Preliminary and incomplete

Data-Rich DSGE and Dynamic Factor Models

MAXYM KRYSHKO[1]
UNIVERSITY OF PENNSYLVANIA

OCTOBER 30, 2009

Abstract: Dynamic factor models and dynamic stochastic general equilibrium (DSGE) models are widely used for empirical research in macroeconomics. The empirical factor literature argues that the co-movement of large panels of macroeconomic and financial data can be captured by relatively few common unobserved factors. Similarly, the dynamics in DSGE models are often governed by a handful of state variables and exogenous processes like preference and/or technology shocks. Boivin and Giannoni (2006) combine a DSGE and a factor model into a data-rich DSGE model, in which DSGE states are factors and factor dynamics are subject to DSGE model implied restrictions. We compare a data-rich DSGE model with a standard New Keynesian core to an empirical dynamic factor model by estimating both on a rich panel of U.S. macroeconomic and financial data compiled by Stock and Watson (2008). We find that the spaces spanned by the empirical factors and by the data-rich DSGE model states are very close. This proximity allows us to propagate monetary policy and technology innovations in otherwise non-structural dynamic factor model to obtain predictions for many more series than a handful of traditional macro variables including measures of real activity, price indices, labor market indicators, interest rate spreads, money and credit stocks, exchange rates.

JEL Classification: C11, C32, E32, E37, E4, E5
Keywords: Data-rich DSGE models; dynamic factor models; Bayesian estimation

---

[1] Correspondence: Department of Economics, University of Pennsylvania, 3718 Locust walk, Philadelphia, PA 19104. E-mail: mkryshko@sas.upenn.edu. Web: http://www.sas.upenn.edu/~mkryshko.
The author thanks his main advisor Frank Schorfheide, thesis committee members Frank Diebold and Jesús Fernández-Villaverde, as well as Flavio Cunha, Cristina Fuentes-Albero, Ed Herbst, Dirk Krueger, Leonardo Melosi, Emanuel Moench, Kevin Song, Sergiy Stetsenko and other participants of Penn Macro lunch and Penn Econometrics lunch for valuable discussions and many useful comments and suggestions.

Data-Rich DSGE and Dynamic Factor Models

1 Introduction

Dynamic factor models (DFM) and dynamic stochastic general equilibrium (DSGE) models are widely used for empirical research in macroeconomics. The traditional areas of DFM application are the construction of coincident and leading indicators (e.g. Stock and Watson 1989, Altissimo et al. 2001) and the forecasting of macro time series (Stock and Watson 1999, 2002a, b; Forni, Hallin, Lippi and Reichlin 2003; Boivin and Ng 2005). DFMs are also in use for the real-time monitoring (Giannone, Reichlin, Small 2008; Aruoba, Diebold, Scotti 2009), in monetary policy applications (e.g. Factor Augmented VAR approach of Bernanke, Boivin, Eliasz 2005, Stock and Watson 2005) and in studying international business cycles (Kose, Otrok, Whiteman 2003, 2008; Del Negro, Otrok 2008). The micro-founded optimization-based DSGE models primarily focus on understanding the sources of business cycle fluctuations and on assessing the importance of nominal rigidities and various types of frictions in the economy. Recently, they appear to have been able to replicate well many salient features of the data (e.g. Christiano, Eichenbaum, Evans 2005; Smets and Wouters 2003, 2007). As a result, the versions of DSGE models extended to open economy and multisector contexts are increasingly used as tools for projections and policy analysis at major central banks (Adolfson et al 2007, 2008; Edge, Kiley and Laforte 2009; Coenen, McAdam, Straub 2008).

The empirical factor literature argues that the co-movement of large panels of macroeconomic and financial data can be captured by relatively few common unobserved factors. Early work by Sargent and Sims (1977) found that the dynamic index model with two indices fits well the real variables in their panel. Giannone, Reichlin, Sala (2004) claim that the number of common shocks, or stochastic dimension of the U.S. economy in their terminology, is two. There has been a lot of theoretical work on developing the more objective criteria to choose the appropriate number of static/dynamic factors in empirical factor models. Based on these criteria, several authors (e.g. Bai, Ng 2007; Hallin, Liska 2007; Stock and Watson 2005) argued for higher number of dynamic factors that drive large US macroeconomic panels – ranging from four to seven.

The dynamics in DSGE models are also often governed by a handful of state variables and exogenous processes like preference and/or technology shocks. Boivin and Giannoni (2006) combine a DSGE and a factor model into a data-rich DSGE model, in which DSGE states are factors and factor dynamics are subject to DSGE model implied restrictions. They argue that the richer information coming from large macroeconomic and financial panel can provide better estimates of the DSGE states and of the structural shocks driving the economy. On top of that, they show that the data-rich DSGE model delivers different estimates of deep structural parameters of the model compared to standard non-data-rich estimation.

If both a data-rich DSGE model and an empirical factor model are taken to the same rich data set, then how similar or different would be the latent empirical factors extracted by a factor model versus the estimated data-rich DSGE model states? Do they span a common space? We ask this question because of three reasons. First, the factor spaces comparison may serve as a useful tool to evaluate a DSGE model. Recent research has shown that misspecification remains a concern to valid inference in DSGE models (Del Negro, Schorfheide, Smets and Wouters 2007 – DSSW hereafter). If a DSGE model is taken to a particular small set of observables, misspecification often manifests itself through the inferior fit. Dynamic factor models usually fit well and perform well in forecasting. So if it turns out that the spaces spanned by two models are close, then it is a good news for a DSGE model. This means that a DSGE model overall captures the sources of co-movement in the large panel of data as sort of a core, and that the differences in fit between a data-rich DSGE model and a DFM are potentially due to restricted factor loadings in the former. Second, it is well known that the latent common components extracted by dynamic factor models from the large panels of data do not mean much in general. That's one of the biggest weaknesses of DFMs. If factor spaces in two models are closely aligned, this facilitates economic interpretation of a dynamic factor model, as the empirical factors become isomorphic to the DSGE model state variables with clear economic meaning. Third, if factor spaces are close, we are able to map the DSGE model concepts into empirical factors every period and therefore propagate the structural shocks in otherwise completely non-structural dynamic factor model to obtain predictions for a broad range of macro series of interest[2]. This way of doing policy analysis is more reliable. This is because on top of the impulse responses derived in the data-rich DSGE model, which might be misspecified, we are able to generate a second set of responses to the same shocks in the context of a factor model that is primarily data-driven and fits better.

We compare a data-rich DSGE model with a standard New Keynesian core to an empirical dynamic factor model by estimating both on a rich panel of U.S. macroeconomic and financial data compiled by Stock and Watson (2008). The estimation involves Bayesian methods.

We find that the spaces spanned by the empirical factors and by the data-rich DSGE model states are very close. This proximity allows us to propagate monetary policy and technology innovations in otherwise non-structural dynamic factor model to obtain predictions for many more series than a handful of traditional macro variables including measures of real activity, price indices, labor market indicators, interest rate spreads, money and credit stocks, exchange rates. We can therefore provide a more complete and comprehensive picture of the effects of monetary policy and technology shocks.

The paper is organized as follows. In Section 2 we present the variant of a dynamic factor model and a data-rich DSGE model to be used in subsequent empirical analysis. Our econometric methodology to estimate two models and also a computational speedup due to Jungbacker and Koopman (2008) are discussed in Section 3. Section 4 describes our data set and transformations. In Section 5 we proceed by

[2] This is similar in spirit to the Factor Augmented VAR approach (originally due to Bernanke, Boivin and Eliasz (2005) and also implemented by Stock and Watson (2005) to study the impact of monetary policy shocks on large panel of macro data) and to the structural factor model of Forni, Giannone, Lippi and Reichlin (2007).

conducting the empirical analysis. We begin by discussing the choice of the prior distributions of model parameters and then briefly describe the posterior estimates of deep structural parameters of the data-rich DSGE model. Second, we analyze the estimated empirical factors and the estimates of the DSGE model state variables, and explore how well they are able to capture the co-movements in the data. Third, we compare the spaces spanned by the latent empirical factors and by the data-rich DSGE model state variables. Finally, we use the proximity of the factor spaces to propagate the monetary policy and technology innovations in otherwise non-structural dynamic factor model to obtain the predictions for the core and non-core macro and financial series of interest. Section 6 concludes.

2 Two Models

In this section, we present two benchmark models the factor spaces of which we will ultimately compare. First, we describe the variant of a dynamic factor model. Then, we present a data-rich DSGE model with a New Keynesian core to be estimated on the same large panel of macro and financial series.

2.1 Dynamic Factor Model

If the forecasting performance is a right guide to choose the appropriate factor model specification, the literature remains rather inconclusive in that respect. For example, Forni, Hallin, Lippi and Reichlin (2003) found supportive results for the generalized dynamic factor specification over the static factor specification, while Boivin and Ng (2005) documented little differences for the competing factor specifications. We choose to work with the version of dynamic factor model as originally developed by Geweke (1977) and Sargent and Sims (1977) and recently used by Stock and Watson (2005).

Let F_t denote the $N \times 1$ vector of common unobserved factors that are related to a $J \times 1$ large ($J \gg N$) panel of macroeconomic and financial data X_t according to the following factor model:

$$X_t = \mathbf{\Lambda} F_t + e_t \tag{1}$$

$$F_t = \mathbf{G} F_{t-1} + \eta_t, \qquad \eta_t \sim iid\ N(\mathbf{0}, \mathbf{Q}) \tag{2}$$

$$e_t = \mathbf{\Psi} e_{t-1} + v_t, \qquad v_t \sim iid\ N(\mathbf{0}, \mathbf{R}), \tag{3}$$

where $\mathbf{\Lambda}$ is the $J \times N$ matrix of factor loadings, e_t are the idiosyncratic errors allowed to be serially correlated, $\mathbf{G}$ is the $N \times N$ matrix that governs common factor dynamics and η_t is the vector of stochastic innovations. The factors and idiosyncratic errors are assumed to be uncorrelated at all leads and lags: $E(F_t e_{i,s}) = 0$, all i, t and s. As in Stock and Watson (2005), we assume that matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{\Psi}$ are diagonal, which implies we have an *exact* dynamic factor model: $E(e_{i,t} e_{j,s}) = 0$, $i \neq j$, all t and s. This is in contrast to *approximate* DFM of Chamberlain and Rothschild (1983) that relaxes this assumption and allows for some correlation across idiosyncratic errors $e_{i,t}$ and $e_{j,t}$, $i \neq j$. As written, the model is already in static form, since data series X_t load only on contemporaneous factors, and not on their lags[3].

2.2 Data-Rich DSGE Model

In any DSGE model, economic agents solve intertemporal optimization problems built from explicit preferences and technology assumptions. Moreover, decision rules of these agents depend upon a number of exogenous stochastic disturbances that characterize uncertainty in economic environment. Equilibrium dynamics of a DSGE model is captured by a system of non-linear expectational difference equations. Standard approach in the literature is to derive a log-linear approximation to this non-linear system around its deterministic steady state and then to solve numerically the resulting linear rational expectations system by one of the available methods[4].

This numerical solution delivers a vector autoregressive process for S_t, the vector collecting all state variables of the DSGE model:

$$S_t = \mathbf{G(\theta)}S_{t-1} + \mathbf{H(\theta)}\varepsilon_t, \qquad \text{where } \varepsilon_t \sim iid\ N(0, \mathbf{Q(\theta)}). \tag{4}$$

The matrices in (4) are functions of structural parameters $\boldsymbol{\theta}$ characterizing preferences and technology in DSGE model. For convenience, we assume that exogenous shocks ε_t are mean-zero normal random variables with diagonal covariance matrix $\mathbf{Q(\theta)}$.

In order to estimate our DSGE on a set of observables $X^T = [X_1, ..., X_T]'$, a state-space representation of the model is usually constructed by augmenting (4) with a number of measurement equations that connect model concepts in S_t to data indicators in vector X_t.

2.2.1 Regular vs Data Rich DSGE Models

Depending on the number of data indicators and on how we connect them to model concepts, we will distinguish regular and data-rich DSGE models. In *regular* DSGE models, the number of observables contained in X_t is usually kept small (most often equal to the number of structural shocks) and model concepts are often assumed to be perfectly measured by a single data indicator[5]. For example, Aruoba and Schorfheide (2009), in money-in-the-utility version of a DSGE model with four structural shocks, specify

[3] In general, a measurement equation is often written as $X_t = \lambda(L) f_t + e_t$, with data loading on current and lagged dynamic factors f_t. However, assuming $\lambda(L)$ has at most *p* lags, and defining $F_t = (f_t', ..., f_{t-p}')'$, we can rewrite it as (1). Here F_t is the vector of static factors as opposed to dynamic factors f_t. To make things simpler, in the model (1)-(3), however, the static and dynamic factors coincide.
[4] Please see Sims (2002), Blanchard and Kahn (1980), Klein (2000), Uhlig (1999), King and Watson (2002).
[5] The underlying reason is to avoid the so called stochastic singularity. The likelihood function for observables X_t with dimension exceeding the number of structural shocks will be degenerate, since according to DSGE model some X_t's can be perfectly (deterministically) predicted from others and this is obviously not true in the data. The solution is to add measurement errors (or theoretical gaps between the model concept and the data indicator) as e.g. in Altug (1989), Sargent (1989), Ireland (2004), or to add more shocks, e.g. Leeper and Sims (1994), Adolfson, Laseen, Linde, Villani (2008).

the following measurement equations for real output $\hat{Y}_t$, inflation $\hat{\pi}_t$, nominal interest rate $\hat{R}_t$ and inverse money velocity $\hat{M}_t - \hat{Y}_t$ (we omit intercept for simplicity):

$$\underbrace{\begin{bmatrix} \text{RealGDP}_t \\ \text{GDP\_Def\_Inflation}_t \\ \text{FedFundsRate}_t \\ \text{IVM\_M2}_t \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 4 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 4 & 0 & \cdots & 0 \\ -1 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}}_{\mathbf{\Lambda}} \cdot \underbrace{\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ \hat{M}_t \\ \vdots \end{bmatrix}}_{S_t} \tag{5}$$

Similarly, Smets and Wouters (2007) estimate a DSGE model with seven structural shocks on seven core U.S. macro variables: again assuming one-to-one model concept-data correspondence and perfect measurement.

Following important contribution of Boivin and Giannoni (2006), *data-rich* DSGE models relax these assumptions and allow for: (i) presence of measurement errors or, alternatively, of terms capturing the theoretical gap between a particular data indicator and a model concept it is supposed to measure; (ii) multiple data indicators $X_{j,t}$ measuring the same model concept $S_{i,t}$, and (iii) many informational data series in X_t that load on all model concepts (and that may contain useful information about the state of the economy). Measurement equation in the data-rich DSGE model for demeaned X_t looks like:

$$\underbrace{\begin{bmatrix} X_t^F \\ \text{--} \\ X_t^S \end{bmatrix}}_{X_t} = \underbrace{\begin{bmatrix} \mathbf{\Lambda_F(\theta)} \\ \text{----} \\ \mathbf{\Lambda_S} \end{bmatrix}}_{\mathbf{\Lambda(\theta)}} S_t + \underbrace{\begin{bmatrix} e_t^F \\ \text{--} \\ e_t^S \end{bmatrix}}_{e_t}, \tag{6}$$

where each data indicator within *core series* X_t^F loads on a single model concept $S_{i,t}$ only (although same $S_{i,t}$ may have several data indicators measuring it), informational *non-core series* X_t^S are related to all state variables S_t, and where measurement errors e_t may be serially correlated, but uncorrelated across different data indicators ($\mathbf{\Psi}$, $\mathbf{R}$ are diagonal):

$$e_t = \mathbf{\Psi} e_{t-1} + v_t, \qquad \text{where } v_t \sim iid\ N(\mathbf{0}, \mathbf{R}). \tag{7}$$

So the state-space representation of the data-rich DSGE model consists of transition equation (4) and measurement equations (6)-(7).

Notice that data-rich DSGE model (4), (6), (7) is very much like dynamic factor model (1)-(3) in which transition of unobserved factors is governed by a DSGE model solution and where some factor loadings are restricted by economic meaning of the DSGE model concepts.

2.2.2 Environment

In this paper, we use relatively standard New Keynesian business cycle model. It is more elaborate than a basic three-equation model used in Woodford (2003), but is "lighter" than models due to Smets and

Wouters (2003, 2007) and Christiano, Eichenbaum and Evans (2005). The model features capital as factor of production, nominal rigidities in price setting, and investment adjustment costs, but it abstracts from wage rigidities, habit formation in consumption and variable capital utilization. Real money stock enters household's utility in additively separable fashion as in Walsh (2003), Sidrauski (1967). In terms of specific version of the model, we draw upon the work by Aruoba and Schorfheide (2009) and their money-in-the-utility specification.

The economy is populated by households, final and intermediate goods producing firms and a Central bank (monetary authority). A representative household works, consumes, saves, holds money balances and accumulates capital. It consumes the final output manufactured by perfectly competitive final good firms. The final good producers produce by combining a continuum of differentiated intermediate goods supplied by monopolistically competitive intermediate goods firms. To manufacture their output, intermediate goods producers hire labor and capital services from households. Also, when optimizing their prices, intermediate goods firms face the nominal price rigidity a la Calvo (1983), and those firms that are unable to re-optimize may index their price to lagged inflation. Monetary policy is conducted by Central bank setting one period nominal interest rate on public debt via Taylor type interest rate feedback rule. Given interest rate, Central bank also supplies enough nominal money balances to meet equilibrium demand from households.

Households

In our environment, there is a continuum of households indexed by $j \in [0;1]$. Each household maximizes the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(x_t(j)) - Ah_t(j) + \frac{\chi_t}{1-\nu_m} \left[\frac{A}{Z_*^{1/(1-\alpha)}} \frac{m_t(j)}{P_t} \right]^{(1-\nu_m)} \right\}, \tag{8}$$

which is additively separable in consumption $x_t(j)$, labor supply $h_t(j)$ and real money balances $m_t(j)/P_t$. Here β stands for discount factor, A denotes disutility of labor, ν_m controls the elasticity of money demand and χ_t is aggregate preference shifter that affects household's marginal utility from holding real money balances[6]. The law of motion for χ_t is:

$$\ln \chi_t = (1-\rho_\chi) \ln \chi_* + \rho_\chi \ln \chi_{t-1} + \varepsilon_{\chi,t}, \qquad \text{where } \varepsilon_{\chi,t} \sim N(0, \sigma_\chi^2) \tag{9}$$

We assume that households are able to trade on a complete set of Arrow-Debreu (A-D) securities which are contingent on all aggregate and idiosyncratic events $\omega \in \Omega$ in the economy. Let $a_{t+1}(j)(\omega)$ denote the quantity of A-D securities (that pay 1 unit of consumption in period $t+1$ in the event ω)

[6] As in Aruoba and Schorfheide (2009), scaling $m_t(j)/P_t$ by a factor $A/Z_*^{1/(1-\alpha)}$ can be viewed as reparameterization of χ_t, in which steady state money velocity remains constant when we move around A and Z_*.

acquired by household j at time t at real price $q_{t+1,t}(j)$. Then the household j's budget constraint in nominal terms is given by:

$$\begin{aligned} &P_t x_t(j) + P_t i_t(j) + b_{t+1}(j) + m_{t+1}(j) + P_t \int_\Omega q_{t+1,t}(j) a_{t+1}(j)(\omega) d\omega = \\ &= P_t W_t h_t(j) + P_t R_t^k k_t(j) + \Pi_t + R_{t-1} b_t(j) + m_t(j) + P_t a_t(j) - T_t \end{aligned} \tag{10}$$

where P_t is the period t price of final good, $i_t(j)$ is investment, $b_t(j)$ and $m_t(j)$ are government bond and money holdings, R_t is the gross nominal interest rate on government bonds, W_t and R_t^k are the real wage and real return on capital earned by household, Π_t stands for the profits from owning the firms, and T_t is the nominal amount of lump-sum taxes paid. Households also accumulate capital $k_t(j)$ according to the following law of motion:

$$k_{t+1}(j) = (1-\delta) k_t(j) + \left[1 - S\left(\frac{i_t(j)}{i_{t-1}(j)}\right)\right] i_t(j), \tag{11}$$

where δ is depreciation rate and $S(\bullet)$ is an adjustment cost function satisfying $S(1)=0$, $S'(1)=0$ and $S''(1)>0$.

Now the problem of each household j is to maximize the utility function (8) subject to budget constraint (10) and capital accumulation equation (11) for all t. Associate Lagrange multipliers $\lambda_t(j)$ and $Q_t(j)$ with constraints (10) and (11), respectively. Then, the First Order Conditions with respect to $x_t(j)$, $h_t(j)$, $m_{t+1}(j)$, $i_t(j)$, $k_{t+1}(j)$ and $b_{t+1}(j)$ are:

$$\lambda_t(j) = \frac{U'(x_t(j))}{P_t} \tag{12}$$

$$\lambda_t(j) = \frac{A}{P_t W_t} \tag{13}$$

$$\frac{U'(x_t(j))}{P_t} = \beta E_t \left\{ \frac{\chi_{t+1}}{P_{t+1}} \left(\frac{A}{Z_*^{1/(1-\alpha)}}\right)^{(1-\nu_m)} \left(\frac{m_{t+1}(j)}{P_{t+1}}\right)^{-\nu_m} + \frac{U'(x_{t+1}(j))}{P_{t+1}} \right\} \tag{14}$$

$$1 = \mu_t(j)\left[1 - S\left(\frac{i_t(j)}{i_{t-1}(j)}\right) - S'\left(\frac{i_t(j)}{i_{t-1}(j)}\right)\frac{i_t(j)}{i_{t-1}(j)}\right] + \beta E_t \left\{ \mu_{t+1}(j) \frac{U'(x_{t+1}(j))}{U'(x_t(j))} S'\left(\frac{i_{t+1}(j)}{i_t(j)}\right)\left[\frac{i_{t+1}(j)}{i_t(j)}\right]^2 \right\} \tag{15}$$

$$\mu_t(j) = \beta E_t \left\{ \frac{U'(x_{t+1}(j))}{U'(x_t(j))} \left(R_{t+1}^k + \mu_{t+1}(j)(1-\delta)\right) \right\} \tag{16}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1}(j))}{U'(x_t(j))} \frac{R_t}{\pi_{t+1}} \right\}, \tag{17}$$

where $\pi_t = P_t / P_{t-1}$ denotes inflation and where we have substituted out Lagrange multiplier $\lambda_t(j)$ with its equivalent expression using marginal utility of consumption and have introduced the normalized shadow price of installed capital $\mu_t(j) = \dfrac{Q_t(j)}{U'(x_t(j))}$.

