

“Personal Influence”: Social Context and Political Competition

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First draft: April 2007

Abstract

We study a model of electoral competition where voters obtain information on candidates' platforms through campaign advertising, and word-of-mouth communication. We show that when the costs of campaign advertising are low, an increase in word-of-mouth communication among voters causes polarization. In particular, the more voters can exchange political information between each other, the more often extremist candidates run in the election and win against moderate candidates.

JEL classification: D72, D83, Z13

Keywords: Social Influence, Political Competition, Campaign Advertising, Polarization

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“The mass do not now take their opinions from dignitaries in Church or State, from ostensible leaders, or from books. Their thinking is done for them by men much like themselves, addressing them or speaking in their name, on the spur of the moment.” J. S. Mill (1869). On Liberty.

“Rational citizens will seek to obtain their free political information from other persons if they can. This expectations seems to be borne out by the existing evidence.” A. Downs (1957). An Economic Theory of Democracy.

1 Introduction

In modern societies a large majority of individuals relies on others in order to obtain most of their political information. Indeed, evidence for the importance of information sharing among voters in shaping individuals’ political choices dates back to the seminal work on personal influence of Lazarsfeld et al. [16], Berelson et al. [2], and Katz and Lazarsfeld [15]. The Columbia sociologists demonstrated the primacy of face-to-face interaction in spreading political information in the society, and documented that this information was more likely to reach people who were still undecided about how they would vote.¹

Interestingly enough, in spite of the predominant role of the mass media in spreading information, recent empirical works show that interpersonal relations are still fundamental in the process of political information sharing and acquisition. For example, in an empirical study of the 1992 American presidential election campaign, Beck et al. [1] conclude that interpersonal discussions outweigh the media in affecting voting behavior and, in a recent study on political disagreement within communication networks, Huckfeldt et al. [14] observe that: “Democratic electorates are composed of individually interdependent, politically interconnected decision makers. [...] they depend on

¹It is worth noticing that the work of the Columbia research agenda is the “existing evidence” Downs is referring to in the quotation. See Downs [7] pages 222 and 229.

one another for political information and guidance [...].”² In light of this evidence, understanding the relationship between social context and individuals’ voting behavior appears of considerable interest. Yet, surprisingly, very little theoretical work has been done so far. The principal contribution of our paper is to propose a tractable model in which interpersonal communication between voters is embedded in a standard model of electoral competition. To the best of our knowledge, our paper is the first to analyze how information sharing through word-of-mouth may affect electoral outcomes.

We consider a citizen-candidate model where the policy space is unidimensional, and there are three groups of citizens: leftists and rightists (henceforth the “partisans”), and independent voters. Independents are decisive in the election and the identity of the median independent voter is ex-ante uncertain. There are two policy-motivated parties, representative of the leftist and rightist groups. Each party selects a candidate that will run in an election. The candidate that wins a simple majority of votes is elected and implements his preferred policy.

Citizens have distance preferences over policy. While partisans always vote for their party, independents cast their votes on the basis of the information they possess about candidates’ policy position. This information depends on their exposure to political advertising. Specifically, independents receive information about candidates’ policy position through two different channels. First, once a party devotes resources to campaign advertising, truthful political information reaches directly a random fraction of independents. We call this information channel *direct exposure*.³ Second, voters obtain information via interpersonal interactions with other voters. We call this channel *con-*

²Apparently, not so much has changed since W. Lippmann’s [17] treatise on public opinion where he states: “Inevitably our opinions cover a bigger space, a longer reach of time, a greater number of things, than we can directly observe. They have, therefore, to be pieced together out of what others have reported and what we can imagine.”

³For evidence about the importance of campaign in providing voters with political information see, e.g., Lodge et al. [18], and Coleman and Manna [6]. See also Zaller [24] for evidence on the effect of media content on policy preferences.

textual exposure. In particular, each independent randomly samples a finite number of other independents, who, in turn, truthfully report the information they have obtained from their direct exposure.⁴ The effectiveness of contextual exposure in spreading political information in the society depends both on the level of contextual exposure and the intensity of direct exposure, which in turn hinges on how much parties spend in electoral campaign.

In this paper we address the following questions. How does contextual exposure affect voters' perception of electoral candidates' ideologies? How does the level of contextual exposure affect political equilibrium outcomes? Does greater contextual exposure increase the likelihood that moderate policies are implemented or, on the contrary, it increases polarization?

Our main results can be summarized as follows. First, depending on the marginal cost of campaign advertising, in equilibrium either parties select extreme candidates and do not disclose any political information, or both parties select a moderate candidate with positive probability and disclose political information only when the candidate is a moderate. Second, when the marginal cost of advertising is low, an increase in the level of contextual exposure, decreases the amount parties spend on campaign advertising. Therefore, voters' beliefs of facing an extreme candidate are reinforced, and the difference between the probability of winning the election when a party selects a moderate as compared to an extremist decreases. As a result, policy-motivated parties select extremists more often. Overall, greater contextual exposure increases the (*ex-ante*) expected probability that an extremist is elected, so that extreme policies are more likely to be implemented. Third, contextual exposure generates heterogeneity in

⁴Our model of acquisition of political information formalizes the idea of “two-step flow communication” which played a central role in the analysis of the Columbia sociologists. They describe the two-step flow communication as a relay function of interpersonal relations, where political information flows directly from mass media to a subset of voters, the “opinion leaders” (which corresponds to direct exposure), and from them to other voters they are in contact with (which corresponds to contextual exposure).

expectations between otherwise equally informed voters. For example, we show that the greater is the contextual exposure of an uninformed voter, the more he expects that parties have selected extremists, when in fact candidates are moderates. Finally, note that a decrease in the marginal cost of advertising has very different effects as compared to an increase in the level of contextual exposure. Indeed, lowering the marginal costs of advertising always increases the amount of campaign advertising and the probability of selecting a moderate candidate.

Our paper is related to two different strands of literature. The first strand focuses on the effects of campaign advertising on electoral competition and voters' welfare. This relatively recent literature can be broadly divided into two groups: models where campaign advertising is assumed to be directly informative (see, e.g., Coate [4], [5]), and models where campaign advertising functions as a signalling device (see, e.g., Prat [20], [21]). Our model belongs to the first group and, in particular, we model electoral competition and direct exposure to political information following Coate [5]. The second strand is represented by the large existing literature on word-of-mouth communication and learning.⁵ In modelling word-of-mouth communication we follow the approach of Ellison and Fudenberg [8], [9], and Galeotti and Goyal [10]. The only paper that we are aware of which incorporates voters' communication in a political economy model is Sinclair [22]. Her interest is in the effects of interpersonal communication on voters' perception of political candidates' competencies. In her model there is no role for campaign advertising, and voters' information about candidates' competencies is exogenously given. She constructs an equilibrium where policy divergence is increasing in the expected benefit that voters derive from competence.

The paper is organized as follows. Section 2 presents the model. Section 3 develops the main results. In Section 4 we provide a generalization of the basic model which allows for the possibility that social ties may vary across individuals. Section 5

⁵For a survey of the existing literature on word-of-mouth communication and local learning see Goyal [12]. For a general treatment of games with local externalities see Galeotti et al. [11].

concludes.

2 Model

Citizens and Parties: Ideologies. There is a continuum of citizens of unit measure. The policy space is unidimensional, and citizens are exogenously divided into three groups: leftists, rightists, and independents. Partisans represent an equal fraction of the population, and their ideology is symmetrically distributed on $[0, m]$ and $[1 - m, 1]$, respectively. The ideology of independents is uniformly distributed on the interval $[\mu - \tau, \mu + \tau]$, where $\tau > 0$, and μ is drawn from a uniform distribution with support $[1/2 - m, 1/2 + m]$. Hence, the identity of the median independent is ex-ante uncertain. We assume that $m < 1/4 - \tau/2$ so that ideologies of independents are always between those of partisans.

