# Electoral Competition with Imperfectly Informed Voters[†]

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**Abstract**

We explore the implications of voter ignorance on policy selection and policy outcomes in a simple model of party competition. For a simple benchmark case, we show that voter ignorance has no effect on the election outcome if the electorate is large. We then examine a model where voters are uncertain about the distribution of voter preferences and about the policy choice of one of the candidates. We characterize the limit equilibria of that model and show that voter ignorance leads to more partisan outcomes. In particular, we show that the Downsian prediction of median preferred outcomes is not robust even if candidates have weak policy preferences and mostly care about winning the election.

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## 1. Introduction

Surveys routinely find that the American electorate is poorly informed about the policy positions of candidates. (See, for example, Berelson, Lazarsfeld and McPhee (1954)). In a 1990/1991 survey only 57% of voters could correctly identify relative ideological positions of the Republican and Democratic parties on a left/right spectrum and only 45% of voters could correctly identify the parties' relative position on federal spending (Delli-Carpini and Keeter (1993), Table 2). The same surveys show that voters are equally ill informed about the electorate itself. For example, the same 1990/1991 survey shows that only 47% of voters correctly identify the party that holds the majority of seats in the Senate (Delli-Carpini and Keeter (1993), Table 2).

In this paper, we explore the implications of voter ignorance on policy selection and policy outcomes in a simple model of party competition. We begin our analysis with a benchmark model in which voters are uncertain about the candidate's policy choice. For that model we show that incomplete information about the policy choice does *not* alter the Downsian prediction: when the electorate is large, the election will fully aggregate information and the median preferred outcome will be chosen with probability close to one.

We then examine a model in which voters are uncertain about the distribution of voter preferences and about the policy choice of candidates. In that model, the election does not aggregate information and, as a result, non-median outcomes emerge. Moreover, a weaker policy preference of candidates may imply more partisan outcomes. Hence, even when candidates are office seekers with a weak preference for partisan policies, the election outcome is not close to the median preferred outcome.

The standard model of party competition (Downs (1957)) considers two candidates who seek to maximize the probability of getting elected. Both candidates choose policies prior to the election. Voters observe the policy choice and vote for the candidate who offers the more attractive policy. The model predicts that the median preferred policy will be implemented. Candidate competition is similar to Bertrand competition in oligopoly models. If one candidate chooses a policy that is median preferred to his opponent's policy then he will win the election. As a result, the Downsian prediction of median preferred outcomes holds even when the candidates have policy preferences.

We consider a very simple and stylized version of a candidate competition model. We assumes that there are two candidates, one candidate (the *committed candidate*) has a fixed and known policy position ($r$) while the other candidate (the *opportunistic candidate*) must choose between a partisan policy (denoted $l$) and a moderate policy (denoted $m$). The opportunistic candidate is motivated both by winning the election and by the resulting policy outcome. In particular, we assume that the opportunistic candidate prefers the partisan to the moderate policy. The median voter prefers $m$ to $r$ and $r$ to $l$. Therefore, the committed candidate's policy is "between" the partisan and the moderate policies of the opportunistic candidate.

In contrast to the standard Downsian model, we assume that voters are imperfectly informed of the opportunistic candidate's policy choice. In particular, we assume that each voter is independently informed of the policy choice with a probability between zero and one. In this simple setting, voter ignorance does not alter the prediction of the candidate competition model: In a large electorate the median preferred moderate policy is implemented with probability close to one (Proposition 1). Hence, despite the fact that only a fraction of voters is informed of opportunistic candidate's policy choice the outcome is *as if* all voters are perfectly informed. This result echoes the information aggregation results in Feddersen and Pesendorfer (1996, 1997).

Our main model adds uncertainty about the distribution of voter preferences to the model above. Hence, we assumes that voters are confronted with two sources of uncertainty: they do not observe the distribution of voter preferences and may be uncertain about the policy choice of a candidate. The parameters are chosen so that for every realization of the distribution of voter preferences the moderate policy $m$ is median preferred with probability close to one in a large electorate.

In contrast to voters, the (opportunistic) candidate learns the distribution of preferences prior to selecting a policy. Hence, the candidate is better informed than voters about the distribution of preference types. To motivate this assumption, note that candidates often take (secret) opinion polls prior to selecting a policy. These opinion polls may provide good information about the distribution of preferences.

In this model, the election does not aggregate information. The reason is that informed voters cannot be pivotal for every realization of the distribution of voter preferences. Therefore, there will be realizations of the distribution of preferences for which the policy choice of the opportunistic candidate has little impact on who gets elected. In those cases, either the partisan policy of the opportunistic candidate or the policy of the committed candidate will be implemented.

Our main result characterizes the limit equilibria when the number of voters goes to infinity. This characterization enables us to derive the following results:

(1) A weaker partisan preference of the opportunistic candidate implies a higher probability that the opportunistic candidate is elected but does *not* imply a lower probability that the partisan policy ($l$) is chosen. We give conditions under which a weaker partisan preference implies a higher probability that the partisan policy is implemented.

(2) We refer to the opportunistic candidate as an "office seeker" if he has a small partisan preference and mostly cares about getting elected. An office seeker wins the election with probability close to one. Hence, an office seeker wins the election with the same probability as in the full information case where all voters learn the policy choice. However, the policy outcome differs from the full information case: when all voters learn the policy choice the median preferred moderate policy $m$ is implemented with probability one; when voters are imperfectly informed of the policy choice the partisan policy $l$ is implemented with significant probability.

(3) If the probability that a voter is informed goes to zero then the probability that the median preferred policy is implemented goes to zero while the probability that the opportunistic candidate is elected stays bounded away from zero. Hence, the opportunistic candidate is elected with positive probability even though the median voter prefers $r$ to $l$ and the opportunistic candidate implements $l$ with probability close to 1 if elected.

To understand the driving force behind these results, note that uninformed voters must form expectations about the opportunistic candidate's policy choice. Since a vote only matters when it is pivotal, the relevant expectations are about the policy choice conditional on that event. In other words, a voter must determine what policy will be chosen if the election will be close. Only in that case does a vote make a difference for the election outcome.

Therefore, the key for the opportunistic candidate's success in our model is his willingness to choose a moderate policy when he expects the election to be close. An office seeker is reluctant to choose the partisan policy and risk losing the election whenever he expects the election to be close. Therefore, conditional on a vote being pivotal an office seeker is expected to choose the partisan policy with a small probability. By contrast, a candidate with a strong preference for the partisan policy is expected to choose the partisan policy even when he risks losing the election with significant probability. As a result, the office seeker receives a much larger share of the uninformed vote than a candidate with a strong partisan preference.

The willingness of a candidates to choose a median preferred policy when the election is close does not imply that this candidate will have a high ex ante probability of choosing the median preferred policy. In fact, the probability that the opportunistic candidate implements the partisan policy is maximal if the candidate is an office seeker. Because the office seeker receives a large share of the uninformed vote he can afford to choose the partisan policy and win the election in situations where the candidate with a strong partisan preference would lose the elections. Conditioning on being pivotal creates a wedge between voting behavior and (unconditional) policy choices. Candidates with a weak policy preference benefit from this effect.

### 1.1 Related Literature

Several authors have examined the robustness of the Downsian prediction by introducing uncertainty about the electorate and policy preferences of candidates. For example, Calvert (1985) analyzes the case where there the two candidates are symmetrically informed of the distribution of voter preferences. Bernhard, Duggan and Squintani (2003) and Chan (2001) analyze the case of asymmetric information. When candidates have policy preferences and are uncertain about the distribution of voter preferences then the policy outcomes may differ from the predicted median's preferred policy. However, if candidates have weak policy preferences and mostly care about winning the election then the

Downsian prediction is robust. The implemented policy will be close to the candidates' estimate of the median preference.

There is a long tradition of models examining the information aggregation properties of elections. The classic result in this area is the Condorcet Jury theorem. See, for example, Young (1988). Traditional jury models assume that voters do not behave strategically. Austen Smith and Banks (1995) and Feddersen and Pesendorfer (1996) analyze models closely related to the jury model under the assumption that voters act strategically. The information aggregation literature assumes that voters are uncertain about a state variable that affects their ranking of candidates. In our context, this corresponds to a situation where the opportunistic candidate's policy is chosen by some exogenous random draw. The difference here is that the candidate's policy choice is a strategic choice. For the benchmark model without aggregate uncertainty we show that the information aggregation result can be extended to this case.

## 2. Ignorant Voters

Two candidates, $a$ and $b$ stand for election. Candidate $a$ is committed to a fixed policy (denoted $r$) while candidate $b$ must choose between a moderate policy (denoted $m$) and a partisan policy (denoted $l$). We denote with $P = \{l, m, r\}$ the set of policies.

The payoff of candidate $b$ is 1 if he is elected and chooses the partisan policy $l$, $\mu \in (0, 1)$ if he is elected and chooses the moderate policy $m$ and 0 if he is not elected.

There are $2n + 1$ voters. Voters are expected utility maximizers whose preference depends on the candidate $b$'s policy choice. We assume that all voters prefer $m$ to $r$ and $r$ to $l$. Hence, all voter's prefer the moderate policy of $b$ to the policy of $a$ and the policy of $a$ to the partisan policy of $b$. We normalize the voters' von Neumann-Morgenstern utility function so that the utility of policy $l$ is zero, the utility of policy $m$ is 1, and the utility of policy $r$ is $\lambda \in T = [0, 1]$ which is the type of the voter. Therefore, if the candidate chooses $m$ with probability $\lambda$ (and $l$ with probability $1 - \lambda$), then the voter type $\lambda$ is indifferent between $a$ and the lottery over policies offered by $b$. Types of voters are drawn according to the probability distribution $F$ on $[0, 1]$. We assume that $F$ admits a continuous density $f$ with $f > 0$.

