

Federal Reserve Bank of Minneapolis  
Research Department Staff Report

Revised March 2003

## Why are Married Women Working So Much?\*

Larry E. Jones

University of Minnesota  
and Federal Reserve Bank of Minneapolis

Rodolfo E. Manuelli

University of Wisconsin

Ellen R. McGrattan

Federal Reserve Bank of Minneapolis  
and University of Minnesota

### ABSTRACT

---

This paper studies the large observed changes in labor supply by married women in the United States over the period from 1950 to 1990, a period when labor supply by single females has hardly changed at all. We investigate the effects of changes in the gender wage gap, technological improvements in the production of non-market goods and potential inferiority of these goods on understanding this change. To this end we use a dynamic general equilibrium model which distinguishes between single and married households. We find that small decreases in the gender wage gap can explain simultaneously the significant increases in the average hours worked by married females and the relative constancy in the hours worked by single females, as well the invariance of male hours over the 1950-1990 period. The two main features of the model that account for the ability of changes to the gender wage gap to match the hours data are: endogenous specialization among married couples and human capital accumulation. We also find that technological improvements in the household have—for realistic values—too small an impact on married female hours and the relative wage of females to males. Some specifications of the inferiority of home goods do match the hours patterns, but have counterfactual predictions for wages and expenditure patterns.

---

\*We thank Elizabeth Caucutt, Nezh Guner, John Kennan, Derek Neal, Richard Rogerson, Ananth Seshadri and Michele Tertilt for useful discussions and the National Science Foundation for financial support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## 1. Introduction

In the last fifty years there have been dramatic changes in the U.S. in the hours allocated to market production as a function of gender and marital status. The most striking fact is the almost threefold increase in hours worked by married women. This has occurred over a period in which married men's hours have shown a small decline and those of single individuals, both females and males, have been virtually unchanged (see Figure 1A). Our objective in this paper is to study the role played by three factors in accounting for the changes in hours: improvements in the technologies used to produce home goods, inferiority of the home good with overall income growth, and decreases in the gender wage gap—defined as the ratio of the wages received by females to those of males.

To this end, we construct a dynamic model of the macroeconomy in which households—which can be either single females, single males or married couples—choose labor supplies, consumption and investment in both human and physical capital. Both market and non-market consumption enter utility and require both quality adjusted time (i.e., hours augmented by human capital) and physical capital to be produced. Single and married individuals interact in aggregate markets for labor, capital, investment and market consumption.

Our results show that improvements in home technologies are not successful in accounting for the data. We find that some forms of satiated utility in home consumption (an extreme form of inferiority) can match the hours data, but brings with it a host of other counterfactual predictions. On the other hand, the narrowing of the gender wage gap can simultaneously account for the changes in hours by all groups.

We show that for technology to have some impact on market hours, home and market goods must be either highly substitutable or highly complementary. Otherwise, a change in home technologies affects only the level of home consumption. If home and market goods are substitutes, as McGrattan, Rogerson, and Wright (1997) and Rupert, Rogerson and Wright (2000) estimate, then improvements in home technologies actually cause market hours by married females to *decrease* rather than increase. The reason is simple: if a married female can produce more efficiently at home, more time is spent in home production. If they are complementary, then hours increase with improvements in home technologies. Even in this case, and even if we take the most extreme favorable version of story that the technological improvements are modeled as reductions in prices of home capital goods (durables and structures), only small labor supply effects will occur. We

find similar difficulties with alternative approaches to modeling improvements in home technologies. Home and market goods must be complements, the improvements must be very large (on the order of a five fold increase over the 40 year period that we study) and general, not limited to the pricing of durable goods. Even with this, the human capital response and resulting increase in female wages fall short of what is seen in the data.

If, on the other hand, we assume that home produced goods are inferior, we find that the pattern of hours changes seen in the data can be reproduced but only with extreme versions of inferiority–satiation. Moreover, only certain forms of satiation (i.e., the timing of who gets satiated when) will simultaneously generate the observed changes by married couples with no change in the behavior of singles. Nevertheless, this approach has difficulties in matching the both the observed changes in the gender wage gap, and the relative constancy of consumer durables, broadly defined, purchases as a percent of GNP.

In contrast, changes in the gender wage gap perform quite well along a variety of dimensions (see Figure 1B for the time series of wages of females relative to males). First, for single females, changes in this gap are similar to changes in the overall level of wages and these have small impacts on labor supply if there is a balanced growth path. This same change implies a large response by married females because they face a different technology set. Married couples —unlike single individuals— can choose to specialize. In our model, the presence of the gender wage gap causes married females to allocate a substantial fraction of their time to home production. Thus, even small changes in male-female wage gap can generate large labor supply responses. Of course, as the allocation of time to market activities by married females increases, the elasticity of response decreases. Thus, in this sense, the model delivers a theory of why married and single females display a different response to changes in wages, and of the time-varying nature of these elasticities.

Changes in the gender wage gap also have implications for human capital investments. Since married couples can partially circumvent the implicit tax on women’s labor associated with the existence of wage gaps by increasing the market hours of males and decreasing the market hours of females, married females accumulate less human capital than either single females or males. Thus, even if they work in the market, married females appear less productive. In response to an increase in relative wages, the optimal —from a private point of view— degree of specialization in home production decreases, and married women respond by increasing their investment in human capital. In the absence of accumulation, their response would be immediate and would lead to a narrowing of

wage differentials, which would be inconsistent with the data. This increase by women in investment in human capital is also consistent with the relative increase in educational attainment by women over the last 30 years.

We conduct sensitivity analyses of our results and find that they are robust to changes in the details about the type of human capital that is included, the bargaining power of women in a household, and who it is that benefits from the existence of the wage gap. Roughly speaking, as long as the change in the sex specific component of wages is comparable to the amount seen in the data, the response by married females matches what the U.S. evidence. If this change is not sex specific (i.e., either it applies to all individuals, or only to married females), the observations cannot be matched by the model.

Throughout, we model the gender wage gap as made up of two distinct pieces, one exogenous and the other endogenous. First, the exogenous element is modeled as sex specific tax rates which are higher for females than for males. Second, in part due to the differences by sex in tax rates, endogenous accumulation decisions vary by sex and marital status, and this also contributes to differences in measured wages. It is the first (exogenous) component that we change in our experiments. Although we do not model the details directly, this approach is consistent with the view that the wage gap (i.e., the sex specific tax component) is a consequence of discrimination, either directly in wages or through the presence of a ‘glass-ceiling.’ Viewed in this light, our results show that even small changes in discrimination over time (of the order of a 6% fall in the tax rate in our benchmark parameterization) give rise to the type of hours changes actually observed in the U. S. since 1950. This could be the result of changes in regulations relating to discriminatory practices or changes in the fundamentals that allow discrimination to appear as an equilibrium phenomenon (see Becker (1971) and Coate and Loury (1993)). What our approach does not rule out is that it is some other change (for example changes in divorce laws) which is driving the observed change in the gender wage gap through its indirect effects on the incentives to invest in unobserved components of human capital.

The results we obtain on the effects of improvements in home technologies are substantially different from those in Greenwood, Seshadri and Yorukoglu (2001). Their model focuses on substitution at the extensive margin (married women either work or not), but also features satiation in home production. Their model performs well in that a calibrated decrease in the price of household durables results in a substantial increase in married women’s labor force participation. Our

approach, which assumes smooth substitution, allows us to disentangle the effects of technological improvements from those of satiation. Our findings suggest that it is the assumption of satiation that is important for their positive results rather than technological improvements per se. Their model also predicts a substantial decrease in married female labor force participation at some point and has the implication that the share of income spent on home durables is ultimately declining. Neither of these predictions matches the data. Finally, they do not consider the effects of the technological change on single individuals.

At the micro level, the pioneering work by Mincer (1962) was a first attempt to explain changes in women's work as driven by the overall increase in wages using a static framework. Using the same principles, but considerably more sophisticated statistical analysis, Smith and Ward (1985) study a model that predicts an increase equal to 58% of the observed change for the period 1950-80, but, as they acknowledge, their model would run into particular trouble in the 80s and 90s when real wage growth was low but female labor force participation increased. Blau (1998) states that "a considerable portion of the change over time in female participation remains 'unexplained' by variables conventionally used in our analyses." Goldin (1990) finds that cohort (or time) effects are more important than standard economic variables. In general, these studies treat married and single females separately and summarize their different response to the same change in wages by indicating that the two groups have different elasticities. In some sense this paper proposes a theory of why the elasticities of female labor supply are so different across marital status and why they have changed so much over time, which may be one reason why time and cohort effects have considerable explanatory value.

Several other fully specified quantitative general equilibrium models have been developed to explain several issues that are related to the economics of the family. We discuss the handful that deal with the issue of female labor supply. Greenwood, Guner, and Knowles (2000a, and 2000b) study a model with endogenous fertility. The model is very successful in replicating the experience of welfare mothers and their children and provides provocative answers to changes in several features of the welfare system. However, from the perspective of female labor supply the model does not perform well. It predicts that the hours worked by married females exceed those of single females by 37%, and that single females work only 60% of the hours worked by single men. Both these implications are at odds with the U.S. evidence. (See Greenwood, Guner, and Knowles (2000b).)

Olivetti (2000) and Caucutt, Guner, and Knowles (2000) investigate the impact of a gender

specific increase in the returns to experience. Olivetti (2000) studies a four period model in which human capital can only be acquired through working. Her model succeeds in predicting an increase in married female market hours. However, from her formulation —and in this she follows the traditional labor literature— the same effects would also have a positive impact in the number of hours worked by single females, and it is difficult to evaluate the impact that differential returns to experience had in the 50s, when married female labor force participation became more significant quantitatively. Caucutt, Guner, and Knowles (2000) would also predict that increases in the returns to experience have a large impact on the hours supplied by single females. In addition, neither paper presents any direct evidence of a sex specific change in the technology that they use to describe learning on the job.

In Section 2 we present a simple static example illustrating the effects we capture with the full model. In Section 3 the dynamic model we study is introduced and in Section 4 we present some of the basic facts that we will use to evaluate alternative hypotheses. In Sections 5, 6 and 7, we study, in turn, the quantitative impacts of improvements in the home technology, the properties of equilibrium when home production is inferior and the effects of changes in wage discrimination. Our results are summarized in Section 8.

## 2. A Simple Static Example

In this section, we lay out a simple static example of labor supply choice in order to build intuition for the results which are coming below. We show that in a standard model of home production labor supply decisions of single females, single males and married couples are independent of changes in the level of technology in both the home and market sectors. These decisions are also shown to be independent of the price of any durable goods used to produce the home good. It is also shown that the labor supply decisions of single individuals are independent of any wage discrimination. However, reductions in the amount of wage discrimination reduce market hours of married males and increase those of married females.

Consider a setting in which all households—single females, single males and pairs— must decide how to allocate their labor endowments across market activities and the production of goods in the home, and how much of their income to allocate to consumption goods and home capital goods. Home production requires the use of both hours and these capital goods. All households face a common set of technological restrictions (i.e., productivities) and each is taxed on labor income

earned in the market sector. Because we will model discrimination as differential tax wedges by sex later in the paper, we allow for the tax faced to differ by sex, but not by marital status. We assume that all households are identical except for marital status to simplify the analysis.

### *Single Households*

In this case, the maximization problem solved by a single female is:

$$\max_{c_{fs}^1, c_{fs}^2, \ell_{fs}^1, \ell_{fs}^2, k_{fs}} \mu \log(c_{fs}^1) + \nu \log(c_{fs}^2) + (1 - \mu - \nu) \log(\ell_{fs})$$

subject to:

$$\begin{aligned} c_{fs}^1 + qk_{fs} &\leq (1 - \tau_f)w\ell_{fs}^1, \\ c_{fs}^2 &\leq Ak_{fs}^\theta (\ell_{fs}^2)^{1-\theta}, \\ \ell_{fs} + \ell_{fs}^1 + \ell_{fs}^2 &= 1, \end{aligned}$$

where  $c_{fs}^1$  and  $c_{fs}^2$  are consumption of the market and home good,  $\ell_{fs}^1$  and  $\ell_{fs}^2$  are hours worked in the market and the home,  $k_{fs}$  is the amount of the home specific capital good purchased and  $q$  is its price,  $w$  is the wage rate,  $A$  is a home specific productivity factor and  $\tau_f$  is the wedge between actual productivity and income for the typical female.<sup>1</sup>

The maximization problem for single males —identified with a subindex  $m$  instead of  $f$ — is similar, with the only difference being in the tax rate faced. As noted above, we will assume that  $(1 - \tau_f) = (1 - \tau_d)(1 - \tau_m)$  where  $\tau_m$  represents the common labor income tax rate and  $\tau_d$  represents the additional wedge faced by a female when there is discrimination in the market activity.

It is straightforward to generalize the problem to allow for sex specific differences in productivities allowing for a rich variety of potential differences in both absolute and comparative advantage across the sexes. Since this will not change any of the results given below, we leave this generalization to the reader.

Letting  $W = (1 - \tau_f)w$ , the solution to this problem is:

$$\ell_{sf}^1 = (\mu + \theta\nu) \frac{W}{(1 - \tau_f)w} = \mu + \theta\nu,$$

$$\begin{aligned}
\ell_{sf}^2 &= (1 - \theta)\nu \frac{W}{(1 - \tau_f)w} = (1 - \theta)\nu, \\
c_{sf}^1 &= \mu W, \\
c_{fs}^2 &= Ak_{fs}^\theta (\ell_{fs}^2)^{1-\theta} = A \left( \theta\nu \frac{W}{q} \right)^\theta ((1 - \theta)\nu)^{1-\theta}, \\
k_{fs} &= \theta\nu \frac{W}{q}.
\end{aligned}$$

Thus, the single female sets the marginal rate of substitution between leisure and consumption equal to the relative, after tax, productivities in the two activities, and capital is chosen so that its marginal product in home production is equal to its cost in terms of consumption. A similar set of equations holds for single males, with the only difference being that  $\tau_m$  appears everywhere (i.e., in  $W$ ) in place of  $\tau_f$ . If it were not for this, it would be immediate that the solutions would be identical.

It is immediate from inspecting these equations that hours used in both the market and the home are independent of  $w$ ,  $A$ ,  $q$  and  $(1 - \tau)$ . These parameters do have an impact on both the levels of consumption and the amount of the home capital good that is purchased. Thus, improvements in technologies do not alter the amount of labor supplied to the market either by single females or single males. It also follows from this that market labor supply of single females and single males will be the same even if there is an additional tax wedge on females due to discrimination (i.e.,  $\tau_d > 0$ ).<sup>2</sup>

In a dynamic setting in which  $w$  and  $A$  are endogenously determined by human capital formation decisions which may differ across the sexes (due to either discrimination or natural productivity differences), analogues of these static first order conditions will still apply and hence much of this reasoning will continue to hold. The main difference is that the levels of consumption and labor supply enter the optimality conditions governing optimal capital accumulation and hence the effects are more complex.

If the utility functions of the two sexes are identical, but are not logarithmic, the results given above need no longer hold. How they are changed depends the elasticity of substitution between home and market goods. For example, if the utility function aggregates home and market goods using a CES aggregator, and home and market goods are substitutes, an increase in productivity in the home (i.e.,  $A$ ) causes both single males and single females to consume more home production and lower market hours (see the Appendix for details). If they are complements, the opposite occurs



causing market hours to increase for both. Similarly, the effects of differences in sex specific tax rates depends on whether home or market goods are substitutes or complements. For example, if home and market goods are substitutes, and females face higher effective rates than males, single female hours supplied to the market will be lower than those of single males, and single females will consume more home goods and less market goods than their male counterparts. This may account for the small but measurable difference in market hours between single males and single females seen in the data. (Single females work slightly less than single males do and this difference has been relatively stable over time.) Of course, the size of these effects will depend on both the changes in relative productivities of the two activities (or the change in sex specific tax rates) and on the degree to which preferences depart from the log specification.