There are two policy-motivated political parties: party L and party R . Party $L(R)$ consists of a representative subgroup of the leftist (rightist) group. A representative of each party is selected to be a candidate in an election and, in the spirit of the citizen-candidate model, the candidate that wins a simple majority of votes is elected and implements her ideology.⁶ For simplicity, we restrict the candidates' type space to be $T = \{e, m\}$, where $e \equiv m/2$. Let $\mathbf{t} = (t_L, t_R) \in T \times T$ be a profile of types, where $t_L \in \{e, m\}$ denotes the ideology of party L 's candidate, and $1 - t_R$ denotes the ideology of party R 's candidate. Henceforth, a candidate is an extremist if her type is $t = e$, while a candidate is a moderate if her type is $t = m$.⁷

Citizens have distance preferences over ideology and, in particular, a citizen with

⁶See Besley and Coate [3], and Osborne and Slivinski [19].

⁷When m is small, the assumption that $t \in \{e, m\}$ is without loss of generality. Indeed, a party, which maximizes the expected utility of its median voter, will never select a candidate that is more extreme than its median member e . Moreover, as the uncertainty about the median voter is sufficiently small, i.e. m is sufficiently small, it is possible to show that a party will never select a candidate with ideology lying in the interior of the interval $[e, m]$.

ideology i gets utility $-|t - i|$ if a candidate of ideology t wins the election. Partisans always vote for their party, while independents cast their votes on the basis of the information they possess about candidates' types.⁸ Independents are *ex-ante* ignorant about candidates' types, but receive information from two sources: parties' advertising (direct exposure) and interaction with other voters (contextual exposure).

Direct Exposure. Each party $j = \{L, R\}$, after having selected its candidate, chooses an amount of resources $x_j \in \mathbb{R}_+$ to spend on campaign advertising. Electoral campaign is truthful and fully informative. In particular, if a party spends x_j , then a random fraction x_j of independents observe party j 's candidate position. The costs of campaign advertising x_j are $C(\alpha, x_j) = \alpha x_j$, where α is a positive constant measuring the efficiency of the advertising technology.⁹

Contextual Exposure. In addition to direct exposure, independents obtain information via word-of-mouth communication. Formally, each independent samples a finite number $k > 0$ of other independents and each sampled independent reports truthfully the information, if any, she has obtained directly by parties' campaign advertising.¹⁰ Hence, k parameterizes the level of contextual exposure among voters, where higher k is equivalent to greater contextual exposure.¹¹ As is common in models of local exter-

⁸For evidence about the fact that partisans tend to be little affected by campaigns, see, e.g., Zaller [23], and Huckfeldt et al. [14].

⁹The assumption of linear advertising costs is made for convenience. Indeed, our results hold for costs function which are increasing and convex in x_j and α , respectively.

¹⁰We are assuming that independents only communicate with other independents. We can easily extend the model to allow independents to sample other voters from the entire population of citizens. In this case, it seems natural to assume that partisans will always report information that is favorable to their preferred party. For example, a leftist partisan will always report that the left candidate is a moderate. In light of this, when an independent hears from one of her acquaintance that the left candidate is a moderate, he will take into consideration that his acquaintance could be a left partisan. Clearly, this would change the specification of the posterior beliefs of independents, but it would not affect the qualitative results obtained in our analysis.

¹¹In Section 4 we generalize our framework to allow for the possibility that social ties may vary

nalities, we take the communication structure, which in our case simply corresponds to the parameter k , as exogenously given.¹²

We analyze the following Bayesian Game. Parties choose simultaneously their own candidate and their advertising intensity conditional on the candidate selected, in order to maximize the expected utility of their median voter. Independents do not observe these choices, but they may be exposed to campaign advertising either directly or via word-of-mouth communication. Based on the information received, independents update their beliefs about candidates' types and cast their vote to maximize their expected utility.

Parties' Strategies and Parties' Utilities. A strategy for party j is a probability distribution over candidates' types and an intensity of campaign advertising for each candidate's type. Formally, let $\sigma_j : T \rightarrow [0, 1]$, where $\sigma_j(t)$ denotes the probability that party j selects a candidate of type t , and $\sigma_j(e) + \sigma_j(m) = 1$. Analogously, let $x_j : T \rightarrow [0, 1]$, where $x_j(t)$ denotes the intensity of campaign advertising of party j when candidate t is selected. A strategy for party j is denoted by $s_j = (\sigma_j, x_j)$; $s = (s_L, s_R)$ denotes a strategy profile.

Let $\pi_L(s|\mathbf{t})$ denote the expected probability of winning of party L , given that the electoral candidates are specified by the profile \mathbf{t} , and that parties are playing according to s . Thus, the expected payoff to party L when its candidate is t_L can be written as follows,

$$U_L(s|t_L) = \sum_{t_R \in \{e, m\}} \sigma_R(t_R) [\pi_L(s|t_L, t_R) (1 - t_R - t_L) - (1 - t_R - e)] - \alpha x_L.$$

across individuals.

¹²Interestingly, Huckfeldt and Sprague [13] point out: "People often choose their associates and the content of their conversations, but each of these choices is, in turn, bounded by an environment that for many purposes must be taken as given rather than chosen." Also, Huckfeldt et al. [14] report that: "[...] individuals exercise limited discretion in the selection of informants. [...] the construction of a communication network occurs with pronounced constraints on supply."

Voting Behavior of Independent Voters. *Ex-post*, the information of an independent about party j 's candidate can be summarized by $I_{k,j} \in T \cup \emptyset$, where $I_{k,j} = t$ means that the independent knows that party j 's candidate is t , while $I_{k,j} = \emptyset$ indicates that the independent is uninformed about party j 's candidate.

Let $\rho_{k,j}(t|I_{k,j}, s)$ denote the belief of an independent that party j 's candidate is t , given $I_{k,j}$ and s . Whenever possible, $\rho_{k,j}(t|I_{k,j}, s)$ is derived using Bayes' rule. Hence, $\rho_{k,j}(t|t, s) = 1$, $\rho_{k,j}(t|t', s) = 0$, for $t \neq t'$, and

$$\rho_{k,j}(t|\emptyset, s) = \frac{\sigma_j(t)(1 - x_j(t))^{k+1}}{\sum_{t' \in T} \sigma_j(t')(1 - x_j(t'))^{k+1}}, \quad (1)$$

for every $t \in T$ such that $\sigma_j(t) > 0$ and $x_j(t) > 0$. We also assume that the equation (1) holds at zero probability events, i.e., when $\sigma_j(t') = 0$ and/or $x_j(t') = 0$.¹³

Each independent votes as if he is pivotal. Hence, an independent with ideology i and information $(I_{k,L}, I_{k,R})$ votes for party L if and only if $i < i_k^*(I_{k,L}, I_{k,R})$, where $i_k^*(I_{k,L}, I_{k,R})$ is the identity of the indifferent independent voter. Given (t_L, t_R) and s , party L 's candidate gets at least half of the independents' votes if and only if $\mu < \mu_L^*(s|t_L, t_R)$.¹⁴ Therefore,

$$\pi_L(s|\mathbf{t}) = \begin{cases} 0 & \text{if } \mu_L^*(s|\mathbf{t}) \leq \frac{1}{2} - m \\ \frac{\mu_L^*(s|\mathbf{t}) + m - \frac{1}{2}}{2m} & \text{if } \mu_L^*(s|\mathbf{t}) \in \left(\frac{1}{2} - m, \frac{1}{2} + m\right) \\ 1 & \text{if } \mu_L^*(s|\mathbf{t}) \geq \frac{1}{2} + m. \end{cases}$$

Political Equilibrium. A political equilibrium consists of (i) parties' strategies, $s^* = (s_L^*, s_R^*)$; (ii) voter belief functions $\rho_{k,j}^*(\cdot)$, $j = L, R$ and indifferent independent voters $i_k^*(\cdot)$ such that

¹³Note that this is a necessary condition for a Bayesian equilibrium to be a sequential equilibrium.