Candidate $b$ does not observe the preference type of individual voters. Voters observe their own type but not the type of other voters. Candidate $b$ chooses a policy $p \in \{l, m\}$. Each voter is independently informed of the policy choices with probability $\delta \in (0, 1]$. If the candidate chooses a mixed action then informed voters observe the realization of the mixed action. Every voter must vote for one of the candidates. The candidate who receives $n + 1$ or more votes wins the election.

A strategy for an uninformed voter specifies for every preference type the probability that the voter votes for candidate $b$. Hence, a strategy for an uninformed voter is a measurable function $\sigma_v : T \to [0, 1]$ where $\sigma_v(\lambda)$ denotes the probability with which an uninformed swing voter of type $\lambda$ votes for $b$. A strategy for $b$ specifies a probability of choosing the moderate policy, denoted $\sigma_b \in [0, 1]$.

We analyze symmetric Nash equilibria in weakly undominated strategies. Hence, we assume that in equilibrium all uninformed voters use the same strategy $\sigma_v$. The assumption that voters choose a weakly undominated strategy implies that informed voters always choose their preferred candidate. Note that all informed voters with type $\lambda \in (0, 1)$ strictly prefer $b$ if $b$ chooses $m$ and strictly prefer $a$ if $b$ chooses $l$. Since $F$ is continuous the types $\lambda = 0$ and $\lambda = 1$ occur with probability 0. Therefore, with probability one there is a unique weakly undominated strategy for an informed voter: vote for $a$ if $b$ chooses $l$ and for $b$ if $b$ chooses $m$.

Below, "equilibrium" refers to a symmetric Nash equilibrium in weakly undominated strategies. Suppressing the strategy of informed voters, an equilibrium can be characterized by a pair $\sigma = (\sigma_b, \sigma_v)$. Every strategy pair induces a probability distribution over outcomes, denoted $\rho \in \Delta(\{l, m, r\})$ where $\rho^p$ denotes the probability that policy $p$ is implemented.

For a fixed $F, \mu, \delta$ satisfying the assumptions above, let $\mathcal{E}_n$ denote the the election game with $2n + 1$ voters and parameters $F, \mu, \delta$. Let $\sigma_n$ denote a strategy profile for the election $\mathcal{E}_n$ and let $\rho_n$ denote the corresponding outcome.

Note that when $\delta = 1$ and hence all voters are informed of $b$'s policy choice then, in equilibrium, $b$ chooses $m$ with probability 1. Proposition 1 characterizes equilibrium outcomes for large electorates. It states that the moderate policy $m$ is implemented with probability close to one in a large electorate. Hence, the fact that voters are imperfectly informed of $b$'s policy choice has no impact on the election outcome if $n$ is large.

**Proposition 1:** *Let $\sigma_n$ be an equilibrium for the election $\mathcal{E}_n$ and let $\rho_n$ denote the corresponding outcome. If $\delta > 0$ then $\rho_n^m \to 1$ as $n \to \infty$.*

**Proof:** It is straightforward to show that a symmetric equilibrium in weakly undominated strategies exists. Here we show that every sequence of equilibria $(\sigma_n)$ implies that the corresponding sequence of probability distributions over outcomes satisfies $\rho_n^m \to 1$.

First note that $\sigma_{bn} > 0$ for all $n$ since for $\sigma_{bn} = 0$ $b$ receives no vote. In that case, a deviation to $m$ strictly increases $b$'s payoff because all informed voters choose $b$ if he chooses $m$. Next, note that if $\sigma_{bn} = 1$ then $b$ receives no vote and $m$ is implemented with probability 1. Hence, it remains to consider the case where (along some subsequence) $\sigma_{bn} \in (0,1)$ for all $n$. If $\sigma_{bn} \in (0,1)$ then for every strategy $\sigma_v$ there is a strictly positive probability that the vote of an uninformed voter is pivotal.

Let $\theta_n$ denote the probability that $b$ chooses $m$ conditional on a vote being pivotal in the election with $n$ voters. In equilibrium, all uninformed voters with types $\lambda < \theta_n$ must vote for 1 while all voters with type $\lambda > \theta_n$ must vote for 2.

Since $b$ chooses both policies with strictly positive probability he must be indifferent between them. Let

$$\begin{aligned}\pi_n^l :=& F(\theta_n)(1-\delta)\\ \pi_n^m :=& F(\theta_n)(1-\delta)+\delta\end{aligned}$$

denote the probability that a randomly selected voter votes for 1 conditional on the policy choice of candidate 1. For $0 < x < 1$, let $B_n(x)$ denote the probability of at least $n+1$ successes from $2n+1$ draws when the probability of success is $x$. Indifference by $b$ implies

$$\mu B_n(\pi_n^l) = B_n(\pi_n^m)$$

We note that this implies that $\pi_n^l \to 1/2$ since $\pi_n^l \geq 1/2+\epsilon$ implies $B_n(\pi_n^l)/B_n(\pi_n^m) \to 1$ and $\pi_n^l \leq 1/2-\epsilon$ implies $B_n(\pi_n^l)/B_n(\pi_n^m) \to 0$. Since $\pi_n^l \to 1/2$ there is $\lambda^* \in [0,1]$ such that

$$\theta_n \to \lambda^* = F^{-1}\left(\frac{1}{2(1-\delta)}\right)$$

It follows that

$$\frac{\sigma_{bn}}{1-\sigma_{bn}}\left(\frac{\pi_n^m(1-\pi_n^m)}{\pi_n^l(1-\pi_n^l)}\right)^n \to \frac{\lambda^*}{1-\lambda^*}$$

Note that

$$\left(\frac{\pi_n^m(1-\pi_n^m)}{\pi_n^l(1-\pi_n^l)}\right)^n \to 0$$

since $\pi_n^l \to 1/2$ and hence we conclude that $\sigma_{bn} \to 1$. Since $\pi_n^m = \pi_n^l + \delta$ and $\pi_n^l \to 1/2$ it follows that $b$ is elected with probability close to one if he chooses $m$ which completes the proof. □

Proposition 1 is related to earlier information aggregation results in Feddersen and Pesendorfer (1997). There it is shown that if we fix the strategy of $b$, then as $n \to \infty$ the probability that $b$ is elected if he chooses $l$ goes to zero and the probability that $b$ is elected if he chooses $m$ goes to one. This result does not imply Proposition 1 because the strategy here is endogenous. Moreover, the conclusion in Proposition 1 is slightly different than the conclusion one would draw from a setting where $b$'s strategy was fixed.

Consider the case where $\delta < 1/2$. In that case, $b$ must mix in equilibrium when $n$ is large. To see this, note that if equilibrium prescribes that $b$ chooses $l$ with probability one then he receives no votes. In that case he would do better by choosing $m$ and thereby receiving the vote of informed voters. If equilibrium prescribes that $b$ chooses $m$ with probability 1 then all uninformed voters must vote for $b$. In that case, $b$ wins the election with probability close to one even if he chooses $l$. It follows that $b$ must be indifferent between choosing $m$ and choosing $l$.

If $b$ is indifferent between choosing $l$ and $m$ then his expected equilibrium vote share if he chooses $l$ (denoted $\pi_n^l$) must converge to 1/2. To see this, first note that if $\pi_n^l$ is bounded above 1/2 for large $n$ then $b$ wins the election with probability close to one when he chooses $l$. This implies that he cannot be indifferent between the two policy choices. When $\pi_n^l$ stays bounded below 1/2 then $b$ wins the election with probability close zero when he chooses $l$ and $n$ is large. In that case, $b$ strictly prefers $m$ to $l$.

Since $\pi_n^l$ converges to 1/2 it follows that conditional on a vote being pivotal a positive fraction of uninformed voters must prefer $b$ to $a$. Hence, conditional on a vote being pivotal the probability that $b$ chooses $m$ must stay bounded away from zero. Note that if $b$ chooses

$l$ then (for large $n$) it is much more likely that a vote is pivotal than when $b$ chooses $m$. This follows because roughly $1/2$ of the electorate votes for $b$ if he chooses $l$ while more than $1/2$ of the electorate vote for $b$ if he chooses $m$. To maintain the incentives for uninformed voters it must therefore be the case that $b$ chooses $l$ with vanishing probability as $n$ goes to infinity.

Hence, $b$ will choose $l$ with strictly positive probability for all $n$. Moreover, conditional on choosing $l$ the probability that $b$ wins stays bounded away from zero. However, the probability that $b$ chooses $l$ must converge to zero as $n \to \infty$ in accordance with Proposition 1.

## 3. Uncertainty about the Electorate

The model in the previous section assumes that the distribution of preferences is known to all players. In this section, we modify the model to allow for uncertainty about the distribution of preferences. The probability distribution over voter preferences depends on a state $s \in S = [0, 1]$.

To simplify the analysis, we introduce partisan voters who prefer one of the candidates irrespective of the policy choice. Each voter is independently assigned a preference type. The probability distribution over types depends on the state and is as follows. With probability $(1 - s)/2$ the voter is a partisan who always prefers $b$; with probability $s/2$ the voter is a partisan who always prefers $a$; and with probability $1/2$ the voter is a swing voter. The preference types of swing voters are drawn independently according to the distribution $F$ on $T = [0, 1]$. We assume that $F$ admits a continuous density $f$ with $f > 0$. As in the previous section, all swing voters receive a utility of 1 if $m$ is implemented and a utility of 0 if $l$ is implemented. The utility of type $\lambda$ when policy $r$ is implemented is $\lambda$. Note that the probability that a voter is a swing voter is independent of the state $s$.

