Optimal Automatic Stabilizers[*]

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October 2017

Abstract

Should the generosity of unemployment benefits and the progressivity of income taxes depend on the presence of business cycles? This paper proposes a tractable model where there is a role for social insurance against uninsurable shocks to income and unemployment, as well as inefficient business cycles driven by aggregate shocks through matching frictions and nominal rigidities. We derive an augmented Baily-Chetty formula showing that the optimal generosity and progressivity depend on a macroeconomic stabilization term. This term pushes for an increase in generosity and progressivity when the level of economic activity is more responsive to social programs in recessions than in booms. A calibration to the U.S. economy shows that taking concerns for macroeconomic stabilization into account raises the optimal unemployment insurance replacement rate by 20 percentage points but has a negligible impact on the optimal progressivity of the income tax. More generally, the role of social insurance programs as automatic stabilizers affects their optimal design.

JEL codes: E62, H21, H30.

Keywords: Counter-cyclical fiscal policy; Redistribution; Distortionary taxes.

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1 Introduction

The usual motivation behind large social welfare programs, like unemployment insurance or progressive income taxation, is to provide social insurance and engage in redistribution. An extensive literature therefore studies the optimal progressivity of income taxes typically by weighing the disincentive effect on individual labor supply and savings against concerns for redistribution and for insurance against idiosyncratic income shocks.[1] In turn, the optimal generosity of unemployment benefits is often stated in terms of a Baily-Chetty formula, which weighs the moral hazard effect of unemployment insurance on job search and creation against the social insurance benefits that it provides.[2]

For the most part, this literature abstracts from aggregate shocks, so that the optimal generosity and progressivity do not take into account business cycles. Yet, from their inception, an auxiliary justification for these social programs was that they were also supposed to automatically stabilize the business cycle.[3] The classic work that focussed on the automatic stabilizers relied on a Keynesian tradition that ignored the social insurance that these programs provide or their disincentive effects on employment. Some recent work brings these two orthogonal literatures together, but so far it has focused on the positive effects of the automatic stabilizers, falling short of computing optimal policies.[4]

The goal of this paper is to answer two classic questions—How generous should unemployment benefits be? How progressive should income taxes be?—but taking into account their automatic stabilizer benefits as well as their social insurance benefits. We present a model in which there is both a role for social insurance as well as aggregate shocks and inefficient business cycles. We introduce unemployment insurance and progressive income taxes as automatic stabilizers, that is, programs that do not directly depend on the aggregate state of the economy even if the aggregate size of the programs changes with the composition of income in the economy. We then solve for the ex ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market

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[1]Mirrlees (1971) and Varian (1980) are classic references, and more recently see Benabou (2002), Conesa and Krueger (2006), Heathcote et al. (2014), Krueger and Ludwig (2013), and Golosov et al. (2016).

[2]See the classic work by Baily (1978) and Chetty (2006).

[3]Musgrave and Miller (1948) and Auerbach and Feenberg (2000) are classic references, while Blanchard et al. (2010) is a recent call for more modern work in this topic.

[4]See McKay and Reis (2016) for a recent model, DiMaggio and Kermani (2016) for recent empirical work, and IMF (2015) for the shortcomings of the older literature.

frictions, and nominal rigidities.

Our first main contribution is to provide a formal, theory-grounded definition of an automatic stabilizer. We show that a business-cycle variant of the Baily-Chetty formula for unemployment insurance and a similar formula for the optimal choice of progressivity of the tax system are both augmented by a new macroeconomic stabilization term. This term equals the expectation of the product of the welfare gain from eliminating economic slack with the elasticity of slack with respect to the replacement rate or tax progressivity. Even if the economy is efficient on average, economic fluctuations may lead to more generous unemployment insurance or more progressive income taxes, relative to standard analyses that ignore the automatic stabilizer properties of these programs. This term captures the automatic stabilizer nature of social insurance programs.

The second contribution is to characterize this macroeconomic stabilization term analytically to understand the different economic mechanisms behind it. Fluctuations in aggregate economic slack, measured by the unemployment rate, the output gap or the job finding rate, can lead to welfare losses through four separate channels. First, they may create a wedge between the marginal disutility of hours worked and the social benefit of work. This inefficiency appears in standard models of inefficient business cycles, and is sometimes described as a result of time-varying markups (Chari et al., 2007; Galí et al., 2007). Second, when labor markets are tight, more workers are employed raising production but the cost of recruiting and hiring workers rises. The equilibrium level of unemployment need not be efficient as hiring and search decisions do not necessarily internalize these tradeoffs. This is the source of inefficiency common to search models (e.g. Hosios, 1990). Third, the state of the business cycle alters the extent of uninsurable risk that households face both in unemployment and income risk. This is the source of welfare costs of business cycles that has been studied by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). Finally, with nominal rigidities, slack affects inflation and the dispersion of relative prices, as emphasized by the new Keynesian business cycle literature (Woodford, 2010; Gali, 2011). Our measure isolates these four effects cleanly in terms of separate additive terms in the condition determining the optimal extent of the social insurance programs.

As for the elasticity of slack with respect to social programs, unemployment benefits and progressive taxes can stabilize the economy even if these policies are themselves not responsive to the business cycle. For one, these policies redistribute across groups who may have different marginal

propensities to consume. In the case of unemployment insurance, the magnitude of this redistribution increases when more people become unemployed in a recession. Moreover, these policies mitigate precautionary savings motives by providing social insurance. Because the risk in pre-tax incomes rises in a recession, the effect of this social insurance on aggregate demand rises as well, so these policies stabilize aggregate demand. We further show that if prices are flexible so aggregate demand matters little, or if monetary policy aggressively stabilizes the business cycle, then little role is left for the social programs to work as stabilizers.

Our third contribution is to investigate the magnitude of the macroeconomic stabilization term and the key mechanisms behind it. We calculate the optimal unemployment replacement rate and tax progressivity and compare these values to what one would find in the absence of aggregate risk. We find a large effect on unemployment insurance: with business cycles, the optimal unemployment replacement rate rises from 41 to 61 percent. However, the level of tax progressivity has little stabilizing effect on the business cycle so the presence of aggregate shocks has almost no effect on the optimal degree of progressivity.

Our analytical results allow us to interpret these numerical results, by allowing us to quantify the tradeoffs between incentives, social insurance and macroeconomic stabilization and the constituent mechanisms of the macroeconomic stabilization term. This highlights the usefulness of the propositions, which are expressed in general terms and involving aggregate endogenous variables, so that they can be used in different models and circumstances while isolating the key forces at hand. Quantitatively, the automatic stabilizer term is large in the case of unemployment benefits, and largely driven by the effect of the business cycle on the extent of uninsurable idiosyncratic risk.

Finally, we use a numerical analysis of the model as a laboratory to explore the roles of some assumptions that we made for analytical tractability. Namely, our baseline results rely on a degenerate wealth distribution for tractability, but we show that allowing for heterogeneity in wealth does not much alter the relative strengths of the tradeoffs so our conclusion about the importance of the macroeconomic stabilization term for the optimal policy is relatively robust.

There are large literatures on the three topics that we touch on: business cycle models with incomplete markets and nominal rigidities, social insurance and public programs, and automatic stabilizers. Our model of aggregate demand has some of the key features of new Keynesian models with labor markets (Gali, 2011) but that literature focuses on optimal monetary policy, whereas we

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study the optimal design of the social insurance system. Our model of incomplete markets builds on McKay and Reis (2016), Ravn and Sterk (2017), and Heathcote et al. (2014) to generate a tractable model of incomplete markets and automatic stabilizers. This simplicity allows us to analytically express optimality conditions for generosity and progressivity, and to, even in a more general case, easily solve the model numerically and so be able to search for the optimal policies. Finally, our paper is part of a surge of work on the interplay of nominal rigidities and precautionary savings, but this literature has mostly been positive whereas this paper's focus is on optimal policy.[5]

On the generosity of unemployment insurance, our work is closest to Landais et al. (2017) and Kekre (2017). They also couch their analysis in terms of the standard Baily-Chetty formula by considering the general equilibrium effects of unemployment insurance. The main difference is that they study benefits as discretionary policy instruments that vary over the business cycle, while we study how the presence of business cycles affects the ex ante fixed level of benefits.[6] Our focus is on automatic stabilizers, an ex ante passive policy, while they consider active stabilization policy, which are quite different questions as the long literature on rules versus discretion in macroeconomics attests.[7] Moreover, our model includes aggregate uncertainty, and we also study income tax progressivity.

On income taxes, our work is closest to Benabou (2002) and Bhandari et al. (2017). Our dynamic heterogeneous-agent model with progressive income taxes is similar to the one in Benabou (2002), but our focus is on business cycles, so we complement it with aggregate shocks and nominal rigidities. Bhandari et al. (2017) is one of the very few studies of optimal income taxes with aggregate shocks. Like us, those authors emphasize the interaction between business cycles and the desire for redistribution.[8] However, they solve for the Ramsey optimal fiscal policy, which adjusts the tax instruments every period in response to shocks, while we choose the ex ante optimal rules for generosity and progressivity. This is consistent with our focus on automatic stabilizers, which are ex ante fiscal systems, rather than counter-cyclical policies.

Finally, this paper is related to the modern study of automatic stabilizers and especially our

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[5]See Oh and Reis (2012); Guerrieri and Lorenzoni (2011); Auclert (2016); McKay et al. (2016); Kaplan et al. (2016); Werning (2015).

[6]See also Mitman and Rabinovich (2011), Jung and Kuester (2015), and Den Haan et al. (2015).

[7]Unlike the United States, where the duration of unemployment benefits is often increased during recessions, in most OECD countries the terms of unemployment insurance programs do not change over the business cycle, as described in http://www.oecd.org/els/soc/.

[8]Werning (2007) also studies optimal income taxes with aggregate shocks and social insurance.

earlier work in McKay and Reis (2016). There, we considered the positive question of how the actual automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one.

The paper is structured as follows. Section 2 presents the model, and section 3 discusses its equilibrium properties. Section 4 derives the macroeconomic stabilization term in the optimality conditions for the two social programs. Section 5 discusses its qualitative properties, the economic mechanisms that it depends on, and its likely sign. Section 6 calibrates the model, quantifies the macro stabilization term and its effects on the optimal automatic stabilizers. Section 7 concludes.

2 The Model

The main ingredients in the model are: uninsurable income and employment risks, social insurance programs, and nominal rigidities so that aggregate demand matters for equilibrium allocations. In the course of presenting the model, we highlight several assumptions that are useful to achieve analytical tractability for now, and which we will discuss and relax later in section 6. Time is discrete and indexed by t.

2.1 Agents and commodities

There are two groups of private agents in the economy: households and firms.

Households are indexed by i in the unit interval, and their type is given by their productivity $\alpha_{i,t} \in \mathbb{R}_0^+$ and employment status $n_{i,t} \in \{0, 1\}$. Every period, an independently drawn share δ dies, and is replaced by newborn households with no assets and productivity normalized to $\alpha_{i,t} = 1$. Households derive utility from consumption, $c_{i,t}$, and publicly provided goods, G_t, and derive disutility from working for pay, $h_{i,t}$, searching for work, $q_{i,t}$, and being unemployed according to the utility function:

$$\mathbb{E}_0 \sum_t \beta^t \left[\log(c_{i,t}) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi (1 - n_{i,t}) \right]. \tag{1}$$

The parameter β captures the joint discounting effect from time preference and mortality risk, while ξ is a non-pecuniary cost of being unemployed.[9]

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[9]If $\hat{\beta}$ is pure time discounting, then $\beta \equiv \hat{\beta}(1 - \delta)$.

The final consumption good is provided by a competitive final goods sector in the amount Y_t that sells for price p_t. It is produced by combining varieties of goods in a Dixit-Stiglitz aggregator with elasticity of substitution $\mu/(\mu - 1)$. Each variety $j \in [0, 1]$ is monopolistically provided by a firm with output $y_{j,t}$ by hiring labor from the households and paying the wage w_t per unit of effective labor.

The mortality risk allows for a stationary cross-sectional distribution of productivity along with permanent shocks. In the analytical sections of the paper, we assume it away, as it plays no significant role in the economics of the analysis.

Assumption 1. *Agents live forever:* $\delta = 0$.

2.2 Asset markets and social programs

Households can insure against mortality risk by buying an annuity, but they cannot insure against risks to their individual skill or employment status. The simplest way to capture this market incompleteness is by assuming that households can only hold a single risk-free annuity, $a_{i,t}$, that has a gross real return R_t.[10]

The net supply of inside assets is zero, while there is a stock of government bonds B_t. Following Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015), the analytical sections use a further strong assumption that will make the distribution of wealth tractable:

Assumption 2. *Households cannot borrow,* $a_{i,t} \geq 0$, *and* $B_t = 0$ *so bonds are in zero supply.*

The government provides two social insurance programs. The first is a progressive income tax such that if $z_{i,t}$ is pre-tax income, the after-tax income is $\lambda_t z_{i,t}^{1-\tau}$. The overall level of taxes determined by $1 - \lambda_t \in [0, 1]$, together with the size of government purchases G_t, pin down the size of the government. The object of our study is instead the automatic stabilizer role of the government, so our focus is on $\tau \in [0, 1]$. This determines the progressivity of the tax system. If $\tau = 0$, there is a flat tax at rate $1 - \lambda_t$, while if $\tau = 1$ everyone ends up with the same after-tax income. In between, a higher τ implies a more convex tax function, or a more progressive income tax system.

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[10]A standard formulation for asset markets that gives rise to these annuities is the following: A financial intermediary sells claims that pay one unit if the household survives and zero units if the household dies, and supports these claims by trading a risk-less bond with return $\tilde{R}$. If a_i are the annuity holdings of household i, the law of large numbers implies the intermediary pays out in total $(1 - \delta) \int a_i di$, which is known in advance, and the cost of the bond position to support it is $(1 - \delta) \int a_i di / \tilde{R}$. Because the risk-less bond is in zero net supply, then the net supply of annuities is zero $\int a_i di = 0$, and for the intermediary to make zero profits, $R_t = \tilde{R}_t / (1 - \delta)$.

The second social program is unemployment insurance. A household qualifies as long as it is unemployed ($n_{i,t} = 0$) and collects benefits that are paid in proportion to what the unemployed worker would earn if she were employed. Suppose the worker's productivity is such that she would earn pre-tax income $z_{i,t}$ if she were employed, then her after-tax unemployment benefit is $b\lambda_t z_{i,t}^{1-\tau}$.[11] Our focus is on the replacement rate $b \in [0, 1]$, with a more generous program understood as having a higher b.[12]

Our goal is to characterize the optimal fixed levels of b and τ set ex ante as automatic stabilizers. These are programs that can automatically stabilize the business cycle without policy intervention, so b and τ do not depend on time or on the state of the business cycle. In this design problem, we are following the tradition in the literature on automatic stabilizers that makes a sharp distinction between built-in properties of programs as opposed to feedback rules or discretionary choices that adjust these programs in response to current or past information.[13]

2.3 Key frictions

There are three key frictions in the economy that create the policy trade-offs that we analyze.

2.3.1 Individual productivity risk

Labor income for an employed household is $\alpha_{it}w_t h_{it}$, where $\alpha_{i,t}$ is idiosyncratic productivity or skill and w_t is the wage per effective unit of labor. The productivity of households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t}\epsilon_{i,t+1} \text{ with } \epsilon_{i,t+1} \sim F(\epsilon; x_t), \tag{2}$$

and where $\int \epsilon dF(\epsilon, x_t) = 1$ for all t, which implies that the average idiosyncratic productivity in the population is constant and equal to one.[14]

The distribution of shocks varies over time so that the model generates cyclical changes in the

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[11]It would be more realistic, but less tractable, to assume that benefits are a proportion of the income the agent earned when she lost her job. But, given the persistence in earnings, both in the data and in our model, our formulation will not be quantitatively too different from this case. Also, in our notation, it may appear that unemployment benefits are not subject to the income tax, but this is just the result of a normalization: if they were taxed and the replacement rate was $\tilde{b}$, then the model would be unchanged and $b \equiv \tilde{b}^{1-\tau}$.

[12]In our model, focusing on the duration of unemployment benefits instead of the replacement rate would lead to similar trade offs, so we refer to b more generally as the generosity of the program.

[13]Perotti (2005) among many others.

[14]Since newborn households have productivity 1, the assumption is that they have average productivity.

distribution of earnings risks, as documented by Storesletten et al. (2004) or Guvenen et al. (2014). We capture this dependence through the aggregate slack in the economy, x_t. A higher x_t implies that the economy is tighter, or that the economy is closer to capacity or booming. Many variables could measure x_t, from the unemployment rate to the output gap. We assume that, if M_t is the job-finding rate per unit of search effort, then:

Assumption 3. *The state of the business cycle is measured by the tightness of the labor market:* $x_t = M_t$.

2.3.2 Employment risk

The second source of risk is employment. We make a strong assumption that unemployment is distributed i.i.d. across households. Given the high (quarterly) job-finding rates in the US, this is not such a poor approximation, and it reduces the state space of the model. At the start of the period, a fraction v of households loses employment and must search to regain employment. Search effort $q_{i,t}$ leads to employment with probability $M_t q_{i,t}$ since the probability of resulting in a match is the same for each unit of search effort. Therefore, if all households make the same search effort, then aggregate hiring will be $v M_t q_t$ and as a result the unemployment rate will be:

$$u_t = v(1 - q_t M_t). \tag{3}$$

Each firm begins the period with a mass $1 - v$ of workers and must post vacancies at a cost to hire additional workers. As in Blanchard and Galí (2010), the cost per hire is increasing in aggregate labor market tightness, which is just equal to the ratio of hires to searchers, or the job-finding rate M_t. The hiring cost per hire is $\psi_1 M_t^{\psi_2}$, denominated in units of final goods where ψ_1 and ψ_2 are parameters that govern the level and elasticity of the hiring costs. Since aggregate hires are the difference between the beginning of period non-employment rate v and the realized unemployment rate u_t, aggregate hiring costs are:

$$J_t \equiv \psi_1 M_t^{\psi_2}(v - u_t). \tag{4}$$

We assume a law of large numbers within the firm so the average productivity of hires is 1.