¹⁴Formally, $i_k^*(I_{k,L}, I_{k,R}) = \frac{1}{2} + \frac{\sum_{t \in T} \rho_{k,L}(t|I_{k,L}, s)t - \sum_{t \in T} \rho_{k,R}(t|I_{k,R}, s)t}{2}$ and $\mu_L^*(s|t_L, t_R) = \sum_{(I_{k,L}, I_{k,R}) \in \{\{t_L\} \cup \emptyset\} \times \{\{t_R\} \cup \emptyset\}} i_k^*(I_{k,L}, I_{k,R}) \Pr(I_{k,L}|s, t_L) \Pr(I_{k,R}|s, t_R)$.

- 1 (s_L^*, s_R^*) are mutual best responses given subsequent voting behavior;
- 2 $\rho_{k,j}^*(\cdot)$ are consistent with s^* for all $j = L, R$, and $i_k^*(\cdot)$ are consistent with $\rho_{k,j}^*(\cdot)$ and s , $j = L, R$.

3 Characterization of Political Equilibrium

Our first result provides a complete characterization of symmetric political equilibria. It turns out that the nature of political equilibria is pinned down by the *independents' misperception* of the types of candidates running for election. Before stating the result formally, we clarify what we mean by independents' misperception.

Consider a symmetric strategy profile, s , where parties randomize between selecting a moderate candidate and an extremist candidate and they advertise with positive intensity only moderate candidates. Suppose the left party selects a moderate; then a fraction $1 - (1 - x(m))^{k+1}$ of independents become informed that the leftist candidate is a moderate. The remaining fraction of uninformed independents will place probability $\rho_k(e|\emptyset, s)$ on the event that the leftist candidate is an extremist, where

$$\rho_k(e|\emptyset, s) = \frac{\sigma(e)}{\sigma(e) + (1 - \sigma(e))(1 - x(m))^{k+1}}.$$

We can then define the probability that a randomly selected independent misperceives a moderate candidate, i.e., the probability that randomly selected independent believes that the candidate is an extremist when in fact the candidate is a moderate, as follows:

$$Q_k[e|m] = (1 - x(m))^{k+1} \rho_k(e|\emptyset, s). \quad (2)$$

Similarly, if party L chooses an extremist, then the probability that an independent voter misperceives an extremist candidate is:

$$Q_k[m|e] = \rho_k(m|\emptyset, s) = 1 - \rho_k(e|\emptyset, s). \quad (3)$$

Adding together (2) and (3) gives us a measure of the probability that a randomly selected independent misperceives the type of the candidate running for election. We

will refer hereafter to

$$\Psi_k \equiv Q_k[e|m] + Q_k[m|e] \quad (4)$$

as the misperception of independents.

We are now ready to provide a complete characterization of symmetric political equilibria.

Proposition 1 *A symmetric political equilibrium always exists and it is unique. For every k , there exists a critical level of the marginal costs of advertising $\alpha^*(k) > 0$ such that*

I If $\alpha \geq \alpha^(k)$ in the symmetric political equilibrium parties always select an extremist candidate and they never advertise, i.e., $\sigma^*(e) = 1$, and $x^*(e) = 0$;*

II If $\alpha < \alpha^(k)$ in the symmetric political equilibrium parties randomize between selecting an extremist candidate and a moderate candidate, and they only advertise moderate candidates. In particular, $x^*(e) = 0$, and $x^*(m)$ and $\sigma^*(e)$ jointly solve:*

$$-\frac{\partial Q_k[e|m]}{\partial x(m)} \frac{2 - 4m + \sigma^*(e) m}{16} = \alpha \quad (5)$$

$$1 - \Psi_k = \frac{4m + 16\alpha x^*(m)}{2 - 3m}. \quad (6)$$

The proof of Proposition 1 can be found in the appendix. Here we provide an intuition. When choosing its political strategy, a party faces a fundamental trade-off. On the one hand, since parties are policy motivated, conditional on winning the election, they enjoy higher utility when selecting an extremist. On the other hand, since independents are decisive, a moderate has a higher chance of winning the election relative to an extremist, and this advantage is higher the lower is the misperception of independents towards a moderate candidate. Clearly, for a given level of contextual exposure, the more a party advertises a moderate, the less independents misperceive the candidate. However, lowering independents' misperception comes at the cost of

an intense campaign. Proposition 1 summarizes how these trade-offs are solved in equilibrium.

For a given level of contextual exposure, if costs of advertising are sufficiently high, parties always choose extremists and never advertise (Part I of Proposition 1). In line with the intuition above, selecting a moderate is profitable only if independents' misperception is sufficiently low, which implies that the party must spend enough resources to advertise her moderate candidate. This is not optimal, when the advertising technology is very inefficient.

In contrast, if the costs of advertising are sufficiently low and one party always selects an extremist, the opponent party will find it profitable to select a moderate candidate and inform the independents about it. Thus, in equilibrium parties have to randomize between the two candidates' types and they will choose to advertise only moderate candidates (Part II of Proposition 1).

Condition (5) requires that parties advertise moderate candidates so that marginal returns from advertising equal the marginal cost α . By (marginally) increasing the resources spent on advertising a moderate, a party lowers the misperception of independents and therefore it increases the probability of winning the election. Such shift in beliefs multiplied by the gains of winning the election constitutes the marginal returns from advertising a moderate.

Condition (6) requires that parties are indifferent between selecting a moderate and an extremist candidate, given the strategy of the opponent party. The RHS of equation (6) represents the expected costs of proposing a moderate instead of an extremist. These costs are composed of the advertising costs and of the policy costs, i.e., the costs of implementing a moderate policy instead of an extreme one. The LHS of equation (6) is the benefit for the party of choosing a moderate instead of an extremist, which is materialized in facing a higher probability of winning the election. Interestingly, this is inversely proportional to the level of independents' misperception. To see why it is the case, suppose that the right party follows the strategy prescribed in the second part of

Proposition 1. The expected probability of winning of the left party when choosing a moderate is $\pi(m) = (1 - \sigma(e)) \pi(m|m) + \sigma(e) \pi(m|e)$, where, abusing notation slightly, $\pi(t|t')$ indicates the probability that the leftist candidate t defeats the rightist candidate t' . Analogously, if the left party chooses an extremist then $\pi(e) = (1 - \sigma(e)) \pi(e|m) + \sigma(e) \pi(e|e)$. Since, $\pi(t|t) = 1/2$ and $\pi(m|e) = 1 - \pi(e|m)$, it follows that

$$\pi(m) - \pi(e) = \frac{1}{2} - \pi(e|m) = \frac{1 - \Psi_k}{8},$$

where the last equality is easily checked.

3.1 The Equilibrium Effect of Contextual Exposure

We now investigate the effect of an increase in contextual exposure on the political outcome. In particular, we explore the equilibrium relation between contextual exposure and policy polarization. In order to do so, we compare the political equilibrium when the level of contextual exposure is k with the political equilibrium when independents communicate with $k+1$ other voters. Since it is the only non trivial situation, we focus on the case $\alpha < \alpha^*(k)$ described in part II of Proposition 1.