The timing of events is as follows. Nature draws a state $s \in S$ according to the distribution $G$. We assume that $G$ admits a continuous density $g$ with $g > 0$. Then, nature independently assigns each voter a preference type according to the probability distribution defined above. Candidates observe the state $s$ but not the preference types of individual voters. Upon observing the state, $b$ chooses a policy $p \in \{l, m\}$. Voters observe their own type but not the type of other voters or the state $s$. Each voter is independently informed of the policy choice of $b$ with probability $\delta \in (0, 1]$. Note that we assume that informed voters observe the *realization* of the policy choice in case $b$ chooses a mixed action. Voters must vote for one of the candidates. The candidate who receives $n + 1$ or more votes wins the election.

A strategy for the uninformed swing voter specifies for every preference type the probability that the voter votes for $b$. Hence, a strategy for an uninformed swing voter is a function $\sigma_v : T \to [0, 1]$ where $\sigma_v(\lambda)$ denotes the probability with which an uninformed swing voter of type $\lambda$ chooses the $b$. A strategy for $b$ specifies a probability distribution over policies for each realization of the state $s$. Hence, a strategy for $b$ is a measurable function $\sigma_b : S \to [0, 1]$ where $\sigma_b(s)$ denotes the probability of choosing the moderate policy (policy $m$.)

We analyze symmetric Nash equilibria in weakly undominated strategies. Therefore, equilibrium strategies of uninformed voters are described by a single strategy $\sigma_v$. As before, informed swing voters (with probability 1) vote for $b$ if $b$ chooses $m$ and for $a$ otherwise. Similarly, partisan voters always choose their preferred candidate. Below, "equilibrium" refers to a symmetric Nash equilibrium in weakly undominated strategies. We suppress the behavior of partisan and informed voters and describe equilibria by a pair $\sigma = (\sigma_b, \sigma_v)$.

The median voter's type depends on the state $s$ and on the distribution of preference types. The law of large numbers guarantees that as the number of voters goes to infinity, the median voter converges (in probability) to a preference type that depends only on the state $s$. We refer to this limit median as the $s-$*median*. (Note that this is true for all states *except* for the states $s = 0, s = 1$ in which the probability that any voter is a partisan for one of the candidates is exactly $1/2$. Since any individual state has probability 0 we may ignore those two states.) If $s \in (0, 1)$ then the $s-$median is the swing voter with type $\lambda$ that satisfies

$$\frac{1-s}{2} + \frac{1}{2}F(\lambda) = \frac{1}{2}$$

which simplifies to

$$\lambda = F^{-1}(s)$$

For a fixed $F, G, u, \mu, \delta$ satisfying the assumptions above, let $\mathcal{E}_n$ denote the the election game with $2n+1$ voters and parameters $F, G, u, \mu, \beta, \delta$. Let $\sigma_n$ denote a strategy profile for the election $\mathcal{E}_n$.

As a simple benchmark, consider the cases where all voters are informed of the candidate's policy choice $\delta = 1$. For $s \in (0,1)$ candidate $b$ expects the $s-$median to be a swing voter. When $n$ is large, it is therefore optimal to choose the moderate policy $m$. As a result, for all $s \in (0,1)$ the probability that $b$ wins the election and implements $m$ converges to one.

**Proposition 2:** *Let $\sigma_n$ be an equilibrium profile for the election $\mathcal{E}_n$ with $\delta = 1$. Then, $\sigma_{bn}(s) \to 1$ for $s > 0$ as $n \to \infty$. For $s \in (0,1)$ the probability that $b$ wins the election converges to one.*

Proposition 2 illustrates how candidate competition can lead to a median preferred outcome in every state. For $0 < s < 1$ the $s-$median is a swing voter and the policy $m$ is implemented with probability close to one when $n$ is large. For $s = 0$ candidate $b$ may choose the partisan policy (if $\mu \leq 1/2$) for large $n$. For $s = 1$ candidate $a$ will win the election with probability 1/2.

Next, we consider the case where voters are asymmetrically informed about the policy choice of $b$. We assume that

$$0 < \delta < 1 \tag{3.1}$$

Our first result establishes that equilibrium strategies of voters and $b$ can be described as cutoff strategies. The strategy $\sigma_v$ is a cutoff strategy if there is a $\lambda \in [0,1]$ such that types $\lambda' < \lambda$ vote for $b$ and types $\lambda' > \lambda$ vote for $a$. The strategy $\sigma_b$ is a cutoff strategy if there is $s \in [0,1]$ such that $b$ chooses $l$ at states $s' < s$ and $m$ at $s' > s$. We call an equilibrium in which voters and candidate $b$ use a cutoff strategy a cutoff equilibrium. A cutoff equilibrium consists of a pair $(\lambda, s)$ where $\lambda$ is the of voters' cutoff and $s$ is the cutoff of $b$. Since the probability of any single $s$ or any single $\lambda'$ is zero, $(\lambda, s)$ is a sufficient description of the strategy profile. Proposition 3 establishes the existence of an equilibrium. It also shows that every equilibrium is a cutoff strategy equilibrium.

**Proposition 3:** *An equilibrium exists. Every equilibrium is a cutoff equilibrium.*

**Proof:** see Appendix.

Note that any swing voter, regardless of whether he is informed or not, assigns strictly positive probability to being pivotal irrespective of the strategy of the other swing voters. Since this is the only event in which a voter can affect the election outcome, the optimal behavior of uninformed voters depends on the probability that $b$ chooses policy $m$ conditional on a vote being pivotal. Let $\theta \in [0,1]$ denote this probability. Recall that for $\lambda < \theta$ the voter strictly prefers $b$ and for $\lambda > \theta$ the voter strictly prefers the $a$. Hence, it follows that in equilibrium uninformed voters use a cutoff strategy with cutoff $\theta$.

Next, we argue that $b$'s best response to any cutoff strategy is also a cutoff strategy. In particular, we will show that given a cutoff strategy for the voters, if it is optimal for $b$ to choose $m$ at state $s$, then the only optimal action for him at higher states is $m$ as well. For $x \in [0,1]$ let $B_n(x)$ be defined as

$$B_n(x) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} x^k (1-x)^{2n+1-k}$$

Hence $B_n(x)$ is the binomial probability of at least $n+1$ successes out of $2n+1$ trials given that the probability of success in each trial is $x$. If success is the event that a single voter will vote for $b$, then $B_n$ becomes the probability of $b$ getting elected. From the perspective of $b$, who knows his own policy choice, the state $s$ and the voters' strategy $\lambda$, the probability of success can be calculated as follows:

$$\begin{aligned} \pi^l(\lambda, s) &= \frac{1-s}{2} + \frac{1-\delta}{2} F(\lambda) \\ \pi^m(\lambda, s) &= \frac{1-s}{2} + \frac{1-\delta}{2} F(\lambda) + \frac{\delta}{2} \\ &= \pi^l(\lambda, s) + \frac{\delta}{2} \end{aligned}$$

Candidate $b$ chooses $l$ if

$$B_n(\pi^l(\lambda, s)) > \mu B_n(\pi^m(\lambda, s))$$

and $m$ if this inequality is reversed. Hence, we need to show that $B_n(\pi^l(s))/B_n(\pi^m(s))$ is decreasing in $s$. We do so by first showing that $B_n$ is log-concave. Since $\pi^m = \pi^l + (1-\lambda)\delta$ with $\pi^m$ a linear and decreasing function of $s$, log-concavity of $B_n$ implies that $B_n(\pi^l(s))/B_n(\pi^m(s))$ is decreasing in $s$. Finally, we use a fixed-point argument to establish the existence of cutoff strategy equilibrium.

Every cutoff strategy pair $(\lambda, s)$ implies a probability distribution over policy outcomes for every states. For a given strategy $(\lambda, s)$ we define with $\rho^p(s')$ the probability that policy $p \in P$ is implemented in state $s'$. Note that

$$\rho^l(s') = \begin{cases} B_n(\pi^l(\lambda, s') & \text{if } s' > s \\ 0 & \text{if } s' > s. \end{cases}$$
$$\rho^m(s') = \begin{cases} B_n(\pi^m(\lambda, s') & \text{if } s' > s \\ 0 & \text{if } s' < s. \end{cases}$$
$$\rho^r(s') = \begin{cases} 1 - B_n(\pi^l(\lambda, s') & \text{if } s' < s \\ 1 - B_n(\pi^m(\lambda, s') & \text{if } s' > s. \end{cases}$$

We refer to $\rho$ as the outcome function corresponding to the strategy $(\lambda, s)$. We define $\bar{\rho}^p := \int_0^1 \rho(s)g(s)ds$ to denote the ex ante probability that policy $p$ is implemented.

In the following we analyze equilibrium strategies when the number of voters is large. We denote with $(\lambda_n, s_n)$ a (cutoff) equilibrium for the election $\mathcal{E}_n$ and with $\rho_n$ the corresponding outcome function. Proposition 4 below characterizes limit points of this sequence.

We say that $s^p(\lambda)$ is the critical state for policy $p$ at cutoff $\lambda$ if

$$\pi^p(\lambda, s) = {}^1\!/_2$$

Hence, the critical state of policy $p \in \{l, m\}$ is the state at which a randomly drawn voter chooses $b$ with probability $1/2$ if $b$ chooses policy $p$. Substituting the definitions of $\pi^l$ and $\pi^m$ from into the definition of a critical state yields

$$s^l(\lambda) = (1-\delta)F(\lambda)$$
$$s^m(\lambda) = (1-\delta)F(\lambda) + \delta$$

Clearly both $s^l$ and $s^m$ are increasing functions of $\lambda$.