We do not take first order conditions with respect to A-D securities holdings $a_{t+1}(j)$ explicitly, because we make use of the result due to Erceg, Henderson and Levin (2000). This result says that under the assumption of complete markets for A-D securities and under the additive separability of labor and money balances in household's utility, the equilibrium price of A-D securities will be such that optimal consumption will not depend on idiosyncratic shocks. Hence, all households will share the same marginal utility of consumption, and given (12), Lagrange multiplier $\lambda_t(j)$ will also be the same across all households: $\lambda_t(j) = \lambda_t$, all *j* and *t*. This implies that in equilibrium all households will choose the same consumption, money and bond holdings, investment and capital. Note that we don't have wage rigidity in this model – therefore the choice of optimal labor will also be same. From now on we can safely drop index j from all equilibrium conditions of households and proceed accordingly.

The first two FOCs could be combined to yield labor supply equation relating real wage to marginal rate of substitution between consumption and labor. (14) is an Euler equation for money holdings, which together with (17) – an Euler equation for bond holdings – implies household's optimal demand for real money balances. Equation (15) determines the law of motion for shadow price of installed capital. If there were no investment adjustment costs, this price will be equal to 1, which is standard in neoclassical growth model. Also note that if we were to have an investment specific technology shock, this shadow price will be equal to relative price of capital in consumption units. Equation (16) is an Euler equation for capital holdings. The shadow cost of purchasing one unit of capital today should be equal to the real return from renting it to firms plus the tomorrow's resale value of capital that has not yet depreciated.

Running a bit ahead, let us define the stochastic discount factor $\Xi^p_{t+1|t}$ that the firms – whose behavior we are going to describe shortly – will use to value streams of future profits:

$$\Xi^p_{t+1|t} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{U'(x_{t+1})}{U'(x_t)} \frac{1}{\pi_{t+1}} \tag{18}$$

Final Good Firms

There is single final good Y_t in our economy manufactured by combining a continuum of intermediate goods $Y_t(i)$ indexed by $i \in [0;1]$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right)^{(1+\lambda)}, \tag{19}$$

where the elasticity of substitution between any goods *i* and *j* is $\frac{1+\lambda}{\lambda}$.

The final good firms purchase intermediate goods in the market, package them into a composite final good, and sell the final good to households. These firms are perfectly competitive and maximize one period profits subject to production function (19), taking as given intermediate goods prices $P_t(i)$ and own output price P_t:

$$\begin{aligned} &\max_{Y_t, Y_t(i)} \quad P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \\ &\text{s.t.} \quad Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} di \right)^{(1+\lambda)} \end{aligned} \tag{20}$$

The first order condition of a representative final good firm is:

$$P_t(i) = P_t Y_t^{\frac{\lambda}{(1+\lambda)}} Y_t(i)^{-\frac{\lambda}{(1+\lambda)}}, \tag{21}$$

which leads to the optimal demand for good *i*:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda)}{\lambda}} Y_t. \tag{22}$$

Since final good firms are perfectly competitive and there is free entry, they earn zero profits in equilibrium:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di = 0, \tag{23}$$

which, together with optimal demand (22), yields the price of final good:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda}} di \right]^{-\lambda}. \tag{24}$$

Intermediate Goods Firms

Our economy is populated by a continuum of intermediate goods firms. Each intermediate goods firm i uses the following technology to produce its output:

$$Y_t(i) = \max\left\{ Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)} - \tilde{F}, 0 \right\}, \tag{25}$$

where $K_t(i)$ is the amount of capital that the firm i rents from households, $H_t(i)$ is the amount of labor input and Z_t is the level of neutral technology evolving according to the law of motion:

$$\ln Z_t = (1-\rho_Z) \ln Z_* + \rho_Z \ln Z_{t-1} + \varepsilon_{Z,t}, \quad \text{where } \varepsilon_{Z,t} \sim N(0, \sigma_Z^2). \tag{26}$$

Parameter α stands for the capital share of production, while parameter $\tilde{F}$ controls the amount of fixed costs in production that guarantee economic profits of the firm will be zero in steady state. Unlike final good producers, we do not allow for free entry or exit on the part of the intermediate goods firms.

All intermediate goods producers are *monopolistically competitive*, in that they take all factor prices (W_t and R_t^k), as well as prices of other firms, as given, but can optimally choose their own price $P_t(i)$ subject to optimal demand (22) for good i from final good firms. Intermediate firms solve a two-stage optimization problem.

In the first stage, the firms hire capital and labor from households to minimize total nominal costs:

$$\begin{aligned} \min_{K_t(i),H_t(i)} \quad & P_t W_t H_t(i) + P_t R_t^k K_t(i) \\ \text{s.t.} \quad & Y_t(i) = \max\left\{ Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)} - \tilde{F}, 0 \right\} \end{aligned} \tag{27}$$

Assuming interior solution, optimality conditions imply ($\eta_t(i)$ is Lagrange multiplier attached to (25)):

$$P_t W_t = \eta_t(i) P_t(i)(1-\alpha) Z_t K_t(i)^{\alpha} H_t(i)^{-\alpha}$$

$$P_t R_t^k = \eta_t(i) P_t(i) \alpha Z_t K_t(i)^{\alpha-1} H_t(i)^{1-\alpha}$$

Take the ratio of two conditions to obtain:

$$\frac{K_t(i)}{H_t(i)} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} \tag{28}$$

If we define aggregate capital stock $K_t = \int_0^1 K_t(i)di$ and aggregate labor $H_t = \int_0^1 H_t(i)di$, integrating both sides of (28) yields:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} H_t \tag{29}$$

Now we can factorize total real variable cost $VC_t(i)$ into real marginal cost MC_t and the variable part of firm i's output $Y_t^{\text{var}}(i) = Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)}$:

$$VC_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)} \right) H_t(i) = \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)} \right) \frac{1}{Z_t} \left(\frac{K_t(i)}{H_t(i)} \right)^{-\alpha} Y_t^{\text{var}}(i) \tag{30}$$

Plugging in the optimal capital labor ratio (28), real marginal cost MC_t turns out to be the same across all intermediate goods firms:

$$MC_t \overset{def}{=} \left(W_t + R_t^k \frac{K_t(i)}{H_t(i)} \right) \frac{1}{Z_t} \left(\frac{K_t(i)}{H_t(i)} \right)^{-\alpha} = \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \frac{W_t^{1-\alpha} \left(R_t^k \right)^{\alpha}}{Z_t} \tag{31}$$

The intuition is that all firms face identical technology shocks and hire inputs at the same factor prices.

In the second stage, all intermediate goods firms have to choose their own price $P_t(i)$ that maximizes total discounted nominal profits subject to demand curve (22). Given optimal choices of inputs from the first stage, the one-period nominal profits of firm i are:

$$\begin{aligned}\Pi_t(i) &= P_t(i)Y_t(i) - P_tW_t\tilde{H}_t(i) - P_tR_t^k\tilde{K}_t(i) = P_t(i)Y_t(i) - P_t\left(MC_tY_t^{\text{var}}(i)\right) = \\ &= \left(P_t(i) - P_tMC_t\right)Y_t(i) - P_tMC_t\tilde{F}\end{aligned} \tag{32}$$

Note that we can ignore the term $P_tMC_t\tilde{F}$ since it doesn't depend on firm's choice.

We assume that intermediate goods firms face *nominal price rigidity* a la Calvo (1983). In each period, a fraction $(1-\zeta)$ of firms can optimize their prices. As in Aruoba and Schorfheide (2009), we modify original Calvo's setup and assume that all other firms cannot adjust their prices and can only index $P_t(i)$ by a geometric weighted average of the fixed rate π_{**} and of previous period's inflation π_{t-1}, with weights $(1-\iota)$ and ι respectively. The corresponding price adjustment factor is:

$$\pi_{t+s|t}^{adj} = \begin{cases} 1, & s = 0 \\ \prod_{l=1}^{s}\left(\pi_{t+l-1}^{\iota}\pi_{**}^{(1-\iota)}\right), & s > 0 \end{cases} \tag{33}$$

The firms allowed to re-optimize must choose the optimal price $P_t^o(i)$ that maximizes discounted value of profits in all states of nature in which the firm faces that price in the future:

$$\begin{aligned}&\max_{P_t^o(i)} \quad \Xi_{t|t}^p(P_t^o(i) - P_tMC_t)Y_t(i) + E_t\left\{\sum_{s=1}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^p(P_t^o(i)\pi_{t+s|t}^{adj} - P_{t+s}MC_{t+s})Y_{t+s}(i)\right\} \\ &\text{s.t.} \quad Y_{t+s}(i) = \left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}} Y_{t+s}, \qquad s = 0,1,2,....\end{aligned} \tag{34}$$

Notice that $\beta^s\Xi_{t+s|t}^p$ is period *t* value of a future dollar for the consumer/household in period *t+s*.

In Appendix A, we show in detail that the first order conditions of the problem (34) boil down to these three equations:

$$f_t^{(1)} = \left(p_t^o\right)^{-\frac{(1+\lambda)}{\lambda}} Y_t + \zeta\beta\left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{1}{\lambda}} E_t\left\{\left(\frac{p_t^o}{p_{t+1}^o\pi_{t+1}}\right)^{-\frac{(1+\lambda)}{\lambda}} \Xi_{t+1|t}^p f_{t+1}^{(1)}\right\} \tag{35}$$

$$f_t^{(2)} = \left(p_t^o\right)^{-\frac{(1+\lambda)}{\lambda}-1} MC_tY_t + \zeta\beta\left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{(1+\lambda)}{\lambda}} E_t\left\{\left(\frac{p_t^o}{p_{t+1}^o\pi_{t+1}}\right)^{-\frac{(1+\lambda)}{\lambda}-1} \Xi_{t+1|t}^p f_{t+1}^{(2)}\right\} \tag{36}$$

$$f_t^{(1)} = (1+\lambda)\, f_t^{(2)}, \tag{37}$$

where we have defined optimal price relative to price level $p_t^o = \frac{P_t^o}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$. Note that we have dropped all indices i because we consider only symmetric equilibrium in which all firms re-optimizing their prices will choose the same price $P_t^o(i) = P_t^o$.

Finally, from (24) and given Calvo pricing, the aggregate price index P_t should evolve as:

$$P_t = \left[(1-\zeta)\left(P_t^o\right)^{-\frac{1}{\lambda}} + \zeta\left(\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}P_{t-1}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{38}$$

and, dividing by P_{t-1} yields:

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta\left(\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda} \tag{39}$$

As is standard in literature, equations (35)-(37) and (39) connect the evolution of inflation to dynamics of real marginal costs and output, and thus imply the New Keynesian Phillips curve.

Monetary and Fiscal Policy

The Central bank sets one period nominal interest rate on public debt via Taylor type interest rate feedback rule responding to deviations of inflation and real output from their target levels:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left(\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{Y_t}{Y_*}\right)^{\psi_2}\right)^{(1-\rho_R)} e^{\varepsilon_{R,t}}, \qquad \text{where } \varepsilon_{R,t} \sim N(0,\sigma_R^2) \tag{40}$$

where R_*, π_* and Y_* are the steady state values of gross nominal interest rate, final good inflation and real final output, respectively. Parameter ρ_R is introduced to control for the degree of interest rate smoothing that we observe in postwar U.S. data. Also, the Central bank supplies enough money balances M_t to meet demand from households, given desired nominal interest rate.

Every period the government spends G_t in real terms to purchase goods in the final goods market, issues nominal bonds B_{t+1} that pay R_t in gross interest next period and collects nominal lump-sum taxes from households T_t. Each period, combined government (Central bank + Treasury) budget constraint is:

$$P_t G_t + R_{t-1}B_t + M_t = T_t + B_{t+1} + M_{t+1} \tag{41}$$

Real government spending is modeled as stochastic fraction of total output (i.e. fiscal policy is passive):

$$G_t = \left(1 - \frac{1}{g_t}\right)Y_t, \tag{42}$$

where g_t is an exogenous process shifting G_t:

$$\ln g_t = (1-\rho_g)\ln g_* + \rho_g \ln g_{t-1} + \varepsilon_{g,t}, \qquad \text{where } \varepsilon_{g,t} \sim N(0,\sigma_g^2). \tag{43}$$

Aggregation

We now derive the aggregate demand condition. To that end, we integrate budget constraints across all households and combine the result with government budget constraint (41), introducing aggregate variables: consumption $X_t = \int_0^1 x_t(j)dj$ and investment $I_t = \int_0^1 i_t(j)dj$:

$$P_t X_t + P_t I_t + P_t G_t = P_t W_t H_t + P_t R_t^k K_t + \Pi_t \tag{44}$$

We can obtain aggregate profits Π_t from intermediate firms' problems:

$$\begin{aligned}\Pi_t &= \int_0^1 P_t(i)Y_t(i)di - P_t W_t \int_0^1 H_t(i)di - P_t R_t^k \int_0^1 K_t(i)di = \\ &= \int_0^1 P_t(i)Y_t(i)di - P_t W_t H_t - P_t R_t^k K_t = \\ &= P_t Y_t - P_t W_t H_t - P_t R_t^k K_t\end{aligned} \tag{45}$$

where the last transformation is by zero profit condition of final good firms. Combine (44) and (45), and divide by P_t to obtain *aggregate demand condition*:

$$X_t + I_t + G_t = Y_t \tag{46}$$

From supply side, the aggregate output of intermediate goods firms $\overline{Y}_t$ is given by:

$$\overline{Y}_t = \int_0^1 Z_t K_t(i)^{\alpha} H_t(i)^{(1-\alpha)} di - \tilde{F} = Z_t \int_0^1 \left(\frac{K_t(i)}{H_t(i)}\right)^{\alpha} H_t(i)di - \tilde{F} = Z_t K_t^{\alpha} H_t^{(1-\alpha)} - \tilde{F}, \tag{47}$$

where we have used the fact that capital/labor ratio is constant across firms. However, from (22):

$$\overline{Y}_t = \int_0^1 Y_t(i)di = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} di \tag{48}$$

Hence, *aggregate supply condition* becomes:

$$Y_t = \frac{1}{D_t}(Z_t K_t^{\alpha} H_t^{1-\alpha} - \tilde{F}), \tag{49}$$

with $D_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}} di$ measuring the extent of aggregate loss of efficiency caused by price dispersion across intermediate goods firms. In Appendix A, we show that aggregate price dispersion D_t evolves according to:

$$D_t = \zeta \left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota} \left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} D_{t-1} + (1-\zeta)\left[\frac{P_t^o}{P_t}\right]^{-\frac{(1+\lambda)}{\lambda}} \tag{50}$$

Equilibrium Conditions and Aggregate Disturbances

We define equilibrium in our economy in a standard way. It is determined by the optimality conditions and laws of motion summarized below:

(1) Households' optimality conditions

$$U'(x_t) = \frac{A}{W_t} \tag{51}$$

$$\frac{U'(x_t)}{P_t} = \beta E_t \left\{ \frac{\chi_{t+1}}{P_{t+1}} \left(\frac{A}{Z_*^{1/(1-\alpha)}} \right)^{(1-\nu_m)} \left(\frac{m_{t+1}}{P_{t+1}} \right)^{-\nu_m} + \frac{U'(x_{t+1})}{P_{t+1}} \right\} \tag{52}$$

$$1 = \mu_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - S'\left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] + \beta E_t \left\{ \mu_{t+1} \frac{U'(x_{t+1})}{U'(x_t)} S'\left(\frac{i_{t+1}}{i_t} \right) \left[\frac{i_{t+1}}{i_t} \right]^2 \right\} \tag{53}$$

$$\mu_t = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \left(R_{t+1}^k + \mu_{t+1}(1-\delta) \right) \right\} \tag{54}$$

$$1 = \beta E_t \left\{ \frac{U'(x_{t+1})}{U'(x_t)} \frac{R_t}{\pi_{t+1}} \right\} \tag{55}$$

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right] i_t \tag{56}$$

$$\Xi_{t+1|t}^p = \frac{U'(x_{t+1})}{U'(x_t)} \frac{1}{\pi_{t+1}} \tag{57}$$

Note that (52) and (55) imply money demand equation[7]:

$$\left(\bar{M}_t \right)^{\nu_m} = \left(\frac{m_{t+1}}{P_t} \right)^{\nu_m} = \frac{\beta R_t}{U'(x_t)(R_t - 1)} E_t \left\{ \left(\frac{A}{Z_*^{1/(1-\alpha)}} \right)^{(1-\nu_m)} \frac{\chi_{t+1}}{\pi_{t+1}^{(1-\nu_m)}} \right\}. \tag{58}$$

(2) Firms' optimality conditions

$$K_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} H_t \tag{59}$$

$$MC_t = \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \frac{W_t^{1-\alpha} \left(R_t^k \right)^{\alpha}}{Z_t} \tag{60}$$

$$f_t^{(1)} = \left(p_t^o \right)^{-\frac{(1+\lambda)}{\lambda}} Y_t + \zeta\beta \left(\pi_t^{\iota} \pi_{**}^{(1-\iota)} \right)^{-\frac{1}{\lambda}} E_t \left\{ \left(\frac{p_t^o}{p_{t+1}^o \pi_{t+1}} \right)^{-\frac{(1+\lambda)}{\lambda}} \Xi_{t+1|t}^p f_{t+1}^{(1)} \right\} \tag{61}$$

[7] We deflate nominal money stock m_{t+1} by P_t (and not P_{t+1}) since it has been chosen in period *t* based on realization of period *t* disturbances. We denote corresponding real money balances by $\bar{M}_{t+1} = m_{t+1}/P_t$.

$$f_t^{(2)} = \left(p_t^o\right)^{-\frac{(1+\lambda)}{\lambda}-1} MC_t Y_t + \zeta\beta\left(\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{(1+\lambda)}{\lambda}} E_t\left\{\left(\frac{p_t^o}{p_{t+1}^o \pi_{t+1}}\right)^{-\frac{(1+\lambda)}{\lambda}-1} \Xi_{t+1|t}^p f_{t+1}^{(2)}\right\} \tag{62}$$

$$f_t^{(1)} = (1+\lambda) f_t^{(2)} \tag{63}$$

$$\pi_t = \left[(1-\zeta)\left(\pi_t p_t^o\right)^{-\frac{1}{\lambda}} + \zeta\left(\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right)^{-\frac{1}{\lambda}}\right]^{-\lambda}, \tag{64}$$

where we have denoted $p_t^o = P_t^o / P_t$ and where equilibrium requires $K_t = k_t$, $H_t = h_t$.

(3) Taylor rule

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*}\right)^{\rho_R}\left(\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{Y_t}{Y_*}\right)^{\psi_2}\right)^{(1-\rho_R)} e^{\varepsilon_{R,t}}, \qquad \text{where } \varepsilon_{R,t} \sim N(0,\sigma_R^2) \tag{65}$$

(4) Aggregate demand and supply

$$X_t + I_t + \left(1 - \frac{1}{g_t}\right)Y_t = Y_t \tag{66}$$

$$Y_t = \frac{1}{D_t}(Z_t K_t^{\alpha} H_t^{1-\alpha} - \tilde{F}) \tag{67}$$

where equilibrium requires that $X_t = x_t$ and $I_t = i_t$, and that:

$$D_t = \zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} D_{t-1} + (1-\zeta)\left[p_t^o\right]^{-\frac{(1+\lambda)}{\lambda}}. \tag{68}$$

(5) Aggregate disturbances (technology, money demand, government spending and monetary policy):

$$\ln Z_t = (1-\rho_Z)\ln Z_* + \rho_Z \ln Z_{t-1} + \varepsilon_{Z,t} \tag{69}$$

$$\ln \chi_t = (1-\rho_\chi)\ln \chi_* + \rho_\chi \ln \chi_{t-1} + \varepsilon_{\chi,t} \tag{70}$$

$$\ln g_t = (1-\rho_g)\ln g_* + \rho_g \ln g_{t-1} + \varepsilon_{g,t}, \tag{71}$$

where it is understood that innovations to the above laws of motion, as well as the monetary policy shock $\varepsilon_{R,t}$, are $iid\, N(0,\sigma_i^2)$ random variables, $i \in \{Z, \chi, g, R\}$.

For convenience, we collect all DSGE model parameters in the vector $\boldsymbol{\theta}$ and stack all innovations in vector $\varepsilon_t = [\varepsilon_{Z,t}, \varepsilon_{\chi,t}, \varepsilon_{g,t}, \varepsilon_{R,t}]'$. We then derive a log-linear approximation to the system of equations (51)-(71) around its deterministic steady state. The resulting linear rational expectations system is solved by method described in Sims (2002).

3 Econometric Methodology

This section discusses estimation techniques for the two models considered in this paper. First, we provide details on a Markov Chain Monte Carlo (MCMC) algorithm to estimate the data-rich DSGE model, including the choice of the prior for factor loadings. Second, we describe the Gibbs Sampler to estimate a dynamic factor model. Finally, we present the novel speedup suggested by Jungbacker and Koopman (2008) that enhances the speed of our Bayesian estimation procedures.