### *Married Couples*

We turn now to the problem of a married couple in this environment. We assume that the bargaining problem within the household is resolved efficiently so that a weighted form of a planner's problem describes the decisions that the couple makes. In this case, the maximization problem solved is:

$$\begin{aligned} \max \quad & \lambda_f [\mu \log(c_{fp}^1) + \nu \log(c_{fp}^2) + (1 - \mu - \nu) \log(\ell_{fp})] \\ & + \lambda_m [\mu \log(c_{mp}^1) + \nu \log(c_{mp}^2) + (1 - \mu - \nu) \log(\ell_{mp})] \end{aligned}$$

subject to:

$$\begin{aligned} c_{fp}^1 + c_{mp}^1 + qk_p &\leq (1 - \tau_m)w\ell_{mp}^1 + (1 - \tau_f)w\ell_{fp}^1, \\ c_{fp}^2 + c_{mp}^2 &\leq Ak_p^\theta (\ell_{fp}^2)^{1-\theta}, \\ \ell_{fp} + \ell_{fp}^1 + \ell_{fp}^2 &= 1, \\ \ell_{mp} + \ell_{mp}^1 &= 1, \end{aligned}$$

where  $c_{fp}^1$  and  $c_{mp}^1$  are the consumption of the market good by the male and female of the pair,  $c_{fp}^2$  and  $c_{mp}^2$  are their consumption levels of the home good,  $\ell_{fp}^1$  and  $\ell_{mp}^1$  are the hours they work in the market, and  $\ell_{fp}^2$  and  $\ell_{mp}^2$  are the hours they work in the home. The remainder of the parameters

are as discussed above. Note that we have maintained the assumption that tax rates are sex specific and will, as above, interpret differences between  $\tau_f$  and  $\tau_m$  as due to the effects of discrimination in the market activity. As in Becker (1991) the solution to this problem is not interior in general since male and female hours are perfect substitutes in both home and market activities. Because of this, there will be specialization within the household. In keeping with what is seen in the data, we will use the first order conditions that result when  $\ell_{mp}^2 = 0$  but that otherwise the solution to the problem is interior.

The solution to the married couple's problem is:

$$\begin{aligned}\ell_{fp}^1 &= [1 - \lambda_f + \lambda_f \mu + \nu(\lambda_f + \theta - 1)] \frac{W}{(1 - \tau_f)w}, \\ \ell_{fp}^2 &= \nu(1 - \theta) \frac{W}{(1 - \tau_f)w}, \\ \ell_{mp}^1 &= [1 - \lambda_m(1 - \mu - \nu)] \frac{W}{(1 - \tau_m)w},\end{aligned}$$

where  $W$  is defined by  $W \equiv (1 - \tau_m)w + (1 - \tau_f)w$ . We have also assumed, for simplicity, that there are no economies of scale in living as a couple. This could be reflected in the example in a variety of ways, but would not affect the results given below.

As in the case of single households, it is immediate to see that changes in  $A$ ,  $q$ , and  $w$  do not affect the household's allocation of hours to any of the activities, leisure, work in the home, or work in the market. As is the case with single agents, there are changes in quantities consumed and in  $k$ , however. The form of these quantity adjustments mirror those for the single agents and will not be included here.

The same is not true with changes in taxes however. If either  $\tau_m$  and  $\tau_f$  are changed holding the other fixed, hours adjust. For example, if  $\tau_m$  is unchanged, but  $\tau_f$  falls, or, equivalently, there is a reduction in discrimination so that  $\tau_d$  falls, it follows that  $\ell_{fp}^1$  increases while  $\ell_{fp}^2$  falls (as does  $\ell_{fp}$ )— the woman works more in the market and less in the home (and consumes less leisure). At the same time,  $\ell_{mp}^1$  falls (and  $\ell_{mp}$  goes up). Thus, in response to a reduction in market discrimination, the woman works more in the market, the man works less. In contrast, if  $\tau_m$  and  $\tau_f$  are both changed proportionally keeping  $(1 - \tau_d) = (1 - \tau_f)/(1 - \tau_m)$  fixed, there is no change in hours.

Summarizing, we see that if utility is logarithmic, changes in technology are neutral for labor

supply decisions for both singles and married couples, whereas reductions in discrimination leave the decisions of singles unchanged but increase married female market hours. For utility specifications differing from log, there will be effects on all agents of changes in technology, even if preferences are homothetic, but their direction will depend on the substitutability between home and market goods and, by continuity, are likely to be small unless the changes are very large or the utility structure deviates greatly from unit elasticity of substitution. In this case, the effect will be present for all agents, single and married, male and female.

### *Inferiority of the Home Good*

There will also be effects of technological change on labor supply if preferences are not homothetic. Since these changes are substantial for some specifications when home goods are inferior, we present a simple version of this phenomenon here. We consider a perturbation on the model above in which households become satiated in  $c^2$  once it is equal to  $c^*$ . Beyond that, the formulation is identical.

We restrict attention to the problem of a single household. Of course, if parameters are such that the solution to the original problem satisfies  $c_{fs}^2 \leq c^*$ , the solution is that presented above. This will hold as long as  $A \left[ \theta \nu \frac{(1-\tau_f)w}{q} \right]^\theta [(1-\theta)\nu]^{1-\theta} \leq c^*$ . This requires a relatively low  $A$ ,  $w$ , and  $(1-\tau_f)$  and a relatively high  $q$ . If this does not hold, the solution is given by  $c_{fs}^2 = c^*$  with  $k_{fs}$  and  $\ell_{sf}^2$  chosen to minimize the cost of producing  $c^*$ . Let  $C(c^*; q, (1-\tau_f)w)$  denote the minimum total cost of producing  $c_{fs}^2 = c^*$ . Then the solution to the household optimization problem is:

$$\begin{aligned} \ell_{fs}^1 &= \frac{1-\mu-\nu}{1-\nu} + \frac{c^*}{A} \left[ \frac{1-\theta}{\theta} \frac{q}{(1-\tau_f)w} \right]^\theta \left[ \frac{1-\mu-\nu}{1-\nu} \frac{1}{1-\theta} - 1 \right], \\ \ell_{fs}^2 &= \frac{c^*}{A} \left[ \frac{\theta}{1-\theta} \frac{(1-\tau)w}{q} \right]^\theta, \\ c_{fs}^1 &= \frac{\mu}{1-\nu} [(1-\tau_f)w - C(c^*; q, (1-\tau_f)w)], \\ k_{fs} &= \frac{\theta}{1-\theta} \frac{(1-\tau)w}{q} \ell_{fs}^2, \\ c_{fs}^2 &= c^*. \end{aligned}$$

In this case, increases in  $w$  and/or decreases in  $\tau_f$  decrease  $\ell_{fs}^2$  but increase  $\ell_{fs}$ . Whether or not  $\ell_{fs}^1$  increases or decreases depends on which is larger,  $(1-\theta)$  or  $(1-\mu-\nu)/(1-\nu)$ . If  $(1-\theta)$  is

larger,  $\ell_{fs}^1$  rises with increases in  $(1 - \tau_f)w$  while the opposite holds if  $(1 - \theta)$  is smaller. Similar results hold for changes in both  $A$  and  $q$ , if  $(1 - \theta)$  is larger than  $(1 - \mu - \nu)/(1 - \nu)$ , increases in  $A$  and/or decreases in  $q$  increase  $\ell_{fs}^1$ . Thus, as is intuitive, what is important is the share of  $\ell$  in the production of  $c^2$  relative to its share in the reduced form utility,  $(1 - \mu - \nu)/(1 - \nu)$ .

Thus, in some cases, satiation gives an alternative route to changes in  $\ell_{fs}^1$ , but as we can see from the example, this effect is present in single households as well as those of married couples. Note however, that as  $A$  or  $w$  rises, or  $q$  falls,  $qk$  falls as a fraction of income.

Although the example we have considered in this section is special, the results are actually more general. In the Appendix, we present an example featuring both quality choices for home production and a glass ceiling on job selection for females. The conclusions are virtually identical.

### 3. General Dynamic Model

In this section we describe a general, aggregate, model. We view this model as complementing the micro evidence, since, of necessity, to capture dynamic effects we cannot model the variety of experiences observed at the individual level. Since married and single females (and males) behave so differently in the data, our theory emphasizes the role of couples in determining labor supply decisions. To this end we follow Greenwood and Hercowitz (1991) and Benhabib, Rogerson and Wright (1991) by assuming that households both produce goods in the home and work in the market. We differ from their analysis by explicitly considering consumption and labor supply of the two partners within a married couple.

We assume that partnerships are composed of one male and one female, and will adopt a dynastic formulation. We will abstract from issues of marriage and divorce and will assume that married couples solve their internal bargaining problem efficiently. Thus, we model the decisions made by individual members of the partnership as being identical to the solution of a weighted utility planner's problem.

The partnership solves,

$$\max \sum_{t=0}^{\infty} \beta^t (1 + \gamma_p)^t [\lambda_f U_f(c_{fpt}^1, c_{fpt}^2, \ell_{fpt}) + \lambda_m U_m(c_{pmt}^1, c_{pmt}^2, \ell_{pmt})] \quad (1)$$

subject to

$$\begin{aligned}
& \sum_{t=0}^{\infty} p_t [c_{fpt}^1 + c_{mpt}^1 + x_{kpt}^1 + q_t x_{kpt}^2 + x_{hfp} + x_{hmp} + x_{\eta fpt} + x_{\eta mpt}] \\
& \leq \sum_{t=0}^{\infty} p_t [(1 - \tau_{kt})r_t + \delta_k \tau_{kt}] k_{pt}^1 + (1 - \tau_{\ell ft}) w_t z_{fpt}^1 + (1 - \tau_{\ell mt}) w_t z_{mpt}^1 + T_{pt}, \\
& c_{fpt}^2 + c_{mpt}^2 \leq A_t^2 F^2(k_{pt}^2, z_{fpt}^2 + z_{mpt}^2), \\
& k_{pt+1}^i \leq [(1 - \delta_k)k_{pt}^i + x_{kpt}^i]/(1 + \gamma_p), \quad i = 1, 2 \\
& h_{gpt+1} \leq [(1 - \delta_h)h_{gpt} + x_{hgpt}]/(1 + \gamma_p), \quad g = f, m, \\
& \eta_{gpt+1} \leq [(1 - \delta_\eta)\eta_{gpt} + x_{\eta gpt}]/(1 + \gamma_p), \quad g = f, m, \\
& z_{gpt}^i \leq \Phi^i(\ell_{gpt}^i, h_{gpt}, \eta_{gpt}), \quad i = 1, 2, \quad g = f, m, \\
& l_{gpt} = 1 - \ell_{gpt}^1 - \ell_{gpt}^2, \quad g = f, m,
\end{aligned}$$

where we follow the same notational convention as in the previous section.

Our formulation is somewhat standard. We have abstracted from any economies of scale at the household level for simplicity, but note that married households do have some benefit directly from the possibility of specialization. The terms  $z_{gpt}^i$  indicate the effective amount of labor allocated to sector  $i$  (1 if market, 2 if non-market) by an individual of gender  $g$  ( $f$  or  $m$ ) who is in a partnership (this is indicated by  $p$ ) at time  $t$ . We allow effective labor to depend on raw hours,  $\ell_{gpt}^i$ , and two forms of human capital,  $h_{gpt}$  and  $\eta_{gpt}$ . The function that maps these inputs into effective labor is indexed by gender, marital status and type of activity. This specification allows for different skills for the production of market (e.g., spreadsheet operations) and non-market goods (e.g., child care services). In addition, it allows us to consider the effects of differential productivity between males and females in the production of some goods. We denote by,  $k_{pt}^i$  the amount of capital devoted to activity  $i$ ,  $i = 1, 2$ . Note that these should be thought of as broad measures of capital goods, for example, including all appliances, autos and the house itself in the production of the home good. Corresponding to this, we want to allow for the relative prices of home capital goods to fall over time and so  $q_t$  denotes the relative price of a home capital good in period  $t$ . The term  $T_{pt}$  is transfers.<sup>3</sup>

The terms  $\tau_{\ell gt}$  capture, as before, tax rates on labor services of a married individual of gender  $g$ . In this aggregate model, this wedge between male and female wages is meant to capture both outright discrimination and other factors (e.g., gender bars, tracking, glass ceilings, changes in the

shadow price of characteristics) that result in lower effective wages for females.<sup>4</sup> This is important because it is the after tax wage rate that will determine the payoff to investments in human capital. There are substantial differences between the *raw wage gap* —which is our driving shock— and the *adjusted wage gap*, which corresponds to what is measured in the data. The latter includes not only the differences captured by  $(1 - \tau_{\ell ft})/(1 - \tau_{\ell mt})$ , but also other differences in characteristics (human capital), both measured and unmeasured that —although endogenous— vary systematically across groups. We assume that labor tax rates do not depend on marital status.

The problem solved by single females (indicated by the subscript  $fs$ ) and single males (with subscript  $ms$ ) are similar to (1), with the obvious changes.

Let  $n_{gs}$  be the number (fraction) of individuals of gender  $g$  ( $f$  or  $m$ ) who are single, and let  $n_p$  be the number (fraction) of partnerships. We will assume that these do not change over time for simplicity. Let a bar over a variable denote economy-wide averages. Thus,  $\bar{k}_t^1 = n_{fs}k_{fst}^1 + n_{ms}k_{mst}^1 + n_pk_{pt}^1$  denotes the aggregate supply of capital, and  $\bar{z}_t^1 = n_{fs}z_{fst}^1 + n_{ms}z_{mst}^1 + n_p(z_{fpt}^1 + z_{mpt}^1)$ , denotes the aggregate supply of effective labor.

We assume that there is a constant returns to scale aggregate production function of market goods given by  $A_t^1 F^1(\bar{k}_t^1, \bar{z}_t^1)$ . Feasibility in the goods market requires that,

$$\begin{aligned} n_{pt}[c_{pt}^1 + x_{kpt}^1 + q_t x_{kpt}^2 + x_{hfp} + x_{\eta fp} + x_{hmp} + x_{\eta mp}] + \\ + n_{ms}[c_{mst}^1 + x_{kmst}^1 + q_t x_{kmst}^2 + x_{hms} + x_{\eta ms}] \\ + n_{fs}[c_{fst}^1 + x_{kfst}^1 + q_t x_{kfst}^2 + x_{hfs} + x_{\eta fs}] + G_t \leq A_t^1 F^1(\bar{k}_t^1, \bar{z}_t^1), \end{aligned}$$

where  $G_t$  denotes government spending in goods and services. In our analysis, we assume that  $G_t$  is a constant fraction of market output.

**DEFINITION 1.** *An equilibrium is a collection of prices  $\{p_t\}, \{r_t\}, \{w_t\}$ , and an allocation (defined as all quantities indexed by type of good, gender and marital status)*

1. *Given prices, the allocation solves (1), and the equivalent problems for singles.*
2. *The allocation is feasible.*

The model we just outlined is too complex to derive interesting quantitative results theoretically. In order to make some progress understanding the effects of changes in technology and wage

discrimination, we use standard numerical techniques to compute equilibrium allocations.

### *Functional Forms and Parameter Choice*

We start with the specification of functional forms we will use in our quantitative analysis. We consider the class of preferences given by  $U^f = U^m = U$  where

$$U = \frac{1}{1-\sigma} \left[ \left( \psi_1 (c^1)^{\psi_2} + (1-\psi_1)(c^2)^{\psi_2} \right)^{(1-\psi_3)/\psi_2} (1-\ell^1-\ell^2)^{\psi_3} \right]^{1-\sigma}.$$

The production function of both types of goods (market and non-market) are assumed to be Cobb-Douglas with the same coefficients for market and non-market goods:

$$F^i(k, z) = A^i k^\theta z^{1-\theta}, \quad i = 1, 2.$$

We assume that the production functions of specific human capital are identical across all categories (gender and marital status) and given by

$$\Phi^i(h, \eta, \ell^i) = (h)^{\kappa_i} (\eta)^{\zeta_i} \ell^i, \quad i = 1, 2.$$

The parameter choices for our base case are in Table 1. We set  $n_p$  to match the fact that roughly 60 percent of the relevant U.S. population was married during the period we study. Values for government spending and tax rates on labor and capital are average postwar values for the United States. The growth rates  $\gamma_p$  and  $\gamma_A$  are long-run trend levels for the United States. The discount factor is chosen so that the trend interest rate is 4 percent.