Proposition 4 characterizes equilibria for large elections. In particular, it shows that the equilibrium cutpoint of candidate $b$ must converge to $s^l(\lambda)$ the critical state of policy $l$. In addition, Proposition 4 provides a simple formula for the equilibrium cutpoint of voters.

**Proposition 4:** *Let $(\lambda_n, s_n)$ be a convergent sequence of equilibria with limit $(\lambda, s)$. Then,*

$$s = s^l(\lambda)$$
$$\lambda = \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + g(s^l(\lambda))(1-\mu)}$$

The first part of Proposition 4 is straightforward. In a large electorate, candidate $b$ wins the election with probability close to one close when $s < s^l(\lambda)$ and $b$ chooses $l$. Since $b$ strictly prefers $l$ to $m$ it follows the $l$ must be chosen for states $s < s^l(\lambda)$. Conversely, if $s > s^l(\lambda)$ then $b$ loses the election with probability close to one if $b$ chooses $l$. Therefore, for states $s > s^l(\lambda)$ policy $m$ is optimal.

The fact that the limit cutpoint for $b$ equals the critical state $s^l(\lambda)$ implies that policy outcomes for a large electorate depend only on the critical states $s^l(\lambda)$ and $s^m(\lambda)$. For states below $s^l(\lambda)$ candidate $b$ is elected and chooses the partisan policy $l$; for states greater than $s^m(\lambda)$ candidate $a$ is elected and policy $r$ will be implemented; for states between the two critical states, candidate $b$ is elected and chooses policy $m$. The following corollary summarizes this implication.

**Corollary:** *Let $(\lambda_n, s_n)$ be a convergent sequence of equilibria with limit $(\lambda, s)$. Let $\rho_n$ be the outcome function corresponding to $(\lambda_n, s_n)$. Then,*

*(i) $\rho_n^l(s') \to 1$ for $0 \leq s' < s^l(\lambda)$ and $\bar{\rho}_n^l \to G(s^l(\lambda))$;*

*(ii) $\rho_n^m(s') \to 1$ for $s^l(\lambda) < s' < s^m(\lambda)$ and $\bar{\rho}_n^m \to G(s^m(\lambda)) - G(s^l(\lambda))$;*

*(iii) $\rho_n^r(s') \to 1$ for $s^m(\lambda) < s' \leq 1$ and $\bar{\rho}_n^r \to 1 - G(s^l(\lambda))$.*

Next, we provide intuition for the characterization of the limit voter cutpoint in Proposition 4. Note that as the number of voters becomes very large, the probability of being pivotal is concentrated around states in which the election is expected to be tied. There are two such states, $s^l(\lambda)$ and $s^m(\lambda)$; Hence, conditional on being pivotal, a voter knows that the state is in one of two small "critical intervals" around the critical states. The inference problem for the uninformed voter therefore reduces to determining the relative likelihoods of $s^l(\lambda)$ and $s^m(\lambda)$ conditional on a vote being pivotal.

Next, consider the incentives for candidate $b$. If the probability of winning with $l$ is less than $\mu$-times the probability of winning with $m$ then $b$ strictly prefers the moderate policy $m$. Therefore, the critical interval around $s^l(\lambda)$ is truncated at the point where the probability of winning drops below $\mu$. (The probability of winning the election with $m$ in a neighborhood of $s^l(\lambda)$ is close to one). Hence, the closer the parameter $\mu$ is to 1 the smaller the critical interval. The key step in the proof is to show that the relative likelihood of the $s^l(\lambda)$ and $s^m(\lambda)$ is related to $\mu$ by the simple formula given in Proposition 4.

We call a value $\theta \in [0, 1]$ that satisfies

$$\theta = \frac{g(s^m(\theta))}{g(s^m(\theta)) + g(s^l(\theta))(1-\mu)}$$

a *critical value.* Proposition 4 establishes that limit points of equilibrium strategies for voters correspond to critical values. It is easy to see that a critical value exists. In general, there may be multiple critical values and hence multiple limit equilibria. In Proposition 5 below, we establish the existence of critical values and show that if $G$ is log-concave then there is a unique critical value.

**Proposition 5:** *(i) The set of critical values is nonempty. (ii) If $g$ is log-concave there is a unique critical value.*

**Proof:** Define the function $h : [0, 1] \to [0, 1]$ as

$$h(\theta) = \frac{g(s^m(\theta))}{g(s^m(\theta)) + g(s^c(\theta))(1-\mu)}$$

Note that $\theta$ is a critical value if and only if it is a fixed-point of $h$. Clearly $s^l$ and $s^m$ are continuous, while $g$ is both continuous and bounded away from 0. Therefore, $h$ has a fixed-point. To ensure that $h$ has a unique fixed-point we show that $h$ is non-increasing whenever $G$ is log-concave. It follows from (2) above that $s^m(\lambda) - s^l(\lambda) = \delta > 0$ does not depend on $\lambda$ and $s^l(\lambda)$ is increasing in $\lambda$, $\frac{g(s^m(\lambda))}{g(s^l(\lambda))}$ is nonincreasing whenever $\frac{g(x+k)}{g(x)}$ is a nonincreasing function of $x$ for $k > 0$, that is, whenever $g$ is log-concave. $\square$

Note that Proposition 4 implies that limit equilibria are fully characterized by the limit cutpoint of voters, $\lambda$. This motivates the following definition.

**Definition:** *For any fixed $F, G, \delta, \mu$ that satisfy the assumptions above, let $\mathcal{E}_n$ denote the the election game with $2n+1$ voters and parameters $F, G, \delta, \mu$. We say that $\lambda$ is a limit equilibrium of $\mathcal{E}_n(\mu)$ if there are equilibria $(\lambda_n, s_n)$ for $\mathcal{E}_n(\mu)$ with $(\lambda_n, s_n) \to (\lambda, s^l(\lambda))$.*

In Proposition 6 below, we analyze how changes in the parameter $\mu$ affect limit equilibra. Note that $\mu$ measures the strength of the partisan preference of candidate $b$. An increase in $\mu$ means that candidate $b$'s preference for a partisan policy becomes weaker. For example, $\mu = 1$ means that the candidate cares only about winning the election but does not care whether he wins with the partisan or the moderate policy. By contrast, $\mu = 0$ means that the candidate is indifferent between loosing the election and winning with the moderate policy but strictly prefers winning with the partisan policy. For any fixed $F, G, \delta$ that satisfy the assumptions above, let $\mathcal{E}_n(\mu)$ denote the the election game with $2n+1$ voters and parameters $F, G, \delta, \mu$.

Proposition 6(i) shows that for $\mu$ close to one the equilibrium in a large electorate has all uninformed voters vote for $b$, i.e., $\lambda$ is close to one. Proposition 6 (ii) assumes that $g$ is log-concave and differentiable and hence the election has a unique limit equilibrium. Under these assumption an increase in $\mu$ (and hence a decrease in the partisan preference of $b$) implies an upward shift in the limit equilibrium cutpoint of voters $\lambda$. Hence, a higher $\mu$ implies that a larger fraction of uninformed voters vote for $b$.

**Proposition 6:** *(i) For every $\epsilon > 0$ there is $\mu < 1$ such that if $\lambda$ is a limit equilibrium of $\mathcal{E}_n(\mu')$ with $\mu' > \mu$ then $\lambda > 1 - \epsilon$. (ii) Assume $g$ is log-concave and differentiable. Let $\lambda$ be the limit equilibrium for $\mathcal{E}_n(\mu)$ and let $\lambda'$ be the limit equilibrium for $\mathcal{E}_n(\mu')$ with $\mu > \mu'$. Then, $\lambda > \lambda'$.*

**Proof:** Part (i) is straightforward using Propositions 2 and 3. It remains to prove part (ii). Since $g$ is log-concave there is a unique critical value $\lambda$ that solves the equation

$$\lambda(1 + R(\lambda)(1 - \mu)) = 1$$

with

$$R(\lambda) = \frac{g((1-\delta)F(\lambda))}{g((1-\delta)F(\lambda) + \delta)}$$

Since $g$ is differentiable and log-concave and $F$ is increasing in $\lambda$ it follows that $R'(\lambda) := \partial R(\lambda)/\partial\lambda \geq 0$. Taking a total derivative therefore yields

$$\frac{d\lambda}{d\mu} = \frac{\lambda R(\lambda)}{1 + R(\lambda(1-\mu) + \lambda R'(\lambda)(1-\mu)} > 0$$

which proves part (ii). □

For the limit equilibrium $\lambda$ the probability that candidate $b$ is elected is given by

$$G(s^m(\lambda)) = G((1-\delta)F(\lambda) + \delta))$$

Therefore, an increase in $\lambda$ implies that candidate $b$ is elected with a higher probability. Hence, Proposition 6(ii) implies that the probability candidate $b$ is elected is monotone in the preference parameter $\mu$. Proposition 6(i) implies that when $\mu$ is sufficiently close to one, then the limit equilibrium $\lambda$ is close to one and therefore the probability that $b$ is elected $(G(s^m(\lambda))$ is close to one. We conclude that if $b$ is an office seeker then he is elected with probability close to 1. Note that this is the same probability with which $b$ is elected in the benchmark where all voters know the policy. However, in contrast to the benchmark case the probability that the partisan policy is chosen does not converge to zero.