As a measure of policy polarization we define the *ex-ante* expected probability that in equilibrium an extremist candidate is elected. This is denoted by Π_k and it is equal to:

$$\Pi_k = \sigma^*(e)^2 + 2\sigma^*(e)(1 - \sigma^*(e))\pi_k(e|m), \quad (7)$$

where the first term is the probability that two extreme candidates compete in the election, and the last term is the probability that an extreme candidate wins against a moderate. The next proposition summarizes the results.¹⁵ We denote by $s_k^* = (\sigma_k^*(e), x_k^*)$ the parties' strategy profile of a symmetric political equilibrium under contextual exposure k .

¹⁵An alternative measure of policy polarization is the probability that in equilibrium an extremist candidate defeats a moderate candidate. Formally, $\pi_k(e|m) = \frac{1}{2} - \frac{1 - \Psi_k}{8}$. The equilibrium impact of an increase in k on Π_k is equivalent to the impact of an increase in k on $\pi_k(e|m)$.

Proposition 2 *There exists $\hat{\alpha}(k) \in (0, \alpha^*(k)]$ such that, for every $\alpha < \hat{\alpha}(k)$, it follows that $x_{k+1}^*(m) < x_k^*(m)$, $\sigma_{k+1}^*(e) > \sigma_k^*(e)$, and $\Pi_{k+1}^* > \Pi_k^*$.*

Proposition 2 establishes that, for sufficiently low marginal cost of advertising, greater contextual exposure decreases the intensity of campaign advertising for moderates, and it increases the probability that extreme candidates run in an election. Ultimately, as independents can exchange political information with more voters, policy polarization increases, i.e., the likelihood that an extremist candidate is elected increases.

We now provide an intuition for this result. An increase in contextual exposure has two effects on parties' incentives to advertise moderates. First, greater contextual exposure increases the marginal returns from advertising because every additional informed independent will spread the information further. Second, greater contextual exposure decreases the marginal returns from advertising because, for the same level of advertising, independents are more likely to obtain information via word-of-mouth communication. Thus, depending on which of the two effects dominate, campaign advertising and contextual exposure are either substitutes or complements. Intuitively, which of the two effects dominates depends on the cost of the advertising technology.

When advertising is relatively cheap, in equilibrium parties find it profitable to massively advertise their moderate candidates. In this case campaign advertising and contextual exposure are substitutes, and an increase in word-of-mouth communication decreases the resources parties spend in campaign advertising.¹⁶ Since campaign advertising decreases, the costs to a party of proposing a moderate instead of an extremist decreases as well, i.e., the RHS of equation 6 decreases. Therefore, in order for parties to be indifferent between candidates, the benefit of selecting a moderate relative to an extremist must decrease. That is, independents' misperception must increase, which is possible only if parties select extremists more often. Overall and surprisingly, greater contextual exposure increases the expected probability that implemented policies are

¹⁶This result is robust to the details of our specific model. See Galeotti and Goyal [10] for a general analysis.

extreme. The political outcome becomes more polarized.

We conclude this section with two remarks. First, not surprisingly, for sufficiently inefficient technology, an increase in the level of contextual exposure can lead to the opposite comparative statics result of Proposition 2. For example, suppose that $\alpha = 0.1$ and $m = 0.1$. For $k = 8$, it is possible to show that in equilibrium parties select extremist candidates with probability $\sigma_8^*(e) = 0.28$, and they advertise moderate candidates with intensity $x_8^*(m) = 0.11$. Consequently, in expectation an extremist candidate is elected with probability $\Pi_8^* = 0.26$. Suppose now that $k = 9$. Simple calculations show that at the new level of contextual exposure parties select extremist less often, $\sigma_9^*(e) = 0.26$, they advertise moderate candidates less, $x_9^*(m) = 0.10$ and, as a result, the political outcome is less polarized, $\Pi_9^* = 0.24$.

Second, the effects of an increase in word-of-mouth communication on equilibrium are rather different from the effects of a decrease in the marginal costs of advertising α (i.e., direct exposure becomes less expensive). Indeed, a decrease in the costs of direct exposure unambiguously increases the intensity of advertising. As an illustration, let $k = 8$ and $m = 0.1$. Figure 1 plots the equilibrium advertising intensity, the probability that parties select an extremist and the ex-ante expected probability that in equilibrium an extremist candidate is elected, for different values of $\alpha \in (0, \alpha^*(8)]$. Intuitively, when direct exposure is cheaper, it is always optimal for parties to advertise their moderate candidate with higher intensity. As a result, parties find it profitable to select extremists candidates less often, and consequently political outcomes becomes less polarized.¹⁷

¹⁷The formal proof of the comparative static result with respect to α is straightforward and therefore omitted.

4 Extension: Heterogeneity in Contextual Exposure

We now extend our framework to allow for the possibility that social ties may vary across individuals. A simple and rather general way to model such heterogeneity is to consider that independents are divided in \bar{k} groups with the interpretation that an independent in group l samples randomly l other citizens among the entire set of independents. Formally, let $I = \{1, 2, \dots, \bar{k}\}$, and let $P : I \rightarrow [0, 1]$, where $P(l)$ gives the fraction among independent voters that sample l other voters. It is readily seen that the model presented in the previous section is one where the distribution P is such that $P(k) = 1$ and $P(l) = 0$, for all $l \in I \setminus \{k\}$, and $k \in I$.

The following proposition generalizes the characterization of symmetric political equilibria presented in Proposition 1 to arbitrary distributions P .

Proposition 3 *A symmetric political equilibrium always exists for any distribution P . In particular, for every P , there exists a $\alpha_P^* > 0$ such that*

I If $\alpha \geq \alpha_P^$ in the unique symmetric political equilibrium parties always select an extremist candidate and they never advertise, i.e., $\sigma^*(e) = 1$, and $x^*(e) = 0$;*

II If $\alpha < \alpha_P^$ in every symmetric political equilibrium parties randomize between selecting an extremist candidate and a moderate candidate and they only advertise moderate candidates. In particular, $x^*(e) = 0$, and $x^*(m)$ and $\sigma^*(e)$ jointly solve:*

$$-\sum_{k=1}^{\bar{k}} P(k) \frac{\partial Q_k[e|m]}{\partial x(m)} \frac{2 - 4m + \sigma^*(e)m}{16} = \alpha \quad (8)$$

$$\sum_{k=1}^{\bar{k}} P(k) [1 - \Psi_k] = \frac{4m + 16\alpha x^*(m)}{2 - 3m}. \quad (9)$$

Similarly to Proposition 1, for high costs of advertising parties select extremists and never advertise, while for small costs of advertising parties randomize between extremists and moderates and advertise only moderate candidates. Figure 2 depicts the

equilibrium for particular values of the parameters. The decreasing curve represents the $\sigma(e)$ which solves equation (8) as a function of $x(m)$, while the increasing curve depicts the $\sigma(e)$ which solves equation (9) as a function of $x(m)$. The intersection of the two schedules pins down the equilibrium values of $\sigma^*(e)$ and $x^*(m)$.

An interesting insight that comes from the generalized model is that, despite the fact that *ex-ante* independents hold the same beliefs, *ex-post* their beliefs are different because of their different level of contextual exposure. In other words, heterogeneity in contextual exposure maps into heterogeneity in expectations between otherwise equally informed voters.