3.1 Estimation of Data-Rich DSGE Model

As discussed in previous section, the state-space representation of our data-rich DSGE model consists of a transition equation of model states S_t and a set of measurement equations relating these states to data X_t:

$$\underbrace{S_t}_{N\times 1} = \underbrace{\mathbf{G(\theta)}}_{N\times N}\underbrace{S_{t-1}}_{N\times 1} + \underbrace{\mathbf{H(\theta)}}_{N\times N_\varepsilon}\underbrace{\varepsilon_t}_{N_\varepsilon \times 1} \tag{72}$$

$$\underbrace{X_t}_{J\times 1} = \underbrace{\mathbf{\Lambda(\theta)}}_{J\times N}\underbrace{S_t}_{N\times 1} + \underbrace{e_t}_{J\times 1} \tag{73}$$

$$e_t = \mathbf{\Psi} e_{t-1} + v_t, \tag{74}$$

where $\varepsilon_t \sim iid\, N(\mathbf{0},\mathbf{Q(\theta)})$, $v_t \sim iid\, N(\mathbf{0},\mathbf{R})$ and where $\mathbf{Q(\theta)}$, $\mathbf{R}$ and $\mathbf{\Psi}$ are assumed diagonal. Essential feature of a data-rich framework is that the panel dimension of data set J is much higher than the number of DSGE model concepts N. For convenience, collect state-space matrices from the measurement equation into $\Gamma = \{\mathbf{\Lambda(\theta)}, \mathbf{\Psi}, \mathbf{R}\}$ and DSGE states-factors into $S^T = \{S_1, S_2, \ldots, S_T\}$. Because of normality of structural shocks ε_t and measurement error innovations v_t, system (72)-(74) is a linear Gaussian state-space model and the likelihood function of data $p(X^T \mid \mathbf{\theta}, \Gamma)$ can be evaluated using Kalman filter.

Following Boivin and Giannoni (2006), we use Bayesian techniques to estimate the unknown model parameters $(\mathbf{\theta}, \Gamma)$. We combine prior $p(\mathbf{\theta},\Gamma) = p(\Gamma \mid \mathbf{\theta})\, p(\mathbf{\theta})$ with likelihood function $p(X^T \mid \mathbf{\theta}, \Gamma)$ to obtain posterior distribution of parameters given data:

$$p(\mathbf{\theta},\Gamma \mid X^T) = \frac{p(X^T \mid \mathbf{\theta},\Gamma)\, p(\mathbf{\theta},\Gamma)}{\int p(X^T \mid \mathbf{\theta},\Gamma)\, p(\mathbf{\theta},\Gamma)\, d\mathbf{\theta}\, d\Gamma} \tag{75}$$

We use Markov Chain Monte Carlo (MCMC) method to estimate posterior density $p(\mathbf{\theta},\Gamma \mid X^T)$ by constructing a Markov Chain with the property that its limiting invariant distribution is our posterior distribution. Similarly to Boivin and Giannoni (2006), and building on the work by An and Schorfheide (2007), Kim and Nelson (2000), Carter and Kohn (1994), Chib and Greenberg (1994), Markov chain is constructed by the Gibbs sampling method with Metropolis-within-Gibbs step to generate draws from posterior distribution $p(\mathbf{\theta},\Gamma \mid X^T)$ and to compute the approximations to posterior means and covariances of parameters of interest.

But before we turn to describing the Gibbs Sampler, we must elaborate on how we connect the DSGE model states to data indicators. This is important, because, unlike Boivin and Giannoni (2006), the link is primarily through the prior on factor loadings $\mathbf{\Lambda(\theta)}$. Recall that we have *core* data series that measure specific model concepts and *non-core* informational variables that are related to all states of DSGE model. Consider the following hypothetical example:

$$\begin{array}{r} \text{core} \left\{ \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \right. \\ \text{non-core} \left\{ \begin{array}{c} \\ \\ \end{array} \right. \end{array} \underbrace{\left[\begin{array}{c} \text{output \#1} \\ \text{output \#2} \\ \text{inflation \#1} \\ \text{inflation \#2} \\ \vdots \\ \hline \text{exchange rate} \\ X_t^{S,\, rest} \end{array} \right]}_{X_t} = \underbrace{\left[\begin{array}{c} \lambda'_{Y1} \\ \lambda'_{Y2} \\ \lambda'_{\pi 1} \\ \lambda'_{\pi 2} \\ \vdots \\ \hline \lambda'_{ER} \\ \mathbf{\Lambda}_S \end{array} \right]}_{\mathbf{\Lambda}(\boldsymbol{\theta})} \cdot \underbrace{\left[\begin{array}{c} \hat{Y}_t \\ \hat{\pi}_t \\ \vdots \end{array} \right]}_{S_t} + \underbrace{\left[\begin{array}{c} e_t^F \\ \hline e_t^S \end{array} \right]}_{e_t} \qquad (76)$$

As a matter of general principle, for each of the core series we center the prior mean of λ's at regular-DSGE-model-implied factor loadings of a corresponding model concept. In example above, this corresponds to the conditional prior for core loadings being:

$$\begin{aligned} p(\lambda_{Y1} \mid \boldsymbol{\theta}) = p(\lambda_{Y2} \mid \boldsymbol{\theta}) = N\left([1,0,0,...,0]', \Omega(\boldsymbol{\theta})\right) \\ p(\lambda_{\pi 1} \mid \boldsymbol{\theta}) = p(\lambda_{\pi 2} \mid \boldsymbol{\theta}) = N\left([0,4,0,...,0]', \Omega(\boldsymbol{\theta})\right). \end{aligned} \qquad (77)$$

This means that in regular DSGE model, the output #1 in data is equal to 1 times output $\hat{Y}_t$ in the model, inflation #1 in data is equal to 4 times inflation $\hat{\pi}_t$ in the model (conversion from quarterly to annual inflation). In data-rich DSGE model, we do not impose $\lambda_{Y,0} = [1,0,0,...,0]'$ and $\lambda_{\pi,0} = [0,4,0,...,0]'$ on loadings λ_Y and λ_π, but instead use them to center the prior means for λ_Y and λ_π. This is different from Boivin and Giannoni (2006), who restrict core factor loadings λ_Y and λ_π to be either $\lambda_{Y,0}$ and $\lambda_{\pi,0}$ or proportional to these.

For non-core series, we center the prior mean of factor loadings at zero vector with identity covariance matrix. In terms of example (76), conditional prior is:

$$p(\lambda_{ER} \mid \boldsymbol{\theta}) = p(\mathbf{\Lambda}'_{S,k} \mid \boldsymbol{\theta}) = N\left([0,0,0,...,0]', \mathbf{I}_N\right), \qquad (78)$$

where sub-index *k* selects one row from matrix $\mathbf{\Lambda}_S$.

Note that prior means for core loadings may in general depend on DSGE model parameters $\boldsymbol{\theta}$. For instance, if core series contain a measure of inverse money velocity IVM_t, then the DSGE model counterpart $\hat{M}_t - \hat{Y}_t$ (real money balances minus real output in logs) depends on state S_t indirectly, say via $\hat{M}_t - \hat{Y}_t = d_{IVM}(\boldsymbol{\theta}) S_t$. As a result, conditional prior for loadings in the IVM measurement equation would be $p(\lambda_{IVM_1} \mid \boldsymbol{\theta}) = N\left(d_{IVM}(\boldsymbol{\theta})', \Omega(\boldsymbol{\theta})\right)$.

Also note that to prevent the data-rich DSGE model from drifting too far away from parameter estimates of a regular DSGE model and to fix the scale of estimated model concepts (we discuss the latter issue in detail in the next section), we make the prior for one of the core series within each core subgroup *perfectly tight*. In example (76), we have two subgroups of core series – output and inflation. This implies, without loss of generality, the perfectly tight prior on loadings in output #1 and inflation #1 equations. Therefore, we write $\mathbf{\Lambda(\theta)}$ to underscore that some loadings will explicitly depend on the DSGE model structural parameters.

Now let us turn to the description of our Gibbs sampler. MCMC implementation for linear Gaussian state-space model (72)-(74) is based on the following conditional posterior distributions:

$$p(\Gamma \mid \boldsymbol{\theta}; X^T) \qquad p(S^T \mid \Gamma, \boldsymbol{\theta}; X^T) \qquad p(\Gamma \mid S^T, \boldsymbol{\theta}; X^T) \qquad p(\boldsymbol{\theta} \mid \Gamma; X^T) \tag{79}$$

The main steps of *Gibbs Sampler* are (we provide full details in Appendix B):

1. Specify initial values $\boldsymbol{\theta}^{(0)}$ and $\Gamma^{(0)}$.
2. Repeat for $g = 1, 2, ..., n_{sim}$

   2.1. Solve DSGE model numerically at $\boldsymbol{\theta}^{(g-1)}$ and obtain matrices $\mathbf{G}(\boldsymbol{\theta}^{(g-1)}), \mathbf{H}(\boldsymbol{\theta}^{(g-1)})$ and $\mathbf{Q}(\boldsymbol{\theta}^{(g-1)})$

   2.2. Draw from $p(\Gamma \mid \boldsymbol{\theta}^{(g-1)}; X^T)$:

   a) Generate unobserved states $S^{T,(g)}$ from $p(S^T \mid \Gamma^{(g-1)}, \boldsymbol{\theta}^{(g-1)}; X^T)$ using Carter-Kohn (1994) forward-backward algorithm;

   b) Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma \mid S^{T,(g)}, \boldsymbol{\theta}^{(g-1)}; X^T)$ by drawing from a complete set of known conditional densities $[\mathbf{R} \mid \mathbf{\Lambda}, \mathbf{\Psi}; \Xi]$, $[\mathbf{\Lambda} \mid \mathbf{R}, \mathbf{\Psi}; \Xi]$ and $[\mathbf{\Psi} \mid \mathbf{\Lambda}, \mathbf{R}; \Xi]$, where $\Xi = \left\{ S^{T,(g)}, \boldsymbol{\theta}^{(g-1)}, X^T \right\}$.

   2.3. Draw DSGE parameters $\boldsymbol{\theta}^{(g)}$ from $p(\boldsymbol{\theta} \mid \Gamma^{(g)}; X^T)$ using Metropolis step:

   a) Propose

   $$\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(g-1)}; \Gamma^{(g)}) \tag{80}$$

   b) Draw $u \sim Uniform(0,1)$ and set

   $$\boldsymbol{\theta}^{(g)} = \begin{cases} \boldsymbol{\theta}^* & \text{if } u \le \alpha(\boldsymbol{\theta}^* \mid \Gamma^{(g)}, \boldsymbol{\theta}^{(g-1)}) \\ \boldsymbol{\theta}^{(g-1)} & \text{otherwise} \end{cases} \tag{81}$$

   where acceptance probability $\alpha(\bullet) = \min\left\{1, r(\boldsymbol{\theta}^{(g-1)}, \boldsymbol{\theta}^*, \Gamma^{(g)})\right\}$ and

   $$r(\boldsymbol{\theta}^{(g-1)}, \boldsymbol{\theta}^*, \Gamma^{(g)}) = \frac{p(\boldsymbol{\theta}^*, \Gamma^{(g)} \mid X^T)}{p(\boldsymbol{\theta}^{(g-1)}, \Gamma^{(g)} \mid X^T)} = \frac{p(X^T \mid \boldsymbol{\theta}^*, \Gamma^{(g)}) p(\Gamma^{(g)} \mid \boldsymbol{\theta}^*) p(\boldsymbol{\theta}^*)}{p(X^T \mid \boldsymbol{\theta}^{(g-1)}, \Gamma^{(g)}) p(\Gamma^{(g)} \mid \boldsymbol{\theta}^{(g-1)}) p(\boldsymbol{\theta}^{(g-1)})}. \tag{82}$$

3. Return $\left\{ \boldsymbol{\theta}^{(g)}, \Gamma^{(g)} \right\}_{g=1}^{n_{sim}}$

Carter-Kohn (1994) algorithm in step 2.2.(a) proceeds as follows. First, it applies Kalman filter to state-space system (72)-(74) to generate filtered DSGE states $\hat{S}_{t|t}$, $t = 1..T$. And then, starting from $\hat{S}_{T|T}$, it rolls back in time along Kalman smoother recursions to draw elements of $S^{T,(g)}$ from a sequence of conditional Gaussian distributions.

The intermediate step to generate DSGE model states $S^{T,(g)}$ is used to facilitate sampling state-space matrices $\Gamma^{(g)}$ in 2.2.(b). Conditional on $S^{T,(g)}$, the elements of matrices $\Gamma^{(g)} = \left\{ \mathbf{\Lambda}^{(g)}, \mathbf{\Psi}^{(g)}, \mathbf{R}^{(g)} \right\}$ are the parameters of simple linear regressions (73)-(74) and we can draw them equation by equation using the approach of Chib and Greenberg (1994). It is a straightforward procedure, since we assume conjugate priors for Γ and conditional posterior densities are all of known functional forms.

To generate DSGE model parameters $\boldsymbol{\theta}^{(g)}$, we introduce Metropolis step 2.3. It is required because density $p(\boldsymbol{\theta} \mid \Gamma; X^T)$ is generally intractable and cannot be easily factorized into known conditionals. We stick to *random-walk version of Metropolis step* in which the proposal density $q(\boldsymbol{\theta}' \mid \boldsymbol{\theta})$ is multivariate Student-t with mean equal to previous draw $\boldsymbol{\theta}^{(g-1)}$ and covariance matrix proportional to inverse Hessian from *regular* DSGE model[8] evaluated at posterior mode.

Under regularity conditions satisfied here for linear Gaussian state-space model, the Markov chain $\{\boldsymbol{\theta}^{(g)}, \Gamma^{(g)}\}$ constructed by Gibbs Sampler above converges to its invariant distribution and, starting from some $g > \overline{g}$, contains draws from posterior distribution of interest $p(\boldsymbol{\theta}, \Gamma \mid X^T)$. Sample averages of these draws (or their appropriate transformations) converge almost surely to respective population moments under our posterior density (Tierney 1994, Chib 2001, Geweke 2005).

Two last implementation issues need to be clarified. To *initialize* our Gibbs Sampler, we first run a regular DSGE model estimation (see footnote 8), compute posterior mean of DSGE model parameters and generate smoothed model states $S^{T,reg}$. Then we take the rich panel of macro and financial series X^T and run equation-by-equation OLS regressions of X_k^T on smoothed DSGE states $S^{T,reg}$ to back out initial values for $\mathbf{\Lambda}$, $\mathbf{\Psi}$ and $\mathbf{R}$.

On a separate note, in measurement equations (73) we keep only the non-redundant state variables of a DSGE model. Because some of the DSGE states are merely linear combinations of the other states, one can interpret this as minimum-state-variable approach in the spirit of McCallum (1983, 1999, 2003). Here, though, the main rationale is to avoid multicollinearity on the right hand side of (73). We always set the corresponding factor loadings in $\mathbf{\Lambda}$ equal to zero.

3.2 Estimation of Dynamic Factor Model

Consider the original dynamic factor model described in section 2.1:

$$X_t = \mathbf{\Lambda} F_t + e_t \tag{83}$$

$$F_t = \mathbf{G} F_{t-1} + \eta_t, \qquad \eta_t \sim iid\ N(\mathbf{0}, \mathbf{Q}) \tag{84}$$

[8] Running a bit ahead, in our empirical analysis this regular DSGE estimation featured the same underlying theoretical DSGE model as in data-rich version, but only four (equal to # of shocks) core observables assumed to have been measured without errors. These core observables are (appropriately transformed) real GDP, GDP deflator inflation, Federal Funds rate and inverse velocity of money based on M2S. See details in Data and Transformations section. Also see the notes to Table D3.

$$e_t = \mathbf{\Psi} e_{t-1} + v_t, \qquad v_t \sim iid\ N(\mathbf{0}, \mathbf{R}). \tag{85}$$

Let us collect the state-space matrices into $\Gamma = \{\mathbf{\Lambda}, \mathbf{\Psi}, \mathbf{R}, \mathbf{G}\}$ and the latent empirical factors into $F^T = \{F_1, F_2, \ldots, F_T\}$. Similarly to the data-rich DSGE model, (83)-(85) is a linear Gaussian state-space model, and we are interested in joint inference about model parameters Γ and latent factors F^T. Unlike in the data-rich DSGE model, though, we no longer have deep structural parameters determining the behavior of matrices in transition equation (84).

We sidestep the problem of proper dimension of factor space by assuming that $\dim(F_t) = N = 6$, the number of non-redundant model concepts in the data-rich DSGE model. In contrast, the dynamic factor literature devoted considerable attention to developing the objective criteria that would determine the proper number of static factors by trading the fit against complexity (Bai and Ng, 2002) and of dynamic factors (e.g. Bai and Ng 2007, Hallin and Liska 2007, Amengual, Watson 2007, Stock and Watson 2005) in DFMs similar to the one above. Our choice, is, however, indirectly supported by work of Stock and Watson (2005) and Jungbacker and Koopman (2008) who, roughly based on these criteria, find seven dynamic and seven static factors driving a similar panel of macro and financial data.

Another problem associated with dynamic factor model (83)-(85) is that the scales and signs of factors F_t and of factor loadings $\mathbf{\Lambda}$ are not separately identified. Regarding scales, take any invertible $N \times N$ matrix $\mathbf{P}$ and notice that the transformed model is observationally equivalent to the original one:

$$X_t = \underbrace{\mathbf{\Lambda}\mathbf{P}^{-1}}_{\tilde{\mathbf{\Lambda}}} \underbrace{\mathbf{P}F_t}_{\tilde{F}_t} + e_t \tag{86}$$

$$\underbrace{\mathbf{P}F_t}_{\tilde{F}_t} = \underbrace{\mathbf{P}\mathbf{G}\mathbf{P}^{-1}}_{\tilde{\mathbf{G}}} \underbrace{\mathbf{P}F_{t-1}}_{\tilde{F}_{t-1}} + \tilde{\eta}_t, \qquad \tilde{\eta}_t \sim iid\ N(\mathbf{0}, \underbrace{\mathbf{P}\mathbf{Q}\mathbf{P}'}_{\tilde{\mathbf{Q}}}) \tag{87}$$

Regarding signs, for the moment think of (83)-(85) as a model with only one factor. Then multiply by -1 the transition equation (84), as well as factor loading and factor itself in measurement equation (83). We obtain the new model, yet observationally equivalent to the original.

We follow the factor literature (e.g. Geweke, Zhu 1996; Jungbacker, Koopman 2008) and make the following normalization assumptions to tell apart factors from factor loadings:

(i) set $\mathbf{Q} = \mathbf{I}_N$ to fix the scale of factors;

(ii) require one loading in $\mathbf{\Lambda}$ to be positive for each factor (sign restrictions);

(iii) normalize some factor loadings in $\mathbf{\Lambda}$ to pin down specific factor rotation.

Denote by $\mathbf{\Lambda_1}$ the upper $N \times N$ block of $\mathbf{\Lambda}$ so that $\mathbf{\Lambda} = [\mathbf{\Lambda}_1'; \mathbf{\Lambda}_2']'$. One way to implement (ii) and (iii) would be to assume that $\mathbf{\Lambda_1}$ is lower triangular (i.e. $\lambda_{ij} = 0$ for $j > i$, $i = 1, 2, ..., N-1$) with strictly positive diagonal $\lambda_{ii} > 0$, $i = \overline{1, N}$ (see Harvey 1989, p.451). However, our data set in estimation, to be described later in section Data and Transformations, will consist of *core* and *non-core* macro and financial series.

Furthermore, within core series we will have four blocks of variables: real output, inflation, nominal interest rate and inverse velocity of money blocks, respectively; each block contains several measures of the same concept. For this reason, we choose another alternative to implement normalizations (ii) and (iii) – the block-diagonal scheme that to some degree exploits group structure of core series in data X_t :

$$\begin{array}{r|cc:cccc}
 & F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\
\hline
\text{Real output \#1} & 1 & 1 & +1 & 0 & 0 & 0 \\
\text{Real output \#2} & 1 & +1 & 1 & 0 & 0 & 0 \\
\text{Real output \#3} & 1 & 1 & 1 & 0 & 0 & 0 \\
\hdashline
\text{Inflation \#1} & 1 & 1 & 0 & 1 & 0 & 0 \\
\text{Inflation \#2} & +1 & 1 & 0 & 1 & 0 & 0 \\
\text{Inflation \#3} & 1 & 1 & 0 & 1 & 0 & 0 \\
\hdashline
\text{Interest rate \#1} & 1 & 1 & 0 & 0 & +1 & 0 \\
\text{Interest rate \#2} & 1 & 1 & 0 & 0 & 1 & 0 \\
\text{Interest rate \#3} & 1 & 1 & 0 & 0 & 1 & 0 \\
\hdashline
\text{IVM \#1} & 1 & 1 & 0 & 0 & 0 & 1 \\
\text{IVM \#2} & 1 & 1 & 0 & 0 & 0 & +1 \\
\text{IVM \#3} & 1 & 1 & 0 & 0 & 0 & 1 \\
\hline
X^{non-core} & 1 & 1 & 1 & 1 & 1 & 1
\end{array} \tag{88}$$

where 1s stand for non-zero elements in $\mathbf{\Lambda}$.

We acknowledge that our block-diagonal scheme imposes some overidentifying restrictions on factor loadings beyond those minimally necessary. However, scheme (88) can also be interpreted as a special case of the appealing dynamic hierarchical factor model of Moench, Ng, and Potter (2008), which – on top of aggregate common factors – introduces intermediate block factors and makes use of the block structure of the data.