For a limit equilibrium $\lambda$, the ex ante probability that the partisan policy is implemented is given by

$$G(s^l(\lambda)) = G((1-\delta)F(\lambda))$$

Proposition 6(ii) implies that this probability is increasing in the parameter $\mu$. Hence, a candidate with a weaker partisan preference will lead to a higher probability of a partisan outcome. When $\mu$ is close to one, then the probability of a partisan outcome is maximal and approximately equal to

$$G(1-\delta)$$

To provide intuition for Proposition 7 consider a limit equilibrium $(\lambda, s)$. This equilibrium has to satisfy the equation

$$\lambda = \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + g(s^l(\lambda))(1-\mu)} \tag{$*$}$$

If $\mu$ is close to 1 then this implies that $\lambda$ must be close to one and hence part (i) follows. For part (ii), log-concavity of $g$ ensures that $g(s^m(\lambda))/g(s^l(\lambda)$ is a non-increasing function of $\lambda$. It is then straightforward to show that the $\lambda$ that solves (*) must be increasing in $\mu$. Hence, the vote share of $b$ among uninformed voters increases in $\mu$.

Next, we analyze the effect of a change in the distribution of swing voter preferences $F$ on the equilibrium outcomes. Let $F_\alpha, \alpha \in [0,1]$ denote a family of probability distributions over preference types $T := [0,1]$. Assume that $F_\alpha(\lambda)$ is increasing and differentiable in $\alpha$. Hence an decrease in $\alpha$ implies a first order stochastically dominant shift in the distribution of preferences. For any fixed $G, \delta$ that satisfy the assumptions above, let $\mathcal{E}_n(\alpha)$ denote the the election game with $2n+1$ voters and parameters $F_\alpha, G, \delta, \mu$.

Proposition 7 says that when swing voter preferences shift towards candidate $b$ then $b$ will win more frequently and choose the partisan policy more frequently.

**Proposition 7:** *Assume $g$ is log-concave and differentiable. Let $\lambda$ be the limit equilibrium for $\mathcal{E}_n(\alpha)$ and let $\lambda'$ be the limit equilibrium for $\mathcal{E}_n(\alpha')$ with $\alpha > \alpha'$. Then, $\lambda \geq \lambda'$ and $s \geq s'$.*

**Proof:** Since $g$ is log-concave there is a unique critical value $\lambda$ that solves the equation

$$\lambda(1 + R(\lambda, \alpha)(1 - \mu)) = 1$$

with

$$R(\lambda, \alpha) := \frac{g((1-\delta)F_\alpha(\lambda))}{g((1-\delta)F_\alpha(\lambda) + \delta)}$$

Since $g$ is differentiable and log-concave and $F_\alpha$ is increasing in $\lambda$ it follows that

$$\partial R/\partial\lambda \geq 0, \partial R/\partial\alpha \leq 0$$

Taking a total derivative therefore yields

$$\frac{d\lambda}{d\alpha} = \frac{-\lambda(1-\mu)\partial R/\partial\alpha}{1 + (1-\mu)(R + \lambda\partial R/\partial\lambda)} \geq 0$$

$\square$

Proposition 7 shows that a shift in the distribution of swing voters' preferences towards candidate $b$ has a similar effect as an increase in $\mu$. It (weakly) increases the probability that an uninformed voter votes for $b$. Therefore the probability of a partisan outcome

$$G((1-\delta)F_{\alpha}(\lambda))$$

is strictly increasing in $\alpha$ and similarly, the probability that $b$ is elected

$$G((1-\delta)F_{\alpha}(\lambda)+\delta)$$

is strictly increasing in $\alpha$.

Next, we consider the effect of $\delta$, the probability that a voter is informed, on the equilibrium outcomes. In general, an increase in $\delta$ has an ambiguous effect on the equilibrium strategies. Recall that $s^m(\lambda)-s^l(\lambda)=\delta$ and therefore in a large electorate the (ex ante) probability that the moderate policy is chosen is bounded above by

$$\max_{s\in[0,1-\delta]}(G(s+\delta)-G(s))$$

It follows that the moderate policy is implemented with probability close to zero if the fraction of informed voters is close to zero. Conversely, the probability that the moderate policy is chosen is bounded below by

$$\min_{s\in[0,1-\delta]}(G(s+\delta)-G(s))$$

For $\delta$ close to one this implies that the moderate policy is chosen with probability close to one.

For any fixed $F,G,\delta$ that satisfy the assumptions above, let $\mathcal{E}_n(\delta)$ denote the the election game with $2n+1$ voters and parameters $F,G,\delta,\mu$. Proposition 8 characterizes the limit equilibrium for small values of $\delta$.

**Proposition 8:** *For every $\epsilon>0$ there is $\delta$ such that if $\lambda$ is a limit equilibrium for $\mathcal{E}_n(\delta')$ with $\delta'<\delta$ then $\left|\lambda-\frac{1}{2-\mu}\right|<\epsilon$.*

**Proof:** Note that $g, s^m, s^l$ are continuous functions on $[0,1]$. Since $s^m(\lambda) - s^l(\lambda) = \delta$ we may choose $\delta$ small enough so that $|R(\lambda) - 1| = |g(s^l(\lambda))/g(s^m(\lambda)) - 1| < \epsilon$ for all $\lambda \in [0,1]$. Therefore

$$|\frac{1}{2-\mu} - \frac{1}{1 + R(\lambda)(1-\mu)}| < \epsilon$$

Proposition 4 then yields the result. $\square$

Proposition 8 says that for small values of $\delta$ the election with a large electorate yields the partisan policy $l$ with probability

$$G(s^l(\lambda)) = G\left(F\left(\frac{1}{2-\mu}\right)\right)$$

and the policy $r$ with probability close to

$$1 - G\left(F\left(\frac{1}{2-\mu}\right)\right)$$

Note that in *every* state $s \in (0,1)$ the median voter strictly prefers $r$ to $l$. Nevertheless, candidate $b$ wins the election with positive probability even though he almost never implements the moderate policy.

In the case where $G$ and $F$ are uniform, the limit equilibrium takes on a particularly simple form. In the uniform case,

$$\begin{aligned}\lambda &= \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + g(s^l(\lambda))(1-\mu)} \\ &= \frac{1}{2-\mu}\end{aligned}$$

and hence the limit equilibrium $\lambda$ depends only on the parameter $\mu$. In this case, the ex ante probability that the moderate policy is implemented is equal to $\delta$, the probability that a voter is informed. Moreover, the probability of a partisan outcome is given by

$$s^l(\lambda) = \frac{1-\delta}{2-\mu}$$

which is decreasing in $\delta$. The probability that $b$ is elected is given by

$$s^m(\lambda) = \frac{1-\delta}{2-\mu} + \delta$$

which is increasing in $\delta$. Finally, note that the equilibrium payoff of $b$ is

$$\frac{1-\delta}{2-\mu}+\delta\mu$$

A straightforward calculation shows that $b$'s payoff is decreasing in $\delta$. Note that candidate $a$'s probability of winning is also decreasing in $\delta$. This follows since the probability of winning for $b$ is increasing in $\delta$ and therefore candidate $a$'s probability of winning is decreasing in $\delta$. Hence, in the uniform case both candidates benefit from an ignorant electorate[1] Note, that the effect of a change in $\delta$ is computed at an ex ante stage, prior to the choice of policies. In section 5, we analyze a model where candidate $b$ can affect $\delta$ at the interim stage after the policy is chosen.

## 4. The Role of Asymmetric Information

To illustrate the role of asymmetric information between voters and candidates we briefly examine a version of the model with symmetric information. Consider a model identical to the model in section 3 but assume that candidates cannot observe the parameter $s$ that determines the distribution of preferences. For simplicity assume that $F$ and $G$ are uniform on $[0,1]$.

In this modified model, the strategy of the candidate cannot depend on $s$. Let $\bar{\sigma}_b \in [0,1]$ denote the probability that candidate $b$ chooses $m$. As before, we consider symmetric equilibria in weakly undominated strategies. It is straightforward to show that the equilibrium strategy of voters is a cutoff strategy $\lambda$. It is equally straightforward to show that there is a unique limit equilibrium $(\lambda, \bar{\sigma}_b)$ for this model with

$$\lambda = \bar{\sigma}_b = \begin{cases} 1 & \text{if } \mu \geq \frac{1-2\delta}{1-\delta} \\ \frac{\delta}{(1-\mu)(1-\delta)} & \text{if } \mu \leq \frac{1-2\delta}{1-\delta} \end{cases}$$

For $\mu \geq \frac{1-2\delta}{1-\delta}$ candidate $b$ chooses $m$ with probability 1 while for $\mu < \frac{1-2\delta}{1-\delta}$ candidate $b$ mixes between $m$ and $l$. In the latter case, the voter cutoff $\lambda$ is chosen so that $b$ is indifferent between $l$ and $m$. In the former case, all uninformed voters vote for $b$. As in the previous section, the voter cutoff is equal to the probability that $b$ chooses $m$ conditional

[1] This assumes that $a$ only cares about winning the election and not about the policy outcome in case $b$ is elected.

on a vote being pivotal. In contrast to the previous model, this probability converges to $\sigma_b$, the probability that $b$ chooses $m$.

As before, a voter is most likely to be pivotal when the state $s$ is in a small interval around $s^l(\lambda)$ or in a small interval around $s^m(\lambda)$. Since $G$ is uniform these intervals are equally likely and conditioning on being pivotal adds no information to the ex ante probability $\bar{\sigma}_b$. For the case of a non-uniform density, conditioning on being pivotal would add information because the density at $s^l(\lambda)$ may not be equal to the density at $s^m(\lambda)$. In contrast to the model with asymmetric information, candidate $b$ cannot condition his behavior on the state $s$ and, as a result, the interval around $s^l(\lambda)$ is not truncated at the point where the probability of winning drops below $\mu$.