Values for the capital share  $\theta$ , the rate of physical depreciation,  $\delta_k$ , and two critical preference parameters,  $\psi_2$  and  $\sigma$ , are the maximum likelihood estimates of McGrattan, Rogerson and Wright (1997) for a model with home production. We set the depreciation rates for human capital,  $\delta_h$  and  $\delta_\eta$ , equal to the depreciation rate for physical capital,  $\delta_k$ , for our benchmark example. Since good estimates for human capital rates are not readily available, this parameter choice will be one focus of our sensitivity analysis.

We choose the remaining preference parameters  $(\psi_1, \psi_3, \lambda_f)$ , two of the elasticities for effective labor  $(\kappa_1, \kappa_2)$ , and the paths of technology and discrimination taxes to achieve several objectives. First, with no change in technology or discrimination, the benchmark parameters yield initial hours of work that match the 1950 hours in Figure 1A for three groups – namely, married females, married

males, and single females – and they yield a relative wage of 51 percent – which is the value we obtain from extrapolating the time series in Figure 1B.<sup>5</sup> Second, we assume that initial leisure of married males is equal to initial leisure of married females. This determines a value for the weight on married female utility,  $\lambda_f$ . This weight turns out to be very low, only 0.062. Because this value is so low, it will be one of the parameters that we focus on when we do sensitivity analysis.

A third objective is to match the time series on relative wages (Figure 1B) in the benchmark simulation with a change in discrimination. To do this and achieve the initial conditions above, we set the initial discrimination tax,  $\tau_{d1950}$ , at 22 percent and set subsequent rates so that the model yields the same time path for relative wages as in Figure 1B.

For the benchmark parameterization, we do not distinguish type- $h$  and type- $\eta$  human capital, and therefore assume that  $\kappa_i = \zeta_i$  in both sectors,  $i = 1, 2$ . We experiment later by assuming no human capital and assuming sector-specific human capital.<sup>6</sup>

We assume that government purchases of goods and services is 20% and redistribute, in a lump sum fashion, any remaining revenue generated. We interpret  $\tau_{\ell mt}$  as the governmentally specified tax rate on labor income and assume that any difference due to discrimination is completely used for redistributive purposes. For simplicity, we assume that this redistribution is equally divided among all agents in the economy. This is consistent with our assumption that although we have modeled discrimination as a tax, it is not being used for revenue generation. Later, we look at alternative specifications of the distribution of revenue.

Finally, since we want to abstract from business cycle frequency effects, we take a period in our model to be five years. Thus,  $t = 0$  corresponds to 1950, and  $t = 10$  corresponds to the year 2000. The calculations that we perform assume that all agents perfectly anticipate the changes that are forthcoming.

For each experiment, we include sensitivity analyses on our results. The parameters used for these alternatives are included in Table 2.

## 4. Background Data

In this section we outline the basic facts about labor supplies, relative wage rates, home capital goods prices and home capital shares that we will compare our model solutions to.

The changes in the levels and composition of hours allocated to market production by sex



and marital status that have occurred since 1950 are notable. The most striking facts are: the average number of hours worked by married women has increased 171% from 8.17 to 22.66 hours per week; the average number of hours worked by married men has decreased from 41.28 to 38.30 (-8%). In contrast to this, the average number of hours worked by single individuals—both males and females— have been relatively stable with single males working slightly more than single females. Both are at a level approximately equal to 70% of those worked by married males and hours for single males show a small decrease over the 40 year period covered in the data, about -4% in total, from 31.58 to 30.24. For single females, the levels were 28.99 hours per week in 1950 and 29.00 in 1990. Finally, we can see that there has been a change in the relative composition of hours by a married couple, with the sum looking more and more like the sum of a single man and a single woman over the period.<sup>7</sup> More precisely an artificial household formed by two single individuals worked approximately 60.5 hours/week in 1950, and about 59.2 hours/week in 1990. On the other hand, the average married couple worked approximately 49.5 hours/week in 1950, but almost 61 hours/week in 1990. (See McGrattan and Rogerson (1998).) These are the observations that we want the model to match as outputs.

Direct measures of changes in productivity in the home are not easy to come by. In the special case that this improvement is realized as cheaper home capital, one part of the evidence is carefully discussed in Greenwood, Seshadri and Yorukoglu (2001). They document that the real price of household appliances decreases at a rate between 3.5% and 8% starting in 1950. They ignore other important categories of home capital, however. Some of these, such as autos, are also important time saving durables used in home production and have seen less dramatic reductions in price. Housing itself has seen virtually no real price reduction. Figure 2A shows the time series of price deflators from NIPA for both a broad and a narrow take on home capital. Durable consumption and residential investment represents about 12% of GDP on average over the 1929 to 2000 period and the price deflator for this category shows a slight decline over the period, but it is not substantial. The other, more narrowly defined, category of household appliances represents about 0.7% of GDP on average over the 1929 to 2000 period and shows a marked decrease in prices over the period with its value in 1990 about 23% of that in 1950. Figure 2B shows the time series of expenditure shares for these two categories. The expenditure share for the durable consumption and residential investment category shows very little change over the period but moves systematically with the cycle. After a short post WWII boom, expenditure share of household appliances drops quickly, returning to its pre-war level, then shows a slow gradual decline over the subsequent 35 years.

The evidence on the size and nature of the gender wage gap has been well documented in the literature, (see Goldin (1990) and (1997)). For example, Blau (1998) finds that a full time working woman with high school education earned about 55% of what a man earned in 1969, that this ratio was relatively flat until the mid 1970's and then rose to about 70% by 1995. A similar change is seen in college graduates. The gender wage gap is a difficult measure to interpret. In principle, it can either measure the direct effects of wage discrimination (the payment of lower wages to one group despite equivalent training and work duties), or differences in unmeasured (by the econometrician) skills that are correlated with sex. These differences in skills themselves could be due to discrimination (e.g., glass ceilings and marriage bars, see Goldin (1990)) or due to other, non-discriminatory, incentives for the development of skills across the sexes (e.g., specialization in the provision of home goods, child care, etc.).

We model the gender wage gap as arising from two distinct sources. The first is wage discrimination in employment which we model as a sex specific tax. This modeling choice is similar to the formulation implied by the Becker (1971) approach to discrimination and can also be interpreted as the shadow price on sex specific constraints on job types (e.g., marriage bars and/or glass ceilings). The second source of wage differences in the model is differences, by sex, in skills (i.e., human capital). Differences by sex in the attainment of these skills is endogenous to the model. The forces driving these differences being partly due to discrimination, partly due to specialization within a married couple. For us then, the lessening of the gender wage gap seen in the data comes from a reduction in this sex specific tax rate. It is difficult to know exactly what this 'discrimination tax' should be or how much it has changed, but Goldin carefully documents several discriminatory practices and the beginning of their decline in the 1950's. Other relevant considerations include the passage of the 19th amendment to the U.S. Constitution in 1919 giving women the right to vote, the introduction of specific regulations against discrimination by sex (e.g., the EEOC, see Goldin), which would have an effect on wage payments by sex in either the Becker (1971) or the Coate and Loury (1993) models of discrimination, and the reduction in union power over the period, which would reduce the amount of effective discrimination in the Becker model. Since there are no direct measures of the size of the relevant tax rate, we will do considerable experimentation below.

## 5. Technological Change in the Home

In this section we study the impact of changes in technology on the allocation of labor by singles and partnerships. There are many ways one could, in principle study the effects of technological change in the model outlined above. This could be done by having sector specific growth rates in market and home activities, for example. Although we do discuss this alternative in the section on sensitivity below, it is problematic in that no direct measurements of the rate of technological change in the home sector. Because of this we focus first on the effect discussed in Greenwood, Seshadri and Yorukoglu (2001), that the price of durables in the home sector has fallen.<sup>8</sup> Corresponding to this, we ask: What is the equilibrium effect of reductions in  $q$  in the budget constraint of the individual households?

A popular explanation of the increase in hours allocated to market work is that improvements in household durables and in the availability of ready made goods (clothes, foodstuffs) “frees up” time from housework. From a theoretical point of view this is not necessarily the case as was shown in Section 2 (see the example in the Appendix also). Increases in productivity can increase or decrease the hours allocated to housework depending on the elasticity of substitution between home and market goods. From an empirical perspective the evidence is mixed. Historians of technology (e.g. Cowan (1983)) argue —using evidence from a number of time-use data studies— that substantial increases in the productivity of labor allocated to home production did not result in decreases in the number of hours of housework, especially during the 1870-1940 period which, it can be argued, saw the largest productivity increases. This could be true because of increases in quantity or quality of home good production (e.g., washing clothes more frequently, cleaning better, etc.) or from changes in demands for doing this work such as moving to suburbs, and/or purchasing larger houses. Economic historians, such as Mokyr (2000), agree with the facts presented by Cowan but differ in the interpretation. Mokyr (2000) argues that several scientific or “knowledge” revolutions induced households to spend more time in housework to increase the quality of home production. Finally, at the other end, Greenwood, Seshadri and Yorukoglu (2001) argue that the diffusion of household durables can account for the increase in female labor force participation.

To give this explanation the best chance for success we use same change in  $q$  as that used in Greenwood, Seshadri and Yorukoglu, that given by appliance prices, a reduction of 77% over the 1950 to 1990 period. As noted above, this is a much more dramatic reduction in prices than what is seen in other household durables (autos and houses themselves for example) and is similar

in magnitude to the reduction of some narrowly defined classes of producer durables.

The results of our computations are shown in Figure 3. As noted above, the best estimates are that home and market goods are substitutes, but in this case, a reduction in  $q$  actually causes married female market hours to fall in contrast to what is seen in the data. Because of this, we focus on examples where home and market goods are complements. The hours series for one of these examples (with  $\psi_2 = -.75$ ) is shown in Figure 3A. As can be seen from the Figure, table, the experiment is successful in that the hours of both single females and single males are unchanged in response to this price reduction. There is a measurable effect on married female work hours, but it is much smaller than the increase in hours seen in the data. Similarly, the change in married male hours is hardly noticeable, again in contrast to what is seen in the data.

Even though human capital is allowed to adjust in response to this change in  $q$ , it does not. Because of this, the relative wage of women to men is unchanged. Again, this is in contrast to what is seen in the data. This is shown in Figure 3B. Figure 3C shows the time path for expenditure shares on home capital. In contrast to what is seen in the data, this share increases significantly and stays high.

Summarizing, the prediction of the model is that in response to the change in the price of durables, hours in the home stay roughly unchanged as do human capital investment decisions. There is a dramatic increase in  $k^2$ , however, mirroring the discussion in Section 2. This can be thought of either as an increase in quantity, or quality of the durables used to produce home goods.

### *Sensitivity Analysis*

Table 3 contains the numerical results for the example discussed above (in the second row) along with the results of several other related experiments. The third row of Table 3 shows the results for an example with even less substitution,  $\psi_2 = -4$ . In this case, married female hours increase more, by 3.2 hours per week, but still significantly less than the 14.3 hours per week in the data. For married males, the change is .8 hours per week, similar in magnitude to the .5 hours seen above, while in the data the corresponding change is 3 hours. Not shown in the table are the results of experiments based on more inclusive notions of home capital goods. Since in those cases, the corresponding reduction in  $q$  is smaller, even smaller changes in hours result.

An alternative way of studying the impacts of improvements of technologies in the home is to

study the effects of increases in  $A^2$  over and above any general technical change. However, in order for this to have a chance at being successful, it is necessary that preferences deviate substantially from the power utility case. The reason for this is simple and it is summarized in the following result

**PROPOSITION 1.** *Let  $U^f = U^m = \frac{[(c^1)^{\psi_1}(c^2)^{1-\psi_1}]^{(1-\psi_3)}(1-\ell^1-\ell^2)^{\psi_3}]^{1-\sigma}}{1-\sigma}$ , (power utility). Then if  $\hat{A}_t^2 = (1+\gamma)A_t^2$ , the new equilibrium has the property that  $\hat{\ell}_{gpt}^i = \ell_{gpt}^i$  and  $\hat{c}_{gpt}^2 = (1+\gamma)c_{gpt}^2$ , and all other variables remain unchanged.*

**Proof.** For any allocation  $\mathbf{z} = \{z_t\}$ , let  $\hat{\mathbf{z}} = \{\hat{z}_t\}$  be the allocation in which the term corresponding to consumption of non-market goods is increased by  $(1+\gamma)$ . Let  $\mathbf{x}$  be the initial equilibrium. Consider the problem solved in equilibrium by a married couple. Holding prices fixed, note that holding all quantities other than  $c_{gpt}^2$  fixed, the set of feasible choices is homogeneous of degree one in  $c_{gpt}^2$ . Thus,  $\hat{\mathbf{x}}$  is a feasible allocation. Given the specification of the utility function, the value of the problem—denoted  $V(\mathbf{x})$ —is such that  $V(\hat{\mathbf{x}}) = (1+\gamma)^\eta V(\mathbf{x})$ , where  $\eta = (1-\psi_1)(1-\psi_3)(1-\sigma)$ . We want to show that  $\hat{\mathbf{x}}$  is maximal in the budget set. Suppose not and let  $\hat{\mathbf{y}} = \{\hat{y}_t\}$  be a preferred allocation that is feasible. Then, scaling down the  $c_{gpt}^2$  coordinate of  $\hat{\mathbf{y}}$  makes the allocation feasible under the original budget set. Let this scaled down allocation be denoted  $\mathbf{y}$ . Then it follows that  $V(\hat{\mathbf{y}}) = (1+\gamma)^\eta V(\mathbf{y})$ . Thus, we have the following inequalities,

$$V(\hat{\mathbf{y}}) = (1+\gamma)^\eta V(\mathbf{y}) > V(\hat{\mathbf{x}}) = (1+\gamma)^\eta V(\mathbf{x}),$$

which implies  $V(\mathbf{y}) > V(\mathbf{x})$ . This contradicts our assumption that  $\mathbf{x}^*$  was maximal in the original budget set. To complete the argument note that under the  $\hat{\mathbf{x}}$  allocation none of the market quantities change and, hence, the original prices still clear all the markets. The problems solved by single females, and single males are handled similarly. ■

This theoretical result implies that, by continuity, changes in the home technology for any specification of preferences near unitary elasticity of substitution between home and market goods must necessarily result in a small effect on hours. As such, it serves as a useful benchmark for what follows.

As above, we only present the case where home and market goods are complements since if they are substitutes, market hours actually fall. For modest changes in  $A^2$  hours worked by married females changed hardly at all in keeping with the conclusion of Proposition 1, and hence we were

forced to examine very large changes in the value of  $A^2$ . In Table 3, we show the results on hours for this change. Again using  $\psi_2 = -0.75$  the size of the change in  $A^2$  that was needed to match the data was to increase it from  $A^2 = 1.0$  to  $A^2 = 5.0$ , over and above our baseline level of technological change. With our baseline growth rate in the market sector of  $\gamma_A = 2\%$ , this corresponds to a growth rate in home productivity of over 5% per year while market productivity grows at only 2% per year. Although the hours data are well matched with only a small change by singles, there are three problems with this simulation. First, for this story to be successful at all, it requires that home and market goods be complements contrary to best estimates. Second, very large changes in technology are required over and above those measured in market productivities. Finally, even in these cases, we see only small effects on the observed wage gap. This last point is important since it is directly related to changes in human capital formation decisions, and as pointed out above, there seems to have been a dramatic shift in the female/male decisions on schooling in the last 50 years.

Our results contrast with those of Greenwood, Seshadri, and Yorukoglu (2001). There are two key differences. First, Greenwood, Seshadri, and Yorukoglu (2001) assume that the labor supply decision is indivisible. Thus, married women are prevented from working part-time. This implies that the elasticity of substitution between home and market goods plays no role in their model. If a household is sufficiently productive then a decrease in the price of a durable that results in adoption on the part of a household “frees up” time—the technology is Leontief—that can only be used to produce either market goods or leisure.