Now, to estimate the model (83)-(85) under normalizing assumptions (i)-(iii), we apply the same line of reasoning as in estimation of the data-rich DSGE model. We construct a Gibbs Sampler that iterates on a complete set of known conditional posterior densities to generate draws from the joint posterior distribution $p(\Gamma, F^T \mid X^T)$ of model parameters $\Gamma = \{\mathbf{\Lambda}, \mathbf{\Psi}, \mathbf{R}, \mathbf{G}\}$ and latent factors F^T :

$$p(F^T \mid \Gamma; X^T) \propto p(F^T \mid \Gamma) p(X^T \mid \Gamma, F^T) \tag{89}$$

$$p(\Gamma \mid F^T; X^T) \propto p(\Gamma) p(F^T \mid \Gamma) p(X^T \mid \Gamma, F^T) \tag{90}$$

The main steps of the Gibbs Sampler are:

1. Specify initial values $\Gamma^{(0)}$ and $F^{T,(0)}$.
2. Repeat for $g = 1, 2, ..., n_{sim}$
   - 2.1. Generate latent factors $F^{T,(g)}$ from $p(F^T \mid \Gamma^{(g-1)}; X^T)$ using Carter-Kohn (1994) forward-backward algorithm;
   - 2.2. Generate state-space parameters $\Gamma^{(g)}$ from $p(\Gamma \mid F^{T,(g)}; X^T)$ by drawing from a complete set of known conditional densities.

3. Return $\left\{\Gamma^{(g)}, F^{T,(g)}\right\}_{g=1}^{n_{sim}}$

Compared to MCMC algorithm for the data-rich DSGE model, this Gibbs sampler is easier and it differs in two key respects:

1. We no longer have complicated Metropolis step, since there is no deep structural parameters $\boldsymbol{\theta}$ coming from the economic model;
2. Inside Γ, we have to draw matrix $\mathbf{G}$ from transition equation of factors (in data-rich DSGE model it was pinned down by numerical solution of a DSGE model given structural parameters $\boldsymbol{\theta}$).

To draw the latent factors F^T from $p(F^T \mid \Gamma; X^T)$, we use the familiar Carter-Kohn (1994) machinery. First, we apply the Kalman filter to linear Gaussian state-space system (83)-(85) to generate filtered latent factors $\hat{F}_{t|t}$, $t=\overline{1,T}$. Then, starting from $\hat{F}_{T|T}$, we roll back in time along the Kalman smoother recursions and generate $F^T = \left\{F_1, F_2, \ldots, F_T\right\}$ by recursively sampling from a sequence of conditional Gaussian distributions: $\left[F_T \mid X^T; \Gamma\right]$, $\left[F_{T-1} \mid F_T, X^{T-1}; \Gamma\right]$, $\ldots\left[F_t \mid F_{t+1}, X^t; \Gamma\right]\ldots\left[F_1 \mid F_2, X_1; \Gamma\right]$, where $X^t = \left\{X_1, \ldots, X_t\right\}$.

To sample from conditional posterior $p(\Gamma \mid F^T; X^T)$, we notice the following: with diagonality of matrices $\boldsymbol{\Psi}$ and $\mathbf{R}$ and conditional on factors F^T, (83) and (85) are a set of standard multivariate linear regressions with AR(1) errors and Gaussian innovations ($k=\overline{1,J}$):

$$X_{k,t} = \boldsymbol{\Lambda}_k' F_t + e_{k,t}, \qquad e_{k,t} = \psi_{kk} e_{k,t-1} + v_{k,t}, \qquad v_{k,t} \sim iid\ N(0, R_{kk}). \tag{91}$$

Hence, under conjugate prior $p(\boldsymbol{\Lambda}, \boldsymbol{\Psi}, \mathbf{R})$, we can apply the insight of Chib and Greenberg (1994) to derive conditional posteriors $\left[\mathbf{R} \mid (\boldsymbol{\Lambda}, \boldsymbol{\Psi}); \mathbf{G}, F^T, X^T\right]$, $\left[\boldsymbol{\Lambda} \mid (\mathbf{R}, \boldsymbol{\Psi}); \mathbf{G}, F^T, X^T\right]$, $\left[\boldsymbol{\Psi} \mid (\boldsymbol{\Lambda}, \mathbf{R}); \mathbf{G}, F^T, X^T\right]$ and to sample accordingly.

What remains to be drawn is the transition matrix $\mathbf{G}$. Given factors F^T, conditional posterior $p(\mathbf{G} \mid (\boldsymbol{\Lambda}, \mathbf{R}, \boldsymbol{\Psi}); F^T, X^T)$ can be derived from a VAR(1) in (84):

$$F_t = \mathbf{G} F_{t-1} + \eta_t, \qquad \eta_t \sim iid\ N(\mathbf{0}, \mathbf{I}_N). \tag{92}$$

We assume the so called Minnesota prior (Doan, Litterman and Sims, 1984; specific version comes from Lubik and Schorfheide, 2005) on transition matrix $\mathbf{G}$ and truncate it to the region consistent with stationarity of (92). We implement our prior by a set of dummy observations that tilt the VAR to a collection of univariate random walks.

3.3 Speedup: Jungbacker, Koopman 2008

Consider the original pure dynamic factor model that we have described in section 2.1. The same argument will in essence apply to a data-rich DSGE model written in the state-space form.

$$\underbrace{X_t}_{J\times 1} = \underbrace{\mathbf{\Lambda}}_{J\times N}\underbrace{F_t}_{N\times 1} + \underbrace{e_t}_{J\times 1} \tag{93}$$

$$\underbrace{F_t}_{N\times 1} = \underbrace{\mathbf{G}}_{N\times N}\underbrace{F_{t-1}}_{N\times 1} + \underbrace{\eta_t}_{N\times 1}, \qquad \eta_t \sim iid\, N(0,\mathbf{Q}) \tag{94}$$

$$\underbrace{e_t}_{J\times 1} = \underbrace{\mathbf{\Psi}}_{J\times J}\underbrace{e_{t-1}}_{J\times 1} + \underbrace{v_t}_{J\times 1}, \qquad v_t \sim iid\, N(0,\mathbf{R}) \tag{95}$$

Here the matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{\Psi}$ are assumed diagonal, which implies we have an "exact" DFM: $E(e_{i,t}e_{j,s}) = 0,\ i \neq j,$ all t and s. First-order dynamics of errors e_t allows us to rewrite the system (93)-(95) in state-space form as follows:

$$\tilde{X}_t = \underbrace{\left[\mathbf{\Lambda} \;\vdots\; -\mathbf{\Psi}\mathbf{\Lambda}\right]}_{\tilde{\mathbf{\Lambda}}}\underbrace{\begin{bmatrix} F_t \\ F_{t-1} \end{bmatrix}}_{\tilde{F}_t} + v_t \tag{96}$$

$$\tilde{F}_t = \underbrace{\begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{G}}}\tilde{F}_{t-1} + \underbrace{\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{H}}}\eta_t, \tag{97}$$

where we denoted $\tilde{X}_t = X_t - \mathbf{\Psi}X_{t-1}$. Collect all the matrices in $\mathbf{\Theta} = \left\{\mathbf{\Lambda}, \mathbf{\Psi}, \mathbf{R}, \tilde{\mathbf{G}}, \tilde{\mathbf{H}}, \mathbf{Q}\right\}$.

Our dynamic factor model (96)-(97) is potentially a high-dimensional object (panel dimension J is high) and therefore the MCMC algorithms outlined in 3.1 and 3.2 spend a lot of time evaluating likelihood with Kalman filter and sampling unobserved factors $\tilde{F}_t$ at every iteration.

To reduce computational costs associated with likelihood-based analysis of (96)-(97), Jungbacker and Koopman (2008) proposed to use the Kalman filter and smoother techniques based on a *lower-dimensional transformation* of original data vector $\tilde{X}_t$. Suppose this transformation is implemented by some $J\times J$ invertible matrix $\mathbf{A}$ such that $\tilde{X}_t^* = \mathbf{A}\tilde{X}_t$, $t = 1..T$. Also, suppose that we partition $\tilde{X}_t^*$ and $\mathbf{A}$ as below:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^L \\ \mathbf{A}^H \end{bmatrix}, \qquad \tilde{X}_t^* = \begin{bmatrix} \tilde{X}_t^L \\ \tilde{X}_t^H \end{bmatrix}, \qquad \text{where } \tilde{X}_t^L = \mathbf{A}^L\tilde{X}_t,\ \tilde{X}_t^H = \mathbf{A}^H\tilde{X}_t, \tag{98}$$

with matrices $\mathbf{A}^L$ and $\mathbf{A}^H$ being $m\times J$ and $(J-m)\times J$, $m < J$.

Jungbacker and Koopman are able to show (Lemma 1, Lemma 2) that you can find a suitable matrix $\mathbf{A}$ such that $\tilde{X}_t^L$ and $\tilde{X}_t^H$ are uncorrelated and only the low-dimensional sub-vector $\tilde{X}_t^L$ depends on factors $\tilde{F}_t$:

$$\begin{aligned} \tilde{X}_t^L &= \mathbf{A}^L\tilde{\mathbf{\Lambda}}\tilde{F}_t + v_t^L, \\ \tilde{X}_t^H &= v_t^H, \end{aligned} \qquad \begin{bmatrix} v_t^L \\ v_t^H \end{bmatrix} \sim iidN\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{\Sigma}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_H \end{bmatrix}\right), \tag{99}$$

where $\mathbf{\Sigma}_L = \mathbf{A}^L \mathbf{R} \mathbf{A}^{L\prime}$ and $\mathbf{\Sigma}_H = \mathbf{A}^H \mathbf{R} \mathbf{A}^{H\prime}$. Moreover, they show that the knowledge of a high-dimensional matrix $\mathbf{A}^H$ and a data vector $\tilde{X}_t^H$ is not required to estimate the unobserved factors $\tilde{F}_t$ and to compute the likelihood of the original DFM.

In terms of matrix $\mathbf{A}^L$, Jungbacker and Koopman prove that it should be of the form:

$$\mathbf{A}^L = \mathbf{C}\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}, \tag{100}$$

for some invertible $m \times m$ matrix $\mathbf{C}$ and $J \times m$ matrix $\bar{\mathbf{\Lambda}}$, columns of which form a basis of column space of $\tilde{\mathbf{\Lambda}}$. In practice, they recommend setting $\bar{\mathbf{\Lambda}} = \tilde{\mathbf{\Lambda}}$ and $\mathbf{C} = \left(\tilde{\mathbf{\Lambda}}'\mathbf{R}^{-1}\tilde{\mathbf{\Lambda}}\right)^{-1}$ in case the matrix of factor loadings $\tilde{\mathbf{\Lambda}}$ has full column rank.

Now that we know $\mathbf{A}^L$ we can sample factors $\tilde{F}_t$ using Carter-Kohn (1994) forward-backward algorithm applied to a lower-dimensional model

$$\tilde{X}_t^L = \mathbf{A}^L \tilde{\mathbf{\Lambda}} \tilde{F}_t + v_t^L, \qquad v_t^L \sim iid\, N(\mathbf{0}, \mathbf{\Sigma}_L) \tag{101}$$

$$\tilde{F}_t = \tilde{\mathbf{G}} \tilde{F}_{t-1} + \tilde{\mathbf{H}} \eta_t, \qquad \eta_t \sim iid\, N(\mathbf{0}, \mathbf{Q}). \tag{102}$$

We can also compute the log-likelihood of data $L(\tilde{X} \mid \mathbf{\Theta})$ as

$$L(\tilde{X} \mid \mathbf{\Theta}) = c + L(\tilde{X}^L \mid \mathbf{\Theta}) - \frac{T}{2}\log\frac{|\mathbf{R}|}{|\mathbf{\Sigma}_L|} - \frac{1}{2}\sum_{t=1}^{T} \hat{v}_t' \mathbf{R}^{-1} \hat{v}_t, \tag{103}$$

where $c = -\frac{1}{2}(J-m)T\log(2\pi)$ and $\hat{v}_t = \tilde{X}_t - \left[\bar{\mathbf{\Lambda}}\left(\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}\bar{\mathbf{\Lambda}}\right)\bar{\mathbf{\Lambda}}'\mathbf{R}^{-1}\right]\tilde{X}_t$. The term $L(\tilde{X}^L \mid \mathbf{\Theta})$ is the log-likelihood of transformed data evaluated by Kalman filter during the forward pass of Carter-Kohn algorithm on low-dimensional model (101)-(102).

In ensuing empirical analysis of a data-rich DSGE model and an empirical DFM, we have applied the Jungbacker-Koopman algorithm presented in this section to improve the speed of computations in both models. To get a sense of CPU time gains, we have also estimated the models – though on fewer draws – without the speedup. We find that the “improved” estimation of the empirical DFM runs 10.5 times faster (a magnitude consistent with CPU gains reported by Jungbacker and Koopman for a DFM of similar size in their study) and the “improved” estimation of the data-rich DSGE model runs 2.5 times faster. Differences in time savings are due to a significant chunk of time that it takes to solve numerically the underlying DSGE model in data-rich DSGE model estimation, a step absent in DFM estimation and not affected by Jungbacker-Koopman speedup.

4 Data and Transformations

To estimate the dynamic factor model and the data-rich DSGE model, we employ a large panel of U.S. quarterly macroeconomic and financial time series compiled by Stock and Watson (2008)[9]. The panel covers 1959:Q1 – 2006:Q4, however, our sample in this paper spans only 1984:Q1 – 2005:Q4. We focus on this later period primarily for two reasons: (i) to avoid dealing with Great Moderation issue[10]; (ii) to concentrate on a period with relatively stable monetary policy regime.

Our data set consists of 12 *core series* that either measure specific DSGE model concepts or are used in DFM normalization scheme (88), and 77 *non-core* informational series that load on all DSGE states (DFM factors) and may contain useful information about the aggregate state of the economy. The core series include three measures of real output (real GDP, index of total industrial production and index of industrial production: manufacturing), three measures of price inflation (GDP deflator inflation, personal consumption expenditure (PCE) deflator inflation, and CPI inflation), three indicators of nominal interest rates (Federal Funds rate, 3-month T-bill rate and the yield on AAA rated corporate bonds), and three series measuring the inverse velocity of money (IVM based on M1 aggregate, M2 aggregate and IVM based on adjusted monetary base). 77 non-core series include the measures of real activity, labor market variables, housing indicators, prices and wages, financial variables (interest rate spreads, exchange rate depreciations, credit stocks, stock returns) and, together with appropriate transformations to eliminate trends, are described in Appendix C.

Most of the core series are computed based on the raw indicators from Stock and Watson (2008) database and from Fred-II database[11] maintained by Federal Reserve Bank of St. Louis (database mnemonics are in italics). To obtain three measures of *real per-capita output*, we take real GDP (*SW2008::GDP251*), total industrial production (*SW2008::IPS10*) and industrial production in manufacturing sector (*SW2008::IPS43*), and divide each series by civilian non-institutional population (*Fred-II::CNP16OV*). We then take natural logarithm and extract the linear trend by an OLS regression. The resulting detrended series are multiplied by 100 to convert them to percentage deviations from respective means. The *inflation* measures are computed as the first difference of the natural logarithm of GDP deflator (*SW2008::GDP272A*), of PCE deflator (*SW2008::GDP273A*), and of Consumer Price Index – All Items (*SW2008::CPIAUCSL*), all multiplied by 400 to get to the annualized percentages. Our indicators of *nominal interest rate* are (i) the effective Federal Funds rate (*SW2008::FYFF*), (ii) 3-month U.S. Treasury bill rate in the secondary market (*SW2008::FYGM3*) and (iii) the yield on Moody's AAA

[9] The data set is available online at:
http://www.princeton.edu/~mwatson/ddisk/hendryfestschrift_replicationfiles_April28_2008.zip

[10] The "Great Moderation" refers to a decline in volatility of output and inflation observed in the U.S. since the mid-1980s until recent financial crisis. For evidence and implications, please see Bernanke (2004), Stock and Watson (2002c), Kim and Nelson (1999), and McConnell and Perez-Quiros (2000). The last two papers attribute a break in the volatility of U.S. GDP growth to 1984:Q1.

[11] Fred-II database is available online at: http://research.stlouisfed.org/fred2/

rated corporate bonds (*SW2008::FYAAAC*). We use simple 3-month average to obtain quarterly annualized interest rates from monthly raw data.

To generate the appropriate *inverse money velocities*, we take three monetary aggregates: sweep-adjusted money stock M1 (*CDJ::M1S*), sweep-adjusted money stock M2 (*CDJ::M2S*) and the monetary base adjusted for changes in reserve requirements (*SW2008::FMFBA*). The sweep-adjusted stocks M1S and M2S are provided by Cynamon, Dutkowsky and Jones (2006)[12] and correct the distortionary impact (on conventional measures M1 and M2) of the financial innovation started in early 1990s. These distortions take the form of underreporting of actual transactions balances and arise because of retail sweep programs and commercial demand deposit sweep programs, in which U.S. banks move a portion of funds from their customer demand deposits or other checkable deposits into instruments with zero reserve requirements. Since our DSGE model does not have any explicit open economy context, we further adjust the monetary base FMFBA by deducting the amount of U.S. dollar currency held physically outside the United States[13]. We take M1S, M2S and the adjusted FMFBA, divide each series by the nominal GDP (*Fred-II::GDP*) to obtain respective inverse velocities of money. For each IVM, we take the natural logarithm of the M/GDP ratio and scale it by 100. Finally, we remove the linear deterministic trend from IVM based on M1S.

Because measurement equations (73) and (83) are modeled without intercepts, we estimate a dynamic factor and a data-rich DSGE models on a demeaned data set. Also, in line with standard practice in factor literature, we standardize each time series so that its sample variance is equal to unity (however, we do not scale the core series when estimating the data-rich DSGE model).

5 Empirical Analysis

The next step in our analysis is to take a dynamic factor model and a data-rich DSGE model to the data using MCMC algorithms described above and to present the empirical results. We begin by discussing the choice of the prior distributions of model parameters and then briefly describe the posterior estimates of deep structural parameters of the data-rich DSGE model. Second, we analyze the estimated empirical factors and the estimates of the DSGE model state variables, and explore how well they are able to capture the co-movements in the data. Third, we compare the spaces spanned by the latent empirical factors and by the data-rich DSGE model concepts. Finally, we use the proximity of the factor spaces to propagate the monetary policy and technology innovations in otherwise non-structural dynamic factor model and obtain the predictions from both models for the core and non-core macro and financial series of interest.

---

[12] Sweep-adjusted money stocks are available online at: http://www.sweepmeasures.com.

[13] Federal Reserve Board: Flow of Funds Accounts of the United States: Z.1 Statistical Release for Mar 12, 2009 (available at http://www.federalreserve.gov/releases/z1/20090312/). Table L.204 “Checkable Deposits and Currency”, line 23 (Rest of the world: Currency), unique identifier: Z1/Z1/FL263025003.Q

5.1 Priors and Posteriors

Since we estimate the DFM (83)-(85) and the data-rich DSGE model (72)-(74) using Bayesian techniques, we have to provide prior distributions for both models' parameters.

Let us first turn to a dynamic factor model. Let $\mathbf{\Lambda}_k$ and R_{kk} be the factor loadings and a variance of the measurement error innovation for the k^{th} measurement equation, $k = 1..J$. Similarly to Boivin and Giannoni (2006) and Kose, Otrok and Whiteman (2008), we assume a joint Normal-InverseGamma prior distribution for $\left(\mathbf{\Lambda}_k, R_{kk}\right)$ so that $R_{kk} \sim IG_2(s_0, \nu_0)$ with location parameter $s_0 = 0.001$ and degrees of freedom $\nu_0 = 3$, and the prior mean of factor loadings is centered around the vector of zeros $\mathbf{\Lambda}_k \mid R_{kk} \sim N(\mathbf{\Lambda}_{k,0}, R_{kk}\mathbf{M}_0^{-1})$ with $\mathbf{\Lambda}_{k,0} = \mathbf{0}$ and $\mathbf{M}_0 = \mathbf{I}_N$. Originally, we allowed for a fairly diffuse $N(0,1)$ prior for the k^{th} measurement equation's autocorrelation Ψ_{kk} , all k. However, because our data transformations are different from the usual Stock and Watson (2008) procedures aimed at achieving approximate stationarity in all series and there could still be series with stochastic trends which we seek to capture with potentially highly persistent dynamic factors, we have decided to make this prior perfectly tight. This implies that all measurement errors are *iid* mean-zero normal random variables. Finally, as explained in Section 3.2, for the factor transition matrix $\mathbf{G}$, we implement a version of Minnesota prior (Lubik and Schorfheide, 2005) and tilt the transition equation (84) to a collection of univariate random walks[14].

We use the Gibbs Sampler presented above to estimate our dynamic factor model. We do not discuss the posterior estimates of DFM parameters here, since we are more interested in comparing factor spaces spanned by the estimated latent factors and by the DSGE model states. However, all the parameter estimates are collected in the technical appendix to this paper available upon request.

In our data-rich DSGE model, we have two groups of parameters – state-space model parameters comprising matrices $\mathbf{\Lambda}$, $\mathbf{\Psi}$ and $\mathbf{R}$, and deep structural parameters $\boldsymbol{\theta}$ of an underlying DSGE model. The prior for the state-space matrices is elicited differently for the core and the non-core data indicators contained in X_t. Regarding the non-core measurement equations, the prior for $\left(\mathbf{\Lambda}_k, R_{kk}\right)$ and for Ψ_{kk} is identical to the one assumed in DFM. In contrast, the prior distribution for the factor loadings in the core measurement equations follows the scheme explained in example (76). Instead of hypothetical "output" and "inflation" groups, we substitute four categories of the core series: real output, inflation, nominal interest rate, and inverse velocity of money, with three specific measures within each category, as described in Data and Transformations section. The joint prior distribution is still Normal-InverseGamma $(\mathbf{\Lambda}_{k,0}, \mathbf{M}_0, s_0, \nu_o)$, but now, for each of the core series, the prior mean of the factor loadings $\mathbf{\Lambda}_{k,0}$ is centered at the regular-DSGE-model-implied factor loadings of a corresponding DSGE model variable

[14] The hyperparameters in actual implementation of the Minnesota prior were set as follows: $\tau = 5$, $d = 0.5$, $w = 1$, $\lambda = 0$, $\mu = 0$. We have also truncated the prior to the region consistent with stationarity of the factor transition equation.