For a given distribution P and equilibrium s^* , let $\Psi_k(s^*|P)$ be the misperception of independents who sample k other voters (defined analogously to 4). The next proposition characterizes the effect of contextual exposure on equilibrium beliefs.

Proposition 4 *Consider a distribution P and assume that $\alpha < \alpha_P^*$. In every symmetric political equilibrium s^* the following holds: $\Psi_{k+1}(s^*|P) < \Psi_k(s^*|P)$, and $\rho_{k+1}(e|\emptyset, s^*) > \rho_k(e|\emptyset, s^*)$, for all $k \in I \setminus \{\bar{k}\}$*

The first part of the proposition establishes that, the greater the contextual exposure of an independent the more precise are (on average) her beliefs about political candidates. This simply follows from the fact that an independent who samples many other voters is more likely to be *ex-post* informed about candidates' ideology. Hence, the misperception of independents is inversely related to their level of contextual exposure.

The second part of the proposition establishes that, conditional on being uniformed, independents who are more exposed to word-of-mouth communication have stronger beliefs that the candidates running for election are extremists. This follows because being uniformed “signals” to the voter that either the candidate is indeed an extremist, or that the candidate is a moderate but none of his political discussants were exposed to campaign advertising. Clearly, the probability that this latter event occurs is decreasing in the level of contextual exposure of a voter. An immediate corollary of these

observations is that when the candidate is indeed a moderate, uninformed independents who are exposed more to word-of-mouth communication have less precise beliefs.

We conclude this section with a simple example which shows that the comparative static results we obtained in the previous section hold also in the generalized model.

Example: Effects of greater contextual exposure on political outcomes. Suppose that $m = 0.2$ and assume that independents sample either one or two other voters, i.e., $P(1) = p$, $P(2) = 1 - p$. Note that a decrease of p implies a first order stochastic dominance shifts in the distribution of contextual exposure.

First, consider a case in which advertising is sufficiently cheap, $\alpha = 0.01$. Figure 3 depicts the political equilibrium for $p = 0.4$ and $p = 0.5$, respectively. In this case, a first order stochastic shifts in the distribution of contextual exposure decreases parties advertising, and increases the likelihood that extremist candidates are selected. As a result, political outcomes are more polarized.

Second, consider a case in which advertising is sufficiently expensive, $\alpha = 0.1$. Figure 4 depicts the political equilibrium for $p = 0.4$ and $p = 0.5$, respectively. In contrast to the previous case, here a first order stochastic shifts decreases the likelihood that extremist candidates are selected. As a result, political outcomes are less polarized.

5 Conclusion

The importance of informal communication in affecting voters' choices is widely documented in economics, political science, and sociology. To the best of our knowledge there is no theoretical model that examines the effects of voters' communication on electoral competition and consequently on political equilibrium outcomes. The main contribution of this paper is to propose a simple and tractable model which is able to provide novel insights on the equilibrium relation between the amount of interpersonal communication among voters and polarization of the electoral outcome.

6 Appendix

The Appendix is organized as follows. We first provide the proof of Proposition 3. The characterization in Proposition 1 then follows from Proposition 3. Next, we complete the proof of Proposition 1 by proving uniqueness of symmetric equilibria. Finally, we provide the proof of Proposition 2 and Proposition 4.

Proof. Proposition 3.

We first characterize symmetric political equilibria in pure strategies. Let $s^* = (s_L^*, s_R^*)$ be part of a symmetric pure-strategy equilibrium. Then s_j^* prescribes to select a candidate $t^* \in T$ with probability 1 and to advertise that candidate with intensity $x^*(t^*)$, $\forall j \in \{L, R\}$. We start by noticing that for s^* to be part of an equilibrium it has to be the case that $x^*(t^*) = 0$. Hence, $U_L(s^*|t^*) = -(1 - m)/2$. There are two possibilities, which we now analyze.

One possibility is that $t^* = m$. Let party L deviate by selecting $t_L = e$. It is easy to see that the best advertising strategy is $x_L(e) = 0$. Let's denote this strategy by \tilde{s}_L . Then, $U_L(\tilde{s}_L, s_R^*|e) = -(2 - 3m)/4 > U_L(s^*|m)$, which contradicts our hypothesis that s^* is an equilibrium.

The other possibility is that $t^* = e$. Let party L deviate by selecting $t_L = m$ and $x_L(m)$; call this strategy \tilde{s}_L . Observe that such deviation is profitable only if $x_L(m) \notin \{0, 1\}$. Thus, assume that $x_L(m) \in (0, 1)$. We now derive the optimal advertising level, given s_R^* , which we denote by $x_L^*(m)$.

To do this, we start by observing that

$$\mu_L^*(\tilde{s}_L, s_R^*|m, e) = \frac{1}{2} + \frac{m}{4} - \frac{m}{4} \sum_{k=1}^{\bar{k}} P(k)(1 - x_L(m))^{k+1},$$

and it is readily seen that $\mu_L^*(\tilde{s}_L, s_R^*|m, e) \in [1/2 - m, 1/2 + m]$ for all $x_L(m) \in (0, 1)$. This fact implies that $\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, for all $x_L(m) \in (0, 1)$. Next, since

$\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, it follows that the expected utility of party L by playing \tilde{s}_L against s_R^* , is,

$$U_L(\tilde{s}_L, s_R^*|m) = \left[\left(\frac{5 - \sum_{k=1}^{\bar{k}} P(k)(1 - x_L(m))^{k+1}}{8} \right) \left(1 - \frac{3}{2}m \right) - (1 - m) \right] - \alpha x_L(m).$$

Hence, the optimal $x_L^*(m) \in (0, 1)$ solves

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x_L^*(m))^k = \frac{16\alpha}{2 - 3m}. \quad (10)$$

Clearly $x_L^*(m)$ is decreasing in α . Moreover, $x_L^*(m) \geq 0$ if and only if $\alpha \leq (2 - 3m)(\hat{k} + 1)/16$, where $\hat{k} = \sum_{k=1}^{\bar{k}} P(k)k$, and $x_L^*(m) = 1$ if and only if $\alpha = 0$. Thus, if $\alpha \geq (2 - 3m)(\hat{k} + 1)/16$, then $x_L^*(m) = 0$ and a pure strategy equilibrium exists.

We can then assume that $\alpha < (2 - 3m)(\hat{k} + 1)/16$. In this case, party L will not deviate from s_L^* if and only if $U_L(s^*|e) \geq U_L(\tilde{s}_L, s_R^*|m)$, where abusing notation \tilde{s}_L prescribes to advertise a moderate candidate with intensity $x_L^*(m)$. The latter inequality is satisfied if and only if

$$\sum_{k=1}^{\bar{k}} P(k)(1 - x_L^*(m))^{k+1} \geq \frac{2 - 7m - 16\alpha}{2 - 3m}. \quad (11)$$

As we shall show later, there exists $\alpha^* \in (0, (2 - 3m)(\hat{k} + 1)/16)$ such that condition (11) holds if and only if $\alpha \geq \alpha^*$. We will also formally derive the expression of α^* . These observations show that a symmetric political equilibrium in pure strategies exists if and only if $\alpha \geq \alpha^*$. Furthermore, in a symmetric pure-strategy political equilibrium each party selects an extremist candidate with probability one and the extremist candidate is never advertised.

We now characterize symmetric mixed-strategy equilibria. For convenience we use the notation $\sigma \equiv \sigma(e)$. First, assume that a symmetric mixed-strategy equilibrium exists, and let $s_j^* = (\sigma^*, x^*(e))$, $j = L, R$, denote the equilibrium strategy profile. It is easy to see that in equilibrium it has to be the case that $x^*(e) = 0$.