Second, since the home technology is assumed to be Leontief, and there are only two options for producing in the home, utility effectively exhibits satiation in the home good in their formulation. This seems to be why their model predicts that eventually married female participation will begin to fall and durables expenditures decline as a fraction of GNP. As we shall see in the next section, this is the driving force behind their results.

## 6. Inferiority of the Home Good

A second type of qualitative explanation for the observed change in married female hours is that the home good is inferior. This, when accompanied by overall income growth, can cause married female home hours to fall freeing up time for more work in the market. In a static setting this change in income could, in principle, cause relatively more effort to be directed at obtaining market goods, and relatively less at home goods. Could this effect be a plausible quantitative

explanation of the regularities seen in U.S. hours and wages?

To address this question, we examined two variations on the model above where the utility function includes inferiority of the home good. The functional forms that we examined were:

$$\begin{aligned}
 V_1(c^1, c^2) &= (\psi_1(c^1)^{\psi_2} + (1 - \psi_1)(c^2)^{\alpha\psi_2})^{(1-\psi_3)/\psi_2}, \text{ with } \alpha \leq 1 \\
 V_2(c^1, c^2) &= \begin{cases} (\psi_1(c^1)^{\psi_2} + (1 - \psi_1)(c^2)^{\psi_2})^{(1-\psi_3)/\psi_2} & \text{if } c^2 < c^* \\ (\psi_1(c^1)^{\psi_2} + (1 - \psi_1)(c^*)^{\psi_2})^{(1-\psi_3)/\psi_2} & \text{if } c^2 > c^* \end{cases}
 \end{aligned}$$

with  $U = \frac{1}{1-\sigma}[(V_i(c^1, c^2))^{1-\psi_3}(1 - \ell^1 - \ell^2)^{\psi_3}]^{1-\sigma}$ ,  $i = 1, 2$ . Thus, when  $\alpha = 1$ ,  $V_1$  is like our base model, but when  $\alpha < 1$ , there is more concavity in the home good than in the market good. The utility function  $V_2$  is even more extreme with strict satiation in the home good.

We also examined two different sources of increases in wealth. These were trend growth in productivity and reductions in prices of capital goods (this version also induces important substitution effects).

What we found is that specifications like that in  $V_1$  are not successful no matter what the source of income growth is. This was true no matter how small we made  $\alpha$ . It did not matter whether the source of growth was technological change overall, or specific to some or all of the capital goods in the model. In all cases, the change in married female labor supply was inconsequential.

Whether or not specifications like those in  $V_2$  work or not depends critically on the choice of  $c^*$ . There is a delicate balancing act, if  $c^*$  is chosen too low, home hours of all households fall, including singles, while their market hours increase. This is not what we see in the data. On the other hand, if  $c^*$  is too large, there is no effect on the market hours of any of the households. There is a range of values for  $c^*$  such that there is a large effect on married couples, but only a small effect on singles. The hours series for one such example are shown in Figure 4A. In this case, we assumed that capital prices are unchanged but that overall productivity grows as in our benchmark parameterization. Both the increase in married female hours and those of married males line up quite well with the data. The same is true for singles, both females and males.

There are three weaknesses of the example however. First and foremost, it requires exactly the right specification of satiation (i.e.,  $c^*$ ) to match the facts. It is difficult to know whether or not this specification is realistic and we know of no independent way of corroborating it. Second,

although the hours data match up well, as can be seen in Figure 4B, even this extreme version only captures about one fourth of the observed change in the wage gap. Finally, as one might guess, one implication that comes along with this specification is that the share of home investment goods in output drops drastically, by a factor of more than 3. Figure 4C shows the time series from the model along with that in the data where the share in output is roughly constant over time.

### *Sensitivity Analysis*

We conducted numerous sensitivity analyses of the examples described above in an attempt to isolate the relative contributions of different sources of income growth and preference specification. What we found was that the effect of any of the sources of income growth gave rise to only small effects when the utility function is of the type in  $V_1$ . In contrast, when utility is given by the form in  $V_2$ , we found that durables prices by themselves gave rise to almost no effect in the absence of overall growth in productivity. Similarly, when durable price reductions were added to the model with productivity growth, again, the marginal effect was quite small. Thus, we conclude that any effect that is present with this specification is only present when we have both strict satiation and overall productivity growth. The declining price of durables seems to only play a minor role.

## **7. Male-Female Wage Differentials**

In this section we study the impact of changes in measures of discrimination in the labor market —given by  $(1 - \tau_{dt}) = (1 - \tau_{\ell ft}) / (1 - \tau_{\ell mt})$ — on labor supply decisions. There is substantial evidence (see Goldin (1990), Blau and Khan (1997) and Blau (1998) for example) that, even controlling for a number of measurable characteristics, female wages are lower than male wages. Moreover, the data indicate that this gap has been narrowing in the last few years. Given the specification that we have chosen, it follows that the gap in wages of women relative to men is given by:

$$\ln \left( \frac{w_{ft}}{w_{mt}} \right) = \ln \left( \frac{1 - \tau_{\ell ft}}{1 - \tau_{\ell mt}} \right) + \kappa_1 \ln \left( \frac{h_{ft}}{h_{mt}} \right) + \zeta_1 \ln \left( \frac{\eta_{ft}}{\eta_{mt}} \right)$$

and hence this gap is made up partly from the direct effects of discrimination (i.e., the differences between  $\tau_{\ell ft}$  and  $\tau_{\ell mt}$ ) and partly from the indirect effects of different human capital accumulation decisions.



As noted above, in the data,  $w_{ft}/w_{mt}$  has risen from about 56% in 1969 to about 69% in 1995. If all relevant skills (i.e.,  $h$  and  $\eta$ ) were perfectly measured and controlled for we would have direct measures of both the level and the change in  $(1 - \tau_{dt})$  that must have occurred over this time period. If, on the other hand,  $h$  represents skills measured by the econometrician (e.g., years of schooling), while  $\eta$  represents other skills that are not adequately measured (e.g., ability to operate spreadsheet software), and if these unmeasured skills differ systematically by sex,  $w_{ft}/w_{mt}$  would be an overestimate of  $(1 - \tau_{dt})$ . Moreover, the change  $w_{ft}/w_{mt}$  would be an overestimate of the true change in discrimination if  $\eta_f/\eta_m$  increases when  $\tau_d$  falls.

We study a version of the model in which the series  $\tau_{dt}$  is calibrated so that the model and data values for the relative wages of women to men match. This series necessarily requires that the value of  $\tau_{dt}$  fall over the time period. To match the observed series of relative wages, we assume a tax rate on women of  $\tau_{\ell ft} = .40$  in 1950 (for comparison, recall that  $\tau_{\ell mt} = .23$ ) and we assume that this falls to  $\tau_{\ell ft} = .35$  by 1995 where it stabilizes.<sup>9</sup> This gives an initial discrimination tax of  $\tau_{d1950} = 1 - (1 - \tau_{\ell f1950})/(1 - \tau_{\ell m1950}) = .22$ , and a final value of  $\tau_{d1995} = .16$ .

Since we do not have direct measure of the  $\tau_{dt}$  series, we will conduct considerable experimentation on this below. Figure 5B shows the time path of relative wages as given by Blau (1998) along with that calculated from our model.

The predictions of the model for the number of hours worked and the comparable census values for the United States are presented in Figure 5A. The model prediction matches the long run behavior of hours worked very accurately, both the change from steady state to steady state and also the path over the last 50 years. In particular, it generates both the large increase by married females and the small decrease by married males that is present in the data. There is also virtually no response by single females to the same change in discrimination over the 1950 to 1990 period. Thus, it does not take large changes in discrimination to mimic the behavior of hours worked by males and females. Indeed, the time path of hours seen in the data is exactly what one would expect from a relatively small change in discrimination.

There are two features of the model that are at odds with the data. First, the hours series from the model for single males are systematically too high throughout the 1950 to 1990 period. Second, the model outcome for single males shows a small but significant downward trend while in the data, they are U-shaped.

The small change in market hours for singles over the 1950 to 1990 period that the model generates is in keeping with the discussion of the static model in Section 2. Thus, the qualitative behavior predicted in the static model with logarithmic preferences continues to hold (approximately) in this dynamic setting even though the static elasticity of substitution between home and market goods is 1.67 and not 1.

The fact that hours in home production are roughly equal for single females and single males and are constant over the experiment is directly reflected in the time paths for home consumption which are also roughly equal and quite stable. This is also in keeping with the static example of Section 2. Although it is not shown here, the behavior of market consumption is more complex. Over time, single female market consumption rises roughly in step with the reduction in effective labor tax rates over the period, a prediction of the static model. However, that is not true for the relationship between single female and single male market consumption. Here, the static model would suggest that the ratio of market consumptions between the two types of single agents would be equal to the ratios of their tax rates. In fact, the single female consumes less than this. The main reason for this is that the existence of discrimination induces a difference in human capital investments which exaggerates the differences in wages and hence, the differences in consumption as well. This is a purely dynamic effect of discrimination.

Is the increase in married female hours in the market coming from leisure or at the cost of home production? As it turns out, the answer to this is that about 33% comes from reduced leisure while 67% comes from reduced work in the home. Indeed, in part because of our assumption that leisure for the two partners is equal in 1950, by 1990 married females are working approximately 9 hours more per week in total than are married males.

As a final point on the equilibrium hours series produced by the model note that, as discrimination is reduced, a married couple looks more and more like a single man and single woman by the end of the transition. That is, as can be seen in Figure 5A, although total market hours for a married couple is substantially less than that for two singles at the beginning of the period (50 hours vs. 60 hours), it is roughly the same by 1995. This is true in the data as well. This is a by-product of the reduced incentives for ‘over-consumption’ of the home good as a tax avoidance strategy by the married couple.

Since this model is successful at matching the hours series, we are led to examine other predictions. One interesting feature of the model is its implications for decisions on human capital investment. There is a substantial difference across the sexes in the investment paths in human capital for both single and married agents. This is directly due to the increased rate of return on human capital accumulation for the woman due to the forecast reduction in tax rates she faces. The time paths for human capital for all agents are shown in Figure 6A. Over time, the decrease in the gender wage gap induces females to invest more in human capital and less in physical capital. Thus, there is a “portfolio reallocation” effect associated with changes in discrimination. The predicted increases are substantial, for married women over 172%, and 36% for single women. There is also a small decrease in male investment (this is only relative to trend, the absolute level does not fall). In some versions of the model (see the next section) the model predicts that human capital of single females will overtake and pass that of single males about the time that the discrimination tax hits zero. Interestingly, this is similar to what has been seen recently in that female college graduation rates in the United States have now surpassed those of males. In contrast, in the versions of the model in Sections 5 and 6, the changes in human capital by females are much smaller with virtually no change in single female stocks.

This change in human capital investments for females has implications for the *composition* of the stock of wealth for all agents in the economy. For a married couple, the share of human capital increases, while the share of physical capital decreases. This change is entirely internally financed by the couple however, with virtually no change over the period in holdings of physical capital. For single females, investment in physical capital decreases dramatically, becoming negative eventually. This decrease, coupled with the increase in their investments in human capital, implies a substantial change in the composition of their “portfolio.” Single males behave in the opposite way. Thus, single females borrow in order to finance investments in human capital. In the model, this is accomplished by a decrease in investment in physical capital. Single males are on the other side of this market. For them, the rate of return on human capital has not increased, and they are happy to lend to single females.

The change in the incentives for human capital accumulation is the property of the model that drives the results on the paths of hours for females that we see. As discrimination falls, wage rates for females would rise even in the absence of any investment. The increased investment in human capital has two effects. The first is that it increases the size of the wage change. The second is that

it exaggerates the increase in the cost of leisure for females over the period of transition. These two effects induce intertemporal substitution of leisure along the transition path. Consequently they to work relatively little early on, with the substantial increases seen in the figure.

The model also has implications for the “marriage premium” for both males and females. We define the male (female) marriage premium as the ratio of hourly wages between married and single males (females). These are shown in Figure 6B. For males, the model predicts a reasonably large increase, wages of single males are about 3% less in 1950, rising to 5% more by 2000 and remaining constant thereafter. In the data, this ratio is also fairly constant, but considerably lower with wages of single males about 20% below that for married males over the entire period. As expected, the differences in relative wages in the model are larger for females. In 1950, the ratio of wages of single females to that of married females is 1.18 and it falls smoothly over the time period, reaching its steady state level of 0.99 by 1990. In the data the corresponding values are 1.15 for 1970 (this is also the value from the model) and 1.02 for 1990. Thus, the model matches this feature of the data fairly well.

Finally, the model has implications for the time path of productivity for the economy. Some economists have argued that part of the productivity slowdown seen in the US in the 1970’s and early 1980’s is due to the increase in female participation (see Baily (1986)). The intuition is straightforward. Females have lower skill levels than their male counterparts due to both discrimination and specialization. As their participation increases, average labor productivity should fall. This argument misses the point that skill acquisition is endogenous, however. The overall effect then depends on which changes faster, female hours or female human capital. The time path of overall labor productivity in the model does indeed fall relative to trend as this argument suggests (but in some parameterizations it actually rises). Thus, the view that the observed path is due to increased female participation is consistent with model of reduced discrimination against women. This also points out that this reduction in productivity relative to trend may actually lead to a welfare improvement, at least for some groups.

### *Sensitivity Analysis*

We turn now to the sensitivity of our results. Some of the modeling choices that we have made are standard since they have counterparts in all dynamic models (e.g., discount factors and preference and production parameters). Others— the choice of welfare weights within a couple,

the specification of effective labor including human capital, the nature of the transfer scheme for distributing the revenue raised by the discrimination tax and the assumption of equal discrimination against married and single females— are more unique to the questions that we are addressing in this paper. We find that generally speaking, our results are robust to alternative specifications. The one exception to this concerns the example where we assume that discrimination affects married women only where it is difficult to match the observed wage series.

Table 4 reports hours of work, relative wages and home investment shares for these alternative versions of the model. The first row contains the statistics from U.S. data and the remaining rows display statistics for the different model parameterizations, the first being the benchmark parameterization discussed above.

In our benchmark case, the weight on married female utility is low ( $\lambda_f = .062$ ). As indicated before, this is necessary in the benchmark example to keep total hours by the female, home plus market, at the same level as her partner. The first experiment reported in Table 4 assumes a higher weight. In order to match the wage series for this parameterization a larger change in  $\tau_d$  was required (see Table 2). In this experiment we assumed that  $\tau_{d2000} = .12$  and hence the reduction was still a relatively modest 10%. Given this however, the changes in hours are very similar to the benchmark case.

In our second experiment, we consider a variation of the model with no human capital. In this case we set  $\kappa_i$  and  $\zeta_i$ ,  $i = 1, 2$ , equal to 0.001 so that the return to human capital is negligible. All other parameters are as in the benchmark example except for the path of  $\tau_d$  which begins at  $\tau_{d1950} = .48$  and falls to  $\tau_{d2000} = .22$  as was required to match the wage series. In this case, it follows that the gap is simply the wedge introduced by the discrimination tax. There is no additional wedge introduced by human capital differences. The smaller gap means that larger changes in discrimination are required to match the relative wage series, but beyond this, the results of the experiment are quite similar. Thus, the role of human capital is quantitatively significant in that with human capital included, only a 6% drop in  $\tau_d$  is required while without it, a drop of 26% is needed. Also included in Table 4 are two further experiments on the role of human capital. In the first we assume that human capital is useful only in the market sector. In the second, we study an example in which the two types of human capital are sector specific. The results are very similar for all variations with the exception of the path  $\tau_d$  required to mimic the gender wage gap.

For the simulations discussed to this point, we assumed that revenues from the discrimination tax were lump-sum rebated in an equal per capita fashion. An alternative hypothesis is that the revenue raised from discrimination against women was used to subsidize some other group of agents. To analyze this possibility, we simulated a specification in which the revenues generated from the discrimination tax were used as a subsidy to married males (cf. Goldin (1990), p. 102). The parameters of the model were recalibrated so as to match initial hours and the time path of the wage gap. For this case, we found that similar changes in the discrimination tax gives nearly identical results as our benchmark.