(real output $\hat{Y}_t$, inflation $\hat{\pi}_t$, nominal interest rate $\hat{R}_t$ or inverse money velocity $\hat{\bar{M}}_t - \hat{Y}_t$), evaluated at the current draw of deep structural parameters $\boldsymbol{\theta}$. The covariance scaling matrix $\mathbf{M}_0$ is assumed diagonal $\mathbf{M}_0 = diag(\boldsymbol{\Omega}(\boldsymbol{\theta}))$, where $\boldsymbol{\Omega}(\boldsymbol{\theta})$ is the unconditional covariance matrix of the DSGE model state variables evaluated at current draw of $\boldsymbol{\theta}$. $\mathbf{M}_0$ is the same across all core measurement equations. This choice implies that the prior will be tighter for the loadings on more volatile DSGE states. A similar approach is pursued in Schorfheide, Sill and Kryshko (2010). The scale s_0 and degrees of freedom ν_0 are the same as in DFM case. Finally, as argued in section 3.1, we make this prior perfectly tight for real GDP, GDP deflator inflation, Federal Funds rate and IVM based on M2S monetary aggregate.

Our choice of prior distribution for the deep structural parameters of a DSGE model broadly follows Aruoba and Schorfheide (2009). A subset of these parameters that are *fixed* in estimation is reported in Table D1. We choose to have a logarithmic utility of household consumption by fixing $\gamma = 1$. We set depreciation rate of capital δ to 0.014, which is the average quarterly ratio of depreciation of fixed assets to the stock of these fixed assets in 1959-2005 (NIPA-FAT11 for stocks, NIPA-FAT13 for depreciation of fixed assets and consumer durables). Steady state annualized inflation rate π_A is fixed at 2.5 percent – the average GDP deflator inflation in our sample. We implicitly impose the Fischer equation and let the steady state annualized real interest rate r_A be equal to 2.84 percent. This value is obtained as the average Federal Funds interest rate in our sample minus π_A. The households' discount factor is therefore $\beta = 1/(1 + r_A/400)$.

We also introduce several normalizations. We normalize to 1 the steady state real output Y_* and steady state money demand shock χ_*. We use average log inverse velocity of money (log[M2S/GDP]) in our sample to pin down $\log(\bar{M}_*/Y_*)$. Finally, as in Aruoba and Schorfheide (2009), we fix $\log(H_*/Y_*)$ to -3.5. This number is derived from the average inverse labor productivity in the data. In our sample, on average a worker produces roughly 33$ of real GDP per one hour. Hence average H/Y in data is $1/33$. From the average share of government spending (consumption plus investment) in nominal GDP, we calibrate g_* to be 1.2.

We also want our data-rich DSGE model to be broadly consistent – in terms of the monetary policy conduct – with the other regular DSGE models estimated on post-1983 data. Therefore, we shut down "data-richness" for a moment and estimate our DSGE model on just three standard observables: real GDP, GDP deflator inflation and Federal Funds rate. The resulting estimates of the Taylor (1993) rule coefficients were: $\psi_1 = 1.82$, $\psi_2 = 0.18$ and $\rho_R = 0.78$. In estimation of the data-rich DSGE model, we set the policy rule coefficients to these values. This procedure is similar in spirit to Boivin and Giannoni (2006) who assume that the policy rate R_t is measured in the data by the Federal Funds rate without an error. This assumption guarantees that the estimated monetary policy rule coefficients will not drift far away from the conventional post-1983 values documented in the literature.

Despite detrending performed on all three measures of real per-capita output, they are still highly persistent. To strike a balance between the observed output persistence and the need to have stationarity in the model, we fix the autocorrelation of the technology shock ρ_Z at 0.98. In the intermediate goods producing sector, we further assume no fixed costs ($\tilde{F}=0$) and the absence of static indexation for non-optimizing firms ($\pi_{**}=1$).

The prior distributions for other parameters are summarized in Table D2. The prior for the steady-state related parameters represents the view that capital share of α in Cobb-Douglas production function of intermediate goods firms is about 0.3 and that the average markup these firms charge is about 15 percent. The prior for the Calvo (1983) probability ζ controlling nominal price rigidity is quite agnostic and spans the range of values consistent with fairly rigid and fairly flexible prices. As in Del Negro and Schorfheide (2008), the prior density for the price indexation parameter ι is close to uniform on a unit interval. Parameter ν_m controlling the interest-rate elasticity of money demand is a priori distributed according to a Gamma distribution with mean 20 and standard deviation 5. Existing literature (e.g. Aruoba, Schorfheide 2009, Levin, Onatsky, Williams and Williams 2005, Christiano, Eichenbaum and Evans 2005) documents fairly large estimates of the money demand elasticity ranging from 10 to 25. The 90 percent interval for the investment adjustment cost parameter S'' spans values that Christiano, Eichenbaum, Evans (2005) find when matching DSGE and vector autoregression impulse response functions. The priors for the parameters determining the exogenous shock processes are taken from Aruoba and Schorfheide (2009). They reflect the belief that the money demand and government spending shocks are quite persistent.

Using the Gibbs Sampler with Metropolis step outlined in section 3.1, we estimate the data-rich DSGE model and report the posterior means and 90% credible intervals of deep structural parameters in the last two columns of Table D3. We find the capital share of output and the average price markup to be in line with estimates from regular – few observables, perfect measurement – DSGE estimation. We find little evidence on dynamic indexation by intermediate goods firms. The implied average duration of nominal price rigidity is about $1/(1-0.797)$ = 4.9 quarters. On the one hand, this is close to what Aruoba and Schorfheide (2009) find in their money-in-the-utility specification of a DSGE model and what Del Negro and Schorfheide (2008) document under the "standard" agnostic prior about nominal price rigidities (their Table 6, p. 1206). On the other hand, this is much higher than the price contracts duration of about 3 quarters found by Smets and Wouters (2007) and Schorfheide, Sill and Kryshko (2010). In the context of a data-rich DSGE model similar to ours, Boivin and Giannoni (2006)'s estimates imply that the firms change prices very slowly – on average once per at least 7 quarters. As anticipated, we have obtained a fairly high elasticity of money demand. Our estimate of ν_m implies that a 100 basis points increase in the interest rate leads to a 3.2 percent decline in real money balances. A very large estimate of the investment adjustment cost parameter, as Aruoba and Schorfheide (2009) argue, has something to do with the need to reduce the volatility of the return to capital and to dampen its effect on marginal costs which in turn affect current inflation through the New Keynesian Phillips curve relationship. This is reasonable given that in our data-rich DSGE model the industrial production measures of real output are more volatile than the GDP-based measure, while the volatilities of inflation measures are fairly similar. The money demand shock χ_t turns out to be highly serially correlated, and the persistence of the government spending shock g_t is high as well, but more moderate. This is hardly surprising as these shocks are now the common factors for a large sub-panel of non-core informational series, many of which are fairly persistent.

5.2 Empirical Factors and Estimated DSGE Model Concepts

Our empirical analysis proceeds by plotting the estimated empirical factors extracted by a dynamic factor model and the estimated DSGE state variables from our data-rich DSGE model. Please note that in addition to these two models, we have also estimated the *regular* DSGE model using standard Bayesian techniques. The underlying theoretical New Keynesian core is the same as in data-rich DSGE model. The difference comes in the measurement equation (73): we keep only 4 core observable data series (real GDP, GDP deflator inflation, Federal Funds interest rate and inverse velocity of money based M2S aggregate), impose the factor loadings as in (5) and assume perfect measurement of all four model concepts (see also the notes to Table D3, p.55).

Figure D1 depicts the posterior means and 90% credible intervals of the estimated data-rich DSGE model states. These include three endogenous variables (model inflation $\hat{\pi}_t$, nominal interest rate $\hat{R}_t$ and real household consumption $\hat{X}_t$) and three structural AR(1) shocks (government spending g_t, money demand χ_t and neutral technology Z_t). It is these states that are included in measurement equation (73) with potentially non-zero loadings. The figure depicts as well the smoothed versions of these same variables in a regular DSGE model estimation derived by Kalman smoother at posterior mean of deep structural parameters.

Four observations stand out. First, all three structural disturbances exhibit large swings and prolonged deviations from zero capturing the persistent low frequency movements in the data. Second, the estimated data-rich DSGE model states are much *smoother* than their counterparts in the regular DSGE model. The intuition is straightforward. In data-rich context, the model states are the common components of a large panel of data, and they have capture well not only a few core macro series (as is the case in regular DSGE model), but also very many non-core informational series.

The third observation is that the money demand shock χ_t appears to be very different in the data-rich versus regular DSGE model estimation. The underlying reason is that in regular DSGE model case it was mainly responsible for capturing the dynamics of inverse money velocity based on M2S in the small 4-series data set. Once we allow for the rich panel of macro and financial observables, χ_t helps explain other series as well (for example, housing variables and non-GDP measures of real output – see Table D5), yet at the cost of the fit for the IVM_M2S. The fourth observation is a counterfactual behavior of government spending shock and real consumption during recessions: the former tends to fall and the latter to rise when times are bad. In reality, of course, it is the other way around: as recession unfolds, the real consumption falls and the government purchases are usually intensified to mitigate the negative impact of recession on aggregate demand. The estimated path of g_t would be ok, however, if we think of it as a general aggregate demand shock not specifically connected to government purchases. In spite of our DSGE model being able to track well the total output dynamics, it cannot discriminate properly the components, in particular $\hat{X}_t$. The solution would seem to be to enlarge the model by incorporating, say, an investment specific technology shock a la Greenwood, Hercowitz and Krusell (1997) and to make the real consumption in the data one of the core observables, as for example is done in Smets and Wouters (2007) and Boivin and Giannoni (2006).

We proceed by discussing the latent empirical factors extracted by our DFM from the same rich data set. Figure D2 plots the posterior means and 90% credible intervals of the estimated factors. First, note that unlike the DSGE model states, these factors have in general *no economic interpretation*. This is less so about factors F3-F6, because of the assumed normalization scheme (88). Second, while factors 3 and 5 indeed look much like the data on real output and nominal interest rate, the factors 4 and 6 – despite the normalization – do not. This shows that the exclusion normalizations favoring certain ex-ante meaning of a particular factor are not a sufficient condition to guarantee this meaning ex-post after estimation. The third observation is that the credible intervals for F1 and F2 – the latent factors common to all macro and financial series in the panel – are not uniformly wide or narrow, as is more or less the case for factors F3-F6. During several years prior to 1990-91 recession, the 90% credible bands for factor F1 expand, and then quickly shrink after recession is over. The same pattern is observed for factor F2 for several years preceding 2001 recession. One interpretation of this finding could be that the volatility of these two factors is not constant over time and follows a regime switching dynamics over the business cycle. Clearly, to have a stronger case, one might like to estimate a DFM on the full postwar sample of available U.S. data.

5.3 How Well Factors Trace Data

Let us now turn to the question of how well the factors and the DSGE states are able to trace the actual data. *A priori* we should expect that the unrestricted dynamic factor model will do a better job on that dimension than the data-rich DSGE model whose cross-equation restrictions might be misspecified and the factor loadings in which might be unduly restricted. And that's indeed what we find and what can be concluded from inspecting Table D4 and Table D5 that present the (posterior mean of) fraction of unconditional variance of data series captured by the empirical factors and by the DSGE model states. On average, the data-rich DSGE model concepts "explain" about 75 percent of variance for the core macro series and 72 percent of variance for the non-core. The latent empirical factors extracted by a DFM are able to account for 95 and 94 percent of variance for core and non-core series, respectively. So overall, the empirical factors capture more than the DSGE states.

More specifically, within core series it is the measures of inflation and of inverse money velocities that are traced relatively poorer than the real output and nominal interest rates in both models. The same picture is observed in the non-core block of series: price and wage inflation measures and the financial variables in both models tend to have a higher fraction of unconditional variance due to measurement errors. In the data-rich DSGE model, the state variables capture about 15 to 25 percent of variance in exchange rate depreciations and stock returns, but about 65 to 85 percent of variance of interest rate spreads and credit stocks. This is not surprising given that our theoretical model does not have New Open Economy Macroeconomics mechanisms (e.g. Lubik and Schorfheide, 2005 or Adolfson, Laseen, Linde, Villani, 2005, 2008) and does not feature financial intermediation (e.g. Bernanke, Gertler, Gilchrist, 1999). In dynamic factor model, these percentages are much higher: the latent factors explain about 97-98 percent of variance of the interest spreads and credit stocks, about 65-82 percent of the variability in exchange rate depreciations and 80-82 percent of stock returns (Table D6). This suggests that our DSGE model is potentially misspecified along this "financial" dimension.

5.4 Comparing Factor Spaces

Up to this point, we have done two things: (i) we have estimated the empirical latent factors in a dynamic factor model and the DSGE states in a data-rich DSGE model; (ii) we have established that both factors and DSGE states are able to explain a significant portion of the co-movement in the rich panel of U.S. macro and financial series. From Figure D1 and Figure D2 we have learned that the factors and the states look quite different, therefore now comes our central question – can the empirical factors and estimated DSGE model concepts span the same factor space? Or, in other words, can we predict the true estimated DFM latent factors from the DSGE model states with a fair amount of accuracy?

Let $F_t^{(pm)}$ and $S_t^{(pm)}$ denote the posterior means of the empirical factors and of the data-rich DSGE model state variables. For each latent factor $F_{i,t}^{(pm)}$, we estimate, by Ordinary Least Squares, the following simple linear regression:

$$F_{i,t}^{(pm)} = \beta_{0,i} + \boldsymbol{\beta}'_{1,i} S_t^{(pm)} + u_{i,t} \tag{104}$$

with mean zero and homoscedastic error term $u_{i,t}$. We report the R^2s for the collection of linear predictive regressions (104) in Table D9. Denoting the OLS estimates by $\hat{\boldsymbol{\beta}}_0 = [\beta_{0,1},..., \beta_{0,N}]'$ and by $\hat{\boldsymbol{\beta}}_1 = [\boldsymbol{\beta}_{1,1},..., \boldsymbol{\beta}_{1,N}]'$, we then construct the predicted empirical factors $\hat{F}_t^{(pm)}$:

$$\hat{F}_t^{(pm)} = \boldsymbol{\beta}_0 + \hat{\boldsymbol{\beta}}_1 S_t^{(pm)} \tag{105}$$

The Figure D3 overlays true estimated DFM factors $F_t^{(pm)}$ versus the DSGE-states-predicted ones $\hat{F}_t^{(pm)}$.

From both Table D9 and Figure D3 we can clearly conclude that the DSGE states predict empirical factors really well and therefore the factor spaces spanned by the DSGE model state variables and by DFM latent factors are very closely aligned. What are the implications of this important finding? First, this implies that a DSGE model indeed captures the essential sources of co-movement in the large panel of data as sort of a core and that the differences in fit between a data-rich DSGE model and a DFM are potentially due to restricted factor loadings in the former. Second, this also implies a greater degree of comfort about propagation of structural shocks to a wide array of macro and financial series – which is the essence of many policy experiments. Third, the proximity of factor spaces facilitates economic interpretation of a dynamic factor model, as the empirical factors are now isomorphic – through the link (105) – to the DSGE model state variables with clear economic meaning.

Regarding the first and second points, we would like to emphasize that our factor spaces comparison is rather informal, approximate tool to evaluate a DSGE model. A more formal and full-fledged approach to DSGE model evaluation is implemented in Del Negro, Schorfheide, Smets and Wouters (2007) where the authors first approximate a DSGE model by a vector autoregression and then systematically relax the DSGE-model-implied cross-equation restrictions documenting how the model fit changes.

5.5 Propagation of Monetary Policy and Technology Innovations

The final and the most appealing implication of the factor spaces proximity in two models is that it allows us to map DSGE model state variables into DFM empirical factors every period and therefore propagate any structural shocks from the DSGE model *in otherwise completely non-structural* dynamic factor model to obtain predictions for a broad range of macro series of interest. Suppose $\boldsymbol{\Lambda}^{dfm-dsge}$ and $\boldsymbol{\Lambda}^{dfm}$ denote the posterior means of factor loadings in the data-rich DSGE model (72)-(74) and in empirical DFM (83)-(85), respectively. Then, for any structural shock $\varepsilon_{i,t}$, we can generate two sets of impulse responses of large panel of data X_t:

$$\left(\frac{\partial X_{t+h}}{\partial \varepsilon_{i,t}}\right)_{dfm-dsge} = \boldsymbol{\Lambda}^{dfm-dsge} \times \frac{\partial S_{t+h}}{\partial \varepsilon_{i,t}} \tag{106}$$

$$\left(\frac{\partial X_{t+h}}{\partial \varepsilon_{i,t}}\right)_{dfm} = \mathbf{\Lambda}^{dfm} \times \frac{\partial F_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{\Lambda}^{dfm}\left[\hat{\boldsymbol{\beta}}_1 \frac{\partial S_{t+h}}{\partial \varepsilon_{i,t}}\right], \tag{107}$$

where $\partial S_{t+h} / \partial \varepsilon_{i,t}$ is computed from the transition equation of the data-rich DSGE model for every horizon $h = 0,1,2,...$ and where we have used the link between S_t and F_t determined by (105).

In what follows we focus on propagating monetary policy ($\varepsilon_{R,t}$) and technology ($\varepsilon_{Z,t}$) innovations in both data-rich DSGE and dynamic factor model to generate predictions for the core and non-core macro series. The corresponding impulse response functions (IRFs) are presented in Figure D4, Figure D5, Figure D6 and Figure D7. It is natural to compare our results to findings in two strands of literature: Factor Augmented Vector Autoregression (FAVAR) literature (e.g. Bernanke, Boivin, Eliasz, 2005; Stock and Watson, 2005) and regular DSGE literature (e.g. Christiano, Eichenbaum, Evans, 2005; Smets and Wouters, 2003, 2007; DSSW 2007; Aruoba and Schorfheide, 2009; Adolfson, Laseen, Linde, and Villani, 2008). In FAVAR studies, we are able to obtain predictions for a rich panel of U.S. data similar to ours, but only of the monetary policy innovations. In regular DSGE literature, one can propagate any structural shocks including monetary policy and technology innovations, but to a limited number of core macro variables (e.g. real GDP, consumption, investment, inflation, interest rate, wage rate and hours worked in Smets and Wouters, 2007). The framework that we propose in this paper is able to deliver on both fronts: we are able to compute the responses of the core and non-core variables to both monetary policy and technology shocks. Moreover, we will have two sets of responses: from the data-rich DSGE model, which might be misspecified, and from the dynamic factor model that is primarily data-driven and fits better. This provides a more complete and comprehensive picture of underlying effects of monetary policy and technology innovations.

One general observation from comparing IRFs should be emphasized from the very beginning. The responses of core variables like real GDP, real consumption and investment, inflation in regular DGSE studies are often hump-shaped matching well the empirical findings from identified VARs. Our IRFs do not have many humps, because the underlying theoretical DSGE model, as presented in section 2.2, abstracts from, say, habit in consumption or variable capital utilization – mechanisms that help get the humps in those often more elaborate models. This, however, can be fixed by replacing the present DSGE model with a more elaborate one.

Let us turn first to the *effects of monetary policy innovation* which are summarized in Figure D4 and Figure D5. Contractionary monetary policy shock corresponds to 0.75 percent (or 75 basis points) increase in the Federal Funds rate. As nominal policy rate rises and the opportunity costs of holding money for households increase, we observe strong liquidity effect associated with falling real money balances. Also, high interest rates make the saving motive and buying more bonds temporarily a more attractive option. This raises households' marginal utility of consumption and discourages current spending in favor of the future consumption. Because the household faces investment adjustment costs and cannot adjust investment quickly, and the government spending in the model is exogenous, the lower consumption leads to a fall in aggregate demand. The firms respond to lower demand in part by contracting real output and in part by reducing the optimal price. Hence the aggregate price level falls, but not as much given nominal rigidities in the intermediate goods producing sector.

Why the monopolistically competitive firms respond to falling demand in part by charging a lower price? The short answer is that because they are able to cut their marginal costs. On the one hand, higher interest rates inhibit investment and the return on capital is falling. On the other hand, firms may now economize on real wages. The market for labor is perfectly competitive, since we assume no wage rigidities. This implies that the real wage is equal to the marginal product of labor, but also that it is equal to the household's marginal rate of substitution between consumption and leisure, as in (51). Since disutility of labor in our model is fixed, and the marginal utility of consumption is higher, the household accepts lower real wage and the firms are able to pass their losses in revenues onto households by reducing own wage bills.

Now given lower marginal costs, the New Keynesian Phillips curve suggests we should observe falling aggregate prices and negative rates of inflation (in terms of a deviation from the steady state inflation). That's what we see in the 2nd column of Figure D4. Notice that channeling the monetary policy shock through the pure dynamic factor model helps correct the so-called "*price puzzle*"[15] for the data-rich-DSGE-model-implied responses of PCE deflator inflation and CPI inflation. Interestingly, a positive response of CPI inflation to monetary policy contraction is also documented in Stock and Watson (2005), despite the fact that they use a data-rich Factor Augmented VAR. It has been argued (e.g. Bernanke, Boivin, Eliasz, 2005) that the rich information set helps eliminate this sort of anomalies.

As can be seen from the 1st column of Figure D4, the response of industrial production (IP) to the monetary policy tightening seems counterfactual compared to FAVAR findings. First, this may have something to do with inherent inertia of IP in responding to monetary policy. It continues to be driven by excessive optimism from the previous phase of the business cycle and it takes time to adjust to new conditions. But once IP falls below the trend, it remains subdued for a long time. Second, this may have something to do with the way the monetary policy shock is identified in FAVAR literature. By construction, in Factor Augmented VAR the industrial production is contained in the list of "slow moving" variables, and the identification of the monetary policy shock is achieved by postulating that it does not affect slow variables contemporaneously. Regarding the responses of real GDP, we document that the data-rich DSGE and DFM models disagree about the magnitude of the contraction. The DFM-implied response is almost negligible implying the costs of disinflation are very small (which is hard to

[15] "Price puzzle" (Sims, 1992) refers to the counterfactual finding in VAR literature that a measure of prices or inflation responds positively to a contractionary monetary policy shock associated with the unexpected increase in the policy interest rate.

believe), whereas the data-rich-DSGE-model-implied response is about minus 0.5 percent – hump shape aside, a value in the ballpark of findings in regular DSGE literature.