In this model of the labor market, there is a surplus in the employment relationship since, on

one side, firms would have to pay hiring costs to replace the worker and, on the other side, a worker who rejects a job must continue searching for a job thereby foregoing wages. This surplus creates a bargaining set for wages, and there are many alternative models of how wages are chosen within this set, from Nash bargaining to wage stickiness, as emphasized by Hall (2005). We assume a convenient wage rule for the analytical results:

Assumption 4. *Wages are set according to the rule:*

$$w_t = \bar{w} A_t (1 - J_t/Y_t) x_t^{\zeta}. \tag{5}$$

The assumption is that the real wage per effective unit of labor depends on three variables, aside from a constant $\bar{w}$. First, it increases proportionately with aggregate effective productivity A_t, as it would in a frictionless model of the labor market. Second, it falls when aggregate hiring costs are higher, so that some of these costs are passed from firms to workers. The justification is that when hiring costs rise, the economy is poorer and this raises labor supply, which the fall in wages exactly offsets. Since these costs are quantitatively small, in reality and in our calibrations, this assumption has little effect in the predictions of the model but allows us to not have to carry this uninteresting wealth effect on labor supply throughout the analysis.[15] Third, when the labor market is tighter, wages rise, with an elasticity of ζ. Nash bargaining models typically lead to a positive dependence between economic activity and wages, while sticky wage models can be approximated by $\zeta = 0$.

Qualitatively, the wage rule does not play a large role in our analysis, but it is useful in simplifying the choice of labor supply on the intensive margin. If labor supply were fixed on the intensive margin, as in most search models of the labor market, then we would not need this assumption. Still, to justify it, Appendix A provides a Nash bargaining protocol that gives rise to a wage rule of this form.[16] Moreover, Appendix A writes a more general wage rule that nests many alternatives, and shows that it would lead to similar, but somewhat more complicated, results. Finally, a notable feature of this wage rule is that the policy parameters do not directly affect wages, although they indirectly affect them through x_t for example. In section 6.5, we allow policy to directly affect wages and show that this lowers the level of benefits, but magnifies their role as automatic stabilizers.

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[15] Moreover, in the special cases of the model studied in section 5, J_t/Y_t is a function of x_t so this term gets absorbed by the next term after a redefinition of ζ.

[16] An implication of Appendix A is that the wage given by (5) always lies within the bargaining set implied by this bargaining protocol.

2.3.3 Nominal rigidities

The firm that produces each variety uses the production function $y_{j,t} = \eta_t^A l_{j,t}$, where $l_{j,t}$ is the effective units of labor hired by the firm and η_t^A is an exogenous productivity shock. Given the structure of the labor market, employed workers set their hours taking the hourly wage as given. We show below they all make the same choice h_t. The firm then chooses how many workers to hire. Marginal cost is then the cost of increasing the number of workers to produce one more unit of output. The firm's marginal cost is:

$$\frac{w_t + \psi_1 M_t^{\psi_2}/h_t}{\eta_t^A}.$$

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. Under flexible prices, the firm would set a constant markup, μ, over marginal cost. The aggregate profits of these firms are distributed among employed workers in proportion to their skill, which can be thought of as representing bonus payments in a sharing economy.

However, individual firms cannot set their price equal to their desired price every period because of nominal rigidities. We assume a simple canonical model of nominal rigidities that captures most of the qualitative insights from New Keynesian economics (Mankiw and Reis, 2010):

Assumption 5. *Every period an i.i.d. fraction θ of firms can set their prices $p_{j,t} = p_t^*$, while the remaining set their price to equal what they expected their optimal price would be: $p_{j,t} = \mathbb{E}_{t-1} p_t^*$.*

2.4 Other government policy

Aside from the two social programs that are the focus of our study, the government also chooses policies for nominal interest rates, government purchases, and the public debt. Starting with the first, we assume a standard Taylor rule for nominal interest rates I_t:

$$I_t = \bar{I} \pi_t^{\omega_\pi} x_t^{\omega_x} \eta_t^I, \tag{6}$$

where $\omega_\pi > 1$ and $\omega_x \geq 0$. The exogenous η_t^I represent shocks to monetary policy.[17]

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[17] As usual, the real and nominal interest rates are linked by the Fisher equation $R_t = I_t / \mathbb{E}_t [\pi_{t+1}]$.

Turning to the second, government purchases follow:

$$G_t = \chi C_t \eta_t^G, \tag{7}$$

where η_t^G are random shocks. Absent these shocks, this rule states that the marginal utility benefit of public goods offsets the marginal utility loss from diverting goods from private consumption. For the baseline case, we assume it holds:

Assumption 6. *The Samuelson (1954) rule holds:* $\eta_t^G = 0$.

Our last assumption is that there are no deficits. It is well known, at least since Aiyagari and McGrattan (1998), that in an incomplete markets economy like ours, changes in the supply of safe assets will affect the ability to accumulate precautionary savings. Deficits or surpluses may stabilize the business cycle by changing the cost of self-insurance. In the same way that we abstracted above from the stabilizing properties of changes in government purchases, this lets us likewise abstract from the stabilizing property of public debt, in order to focus on our two social programs.[18] Letting $z_{i,t}$ denote the income of household i should they be employed:

Assumption 7. *The government runs a balanced budget by adjusting* λ_t:

$$G_t = \int n_{i,t} \left(z_{i,t} - \lambda_t z_{i,t}^{1-\tau} \right) - (1 - n_{i,t}) b \lambda_t z_{i,t}^{1-\tau} di. \tag{8}$$

3 Equilibrium and the role of policy

Our model combines idiosyncratic risk, incomplete markets, and nominal rigidities, and yet it is structured so as to be tractable enough to analytically investigate optimal policy. An aggregate equilibrium is a solution for 17 endogenous variables using a system of equations summarized in Appendix B.4, together with the exogenous processes η_t^A, η_t^G, and η_t^I. This section highlights how the special assumptions that we flagged in the previous section, with their virtues and limitations, lead to analytical results and make transparent the role for social insurance policy and the distortions it creates.

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[18] In previous work (McKay and Reis, 2016), we found that allowing for deficits and public debt had little effect on the effectiveness of stabilizers. This is because, in order to match the concentration of wealth in the data, almost all of the public debt is held by richer households who are already close to fully self insured. The same will turn out to be the case in this economy, as we will later show.

3.1 Inequality and heterogeneity

The following result follows directly from Assumption 2 and plays a crucial role in simplifying the analysis:

Lemma 1. *All households choose the same asset holdings, hours worked, and search effort, so $a_{i,t} = 0$, $h_{i,t} = h_t$, and $q_{i,t} = q_t$ for all i.*

To prove this result, note that the decision problem of a household searching for a job at the start of the period is:

$$V^s(a, \alpha, \mathcal{S}) = \max_q \left\{ MqV(a, \alpha, 1, \mathcal{S}) + (1 - Mq)V(a, \alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \tag{9}$$

where we used $\mathcal{S}$ to denote the collection of aggregate states. The decision problem of the household at the end of the period is:

$$V(a, \alpha, n, \mathcal{S}) = \max_{c, h, a' \geq 0} \left\{ \log c - \frac{h^{1+\gamma}}{1 + \gamma} + \chi \log(G) - \xi(1 - n) + \right.$$

$$\left. \beta \mathbb{E}\left[(1 - \upsilon)V(a', \alpha', 1, \mathcal{S}') + \upsilon V^s(a', \alpha', \mathcal{S}') \right] \right\}, \tag{10}$$

$$\text{subject to: } a' + c = Ra + \lambda \left(n + (1 - n)b \right) \left[\alpha(wh + d) \right]^{1-\tau}. \tag{11}$$

Starting with asset holdings, since no agent can borrow and bonds are in zero net supply, then it must be that $a_{i,t} = 0$ for all i in equilibrium because there is no gross supply of bonds for savers to own. Turning to hours worked, the intra-temporal labor supply condition for an employed household is:

$$c_{i,t} h_{i,t}^\gamma = (1 - \tau)\lambda_t z_{i,t}^{-\tau} w_t \alpha_{i,t}, \tag{12}$$

where the left-hand side is the marginal rate of substitution between consumption and leisure, and the right-hand side is the after-tax return to working an extra hour to raise income $z_{i,t}$. More productive agents want to work more. However, they are also richer and want to consume more. The combination of our preferences and the budget constraint imply that these two effects exactly

cancel out so that in equilibrium all employed households work the same hours:

$$h_t^\gamma = \frac{(1-\tau)w_t}{w_t h_t + d_t},$$ (13)

where d_t is aggregate dividends per employed worker.[19]

Finally, the optimality condition for search effort is:

$$q_{i,t}^\kappa = M_t \left[V(a_{i,t}, \alpha_{i,t}, 1, \mathcal{S}) - V(a_{i,t}, \alpha_{i,t}, 0, \mathcal{S}) \right].$$ (14)

Intuitively, the household equates the marginal disutility of searching on the left-hand side to the expected benefit of finding a job on the right-hand side, which is the product of the job-finding probability M_t and the increase in value of becoming employed. Appendix B.1 shows that this increase in value is independent of $\alpha_{i,t}$. The key assumption that ensures this is that unemployment benefits are indexed to income $z_{i,t}$ so the after-tax income with and without a job scales with idiosyncratic productivity in the same way. This then implies that $q_{i,t}$ is the same for all households, finishing the proof.

The lemma clearly limits the scope of our study. We cannot speak to the effect of policy on asset holdings, and differences in labor supply are reduced to having a job or not, which ignores diversity in part-time jobs and overtime. At the same time, it has the substantial payoff of implying that $\mathcal{S}$ contains only aggregate variables, so we do not need to keep track of the cross-sectional distribution of wealth to characterize an equilibrium. Thus, our model can be studied analytically and numerical solutions are easy to compute. Moreover, arguably the social programs that we study are more concerned with income inequality, rather than wealth inequality, and the vast majority of studies of the automatic stabilizers also ignores any direct effects of wealth inequality (as opposed to income inequality) on the business cycle.

Even though there is no wealth inequality, there is a rich distribution of income and consumption driven by heterogeneity in employment status $n_{i,t}$ and skill $\alpha_{i,t}$ in our model. In section 6, we are able to fit the more prominent features of income inequality in the United States by parameterizing the distribution $F(\epsilon, x)$. Moreover, in our model, there is a rich distribution of individual prices and output across firms, $(p_{j,t}, y_{j,t})$, driven by nominal rigidities. And finally, the exogenous

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[19]To derive this, substitute $z_{i,t} = \alpha_{i,t}(w_t h_{i,t} + d_t)$ and $c_{i,t} = \lambda_t z_{i,t}^{1-\tau}$ into (12).

aggregate shocks to productivity, monetary policy, and government purchases, $(\eta_t^A, \eta_t^I, \eta_t^G)$, affect all of these distributions, which therefore vary over time and over the business cycle. In spite of the simplifications and their limitations, our model still admits a rich amount of inequality and heterogeneity.

3.2   Quasi-aggregation and consumption

Define $\tilde{c}_t$ as the consumption of the average-skilled ($\alpha_{i,t} = 1$), employed agent. This is related to aggregate consumption, C_t, according to (see Appendix B.2):

$$\tilde{c}_t = \frac{C_t}{\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right](1 - u_t + u_t b)}. \tag{15}$$

Funding higher replacement rates requires larger taxes on those employed, so it reduces their consumption. Likewise, the amount of revenue raised by the progressive tax system depends on the distribution of income as summarized by $\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]$. More dispersed incomes generate higher revenues and allow for lower taxes for a given level of income.

The next property that simplifies our model is proven in Appendix B.2.

Lemma 2. *Aggregate consumption dynamics follows a modified Euler equation:*

$$\frac{1}{\tilde{c}_t} = \beta R_t \, \mathbb{E}_t\left\{\frac{1}{\tilde{c}_{t+1}} Q_{t+1}\right\}, \tag{16}$$

$$\text{with: } Q_{t+1} \equiv \left[(1 - u_{t+1}) + u_{t+1} b^{-1}\right] \mathbb{E}\left[\epsilon_{i,t+1}^{-(1-\tau)}\right]. \tag{17}$$

and equation (15)*.*

Without uncertainty on individual productivity or unemployment, $Q_{t+1} = 1$, so equation (16) becomes the standard Euler equation from intertemporal choice stating that expected consumption growth is inversely related to the product of the discount factor and the real interest rate. The variable Q_{t+1} captures how uninsurable risk affects aggregate consumption dynamics through precautionary savings motives. The more uncertain is income, the larger is Q_{t+1} and so the larger are savings motives leading to steeper consumption growth. A more generous unemployment insurance system and a more progressive income tax lower the dispersion of after-tax income growth

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and reduce the effect of this Q_{t+1} term. This Euler equation is the key equation through which precautionary savings motives determine the fluctuations in output.

3.3 Policy distortions and redistribution over the business cycle

Social policies not only affect aggregate consumption, but also all individual choices in the economy, introducing both distortions and redistribution.

Combining the optimality condition for hours with our Assumption 4 on the wage rule gives (see Appendix B.3):

$$h_t = [\bar{w}(1 - \tau)]^{\frac{1}{1+\gamma}} x_t^{\frac{\zeta}{1+\gamma}}. \tag{18}$$

A more progressive income tax lowers hours worked by increasing the ratio of the marginal tax rate to the average tax rate.

Moving to search effort, Appendix B.3 shows that:

$$q_t^\kappa = M_t \left[\xi - \frac{h_t^{1+\gamma}}{1 + \gamma} - \log(b) \right]. \tag{19}$$

This states that the marginal disutility of searching for a job is equal to the probability of finding a job times the increase in utility of having a job. This utility gain is equal to the difference between the non-pecuniary cost of unemployment and the disutility of working, minus the loss in utility units of reducing consumption by a factor b. More generous benefits therefore lower search effort. Intuitively, they lower the value of finding a job, so less effort is expended looking for one.[20]

The distribution of consumption in the economy is given by a relatively simple expression:

$$c_{i,t} = \left[\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b) \right] \tilde{c}_t. \tag{20}$$

The expression in brackets shows that more productive and employed households consume more, as expected. Combined with $\tilde{c}_t$, this formula also shows how social policies redistribute income and equalize consumption. A higher b requires larger contributions from all households, lowering $\tilde{c}_t$, but

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[20]Equations (18) and (19) show why Assumption 3, that the tightness of the labor market measures the state of the business cycle, is not particularly strong. Since (h_t, q_t) are functions of only M_t and parameters, the unemployment rate and the output gap (the difference between actual output and that which arises with flexible prices) are also functions of M_t as the single endogenous variable.

only increases the term in brackets for unemployed households. Therefore, it raises the consumption of the unemployed relative to the employed. In turn, a higher τ lowers the cross-sectional dispersion of consumption because it reduces the income of the rich more than that of the poor. The state of the business cycle affects the extent of the redistribution by driving both unemployment and the cross-sectional distribution of productivity risk.

Finally, social programs also affect price dispersion and inflation. Recalling that average labor productivity is $A_t \equiv Y_t/[h_t(1 - u_t)]$, then integrating over the individual production functions and using the demand for each variety it follows immediately that $A_t = \eta_t^A/S_t$ where the new variable $S_t \geq 1$ is price dispersion:

$$S_t \equiv \int (p_t(j)/p_t)^{\mu/(1-\mu)} \, dj = \left(\frac{p_t^*}{p_t}\right)^{\mu/(1-\mu)} \left[\theta + (1 - \theta)\left(\frac{\mathbb{E}_{t-1}p_t^*}{p_t^*}\right)^{\mu/(1-\mu)}\right], \tag{21}$$

where the last equality follows from Assumption 5. Nominal rigidities lead otherwise identical firms to charge different prices, and this relative-price dispersion lowers productivity and output in the economy. The social insurance system will alter the dynamics of aggregate demand leading to different dynamics for nominal marginal costs, inflation, and price dispersion.

4 Optimal policy and insurance versus incentives

All agents in our economy are identical ex ante, making it natural to take as the target of policy the utilitarian social welfare function. Using equation (20) and integrating the utility function in equation (1) gives the objective function for policy $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t$, where period-welfare is:

$$W_t = \mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) + u_t \log b - \log\left(1 - u_t + u_t b\right)$$
$$+ \log(C_t) - (1 - u_t)\frac{h_t^{1+\gamma}}{1 + \gamma} - \upsilon\frac{q_t^{1+\kappa}}{1 + \kappa} + \chi \log(G_t) - \xi u_t. \tag{22}$$

The first line shows how inequality affects social welfare. Productivity differences and unemployment introduce costly idiosyncratic risk, which is attenuated by the social insurance policies. The second line captures the usual effect of aggregates on welfare. While these would be the terms that would survive if there were complete insurance markets, recall that the incompleteness of markets also affects the evolution of aggregates, as we explained in the previous section.

The policy problem is then to pick b and τ once and for all to maximize equation (22) subject to the equilibrium conditions.

4.1 Optimal unemployment insurance

Appendix C derives the following optimality condition for b:

Proposition 1. *The optimal choice of the generosity of unemployment insurance b satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_t \left(\frac{1}{b} - 1 \right) \left. \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{x,q} + \left. \frac{\partial \log \tilde{c}_t}{\partial \log u_t} \right|_x \left. \frac{\partial \log u_t}{\partial b} \right|_x + \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = 0. \qquad (23)$$

Equation (23) is closely related to the Baily-Chetty formula for optimal unemployment insurance. The first term captures the social insurance value of changing the replacement rate. It is equal to the percentage difference between the marginal utility of unemployed and employed agents times the elasticity of the consumption of the unemployed with respect to the benefit. If unemployment came with no differences in consumption, this term would be zero, and likewise if giving higher benefits to the unemployed had no effect on their consumption. But as long as employed agents consume more, and raising benefits closes some of the consumption gap, then this term will be positive and call for higher unemployment benefits.

The second term gives the moral hazard cost of unemployment insurance. It is equal to the product of the elasticity of the consumption of the employed with respect to the unemployment rate, which is negative, and the elasticity of the unemployment rate with respect to the benefit that arises out of reduced search effort. Higher replacement rates induce agents to search less, which raises equilibrium unemployment, and leads to higher taxes to finance benefits.

In the absence of general equilibrium effects, these would be the only two terms as they are derivatives keeping the state of the business cycle x fixed. They capture the standard trade-off between insurance and incentives in the literature but now averaged across time. With business cycles and general equilibrium effects, there is an extra macroeconomic stabilization term. The larger this term is, the more generous optimal unemployment benefits should be. We explain this shortly, but first, we turn to the income tax.

4.2 Optimal progressivity of the income tax

Appendix C shows the following:

Proposition 2. *The optimal progressivity of the tax system τ satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{\text{Cov}\left(\alpha_{i,0}^{1-\tau}, \log \alpha_{i,0}\right)}{\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]} + \frac{\beta}{1-\beta} \frac{\text{Cov}\left(\epsilon_{i,t+1}^{1-\tau} \log \epsilon_{i,t+1}\right)}{\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]} \\ -\left(\frac{A_t}{C_t} - h_t^{\gamma}\right) \frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)} + \left. \frac{\partial \log \tilde{c}_t}{\partial \log u_t}\right|_x \left. \frac{\partial \log u_t}{\partial \tau}\right|_x \\ + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} \end{array} \right\} = 0. \tag{24}$$

The three rows again capture the trade-offs between insurance, incentives, and macroeconomic stabilization, respectively. Starting with the first, row, the first term gives the welfare benefit of redistributing already existing differences in income, as captured by the initial dispersion of skills. The second term gives the welfare benefits of reducing the dispersion in after-tax incomes due to skill shocks that he household is exposed to in the future. Both terms in the first row have a similar structure and are both positive.[21]

The second row gives the incentive costs of raising progressivity. The first term on the row is the labor wedge, the gap between the marginal product of labor and the marginal disutility of labor. More progressive taxes raise the wedge by discouraging labor supply, as explained earlier. The second term reflects the effect of the tax system on the unemployment rate taking slack as given. The tax system affects the relative rewards to being employed and therefore alters household search effort and the unemployment rate.