Given a profile $s = (s_L, s_R)$ with $x_j(e) = 0$, $j = L, R$, we have that

$$U_L(s|e) = \sigma_R(1 - m)[\pi_L(s|e, e) - 1] + (1 - \sigma_R) \left(1 - \frac{3m}{2}\right) [\pi(s|e, m) - 1],$$

and

$$\begin{aligned} U_L(s|m) &= \sigma_R \left[\pi_L(s|m, e) \left(1 - \frac{3m}{2}\right) - (1 - m) \right] + \\ &+ (1 - \sigma_R) \left[\pi_L(s|m, m)(1 - 2m) - \left(1 - \frac{3m}{2}\right) \right] - \alpha x_L(m). \end{aligned} \quad (12)$$

Moreover,

$$\begin{aligned} \frac{\partial \pi_L(s|m, e)}{\partial x_L(m)} &= \frac{\partial \pi_L(s|m, m)}{\partial x_L(m)} = \\ &= \frac{1}{8} \sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x_L(m))^k [\rho_{k,L}(e|\emptyset, s)]^2. \end{aligned}$$

Hence, in a symmetric equilibrium,

$$\frac{\partial U_L(m, x_L(m); s_R)}{\partial x_L(m)} \Big|_{s^*} = 0,$$

if and only if

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)(1 - x^*(m))^k [\rho_k(e|\emptyset, s^*)]^2 = \frac{16\alpha}{2 - 4m + \sigma m}, \quad (13)$$

where

$$\rho_k(e|\emptyset, s^*) = \frac{\sigma}{\sigma + (1 - \sigma)(1 - x^*(m))^{k+1}}. \quad (14)$$

It is easy to verify that condition (13) is equivalent to condition (8) stated in Proposition 3.

Next, in a symmetric mixed-strategy political equilibrium it has to be the case that each party is indifferent between selecting a moderate candidate and selecting an extremist candidate, i.e., $U_L(s^*|e) = U_L(s^*|m)$. Since in a symmetric equilibrium we

have that

$$\begin{aligned}\pi_L(s^*|e, e) &= \pi(s^*|m, m) = \frac{1}{2} \\ \pi(s^*|e, m) &= \frac{1}{2} + \frac{1}{4m} \sum_{k=1}^{\bar{k}} P(k)(\bar{t}_k - m)(1 - (1 - x^*(m))^{k+1}) \\ \pi(s^*|m, e) &= 1 - \pi(s^*|e, m),\end{aligned}$$

where

$$\bar{t}_k = m - \frac{m}{2} \rho_k(e|\emptyset, s^*),$$

it follows that $U_L(s^*|e) = U_L(s^*|m)$ if and only if

$$\sum_{k=1}^{\bar{k}} P(k) \rho_k(e|\emptyset, s^*) (1 - (1 - x^*(m))^{k+1}) = \frac{4m + 16\alpha x}{2 - 3m}. \quad (15)$$

Note that condition (15) is equivalent to condition (9) stated in Proposition 3. We have then proved that if a symmetric mixed-strategy equilibrium exists then it is characterized by conditions (8) and (9).

The third step of the proof is to show existence of equilibria. Here, we start by showing that a mixed strategy equilibrium exists if and only if $\alpha < \alpha^*$, and we determine the value of α^* . For convenience, we use the notation $p = 1 - x(m)$, hereafter.

Define

$$f(\sigma, p) = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^k \sigma^2 (2 - 4m + \sigma m)}{[\sigma + (1 - \sigma)p^{k+1}]^2},$$

and note that the equilibrium condition (13) holds if and only if (σ, p) are such that $f(\sigma, p) = 16\alpha$. The following properties of $f(\cdot, \cdot)$ will prove useful for the proof.

Property 1: $f(0, p) = 0$;

Property 2: $f(1, p) = \sum_{k=1}^{\bar{k}} P(k)(k+1)p^k(2 - 3m)$ and it is increasing in p ;

Property 3:

$$\frac{\partial f(\sigma, p)}{\partial \sigma} = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^k \sigma [2p^{k+1}(2-4m+\sigma m) + \sigma m(\sigma + (1-\sigma)p^{k+1})]}{[\sigma + (1-\sigma)p^{k+1}]^3} > 0.$$

Properties 1, 2, and 3 imply that $\tilde{\sigma}(p) : f(\tilde{\sigma}(p), p) = 16\alpha$ is a well defined function of p for all $p \in [\underline{p}, 1]$, where \underline{p} solves $f(1, \underline{p}) = 16\alpha$, i.e.,

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)\underline{p}^k = \frac{16\alpha}{2-3m}.$$

Note that $\underline{p} \in (0, 1)$ if and only if $\alpha < (2-3m)(\hat{k}+1)/16$.

We now study how $\tilde{\sigma}(p)$ behaves in $p \in [\underline{p}, 1]$. The following properties of $\tilde{\sigma}(\cdot)$ are useful:

Property 4: $\tilde{\sigma}(\underline{p}) = 1$;

Property 5: $\tilde{\sigma}(1) \in (0, 1)$ solves

$$\tilde{\sigma}(1)^2(2-4m+\tilde{\sigma}(1)m) = \frac{16\alpha}{\hat{k}+1};$$

Property 6: $\partial f(p, \sigma)/\partial p$ may change sign only once, and

$$\frac{\partial f(\sigma, p)}{\partial p} \Big|_{\underline{p}, \tilde{\sigma}(\underline{p})} > 0.$$

Note that Property 6 follows from inspection of

$$\frac{\partial f(\sigma, p)}{\partial p} = \sum_{k=1}^{\bar{k}} P(k) \frac{(k+1)p^{k-1}\sigma^2(2-4m+\sigma m)(k\sigma - p^{k+1}(1-\sigma)(k+2))}{[\sigma + (1-\sigma)p^{k+1}]^3},$$

and Property 4.

Using the implicit function theorem and invoking properties 3, 4, 5, and 6 it follows that $\tilde{\sigma}(p)$ is either always decreasing in p for all $p \in [\underline{p}, 1]$, or there exists a $\tilde{p} > \underline{p}$ such that $\tilde{\sigma}(p)$ is decreasing in p for all $p \in [\underline{p}, \tilde{p}]$, while it is increasing in p for all $p \in (\tilde{p}, 1]$.

We now define

$$g(\sigma, p) = \sum_{k=1}^{\bar{k}} P(k) \frac{(1 - p^{k+1})\sigma}{\sigma + (1 - \sigma)p^{k+1}} + \frac{16\alpha(1 - p)}{2 - 3m},$$

so that the equilibrium condition (15) holds if and only if $g(\sigma, p) = 4m/(2 - 3m)$. The following properties of $g(\cdot, \cdot)$ are useful:

Property 1': $g(0, p) = -16\alpha p/(2 - 3m)$;

Property 2': $g(1, p) = \sum_{k=1}^{\bar{k}} P(k)(1 - p^{k+1})$ is decreasing in p for all $p \geq \underline{p}$;

Property 3':

$$\frac{\partial g(\sigma, p)}{\partial \sigma} = \sum_{k=1}^{\bar{k}} P(k) \frac{(1 - p^{k+1})p^{k+1}}{[\sigma + (1 - \sigma)p^{k+1}]^2} > 0;$$

Property 4': $\partial g(\sigma, p)/\partial p < 0$, which is easily checked.