If we look at the effects of the monetary policy tightening on non-core macro and financial variables (Figure D5), they complete the picture for the core series with details. The real activity measures such as real consumption of durables, real residential investment and housing starts broadly decline. Prices go down as well; in particular we observe negative rates of commodity price inflation and investment deflator inflation. The measures of employment fall (e.g. employment in services sector) indicating tensions in the labor market, while the unemployment gains momentum with a lag before eventually returning to normal. The interest rate spreads (for instance, 6-month over 3-month Treasury bill rate) widen considerably, reflecting tighter money market conditions and increased liquidity risks and credit risks. Consumer credit is contracted, in part due to lower demand from borrowers facing higher interest rates and in part owing to reduced availability of funds. Dollar appreciates, reflecting intensified capital inflows lured by higher returns in domestic financial market. As a result, both export and import price indices fall, thereby translating – according to the magnitudes in Figure D5 – into a deterioration of the U.S. terms of trade.

Broadly speaking, the reported results are very similar to FAVAR findings of Bernanke, Boivin and Eliasz (2005) and Stock and Watson (2005). Except for the humps, they also accord well with the monetary policy effects on the core variables documented in regular DSGE literature. On top of that, the responses of the non-core variables seem to provide a reasonable and consistent picture of monetary tightening as well.

We plot the *effects of a positive technology innovation* in Figure D6 (core series) and Figure D7 (non-core series). Following the positive TFP shock, the real output broadly increases (although there is a disagreement between DFM and data-rich DSGE model as to the response of real GDP), as our economy becomes more productive and the firms find it optimal to produce more. New demand comes primarily from the higher capital investment reflecting much better future return on capital, and also from additional household consumption fueled by greater income. The higher output on the supply side plus improved efficiency implies a downward pressure on prices. Through the lenses of New Keynesian Phillips curve, the current period inflation is positively related to expected future inflation and to current marginal costs. A positive technology shock has raised production efficiency and reduced the current marginal costs (elevated real wage resulting from increased labor demand was not enough to prevent that). However, because technology innovation is very persistent, the firms expect future marginal costs and thus future inflation to be lower as well. This anticipation effect, coupled with currently low marginal costs, leads to prices falling now, as is evident from the column 2 of the Figure D6.

The increase in real output above steady state and the fall of inflation below target level, under estimated monetary policy Taylor rule, requires the Fed to move the policy rate in opposite directions. The fact that the Fed actually lowers the policy rate means that the falling prices effect dominates, with other interest rates following the course of the Federal Funds rate (column 3, Figure D6). Declining interest rates boost the real output even more, which in turn raises further the return on capital. As the positive impact of technological innovation dissipates, this higher return, through the future marginal costs channel, fuels inflationary expectations that ultimately translate into contemporaneous upward price pressures. The Fed reacts by increasing the policy rate, which explains the observed hump in the interest rate IRF. Given temporarily lower interest rates, the households choose to hold, with some lag, relatively higher real money balances (from column 4, Figure D6, this applies more to M1S and monetary base, and less to M2S aggregate that comprises a hefty portion of interest-bearing time deposits). A part of the growing money demand comes endogenously from the elevated level of economic activity.

These results – both in terms of the magnitudes and shapes of responses – align fairly closely with findings in regular DSGE literature (e.g. Smets and Wouters, 2007; Aruoba, Schorfheide, 2009; DSSW 2007).

The responses of the non-core macroeconomic series (Figure D7) appear to enrich the story for core variables with additional details. Following positive technology innovation, the subcomponents of real GDP (real consumption of durables, real residential investment) or components of industrial production (e.g. production of business equipment) generally expand (although there is weaker agreement between DFM and data-rich DSGE model predictions). Measures of employment (e.g. employment in services sector) increase. This is however in contrast to Smets and Wouters (2003) and Adolfson, Laseen, Linde, Villani (2005) who find on European data that the employment actually falls after positive stationary TFP shock. As marginal costs fall, commodity price inflation (P_COM) and investment deflator inflation (PInv_GDP) follow the overall downward price pressures trend. The interest rate spreads (SFYGM6) shrink, in part reflecting the lower level of perceived risks, while credit conditions ease leading to growth in business loans. Despite the interest rates being below average for a prolonged period of time, the dollar appreciates, but by less than after the monetary tightening. Finally, the real wage (RComp_Hour) increases, while the average hours worked (Hours_AVG) decline. The rise in real wage and the initial fall in hours worked are in line with evidence documented by Smets and Wouters (2007). However, the subsequent dynamics of hours are quite different: in Smets and Wouters the hours turn significantly positive after about two years. Here they stay below steady state for much longer. This may have something to do with a greater amount of persistence in the technology process in our model.

6 Conclusions

In this paper, we have compared a data-rich DSGE model with a standard New Keynesian core to an empirical dynamic factor model by estimating both on a rich panel of U.S. macroeconomic and financial indicators compiled by Stock and Watson (2008). We have established that the spaces spanned by the empirical factors and by the data-rich DSGE model states are very closely aligned.

What can we learn from that? First, this implies that a DSGE model indeed captures the essential sources of co-movement in the data and that the differences in fit between a data-rich DSGE model and a DFM are potentially due to restricted factor loadings in the former. Second, this also implies a greater degree of comfort about propagation of structural shocks to a wide array of macro and financial series. Third, the proximity of factor spaces facilitated economic interpretation of a dynamic factor model, as the empirical factors become isomorphic to the DSGE model state variables with clear economic meaning.

Most importantly, the proximity of factor spaces in two models has allowed us to propagate the monetary policy and technology innovations in otherwise completely non-structural dynamic factor model to obtain predictions for many more series than a handful of traditional macro variables including measures of real activity, price indices, labor market indicators, interest rate spreads, money and credit stocks, exchange rates. The responses of these non-core variables therefore provided a more complete and comprehensive picture of the effects of monetary policy and technology shocks and may serve as a check on empirical plausibility of a DSGE model.

References

Adolfson, Malin, Laseén, Stefan, Linde, Jesper and Mattias Villani (2005), "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," *Sveriges Riksbank Working Paper Series*, #179, March

Adolfson, Malin, Laseén, Stefan, Linde, Jesper and Mattias Villani (2007), "Forecasting Performance of an Open Economy Dynamic Stochastic General Equilibrium Model," *Econometric Reviews*, 26, pp. 289-328

Adolfson, Malin, Laseén, Stefan, Linde, Jesper and Mattias Villani (2008), "Evaluating an Estimated New Keynesian Small Open Economy Model," *Journal of Economic Dynamics and Control*, 32, pp. 2690-2721

Altissimo, F., Bassanetti, A., Cristadoro R., Forni, Mario, Hallin, Marc, Lippi, Marco, and Lucrezia Reichlin (2001), "EuroCOIN: A Real Time Coincident Indicator of the Euro Area Business Cycle," *Center for Economic Policy Research Discussion Paper 3108*

Altug, Sumru (1989), "Time-to-Build and Aggregate Fluctuations: Some New Evidence," *International Economic Review*, 30 (4), pp. 889-920

Amengual, D. and Mark Watson (2007), "Consistent Estimation of the Number of Dynamic Factors in a Large N and T Panel," *Journal of Business and Economic Statistics*, pp. 91-96

An, Sungbae and Frank Schorfheide (2007), "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26, pp. 211-219

Aruoba, S. Borağan, Diebold, Francis and Chiara Scotti (2009), "Real-Time Measurement of Business Conditions," *Journal of Business and Economic Statistics*, 24 (4), pp. 417-427 (October)

Aruoba, S. Borağan and Frank Schorfheide (2009), "Sticky Prices versus Monetary Frictions: An Estimation of Policy Trade-offs," *NBER Working Paper No 14870*

Bai, Jushan and Serena Ng (2002), "Determining the Number of Factors in Approximate Factor Models," *Econometrica*, 70 (1), pp. 191-221

Bai, Jushan and Serena Ng (2007), "Determining the Number of Primitive Shocks in Factor Models," *Journal of Business and Economic Statistics*, 25 (1), pp. 52-60

Bernanke, Ben S. (2004), "The Great Moderation," Federal Reserve Board, Remarks by Governor Ben S. Bernanke, February 20, 2004 (available online at http://www.federalreserve.gov/BOARDDOCS/SPEECHES/2004/20040220/default.htm)

Bernanke, Ben, Boivin, Jean and Piotr Eliasz (2005), "Measuring the Effects of Monetary Policy: A Factor Augmented Vector Autoregressive (FAVAR) Approach," *Quarterly Journal of Economics*, 120, pp. 387-422

Bernanke, Ben, Gertler, Mark and Simon Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. Taylor, M. Woodford (eds.), *Handbook of Macroeconomics*, Vol. 1, Chapter 21, pp. 1341-1393

Blanchard, Olivier and Charles Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 48 (5), pp. 1305-1311

Boivin, Jean and Marc Giannoni (2006), "DSGE Models in a Data-Rich Environment," *Manuscript*, HEC Montreal and Columbia University

Boivin, Jean and Serena Ng (2005), "Understanding and Comparing Factor Based Macroeconomic Forecasts," *International Journal of Central Banking*, 1, pp. 117-152

Calvo, Guillermo (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12 (3), pp. 383-398

Carter, Christopher and Robert Kohn (1994), "On Gibbs Sampling for State Space Models," *Biometrika*, 81 (3), pp. 541-553

Chamberlain, Gary and M. Rothschild (1983), "Arbitrage, Factor Structure and Mean-Variance Analysis in Large Asset Markets," *Econometrica*, 70, pp. 191-221

Chib, Siddhartha (2001), "Markov Chain Monte Carlo Methods: Computation and Inference", in J.J. Heckman and E. Leamer (eds.), *Handbook of Econometrics*. Vol. 5, Elsevier Science BV, Chapter 57

Chib, Siddhartha and Edward Greenberg (1994), "Bayes Inference in Regression Models with ARMA (p, q) Errors," *Journal of Econometrics*, 64, pp. 183-206

Christiano, Lawrence, Eichenbaum, Martin and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113 (1), pp. 1-45

Coenen, Günter, McAdam, Peter and Roland Straub (2008), "Tax Reform and Labour-Market Performance in the Euro Area: A Simulation-Based Analysis Using the New Area-Wide Model," *Journal of Economic Dynamics and Control*, 32, pp. 2543-2583

Cynamon, Barry, Dutkowsky, Donald and Barry Jones (2006), "Redefining the Monetary Aggregates: A Clean Sweep," *Eastern Economist Journal*, 32 (4), pp. 661-672

Del Negro, Marco and Christopher Otrok (2008), "Dynamic Factor Models with Time-Varying Parameters. Measuring Changes in International Business Cycles," *Federal Reserve Bank of New York Staff Report*, #325

Del Negro, Marco and Frank Schorfheide (2008), "Forming Priors for DSGE Models (And How It Affects the Assessment of Nominal Rigidities," *Journal of Monetary Economics*, 55, pp. 1191-1208

Del Negro, Marco, Schorfheide, Frank, Smets, Frank and Raf Wouters (2007), "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, 25, pp. 123-162

Doan, Thomas, Litterman, Robert and Christopher Sims (1984), "Forecasting and Conditional Projections Using Realistic Prior Distributions," *Econometric Reviews*, 3 (4), pp. 1-100

Edge, Rochelle, Kiley, Michael and Jean-Philippe Laforte (2009), "A Comparison of Forecast Performance between Federal Reserve Staff Forecasts, Simple Reduced-Form Models, and a DSGE Model," *Finance and Economics Discussion Series 2009-2010*, Federal Reserve Board of Governors

Erceg, C., Henderson, D. and Andrew Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46 (2), pp. 281-313

Forni, Mario, Giannone, Domenico, Lippi, Marco and Lucrezia Reichlin (2007), "Opening the Black Box: Structural Factor Models with Large Cross-Sections," *Manuscript*, Center for Economic and Policy Research (CEPR)

Forni, Mario, Hallin, Marc, Lippi, Marco, and Lucrezia Reichlin (2003), "Do Financial Variables Help Forecasting Inflation and Real Activity in the Euro Area?" *Journal of Monetary Economics*, 50, pp. 1243-1255

Geweke, John (1977), "The Dynamic Factor Analysis of Economic Time Series," in D.J. Aigner and A.S. Goldberger (eds.), *Latent Variables in Socio-Economic Models*, Amsterdam, North-Holland Publishing, Ch. 19

Geweke, John (2005), *Contemporary Bayesian Econometrics and Statistics*, Hoboken, New Jersey: John Wiley and Sons, Inc

Geweke, John and Guofu Zhu (1996), "Measuring the Pricing Error of the Arbitrage Pricing Theory," *Review of Financial Studies*, 9 (2), pp. 557-587

Giannone, Domenico, Reichlin, Lucrezia, and Luca Sala (2004), "Monetary Policy in Real Time," *NBER Macroeconomics Annual 2004*, pp. 161-200

Giannone, Domenico, Reichlin, Lucrezia, and D. Small (2008), "Nowcasting: The Real-Time Informational Content of Macroeconomic Data," *Journal of Monetary Economics*, 55, pp. 665-676

Greenwood, Jeremy, Hercowitz, Z. and Per Krusell (1997), "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 83, pp. 342-362

Hallin, Marc and Roman Liška (2007), "Determining the Number of Factors in General Dynamic Factor Model," *Journal of the American Statistical Association*, 102, pp. 603-617

Harvey, Andrew (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge: Cambridge University Press

Ireland, Peter (2004), "A Method for Taking Models to the Data," *Journal of Economic Dynamics and Control*, 28 (6), pp. 1205-1226

Jungbacker, Borus and Siem Jan Koopman (2008), "Likelihood-Based Analysis for Dynamic Factor Models," *Manuscript*, Department of Econometrics, VU University Amsterdam

Kim, Chang-Jin and Charles Nelson (1999), "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *The Review of Economics and Statistics,* 81, pp. 608 – 616

Kim, Chang-Jin and Charles Nelson (2000), *State-Space Models with Regime Switching,* Cambridge, MA: MIT Press

King, Robert and Mark Watson (2002), "System Reduction and Solution Algorithms for Solving Linear Difference Systems under Rational Expectations," *Computational Economics*, 20, pp. 57-86

Klein, Paul (2000), "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model," *Journal of Economic Dynamics and Control*, 24, pp. 1405-1423

Kose, M. Ayhan, Otrok, Christopher, and Charles Whiteman (2003), "International Business Cycles: World, Region and Country Specific Factors," *American Economic Review*, 93 (4), pp. 1216-1239

Kose, M. Ayhan, Otrok, Christopher, and Charles Whiteman (2008), "Understanding the Evolution of World Business Cycles," *Journal of International Economics*, 75, pp. 110-130

Leeper, Eric and Christopher Sims (1994), "Toward a Modern Macroeconomic Model Usable for Policy Analysis," *NBER Macroeconomics Annual 1994*, pp. 81-117

Levin, Andrew, Onatsky, Alexei, Williams, John and Noah Williams (2005), "Monetary Policy under Uncertainty in Micro-Founded Macroeconometric Models," *NBER Macroeconomic Annual 2005*, pp. 229-287

Lubik, Thomas and Frank Schorfheide (2005), "A Bayesian Look at New Open Economy Macro-ecnomics," *NBER Macroeconomics Annual 2005*, pp. 313–366

McCallum, Bennett T. (1983), "On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics*, 11 (March), pp. 139-168

McCallum, Bennett T. (1999), "Role of the Minimal State Variable Criterion in Rational Expectations Models," *International Tax and Public Finance*, 6 (November), pp. 621-639

McCallum, Bennett T. (2003), "The Unique Minimum State Variable RE Solution is e-Stable in All Well Formulated Linear Models," *NBER Working Paper No 9960*, September

McConnell, Margaret and Gabriel Perez-Quiros (2000), "Output Fluctuations in the United States: What has Changed since the Early 1980's," *American Economic Review*, Vol. 90, No. 5, pp. 1464-1476

Moench, Emanuel, Ng, Serena and Simon Potter (2008), "Dynamic Hierarchical Factor Models," *Manuscript*, Federal Reserve Bank of New York and Columbia University

Sargent, Thomas (1989), "Two Models of Measurements and the Investment Accelerator," *Journal of Political Economy*, 97, pp. 251-287

Sargent, Thomas and Christopher Sims (1977), "Business Cycle Modeling Without Pretending to Have too Much A-priori Economic Theory," in C.Sims et al. (eds.), *New Methods in Business Cycle Research*, Federal Reserve Bank of Minneapolis, Minneapolis

Schorfheide, Frank, Sill, Keith and Maxym Kryshko (2010), "DSGE Model Based Forecasting of Non-Modeled Variables," *International Journal of Forecasting* (forthcoming, July)

Sidrauski, Miguel (1967), "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review*, 57 (2), pp. 534-544

Sims, Christopher (1992), "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy," *European Economic Review*, 36 (5), pp. 975-1000

Sims, Christopher (2002), "Solving Rational Expectations Models," *Computational Economics*, 20, pp. 1-20

Smets, Frank and Raf Wouters (2003), "An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area," *Journal of the European Economic Association*, 1, pp. 1123-1175

Smets, Frank and Raf Wouters (2007), "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, pp. 586-606

Stock, James and Mark Watson (1989), "New Indices of Coincident and Leading Economic Indicators," *NBER Macroeconomics Annual 1989*, pp. 351-394

Stock, James and Mark Watson (1999), "Forecasting Inflation," *Journal of Monetary Economics*, 44, pp. 293-335

Stock, James and Mark Watson (2002a), "Macroeconomic Forecasting Using Diffusion Indexes," *Journal of Business and Economic Statistics*, 20, pp. 147-162

Stock, James and Mark Watson (2002b), "Forecasting Using Principal Components from a Large Number of Predictors," *Journal of the American Statistical Association*, 97, pp. 1167-1179

Stock, James and Mark Watson (2002c), "Has the Business Cycle Changed and Why?" *NBER Working Paper No 9127*, September

Stock, James and Mark Watson (2005), "Implications of Dynamic Factor Models for VAR Analysis," *NBER Working Paper No 11467*

Stock, James and Mark Watson (2008), "Forecasting in Dynamic Factor Models Subject to Structural Instability," in *The Methodology and Practice of Econometrics, A Festschrift in Honour of Professor David F. Hendry,* Jennifer Castle and Neil Shephard (eds), Oxford: Oxford University Press (http://www.princeton.edu/~mwatson/papers/hendryfestschrift_stockwatson_April282008.pdf)

Taylor, John (1993), "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195-214

Tierney, Luke (1994), "Markov Chains for Exploring Posterior Distributions," *Annals of Statistics*, 22, pp. 1701-1762

Uhlig, Harald (1999), "A Toolkit for Analyzing Non-Linear Dynamic Stochastic Models Easily," in Ramon Marimon and Andrew Scott (eds), *Computational Methods for the Study of Dynamic Economies,* Oxford University Press, New York, pp. 30-61

Walsh, Carl (2003), *Monetary Theory and Policy*, 2nd edition, Cambridge, MA: MIT Press

Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press

Appendix A. DSGE Model

Appendix A1. First Order Conditions of Intermediate Goods Firm

Monopolistically competitive intermediate goods producer i, which is allowed to re-optimize, chooses the optimal price $P_t^o(i)$ that maximizes discounted stream of profits subject to optimal demand from final good producers:

$$\begin{aligned} &\max_{P_t^o(i)} \quad E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^{p}\,(P_t^o(i)\pi_{t+s|t}^{adj}-P_{t+s}MC_{t+s})Y_{t+s}(i)\right\} \\ &\text{s.t.} \quad Y_{t+s}(i)=\left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{t+s}, \qquad s=0,1,2,.... \end{aligned} \tag{108}$$

First, obtain an expression for $\dfrac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}$:

$$\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}=-\frac{(1+\lambda)}{\lambda}\left[\frac{P_t^o(i)\pi_{t+s|t}^{adj}}{P_{t+s}}\right]^{-\frac{(1+\lambda)}{\lambda}-1}\frac{\pi_{t+s|t}^{adj}}{P_{t+s}}Y_{t+s}=-\left(\frac{1+\lambda}{\lambda}\right)\frac{Y_{t+s}(i)}{P_t^o(i)}. \tag{109}$$

Now the first order condition for the problem (108), where we will plug optimal demand $Y_{t+s}(i)$ into the objective function and assume interior solution, is:

$$E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^{p}\left[\pi_{t+s|t}^{adj}\left(Y_{t+s}(i)+P_t^o(i)\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}\right)-P_{t+s}MC_{t+s}\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}\right]\right\}=0. \tag{110}$$

Consider expression inside square brackets:

$$\begin{aligned} [\cdots] &= \pi_{t+s|t}^{adj}Y_{t+s}(i)+\left(P_t^o(i)\pi_{t+s|t}^{adj}-P_{t+s}MC_{t+s}\right)\frac{\partial Y_{t+s}(i)}{\partial P_t^o(i)}= \\ &= \pi_{t+s|t}^{adj}Y_{t+s}(i)-\left(P_t^o(i)\pi_{t+s|t}^{adj}-P_{t+s}MC_{t+s}\right)\frac{(1+\lambda)}{\lambda}\frac{Y_{t+s}(i)}{P_t^o(i)}= \\ &= \frac{Y_{t+s}(i)}{P_t^o(i)}\left(P_t^o(i)\pi_{t+s|t}^{adj}-\frac{(1+\lambda)}{\lambda}\left(P_t^o(i)\pi_{t+s|t}^{adj}-P_{t+s}MC_{t+s}\right)\right)= \\ &= \frac{1}{\lambda}\frac{Y_{t+s}(i)}{P_t^o(i)}\left((\lambda-1-\lambda)P_t^o(i)\pi_{t+s|t}^{adj}+(1+\lambda)P_{t+s}MC_{t+s}\right). \end{aligned}$$

Cancelling out $1/\lambda\neq 0$ and multiplying (110) by -1, we could rewrite the FOC as follows:

$$E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi_{t+s|t}^{p}\frac{Y_{t+s}(i)}{P_t^o(i)}\left[P_t^o(i)\pi_{t+s|t}^{adj}-(1+\lambda)P_{t+s}MC_{t+s}\right]\right\}=0. \tag{111}$$

Remark 1: Since for $s>0$, $\pi_{t+s|t}^{adj}=\prod_{l=1}^{s}\pi_{t+l-1}^{\iota}\pi_{**}^{(1-\iota)}$ and $\pi_{(t+1)+s|(t+1)}^{adj}=\prod_{l=1}^{s}\pi_{(t+1)+l-1}^{\iota}\pi_{**}^{(1-\iota)}=\pi_{t+1}^{\iota}\pi_{t+2}^{\iota}...\pi_{t+s}^{\iota}\pi_{**}^{(1-\iota)s}$,

it follows that $\pi_{t+(s+1)|t}^{adj}=\prod_{l=1}^{s+1}\pi_{t+l-1}^{\iota}\pi_{**}^{(1-\iota)}=\pi_t^{\iota}\pi_{t+1}^{\iota}\pi_{t+2}^{\iota}...\pi_{t+s}^{\iota}\pi_{**}^{(1-\iota)(s+1)}=\left[\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right]\pi_{(t+1)+s|(t+1)}^{adj}$.