In the model with symmetric information a higher $\mu$ yields more moderate policy outcomes. In particular, for $\mu$ high enough we get the Downsian prediction that $m$ is chosen with probability close to one. This is true for any continuous $F, G$ with densities $f, g > 0$. Moreover, for the case of a uniform $G$, the probability that $b$ chooses $m$ is equal to the probability that $b$ chooses $m$ conditional on a vote being pivotal. If $G$ has a log-concave density then both probabilities are increasing in $\mu$. Hence, office seekers choose the moderate policy with a high probability ex ante and conditional on a vote being pivotal.

By contrast, in the corresponding model with asymmetric information the (ex ante) probability that $b$ chooses $m$ is

$$1 - \frac{1-\delta}{2-\mu}$$

while the probability that $b$ chooses $m$ conditional on a vote being pivotal is

$$\frac{1}{2-\mu}$$

Hence, the ex ante probability that $b$ chooses $m$ is decreasing in $\mu$ while the probability that $b$ chooses $m$ conditional on a vote being pivotal increases in $\mu$. The asymmetric information between voters and candidates creates a wedge between the two probabilities that prevents median preferred outcomes even when candidates have weak partisan preferences.

Let $(\lambda, s)$ denote a limit equilibrium for the model analyzed in the previous section (for a general $F, G$). The strategy $s$ of candidate $b$ implies that for states $s' > s$ the candidate chooses $m$ and for states $s' < s$ the candidate chooses $l$. Hence, the ex ante probability that $b$ chooses $m$ is $1 - G(s)$. The corresponding probability that candidate $b$ chooses $m$ conditional on a vote being pivotal is equal to $\lambda$, the voter cutoff.

The following proposition shows that if $b$ has a weak partisan preference then $\lambda > 1 - G(s)$.

**Proposition 9:** *(i) There is $\bar{\mu} < 1$ such that if $\lambda$ is a limit equilibrium for $\mathcal{E}(\mu)$ for $\mu > \bar{\mu}$ then $\lambda > 1 - G(s^l(\lambda))$. (ii) If $g$ is log-concave and differentiable then there is $\bar{\mu} \in [0, 1)$ such that if $\lambda$ is a limit equilibrium for $\mathcal{E}(\mu)$ then $\lambda > G(s^l(\lambda)$ for $\mu > \bar{\mu}$ and $\lambda < G(s^l(\lambda)$ for $\mu < \bar{\mu}$.*

For the case of uniform $F, G$ we have

$$\lambda > 1 - G(s^l(\lambda))$$

if

$$\mu > \delta$$

Hence, if candidate $b$'s partisan preference is weak relative to the fraction of informed voters then $b$ receives a larger share of the uninformed vote than his ex ante strategy would suggest.

## 5. Control of Information

This section considers a situation where $b$ can control the information about policy choices. As in the previous section, we assume that $a$ has a fixed policy $r$ and $b$ chooses a policy $p \in \{l, m\}$. In addition, $b$ chooses the probability $\delta \in \{\underline{\delta}, \bar{\delta}\}$ with $0 < \underline{\delta} < \bar{\delta} < 1$ with which voters are informed of the policy choice. We assume that the choice of $\delta$ is not observed by voters.

One interpretation of this model is the following. Suppose $b$ runs two campaign commercials. One commercial is uninformative about the policy choice while the other commercial is informative. The candidate runs a certain fixed number of commercials but must choose what proportion of the commercials are informative. Voters sample one (or more) of the commercials at random. If a voters has the informative commercial in his sample he observes the policy choice. The voter remains uninformed if the informative commercial is not in the sample.

All swing voters strictly prefer $m$ to $r$. Since voters never use weakly dominated strategies this implies that $b$ will choose $\delta = \bar{\delta}$ whenever he chooses the moderate policy $m$. Note also that all swing voters prefer $r$ to $l$. Therefore choosing $\bar{\delta}$ when the policy $l$ is chosen cannot be optimal unless all uninformed swing voters vote for $a$. If all uninformed swing voters vote for $a$ then the choice of $\delta$ does not affect voting behavior when $a$ chooses $l$. Therefore, the analysis below suppresses the choice of $\delta$ and assumes that the probability that a voter is informed of the policy choice is $\underline{\delta}$ if $b$ chooses $l$ and $\bar{\delta}$ if $b$ chooses $m$.

It is straightforward to adapt the analysis of the previous section to this case. The critical states $s^l, s^m$ are defined as follows:

$$s^l(\lambda) = F(\lambda)(1 - \underline{\delta})$$
$$s^m(\lambda) = F(\lambda) + \bar{\delta}(1 - F(\lambda))$$

With this modified definition of critical states, Proposition 4 is unchanged. Proposition 5' below is the modified version of Proposition 3 for this case.

**Proposition 4′:** *Let $(\lambda_n, s_n)$ be a convergent sequence of equilibria and let $(\lambda, s)$ be the limit. Then,*

$$s = s^l(\lambda)$$
$$\lambda = \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + (1-\mu)g(s^l(\lambda))\frac{1-\underline{\delta}}{1-\bar{\delta}}}$$

Consider the case where $\mu$ is close to one. As before this implies that $b$ wins the election with probability close to one unless there is a partisan majority for candidate 2. In addition, in this case $b$ will choose the partisan policy with probability close to one if elected. Hence, the cutoff point $s$ that defines $b$'s strategy is close to one when $n$ is large.

**Proposition 10:** *For every $\epsilon > 0$ there is $\bar{\mu}$ such that if $(\lambda, s)$ is the limit of a sequence of equilibria for $\mathcal{E}_n(\mu)$ with $\mu > \bar{\mu}$ then $\lambda > 1 - \epsilon, s > 1 - \underline{\delta} - \epsilon$.*

Proposition 10 says that the equilibrium cutoff $s$ is close to $1 - \underline{\delta}$ for $\mu$ close to one. Hence, candidate $b$ chooses the partisan policy with probability

$$G(1 - \underline{\delta})$$

If $\underline{\delta}$ is close to zero then this implies that $b$ chooses the partisan policy with probability close to one and wins the election in nearly all states. Candidate $b$ chooses the moderate policy only when the state is close to 1 and approximately half the voters in a large electorate are partisans for $a$.

In this case, candidate $b$ is elected with probability close to one but chooses the moderate policy with probability close to zero. All swing voters vote for $b$ even though they strictly prefer policy $r$ to policy $l$.

Proposition 10 shows that an office seeker will choose to run an uninformative campaign (i.e., choose $\delta$ close to zero) for most realizations of the distribution of the electorate and choose the partisan policy. Of course, uninformed voters take this into account. The terms $\frac{1-\delta}{1-\overline{\delta}}$ reflects this effect. However, for $\mu$ close to one this term has little effect on $\lambda$ and hence has little effect on the probability that $b$ is elected.

## 6. Conclusion

We analyze how candidate competition is altered when not all voters are informed of the candidate's policy choice. We show that when a candidate is an office seeker with a weak partisan preference then the incomplete information of voters will induce him to choose more partisan policies. At the same time the candidate will be treated by uninformed voters as if he had chosen the moderate (median preferred) policy.

One consequence of this effect is that candidates have little incentive to spend resources to inform voters of their policy choices. For the uniform example in section 3 we show that candidates payoffs are decreasing in $\delta$ the probability that a voter is informed. For the general case this comparative static is ambiguous. However, for the case of an office seeker (with $\mu$ close to one) it is clear that there the opportunistic candidate has little to gain from an increase in $\delta$. As long as voters are convinced that a candidate will "do what it takes" to get elected, the chances of getting elected are not affected by the level of ignorance among voters. At the same time, a less well informed electorate allows the candidate to choose policies that closer match his policy preference.

## 7. Appendix

**Lemma 1:** *(i)* $B_n(x) = \frac{\int_0^x \theta^n(1-\theta)_n d\theta}{\int_0^1 \theta^n(1-\theta)^n d\theta}$; *(ii)* $B_n$ *is log-concave.*

**Proof:** (i) The binomial theorem implies that

$$\begin{aligned}\int_0^x \theta^n(1-\theta)^n d\theta &= \int_0^x (\theta-\theta^2)^n d\theta = \int_0^x \sum_{k=0}^n \binom{n}{k}(-1)^k\theta^{n+k} \\ &= \sum_{k=0}^n \binom{n}{k}\frac{x^{n+k+1}}{n+k+1}(-1)^k\end{aligned} \tag{A1}$$

Next, we show that

$$B_n(x) = \frac{(2n+1)!}{n!n!}\sum_{k=0}^n \binom{n}{k}\frac{x^{n+k+1}}{n+k+1}(-1)^k \tag{A2}$$