Finally, the third row captures the concern for macroeconomic stabilization in a very similar way to the term for unemployment benefits. A larger stabilization term in (24) justifies a larger labor wedge and therefore a more progressive tax.

4.3 The macroeconomic stabilization term

The two previous propositions clearly isolate the automatic-stabilizing role of the social insurance programs in a single term. It equals the product of the welfare benefit of changing slack and the response of slack to policy. If business cycles are efficient, the macroeconomic stabilization term

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[21]Each of the terms involves the covariance of two increasing functions of a single random variable, which is positive if the underlying random variable has positive variance. The denominators are positive because α_i and ϵ_i take positive values.

is zero. That is, if the economy is always at an efficient level of slack, so that $dW_t/dx_t = 0$, then there is no reason to take macroeconomic stabilization into account when designing the stabilizers. Intuitively, the business cycle is of no concern for policymakers in this case.

Even if business cycles are efficient on average or the stabilizers have no effect on the average level of slack, the stabilizers can still have stabilization benefits. This is because:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \mathbb{E}_0 \left[\frac{dW_t}{dx_t} \right] \mathbb{E}_0 \left[\frac{dx_t}{db} \right] + \mathrm{Cov} \left[\frac{dW_t}{dx_t}, \frac{dx_t}{db} \right] \right\}, \qquad (25)$$

so that even if $\mathbb{E}_0 \left[\frac{dW_t}{dx_t} \right] \mathbb{E}_0 \left[\frac{dx_t}{db} \right] = 0$, a positive covariance term would still imply a positive aggregate stabilization term and an increase in benefits (or more progressive taxes). Our model therefore provides a sharp definition of the the hallmark of a social policy that serves as an automatic stabilizer: it stimulates the economy more in recessions, when slack is inefficiently high. The stronger this effect, the larger the program should be. In the next section, we discuss the sign of this covariance and what affects it.

5   Inspecting the macroeconomic stabilization term

Understanding the automatic stabilizer nature of social programs requires understanding separately the effect of slack on welfare, dW_t/dx_t, and the effect of the social policies on slack, dx_t/db and $dx_t/d\tau$. Instead of trying to measure the covariance between these two unobservables in the data, a daunting task, we proceed to characterize their structural determinants in terms of familiar economic channels that have been measured elsewhere.

5.1   Slack and welfare

There are five separate channels through which the business cycle may be inefficient in our model, characterized in the following result:

Proposition 3. *The effect of macroeconomic slack on welfare can be decomposed into:*

$$\frac{dW_t}{dx_t} = \underbrace{(1-u_t)\left[\frac{A_t}{C_t} - h_t^\gamma\right]\frac{dh_t}{dx_t}}_{labor\text{-}wedge} - \underbrace{\frac{Y_t}{C_tS_t}\frac{dS_t}{dx_t}}_{price\text{-}dispersion} + \underbrace{\frac{1}{C_t}\frac{\partial C_t}{\partial u_t}\bigg|_x\frac{du_t}{dx_t} - \frac{1}{C_t}\frac{\partial J_t}{\partial x_t}\bigg|_u}_{extensive\text{-}margin} \tag{26}$$

$$\underbrace{-\left(\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right)\frac{\partial u_t}{\partial x_t}\bigg|_q + \frac{1-b}{1-u_t+u_tb}\frac{du_t}{dx_t}}_{unemployment\text{-}risk} + \underbrace{\frac{\beta}{1-\beta}\frac{d}{dx_t}\int\log\left(\frac{\epsilon^{1-\tau}}{\int\epsilon^{1-\tau}dF(\epsilon,x_t)}\right)dF(\epsilon,x_t)}_{income\text{-}risk}$$

The first term captures the effect of the labor wedge or markups. In the economy, A_t/C_t is the marginal product of an extra hour worked in utility units, while h_t^γ is the marginal disutility of working. If the first exceeds the second, the economy is underproducing, and increasing hours worked would raise welfare.

The second term captures the effect of slack on price dispersion. Because of nominal rigidities, aggregate shocks will lead to price dispersion. In that case, changes in aggregate slack will affect inflation, via the Phillips curve, and so price dispersion. This is the conventional welfare cost of inflation in new Keynesian models.

The third and fourth terms capture the standard extensive margin trade-off in models with costly matching. On the one hand, tightening the labor market lowers unemployment and raises consumption. On the other hand, it increases hiring costs. If $\partial C_t/\partial u_t|_x\, du_t/dx_t > \partial J/\partial x_t$, welfare rises as the labor market gets tighter.[22]

The terms in the second line of equation (26) fix aggregate consumption and focus on inequality and its effect on welfare. If the extent of income risk is cyclical, which the literature since Storeslet-ten et al. (2004) has demonstrated, then raising economic activity reduces income risk and so raises welfare. In our model, there is both unemployment and income risk, so this works through two channels.

The fourth and fifth term capture the effect of slack on unemployment risk. For a given aggregate consumption, more unemployment has two effects on welfare. First, there are more unemployed who consume a lower amount. The term $\xi - \log b - h_t^{1+\gamma}/(1+\gamma)$ is the utility loss from becoming unemployed. Second, those who are employed consume a larger share (dividing the pie among fewer

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[22]The partial derivative of C_t with respect to u_t given x_t is defined mathematically in Appendix C. It is the gain in consumption from putting more people to work but without changing wages, hours on the intensive margin, price dispersion, or the other consequences of changing x_t.

employed people). These are the two effects of unemployment risk.

The sixth and final term captures the effect of slack on the distribution of skill shocks. Slack affects welfare by changing the distribution $F(\epsilon, x_t)$ and we will emphasize pro-cyclical skewness of the distribution. By the concavity of the log function, a more negatively skewed $F(.)$ results in more welfare losses.

5.2 Three special cases

To better understand these different channels of welfare effects, and link them to the literature before us, we consider three special cases that correspond to familiar models of fluctuations.

5.2.1 Frictional unemployment

Consider the special case where prices are flexible ($\theta = 1$), there is no productivity risk ($\text{Var}(\epsilon) = 0$), and labor supply does not vary on the intensive margin because hours worked are constant ($\gamma = \infty$). The only source of inequality is then unemployment, due to the costly process of search and matching. Therefore, equation (26) becomes:

$$\frac{dW_t}{dx_t} = \frac{1}{C_t}\frac{dC_t}{du_t}\frac{du_t}{dx_t} - \frac{1}{C_t}\frac{\partial J_t}{\partial x_t}\bigg|_u - \left(\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right)\frac{\partial u_t}{\partial x_t}\bigg|_q + \frac{1-b}{1-u_t+u_t b}\frac{du_t}{dx_t}. \qquad (27)$$

as only the extensive margin effect and the unemployment risk are now present.

In this special case, our model is close to the one in Landais et al. (2017). They discuss the macroeconomic effects of unemployment benefits from the perspective of their effect on labor market tightness by changing the worker's bargaining position and wages on the one hand and, on the other hand, their impact on dissuading search effort.

5.2.2 Real business cycle effects

Next, we consider the case of flexible prices ($\theta = 1$), constant search effort ($\kappa = \infty$), an exogenous job finding rate (M_t exogenous), and a log-normal $F(\epsilon, x_t)$ with variance of $\log \epsilon$ given by $\sigma^2(x_t)$ and mean $-0.5 \times \sigma^2(x_t)$.[23] With nominal rigidities and search removed, what is left is the labor

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[23]When M_t is constant we need to define slack differently from $x_t = M_t$. In this case, the role of x_t is to change the wage and change labor supply on the intensive margin. The wage will need to adjust to clear the labor market as in the three-equation New Keynesian model and then the wage rule, equation (5), becomes the definition of x_t.

wedge and the effect of cyclical income risk on welfare, so equation (26) simplifies to

$$\frac{dW_t}{dx_t} = (1 - u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{\beta}{1 - \beta} (1 - \tau)^2 \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2}. \tag{28}$$

In this case, our paper fits into the standard analysis of business cycles in Chari et al. (2007) through the first term, and into the study the costs of business cycles due to income inequality emphasized by Krebs (2003) through the second term.

5.2.3 Aggregate demand effects

Traditionally, the literature on automatic stabilizers has focussed on aggregate demand effects following a Keynesian tradition. When there is no productivity risk ($\text{Var}(\epsilon) = 0$), job search effort is constant ($\kappa = \infty$) and the labor market's matching frictions are constant (M_t is constant), equation (26) simplifies to:

$$\frac{dW_t}{dx_t} = (1 - u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t},$$

so only the markup effects are present, both through the labor wedge and through price dispersion.

Appendix D.2 shows that a second-order approximation of W_t around the flexible-price, socially-efficient level of aggregate output Y_t^* and consumption C_t^* transforms this expression into

$$\frac{dW_t}{dx_t} = \left(\frac{Y_t^*}{C_t^*} \right) \left[\left(\frac{1}{C_t^*} + \frac{\gamma}{Y_t^*} \right) \left(\frac{Y_t^* - Y_t}{Y_t^*} \right) \frac{dY_t}{dx_t} + \left(\frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1} \right) \left(\frac{\mathbb{E}_{t-1} p_t - p_t}{\mathbb{E}_{t-1} p_t} \right) \frac{dp_t}{dx_t} \right],$$

In this case, our model fits into the new Keynesian framework with unemployment developed in Blanchard and Galí (2010) or Gali (2011). Raising slack affects the output gap and the price level, through the Phillips curve, and this affects welfare through the two conventional terms in the expression. The first is the effect on the output gap, and the second the effect on surprise inflation. These are the two sources of welfare costs in this economy.

5.3 Social programs and slack

We now turn attention to the second component of the macroeconomic stabilization term, either dx_t/db in the case of unemployment benefits, or $dx_t/d\tau$ in the case of tax progressivity. Obtaining

analytical expressions for them is hard, unless extra assumptions are made. In Appendix E, we characterize these terms by assuming that there is only aggregate uncertainty at date 0, that household job search is exogenous, and that the income-risk distribution is log-normal. Here, we discuss briefly the lessons from this very special case, which prove to be robust in our numerical analysis.

Starting with dx_t/db, there are two direct effects of raising unemployment benefits on economic slack. The first is a classic redistribution effect: aggregate demand increases with an increase in unemployment benefits because the unemployed have a high marginal propensity to consume. The second is a precautionary effect: because unemployment benefits provide social insurance, they lower uncertainty about the future, which reduces precautionary savings motive, and pushes up aggregate demand today. Both of these effects become stronger in recessions, the first because there are more unemployed receiving benefits, and the second because the risk of unemployment and the corresponding precautionary savings motive rise during recessions. Unemployment benefits therefore stimulate more during recessions, satisfying our criteria for an automatic stabilizer.

Turning to $dx_t/d\tau$, there is also a potentially important precautionary effect. When households face uninsurable productivity risk, a progressive tax system will raise aggregate demand by reducing the precautionary savings motive. This effect is counter-cyclical if risk increases in a recession, as has been documented by Guvenen et al. (2014).

Like all fiscal policy, the effectiveness of the automatic stabilizers on economic activity further depends on two other factors. The first one is the aggressiveness of monetary policy. A booming economy leads to higher nominal interest rates, both directly via the Taylor rule and indirectly via higher inflation. With nominal rigidities, this raises the real interest rate, which dampens the effectiveness of any policy on equilibrium slack. An extreme example of this is when the economy is at the zero lower bound, which magnifies the effectiveness of the automatic stabilizers.[24] The second factor is the extent of the counter-cyclicality of income risk. In response to a reduction in aggregate demand, labor market tightness falls, leading to an increase in risk, and an increase in the precautionary savings motive. This further reduces aggregate demand amplifying the business cycle.[25]

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[24] McKay and Reis (2016) and Kekre (2017) show that automatic stabilizers and unemployment benefits, respectively, have stronger stimulating effects when the economy is at the zero lower bound.

[25]Similar reinforcing dynamics arise out of unemployment risk in Ravn and Sterk (2017), Den Haan et al. (2015), and Heathcote and Perri (2015).

5.4 Summary and likely sign

To summarize, there are two main channels through which unemployment benefits or income tax progressivity raise aggregate demand and thereby eliminate slack. These channels are redistribution and social insurance, and both are increasing in the unemployment rate. As unemployment and income risks are counter-cyclical, these forces push for counter-cyclical elasticities of slack to the social programs, since they dampen the counter-cyclical fluctuations in the precautionary savings motive. If business cycles are inefficient in the sense that tightness is inefficiently low in a recession, then we expect a positive covariance between dW_t/dx_t and the elasticities of tightness with respect to policy. This positive covariance implies a positive aggregate stabilization term and more generous unemployment benefits and a more progressive tax system even if the business cycle is efficient on average.

6 Quantitative analysis

We have shown that the presence of business cycles leads to a macroeconomic stabilization term in the determination of the optimal generosity of unemployment insurance and the progressivity of income taxes, and that this term likely makes these programs more generous and progressive, respectively. We now turn to numerical solutions in order to achieve three goals. First, we ask whether the macroeconomic stabilization term is quantitatively significant. Second, we use the analytical formulas presented above to understand the mechanisms driving the numerical results, and to show that our three propositions can be applied to make sense of the results from more complicated models.

The third goal is to evaluate the seven assumptions that we made in section 2 for analytical tractability. Three of these cannot be kept while taking the model to fit the data. To generate reasonable business cycles, we now allow for mortality, government spending shocks, and nominal rigidity that lasts for more than one period through a Calvo pricing model. Therefore, we dispense with Assumptions 1, 5, and 6. We also dispense with Assumption 3, and use the employment rate as a measure of slack: $x_t = (1 - u_t)/(1 - \bar{u})$. This maters little to the results because employment and the job finding rate are highly correlated in the model, but this alternative assumption makes the calibration more transparent because the unemployment rate is easily measured. The remaining

three assumptions—the wage rule, no public deficits, and no gross savings—are important also for numerical tractability, so we maintain them for now, and then relax them one by one in section 6.5 to discuss their roles.

6.1 Calibration and solution of the general model

We solve the model using global methods, as described in Appendix F.2, so that we can accurately compute social welfare, assuming that the economy starts at date 0 at the deterministic steady state. We then numerically search for the values of b and τ that maximize the social welfare function, and compare these with the maximal values in a counterfactual economy without aggregate shocks, but otherwise identical.

Table 1 shows the calibration of the model, dividing the parameters into different groups. The first group has parameters set ex ante to standard choices in the literature. Only the last one deserves some explanation. ψ_2 is the elasticity of hiring costs with respect to labor market tightness, and we set it at 1 as in Blanchard and Galí (2010), in order to be consistent with an elasticity of the matching function with respect to unemployment of 0.5 as suggested by Petrongolo and Pissarides (2001).

Panel B contains parameters individually calibrated to match time-series moments. For the preference for public goods, we target the observed average ratio of government purchases to GDP in the US in 1984-2007. For the monetary policy rule, we use OLS estimates of equation (6). The parameter υ determines the probability of not having a job in our model, which we set equal to the sum of quarterly job separation rate, which we construct following Shimer (2012), and the average unemployment rate. Finally, we estimate a version of the income innovation process specified in equation (2) using a mixture of normals as a flexible parameterization of the distribution $F(\epsilon'; \cdot)$. Two of the mixture components shift with the unemployment rate to match the observed pro-cyclical skewness of earnings growth rates documented by Guvenen et al. (2014). This parametric income process is similar to the one in McKay (2017) and Guvenen and McKay (2017) and Appendix F.1 provides additional details.[26] As a check on our calibration, the model implies a cross-sectional variance of log consumption of 0.40, while in 2005 CEX data the variance of log consumption of non-durables was around 0.35 (Heathcote et al., 2010).

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[26]We include unemployment fluctuations in the income process we simulate to match the empirical moments so the contribution of unemployment to observed changes in income distributions is accounted for.

Symbol	Value	Description	Target
Panel A. Parameters chosen ex ante			
δ	1/200	Mortality rate	50 year expected lifetime
$\frac{\mu}{\mu-1}$	6	Elasticity of substitution	Basu and Fernald (1997)
θ	1/3.5	Prob. of price reset	Klenow and Malin (2010)
$1/\gamma$	1/2	Frisch elasticity	Chetty (2012)
ψ_2	1	Elasticity of hiring cost	Blanchard and Galí (2010)
Panel B. Parameters individually calibrated			
χ	0.262	Preference for public goods	$G/Y = 0.207$
ω_π	1.66	Mon. pol. response to π	Estimated Taylor rule
ω_u	0.133	Mon. pol. response to u	Estimated Taylor rule
υ	0.153	Job separation rate	Average value
$F(\epsilon,.)$	mix-normals	Productivity-risk process	Guvenen and McKay (2017)
Panel C. Parameters jointly calibrated to steady-state moments			
β	0.977	Discount factor	3% annual real interest rate
$\bar{w}$	0.809	Average wage	Unemployment rate = 6.1%
ψ_1	0.0309	Scale of hiring cost	Recruiting costs of 3% of pay
$1/\kappa$	0.0699	Search effort elasticity	$d\log u/d\log b\vert_x = 0.5$
ξ	0.230	Pain from unemployment	Leisure benefit of unemployment
Panel D. Parameters jointly calibrated to volatilities			
ρ	0.9	Autocorrelation of shocks	0.9
StDev(η^A)	0.784%	TFP innovation	StDev(u_t) = 1.59%
StDev(η^I)	0.265%	Monetary policy innovation	StDev($Y\vert\eta^A$) = StDev($Y\vert\eta^I$)
StDev(η^G)	4.63%	Government purchases innovation	StDev(G_t/Y_t) = 1.75%
ζ	1.68	Elasticity of wage w.r.t. x	StDev(h_t)/ StDev($1 - u_t$) = 0.568
Panel E: Automatic stabilizers			
b	0.810	UI replacement rate	Rothstein and Valletta (2017)
τ	0.151	Progressivity of tax system	Heathcote et al. (2014)

Table 1: Calibrated parameter values and targets

Panel C has parameters chosen jointly to target a set of moments. We target the average unemployment rate between 1960 to 2014 and recruiting costs of 3 percent of quarterly pay, consistent with Barron et al. (1997). The parameter κ controls the marginal disutility of effort searching for a job, and we set it to target a micro-elasticity of unemployment with respect to benefits of 0.5 as reported by Landais et al. (2017). Last in the panel is ξ, the non-pecuniary costs of unemployment. In the model, the utility loss from unemployment is $\log(1/b) - h^{1+\gamma}/(1 + \gamma) + \xi$, reflecting the loss in consumption, the gain in leisure, and other non-pecuniary costs of unemployment. We set $\xi = h^{1+\gamma}/(1 + \gamma)$ in the steady state of our baseline calibration so that the benefit of increased leisure in unemployment is dissipated by the non-pecuniary costs.