Remark 2: Since for $s>0$, $\Xi^{p}_{t+s|t}=\frac{\lambda_{t+s}}{\lambda_t}$ and so $\Xi^{p}_{t+(s+1)|t}=\frac{\lambda_{t+s+1}}{\lambda_t}$, it follows that $\Xi^{p}_{(t+1)+s|(t+1)}=\frac{\lambda_{t+1+s}}{\lambda_{t+1}}\frac{\lambda_t}{\lambda_t}=$ $=\Xi^{p}_{t+(s+1)|t}\Big/\Xi^{p}_{t+1|t}$ and that $\Xi^{p}_{t+(s+1)|t}=\Xi^{p}_{t+1|t}\Xi^{p}_{(t+1)+s|(t+1)}$.

Remark 3: Notice that given expression for an optimal demand for good i in (108), $Y_{t+(s+1)}(i)\neq Y_{(t+1)+s}(i)$. However, using result from Remark 1, we obtain:

$$Y_{t+(s+1)}(i)=\left[\frac{P^{o}_{t+1}(i)}{P^{o}_{t+1}(i)}\frac{P^{o}_{t}(i)\pi^{adj}_{t+(s+1)|t}}{P_{t+(s+1)}}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{t+(s+1)}=\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\left[\frac{P^{o}_{t+1}(i)}{P_{(t+1)+s}}\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]\pi^{adj}_{(t+1)+s|(t+1)}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{t+1+s}=$$

$$=\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{(t+1)+s}(i)$$

To express FOC (111) recursively, we define two auxiliary variables:

$$f^{(1)}_t=E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi^{p}_{t+s|t}Y_{t+s}(i)\pi^{adj}_{t+s|t}\right\}\tag{112}$$

$$f^{(2)}_t=E_t\left\{\sum_{s=0}^{\infty}(\zeta\beta)^s\,\Xi^{p}_{t+s|t}Y_{t+s}(i)\frac{P_{t+s}}{P^{o}_{t}(i)}MC_{t+s}\right\},\tag{113}$$

so that FOC becomes:

$$f^{(1)}_t=(1+\lambda)\,f^{(2)}_t.\tag{114}$$

Recalling that $\Xi^{p}_{t|t}=1$, $\pi^{adj}_{t|t}=1$ and using results from Remarks 1, 2 and 3, we can rewrite (112) as:

$$f^{(1)}_t=Y_t(i)+E_t\left\{\sum_{k=0}^{\infty}(\zeta\beta)^{k+1}\,\Xi^{p}_{t+(k+1)|t}Y_{t+(k+1)}(i)\pi^{adj}_{t+(k+1)|t}\right\}=$$

$$=Y_t(i)+(\zeta\beta)E_t\left\{\sum_{k=0}^{\infty}(\zeta\beta)^k(\Xi^{p}_{t+1|t}\Xi^{p}_{(t+1)+k|(t+1)})\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{(t+1)+k}(i)\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]\pi^{adj}_{(t+1)+k|(t+1)}\right\}=$$

$$=Y_t(i)+\zeta\beta\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]^{-\frac{1}{\lambda}}E_t\left\{\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\Xi^{p}_{t+1|t}\sum_{k=0}^{\infty}(\zeta\beta)^k\,\Xi^{p}_{(t+1)+k|(t+1)}Y_{(t+1)+k}(i)\pi^{adj}_{(t+1)+k|(t+1)}\right\}=$$

$$=\left[\frac{P^{o}_{t}(i)}{P_t}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_t+\zeta\beta\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]^{-\frac{1}{\lambda}}E_t\left\{\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\Xi^{p}_{t+1|t}f^{(1)}_{t+1}\right\}.\tag{115}$$

Similarly, the recursion for $f^{(2)}_t$ becomes:

$$f^{(2)}_t=Y_t(i)\frac{P_tMC_t}{P^{o}_{t}(i)}+(\zeta\beta)E_t\left\{\sum_{k=0}^{\infty}(\zeta\beta)^k(\Xi^{p}_{t+1|t}\Xi^{p}_{(t+1)+k|(t+1)})\left[\frac{P^{o}_{t}(i)}{P^{o}_{t+1}(i)}\right]^{-\frac{(1+\lambda)}{\lambda}}\times\right.$$

$$\left.\times\left[\pi^{\iota}_{t}\pi^{(1-\iota)}_{**}\right]^{-\frac{(1+\lambda)}{\lambda}}Y_{(t+1)+k}(i)\frac{P_{t+1+k}}{P^{o}_{t}(i)}MC_{t+1+k}\frac{P^{o}_{t+1}(i)}{P^{o}_{t+1}(i)}\right\}=$$

$$=\left[\frac{P_t^o(i)}{P_t}\right]^{-\frac{(1+\lambda)}{\lambda}-1} MC_tY_t+\zeta\beta\left[\pi_t^{\iota}\pi_{**}^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}} E_t\left\{\left[\frac{P_t^o(i)}{P_{t+1}^o(i)}\right]^{-\frac{(1+\lambda)}{\lambda}-1}\Xi_{t+1|t}^p f_{t+1}^{(2)}\right\}. \qquad (116)$$

Equilibrium conditions (115), (116) and (114) correspond to equations (35), (36) and (37) in the main text.

Appendix A2. Evolution of Price Dispersion

Aggregate price dispersion across intermediate goods firms is captured by variable $D_t=\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}di$.

By properties of Calvo pricing, $P_t(i)$ is equal to optimal price P_t^o with probability $1-\zeta$ (optimizing firms) and is equal to $\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]P_{t-1}(i)$ with probability ζ (non-optimizing firms). Therefore, by definition of D_t we have:

$$D_t=\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}di=(1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}+\zeta\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}}\int_0^1\left(\frac{P_{t-1}(i)}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}di=$$

$$=(1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}+\zeta\left[\pi_{t-1}^{\iota}\pi_{**}^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}}\left(\frac{P_{t-1}}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}\int_0^1\left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\frac{(1+\lambda)}{\lambda}}di$$

The last line implies:

$$D_t=(1-\zeta)\left(\frac{P_t^o}{P_t}\right)^{-\frac{(1+\lambda)}{\lambda}}+\zeta\left[\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota}\left(\frac{\pi_{**}}{\pi_t}\right)^{(1-\iota)}\right]^{-\frac{(1+\lambda)}{\lambda}}D_{t-1} \qquad (117)$$

Appendix A3. Steady State and Log-Linearized Equilibrium Conditions

In what follows we specialize the household's utility to be constant-relative-risk-aversion function:

$$U(x_t)=B\frac{x_t^{1-\gamma}}{1-\gamma}.$$

In addition, for any generic variable V_t the corresponding "star" variable V_* denotes its steady state value and "hat" variable stands for log-deviation from steady state: $\hat{V}_t=\ln(V_t/V_*)$

Steady State Conditions

$$R_*=\frac{\pi_*}{\beta}$$

$$R_*^k=\frac{1}{\beta}+\delta-1$$

$$p_*^o = \left(\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta} \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{1-\iota}{\lambda}} \right)^{-\lambda}$$

$$D_* = \frac{1-\zeta}{1-\zeta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1+\lambda)}{\lambda}(1-\iota)}} \left(p_*^o \right)^{-\frac{(1+\lambda)}{\lambda}}$$

$$\overline{Y}_* = Y_* D_*$$

$$K_* = \frac{\alpha p_*^o (\overline{Y}_* + \tilde{F})}{(1+\lambda) R_*^k} \left(\frac{1-\zeta\beta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1+\lambda)}{\lambda}(1-\iota)}}{1-\zeta\beta \left(\frac{\pi_{**}}{\pi_*} \right)^{-\frac{(1-\iota)}{\lambda}}} \right)$$

$$Z_* = \frac{(\overline{Y}_* + \tilde{F})}{K_*^{\alpha} H_*^{1-\alpha}}$$

$$I_* = \delta K_*$$

$$W_* = \frac{1-\alpha}{\alpha} \frac{K_*}{H_*} R_*^k$$

$$X_* + I_* + \left(1 - \frac{1}{g_*} \right) Y_* = Y_*$$

$$A = \frac{1}{\overline{M}_*} \left(\frac{\chi_* W_* \pi_*^{\nu_m}}{(R_* - 1) Z_*^{\frac{1-\nu_m}{1-\alpha}}} \right)^{\frac{1}{\nu_m}}$$

$$B X_*^{-\gamma} = \frac{A}{W_*}$$

Log-Linearized Equilibrium Conditions

Households

$$\hat{W}_t = \gamma \hat{X}_t$$

$$\hat{I}_t = \frac{1}{1+\beta} \hat{I}_{t-1} + \frac{\beta}{1+\beta} \hat{I}_{t+1} + \frac{1}{S''(1+\beta)} \hat{\mu}_t$$

$$-\gamma \hat{X}_t = -\gamma \hat{X}_{t+1} + (\hat{R}_t - \hat{\pi}_{t+1})$$

$$\hat{\mu}_t - \gamma \hat{X}_t = \beta(1-\delta) \hat{\mu}_{t+1} - \gamma \hat{X}_{t+1} + \beta R_*^k \hat{R}_{t+1}^k$$

$$\hat{K}_{t+1} = (1-\delta) \hat{K}_t + \delta \hat{I}_t$$

$$\hat{\Xi}_{t|t-1}^p = -\gamma (\hat{X}_t - \hat{X}_{t-1}) - \hat{\pi}_t$$

$$\nu_m \hat{\bar{M}}_{t+1} = \gamma \hat{X}_t + \nu_m \hat{\chi}_{t+1} - (1-\nu_m)\hat{\pi}_{t+1} - \frac{1}{R_* - 1}\hat{R}_t$$

Firms

$$\hat{K}_t = \hat{H}_t + \hat{W}_t - \hat{R}_t^k$$

$$\hat{MC}_t = (1-\alpha)\hat{W}_t + \alpha \hat{R}_t^k - \hat{Z}_t$$

$$\hat{f}_t^{(1)} = \hat{f}_t^{(2)}$$

$$\hat{f}_t^{(1)} = (1-C_1)\left(-\frac{1+\lambda}{\lambda}\hat{p}_t^o + \hat{Y}_t\right) + C_1\left(-\frac{\iota}{\lambda}\hat{\pi}_t + \frac{1+\lambda}{\lambda}[-\hat{p}_t^o + \hat{\pi}_{t+1} + \hat{p}_{t+1}^o] + \hat{\Xi}_{t+1|t}^p + \hat{f}_{t+1}^{(1)}\right)$$

$$\hat{f}_t^{(2)} = (1-C_2)\left(-\left(\frac{1+\lambda}{\lambda}+1\right)\hat{p}_t^o + \hat{Y}_t + \hat{MC}_t\right) + C_2\left(-\frac{\iota(1+\lambda)}{\lambda}\hat{\pi}_t + \left(\frac{1+\lambda}{\lambda}+1\right)[-\hat{p}_t^o + \hat{\pi}_{t+1} + \hat{p}_{t+1}^o] + \hat{\Xi}_{t+1|t}^p + \hat{f}_{t+1}^{(2)}\right)$$

$$\hat{p}_t^o = (C_3 - 1)\hat{\pi}_t - C_3 \iota \zeta \left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}} \hat{\pi}_{t-1},$$

$$\text{where } C_1 = \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}},\ C_2 = \zeta\beta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1+\lambda}{\lambda}(1-\iota)},\ C_3 = \frac{1}{1-\zeta}\left(p_*^o\right)^{\frac{1}{\lambda}}.$$

Taylor Rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_1 \hat{\pi}_t + \psi_2 \hat{Y}_t) + \varepsilon_{R,t}$$

Aggregate Demand and Supply

$$\hat{\bar{Y}}_t = \hat{Y}_t + \hat{D}_t$$

$$\hat{\bar{Y}}_t = \left(1 + \frac{\tilde{F}}{\bar{Y}_*}\right)(\hat{Z}_t + \alpha \hat{K}_t + (1-\alpha)\hat{H}_t)$$

$$\hat{D}_t = \left(-\frac{p_*^0}{D_*}\frac{1+\lambda}{\lambda}(1-\zeta)\right)\hat{p}_t^o + \zeta\left(\frac{\pi_{**}}{\pi_*}\right)^{-\frac{1+\lambda}{\lambda}(1-\iota)}\left(\hat{D}_{t-1} + \frac{1+\lambda}{\lambda}\hat{\pi}_t - \frac{\iota(1+\lambda)}{\lambda}\hat{\pi}_{t-1}\right)$$

$$\hat{Y}_t = \frac{X_*}{X_* + I_*}\hat{X}_t + \frac{I_*}{X_* + I_*}\hat{I}_t + \hat{g}_t$$

Aggregate Disturbances

$$\hat{Z}_t = \rho_Z \hat{Z}_{t-1} + \varepsilon_{Z,t}, \qquad \varepsilon_{Z,t} \sim iid\ N(0, \sigma_Z^2)$$

$$\hat{\chi}_t = \rho_\chi \hat{\chi}_{t-1} + \varepsilon_{\chi,t}, \qquad \varepsilon_{\chi,t} \sim iid\ N(0, \sigma_\chi^2)$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}, \qquad \varepsilon_{g,t} \sim iid\ N(0, \sigma_g^2)$$

$$\varepsilon_{R,t} \sim iid\ N(0, \sigma_R^2)$$

Appendix B. Markov Chain Monte Carlo Algorithms

Appendix B. Estimation Procedure: Details of the Markov Chain Monte Carlo Algorithms

(to be completed)

Appendix C. Data: Description and Transformations

#	Short Name	SW Mnemonic	Trans Code	Description

Core Series

#	Short Name	SW Mnemonic	Trans Code	Description
	Real Output			
1.	RGDP		4	Real Per-capita Gross Domestic Product
2.	IP_TOTAL		4	Per-capita Industrial Production Index: Total
3.	IP_MFG		4	Per-capita Industrial Production Index: Manufacturing
	Inflation			
4.	PGDP		4	GDP Deflator Inflation
5.	PCED		4	Personal Consumption Expenditure Deflator Inflation
6.	CPI_ALL		4	Consumer Price Index (All Items) Inflation
	Nominal Interest Rate			
7.	FedFunds		4	Interest Rate: Federal Funds (effective), % per annum
8.	TBill_3m		4	Interest Rate: U.S. Treasury bills, secondary market, 3 month, % per annum
9.	AAABond		4	Bond Yield: Moody's AAA Corporate, % per annum
	Inverse Velocity of Money (M/Y)			
10.	IVM_M1S_det		4	Inverse Velocity of Money based on M1S aggregate
11.	IVM_M2S		4	Inverse Velocity of Money based on M2S aggregate
12.	IVM_MBase_bar		4	Inverse Velocity of Money based on adjusted Monetary Base

Non-Core Series

#	Short Name	SW Mnemonic	Trans Code	Description
	Output and Components			
1.	IP_CONS_DBLE	IPS13	3*	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
2.	IP_CONS_NONDBLE	IPS18	3*	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
3.	IP_BUS_EQPT	IPS25	3*	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
4.	IP_DBLE_MATS	IPS34	3*	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
5.	IP_NONDBLE_MATS	IPS38	3*	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
6.	IP_FUELS	IPS306	3*	INDUSTRIAL PRODUCTION INDEX - FUELS
7.	PMP	PMP	0	NAPM PRODUCTION INDEX (PERCENT)
8.	RCONS	GDP252	3*	Real Personal Consumption Expenditures, Quantity Index (2000=100) , SAAR
9.	RCONS_DUR	GDP253	3*	Real Personal Consumption Expenditures - Durable Goods , Quantity Index (2000=100), SAAR
10.	RCONS_SERV	GDP255	3*	Real Personal Consumption Expenditures - Services, Quantity Index (2000=100) , SAAR
11.	REXPORTS	GDP263	3*	Real Exports, Quantity Index (2000=100) , SAAR
12.	RIMPORTS	GDP264	3*	Real Imports, Quantity Index (2000=100) , SAAR
13.	RGOV	GDP265	3*	Real Government Consumption Expenditures & Gross Investment, Quantity Index (2000=100), SAAR
	Labor Market			
14.	EMP_MINING	CES006	3*	EMPLOYEES, NONFARM - MINING
15.	EMP_CONST	CES011	3*	EMPLOYEES, NONFARM - CONSTRUCTION
16.	EMP_DBLE_GDS	CES017	3*	EMPLOYEES, NONFARM - DURABLE GOODS
17.	EMP_NONDBLES	CES033	3*	EMPLOYEES, NONFARM - NONDURABLE GOODS
18.	EMP_SERVICES	CES046	3*	EMPLOYEES, NONFARM - SERVICE-PROVIDING
19.	EMP_TTU	CES048	3*	EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES
20.	EMP_WHOLESALE	CES049	3*	EMPLOYEES, NONFARM - WHOLESALE TRADE
21.	EMP_RETAIL	CES053	3*	EMPLOYEES, NONFARM - RETAIL TRADE
22.	EMP_FIRE	CES088	3	EMPLOYEES, NONFARM - FINANCIAL ACTIVITIES
23.	EMP_GOVT	CES140	3	EMPLOYEES, NONFARM - GOVERNMENT
24.	URATE_ALL	LHUR	0	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
25.	U_DURATION	LHU680	0	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
26.	U_L5WKS	LHU5	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
27.	U_5_14WKS	LHU14	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
28.	U_M15WKS	LHU15	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
29.	U_15_26WKS	LHU26	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
30.	U_M27WKS	LHU27	3	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
31.	HOURS_AVG	CES151	0	AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING
	Housing			
32.	HSTARTS_NE	HSNE	1	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
33.	HSTARTS_MW	HSMW	1	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
34.	HSTARTS_SOU	HSSOU	1	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
35.	HSTARTS_WST	HSWST	1	HOUSING STARTS:WEST (THOUS.U.)S.A.

35.	HSTARTS_WST	HSWST	1	HOUSING STARTS:WEST (THOUS.U.)S.A.
36.	RRESINV	GDP261	3*	Real Gross Private Domestic Investment - Residential, Quantity Index (2000=100), SAAR
	Financial Variables			
37.	SFYGM6	Sfygm6	0	fygm6-fygm3 fygm6: INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA) fygm3: INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
38.	SFYGT1	Sfygt1	0	fygt1-fygm3 fygt1: INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
39.	SFYGT10	Sfygt10	0	fygt10-fygm3 fygt10: INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
40.	SFYBAAC	sFYBAAC	0	FYBAAC-Fygt10 FYBAAC: BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
41.	BUS_LOANS	BUSLOANS	3	Commercial and Industrial Loans at All Commercial Banks (FRED) Billions $ (SA)
42.	CONS_CREDIT	CCINRV	3*	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
43.	DLOG_EXR_US	EXRUS	2	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
44.	DLOG_EXR_CHF	EXRSW	2	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
45.	DLOG_EXR_YEN	EXRJAN	2	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
46.	DLOG_EXR_GBP	EXRUK	2	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
47.	DLOG_EXR_CAN	EXRCAN	2	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
48.	DLOG_SP500	FSPCOM	2	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
49.	DLOG_SP_IND	FSPIN	2	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
50.	DLOG_DJIA	FSDJ	2	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
	Investment, Inventories, Orders			
51.	NAPMI	PMI	0	PURCHASING MANAGERS' INDEX (SA)
52.	NAPM_NEW_ORDRS	PMNO	0	NAPM NEW ORDERS INDEX (PERCENT)
53.	NAPM_VENDOR_DEL	PMDEL	0	NAPM VENDOR DELIVERIES INDEX (PERCENT)
54.	NAPM_INVENTORIES	PMNV	0	NAPM INVENTORIES INDEX (PERCENT)
55.	RINV_GDP	GDP256	3*	Real Gross Private Domestic Investment, Quantity Index (2000=100) , SAAR
56.	RNONRESINV_STRUCT	GDP259	1	Real Gross Private Domestic Investment - Nonresidential - Structures, Quantity Index (2000=100), SAAR
57.	RNONRESINV_BEQUIPT	GDP260	3*	Real Gross Private Domestic Investment - Nonresidential - Equipment & Software
	Prices and Wages			
58.	RAHE_CONST	CES277R	3*	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION (CES277/PI071)
59.	RAHE_MFG	CES278R	3	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG (CES278/PI071)
60.	P_COM	PSCCOMR	2	Real SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) (PSCCOM/PCEPILFE) PSCCOM: SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) PCEPILFE: PCE Price Index Less Food and Energy (SA) Fred
61.	P_OIL	PW561R	2	PPI Crude (Relative to Core PCE) (pw561/PCEPiLFE) pw561: PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
62.	P_NAPM_COM	PMCP	2	NAPM COMMODITY PRICES INDEX (PERCENT)
63.	RCOMP_HOUR	LBPUR7	1*	REAL COMPENSATION PER HOUR,EMPLOYEES:NONFARM BUSINESS(82=100,SA)
64.	ULC	LBLCPU	1*	UNIT LABOR COST: NONFARM BUSINESS SEC (1982=100,SA)
65.	PCED_DUR	GDP274A	2	Personal Consumption Expenditures: Durable goods Price Index
66.	PCED_NDUR	GDP275A	2	Personal Consumption Expenditures: Nondurable goods Price Index
67.	PCED_SERV	GDP276A	2	Personal Consumption Expenditures: Services Price Index
68.	PINV_GDP	GDP277A	2	Gross private domestic investment Price Index
69.	PINV_NRES_STRUCT	GDP280A	2	GPDI Price Index: Structures
70.	PINV_NRES_EQP	GDP281A	2	GPDI Price Index: Equipment and software Price Index
71.	PINV_RES	GDP282A	2	GPDI Price Index: Residential Price Index
72.	PEXPORTS	GDP284A	2	GDP: Exports Price Index
73.	PIMPORTS	GDP285A	2	GDP: Imports Price Index
74.	PGOV	GDP286A	2	Government consumption expenditures and gross investment Price Index
	Other			
75.	UTL11	UTL11	0	CAPACITY UTILIZATION - MANUFACTURING (SIC)
76.	UMICH_CONS	HHSNTN	1	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)
77.	LABOR_PROD	LBOUT	1*	OUTPUT PER HOUR ALL PERSONS: BUSINESS SEC(1982=100,SA)

Notes: Transformation codes: 0 – nothing; 1 – log(); 2 – dlog(); 3 – log of the ratio of subaggregate to aggregate; 4 – transformation described in the main text, p. 27. Asterisk (*) indicates the transformed variable has been further linearly detrended.