Our benchmark simulation assumes that there is equal discrimination against both married and single females. In fact, many of the discriminatory practices that have been documented (e.g., marriage bars, etc., see Goldin (1990)), seem to exhibit more discrimination against married women than single ones. Because of this, we consider an example in which we set the discriminatory taxes for single females equal to zero, with the other parameters held fixed at their benchmark levels. This experiment is the last row in Table 4. In this case, no time path for  $\tau_d$  could be found that replicates the time series of the gender wage gap. Hence we use the path from our benchmark parameterization. We find that this change significantly increases the relative wage of females in all periods (it is .68 in the model now as opposed to .56) and leads to higher hours for all groups except married males. There is still a substantial change in married female hours from the reduction in discrimination, however, from 11.2 to 22.8 hours per week.

In sum, the results reported in our benchmark example seem to be fairly robust both to changes in the parameters and the details of the treatment of the discrimination tax. As long as the levels and changes in  $\tau_d$  are chosen so as to match the observed path of the gender wage gap, the time series for hours matches those seen in the data. The size and the change of  $\tau_d$  that is needed for this depends critically on the role of human capital however.

## 8. Conclusions

In this paper we have shown that in a dynamic general equilibrium model family labor supply and human capital decisions, the response to a small change in the gender wage gap is for married females to increase labor supply substantially while those of single females change only slightly. This is similar to what has been observed in the U.S. over the last fifty years. We explored two other candidate explanations for these changes using the same model—improvements in the technologies

used to produce home goods, and inferiority of the home good with overall productivity growth—and found that they face considerable difficulties.

Changes in the wage gap do not have large effects on singles given a specification in which income and substitution effects cancel each other. This characterizes the situation of both single males and females. However, for partnerships there is another margin in which the partners can move: endogenous specialization allows married females to partially avoid the discrimination tax. Thus, changes in the male/female wage gap induce substantial reallocations within the family, even though overall changes in the level of wages has a small impact on labor supply. This implies that married females respond to changes in discrimination by substantially changing their market hours.

Our results suggest that changes in the rate of productivity growth in the home production sector are not as successful explaining the U.S. historical experience for two reasons. First, for hours of married females to increase, we need to assume that market and home goods are complementary, in contrast with the best estimates. Second, even in this case, to match the increases in the number of hours worked by married females, the model requires exceptionally large increases in the productivity in household activities. The impact of the declining relative price of home capital was found to be relatively minor, with the model predicting that this will be met with increases in the quantity and quality of home production. Similarly, stories based on inferiority of home goods face difficulties. Even with the extreme versions of this specification that are needed to match the hours pattern, the model produces counterfactual predictions about expenditure shares on home durables. It also has difficulty in tracking the observed pattern of the gender wage gap.

We view our model a time changing discrimination tax as a first step. We have ignored the effects of uncertainty about returns to human capital and the permanence of marriage (divorce). In addition, we have taken the decision to form partnerships as exogenous. In ongoing work we study both the impact of uncertainty about marital status on accumulation decisions and the effects of endogenizing the marriage decision. We conjecture that including these features will not change the basic conclusions we have reached here, but will improve the models overall fit, especially for the behavior of single households. This is because much of what is at odds with the data in the current version of the model comes from intertemporal trading between single males and females conducted under the assumption that marital status will not change. When this is no longer true we expect that the predicted downward trend in single male hours (for example) will largely disappear.

## Appendix. Technological Shocks, Occupational Choice and the Gender Wage Gap

In this appendix, we discuss a series of simple static models for both single households and partnerships to illustrate the role played by the elasticity of substitution between market and home produced goods and to discuss the sense in which changes in “glass ceiling” type of policies and technological change in the production of market goods have effects that are similar to changes in the discrimination tax discussed in Section 7.

In order to accommodate changes in the quality of home produced goods—an effect discussed by Mokyr (2000)— we generalize the specification of the home production technology. We will assume that there is only one market produced good,  $c$ , and a variety of home-produced goods. Let  $y_i$  be the quantity produced of good  $i$ , where  $i \in [0, \infty)$ . However, at any given time, there is a finite measure of goods produced. Given our specification, it is without loss of generality that we choose the set of produced goods to be  $[0, z]$  where  $z$  is chosen by the household. Let’s assume that—independently of the amount produced—the household has to purchase one unit of a market good to produce good  $i$ . The price of the market good is  $q_i$  and its productivity is  $a_i$ . In what follows we interpret this good as a household durable, although in a static framework there is no meaningful distinction between durables and non-durables. If  $\ell_i$  units of labor are allocated to produce good  $i$ , output is

$$y_i = a_i m(s, a_i) \ell_i^\alpha, \quad i \in [0, z], \quad 0 < \alpha < 1,$$

where  $m(s, a_i)$  is interpreted as a correction factor that depends on both the occupation of the individual producing the home goods,  $s$ , and the quality of the capital goods used in home production,  $a_i$ . It is assumed that  $m_s < 0$ , and  $m_a > 0$ . The interpretation is simple: individuals in more demanding (high  $s$ ) occupations cannot effectively organize the time they use to produce home goods; thus, relative to individuals in simple occupations, they are less productive at home.

Let the aggregate home produced good,  $y$ , be given by,

$$y = \left[ \int_0^z (a_i m(s, a_i) \ell_i^\alpha)^{1/\rho} di \right]^\rho \quad \rho \geq 1.$$

In addition, the time constraint requires that

$$\int_0^z \ell_i di + \ell^1 \leq 1.$$



To simplify we assume that all goods have the same technology and, hence, we look at symmetric allocations. Thus, it follows that,

$$\begin{aligned}\ell_i &= \frac{n}{z}, & \ell^1 &= 1 - n \\ y &= z^\rho am(s, a) \left(\frac{n}{z}\right)^\alpha = am(s, a) n^\alpha z^{\rho-\alpha}, \\ c &= w(s)\ell^1 - qz.\end{aligned}$$

The last term of the household's budget constraint,  $qz$ , just captures the cost of purchasing (in this static model it is really renting) one unit of capital goods per type of home good produced. Even though the results hold more generally, it is easier if we assume that the job wage profile is linear. Thus,  $w(s) = ws$ . In this interpretation, high  $s$  jobs are better, and an increase in  $w$  corresponds to an overall increase in wages for both males and females.

### Single Households

Since we restrict ourselves to symmetric allocations, it follows that single households solve the following problem:

$$\max_{n, z, s} U[ws(1 - n) - qz, am(s, a)n^\alpha z^{\rho-\alpha}].$$

The first order conditions are

$$\begin{aligned}n : & \quad U_c ws = U_x \alpha am(s, a) n^{\alpha-1} z^{\rho-\alpha} \\ z : & \quad U_c q = U_x (\rho - \alpha) am(s, a) \alpha n^\alpha z^{\rho-\alpha-1} \\ s : & \quad U_c w(1 - n) + U_x am_s(s, a) n^\alpha z^{\rho-\alpha} = 0.\end{aligned}$$

It follows from these conditions that the number of hours allocated to housework and the variety of home-produced goods are related according to

$$n = \frac{\alpha}{\rho - \alpha} \frac{qz}{w(s)}.$$

To highlight the role played by the elasticity of substitution we assume

$$U(c, y) = [\phi c^{-\psi} + (1 - \phi)y^{-\psi}]^{-1/\psi}.$$

Thus we assume that the elasticity of substitution between home and market goods is  $\frac{1}{1+\psi} > 0$ . Manipulation of the conditions result in the following condition

$$\frac{1 - \ell^1}{(1 - (1 - \ell^1)\frac{\rho}{\alpha})^{1+\psi}} = \frac{\alpha(1 - \phi)}{\phi} [w(s)^{\rho-\alpha-1} q^{\alpha-\rho} a m(s, a) (1 - \ell^1)^\rho]^{-\psi}. \quad (\text{A1})$$

Equation (A1) gives the optimal choice of  $\ell^1$  conditional on the choice of occupation,  $s$ . We assume for now that females face a “glass ceiling” type of policy and that their unconstrained choice of  $s$ ,  $s^f$ , is greater than the institutionally capped feasible occupation,  $\bar{s}$ . Thus, we set  $s = \bar{s}$ . It turns out that the predictions of the model depend on the degree of preference for variety,  $\rho$ , and the elasticity of substitution. In the borderline case in which  $\rho = 1 + \alpha$  the model delivers the result that hours worked are independent of wages. Increases in the productivity of the market goods used to produce the home good,  $a$ , or decreases in its price,  $q$ , result in increases in the number of hours worked if home and market goods are complements, and decreases if they are substitutes.

In the case  $\rho > 1 + \alpha$ , we obtain that if home and market goods are complements (substitutes) the model predicts (i) increases in wages,  $w$ , and in the productivity of household durables,  $a$ , increase (decrease) labor supply; and (ii) decreases in the price of household durables,  $q$ , increase (decrease) labor supply.

Thus, to ensure that the model delivers, as an optimal response, no changes in the number of hours worked by single males and females in the face of substantial changes in real wages and the price of household durables, we assume that  $\psi = 0$ . Thus, the utility function is assumed to be of the form

$$U(c, y) = c^\phi y^{(1-\phi)}.$$

In this case, the equilibrium choices for an  $s$ -constrained household (single females) are

$$\begin{aligned} \ell_s^1 &= \frac{\phi + (1 - \phi)(\rho - \alpha)}{\phi + (1 - \phi)\rho}, \\ z_s &= \frac{(1 - \phi)(\rho - \alpha) w(\bar{s})}{\phi + (1 - \phi)\rho q}. \end{aligned}$$

It follows that changes in base wages,  $w$ , the level of the glass ceiling,  $\bar{s}$ , the productivity of household durables,  $a$ , or their price,  $q$ , do not affect the number of hours worked. Increases in wages and the level of the “glass ceiling,” (increases in  $\bar{s}$ ) as well as decreases in the price of durables result

in more variety (or quality) of home goods. This, to some extent, captures Mokyr’s (2000) claims that some developments—which we capture in lower values of  $q$ —lead households to increase the quality (our  $z$ ) of the non-market goods that they produced.<sup>10</sup>

## Partnerships

How do wage and price changes affect married couples? We assume—as before—that the utility function is Cobb-Douglas and that the home produced good is shared according to some fixed rule.<sup>11</sup> Since males are unconstrained we assume that they choose the highest available occupation which we denote by  $s_m$ .<sup>12</sup> We also assume that the equilibrium choices coincide with the solution of a weighted planner’s problem with the weights  $(\lambda_f, \lambda_m)$  adding up to 1. The equilibrium levels of labor supply and quality-variety choice by a married female are given by

$$\begin{aligned}\ell_{fp}^1 &= \frac{\phi + (1 - \phi)(\rho - \alpha)\kappa}{\phi + (1 - \phi)\rho\kappa} - \frac{(1 - \phi)\alpha\kappa}{\phi + (1 - \phi)\rho\kappa} \frac{w(s_m)}{w(\bar{s})}, \\ z_p &= \frac{(1 - \phi)(\rho - \alpha)\kappa}{\phi + (1 - \phi)\rho\kappa} \left[ \frac{w(s_m) + w(\bar{s})}{q} \right],\end{aligned}$$

where

$$\kappa \equiv \frac{1 + \left(\frac{\lambda_f}{\lambda_m}\right)^{\frac{1}{1-\phi}+1}}{1 + \left(\frac{\lambda_f}{\lambda_m}\right)^{\frac{1}{1-\phi}}}.$$

There are four important properties of the model that are captured in the equations above. First, for married females it is only their wage rate *relative to that of their male partners* that affects labor supply. In our linear case ( $w(s) = ws$ ) only the ratio  $s_m/\bar{s}$  matters. Overall increases in wages have no impact on female labor supply. Thus, the model is consistent with low elasticities of labor supply with respect to overall wage increases. Second, in the linear case, we can interpret the ratio  $\bar{s}/s_m$  as the “discrimination tax” that we modeled in section 4. To see this, let the marginal product of labor be denoted by  $w'$ . Then, the after tax wage rate for males is  $w(s_m) = (1 - \tau_{\ell m})w's_m$ , and

$$\frac{w(\bar{s})}{w(s_m)} = \frac{(1 - \tau_{\ell m})w'\bar{s}}{(1 - \tau_{\ell m})w's_m} = \frac{\bar{s}}{s_m}.$$

Thus, the “wedge” between female and male wages is associated with restrictions on the type of occupations that women have access to. Third, the model implies that increases in overall wages,

$w$ , or decreases in the price of household durables,  $q$ , induce the partnership to produce a larger variety (quality) of home goods but they have no labor supply effect. Finally, the model emphasizes that it is the gap between the male and female occupation that matters. Thus, it is possible that increases in  $\bar{s}$  are not necessarily associated with a decrease in the discrimination tax in the sense we use it in this paper. To see this, consider the case in which —due to technological change in the market sector— males want to increase their occupational index to  $s_{m'}$ . Even if that process is accompanied by a relaxation of the “glass ceiling” to  $\bar{s}'$ , it is still possible that the ratio  $\bar{s}'/s_{m'}$  is lower than  $\bar{s}/s_m$ , and this is consistent with more *effective* discrimination.

In summary, this discussion shows that to capture the relative constancy of the number of hours worked by single individuals a power specification of preferences (or something close to it) is necessary. In addition, it shows that in a model of endogenous occupational choice, “glass ceiling” type of policies have the same impact as the discrimination tax in Section 7.

## Notes

<sup>1</sup>Alternatively,  $k$  can be interpreted as the quality of an indivisible durable good that the household purchases. Under this interpretation, higher quality durables provide more services and cost correspondingly more, with the slope of the price/quality tradeoff given by  $q$ . All results below concerning a reduction in  $q$  are then interpreted as shifting the entire curve down in a proportional fashion. Similar results are obtained when the cost of durable of type  $k$  is given by  $qk^\varsigma$  with  $\varsigma > 1$ , and  $q$  is lowered.

<sup>2</sup>Note that the results given here continue to hold even if the share parameters are different between the types of agents. For example, if  $\nu_{fs} > \nu_{ms}$ , single females will devote more hours to the production of home goods than will single males. Even in this more general case, changes in technology will not affect overall hours devoted to market activities by these two groups.

<sup>3</sup>>From a formal point of view, the excess revenue of the tax imposed on female hours needs to be allocated. In our model, we rebate these amounts to the agents in the economy in a lump sum fashion. The last term,  $T_{pt}$  captures these transfers in addition to any excess revenue over and above government purchases of goods and services. We do some experimentation with this expenditure rule in our sensitivity analysis.

<sup>4</sup>See Section 6 for some alternative specifications of technologies and institutions that result in wedges between male and female wages.

<sup>5</sup>We had difficulty matching the 1950 hours for all four groups and the transitional path of relative wages in our benchmark simulations.

<sup>6</sup>It is difficult to know how large the effects of human capital should be in the model. For example, under the assumption that,  $\log h = Ed$ , the number of years of education, and  $\log \eta = Exp$  the number of years of experience the estimates from Mincer style regressions of  $\kappa_1$  are around 0.10, while those of  $\zeta_1$  are around 0.05, or about .15 in total (cf. Bils and Klenow (2000)). These values are lower than those we use in our simulation. On the other hand, if we instead assume that  $h = Ed$ , the number of years of education, and  $\eta = Exp$ , the corresponding estimate of  $\kappa_1 + \zeta_1$  is near one. Thus, our parameterization lies between these two extremes.

<sup>7</sup>We consider the average for the 25-64 age group.

<sup>8</sup>This type of change is equivalent to a formulation with a multisector model with each producing a different market good, but with the same CRS production function, and sector specific changes in that technology.

<sup>9</sup>The exact sequence of tax rates we use is: .4, .397, .394, .392, .389, .386, .383, .375, .368, .360, .353, .353, .... Note that this implies a fairly slow reduction in discrimination between 1950 and 1975 with an acceleration occurring after that.

<sup>10</sup>The condition for the optimal choice of occupation requires that  $\frac{m_s(s,a)s}{m(s,a)} = \frac{\phi+(1-\phi)(\rho-\alpha)}{(1-\phi)\alpha}$ .