Panel D calibrates the three aggregate shocks in our model that perturb productivity, monetary policy, and public expenditures. In each case, the exogenous process is an AR(1) in logs with common autocorrelation. We set the variances to match three targets: (i) the standard deviation of the unemployment rate, (ii) the standard deviation of G_t/Y_t, and (iii) equal contributions of productivity and monetary shocks to the variance of aggregate output. We set $\zeta = 1.68$ to match the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio.

Finally, panel E has the baseline values for the automatic stabilizers. For τ we adopt the estimate of 0.151 from Heathcote et al. (2014). In calibrating b we target the observed degree of insurance that households have against unemployment shocks, as measured by the change in consumption upon unemployment. We set $b = 0.81$, consistent with a 19% decline in consumption when unemployed since the literature has found consumption changes between 16% and 21%.[27] These calibrated values for b and τ do not directly enter our analysis of optimal policy but are used to jointly calibrate the other structural parameters of the model.

As a check on the model's performance, the standard deviation of hours, output, and inflation in the mode are 0.75%, 1.68%, and 0.65%. The equivalent moments in the US data 1960-2014 are 0.84%, 1.32%, and 0.55%.

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[27]See Stephens Jr (2004), Aguiar and Hurst (2005), Saporta-Eksten (2014), and Chodorow-Reich and Karabarbounis (2016).
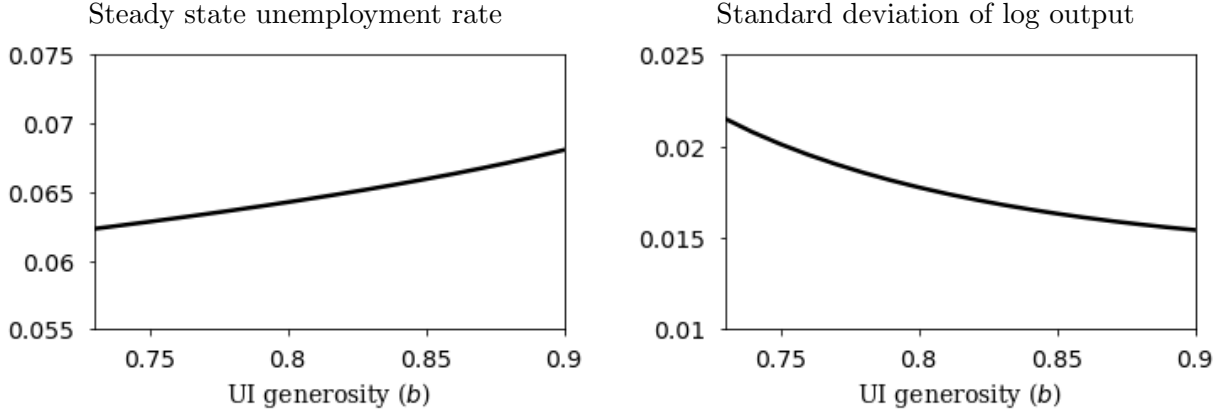
Figure 1: Effects of changing b for $\tau = 0.26$

6.2 Optimal automatic stabilizers

Our first main quantitative result is that aggregate shocks increase the optimal b to 0.853 from 0.773 in the absence of aggregate shocks. Converting the values for b into pre-tax UI replacement rates based on a two-earner household, then we have an optimal replacement rate of 61% with aggregate shocks as compared to 41% percent without.[28]

Figure 1 provides a first hint for why this effect is so quantitatively large. It shows the effects of raising b on the steady state unemployment rate and on the volatility of log output. Raising the generosity of unemployment benefits hurts the incentives for working, so unemployment rises somewhat. However, it has a strong macroeconomic stabilizing effect.

Our second main quantitative result is that the optimal tax progressivity is almost unchanged at 0.260 relative to 0.267 without aggregate shocks. The left panel of figure 2 shows the steady state level of output as a log deviation from the level under the optimal policy without aggregate shocks. There is a strong negative effect of progressivity on output that works through the disincentive effect on labor supply. At the optimal policy without aggregate shocks, the welfare loss from reducing aggregate output is balanced by the welfare gain from insuring productivity risk. When we introduce aggregate shocks, the level of tax progressivity has almost no effect, and actually a slight positive effect, on the volatility of the business cycle, as shown in the right-panel of figure 2. Therefore, there is no stabilization benefit of raising progressivity.

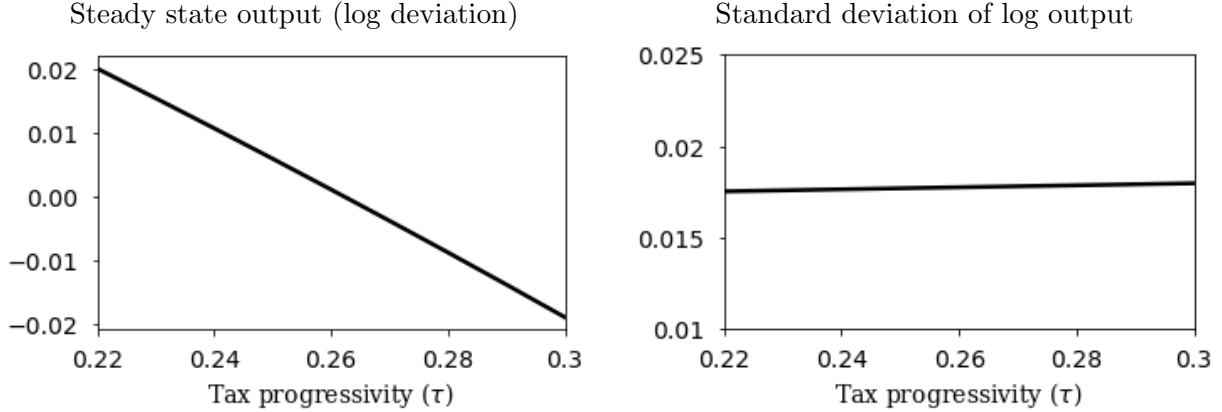[28]The conversion we apply here is to solve for x in $(x/2 + 1/2)^{1-\tau} = b$.

Figure 2: Effects of changing τ for $b = 0.80$. The left panel shows log output as a deviation from the value at $\tau = 0.267$. The right panel uses the same vertical axis as figure 1 for comparison.

6.3   Using the analytical propositions to understand the numerical results

What drives the large automatic stabilization role for unemployment benefits, but not for income tax progressivity? Our analytical results provide guidance on the key economic channels at play.

Figure 3 uses Proposition 1 and Proposition 3 to understand why the optimal b rises in the presence of business cycle risk. The curves labelled "Insurance", "Incentives", and "Macro stabilization" plot each of the the three terms within the curly brackets in Proposition 1. These terms are the marginal welfare gains and losses from increasing b. Under the conditions of the proposition, adding the three terms and setting to zero gave the optimal level of b. Strikingly, the figure confirms that the even though the assumptions for the propositions do not hold exactly in this economy, the sum of three terms is very close to zero at the optimal b. Proposition 1 provides a numerically good approximation.[29]

Moreover, Figure 3 shows that, as b rises, the incentive costs worsen, because the value of working gets closer to the value of unemployment, and the insurance gain diminishes, as the consumption (and therefore marginal utilities) of employed and unemployed become closer. In the absence of the macro stabilization term, the optimal b would be the one at which these two terms balance, which is near $b = 0.75$. Yet, the macro stabilization term is positive and large, so the actual optimal b is much larger. This role of the program, which has been neglected so far, is as important as

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[29]There are two reasons that the proposition does not hold exactly. First, we now allow for government spending shocks so the Samuelson (1954) rule does not hold and changes in x_t affect welfare by changing G_t in the presence of a wedge between $1/C_t$ and χ/G_t. Second, to compute a numerical solution we set the mortality rate to $\delta = 0.005$.
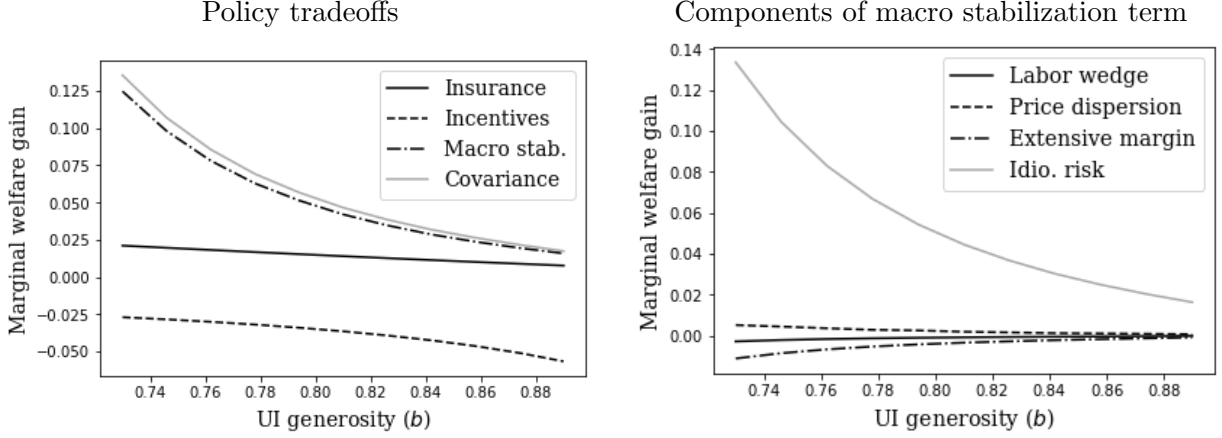
Policy tradeoffs — Components of macro stabilization term

Figure 3: Marginal welfare gain from changing b for $\tau = 0.26$. The quantities in the left panel correspond to the terms in Proposition 1. The covariance term shows that the macroeconomic stabilization term is driven by the covariance term in equation (25). The terms in the right panel correspond to $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW_t}{dx_t} \frac{dx_t}{db}$ with $\frac{dW_t}{dx_t}$ broken into components as in Proposition 3. Both figures are scaled to units of consumption equivalent welfare per unit change in b.

the incentives and redistribution roles that the literature has emphasized instead. A concern for automatic stabilizers makes the unemployment insurance system significantly more generous.

The third and last striking result from Figure 3 is that the macroeconomic stabilization term is driven by the covariance term in equation (25), not by the steady state inefficiency in the economy. The benefits from stabilization do not come from closing the average gap between the level of activity and its optimal level, but rather from attenuating the amplitude of the business cycle.

The right panel of Figure 3 unpacks the sources of the business-cycle stabilization benefits in terms of the different sources of inefficient fluctuations that we characterized in Proposition 3. Each curve in the figure corresponds to a component of the marginal welfare gain or loss from reducing slack displayed in Proposition 3. The dominant component is clearly the reduction in idiosyncratic risk that results from a higher b. By stabilizing the economy, more generous unemployment benefits reduce the risk that households face in their pre-government incomes. This channel is distinct from the insurance benefit, which is the smoothing of post-government income for a given risk to pre-government income. The other components that are plotted in the figure, which reflect benefits and losses in aggregate efficiency are all small in contrast. The inefficient utilization of labor on the extensive margin is in fact negative, as raising b raises the unemployment rate on average.

Raising the progressivity of the income tax instead has a small macroeconomic stabilization

		b		τ		replacement rate	
	Aggregate shocks.................	without	with	without	with	without	with
(i)	Baseline	0.773	0.853	0.267	0.260	0.41	0.61
(ii)	Flexible prices	0.773	0.766	0.267	0.265	0.41	0.39
(iii)	Aggressive monetary policy	0.773	0.796	0.267	0.268	0.41	0.46
(iv)	Acyclical purchases	0.773	0.853	0.267	0.261	0.41	0.61
(v)	Volatile business cycle	0.773	0.855	0.267	0.256	0.41	0.62

Table 2: Optimal policies under alternative specifications. Replacement rates are calculated based on two-worker households as described in footnote 28.

benefit for two related reasons. First, unlike unemployment risk, the cyclical fluctuations in the income risk innovations are modest, even if they have large welfare effects by virtue of being permanent. Therefore, there is less to be gained by stabilizing the precautionary motive that results from these risks. Second, to make a meaningful difference to the precautionary motive would require a large change in τ. Yet, this has a large disincentive cost. As a result, unemployment benefits are both a more effective and a less costly tool to perform macroeconomic stabilization than raising the progressivity of the income tax.

This comparison across the tools also explains why the optimal τ falls with aggregate shocks. This is not because progressivity does not have a macro stabilization role, but because unemployment insurance is more effective. Our finding of a lower τ in the presence of business cycle risk results from our joint optimization over b and τ. If we hold b fixed and optimize only over τ, then business cycles lead to a slightly higher τ.

6.4 Interactions

The discussion so far noted that the automatic stabilizers interact with other policies, with aggregate demand, and with shocks driving inefficient business cycles. Table 6.4 investigates these interactions numerically. Row (i) repeats our baseline results for comparison.

Row (ii) shows that macroeconomic stabilization concerns have almost no effect on the optimal b and τ when prices are flexible. This confirms the important role of aggregate demand and inefficient business cycles.

Rows (iii) and (iv) investigate the interaction with other policies. Row (iii) increases the coefficient on inflation in the monetary policy rule to 2.50 from 1.66. With this more aggressive

monetary policy rule there is less of a need for fiscal policy to manage aggregate demand. Therefore, stabilization plays a smaller role in the design of the optimal social insurance system than in the baseline calibration. Moreover, flexible prices or aggressive monetary policy make the real interest rate responds strongly to changes in slack. The elasticities of slack with respect to the social programs is small leading to a small automatic-stabilizer role.

Row (iv) changes instead the policy rule for government purchases. Our baseline specification, following the Samuelson rule, makes government purchases pro-cyclical. Row (iv) considers instead acyclical government purchases, as we observe in the data, by using instead the rule: $G_t = \bar{G}\eta_t^G$. While the pro-cyclical rule amplifies the business cycle and leaves a larger role for the automatic stabilizers, the effect is quantitatively minor.

Rows (v) shows that the calibration of the aggregate shocks is not crucial to our results. Row (v) raises the standard deviations of the productivity and monetary policy shocks by 25%. As one might expect, when the aggregate shocks are more volatile, stabilization plays a larger role in the choice of b and τ, but the difference is minor. What is important for our results is the degree of internal amplification of shocks through precautionary savings effects on aggregate demand. While the results are sensitive to the degree of price flexibility and the monetary policy rule, they are not so sensitive to the exact nature of the shocks hitting the economy.

6.5 Evaluating our simplifying assumptions

We have maintained three simplifying assumptions for tractability: a particular wage rule, a balanced government budget, and assets in zero gross supply leading to no private savings in equilibrium. We now relax each of these in turn.

6.5.1 The wage rule

There are two features of wages that are especially important for the effectiveness of the stabilizers. The first is the cyclicality of wages, since this affects the amplitude of the business cycle. The second is whether there is a direct response of wages to changes in policy, as changes in benefits or take-home pay affect that bargaining power of workers.

In our wage rule, the parameter ζ captures the elasticity of wages with respect to slack. Making the wage more cyclical makes the intensive margin of labor supply more volatile while the unemploy-

ment rate (the extensive margin) becomes less volatile. With less variability in the unemployment rate there is less amplification through the precautionary savings motive, and so less value in using the automatic stabilizers to stabilize the business cycle. When we double ζ, the optimal b is now only 0.804, and increases with business cycles only by 0.031. While this change reduces the role of the automatic stabilizers, it implies a counterfactually low level of unemployment volatility.

In the baseline specification, the steady state wage barely changes across policy regimes, since policy only affects the costs of hiring in the term $1 - J_t/Y_t$, which is quite small in practice. This specification is consistent with empirical work that fails to find an effect of unemployment benefits on wages.[30] However, some studies have found large effects of unemployment benefits on the equilibrium unemployment rate possibly reflecting general equilibrium effects operating through wages (Hagedorn et al., 2013). We explore how our analysis is affected when the steady state wage is increasing in the unemployment benefit, by assuming that $\bar{w}$ has an elasticity of 0.1 with respect to b.

With a positive wage elasticity, the steady state unemployment rate increases with the benefits more strongly than before. Higher benefits raise wages, which lowers the incentives for firms to hire workers, leading to a lower job-finding rate. Moreover, the incentives for searching for a job fall both because of the usual moral hazard effect and due to the decline in the job-finding rate. With a positive wage elasticity, the greater sensitivity of the steady state unemployment rate to the unemployment benefit leads to much lower optimal benefits in the absence of aggregate shocks: 0.53 as opposed to 0.77 in our baseline.

However, when there are aggregate shocks, low benefits lead to strong de-stabilizing dynamics because of the precautionary savings motive. Because there is little social insurance, the precautionary savings motive is stronger and more cyclical, which leads to stronger internal amplification of shocks so fluctuations become costlier. The automatic-stabilizer nature of unemployment benefits is therefore substantially stronger. As a result, in this case we find that aggregate shocks lead to a large change in the optimal benefit relative to what is optimal in steady state: the optimal b rises from 0.53 to 0.73 with aggregate shocks. Therefore, a positive wage elasticity with respect to benefits leads to lower replacement rates overall, but a larger effect of the business cycle on the optimal policy.

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[30]Card et al. (2007), Van Ours and Vodopivec (2008), Lalive (2007), and Johnston and Mas (2015) find no evidence that UI generosity affects earnings upon re-employment.