Observe that

$$\begin{aligned}B_n(x) &= \sum_{k=n+1}^{2n+1}\binom{2n+1}{k}x^k(1-x)^{2n+1-k} \\ &= \sum_{k=n+1}^{2n+1}\sum_{m=0}^{2n+1-k} x^{k+m}(-1)^m\binom{2n+1-k}{m}\binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1} x^t \sum_{k=n+1}^{t}\binom{2n+1-k}{t-k}(-1)^{t-k}\binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1}\frac{(2n+1)!\,x^t}{(2n+1-t)!}\sum_{m=0}^{t-(n+1)}\frac{(-1)^m}{m!(t-m)!} \\ &= \sum_{t=n+1}^{2n+1}\frac{(2n+1)!\,x^t}{(2n+1-t)!}\frac{(-1)^{t-(n+1)}}{t!}\frac{(t-1)!}{(t-(n+1))!n!} \\ &= \frac{(2n+1)!}{n!n!}\sum_{t=n+1}^{2n+1}\frac{(-1)^{t-(n+1)}n!\,x^t}{(2n+1-t)!(t-(n+1))!\,t} \\ &= \frac{(2n+1)}{n!n!}\sum_{k=0}^{n}\binom{n}{k}\frac{x^{n+k+1}}{n+k+1}(-1)^k\end{aligned}$$

The second equality follows from the binomial theorem. The fifth equality follows from the following identity (see Feller (1967) pg 65:)

$$\binom{a}{k} - \binom{a}{k-1} + - \ldots \binom{a}{0} = \binom{a-1}{k}$$

We conclude from $(A1)$ and $(A2)$ that

$$A \cdot \frac{\int_0^x \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = B_n(x)$$

for some constant $A > 0$. Clearly, $A = 1$ since

$$1 = B_n(1) = A \cdot \frac{\int_0^1 \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = A$$

which proves part (i)

(ii) We must show that

$$\frac{d}{dx}\left(\frac{B_n'(x)}{B_n(x)}\right) < 0 \qquad (A3)$$

Substituting the left hand side expression from part (i) and computing the derivative, straightforward computation shows that inequality (A3) is equivalent to

$$n(1-2x)\left(\int_0^x \theta^n (1-\theta)^n d\theta\right) - x^{n+1}(1-x)^{n+1} < 0 \qquad (A4)$$

For $x \geq 1/2$ the inequality is obviously correct. To see that it holds for $x < 1/2$ note that for $x < 1/2$ we have

$$(1-2x)\left(\int_0^x \theta^n (1-\theta)^n\right) \leq \left(\int_0^x \theta^n (1-\theta)^n (1-2\theta) d\theta\right) = \frac{x^{n+1}(1-x)^{n+1}}{n+1} \qquad (A5)$$

Substituting $(A5)$ into $(A4)$ proves part (ii). □

**Lemma 2:** *Let $\epsilon > 0$ and $1 \geq b \geq 1/2 + \epsilon > a \geq 0$. Define $\alpha := \max\{a, 1/2 - \epsilon\}, \beta := 1/2 + \epsilon$. Then,*

$$\frac{\int_\alpha^\beta x^n (1-x)^n dx}{\int_a^b x^n (1-x)^n dx} \to 1$$

**Proof:** Straightforward. □

### 7.1 Proof of Proposition 3

In the text, we have shown that in any equilibrium the voters must use a cutoff strategy. Next, we show that $b$'s best response to any cutoff strategy is also a cutoff strategy.

To see that a best response by $b$ to a cutoff strategy by voters is a cutoff strategy note that $b$ chooses $l$ if

$$\frac{B_n(\pi^m(\lambda, s))}{B_n(\pi^l(\lambda, s))} < \mu$$

and $m$ if this inequality is reversed. To show that this yields a cutoff strategy, it suffices to show that $\frac{B_n(\pi^m(\lambda,s))}{B_n(\pi^l(\lambda,s))}$ is increasing in $s$ or equivalently

$$\ln B_n(\pi^m(\lambda, s)) - \ln B_n(\pi^l(\lambda, s))$$

is increasing in $s$. Recall that $\pi^l(\lambda, s)$ is a decreasing linear function of $s$ and $\pi^m(\lambda, s) = \pi^l(\lambda, s) + \frac{\delta}{2}$. Lemma 1 shows that $\ln B_n$ is concave and therefore $\ln B_n(x + \delta) - \ln B_n(x)$ is decreasing in $x$. Hence, $\ln B_n(\pi^m(\lambda, s)) - \ln B_n(\pi^l(\lambda, s))$ is increasing in $s$.

Let $h : T \times S \to T$ be defined as

$$h(\lambda, s) := \left[1 + \frac{\int_0^s \pi^l(\lambda, s)^n (1 - \pi^l(\beta, s))^n g(s) ds}{\int_s^1 \pi^m(\lambda, s)^n (1 - \pi^m(\lambda, s))^n g(s) ds}\right]^{-1}$$

Note that $h$ is continuous. The cutoff $h(\lambda, s)$ describes the optimal cutoff of a voter if other voters are using the symmetric cutoff strategy $\lambda$ and $b$ is using the cutoff strategy $s$.

Let $k : T \to S$ be defined as follows:

$$k(\lambda) := \begin{cases} 1 & \text{if } \frac{B_n(\pi^l(\lambda,1)))}{B_n(\pi^m(\lambda,1))} > \mu \\ 0 & \text{if } \frac{B_n(\pi^l(\lambda,0)))}{B_n(\pi^m(\lambda,0))} < \mu \\ \{s | \frac{B_n(\pi^l(\lambda,s)))}{B_n(\pi^m(\lambda,s)))} = \mu\} & \text{otherwise.} \end{cases}$$

The cutoff strategy with cutoff $k(\lambda)$ is the best response of $b$ to the cutoff strategy $\lambda$ by voters. Note that $k$ is well defined since $\frac{B_n(\pi^l(\lambda,s)))}{B_n(\pi^m(\lambda,s)))}$ is decreasing and continuous in $s$. We conclude that $(h, k) : S \times T \to S \times T$ has a fixed point. This proves that there is an equilibrium in cutoff strategies. $\square$

### 7.2 Proof of Proposition 4

**Lemma 3:** *Let* $(\lambda_n, s_n)$ *be a convergent sequence of equilibria with limit* $(\lambda, s)$*. Then,* $s = s^l(\lambda)$.

**Proof:** Let $s' < s^l(\lambda)$. Then there is $\epsilon > 0$ such that $\pi^l(\lambda_n, s') \geq 1/2 + \epsilon$ for $n$ sufficiently large. This implies that $b$ wins the election with probability close to one if he chooses policy $l$. Since $\mu < 1$ this implies that $l$ must be the unique optimal choice and hence $s > s'$. We conclude that $s \geq s^l(\lambda)$.

If $s^l(\lambda) > s' > s^m(\lambda)$ then there is $\epsilon > 0$ such that $\pi^m(\lambda_n, s') > 1/2+\epsilon$ and $\pi^l(\lambda_n, s') < 1/2 - \epsilon$. This implies that $b$ wins with probability close to one if he chooses $m$ but loses with probability close to one if he chooses $l$. Since $0 < \mu$ it follows that the unique optimal choice is $m$. It follows that $s \leq s^l(\lambda)$. This shows that $s = s^l(\lambda)$. □

**Lemma 4:** *Let* $(\lambda_n, s_n)$ *be a convergent sequence of equilibria with limit* $(\lambda, s)$*. Then,* $0 < s_n < 1$ *for large* $n$ *and* $B_n(\pi^l(\lambda_n, s_n)) \to \mu$*.*

**Proof:** (i) We first show that there is $\bar{n}$ such that $0 < s_n < 1$ for $n > \bar{n}$. Note that if $s_n = 1$ then $\lambda_n = 0$ (conditional on a vote being pivotal $b$ chooses $l$ with probability 1). Further note that $s^l(0) = 1$. Therefore, by Lemma 3 there can be no subsequence of equilibria with $s_n = 1$. If $s_n = 0$ then $\lambda = 1$. Note that $s^l(1) = 1 - \delta > 0$. Therefore, by Lemma 3 there can be no subsequence of equilibria with $s_n = 0$.

(ii) For $0 < s_n < 1$ we must have

$$\mu B_n(\pi^m(\lambda_n, s_n)) = B_n(\pi^l(\lambda_n, s_n))$$

This follows since $\pi^p(\lambda, s)$ is continuous in $s$ for all $n$. Further observe that $\pi^m = \pi^l + \delta/2$ with $\pi^l(\lambda, s) = 1/2$. Therefore, it follows that for large $n$, $\pi^m(\lambda_n, s_n) > 1/2 + \epsilon$ for some $\epsilon > 0$ and hence $B_n(\pi^m(\lambda_n, s_n)) \to 1$. This yields the Lemma. □

Observe that $\pi^p \in (0, 1)$ for all $\lambda, s$ and hence there is a positive probability that a vote is pivotal. Let $h_n : T \times S \to T$ be defined as

$$h_n(\lambda, s) := \left[1 + \frac{\int_0^s \pi^l(\lambda, s)^n (1 - \pi^l(\lambda, s))^n g(s) ds}{\int_s^1 \pi^m(\lambda, s)^n (1 - \pi^m(\lambda, s))^n g(s) ds}\right]^{-1}$$

Note that $h_n$ is the probability that a vote is pivotal as a function of the cutoff strategies $(\lambda, s)$.

**Lemma 4:** *Let $(\lambda_n, s_n)$ be a sequence of equilibria converging to $(\lambda, s)$. Then, $0 < \lambda < 1$.*

**Proof:** First, we show that $\lambda_n = h_n(\lambda_n, s_n) \leq 1 - \epsilon$ for some $\epsilon > 0$ and $n$ sufficiently large. If $\lambda \leq 1/2$ for large $n$ we are done. Hence, assume that $\lambda > 1/2$ and hence $\lambda_n > 1/2$ for large $n$. Since $g > 0$, there is a constant $c > 0$ such that

$$\begin{aligned}
\frac{1 - h_n(\lambda_n, s_n)}{h_n(\lambda_n, s_n)} &\geq c \cdot \frac{\int_0^{s_n} \pi^l(\lambda_n, s)^n (1 - \pi^l(\lambda_n, s))^n ds}{\int_{s_n}^1 \pi^m(\lambda_n, s)^n (1 - \pi_m(\lambda_n, s))^n ds} \\
&\geq c \cdot \frac{\int_{\pi^l(\lambda_n, 0)}^{\pi^l(\lambda_n, s_n)} x^n (1 - x)^n ds}{\int_0^1 x^n (1 - x)^n ds}
\end{aligned}$$

The last inequality is derived by a change of variables using the fact that $\pi^m = \pi^l + \delta/2$ and $\pi^l$ is a linear increasing function of $s$.