This implies that increases in  $a$  result in increases in  $s$ . Thus, for single households, productivity increases in household durables result in higher occupations for males (unconstrained) but not for females (constrained), with no labor supply effects in either case.

<sup>11</sup>This formulation is flexible enough to allow for home produced goods to be pure public goods.

<sup>12</sup>It is simple to include a cost of acquiring skills so that the occupation of married males can be endogenized. This, however, does not add much to the argument.

## References

- [1] Aiyagari, S. R., Greenwood, J. and N. Guner, (2000), "On the State of the Union," **Journal of Political Economy**, Vol. 108, pp: 213-244.
- [2] Baily, Martin N., (1986), "What Has Happened to Productivity Growth?," *Science*, October, vol. 234, pp: 443-451.
- [3] Becker, Gary S., (1971), **The Economics of Discrimination**, 2nd Ed., University of Chicago Press, Chicago.
- [4] Becker, Gary S., (1991), **A Treatise on the Family**, (enlarged edition), Cambridge, MA, Harvard University Press.
- [5] Benhabib, Jess, Richard Rogerson, and Randall Wright, (1991), "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," **Journal of Political Economy**, Vol. 99, pp: 1166-87.
- [6] Bergstrom, Theodore C., (1997), "A Survey of Theories of the Family," in M. Rosenzweig and O. Stark (eds.) **Handbook of Population Economics**, Elsevier.
- [7] Bils, Mark and Peter Klenow, (2000), "Does Schooling Cause Growth?," **American Economic Review** Vol. 90, No. 5, December, pp:1160-1183.
- [8] Blau, Francine, D, 1998, "Trends in the Well-Being of American Women, 1970-1985," **Journal of Economic Literature**, Vol XXXVI, pp: 112-165, March.
- [9] Brien, Michael J., Lee A. Lillard, and Steven Stern, (1999), "Cohabitation, Marriage, and Divorce in a Model of Match Quality," University of Virginia Working Paper, January.
- [10] Campbell, J. and S. Ludvigson, (2000), "Elasticities of Substitution in Real Business Cycle Models with Home Production," Harvard Institute of Economic Research, Discussion Paper No. 1900, June.
- [11] Caucutt, E., Guner, N. and J. Knowles, (2000), "The Timing of Births: A Marriage Market Analysis," (work in progress)
- [12] Cherlin, Andre J., (1992), **Marriage, Divorce, Remarriage**, Harvard University Press, Cambridge, Mass.

- [13] Chiappori, Pierre-Andre, (1992), "Collective Labor Supply and Welfare," **Journal of Political Economy**, Vol. 100, pp: 437-67.
- [14] ———, (1997), "Introducing Household Production in Collective Models of Labor Supply," **Journal of Political Economy**, Vol. 105, No. 1, pp: 191-209.
- [15] Coate, S. and G. Loury, (1993), "Will Affirmative Action Policies Eliminate Negative Stereotypes?," **American Economic Review**; Vol. 83, No. 5, pp: 1220-1240, December.
- [16] Cogley, Timothy J. and James Nason, (1995), "Output Dynamics in Real-Business-Cycle Models," **American Economic Review**; Vol. 85, No. 3, pp: 492-511, June.
- [17] Cooley, Thomas, F., 1995, **Frontiers of Business Cycle Research**, Princeton University Press, Princeton, New Jersey.
- [18] Cowan, R. S., (1983), **More Work for Mother**, Basic Books, .
- [19] Cubeddu, Luis and Jose-Victor Rios-Rull, (1997), "Marital Risk and Capital Accumulation," Federal Reserve Bank of Minneapolis Staff Report, June.
- [20] Suzanne Duryea and Miguel Szekely, (1998), "Labor Markets in Latin America: A Supply-Side Story," Inter-American Development Bank. Office of the Chief Economist. Working Paper 374.
- [21] Goldin, Claudia, (1990), **Understanding the Gender Gap: An Economic History of American Women**, Oxford University Press.
- [22] ———, (1997), "Career and Family: College Women Look at the Past," in F. Blau and R. Ehrenberg (eds) **Gender and Family Issues in the Workplace**, Russell Sage Foundation,
- [23] Gray, Jeffrey S., (1998), "Divorce-Law Changes, Household Bargaining, and Married Women's Labor Supply," **American Economic Review**, Vol. 88, No. 3, pp: 628-642, June.
- [24] Greenwood, Jeremy, and Zvi Hercowitz, (1991), "The Allocation of Capital and Time Over the Business Cycle," **Journal of Political Economy**, Vol. 99, pp: 1188-1214.
- [25] Greenwood, J., Guner, N. and J. Knowles, (2000a), "Women on Welfare: A Macroeconomic Analysis," **American Economic Review**, Vol. 90, No. 2, pp: 383-388.
- [26] Greenwood, J., Guner, N. and J. Knowles, (2000b), "A Macroeconomic Analysis of Marriage, Fertility and the Distribution of Income," University of Rochester working paper.



- [27] Greenwood, J., Seshadri, A. and M. Yorukoglu, (2001), “Engines of Liberation,” University of Rochester working paper.
- [28] Hill, Martha S., (1985), “Patterns of Time Use,” in F. Thomas Juster and Frank P. Stafford (eds), **Time, Goods and Well-Being**, Institute for Social Research, The University of Michigan.
- [29] Johnson, William R. and Jonathan Skinner, (1986), “Labor Supply and Marital Separation,” **American Economic Review**, Vol. 76, Issue 3, pp: 455-69, June.
- [30] Juhn, Chinhui and Kevin M. Murphy, (1997), “Wage Inequality and Family Labor Supply,” **Journal of Labor Economics**, Vol. 15, No. 1, Pt. 1, pp: 72-97.
- [31] McGrattan, Ellen, Richard Rogerson, and Randall Wright, (1997), ” An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy,” **International Economic Review**, Vol. 38, No. 2, May, pp: 267-90.
- [32] McGrattan, Ellen and Richard Rogerson, (1998), “Changes in Hours Worked Since 1950,” **Federal Reserve Bank of Minneapolis Quarterly Review**, Winter.
- [33] Mincer, Jacob, (1962), “Labor Force Participation of Married Women,” in H.G. Lewis (ed) **Aspects of Labor Economics**, Princeton University Press (for NBER).
- [34] Mokyr, Joel, (2000), “Why Was There More Work for Mother? Knowledge and Household Behavior, 1870-1945,” working paper, Northwestern University.
- [35] Mortensen, Dale, (1988), “Matching: Finding a Partner for Life or Otherwise,” **American Journal of Sociology**, Vol. 94, Supplement, pp: S215-S240.
- [36] Newmark, David and Andrew Postlewaite, (1998), “Relative Income Concerns and the Rise in Married Women’s Employment,” **Journal of Public Economics**, 70(1), pp: 157-83, October 1998.
- [37] Olivetti, Claudia, (2001), ” Changes in Women’s Hours of Work: The Effect of Changing Returns to Experience,” Department of Economics, Boston University, December.
- [38] O’Neill, June, and Solomon Polachek, (1993), “Why the Gender Gap in Wages Narrowed in the 1980s,” **Journal of Labor Economics**, Vol 11, No. 1, Pt. 1, pp: 205-228 .

- [39] Pencavel, John, (1998), "The Market Work Behavior and Wages of Women: 1975-1994," Department of Economics, Stanford University, January.
- [40] Regalia, F. and V. Rios-Rull, (1999), "What Accounts for the Increase in Single Households and the Stability of Fertility?" Universitat Pompeu Fabra working paper.
- [41] Rupert, P., R. Rogerson, and R. Wright, (2000), "Homework in Labor Economics: Household Production and Intertemporal Substitution," **Journal of Monetary Economics**, Vol. 46, No. 3, pp: 557-580.
- [42] Smith, James P. and Michael P. Ward, (1985), "Time Series Growth in the Female labor Force," **Journal of Labor Economics**, Vol. 3, No. 1, Pt. 2, pp: S59-S90.
- [43] Smith, James P. and Michael P. Ward, 1989, "Women in the Labor Market and in the Family," **Journal of Economic Perspectives**, Vol. 3, Issue 1, pp: 9-23, Winter.
- [44] Weiss, Yoram, (1997), "The Formation and Dissolution of Families: Why Marry? Who Marries Whom? And What Happens Upon Divorce?," in M. Rosenzweig and O. Stark (eds.) **Handbook of Population Economics**, Elsevier.
- [45] Weiss, Yoram and Robert J. Willis, 1997, "Match Quality, New Information, and Marital Resolution," in **Journal of Labor Economics**, Vol. 15, No. 1, Pt. 1, pp: S293-S329.

TABLE 1. BENCHMARK PARAMETER VALUES

Description	Formula	Benchmark Value
Fraction married	$2n_p$	.6
Government tax rates & spending		
Labor tax rate	$\tau_\ell$	.23
Capital tax rate	$\tau_k$	.5
Government spending share	$G/F^1$	.2
Annual growth rates		
Population growth	$\gamma_p$	1%
Technological growth	$\gamma_A$	2%
Annual discount factor	$\beta$	1.017
Capital share	$\theta$	.22
Annual depreciation rates		
Physical capital	$\delta_k$	8%
Type- $h$ human capital	$\delta_h$	8%
Type- $\eta$ human capital	$\delta_\eta$	8%
Preferences, $U = \frac{1}{1-\sigma} \{ \psi_1 (c^1)^{\psi_2} + (1 - \psi_1) (c^2)^{\psi_2} \}^{\frac{(1-\psi_3)}{\psi_2}} l^{\psi_3} \}^{1-\sigma}$		
Weight on market consumption	$\psi_1$	.682
Market-home substitution parameter	$\psi_2$	.429
Weight on leisure	$\psi_3$	.557
Intertemporal substitution parameter	$\sigma$	6.783
Weight on female in joint utility, $\lambda_f U^f + \lambda_m U^m$	$\lambda_f$	.062
Effective market labor, $z^1 = h^{\kappa_1} \eta^{\zeta_1} \ell^1$		
Elasticity with respect to $h$	$\kappa_1$	.243
Elasticity with respect to $\eta$	$\zeta_1$	.243
Effective home labor, $z^2 = h^{\kappa_2} \eta^{\zeta_2} \ell^2$		
Elasticity with respect to $h$	$\kappa_2$	.166
Elasticity with respect to $\eta$	$\zeta_2$	.166
Initial discrimination tax	$\tau_d$	.22

TABLE 2. NUMERICAL EXPERIMENTS

Description	Experiment	Non-benchmark parameters
Cheaper Home Investment with Moderate elasticity	$q$ falls, $q_{2000}/q_{1950} = .23$	$\tau_{d,t} = .27$ $\psi_2 = -.75$
Larger elasticity		$\psi_2 = -4$
Improved Home Technology	$A^2$ rises, $A_{2000}^2/A_{1950}^2 = 5$	$\psi_2 = -.75, \lambda_f = .2, \psi_1 = .68, \psi_3 = .52, \kappa_1 = \zeta_1 = .097, \kappa_2 = \zeta_2 = .001$
Inferior Home Goods	$c^2 \leq c^*$ binds	$c^* = .049$
Fall in Discrimination with Benchmark parameters	$\tau_d$ falls $\tau_{d,2000}/\tau_{d,1950} = .72$	
Equal utility weights	$\tau_{d,2000}/\tau_{d,1950} = .53$	$\lambda_f = .5, \psi_1 = .63, \psi_3 = .65, \kappa_1 = \zeta_1 = .23, \kappa_2 = \zeta_2 = .31$
No human capital	$\tau_{d,2000}/\tau_{d,1950} = .46$	$\kappa_i = \zeta_i = .001, \tau_{d,1950} = .48, \psi_1 = .71, \psi_3 = .45$
Market-only human capital	$\tau_{d,2000}/\tau_{d,1950} = .47$	$\kappa_2 = \zeta_2 = .001, \kappa_1 = \zeta_1 = .047, \tau_{d,1950} = .42, \psi_1 = .76, \psi_3 = .47$
Sector-specific capital	$\tau_{d,2000}/\tau_{d,1950} = .50$	$\kappa_2 = \zeta_1 = .001, \kappa_1 = \zeta_2 = .1, \tau_{d,1950} = .40, \psi_1 = .72, \psi_3 = .47$
Married males subsidized	$\tau_{d,2000}/\tau_{d,1950} = .50$	$\psi_1 = .62, \psi_3 = .58, \kappa_1 = \zeta_1 = .24, \kappa_2 = \zeta_2 = .20$
No singles discrimination	$\tau_{d,2000}/\tau_{d,1950} = .72$	$\tau_{i,fs} = \tau_{i,ms}$

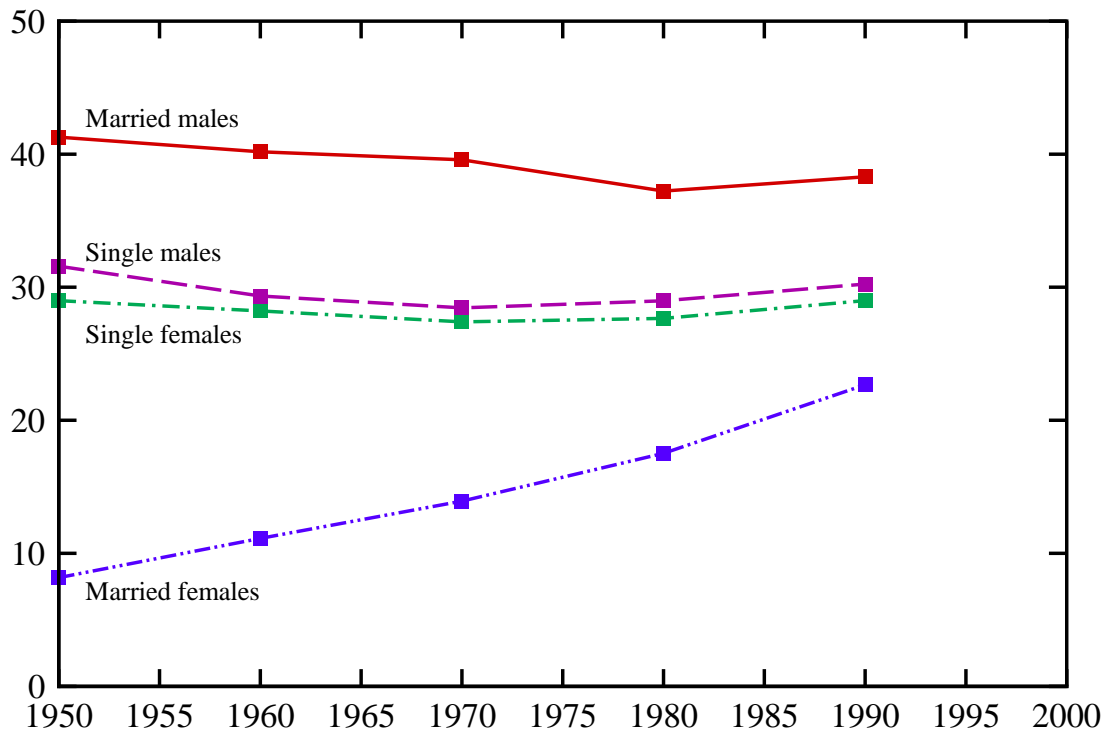
TABLE 3. EFFECTS OF CHANGES IN TECHNOLOGY

DATA	1950 RESULTS					1990 RESULTS					
	HOURS PER WEEK			Wage RATIO (%)	SHARE, HOME INV. (1950=100)	HOURS			Wage RATIO (%)	SHARE, HOME INV. (1950=100)	
MODEL:	MF	MM	SF	SM		MF	MM	SF	SM		
Cheaper Home Investment	8.2	41.3	29.0	31.6	51	22.7	38.3	29.0	30.2	69	85
with Moderate Elasticity	13.1	41.7	30.3	34.0	44	15.4	41.2	30.1	34.2	45	122
with Larger Elasticity	19.4	40.7	30.5	33.9	47	22.6	39.9	30.6	34.6	48	109
Inferior Home Goods	9.9	41.3	28.9	33.9	51	16.6	38.9	27.1	32.8	55	87
Improved Home Technology	5.5	39.9	27.1	27.6	59	22.7	38.1	30.8	32.2	68	76