6.5.2 Budget deficits

The baseline analysis assumed a balanced budget, which was achieved by having the average tax rate respond to any shocks to fiscal revenues or expenditures. Because of the way in which we specified the tax function, it is the progressivity (τ) rather than the level (λ_t) that drives the willingness to work or consume, as shown in section 3. Still, one may worry that public deficits during recessions may, through an effect on aggregate demand, further enhance the effectiveness of the stabilizers. To allow for this possibility, while still maintaining the aggregation result that there is a zero gross supply of domestic assets, we now allow for the government to borrow from foreigners. In particular, we assume the government can borrow at a world interest rate $R^* = 1.03^{1/4}$, so the government budget constraint now is:

$$
G_t + R^* B_t = \int n_{i,t} \left(z_{i,t} - \lambda_t z_{i,t}^{1-\tau} \right) - (1 - n_{i,t}) \, b \lambda_t z_{i,t}^{1-\tau} di + B_{t+1}
$$

The fiscal policy rule for taxes is now not to balance the budget at every date, but rather to gradually pay for the public debt:[31]

$$
\lambda_t = \bar{\lambda} \left(\frac{\lambda_t^*}{\bar{\lambda}} \right)^{-\ell_\lambda} - \ell_B \frac{B_t}{\bar{Y}},
$$

where λ_t^* is the level of λ_t that results in a balanced budget in period t and $1 - \bar{\lambda}$ is the steady state level of taxes. We set the parameter $\ell_\lambda = 0.5$ to match the standard deviation of public deficits to GDP, which is 0.9%. The parameter $\ell_B = 0.1$ makes the public debt stationary, and its value is set to match the high persistence of the observed debt-to-GDP ratio.

Allowing for foreign borrowing has only a minor impact on the results. The optimal b without business cycles now is 0.773 and the optimal b with business cycles is 0.852, while the respective numbers for τ are 0.267 and 0.263. While a public deficit results in a current account deficit that is capable of stabilizing aggregate consumption, it does not necessarily stabilize production or employment. Because idiosyncratic risk, which is crucial for our welfare considerations, is driven by labor demand, the deficits do not reduce the need to stabilize production. While deficits are helpful in stabilizing aggregate consumption, that is not the main rationale for stabilizing the economy.

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[31]Recall that after tax income is $\lambda_t z_{i,t}^{1-\tau}$ where $z_{i,t}$ is pre-tax income so the average tax rate is decreasing in λ.

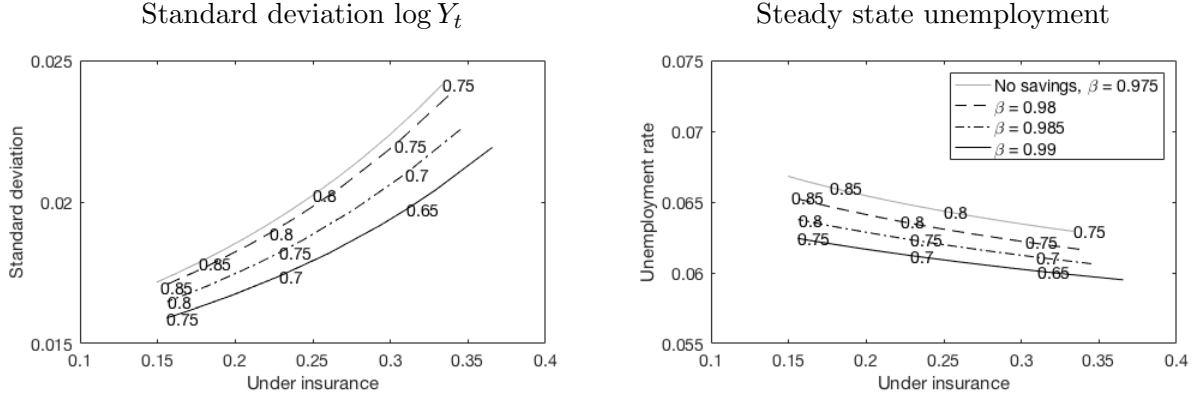Standard deviation log Y_t — Steady state unemployment

Figure 4: Standard deviation of log output and steady state unemployment rate as functions of insurance against unemployment shocks for different levels of savings. Insurance is defined as the percentage difference between $\mathbb{E}[u'(c)|\text{unemployed}]$ and $\mathbb{E}[u'(c)|\text{employed}]$ in steady state. The numbers labeling the lines give the value of b that corresponds to that level of insurance.

6.5.3 Aggregate borrowing and saving

Finally, we drop the last assumption that there are no assets in gross supply. This was important so far to keep both the analytical and the numerical models tractable, since we did not have to keep track of a changing wealth distribution. The key new assumption is that now there is a constant and policy-invariant positive stock of government debt that private households can hold to self insure against unemployment shocks, so now there is a distribution of wealth that changes over time and responds to policy. The level of taxes adjusts to keep this stock of debt constant, so there are no deficits, which were studied in the previous sub-section. Appendix G provides a detailed description of the equilibrium conditions of the extended model, which we solve using the Reiter (2009) method, which gives a non-linear solution with respect to idiosyncratic state variables, but a linearized solution with respect to aggregate states. To keep the computational burden manageable, we focus on unemployment benefits and unemployment risk by shutting off the productivity risk.

The left panel of Figure 4 shows how the volatility of the economy in terms of the standard deviation of log output is related to the steady state level of the insurance against unemployment that households have. This is measured in the horizontal axis in terms of the average difference in marginal utilities between the employed and the unemployed, so a larger difference in marginal utilities corresponds to less insurance. Moving along a curve in the figure varies the generosity of unemployment benefits, indicated by the labels, and so the insurance that households have against unemployment, while holding the aggregate level of savings constant. Each curve corresponds to

a different level of total savings in the economy, obtained by varying the discount factor β, while keeping fixed the annual interest rate at 2%. The right panel of Figure 4 shows a similar plot but with the steady state unemployment rate on the vertical axis.

The left panel of Figure 4 shows that, for a given level of insurance (a point on the x-axis) the slopes of the volatility lines are quite similar across levels of savings. Therefore, the marginal benefit in terms of stabilizing the economy from providing more insurance through the UI system changes little across levels of savings. In turn, the right panel shows that the same applies with respect to the marginal incentive cost, since the increase in the unemployment rate reflected in the slopes of the curves again barely changes with the level of savings. Therefore, the macroeconomic stabilization term will be similar to our baseline, and so will be the effect of business cycles on optimal unemployment insurance.

Calculating the optimal b in the model with savings turns out to not be too informative because, since we now omit the time-varying productivity risk, we are leaving out a key determinant of welfare. Instead, we pick b to maximize an ad hoc objective function: $V(b) + \mathcal{V}_\sigma \sigma(b)$, where $V(b)$ is steady state welfare, $\sigma(b)$ is the standard deviation of log output and $\mathcal{V}_\sigma$ is a parameter that stands in for the value of output stabilization that comes from time-varying productivity risk in the baseline model. We set $\mathcal{V}_\sigma$ so that if there is no savings (and no productivity risk), then the optimal b and insurance level match the optimum in our baseline model.

Table 3 shows the results from maximizing this objective function. The first interesting lesson is that, with savings but no business cycles, the optimal unemployment benefit is less generous. Agents partly self-insure now, so they do need as much social insurance. Second, introducing business cycles makes a larger difference to the optimal b and the extent of under-insurance with savings than without them. While savings reduce the marginal benefit of social insurance, they also reduce the disincentive effect of benefits on working. As Figure 4 highlighted, the end effect is that the trade-off between insurance and incentives ends up being quite stable in spite of savings.

7   Conclusion

Policy debates take as given that there are stabilizing benefits of unemployment insurance and income tax progressivity, but there are few systematic studies of what factors drive these benefits and how large they are. In contrast, the study of these social programs in the academic literature

	Without aggregate shocks		With aggregate shocks	
	b	Under-insurance	b	Under-insurance
Baseline	0.773	0.294	0.853	0.172
No savings, ad hoc objective	0.780	0.283	0.853*	0.172*
Savings, ad hoc objective	0.600	0.406	0.741	0.168

Table 3: Optimal policies with and without savings. Under-insurance is defined as the percentage difference between $\mathbb{E}[u'(c)|\text{unemployed}]$ and $\mathbb{E}[u'(c)|\text{employed}]$ in steady state. * indicates that the ad hoc objective function is designed to match the optimal policy in the baseline model.

rarely takes into account this macroeconomic stabilization role, instead treating it as a fortuitous side benefit.

This paper tries to remedy this situation. Our theoretical study provides a theoretical characterization of what is an automatic stabilizer. In general terms, an automatic stabilizer is a fixed policy for which there is a positive covariance between the effect of slack on welfare, and the effect of the policy on slack. If a policy tool has this property of stimulating the economy more in recessions when slack is inefficiently high, then its role in stabilizing the economy calls for expanding the use of the policy beyond what would be appropriate in a stationary environment. Overall, we found that the role of social insurance programs as automatic stabilizers affects their optimal design and, in the case of unemployment insurance, it can lead to substantial differences in the generosity of the system.

Our focus on the automatic stabilizing nature of existing social programs led us to take a Ramsey approach to the ex ante design of fiscal policy. Future work might explore how these forces affect the design of the social insurance system from a Mirrleesian perspective. Another question is how these fiscal policy programs can adjust to the state of the business cycle, taking into account measurement difficulties, time inconsistency, political economy, and other challenges of implementing state-dependent stabilization policy. There is already some progress on these two topics, and hopefully our analysis will provide some insights to guide their further development.

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Appendix

This appendix contains five sections, which: describe the role of the wage rule introduced in section 2; provide auxiliary steps to some results stated in section 3; prove the propositions in sections 4; derive the AD-AS apparatus and prove the propositions in section 5; and describe the methods used to solve the model in section 6.

A   The wage rule

This section first describes a bargaining protocol that gives rise to the wage rule we assumed, and next modifies our analytical results to allow for a non-isoelastic wage rule.

A.1   Nash bargaining protocol and our wage rule

This appendix presents a Nash bargaining protocol that gives rise to our wage rule. To define a Nash bargaining solution we have to define the outside options of the firm and the worker. If bargaining breaks down, we assume that the firm can obtain a different worker by paying the hiring cost again and the worker can meet another firm but loses some of the period's time endowment. Let z_t be the amount of time lost if bargaining breaks down. One could assume that z_t is a constant parameter (e.g. it takes a given amount of time to meet a new firm once bargaining breaks down) or one could assume that it is easier to find a new firm when the labor market is tight or more productive $z_t = \bar{z}A^{-1}M_t^{-\iota}$. The worker chooses total market time, h, before bargaining begins. If bargaining breaks down, we assume the next employer pays the equilibrium wage (i.e. we do not consider bargaining in that relationship). We assume that employed workers of a given skill are able to pool the gains from bargaining between themselves (in equilibrium they all negotiate the same wage so there are no transfers, but this means that bargaining does not take into account the curvature of the individual utility function or the non-linearity of the tax system).

The surplus to a worker of an offer that pays w when the market wage is $\bar{w}$ is

$$V(w, \bar{w}, \alpha) = \mu_V(\alpha)\alpha \left[wh - \bar{w}(h - z)\right],$$

where $\mu_V(\alpha)$ is the marginal valuation of resources by the employed workers of skill α. The surplus

to the firm is

$$J(w, \bar{w}, \alpha) = \mu_J \alpha \left[\psi_1 M^{\psi_2} + \bar{w} h - wh \right],$$

where μ_J is the marginal valuation of resources by the firm. In the firm's surplus we assume hiring costs are proportional to α, which is consistent with our assumption that hiring an effective unit of labor costs $\psi_1 M^{\psi_2}$ because the average α in the employed and unemployed populations is always constant at one.

The generalized Nash bargaining solution satisfies

$$\max_w V(w, \bar{w}, \alpha)^{\aleph} J(w, \bar{w}, \alpha)^{1-\aleph}$$

where $\aleph$ is the worker's bargaining power. The first order condition at $w = \bar{w}$ is

$$\frac{\aleph}{1 - \aleph} \mu_J \psi_1 M^{\psi_2} h \mu_V(\alpha) \alpha^2 = \mu_J h \mu_V(\alpha) wz \alpha^2$$

$$\frac{\aleph}{1 - \aleph} \psi_1 M^{\psi_2} = wz$$

Substituting $z = A^{-1} M^{-\iota}$

$$w = \frac{1}{\bar{z}} \frac{\aleph}{1 - \aleph} \psi_1 A M^{\psi_2 + \iota} \tag{29}$$

Now set $\bar{z}$ and $\aleph$ such that $\frac{1}{\bar{z}} \frac{\aleph}{1-\aleph} \psi_1 = \bar{w} \bar{M}^{-\zeta}$ and set ι such that $\psi_2 + \iota = \zeta$. The result is that this bargaining protocol gives rise to our wage rule except for the $(1 - J/Y)$ term, which is quantitatively innocuous. If z_t fluctuates in a more complicated manner, we can derive a wage rule that includes the $(1 - J/Y)$ term.

A.2   The role of the wage rule in our analysis

We assumed that the wage was determined by equation (5). A more general specification uses a generic wage function:

$$w_t = w(\eta_t, x_t, b, \tau),$$

that maps the aggregate shocks, slack (or labor market tightness), and policy parameters into a wage.

In this more general case, we can write h_t as:

$$h_t = \left\{ (1-\tau) \left[\frac{\eta_t^A}{S(x_t)} \left(1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t^A, x_t, b, \tau) \right\}^{1/(1+\gamma)}$$

where $S(x_t)$ is the level of price dispersion associated with that level of slack. Using the generic wage rule and equations (3), (4), (19), (21), (36), and (38), we can write J_t/Y_t as a function of (η_t, x_t, b, τ) to yield:

$$h_t = \{ (1-\tau) H(\eta_t, x_t, b, \tau) \}^{1/(1+\gamma)} ,$$

where $H_t \equiv \left[\frac{\eta_t}{S(x_t)} \left(1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t, x_t, b, \tau)$. The purpose of assuming the wage rule in equation (5) in the paper is that H_t simplifies to x_t^ζ.

However, not making this simplification, and so carrying the new $H(.)$ term along only adds to our results a few additional terms. To start, in addition to those terms that appear in equation (23) (Proposition 1), we would now need to add:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1-u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \left. \frac{\partial H_t}{\partial b} \right|_x .$$

Similarly, we would need to modify equation (24) (Proposition 2) to include a term:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1-u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \left. \frac{\partial H_t}{\partial \tau} \right|_x .$$

Both of these terms relate to the adjustment of hours on the intensive margin. Intuitively, the wage plays two roles in the economy: it determines the marginal cost of the firms and therefore the incentives for hiring and it determines the incentives for labor supply on the intensive margin. The first effect is captured in our analysis through dx_t/db. Using our wage rule, the second effect is captured through $\frac{dh_t}{dx_t} \frac{dx_t}{db}$. A more general wage rule brings in these new effects of policy on intensive margin labor supply. A different wage rule would also result in different values for dx_t/db and $dx_t/d\tau$, but these are not qualitatively different considerations than the ones we focus on

already.

B Additional steps for deriving the results in section 3

B.1 The value of employment

In equilibrium, $a_{i,t} = 0$, and search effort is determined by comparing the value of working and not working according to equation (14). This section of the appendix derives the two key steps that make this difference independent of the household's type, so that all households choose the same search effort.

Lemma 3. *Suppose the household's value function has the form*

$$V(\alpha, n, \mathcal{S}) = V^{\alpha}(\alpha, \mathcal{S}) + V^{n}(a, n, \mathcal{S})$$

for some functions V^{α} and V^{n} where $\mathcal{S}$ is the aggregate state. The choice of search effort is then the same for all searching households regardless of α.

Proof: The household's search problem is

$$V^{s}(\alpha, \mathcal{S}) = \max_{q} \left\{ MqV(\alpha, 1, \mathcal{S}) + (1 - Mq)V(\alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

Substitute for the value functions to arrive at

$$V^{s}(\alpha, \mathcal{S}) = \max_{q} \left\{ Mq \left[V^{\alpha}(\alpha, \mathcal{S}) + V^{n}(a, 1, \mathcal{S}) \right] + (1 - Mq) \left[V^{\alpha}(\alpha, \mathcal{S}) + V^{n}(a, 0, \mathcal{S}) \right] - \frac{q^{1+\kappa}}{1 + \kappa} \right\}$$

$$V^{s}(\alpha, \mathcal{S}) = V^{\alpha}(\alpha, \mathcal{S}) + \max_{q} \left\{ MqV^{n}(1, \mathcal{S}) + (1 - Mq)V^{n}(0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \tag{30}$$

where we have brought $V^{\alpha}(\alpha, \mathcal{S})$ outside the max operator as it appears in an additively separable manner. As there is no α inside the max operator, the optimal q is independent of α. □

Lemma 4. *The household's value function can be written as*

$$V(\alpha, n, \mathcal{S}) = \frac{1 - \tau}{1 - \beta} \log(\alpha) + n \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1 + \gamma} + \xi \right] + \bar{V}(\mathcal{S}) \tag{31}$$

for some function $\bar{V}$.

Proof. Suppose that the value function is of the form given in (31). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman equation is in this class by the contraction mapping theorem. V^s will then be

$$V^s(\alpha, \mathcal{S}) = \frac{1-\tau}{1-\beta} \log(\alpha) + Mq^* \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi\right] - \frac{q^{*1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S})$$

and the choice of q^* is independent of α by Lemma 3. Regardless of employment status, the continuation value is

$$(1-\upsilon)V(\alpha', 1, \mathcal{S}') + \upsilon V^s(\alpha', \mathcal{S}')$$

$$= (1-\upsilon)\left[\frac{1-\tau}{1-\beta}\log(\alpha') + \left[\log\frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi\right] + \bar{V}(\mathcal{S}')\right]$$

$$+ \upsilon\left[\frac{1-\tau}{1-\beta}\log(\alpha') + M'q^{*'}\left[\log\frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi\right] - \frac{q^{*'1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}')\right]$$

$$= \frac{1-\tau}{1-\beta}\log(\alpha') + g(\mathcal{S}'),$$

where

$$g(\mathcal{S}) \equiv (1 - \upsilon + \upsilon M'q^{*'})\left[\log\frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma}\right] - (1-\upsilon)\frac{q^{*'1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}').$$

Turning to the Bellman equation, we have

$$V(\alpha, n, \mathcal{S}) = \log\left[\lambda(n + (1-n)b)\left(\alpha(wh + d)\right)^{1-\tau}\right] - n\frac{h^{1+\gamma}}{1+\gamma} - (1-n)\xi + \beta\mathbb{E}\left[\frac{1-\tau}{1-\beta}\log(\alpha') + g(\mathcal{S}')\right]$$

$$= \frac{1-\tau}{1-\beta}\log(\alpha) + n\left[\log\frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi\right]$$

$$+ \log\left[\lambda(wh + d)^{1-\tau}\right] + \log b - \xi + \beta\frac{1-\tau}{1-\beta}\mathbb{E}\left[\log(\epsilon')\right] + \beta\mathbb{E}\left[g(\mathcal{S}')\right].$$

Finally, $\bar{V}(\mathcal{S})$ is given by the second row of the expression above. □

B.2 Proof of lemma 2

First, the Euler equation for a household is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E}\left[\frac{1}{c_{i,t+1}}\right].$$

as usual. At the same time, as we showed in the text, the consumption of an individual is

$$c_{i,t} = \lambda_t \alpha_{i,t}^{1-\tau} \left(w_t h_t + d_t \right)^{1-\tau} (n_{i,t} + (1 - n_{i,t})b). \tag{32}$$

Replacing individual consumption into the Euler equation, and rewriting gives:

$$\frac{1}{\lambda_t \left(w_t h_t + d_t \right)^{1-\tau}} \geq \beta R_t \mathbb{E}_t \left\{ \frac{(1 - u_{t+1}) + u_{t+1} b^{-1}}{\lambda_{t+1} \left(w_{t+1} h_{t+1} + d_{t+1} \right)^{1-\tau}} \right\} \mathbb{E}_t \left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] [n_{i,t} + (1 - n_{i,t})b] . \tag{33}$$

Notice that $\mathbb{E} \left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] = \mathbb{E} \left[\epsilon_{i,t+1}^{\tau-1} \right]$ is common across households. This Euler equation only differs across households due to the final term involving $n_{i,t}$. Assuming the unemployment benefit replacement rate is less than one, this term will be larger for employed than unemployed so in equilibrium all unemployed will be constrained and the Euler equation will hold with equality for all employed.[32]

Summing equation (32) across households gives:

$$C_t = \lambda_t \left(w_t h_t + d_t \right)^{1-\tau} (1 - u_t + u_t b) \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] . \tag{34}$$

Solve (34) for $\lambda_t \left(w_t h_t + d_t \right)^{1-\tau}$ and substitute it into (32) to arrive at:

$$c_{i,t} = \alpha_{i,t}^{1-\tau}(n_{i,t} + (1 - n_{i,t})b) \frac{C_t}{(1 - u_t + u_t b) \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]} . \tag{35}$$

Equations (15) and (20) follow from (35) and the definition of $\tilde{c}_t$.