Properties 1', 2', and 3' imply that $\bar{\sigma}(p) : g(\bar{\sigma}(p), p) = 4m/(2 - 3m)$ is a well defined function of p for all $p \in (0, \bar{p}]$, where $\bar{p} : g(1, \bar{p}) = 4m/(2 - 3m)$, i.e.

$$\sum_{k=1}^{\bar{k}} P(k)(1 - \bar{p}^{k+1}) = \frac{4m + 16\alpha(1 - \bar{p})}{2 - 3m}. \quad (16)$$

It is easy to check that $\bar{p} \in (0, 1)$. Furthermore, using the implicit function theorem and invoking properties 3' and 4' it follows that $\bar{\sigma}(p)$ is increasing in p , for all $p \in (0, \bar{p}]$. Also, $\bar{\sigma}(\bar{p}) = 1$ and $\bar{\sigma}(\underline{p}) < 1$.

Summarizing $\tilde{\sigma}(p)$ is first decreasing and then possibly increasing in p and it is defined for all $p \in [\underline{p}, 1]$, while $\bar{\sigma}(p)$ is increasing in p and it is defined for all $p \in (0, \bar{p}]$. Furthermore, $\tilde{\sigma}(\underline{p}) = 1$, while $\bar{\sigma}(\underline{p}) < 1$. Since a symmetric mixed strategy equilibrium is given by p^* and σ^* such that $\sigma^* = \tilde{\sigma}(p^*) = \bar{\sigma}(p^*)$, an equilibrium exists if and only

if $\underline{p} < \bar{p}$. This holds, if and only if $\alpha < \alpha^*$, where α^* is the unique solution of

$$\sum_{k=1}^{\bar{k}} P(k)(k+1)\underline{p}^k = \frac{16\alpha^*}{2-3m} \quad (17)$$

$$\sum_{k=1}^{\bar{k}} P(k)(1-\underline{p}^{k+1}) = \frac{4m+16\alpha^*(1-\underline{p})}{2-3m}. \quad (18)$$

Note that $\alpha^* \in \left(0, (2-3m)(\widehat{k}+1)/16\right)$, and that if $\alpha = \alpha^*$, then $\underline{p} = \bar{p}$. In this latter case, $\sigma^* = 1$ and condition (11) holds with equality. Similarly, for all $\alpha > \alpha^*$ condition 11 holds with strict inequality. We have proved that a symmetric equilibrium always exists and that if $\alpha > \alpha^*$ there is a unique equilibrium in pure strategy, otherwise there exists an equilibrium in mixed strategy. This concludes the proof of Proposition 3. ■

Proof. Proposition 1 The characterization of equilibria in Proposition 1 is a special case of Proposition 3. So, to complete the proof of Proposition 1 we only need to show that when P is such that $P(k) = 1$ and $P(l) = 0$, for all $l \in I \setminus \{k\}$, and $k \in I$, there exists a unique symmetric equilibrium. If $\alpha \geq \alpha^*$ the claim is true for arbitrary P , so it also holds in this special case.

Assume then that $\alpha < \alpha^*$, and recall that $p = 1 - x(m)$ and $\sigma(e) = \sigma$. We can rewrite the equilibrium condition (6) as follows:

$$\sigma = \frac{p^{k+1}(4m+16\alpha(1-p))}{(1-p^{k+1})(2-7m-16\alpha(1-p))}, \quad (19)$$

and we know from the proof of Proposition 3 that, in equilibrium, σ is increasing in p .

Equilibrium condition (5) is equivalent to:

$$(2-4m+\sigma m)(k+1)p^k \rho_k^2(e|\emptyset, s^*) = 16\alpha, \quad (20)$$

where σ is given by expression (19). To establish uniqueness is then sufficient to prove that, in equilibrium, the LHS of (20) is increasing in p (where we must take into account that σ is a function of p).

To see this note that since in equilibrium σ is increasing in p it follows that $(2 - 4m + \sigma m)(k + 1)p^k$ is also increasing in p . Therefore, it is enough to show that in equilibrium $\rho_k(e|\emptyset, s^*)$ is also increasing in p .

In order to prove this, we first use condition (6) to rewrite the expression of $\rho_k(e|\emptyset)$ in equilibrium, and we obtain

$$\rho_k(e|\emptyset, s^*) = \frac{4m + 16\alpha(1 - p^*)}{(2 - 3m)(1 - p^{*k+1})}.$$

Therefore,

$$\frac{d\rho_k(e|\emptyset, s^*)}{dp} = \frac{(k + 1)p^{*k}(4m + 16\alpha(1 - p^*))}{(2 - 3m)(1 - p^{*k+1})^2} - \frac{16\alpha}{(2 - 3m)(1 - p^{*k+1})} > 0$$

if and only if

$$\frac{(k + 1)p^{*k}(4m + 16\alpha(1 - p^*))}{(1 - p^{*k+1})} - 16\alpha > 0. \quad (21)$$

Note that the equilibrium condition (20) is the same as

$$(2 - 4m + \sigma m)(k + 1)p^{*k} \left(\frac{4m + 16\alpha(1 - p^*)}{(2 - 3m)(1 - p^{*k+1})} \right)^2 = 16\alpha,$$

which implies that

$$\frac{(k + 1)p^{*k}[4m + 16\alpha(1 - p^*)]}{(2 - 3m)(1 - p^{*k+1})} = \frac{16\alpha(2 - 3m)}{(2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*))}.$$

Using the last equation, we have that inequality (21) is satisfied if and only if:

$$(1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*)) > 0,$$

which is always satisfied for every $\alpha < \alpha^*$. Indeed,

$$\begin{aligned} & (1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*)) > \\ & (2 - 3m) \left((1 - p^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - p^*)) \right), \end{aligned}$$

because $(1 - p^{*k+1})(2 - 3m)^2 - (2 - 4m + \sigma m)(4m + 16\alpha(1 - p^*))$ is decreasing in σ .

Further,

$$\begin{aligned} & (2 - 3m) \left((1 - p^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - p^*)) \right) \\ & > (2 - 3m) \left((1 - \bar{p}^{*k+1})(2 - 3m) - (4m + 16\alpha(1 - \bar{p}^*)) \right) = 0, \end{aligned}$$

because for all $\alpha < \alpha^*$ and for all $p \in [p, \bar{p}]$, the LHS of the inequality is decreasing in p , and the last equality follows by the definition of \bar{p} (see equation (16)). ■

Proof. Proposition 2 Let $A = 4m + 16\alpha(1 - p)$, $B = 2 - 7m - 16\alpha(1 - p)$ and $C = 2 - 4m + \sigma m$. Recall that in equilibrium

$$f(p, \sigma(p)) - \alpha = \frac{(k+1)p^k A^2 C}{(2-3m)^2(1-p^{k+1})} - 16\alpha = 0$$

$$\sigma(p) = \frac{Ap^{k+1}}{B(1-p^{k+1})}.$$

We start by showing that if k increases then p increases, i.e., $x(m)$ decreases. We first derive the following expressions:

$$\frac{\partial f(p, \sigma(p))}{\partial k} = \frac{A^2 p^k \left[C + C(k+1) \ln(p) + (k+1)m \frac{\partial \sigma(p)}{\partial k} + \frac{2(k+1)Cp^{k+1} \ln(p)}{(1-p^{k+1})} \right]}{(2-3d)^2(1-p^{k+1})^2}$$

$$\frac{\partial \sigma(p)}{\partial k} = \frac{Ap^{k+1}}{B(1-p^{k+1})^2} \ln(p).$$

It is easy to see that for p sufficiently low then $\partial f(p, \sigma(p))/\partial k < 0$. Therefore, there exists $\hat{\alpha} > 0$ such that for all $\alpha < \hat{\alpha}$, in equilibrium, $\partial f(p, \sigma(p))/\partial k < 0$. Since we know that in equilibrium $\partial f(p, \sigma(p))/\partial p > 0$, using the implicit function theorem it follows that for all $\alpha < \hat{\alpha}$, if k increases then p increases, i.e., $x(m)$ decreases.