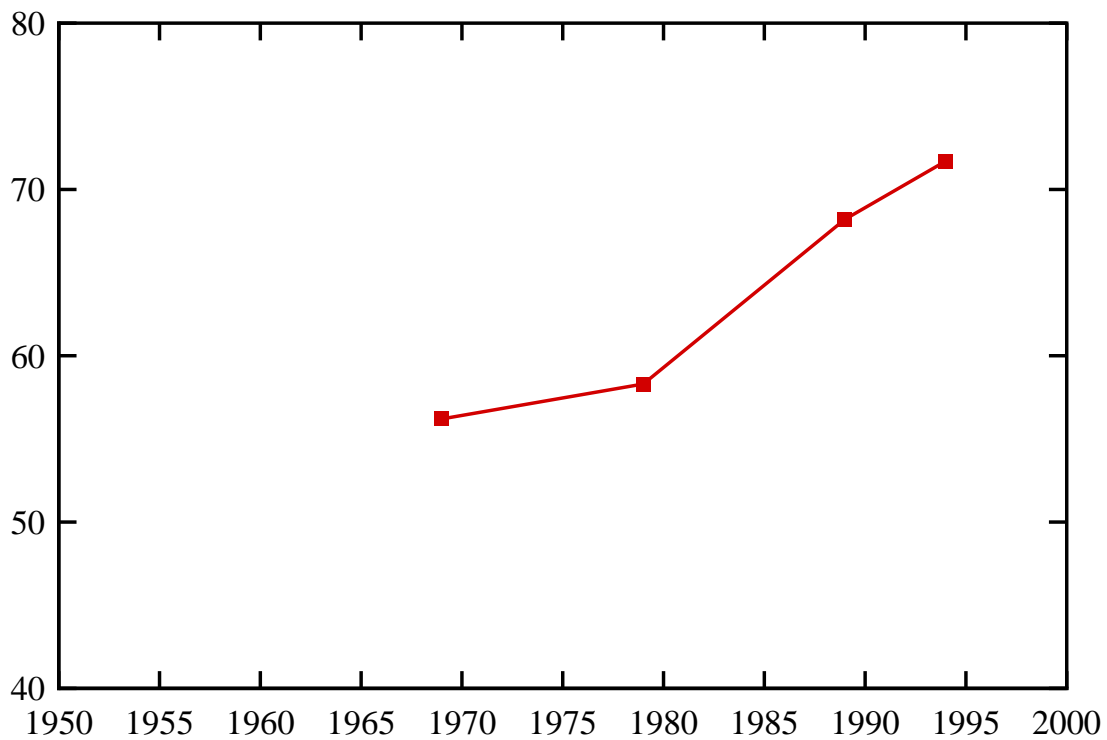
TABLE 4. EFFECTS OF FALL IN DISCRIMINATION

DATA	1950 RESULTS					1990 RESULTS					
	HOURS PER WEEK			Wage RATIO (%)	SHARE, HOME INV. (1950=100)	HOURS			Wage RATIO (%)	SHARE, HOME INV. (1950=100)	
MODEL:	MF	MM	SF	SM		MF	MM	SF	SM		
Benchmark	8.2	41.3	29.0	31.6	51	22.7	38.3	29.0	30.2	69	85
Equal utility weights	8.0	41.1	28.0	34.1	51	21.5	36.9	30.0	31.7	69	95
No human capital	8.2	41.3	27.1	32.8	52	17.1	39.3	30.0	30.9	68	98
Market-only human capital	8.4	41.4	25.1	35.1	52	20.4	38.2	31.9	32.7	68	84
Sector-specific capital	8.2	41.3	27.1	35.9	50	23.1	37.6	32.6	33.2	68	84
Married males subsidized	8.2	41.3	27.9	35.9	51	22.7	37.6	32.8	33.4	68	84
No singles discrimination	8.0	41.1	27.3	33.2	51	19.3	38.4	30.8	30.1	68	94
	10.0	41.1	34.2	34.2	69	22.6	37.7	32.7	32.7	82	96

Figure 1.  
U.S. Hours and Wages

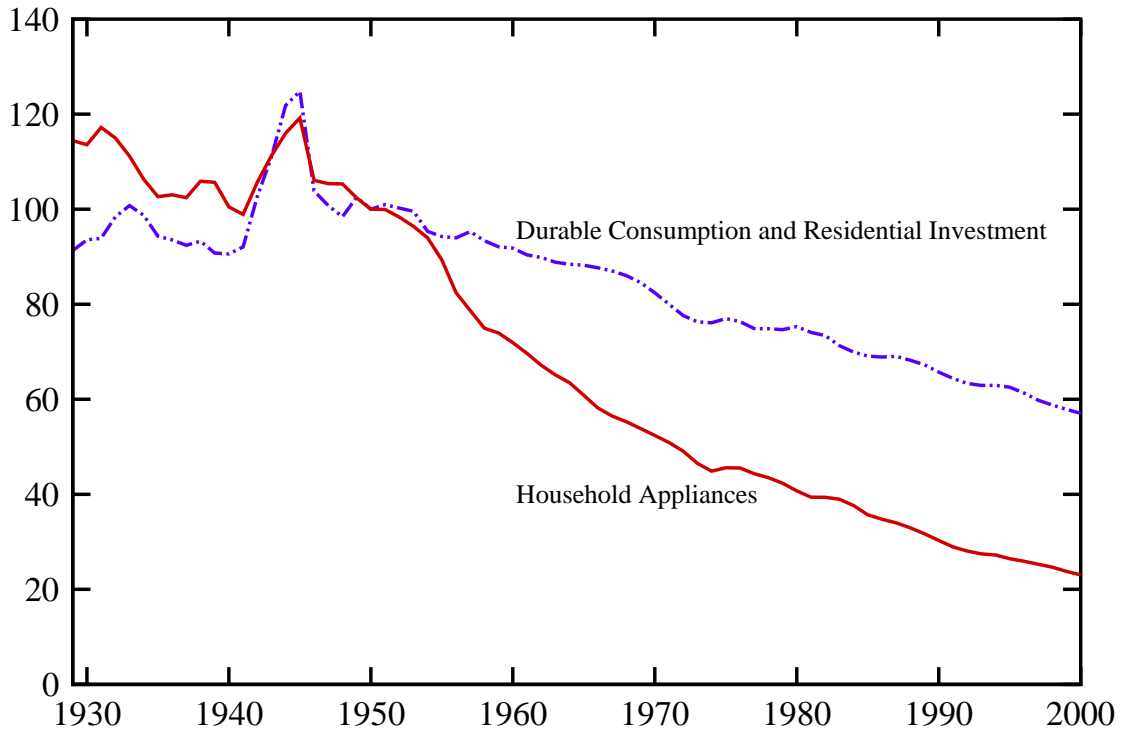


A. Market Hours of Work Per Week

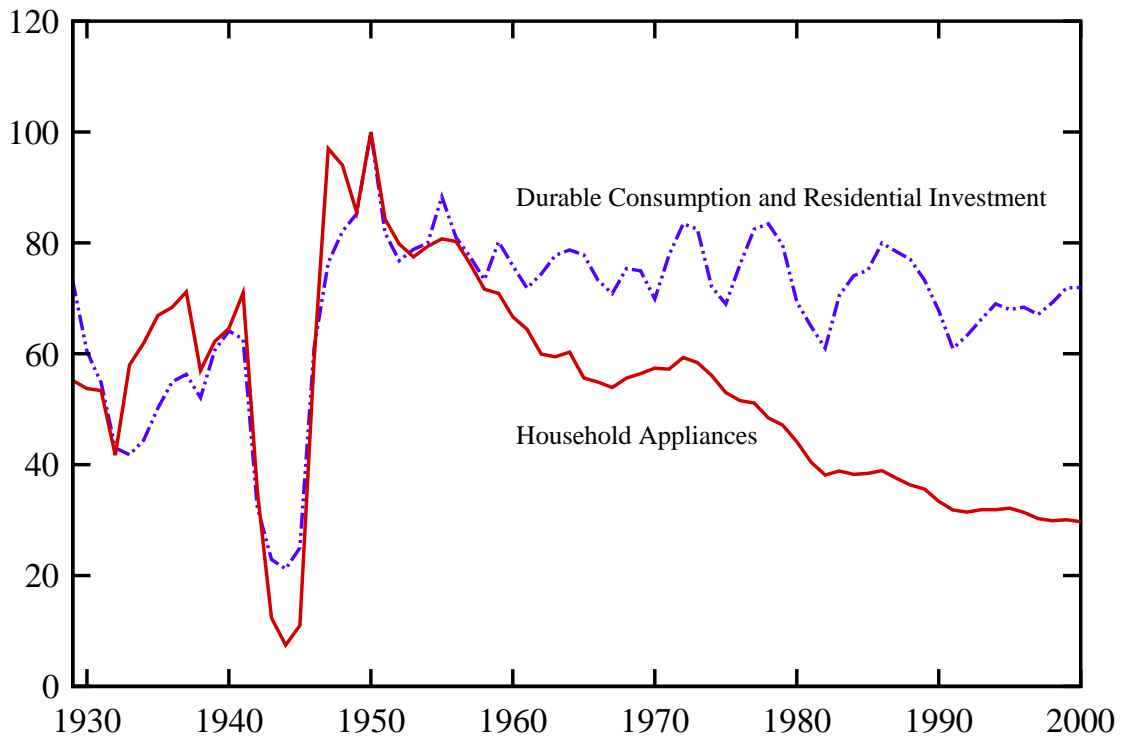


B. Ratio of Female to Male Wage Rate (%)

Figure 2.  
U.S. Deflators and Expenditure Shares

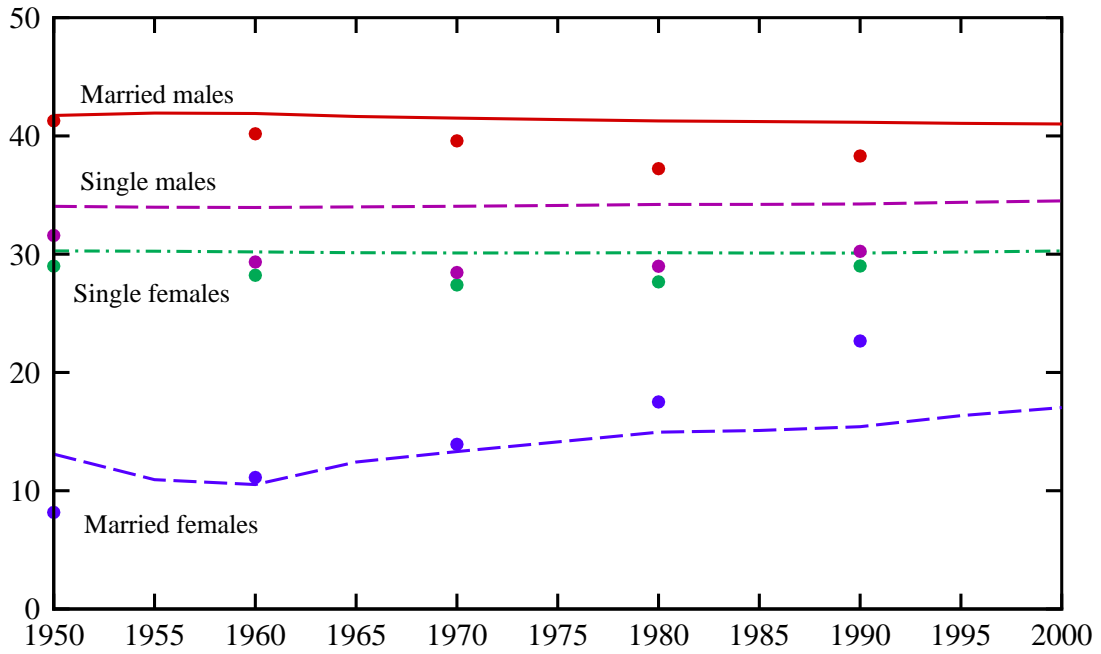


A. Ratio of Home Investment Deflators to GDP Deflator (1950=100)

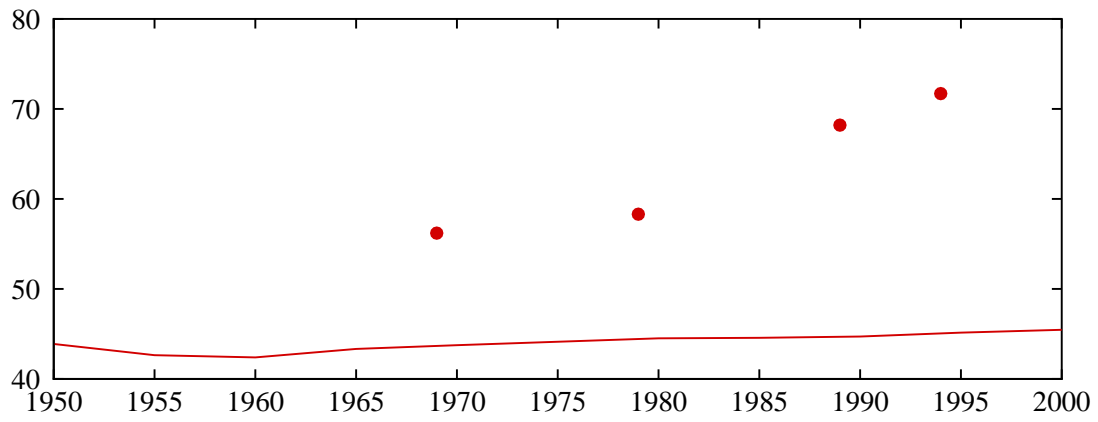


B. Home Investment Expenditure Shares (1950=100)

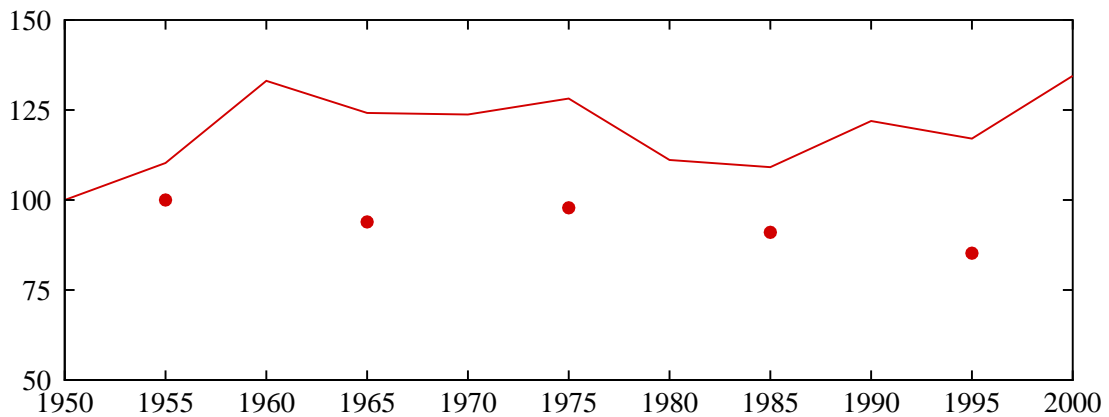
Figure 3.  
 Predictions of Model with Falling Home Capital Price  
 (Lines= model results, Dots= observations)