To derive equations (16) and (17), solve (34) for $\lambda_t \left(w_t h_t + d_t \right)^{1-\tau}$ and substitute it into (33) holding with equality and $n_{i,t} = 1$ to arrive at:

$$\frac{(1 - u_t + u_t b) \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]}{C_t} = \beta R_t \mathbb{E}_t \left\{ \left[(1 - u_{t+1}) + u_{t+1} b^{-1} \right] \frac{(1 - u_{t+1} + u_{t+1} b) \mathbb{E}_i \left[\alpha_{i,t+1}^{1-\tau} \right]}{C_{t+1}} \right\} \mathbb{E} \left[\epsilon_{i,t+1}^{\tau-1} \right]$$

---

[32]Here we follow Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint $a' = 0$ so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See Krusell et al. (2011) for further discussion of this point.

and substitute in $\tilde{c}_t$ using (15):

$$\frac{1}{\tilde{c}_t} = \beta R_t \mathbb{E}_t \left\{ \left[(1 - u_{t+1}) + u_{t+1} b^{-1} \right] \frac{1}{\tilde{c}_{t+1}} \right\} \mathbb{E} \left[\epsilon_{i,t+1}^{\tau-1} \right].$$

Rearranging gives eqations (16) and (17) as $\mathbb{E} \left[\epsilon_{i,t+1}^{\tau-1} \right]$ is known at date t.

B.3 Optimal hours worked and search effort

We start by deriving equation (18). Using labor market clearing and the definition of A_t we can write the aggregate production function as

$$Y_t = A_t h_t (1 - u_t). \tag{36}$$

Output net of hiring costs is paid to employed workers in the form of wage and dividend payments. As the average $\alpha_{i,t}$ is equal to one we have

$$Y_t - J_t = (w_t h_t + d_t)(1 - u_t). \tag{37}$$

Multiply both sides of (36) by $(Y_t - J_t)/Y_t$ and substitute for $Y_t - J_t$ using (37) to give:

$$w_t h_t + d_t = A_t h_t \frac{Y_t - J_t}{Y_t}.$$

Substitute this for $w_t h_t + d_t$ in equation (13) to arrive at

$$h_t^\gamma = \frac{(1 - \tau) w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}. \tag{38}$$

Finally, use the wage rule in equation (5) to arrive at (18).

We turn next to derive optimal search effort in equation (19). By Lemma 4, the results of Lemma 3 can be applied. Proceed from equation (30) using the functional form for the value function in Lemma 4. This leads to

$$\max_q \left\{ M q \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1 + \gamma} + \xi \right] - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

The first order condition yields equation (19).

B.4 Equilibrium definition

We first state some addition equilibrium conditions and then state a definition of an equilibrium. The price-setting first order condition is

$$p_t^* = \mu \left(\frac{w_s h_s + \psi_1 M_s^{\psi_2}}{A_s h_s} \right) \tag{39}$$

for firms with full information and $\mathbb{E}_{t-1}(p_t^*)$ for others. Inflation satisfies

$$p_t^{\frac{1}{1-\mu}} = \theta p_t^{*(\frac{1}{1-\mu})} + (1-\theta) \mathbb{E}_{t-1}(p_t^*)^{\frac{1}{1-\mu}} \tag{40}$$

and price dispersion evolves according to

$$S_t = \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \left[\theta + (1-\theta) \left(\frac{\mathbb{E}_{t-1} p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} \right].. \tag{41}$$

The aggregate resource constraint is:

$$Y_t - J_t = C_t + G_t. \tag{42}$$

The Fisher equation is:

$$R_t = I_t / \mathbb{E}_t [\pi_{t+1}] . \tag{43}$$

The link between $\tilde{c}_t$ and C_t depends on $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$. This evolves according to:

$$\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] = (1-\delta) \mathbb{E}_i \left[\alpha_{i,t-1}^{1-\tau} \right] \mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau} \right] + \delta.. \tag{44}$$

An equilibrium of the economy can be calculated from a system equations in 17 variables and three exogenous processes. The variables are

$$C_t, \tilde{c}_t, u_t, \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, q_t, M_t.$$

And the equations are: (3), (4), (5), (6), (7), (15) (16), (17), (18), (19), (36), (39), (40), (41), (42), (43), and (44). The exogenous processes are η_t^A, η_t^G, and η_t^I.

In the quantitative model with Calvo pricing, equations (39), (40), and (41) become

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s h_s + \psi_1 M_s^{\psi_2}\right)/(A_s h_s)}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}$$

$$\pi_t = \left[(1-\theta)/\left[1 - \theta\left(\frac{p_t^*}{p_t}\right)^{1/(1-\mu)}\right]\right]^{1-\mu}$$

$$S_t = (1-\theta) S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta\left(\frac{p_t^*}{p_t}\right)^{\mu/(1-\mu)}.$$

where p_t^*/p_t is the relative price chosen by firms that adjust their price in period t.

C   Proofs for section 4

C.1   Skill dispersion without mortality

Lemma 5. *Under no mortality, $\delta = 0$:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_i \left[\log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right)\right]$$

$$= \frac{1}{1-\beta}\left[\mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right)\right] + \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{\beta^{t+1}}{1-\beta}\left[\mathbb{E}_i \log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right)\right].$$

Proof: When there is no mortality, $\delta = 0$, we can compute the cumulative welfare effect of a change in $F(\epsilon_{i,t+1}, x_t)$ including the effects on current and future skill dispersion. In particular

$$\mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) = \mathbb{E}_i \log\left(\alpha_{i,t-1}^{1-\tau} \epsilon_{i,t}^{1-\tau}\right)$$

$$= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau} \epsilon_{i,1}^{1-\tau} \cdots \epsilon_{i,t}^{1-\tau}\right)$$

$$= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) + \mathbb{E}_i \log\left(\epsilon_{i,1}^{1-\tau}\right) + \cdots + \mathbb{E}_i \log\left(\epsilon_{i,t}^{1-\tau}\right).$$

Similarly

$$\log \left(\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] \right) = \log \left(\mathbb{E}_i \left[\alpha_{i,t-1}^{1-\tau} \right] \mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau} \right] \right)$$

$$= \log \left(\mathbb{E}_i \left[\alpha_{i,0}^{1-\tau} \right] \mathbb{E}_i \left[\alpha_{i,1}^{1-\tau} \right] \cdots \mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau} \right] \right)$$

$$= \log \left(\mathbb{E}_i \left[\alpha_{i,0}^{1-\tau} \right] \right) + \log \left(\mathbb{E}_i \left[\alpha_{i,1}^{1-\tau} \right] \right) + \cdots + \log \left(\mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau} \right] \right)$$

Notice that in this no-mortality case, the date-t loss from skill dispersion can be written as:

$$\mathbb{E}_i \log \left(\alpha_{i,t}^{1-\tau} \right) - \log \left(\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] \right) = \mathbb{E}_i \log \left(\alpha_{i,0}^{1-\tau} \right) - \log \left(\mathbb{E}_i \left[\alpha_{i,0}^{1-\tau} \right] \right) + \sum_{s=1}^{t} \left[\mathbb{E}_i \log \left(\epsilon_{i,s}^{1-\tau} \right) - \log \left(\mathbb{E}_i \left[\epsilon_{i,s}^{1-\tau} \right] \right) \right].$$

Finally, take the expected discounted sum of this expression and rearrange to prove the result. $\square$

Lemma 6. *For a random variable X,*

$$\frac{d}{d\tau} \left\{ \mathbb{E} \left[\log \left(X^{1-\tau} \right) \right] - \log \left(\mathbb{E} \left[X^{1-\tau} \right] \right) \right\} = \frac{\mathrm{Cov} \left(X^{1-\tau}, \log X \right)}{\mathbb{E} \left[X^{1-\tau} \right]}$$

Proof:

$$\frac{d}{d\tau} \left\{ \mathbb{E} \left[\log \left(X^{1-\tau} \right) \right] - \log \left(\mathbb{E} \left[X^{1-\tau} \right] \right) \right\} = -\mathbb{E} \left[\log \left(X \right) \right] + \frac{\mathbb{E} \left[X^{1-\tau} \log X \right]}{\mathbb{E} \left[X^{1-\tau} \right]}$$

$$= -\mathbb{E} \left[\log \left(X \right) \right] + \frac{\mathbb{E} \left[X^{1-\tau} \right] \mathbb{E} \left[\log X \right] + \mathrm{Cov} \left(X^{1-\tau}, \log X \right)}{\mathbb{E} \left[X^{1-\tau} \right]}$$

$$= \frac{\mathrm{Cov} \left(X^{1-\tau}, \log X \right)}{\mathbb{E} \left[X^{1-\tau} \right]}$$

$\square$

C.2   Proof of Proposition 1

For this proof, in addition to the social welfare function, (22), the relevant equations of the model are (3), (36), (4), (42), (19), and (18).

The first-order condition of the social welfare function with respect to b is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} + \frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \bigg|_x - v q_t^\kappa \frac{\partial q_t}{\partial b} \bigg|_x + \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = 0. \tag{45}$$

The first two terms in (45) can be expressed as

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left(\frac{1}{b} - 1 + 1 - \frac{1}{1 - u_t + u_t b} \right)$$

$$= u_t \left(\frac{1}{b} - 1 \right) \left(1 - \frac{u_t b}{1 - u_t + u_t b} \right)$$

and note that

$$\left. \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{x,q} = \frac{\partial}{\partial \log b} \log \left(\frac{bC_t}{\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] (1 - u_t + u_t b)} \right) \tag{46}$$

$$= 1 - \frac{u_t b}{1 - u_t + u_t b}$$

where the partial derivative on the right hand side of (46) is with respect to b alone.[33] So, we have:

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left(\frac{1}{b} - 1 \right) \left. \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{x,q} \tag{47}$$

For the third and fourth terms in (45), start by noting that:

$$\frac{dW_t}{du_t} = \log b + \frac{1-b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t} + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \tag{48}$$

$$= \left(\log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \right) + \left. \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \right|_x \tag{49}$$

where

$$\left. \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \right|_x = \frac{1-b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t}.$$

Then, it follows that:

$$\left. \frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \right|_x - \upsilon q_t^\kappa \left. \frac{\partial q_t}{\partial b} \right|_x = \left[\left(\log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi \right) + \left. \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \right|_x \right] \left. \frac{\partial u_t}{\partial b} \right|_x - \upsilon q_t^\kappa \left. \frac{\partial q_t}{\partial b} \right|_x.$$

---

[33]As $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$ is an endogenous state that depends on the history of x, we are taking the partial derivative holding fixed this history.

Using equation (19), this becomes

$$
\begin{aligned}
\frac{dW_t}{du_t} \left.\frac{\partial u_t}{\partial b}\right|_x - \upsilon q_t^\kappa \left.\frac{\partial q_t}{\partial b}\right|_x &= \left[\left(\log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi\right) + \frac{1}{\tilde{c}_t}\left.\frac{\partial \tilde{c}_t}{\partial u_t}\right|_x\right]\left.\frac{\partial u_t}{\partial b}\right|_x + \left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{du_t}{dq_t}\left.\frac{\partial q_t}{\partial b}\right|_x \\
&= \frac{1}{\tilde{c}_t}\left.\frac{\partial \tilde{c}_t}{\partial u_t}\right|_x \left.\frac{\partial u_t}{\partial b}\right|_x \\
&= \left.\frac{\partial \log \tilde{c}_t}{\partial \log u_t}\right|_x \left.\frac{\partial \log u_t}{\partial b}\right|_x .
\end{aligned}
\tag{50}
$$

Substituting (47) and (50) into (45) yields the result.

C.3 Proof of Proposition 2

First we use Lemma 5 to substitute for $\mathbb{E}_0 \sum_{t=0}^\infty \beta^t \mathbb{E}_i \left[\log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right)\right]$ in the social welfare function. Then we proceed as in the proof of Proposition 1. The first-order condition of the social welfare function with respect to τ is:

$$
\frac{\text{Cov}\left(\alpha_{i,0}^{1-\tau}, \log \alpha_{i,0}\right)}{\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right](1-\beta)} + \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \left\{\frac{\beta}{1-\beta}\frac{\text{Cov}\left(\epsilon_{i,t+1}^{1-\tau}, \log \epsilon_{i,t+1}^{1-\tau}\right)}{\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]} + \frac{dW_t}{dh_t}\left.\frac{\partial h_t}{\partial \tau}\right|_x + \frac{dW_t}{dx_t}\frac{dx_t}{d\tau}\right\} = 0,
\tag{51}
$$

where we have used Lemma 6 twice. From (22), (36), and (42) we have

$$
\frac{dW_t}{dh_t} = \frac{dW_t}{du_t}\frac{du_t}{dq_t}\frac{dq_t}{dh_t} + \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma - \upsilon q_t^\kappa \frac{dq_t}{dh_t}.
$$

From (19) we have

$$
\left.\frac{\partial h_t}{\partial \tau}\right|_x = -\frac{h_t}{(1-\tau)(1+\gamma)}
$$

combining these and using (19), (49), and $du_t/dq_t = -\upsilon M_t$ we arrive at

$$
\begin{aligned}
\frac{dW_t}{dh_t}\frac{\partial h_t}{\partial \tau}\bigg|_x &= \frac{dW_t}{du_t}\frac{du_t}{dq_t}\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial \tau}\bigg|_x + \frac{A_t(1-u_t)}{C_t}\frac{\partial h_t}{\partial \tau}\bigg|_x - (1-u_t)h_t^\gamma\frac{\partial h_t}{\partial \tau}\bigg|_x - \upsilon q_t^\kappa\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial \tau}\bigg|_x \\
&= \frac{1}{\tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial u_t}\bigg|_x\frac{du_t}{dq_t}\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial \tau}\bigg|_x - \left[\frac{A_t h_t(1-u_t)}{C_t} - (1-u_t)h_t^{1+\gamma}\right]\frac{1}{(1-\tau)(1+\gamma)} \\
&= \frac{\partial \log \tilde{c}_t}{\partial \log u_t}\bigg|_x\frac{\partial \log u_t}{\partial \tau}\bigg|_x - \left[\frac{A_t}{C_t} - h_t^\gamma\right]\frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)}.
\end{aligned}
$$

Substituting this into (51) yields the desired result.

