigma}|L)/f(\bar{\sigma}|H)$. By belief consistency, $\frac{\beta_n(x_n)}{1-\beta_n(x_n)}$ is bounded away from zero. Consequently $Z(x_n, \sigma_n^*) > 0$ for n sufficiently large. A contradiction then follows as in Claim 3. ∎