Optimal behavior requires that $\pi^l(\lambda_n, s^n) \leq b_n$ where $B_n(b_n) = \mu$. By the strong law of large numbers, $b^n \to 1/2$. Let $a_n := \pi^l(\lambda_n, 0) = 1/2 + F(\lambda_n)(1 - \delta)/2$ and note that $a_n > 1/2$ for $n$ large since $\lambda_n \geq 1/2$. We conclude that

$$\begin{aligned}
\frac{\int_{\pi^p(\lambda_n, s^n)}^{\pi^p(\lambda_n, 1)} x^n (1 - x)^n ds}{\int_0^1 x^n (1 - x)^n ds} &= \frac{\int_{b_n}^{a_n} x^n (1 - x)^n dx}{\int_0^1 x^n (1 - x)^n dx} \\
&\geq \frac{1}{2} \frac{\int_{b^n}^1 x^n (1 - x)^n dx}{\int_0^1 x^n (1 - x)^n dx} \\
&= (1 - \mu)/2
\end{aligned}$$

The first inequality follows from Lemma 2 and the last equality follows from Lemma 1. Since $(1 - \mu)/2 > 0$ we conclude that $1 - h(\lambda_n, s_n)$ is bounded away from 0 and hence $h(\lambda_n, s_n)$ is bounded away from 1 for all $n$. This shows that $\lambda < 1 - \epsilon$ for some $\epsilon > 0$.

Next we show that $\lambda > \epsilon$ for some $\epsilon > 0$. Let $a'_n := \pi^m(\lambda_n, 1)$ and note that $a'_n = 1/2 - (1 - \delta)F(1 - \lambda_n)/2$. Since $\lambda_n$ stays bounded away from 1 we conclude that there is $\epsilon > 0$ such that $a'_n < 1/2 - \epsilon$ for large $n$. Let $b'_n := \pi^m(\lambda_n, s_n)$ and note that $b'_n = \pi^l(\lambda_n, s_n) + \delta/2$. Since $\pi^l(\lambda_n, s_n) \to 1/2$ we conclude that there is $\epsilon > 0$ such that $b'_n \geq 1/2 + \epsilon$ for some $\epsilon > 0$.

By an analogous argument to the one above, we conclude that there is a constant $c' > 0$ such that for large $n$

$$\frac{1-h_n(\lambda_n,s_n)}{h_n(\lambda_n,s_n)} \leq c' \cdot \frac{\int_0^1 x^n(1-x)^n ds}{\int_{a'_n}^{b'_n} x^n(1-x)^n ds} \geq c'/2$$

where the last inequality follows from Lemma 2. This shows that $h(\lambda_n, s_n)$ stays bounded away from zero for all $n$ and therefore $\lambda > 0$. $\square$

**Lemma 5:** *Let $(\lambda_n, s_n)$ be a sequence of equilibria and converging to $(\lambda, s)$. Then,*

$$\lambda = \frac{g(s^m(\lambda))}{g(s^m(\lambda)) + g(s^l(\lambda))(1-\mu)}$$

**Proof:** Note that

$$\alpha_n := \frac{1-\lambda_n}{\lambda_n} = \frac{\int_{s_n}^1 \pi^l(\lambda_n,s)^n(1-\pi^l(\lambda_n,s))^n g(s)}{\int_0^{s_n} \pi^m(\lambda_n,s)^n(1-\pi^m(\lambda_n,s))^n g(s)}$$

To prove the Lemma, we must show that

$$\alpha_n \to (1-\mu)\frac{g(s^l(\lambda))}{g(s^m(\lambda))}$$

Let $a_n := \pi^l(\lambda_n, s_n), b_n := \pi^m(\lambda_n, s_n), \bar{a}_n := \pi_l(\beta_n, 0), \underline{b}_n := \pi_m(\lambda_n, 1)$. By Lemma 4 and the fact that $\delta > 0$ there is $\epsilon > 0$ such that $\bar{a}_n \geq 1/2 + \epsilon$ and $\underline{b}_n \leq 1/2 - \epsilon$ and $b_n \geq 1/2 + \epsilon$ for large $n$. Moreover, $a_n \to 1/2$ by Proposition 3.

A change of variables yields

$$\alpha_n = \frac{\int_{\bar{a}_n}^{a_n} x^n(1-x)^n \hat{g}_n^l(x)dx}{\int_{\underline{b}_n}^{b_n} (1-x)^n x^n \hat{g}_n^m(x)dx}$$

where $\hat{g}_n^p(x) = g(s : \pi^p(\lambda_n, s) = x)$.

**Step 1:**

$$\frac{\int_{a_n}^{\bar{a}_n} x^n(1-x)^n \hat{g}_n^l(x)dx}{g(s_n)\int_{a_n}^1 x^n(1-x)^n} \to 1$$

**Proof:** Since $a_n \to 1/2$ and $\bar{a}_n > 1/2 + \epsilon'$ for some $\epsilon' > 0$ we can apply Lemma 2 to conclude that for every $0 < \epsilon \leq \epsilon'$

$$\frac{\int_{a_n}^{\bar{a}_n} x^n(1-x)^n \hat{g}_n^l(x)dx}{\int_{a_n}^{a_n+\epsilon} x^n(1-x)^n \hat{g}_n^l(x)dx} \to 1$$

Since $g$ is continuous it follows that $\hat{g}_n$ is equicontinuous. Moreover, $\hat{g}_n(a_n) = g(s_n) \to g(s)$. This implies that

$$\frac{\int_{a_n}^{\bar{a}_n} x^n(1-x)^n \hat{g}_n^l(x)dx}{g(s)\int_{a_n}^{a_n+\epsilon} x^n(1-x)^n dx} \to 1$$

which (again by Lemma 2) implies that

$$\frac{\int_{a_n}^{\bar{a}_n} x^n(1-x)^n \hat{g}_n^l(x)dx}{g(s)\int_0^1 x^n(1-x)^n dx} \to 1$$

proving Step 1.

**Step 2:**

$$\frac{\int_{\underline{b}_n}^{b_n} x^n(1-x)^n \hat{g}_n^m(x)dx}{g(s^m(\lambda))\int_0^1 x^n(1-x)^n dx} \to 1$$

**Proof:** Note that $\underline{b}_n \leq 1/2 - \epsilon, b_n \geq 1/2 + \epsilon$ for some $\epsilon > 0$. Therefore, we can apply Lemma 1 to yield

$$\frac{\int_{\underline{b}_n}^{b_n} x^n(1-x)^n \hat{g}_n^m(x)dx}{\int_{1/2-\epsilon}^{1/2+\epsilon} x^n(1-x)^n \hat{g}_n^m(x)dx} \to 1$$

Since $\hat{g}_n$ is equicontinuous and $\hat{g}_n^m(1/2) \to g(s^m(\lambda))$ it follows that

$$\frac{\int_{\underline{b}_n}^{b_n} x^n(1-x)^n \hat{g}_n^m(x)dx}{g(s^m(\lambda))\int_{1/2-\epsilon}^{1/2+\epsilon} x^n(1-x)^n dx} \to 1$$

Applying Lemma 1 again yields step 2.

Steps 1 and 2 imply that

$$\alpha_n \to \frac{g(s)\int_{a_n}^1 x^n(1-x)^n dx}{g(s^m(\lambda))\int_0^1 (1-x)^n x^n dx}$$

By Lemma 1,

$$\frac{\int_{a_n}^1 x^n(1-x)^n dx}{\int_0^1 (1-x)^n x^n dx} = 1 - B_n(a_n)$$

By Lemma 3, $B_n(\lambda_n) \to \mu$. Therefore, we conclude that

$$\alpha_n \to (1-\mu)\frac{g(s)}{g(s^m(\lambda))}$$

which completes the proof of the lemma. $\square$

## References

1. Austen-Smith D. and J. Banks, (1995) "Information Aggregation, Rationality and the Condorcet Jury Theorem", American Political Science Review, 90, 34-45.

2. Berelson, B. R., P. Lazarsfeld, and W. N. McPhee. 1954. *Voting: A Study of Opinion Formation in a Presidential Campaign.* Chicago: University of Chicago Press.

3. Bernhardt, Duggan and Squintani (2003), "Electoral Competition with Privately Informed Candidates", mimeo, University of Rochester.

4. Calvert, R. (1985), "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." American Journal of Political Science 29 (February 1985), pp. 69-95.

5. Chan, J. (2001), "Electoral Competition with Private Information", mimeo, Johns Hopkins University.

6. Delli Carpini, M. X., and S. Keeter (1993) "Measuring Political Knowledge: Putting First Things First," *American Journal of Political Science* 37:1179-1206.

7. Downs, A., *An Economic Theory of Democracy*, New York, Harper, 1957.

8. Feddersen, T and W. Pesendorfer, (1996) "The Swing Voter's Curse", American Economic Review 86, 381-398.

9. Young, P., (1988) Condorcet's Theory of Voting, American Political Science Review, 82, 1231-1244.