D   Proofs for section 5

D.1   Proof of Proposition 3

Proceeding as in the proof of Proposition 2 we have

$$
\begin{aligned}
\frac{dW_t}{dx_t} = &\frac{dW_t}{du_t}\frac{du_t}{dx_t} - \upsilon q_t^\kappa\frac{dq_t}{dx_t} + \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\frac{\partial J_t}{\partial x_t}\bigg|_u \\
&- \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t} + \frac{\beta}{1-\beta}\frac{d}{dx_t}\left[\mathbb{E}_i \log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right)\right]
\end{aligned}
\tag{52}
$$

where dW_t/du_t is given by (49). We rearrange (52) to arrive at the desired result. First, note that:

$$
\frac{du_t}{dx_t} = -\upsilon q_t\frac{dM_t}{dx_t} - \upsilon M_t\frac{dq_t}{dx_t}.
\tag{53}
$$

Using this, (19), and (49), equation (52) then becomes

$$
\begin{aligned}
\frac{dW_t}{dx_t} = &\left(-\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right) + \frac{1}{\tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial u_t}\bigg|_x\right)\left(-\upsilon q_t\frac{dM_t}{dx_t} - \upsilon M_t\frac{dq_t}{dx_t}\right) - \upsilon M_t\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{dq_t}{dx_t} \\
&+ \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\frac{\partial J_t}{\partial x_t}\bigg|_u - \frac{\beta}{1-\beta}(1-\tau)^2\frac{d}{dx_t}\frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t} \\
= &\upsilon q_t\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{dM_t}{dx_t} - \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t} + \frac{1}{\tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial u_t}\bigg|_x\frac{du_t}{dx_t} \\
&+ \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\frac{\partial J_t}{\partial x_t}\bigg|_u + \frac{\beta}{1-\beta}\frac{d}{dx_t}\left[\mathbb{E}_i \log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right)\right].
\end{aligned}
$$

This can be rearranged to the desired result by making use of

$$\frac{1}{\tilde{c}_t}\frac{\partial \tilde{c}_t}{\partial u_t}\bigg|_x = \frac{1-b}{1-u_t+u_tb} - \frac{A_th_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t} = \frac{1}{C_t}\frac{\partial C_t}{\partial u_t}\bigg|_x + \frac{1-b}{1-u_t+u_tb},$$

$$\mathbb{E}_i \log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right) = \int \log\left(\frac{\epsilon^{1-\tau}}{\int \epsilon^{1-\tau}dF(\epsilon,x_t)}\right)dF(\epsilon,x_t),$$

and

$$-vq_t\frac{dM_t}{dx_t} = \frac{\partial u_t}{\partial x_t}\bigg|_q.$$

D.2 Deriving the special case in section 5.2.3

We can normalize $\mathbb{E}_{t-1}p_t = 1$. A fraction θ of firms set the price p_t^* and the remaining set the price $\mathbb{E}_{t-1}p_t^* = 1$. From the definition of the price index we can solve for

$$\left(\frac{p_t^*}{p_t}\right)^{\mu/(1-\mu)} = \left(\frac{1-(1-\theta)p_t^{-1/(1-\mu)}}{\theta}\right)^{\mu}.$$

Substituting into (21) we arrive at

$$S_t = (1-\theta)p_t^{\mu/(\mu-1)} + \theta^{1-\mu}\left(1-(1-\theta)p_t^{1/(\mu-1)}\right)^{\mu}.$$

This makes clear that S_t is a function of p_t. Differentiating this function we arrive at

$$S'(1) = (1-\theta)\frac{\mu}{\mu-1}\left[p_t^{1/(\mu-1)} - \theta^{1-\mu}\left(1-(1-\theta)p_t^{1/(\mu-1)}\right)^{\mu-1}p_t^{(2-\mu)/(\mu-1)}\right] = 0$$

$$S''(1) = \frac{1-\theta}{\theta}\frac{\mu}{\mu-1}.$$

Next, rewrite the welfare function as

$$W_t = \log(C_t) - (1-u_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \chi\log(G_t) + \text{t.i.p.},$$

$$C_t = Y_t - J_t - G_t,$$

$$Y_t = \frac{\eta_t^A}{S(p_t)}h_t(1-u_t)$$

$$G_t = \chi C_t.$$

Observe that u_t and J_t are exogenous in this case as M_t and q_t are exogenous. Use the production function to rewrite h_t in terms of Y_t

$$h_t = \frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}.$$

Now rewrite W_t in terms of Y_t and p_t

$$W_t = (1 + \chi) \log\left(\frac{Y_t - J_t}{1 + \chi}\right) - \frac{1 - u_t}{1 + \gamma}\left(\frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}\right)^{1+\gamma}.$$

With flexible prices we have $p = 1$ and $S = 1$. The flexible-price, socially-efficient level of output, Y^*, satisfies

$$\frac{\eta^A}{Y_t^* - J_t - G_t} = \left(\frac{Y_t^*}{\eta_t^A(1 - u_t)}\right)^{\gamma}.$$

Treating W_t as a function of Y_t and p_t, we take a second-order Taylor approximation around the point $(Y_t^*, 1)$. The first-derivatives are

$$W_Y(Y_t^*, 1) = \frac{1}{Y_t - J_t - G_t} - \left(\frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}\right)^{\gamma} \frac{S(p_t)}{\eta_t^A}$$

$$W_p(Y_t^*, 1) = -(1 - u_t)\left(\frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}\right)^{\gamma} \frac{Y_t}{\eta_t^A(1 - u_t)} S'(p_t)$$

and both are zero, the former because Y^* is optimal and the latter because $S'(1) = 0$. The second derivatives are

$$W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2} - \gamma(Y_t^*)^{\gamma-1}\left(\frac{S(p_t)}{\eta_t^A}\right)^{1+\gamma}\left(\frac{1}{1 - u_t}\right)^{\gamma},$$

$$W_{Yp}(Y_t^*, 1) = -\left(\frac{Y_t^*}{1 - u_t}\right)^{\gamma}\left(\frac{1}{\eta_t^A}\right)^{1+\gamma}(1 + \gamma)S(p_t)^{\gamma}S'(p_t),$$

$$W_{pp}(Y_t^*, 1) = -(1 - u_t)\left(\frac{Y_t^*}{\eta_t^A(1 - u_t)}\right)^{1+\gamma}\left[\gamma S(p_t)^{\gamma-1}S'(p_t) + S(p_t)^{\gamma}S''(p_t)\right].$$

Using $S(1) = 1, S'(1) = 0$, the expression for $S''(1)$ above, and the optimality condition for Y_t^* we

arrive at

$$W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2}\left[1 + \gamma\frac{C_t^*}{Y_t^*}\right],$$

$$W_{Yp}(Y_t^*, 1) = 0,$$

$$W_{pp}(Y_t^*, 1) = -\frac{Y_t^*}{C_t^*}\frac{1-\theta}{\theta}\frac{\mu}{\mu-1}.$$

By Taylor's theorem we can write

$$W(Y_t, p_t) \approx \frac{1}{2}W_{YY}(Y_t^*, 1)(Y_t - Y_t^*)^2 + \frac{1}{2}W_{pp}(Y_t^*, 1)(p_t - 1)^2$$

Observe that Y and p are functions of x. So when we differentiate with respect to x we arrive at

$$\frac{dW_t}{dx_t} \approx W_{YY}(Y_t^*, 1)(Y_t - Y_t^*)\frac{dY_t}{dx_t} + W_{pp}(Y_t^*, 1)(p_t - 1)\frac{dp_t}{dx_t}.$$

Substituting for W_{YY} and W_{pp} and rearranging yields the result.

E The link from the stabilizers to slack

As stated in the text, assume now that: (i) aggregate shocks only occur at date 0 and there is no aggregate uncertainty after that, (ii) the household search effort is exogenous and constant ($\kappa = \infty$), and (iii) the income-risk distribution $F(.)$ is log-normal with $\text{Var}(\log \epsilon) = \sigma^2(x)$ and $\mathbb{E}(\log \epsilon) = -0.5\sigma(x)^2$. With these simplifications, the household decisions are reduced to consumption and hours worked.

The first step in the analysis is to express the equilibrium in date 0 in terms of as few variables as possible, namely x_0. First, it follows from equations (3), (4), (5), and (36) that we can express J, u, Y, and w as functions of x and non-policy parameters. It then follows that $j \equiv J/Y$ is a function of x. Second, a firm that sets its price in period 0 will set the optimal markup over current marginal cost because all prices will be re-optimized in subsequent periods. Therefore the optimal relative price for a firm that updates in period 0 is $\mu\left(w_0 + \psi_1 M_0^{\psi_2}/h(x_0, \tau)\right)/A_0$ where $h(x, \tau)$ is given by (18). The price level and inflation rate in period 0 are therefore functions of x_0 and τ. Third, it follows from (6) that the same is true of the nominal interest rate I_0 and S_0. The Fisher
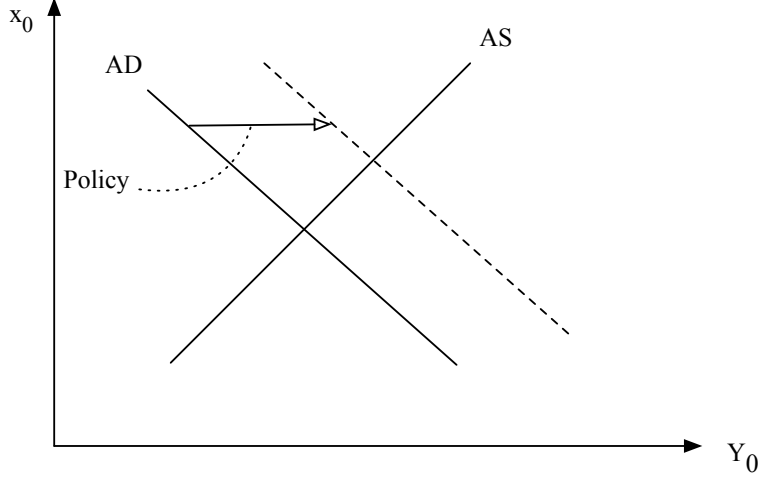
Figure 5: Determination of x_0.

equation (43) and the assumption that the central bank sets inflation to zero in period 1 implies that R_0 is also a function of x_0 and τ.

The next step is to derive an "aggregate demand curve" from the aggregate Euler equation. Using equations (15), (16), and (17) along with the log-normal income process we arrive at:

$$\frac{(1 - u_t + u_t b)}{C_t} = \beta R_t e^{\sigma_\epsilon^2 (x_t)(1-\tau)^2} \mathbb{E}_t \left[\frac{\left(1 - u_{t+1}(1-b)\right)\left(1 - u_{t+1}(1-b^{-1})\right)}{C_{t+1}} \right].$$

For $t \geq 0$ the expectation in the Euler equation disappears because outcomes in $t + 1$ are deterministic. We can re-write the Euler equation as

$$C_0 = \frac{1 - u_0 + u_0 b}{\beta R_0(x_0)} \frac{C_1}{\left(1 - u_1(1-b)\right)\left(1 - u_1(1-b^{-1})\right)} e^{-\sigma_\epsilon^2 (x_0)(1-\tau)^2}.$$

Next, recall from market clearing that: $Y_0(1 - j_0) = C_0(1 + \chi \eta_0^G)$ Replacing out C_0 gives:

$$Y_0 = \frac{1 + \chi \eta_0^G}{1 + \chi \eta_1^G} \frac{1 - j_1}{1 - j_0} \frac{1 - u_0 + u_0 b}{\beta R(x_0)} \frac{Y_1}{\left(1 - u_1(1-b)\right)\left(1 - u_1(1-b^{-1})\right)} e^{-\sigma_\epsilon^2 (M_0)(1-\tau)^2}. \quad (54)$$

This provides an "AD" curve: a negative relation between x_0 and Y_0 that came from combining the aggregate Euler equation, the monetary rule, and the Phillips curve.

The aggregate production function, (36), is the "AS" curve: a positive relation between x_0 and Y_0. Combing AD and AS in figure 5, the equilibrium is at the intersection of the two curves.

The impact of the social policies on equilibrium slack depends on two features of the diagram. First, how much does a change in policy horizontally shift the AD curve? A bigger shift in AD will lead to a larger effect on slack. Second, what are the slopes of the two curves? If both the AD and the AS are steeper, then a given horizontal shift in the AD brought about by the policies leads to a larger change in slack at the new equilibrium. We discuss shifts and slopes separately.

E.1 The general equilibrium

Before proceeding, we solve for the intersection of the two curves, the equilibrium of the economy. First, the AD curve depends on Y_1. Under flexible prices, the price-setting first-order condition implies

$$w(x_t) + \frac{\psi_1 x_t^{\psi_2}}{h(x_t, \tau)} = \frac{\eta_t^A}{\mu}.$$

This equation implies x_t is a function of η_t^A, τ, and non-policy parameters when prices are flexible. We call this function $x^{\text{Flex}}(\eta_t^A, \tau)$. Y_1 can be calculated from (36) with $x_1 = x^{\text{Flex}}(\eta_1^A, \tau)$ because all prices are flexible in period 1. This leads to

$$Y_1 = \eta_1^A(1 - u(x_1))\left[\bar{w}(1 - \tau)\right]^{\frac{1}{1+\gamma}} (x_1)^{\frac{\zeta}{1+\gamma}} \tag{55}$$

Note that x_1 is given by $x^{\text{Flex}}(\eta_1^A, \tau)$ so (55) makes Y_1 independent of x_0 and b.

We now set AS = AD. Specifically, equations (54) and (36) lead to two expressions for Y_0. Taking the log of equations (54) and (36) and set them equal to one another yields an implicit solution for x_0:

$$\log\left(\frac{1 - \chi\eta_0^G}{1 - \chi\eta_1^G}\right) + \log(1 - j_1) - \log(1 - j_0) \tag{56}$$

$$+ \log\left(1 - u_0 + u_0 b\right) - \log\beta - \log\left(R(x_0, \tau)\right) + \log\left(Y_1(x_1^{\text{Flex}}, \tau)\right) - \log\left(1 - u_1(1 - b)\right)$$

$$- \log\left(1 - u_1(1 - b^{-1})\right) - \sigma_\epsilon^2(x_0)\left(1 - \tau\right)^2 - \log\left(\eta_0^A\right) + \log\left(S_0\right) - \log\left(h_0\right) - \log\left((1 - u_0)\right) = 0.$$

E.2 Unemployment benefits and slack

The first result is:

Proposition 4. *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log b} = \Lambda^{-1} \left[\frac{u_0 b}{1 - u_0 + u_0 b} + \frac{u_1 b^{-1}}{1 - u_1(1 - b^{-1})} - \frac{u_1 b}{1 - u_1(1 - b)} \right] \tag{57}$$

where Λ is defined below in lemma 3.

Proof. x_0 is implicitly defined by (56). Using the implicit function theorem we obtain

$$\frac{dx_0}{db} = - \frac{\frac{u_0}{1 - u_0 + u_0 b} - \frac{u_1}{1 - u_1(1-b)} + \frac{u_1 b^{-2}}{1 - u_1(1-b^{-1})}}{-\frac{1-b}{1-u_0+u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1-\tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1-u_0}\frac{du_0}{dx_0} + \frac{1}{1-j_0}\frac{dj_0}{dx_0}}$$

As an elasticity we have

$$\frac{b}{x_0}\frac{dx_0}{db} = \frac{\frac{u_0 b}{1 - u_0 + u_0 b} - \frac{u_1 b}{1 - u_1(1-b)} + \frac{u_1 b^{-1}}{1 - u_1(1-b^{-1})}}{\frac{d\log R_0}{d\log x_0} + (1-\tau)^2\sigma_\epsilon^2(x_0)\frac{d\log\sigma_\epsilon^2(x_0)}{d\log x_0} + \frac{1-b}{1-u_0+u_0 b}u_0\frac{d\log u_0}{d\log x_0} - \frac{d\log S_0}{d\log x_0} + \frac{d\log h_0}{d\log x_0} + \frac{d\log(1-u_0)}{d\log x_0} + \frac{d\log(1-j_0)}{d\log x_0}}$$
$$\tag{58}$$

$\square$

There are two direct effects of raising unemployment benefits on economic slack. The first shows the usual logic of the unemployment insurance system as an automatic stabilizer based on redistribution: aggregate demand responds more strongly to benefits if there are more unemployed workers. This effect arises because the unemployed have a high marginal propensity to consume so the extra benefits lead to an increase in demand.

Second, there is an additional effect coming from expected marginal utility in the future, which depends on the unemployment rate in the next period ($t = 1$). Expected marginal utility is determined by two components. The first is the social insurance that UI provides, lowering uncertainty and precautionary savings and so pushing up aggregate demand today. The second is the higher taxes in order to finance the higher benefits, which reduce consumption. If the former component dominates, then the two effects, captured by the expression in square brackets, shift the AD curve rightwards. As the unemployment rate increases in recessions, both of these effects point towards a counter-cyclical elasticity.

For the second component to dominate requires that expected marginal utility rises with u_1. For unemployment rates less than 50%, this is true. The proof is as follows: start with $\frac{u_0 b}{1 - u_0(1-b)}$,

the derivative with respect to u_0 is

$$\frac{b(1 - u_0(1 - b)) + u_0 b(1 - b)}{[1 - u_0(1 - b)]^2} = \frac{b}{[1 - u_0(1 - b)]^2} > 0.$$

Next turn to $\frac{bu_1(1-u_1)(b^{-2}-1)}{[1+u_1(b^{-1}-1)][1+u_1(b-1)]}$. We start by dividing by $b(b^{-2} - 1)$, which is positive for $b \in (0, 1)$. We will then show the derivative with respect to u_1 is positive

$$\frac{1 - 2u_1}{\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\}^2} [1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]$$
$$- \frac{u_1(1 - u_1)}{\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\}^2} [(b^{-1} - 1)(1 + u_1(b - 1)) + (b - 1)(1 + u_1(b^{-1} - 1))] > 0$$

multiply both sides by $\left\{[1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)]\right\}^2$

$$[1 - 2u_1][1 + u_1(b^{-1} - 1)][1 + u_1(b - 1)] - u_1(1 - u_1)[(b^{-1} - 1)(1 + u_1(b - 1)) + (b - 1)(1 + u_1(b^{-1} - 1))] > 0$$

Rearranging

$$(1 - 2u_1)[1 + (u_1 - u_1^2)(b^{-1} + b - 2)] - u_1(1 - u_1)(b^{-1} + b - 2)(1 - 2u_1) > 0$$

divide both sides $(1 - 2u_1)$ (recall $u_1 < 1/2$) and rearrange

$$1 + u_1(1 - u_1)(b^{-1} + b - 2) - u_1(1 - u_1)(b^{-1} + b - 2) = 1 > 0.$$

The overall effect on tightness is tempered by the slopes of AD and AS, through the term Λ. We explain this effect below, but since it is common to the effect of more progressive taxes, we turn to that first.

E.3 Tax progressivity and slack

The next result is:

Proposition 5. *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log \tau} = \Lambda^{-1}\left[2\sigma_\epsilon^2(x_0)(1 - \tau)\tau - \left.\frac{\partial \log R_0}{\partial \log \tau}\right|_x + \left.\frac{\partial \log S_0}{\partial \log \tau}\right|_x + \frac{d \log Y_1}{d \log x_1}\frac{d \log x_1}{d \log \tau}\right] \tag{59}$$

where Λ is defined below in lemma 3.

Proof. Using the implicit function theorem to differentiate (56) we obtain

$$
\frac{dx_0}{d\tau} = -\frac{2\sigma_\epsilon^2(x_0)(1-\tau) + \frac{1}{Y_1}\frac{1}{1+\gamma}\frac{Y_1}{1-\tau} - \frac{1}{h_0}\frac{1}{1+\gamma}\frac{h_0}{1-\tau} + \frac{1}{S_0}\frac{\partial S_0}{\partial \tau}\big|_x - \frac{1}{R_0}\frac{\partial R_0}{\partial \tau}\big|_x + \frac{1}{Y_1}\frac{dY_1}{dx_1}\frac{dx^{\text{Flex}}_{(A_1,\tau)}}{d\tau}}{-\frac{1-b}{1-u_0+u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1-\tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1-u_0}\frac{du_0}{dx_0} + \frac{1}{1-j_0}\frac{dj_0}{dx_0}}
$$

$$
= -\frac{2\sigma_\epsilon^2(x_0)(1-\tau) - \frac{1}{R_0}\frac{\partial R_0}{\partial \tau}\big|_x + \frac{1}{S_0}\frac{\partial S_0}{\partial \tau}\big|_x + \frac{1}{Y_1}\frac{dY_1}{dx_1}\frac{dx^{\text{Flex}}_{(A_1,\tau)}}{d\tau}}{-\frac{1-b}{1-u_0+u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1-\tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1-u_0}\frac{du_0}{dx_0} + \frac{1}{1-j_0}\frac{dj_0}{dx_0}}
$$

As an elasticity we have

$$
\frac{\tau}{x_0}\frac{dx_0}{d\tau} = \frac{2\sigma_\epsilon^2(x_0)(1-\tau)\tau - \frac{\partial \log R_0}{\partial \log \tau}\big|_x + \frac{\partial \log S_0}{\partial \log \tau}\big|_x + \frac{d\log Y_1}{d\log x_1}\frac{d\log x_1}{d\log \tau}}{\frac{d\log R_0}{d\log x_0} + (1-\tau)^2\sigma_\epsilon^2(x_0)\frac{d\log \sigma_\epsilon^2(x_0)}{d\log x_0} + \frac{1-b}{1-u_0+u_0 b}u_0\frac{d\log u_0}{d\log x_0} - \frac{d\log S_0}{d\log x_0} + \frac{d\log h_0}{d\log x_0} + \frac{d\log(1-u_0)}{d\log x_0} + \frac{d\log(1-j_0)}{d\log x_0}}
$$

$$\tag{60}$$

$\square$

The first term in between square brackets reflects the effect of social insurance on aggregate demand. When households face uninsurable skill risk, a progressive tax system will raise aggregate demand by reducing the precautionary savings motive. This term will be counter-cyclical if risk increases in a recession as has been documented by Guvenen et al. (2014).