Next, we show that if k increases then σ increases. To see this note that:

$$\frac{d\sigma}{dk} = \frac{\partial f(p, \sigma(p))}{\partial p} \frac{\partial \sigma(p)}{\partial k} - \frac{\partial f(p, \sigma(p))}{\partial k} \frac{\partial \sigma(p)}{\partial p},$$

where

$$\frac{\partial f(p, \sigma(p))}{\partial p} = \frac{(k+1)p^{k-1} \left[kCA^2 - 32pAC\alpha + mpA^2 \frac{\partial \sigma(p)}{\partial p} + \frac{2CA^2(k+1)p^{k+1}}{(1-p^{k+1})} \right]}{(2-3m)^2(1-p^{k+1})^2}$$

$$\frac{\partial \sigma(p)}{\partial p} = \frac{A(k+1)p^k}{B(1-p^{k+1})^2} - \frac{16\alpha(2-3m)p^{k+1}}{B^2(1-p^{k+1})}.$$

Using these expressions, it follows that:

$$\lim_{p \rightarrow 0} \frac{d\sigma}{dk} = \lim_{p \rightarrow 0} \left[-\frac{(k+1)CA^3}{B(2-3m)^2(1-p^{k+1})^4} p^{2k} \ln(p) \right] = 0^+,$$

which implies that for low α (i.e. low p), an increase in k increases σ .

We finally show that for sufficiently small α ,

$$\Pi_k = \sigma^{*2} + 2\sigma^*(1 - \sigma^*)\pi_k(e|m),$$

where $\pi_k(e|m) = 1/2 - (1 - \Psi_k)/8$, is increasing in k . Note that for small α , $\sigma < 1/2$ and therefore Π_k is increasing in σ , keeping constant $\pi_k(e|m)$. So, it is sufficient to show that $\pi_k(e|m)$ is increasing in k , which is equivalent to show that $(1 - \Psi_k)$ is decreasing in k . This follows immediately by equilibrium condition (6). Indeed, we know that if k increases, then $x(m)$ decreases (and so the RHS of condition (6) decreases), which implies that in the new equilibrium $(1 - \Psi_k)$ must decrease. This concludes the proof of the proposition. ■

Proof. Proposition 4 To prove the first part of the proposition, recall that

$$\Psi_k(s^*|P) = 1 - \frac{(1 - (1 - x(m))^{k+1})\sigma}{\sigma + (1 - \sigma)(1 - x(m))^{k+1}},$$

and therefore,

$$\frac{\partial \Psi_k(s^*|P)}{\partial k} = \frac{\sigma(1 - x(m))^{k+1}}{[\sigma + (1 - \sigma)(1 - x(m))^{k+1}]^2} \ln(1 - x) < 0.$$

The second part follows by noticing that in equilibrium $x(e) = 0$ and this implies that

$$\rho_k(e|\emptyset, s^*) = \frac{\sigma}{\sigma + (1 - \sigma)(1 - x(m))^{k+1}},$$

which is clearly increasing in k . ■

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Figure 1. Comparative Static on Advertising Costs
 $\alpha = 0 \dots 0.15, m=0.1, k=8$

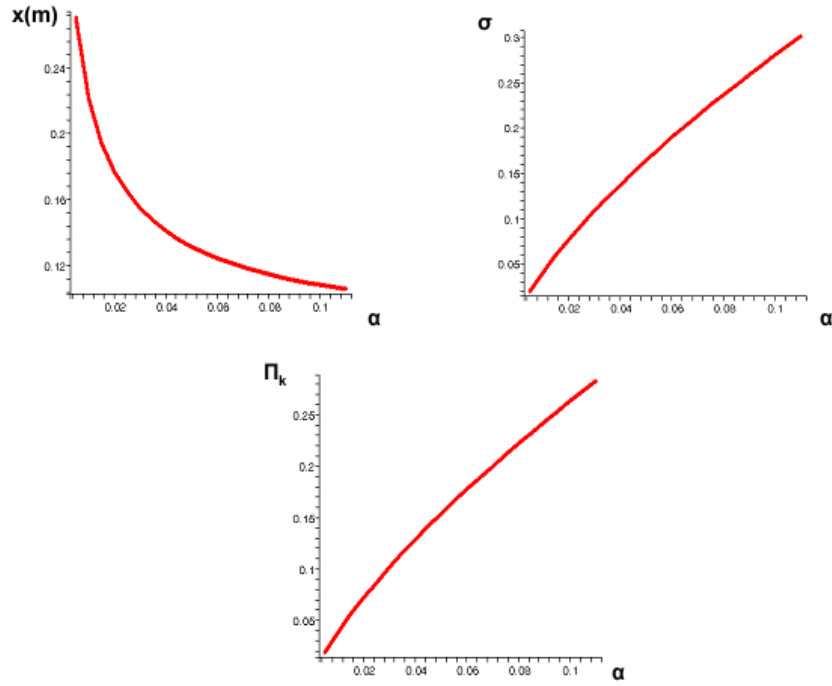


Figure 2. Equilibrium under Heterogeneity in Contextual Exposure: $P(1)=0.5, P(2)=0.5, \alpha = 0.01, m=0.2$

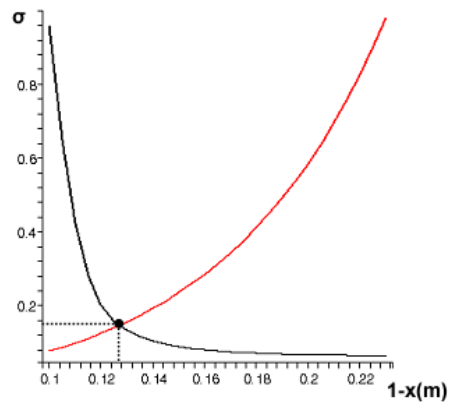


Figure 3. FOSD shift in Contextual Exposure:
 $P(1)=p, P(2)=1-p, \alpha=0.01, m=0.2$

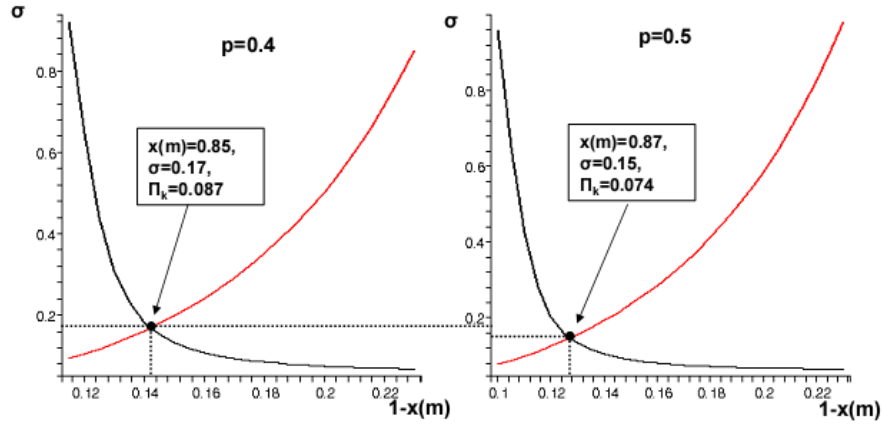


Figure 4. FOSD shift in Contextual Exposure:
 $P(1)=p, P(2)=1-p, \alpha=0.1, m=0.2$