A. Market Hours of Work Per Week



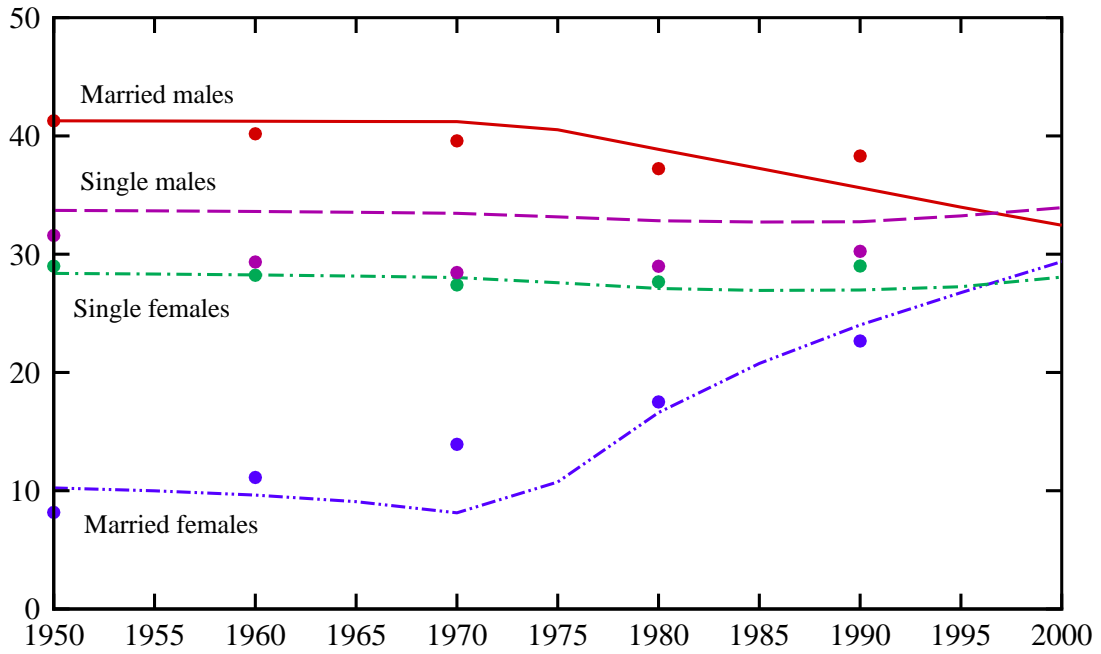
B. Ratio of Female to Male Wage Rate (%)



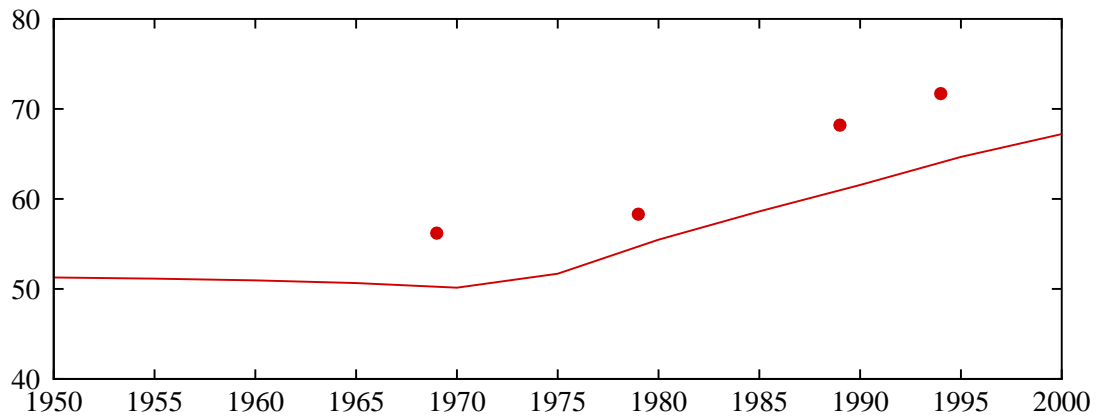
C. Ratio of Home Investment to Market Output (1950=100)



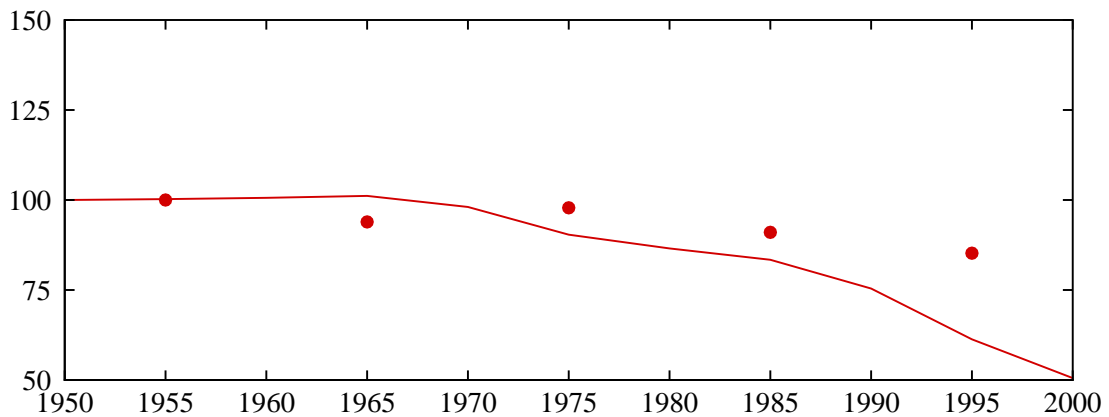
Figure 4.  
 Predictions of Model with Inferior Home Goods  
 (Lines= model results, Dots= observations)



A. Market Hours of Work Per Week

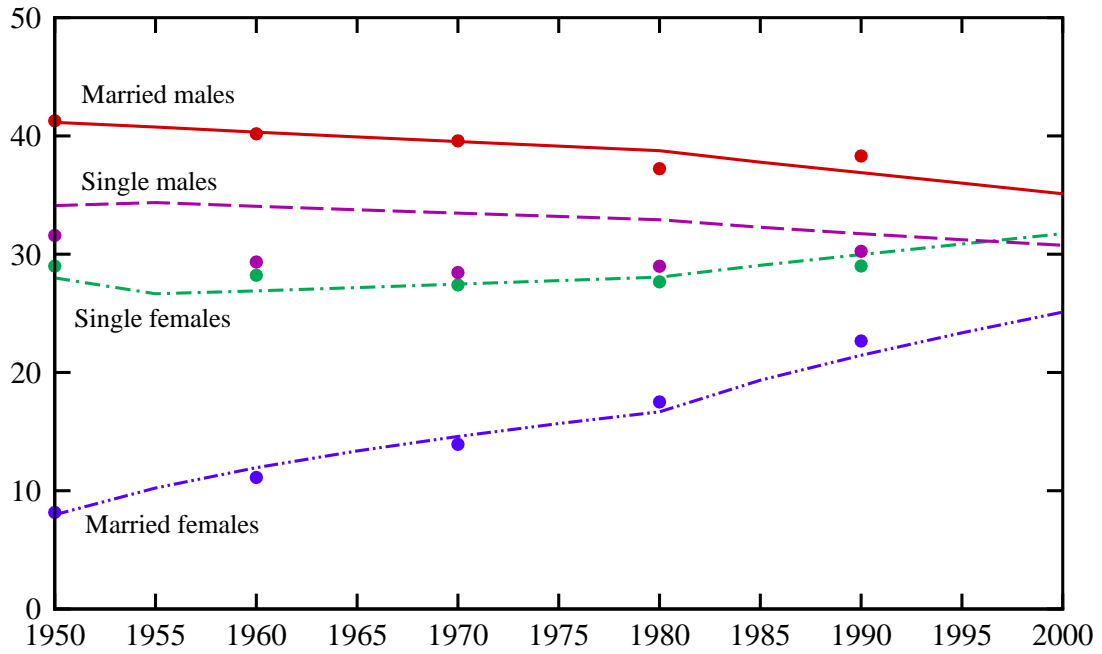


B. Ratio of Female to Male Wage Rate (%)

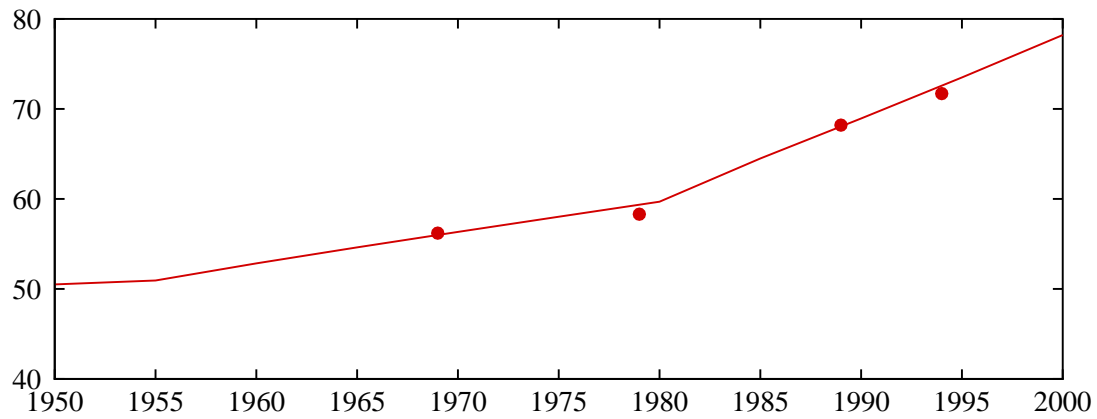


C. Ratio of Home Investment to Market Output (1950=100)

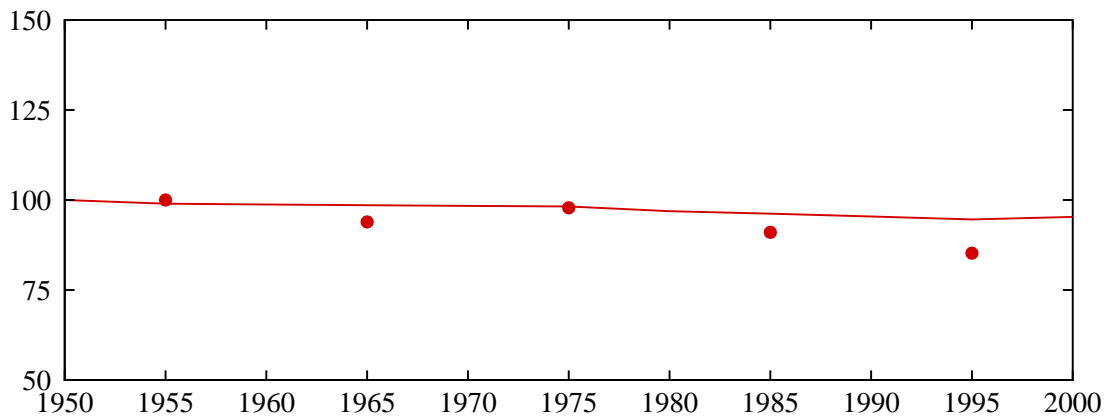
Figure 5.  
 Predictions of Model with Changes in Discrimination  
 (Lines= model results, Dots= observations)



A. Market Hours of Work Per Week

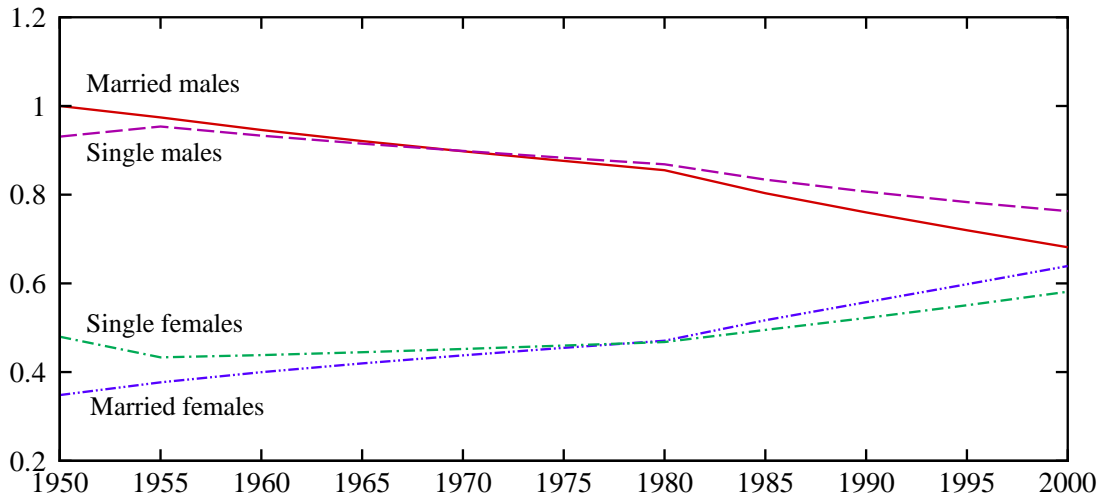


B. Ratio of Female to Male Wage Rate (%)

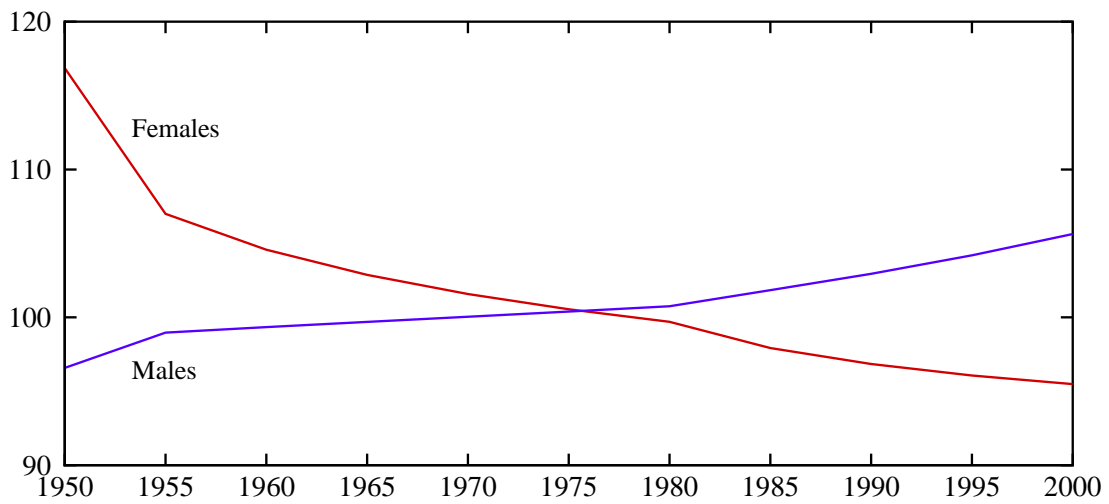


C. Ratio of Home Investment to Market Output (1950=100)

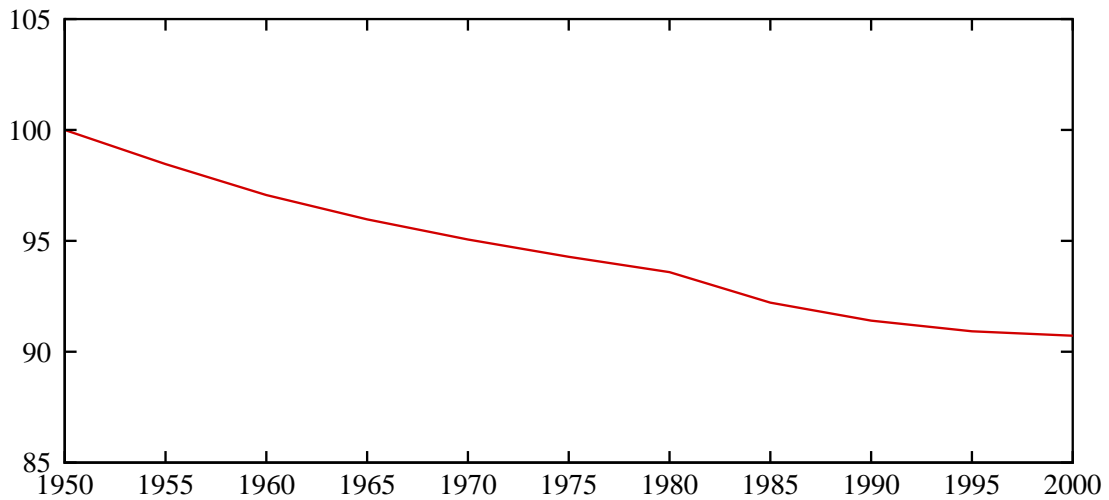
Figure 6.  
 Auxiliary Predictions of Model with Discrimination



A. Human Capital Relative to Married Males in 1950



B. Ratio of Single to Married Wage Rate by Sex (%)



C. Average Market Productivity Relative to Trend (1950=100)