The remaining terms inside the brackets reflect the effect of a change in τ on marginal cost holding x_t fixed. These terms arise because hiring costs are spread over the hours of the worker so that even at given levels of w_t and M_t, a more progressive tax would lower hours worked per worker and raise hiring costs per hour worked. If labor supply were fixed on the intensive margin, these terms would not be present. With a choice of hours of work, we see that higher marginal costs put upward pressure on prices which is reflected in real interest rates via the monetary policy rule and in price dispersion. For similar reasons, an increase in τ puts upward pressure on marginal cost in period 1. As prices are flexible in period 1 this force leads to a reduction in labor market tightness in period 1 so that marginal cost is again equal to the inverse of the desired markup. Because these effects all operate through rescaling the hiring costs, which are themselves small, they are not likely to be important quantitatively.

E.4 Slopes of AS and AD

Finally, we turn to Λ, which reflects the slopes of the AS and AD curves. As the previous two propositions showed, a larger Λ attenuates the effect of the social policies on slack, because it makes both AS and AD flatter. The next lemma describes what determines it:

Lemma 7. *Under the assumptions of section 5.3:*

$$
\begin{aligned}
\Lambda = & \frac{d \log R_0}{d \log x_0} + (1 - \tau)^2 \sigma_\epsilon^2(x_0) \frac{d \log \sigma_\epsilon^2(x_0)}{d \log x_0} + \frac{1 - b}{1 - u_0 + u_0 b} u_0 \frac{d \log u_0}{d \log x_0} \\
& + \frac{d \log h_0}{d \log x_0} + \frac{d \log(1 - u_0)}{d \log x_0} + \frac{d \log(1 - J_0/Y_0)}{d \log x_0} - \frac{d \log S_0}{d \log x_0}
\end{aligned}
\tag{61}
$$

Proof. See (58) and (60). $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

The first line has the three terms that affect the slope of the AD curve. The first effect involves the real interest rate. A booming economy leads to higher nominal interest rates, both directly via the Taylor rule and indirectly via higher inflation. With nominal rigidities, this raises the real interest rate, which dampens the effectiveness of any policy on equilibrium slack. In other words, a more aggressive monetary policy rule (or more flexible prices) makes AD flatter and so attenuates the effectiveness of social policies. An extreme example of this is when the economy is at the zero lower bound, which magnifies the effectiveness of the automatic stabilizers.[34]

The second term refers to income risk. If risk is counter-cyclical, this term is negative, so it makes social programs more powerful in affecting slack. The reason is that there is a destabilizing precautionary savings motive that amplifies demand shocks. In response to a reduction in aggregate demand, labor market tightness falls, leading to an increase in risk and an increase in the precautionary savings motive and so a further reduction in aggregate demand. These reinforcing effects make the AD curve steeper so that social policies become more effective.

The third term reflects the impact of economic expansions on the number of employed households, who consume more than unemployed households. Therefore, aggregate demand rises as employment rises in a tighter labor market. This consumption multiplier makes the AD curve steeper and increases the effectiveness of social programs.

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[34] McKay and Reis (2016) and Kekre (2017) show that automatic stabilizers and unemployment benefits, respectively, have stronger stimulating effects when the economy is at the zero lower bound.

The second line in the lemma has the four terms that affect the slope of the AS. Increasing slack raises hours worked or employment, this makes the AS flatter as output increases by relatively more, so it raises Λ and attenuates the effect of social programs. This occurs net of hiring costs, since the fact that they increase with slack works in the opposite direction. Finally, if in a booming economy the efficiency loss from price dispersion increases, so S increases, then the AS is likewise steeper and Λ is lower.

F   Description of methods for section 6

F.1   Estimated income process

The material in this appendix describes an implementation of the procedure of Guvenen and McKay (2017).

The income process is as follows: $\alpha_{i,t}$ evolves as in (2). Earnings are given by $\alpha_{i,t}w_t$ when employed and zero when unemployed. Notice that here we normalize $h_t = 1$ and subsume all movements in h_t into w_t. While this gives a different interpretation to w_t it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\ N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\ N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\ N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 \end{cases}$$

The tails of F move over time as driven by the latent variable x_t such that

$$\mu_{1,t} = \bar{\mu}_t,$$
$$\mu_{2,t} = \bar{\mu}_t + \mu_2 - x_t,$$
$$\mu_{3,t} = \bar{\mu}_t + \mu_3 - x_t,$$
$$\mu_{4,t} = \bar{\mu}_t.$$

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where $\bar{\mu}_t$ is a normalization such that $\mathbb{E}_i[exp\{\epsilon_{i,t+1}\}] = 1$ in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by Guvenen et al. (2014) and Guvenen et al. (2015). Our objective function is a weighted sum of the squared difference between the model-implied and empirical moments.

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for x_t and w_t. We assume that these series are linearly related to observable labor market indicators (for details see McKay, 2017). Call the weights in these linear relationships β. We then search over the parameters P, μ, σ, and β subject to the restrictions $P_2 = P_3$ and $\sigma_2 = \sigma_3$.

Guvenen et al. (2014) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in the top panel of figure 6. The estimated β implies a time-series for x_t which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure 6. We regress this time-series on the unemployment rate and find a coefficient of 16.7.[35] The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation σ_4 is estimated to be very large and this contributes to the high kurtosis of the earnings growth distribution. In particular, omitting this component leads to a substantially smaller τ as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation

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[35] We regress this estimated time series x_t on the unemployment rate, which we smooth with an HP filter with smoothing parameter 100,000. If we call this regression function f, we then proceed with $F(\epsilon'; f(u))$.
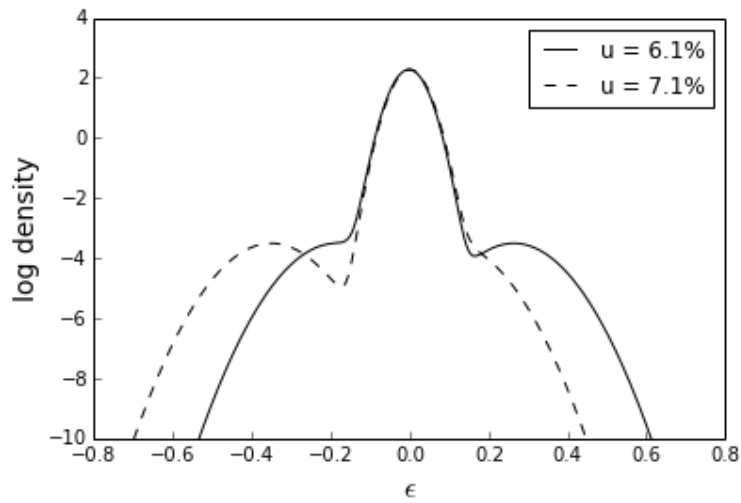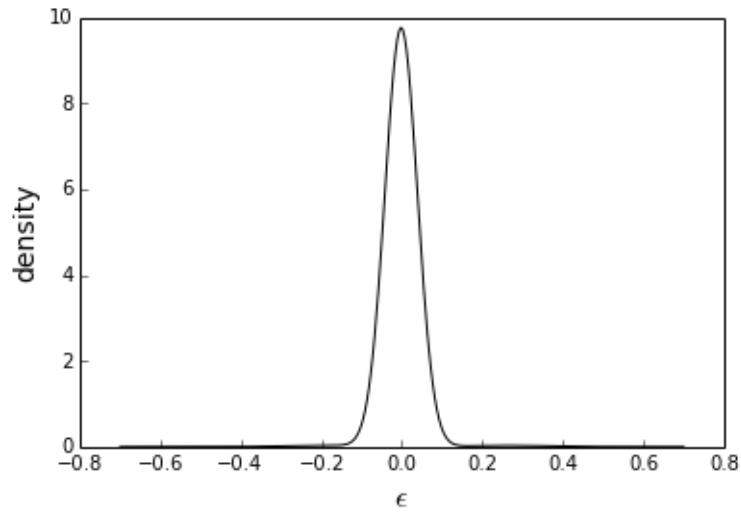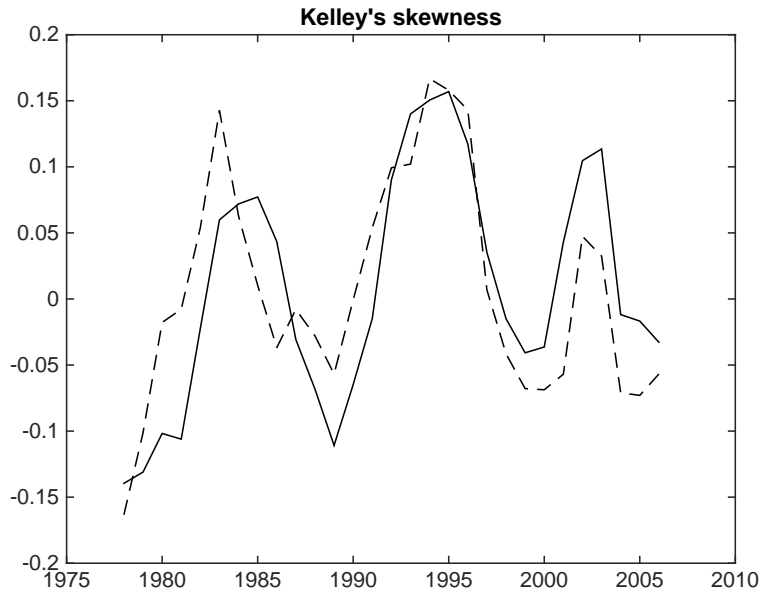
Figure 6: Properties of $F(\epsilon)$.

distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) =\sim \begin{cases} N(\mu_{1,t}, 0.0403) & \text{with prob. } 0.9855, \\ N(\mu_{2,t}, 0.0966) & \text{with prob. } 0.00727, \\ N(\mu_{3,t}, 0.0966) & \text{with prob. } 0.00727 \end{cases}$$

with

$$\mu_{1,t} = \bar{\mu}_t,$$

$$\mu_{2,t} = \bar{\mu}_t + 0.266 - 16.73(u_t - u^*),$$

$$\mu_{3,t} = \bar{\mu}_t - 0.184 - 16.73(u_t - u^*),$$

where u^* is the steady state is unemployment rate in our baseline calibration. The bottom panels of figure 6 show the density of ϵ and how it changes with an increase in the unemployment rate.

F.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2}/h_s\right) /A_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}.$$

Define p_t^A as

$$p_t^A = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2}/h_s\right) /A_s$$

and p_t^B as

$$p_t^B = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s$$

such that

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}.$$

p_t^A and p_t^B can be rewritten as

$$p_t^A = \mu Y_t \left(w_t + \psi_1 M_s^{\psi_2}/h_s \right) / A_t + (1-\theta)\mathbb{E}_t \left[\left(\frac{I_t}{\pi_{t+1}} \right)^{-1} \pi_{t+1}^{-\mu/(1-\mu)} p_{t+1}^A \right] \tag{62}$$

$$p_t^B = Y_t + (1-\theta)\mathbb{E}_t \left[\left(\frac{I_t}{\pi_{t+1}} \right)^{-1} \pi_{t+1}^{-1/(1-\mu)} p_{t+1}^B \right]. \tag{63}$$

The procedure we use builds on the method proposed by Maliar and Maliar (2015) and their application to solving a New Keynesian model. We first describe how we solve the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are six state variables that evolve according to

$$\mathbb{E}_i \left[\alpha_{i,t+1}^{1-\tau} \right] = (1-\delta)\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] \mathbb{E}_i \left[\epsilon_{i,t+1}^{1-\tau} u_t \right] + \delta$$

$$\mathbb{E}_i \left[\log \alpha_{i,t+1} \right] = (1-\delta) \left[\mathbb{E}_i \left[\log \alpha_{i,t} \right] + \mathbb{E}_i \left[\log \epsilon i, t+1 | u_t \right] \right]$$

$$S_{t+1}^A = S_t$$

$$\log \eta_{t+1}^A = \rho^A \log \eta_t^A + \varepsilon_{t+1}^A$$

$$\log \eta_{t+1}^G = \rho^G \log \eta_t^G + \varepsilon_{t+1}^G$$

$$\log \eta_{t+1}^I = \rho^I \log \eta_t^I + \varepsilon_{t+1}^I,$$

where S^A is the level of price dispersion in the previous period and the ε terms are i.i.d. normal innovations.

There are five variables that we approximate with complete second-order polynomials in the state: $(1/C_t)$, p_t^A, p_t^B, J_t, and V_t, where V_t is the value of the social welfare function. We use (15) and (16) to write the Euler equation in terms of C_t and this equation pins down $1/C_t$. p_t^A and p_t^B satisfy (62) and (63). V_t satisfies

$$V_t = W_t + \beta \, \mathbb{E}_t \left[V_{t+1} \right].$$

J_t satisfies $J_t = \psi_1 M_t^{\psi_2}(v - u_t)$. Abusing language slightly, we will refer to these variables that we approximate with polynomials as "forward-looking variables."

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated t. We call these the "static" variables.

To summarize, let $\mathcal{S}_t$ be the state variables, $\mathcal{X}_t$ be the forward-looking variables, and $\mathcal{Y}_t$ be the

static variables. The three blocks of equations are

$$\mathcal{S}' = \mathcal{G}^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$$

$$\mathcal{X} = \mathbb{E}\, \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$$

$$\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$$

where $\mathcal{G}^{\mathcal{S}}$ are the state-transition equations, $\mathcal{G}^{\mathcal{X}}$ are the forward-looking equations and $\mathcal{G}^{\mathcal{Y}}$ are the state equations. Let $\mathcal{X} \approx \mathcal{F}(\mathcal{S}, \Omega)$ be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by Ω. We then operationalize the equations as follows: given a value for $\mathcal{S}$, we calculate $\mathcal{X} = \mathcal{F}(\mathcal{S}, \Omega)$ and $\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$. We then take an expectation over ε' using Gaussian quadrature. For each value of ε' in the quadrature grid, we compute $\mathcal{S}' = G^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$, $\mathcal{X}' = \mathcal{F}(\mathcal{S}', \Omega)$ and $\mathcal{Y}' = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}', \mathcal{X}')$. We can now evaluate $\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$ for this value of ε' and looping over all the values in the quadrature grid we can compute $\hat{\mathcal{X}} = \mathbb{E}\, \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$. $\hat{\mathcal{X}}$ will differ from the value of $\mathcal{X}$ that was obtained initially from $\mathcal{F}(\mathcal{S}, \Omega)$. We repeat these steps for all the values of $\mathcal{S}$ in our grid for the aggregate state space. We then adjust the coefficients Ω part of the way towards those implied by the solutions $\hat{\mathcal{X}}$. We then iterate this procedure to convergence of Ω.

Evaluating some of the equations of the model involves taking integrals against the distribution of idiosyncratic skill risk $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t - u^*)$. We do this using Gaussian quadrature within each of the components of the mixture distribution. We compute expectations over aggregate shocks using Gaussian quadrature as well.

We use a two-step procedure to construct the grid on the aggregate state space. We have six aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited by simulations of the solution. We create a box of policy parameters $[b_L, b_H] \times [\tau_L, \tau_H]$. For each of the four corners of this box, we use the procedure of Maliar and Maliar (2015) to construct a grid and solve the model. This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us four grids, which we then merge and eliminate nearby points using the techniques of Maliar and Maliar (2015). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.

G Extended model with savings

In the extension with savings, the household's problem is (we omit aggregate states for simplicity of notation)

$$V(a, n) = \max_{c, a', h} \left\{ \log c - \frac{h^{1+\gamma}}{1+\gamma} + \beta \, \mathbb{E} \left[(1-v)V(a', 1) + v V^S(a') \right] \right\}$$

such that

$$c + a' = Ra + (n + (1-n)b) \, \lambda \, (wh + d)^{1-\tau},$$

where for an employed individual h is a choice and for an unemployed worker h should be replaced by $h(a)$, which is the equilibrium decision rule of employed workers. The value of entering the period without a match is

$$V^S(a) = \max_q \left\{ MqV(a, 1) + (1 - Mq)V(a, 0) - \frac{q^{1+\kappa}}{1+\kappa} \right\}.$$

The relevant efficiency conditions are

$$\beta R \, \mathbb{E} \left[(1 - v + v M'q') \frac{1}{c(a', 1)} + v(1 - M'q') \frac{1}{c(a', 0)} \right] = \frac{1}{c(a, n)}$$

$$\lambda \, (1 - \tau) \, (wh + d)^{-\tau} \, w = c(a, n)h(a)^\gamma$$

$$M \left[V(a, 1) - V(a, 0) \right] = q(a)^\kappa.$$

Let $\Phi(a)$ be the distribution of households over beginning of period assets (before interest). Let $N(a, M) \equiv 1 - v + vq(a)M$ be the probability that a worker with assets a is employed. Define $\mathcal{H}$ as aggregate hours worked per employed worker and $\mathcal{Q}$ as average search effort. Aggregate quantities, are then given by

$$C = \int c(a, 1)N(a, M) + c(a, 0) \left[1 - N(a, M) \right] d\Phi(a) \tag{64}$$

$$\mathcal{H} = \int h(a)N(a, M)d\Phi(a) / \int N(a, M)d\Phi(a) \tag{65}$$

$$\mathcal{Q} = \int q(a)d\Phi(a). \tag{66}$$

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Given the definition of $\mathcal{H}$, total labor input is $\mathcal{H}(1-u)$.

An equilibrium of the economy can be calculated from a system equations in 14 variables, three exogenous processes, the solution to the household's problem, which gives rise to policy rules for consumption, work effort, and search effort, and the dynamics of the distribution of wealth $\Phi(a)$. The variables are

$$C_t, u_t, R_t, I_t, \pi_t, Y_t, G_t, \mathcal{H}_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, \text{II}_t, M_t.$$

And the equations are: (3), (4), (5), (40), (6), (7), (41), (36), (42), (43), (39), (64), (65), (66). The exogenous processes are η_t^A, η_t^G, and η_t^I.