Source of data: Stock and Watson (2008), "Forecasting in Dynamic Factor Models Subject to Structural Instability," available online at http://www.princeton.edu/~mwatson/ddisk/hendryfestschrift_replicationfiles_April28_2008.zip

Full sample available: 1959:Q1-2006:Q4. Sample used in estimation: 1984:Q1-2005:Q4.

All series available at monthly frequency have been converted to quarterly by simple averaging in native units.

Appendix D. Tables and Figures

Table D1. Data-Rich DSGE Model: Parameters Fixed During Estimation - Calibration and Normalization

Parameter Name	Mnemonics	Value
Depreciation rate	δ	0.014
Risk aversion in HH utility function	γ	1
Money demand shock in steady state	χ_*	1
Share of govt spending in steady state	g_*	1.2
Fixed costs in production	F	0
MP rule: response to inflation	ψ_1	1.82
MP rule: response to output gap	ψ_2	0.18
MP rule: int rate smoothing parameter	ρ_R	0.78
Persistence: TFP shock	ρ_Z	0.98
Steady state inflation (in % pa)	π_A	2.5
Steady state real interest rate (in % pa)	r_A	2.84
Price indexation parameter	π_{**}	1
Steady state real GDP	Y_*	1
Log inverse velocity of money in SS	$\log(M_* / Y_*)$	0.778
Steady state of log average inverse labor productivity	$\log(H_* / Y_*)$	–3.5
Transformations: $\beta = \dfrac{1}{1 + r_A / 400}$; $\pi_* = 1 + \dfrac{\pi_A}{400}$		

Table D2. Data-Rich DSGE Model: Prior Distributions

Parameter Name		Domain	Density	Para 1	Para 2
Firms					
Share of capital	α	[0;1)	Beta	0.3	0.025
Average economy wide markup	λ	$R+$	Gamma	0.15	0.01
$1-\zeta$ prob of reoptimizing firm's price	ζ	[0;1)	Beta	0.6	0.15
Indexation parameter	ι	[0;1)	Beta	0.5	0.25
Households					
Elasticity of money demand	ν_m	$R+$	Gamma	20	5
Investment adjustment cost parameter	S''	$R+$	Gamma	5.0	2.5
Shocks					
Persistence: govt spending process	ρ_g	[0;1)	Beta	0.8	0.1
Persistence: money demand shock	ρ_χ	[0;1)	Beta	0.8	0.1
Stdev: govt spending process	σ_g	$R+$	InvGamma	1	4
Stdev: money demand shock	σ_χ	$R+$	InvGamma	1	4
Stdev: monetary policy shock	σ_R	$R+$	InvGamma	0.5	4
Stdev: TFP shock	σ_Z	$R+$	InvGamma	1	4

Notes: Para 1 and Para 2 are (i) the means and the standard deviations for Beta, Gamma, and Normal distributions; (ii) the upper and the lower bound of support for the Uniform distribution; (iii) s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma \mid s,\nu) \propto \sigma^{-\nu-1}\exp(-\nu s^2/2\sigma^2)$.

Table D3. Data-Rich DSGE Model: Posterior Estimates

		Regular DSGE model		Data-Rich DSGE model	
Parameter Name		**Mean**	**90% CI**	**Mean**	**90% CI**
Firms					
Share of capital	α	0.282	[0.269, 0.296]	0.2766	[0.266, 0.292]
Average economy wide markup	λ	0.15	[0.133, 1.166]	0.134	[0.117, 0.154]
$1-\zeta$ prob of reoptimizing firm's price	ζ	0.759	[0.709, 0.809]	0.797	[0.777, 0.819]
Indexation parameter	ι	0.05	[0.00, 0.101]	0.0326	[0.001, 0.0636]
Households					
Elasticity of money demand	ν_m	25.943	[19.581, 31.65]	23.199	[17.13, 31.27]
Investment adjustment cost parameter	S''	11.079	[6.299, 15.683]	30.754	[26.506, 35.29]
Shocks					
Persistence: govt spending process	ρ_g	0.886	[0.85, 0.92]	0.870	[0.839, 0.909]
Persistence: money demand shock	ρ_χ	0.974	[0.958, 0.992]	0.961	[0.936, 0.981]
Stdev: govt spending process	σ_g	1.227	[1.062, 1.388]	0.851	[0.605, 1.238]
Stdev: money demand shock	σ_χ	0.865	[0.757, 0.972]	0.396	[0.327, 0.464]
Stdev: monetary policy shock	σ_R	0.199	[0.175, 0.223]	0.2404	[0.211, 0.275]
Stdev: TFP shock	σ_Z	0.557	[0.471, 0.639]	0.375	[0.322, 0.439]
Implied Slope of NK Phillips Curve	κ	0.0745		0.0517	

Notes: Results labeled "Regular DSGE model" refer to the standard Bayesian estimation of the same underlying theoretical DSGE model as presented in the paper, but only on 4 core observable data series (real GDP, GDP deflator inflation, Federal Funds rate and inverse velocity of money based on M2S aggregate) assumed to be perfectly measured. In terms of the state-space representation (72)-(74), this means that the vector of data X_t contains just these 4 core observables, the factor loadings $\mathbf{\Lambda}$ are restricted as in (5), and there are no measurement errors e_t.

Table D4. Pure DFM: Fraction of Unconditional Variance Captured by Factors

iid Measurement Errors; Dataset = DFM3.txt
<u>on average</u>, 100K draws, 20K burn-in

	All Factors	Error term
Core Variables	**0.948**	**0.052**
Real output	0.993	0.007
Inflation	0.896	0.104
Interest rates	0.990	0.010
Money velocities	0.914	0.086
Non-Core Variables	**0.941**	**0.059**
Output and components	0.982	0.018
Labor market	0.981	0.019
Investment, inventories, orders	0.986	0.014
Housing	0.970	0.030
Prices and wages	0.908	0.092
Financial variables	0.854	0.146
Other	0.973	0.027

Table D5. Data-Rich DSGE Model: Fraction of Unconditional Variance Captured by Model Concepts

iid Measurement Errors; Dataset = DFM3.txt
<u>on average</u>, 20K draws, 4K burn-in

	GOV	CHI	MP	Z	All Shocks	Error term
Core Variables	**0.05**	**0.08**	**0.06**	**0.56**	**0.749**	**0.251**
Real output	0.14	0.21	0.03	0.48	0.852	0.148
Inflation	0.01	0.02	0.01	0.70	0.733	0.267
Interest rates	0.01	0.00	0.15	0.76	0.925	0.075
Money velocities	0.07	0.09	0.04	0.29	0.489	0.512
Non-Core Variables	**0.09**	**0.13**	**0.06**	**0.45**	**0.719**	**0.281**
Output and components	0.07	0.27	0.08	0.45	0.873	0.127
Labor market	0.19	0.14	0.06	0.46	0.848	0.152
Investment, inventories, orders	0.10	0.13	0.02	0.63	0.882	0.118
Housing	0.04	0.26	0.07	0.42	0.794	0.206
Prices and wages	0.03	0.05	0.04	0.45	0.568	0.432
Financial variables	0.06	0.03	0.05	0.32	0.451	0.549
Other	0.02	0.12	0.09	0.64	0.866	0.134

Table D6. Pure DFM: Unconditional Variance Captured by Factors

iid Measurement Errors; Dataset = DFM3.txt
on average, 100K draws, 20K burn-in

Algorithm: Jungbacker-Koopman
Identification: Scheme 2 - Block Diagonal

	F1	F2	F3	F4	F5	F6	All Factors	Measurement Error
Real GDP	0.119	0.142	0.301	0.160	0.115	0.148	0.984	0.016
IP_Total	0.137	0.105	0.343	0.135	0.113	0.164	0.996	0.004
IP_MFG	0.131	0.105	0.350	0.136	0.114	0.162	0.997	0.003
GDP Def inflation	0.147	0.173	0.166	0.169	0.110	0.142	0.907	0.094
PCE Def inflation	0.148	0.177	0.168	0.173	0.110	0.145	0.921	0.079
CPI ALL Inflation	0.130	0.167	0.159	0.166	0.102	0.138	0.862	0.138
FedFunds	0.135	0.169	0.185	0.169	0.186	0.148	0.993	0.008
3m T-Bill rate	0.136	0.166	0.185	0.168	0.189	0.148	0.991	0.009
AAA Bond yield	0.118	0.114	0.192	0.150	0.267	0.147	0.988	0.012
IVM_M1S_det	0.117	0.164	0.149	0.151	0.097	0.130	0.808	0.193
IVM_M2S	0.206	0.141	0.197	0.145	0.114	0.192	0.994	0.006
IVM_MBASE_bar	0.197	0.154	0.175	0.146	0.116	0.152	0.940	0.060
IP_CONS_DBLE	0.134	0.139	0.217	0.159	0.121	0.169	0.938	0.062
IP_CONS_NONDBLE	0.133	0.115	0.253	0.142	0.149	0.201	0.992	0.008
IP_BUS_EQPT	0.161	0.142	0.199	0.191	0.134	0.157	0.984	0.017
IP_DBLE_MATS	0.135	0.110	0.226	0.154	0.137	0.233	0.994	0.006
IP_NONDBLE_MATS	0.147	0.133	0.175	0.185	0.113	0.242	0.996	0.004
IP_FUELS	0.147	0.144	0.212	0.175	0.133	0.149	0.959	0.041
PMP	0.145	0.146	0.216	0.170	0.143	0.170	0.989	0.011
UTL11	0.141	0.181	0.184	0.183	0.143	0.165	0.997	0.003
RAHE_CONST	0.147	0.152	0.192	0.167	0.121	0.180	0.958	0.042
RAHE_MFG	0.166	0.137	0.184	0.149	0.120	0.228	0.983	0.017
EMP_MINING	0.130	0.118	0.211	0.210	0.123	0.169	0.960	0.040
EMP_CONST	0.153	0.141	0.193	0.166	0.112	0.234	0.998	0.002
EMP_DBLE_GDS	0.201	0.140	0.203	0.160	0.133	0.160	0.996	0.004
EMP_NONDBLES	0.158	0.120	0.183	0.183	0.116	0.236	0.995	0.005
EMP_SERVICES	0.164	0.155	0.211	0.141	0.126	0.201	0.997	0.003
EMP_TTU	0.140	0.159	0.184	0.173	0.139	0.176	0.971	0.029
EMP_WHOLESALE	0.144	0.167	0.168	0.142	0.114	0.145	0.879	0.121
EMP_RETAIL	0.162	0.157	0.177	0.163	0.143	0.164	0.967	0.033
EMP_FIRE	0.219	0.142	0.181	0.160	0.121	0.156	0.979	0.021
EMP_GOVT	0.150	0.135	0.266	0.137	0.152	0.155	0.996	0.004
URATE_ALL	0.124	0.175	0.255	0.157	0.141	0.141	0.993	0.007
U_DURATION	0.135	0.143	0.197	0.223	0.116	0.183	0.997	0.003
U_L5WKS	0.128	0.144	0.201	0.211	0.142	0.169	0.995	0.005
U_5_14WKS	0.145	0.143	0.195	0.167	0.154	0.163	0.966	0.034
U_M15WKS	0.132	0.153	0.198	0.218	0.121	0.177	0.998	0.002
U_15_26WKS	0.123	0.153	0.196	0.190	0.160	0.155	0.976	0.024
U_M27WKS	0.136	0.149	0.196	0.218	0.113	0.184	0.997	0.003
HOURS_AVG	0.151	0.147	0.207	0.163	0.145	0.178	0.991	0.009
HSTARTS_NE	0.132	0.135	0.193	0.173	0.154	0.175	0.962	0.038
HSTARTS_MW	0.118	0.121	0.240	0.163	0.155	0.145	0.942	0.058

HSTARTS_MW	0.118	0.121	0.240	0.163	0.155	0.145	0.942	0.058
HSTARTS_SOU	0.133	0.121	0.194	0.240	0.119	0.183	0.990	0.010
HSTARTS_WST	0.128	0.143	0.190	0.223	0.120	0.180	0.982	0.018
SFYGM6	0.138	0.143	0.201	0.167	0.152	0.168	0.970	0.030
SFYGT1	0.133	0.139	0.189	0.164	0.191	0.160	0.976	0.025
SFYGT10	0.150	0.197	0.182	0.160	0.132	0.153	0.974	0.026
SFYBAAC	0.151	0.188	0.178	0.170	0.129	0.171	0.988	0.012
BUS_LOANS	0.140	0.138	0.189	0.199	0.167	0.154	0.986	0.014
CONS_CREDIT	0.140	0.145	0.184	0.176	0.123	0.208	0.976	0.024
P_COM	0.139	0.133	0.189	0.151	0.112	0.150	0.874	0.126
P_OIL	0.117	0.121	0.181	0.139	0.104	0.130	0.792	0.208
P_NAPM_COM	0.138	0.128	0.197	0.147	0.125	0.148	0.882	0.118
DLOG_EXR_US	0.127	0.107	0.141	0.121	0.095	0.118	0.709	0.291
DLOG_EXR_CHF	0.107	0.100	0.135	0.112	0.090	0.111	0.655	0.345
DLOG_EXR_YEN	0.128	0.125	0.168	0.134	0.126	0.134	0.814	0.186
DLOG_EXR_GBP	0.098	0.095	0.129	0.111	0.088	0.105	0.626	0.374
DLOG_EXR_CAN	0.136	0.130	0.160	0.142	0.126	0.132	0.825	0.175
DLOG_SP500	0.133	0.136	0.171	0.138	0.111	0.137	0.827	0.173
DLOG_SP_IND	0.129	0.139	0.167	0.138	0.110	0.136	0.819	0.181
DLOG_DJIA	0.128	0.126	0.174	0.134	0.111	0.133	0.807	0.193
UMICH_CONS	0.142	0.121	0.246	0.142	0.130	0.167	0.949	0.051
NAPMI	0.144	0.149	0.219	0.173	0.140	0.170	0.994	0.006
NAPM_NEW_ORDRS	0.146	0.146	0.214	0.169	0.139	0.170	0.983	0.017
NAPM_VENDOR_DEL	0.142	0.147	0.222	0.170	0.137	0.168	0.985	0.015
NAPM_INVENTORIES	0.137	0.155	0.211	0.176	0.145	0.161	0.985	0.015
RCONS	0.172	0.144	0.187	0.175	0.127	0.177	0.982	0.018
RCONS_DUR	0.141	0.118	0.203	0.175	0.114	0.230	0.980	0.020
RCONS_SERV	0.139	0.134	0.186	0.202	0.115	0.214	0.990	0.010
RINV_GDP	0.153	0.125	0.225	0.155	0.145	0.192	0.995	0.005
RNONRESINV_STRUCT	0.165	0.138	0.187	0.153	0.118	0.224	0.984	0.016
RNONRESINV_BEQUIPT	0.141	0.168	0.185	0.198	0.128	0.156	0.976	0.024
RRESINV	0.176	0.155	0.182	0.186	0.128	0.150	0.977	0.023
REXPORTS	0.152	0.130	0.177	0.226	0.117	0.192	0.993	0.007
RIMPORTS	0.129	0.106	0.236	0.149	0.137	0.222	0.978	0.022
RGOV	0.203	0.133	0.207	0.141	0.138	0.171	0.994	0.006
LABOR_PROD	0.173	0.144	0.175	0.199	0.115	0.166	0.972	0.028
RCOMP_HOUR	0.183	0.161	0.190	0.153	0.123	0.177	0.987	0.014
ULC	0.134	0.151	0.187	0.225	0.122	0.170	0.989	0.011
PCED_DUR	0.135	0.133	0.178	0.174	0.181	0.150	0.950	0.050
PCED_NDUR	0.133	0.152	0.174	0.163	0.108	0.136	0.866	0.134
PCED_SERV	0.131	0.117	0.200	0.139	0.134	0.144	0.865	0.135
PINV_GDP	0.154	0.162	0.174	0.176	0.116	0.142	0.925	0.075
PINV_NRES_STRUCT	0.129	0.165	0.189	0.177	0.137	0.149	0.945	0.055
PINV_NRES_EQP	0.172	0.129	0.182	0.151	0.113	0.149	0.897	0.103
PINV_RES	0.121	0.135	0.191	0.173	0.110	0.140	0.870	0.130
PEXPORTS	0.164	0.147	0.204	0.170	0.123	0.155	0.963	0.037
PIMPORTS	0.149	0.142	0.192	0.162	0.117	0.144	0.906	0.094
PGOV	0.122	0.125	0.156	0.140	0.111	0.124	0.778	0.222

Notes: Please see the Appendix C. Data: Description and Transformations for the corresponding mnemonics of data indicators reported here.

Table D7. Data-Rich DSGE Model: Fraction of Unconditional Variance Captured by Model Concepts

iid Measurement Errors; Dataset = DFM3.txt
<u>on average</u>, 20K draws, 4K burn-in

Algorithm: Jungbacker-Koopman

	GOV	CHI	MP	Z	All Shocks	Measurement Error
Real GDP	0.081	0.000	0.040	0.648	0.770	0.230
IP_Total	0.167	0.308	0.021	0.395	0.891	0.110
IP_MFG	0.166	0.317	0.020	0.392	0.894	0.106
GDP Def inflation	0.011	0.000	0.011	0.789	0.811	0.189
PCE Def inflation	0.004	0.035	0.003	0.703	0.745	0.255
CPI ALL Inflation	0.005	0.031	0.006	0.600	0.642	0.358
FedFunds	0.004	0.000	0.135	0.817	0.956	0.044
3m T-Bill rate	0.007	0.003	0.160	0.788	0.958	0.042
AAA Bond yield	0.013	0.008	0.168	0.672	0.861	0.139
IVM_M1S_det	0.055	0.174	0.016	0.404	0.648	0.352
IVM_M2S	0.042	0.063	0.003	0.071	0.178	0.822
IVM_MBASE_bar	0.099	0.031	0.104	0.406	0.639	0.361
IP_CONS_DBLE	0.051	0.090	0.018	0.650	0.810	0.190
IP_CONS_NONDBLE	0.151	0.551	0.025	0.109	0.836	0.164
IP_BUS_EQPT	0.259	0.103	0.106	0.407	0.874	0.126
IP_DBLE_MATS	0.069	0.677	0.024	0.131	0.901	0.099
IP_NONDBLE_MATS	0.060	0.229	0.028	0.645	0.962	0.038
IP_FUELS	0.081	0.136	0.044	0.457	0.718	0.282
PMP	0.085	0.046	0.014	0.702	0.848	0.153
UTL11	0.010	0.002	0.066	0.913	0.991	0.010
RAHE_CONST	0.131	0.010	0.035	0.566	0.742	0.258
RAHE_MFG	0.116	0.024	0.124	0.651	0.915	0.085
EMP_MINING	0.055	0.030	0.007	0.596	0.688	0.312
EMP_CONST	0.094	0.190	0.134	0.546	0.964	0.037
EMP_DBLE_GDS	0.137	0.272	0.177	0.381	0.967	0.034
EMP_NONDBLES	0.035	0.117	0.186	0.609	0.947	0.053
EMP_SERVICES	0.111	0.400	0.069	0.379	0.958	0.042
EMP_TTU	0.012	0.320	0.011	0.399	0.743	0.258
EMP_WHOLESALE	0.011	0.020	0.056	0.248	0.335	0.665
EMP_RETAIL	0.011	0.237	0.059	0.455	0.761	0.239
EMP_FIRE	0.022	0.150	0.111	0.501	0.784	0.216
EMP_GOVT	0.162	0.237	0.016	0.467	0.882	0.118
URATE_ALL	0.175	0.056	0.014	0.619	0.864	0.136
U_DURATION	0.656	0.149	0.015	0.147	0.967	0.033
U_L5WKS	0.384	0.051	0.031	0.463	0.928	0.072
U_5_14WKS	0.143	0.033	0.011	0.523	0.710	0.290
U_M15WKS	0.575	0.099	0.018	0.284	0.977	0.023
U_15_26WKS	0.096	0.006	0.043	0.715	0.859	0.141
U_M27WKS	0.664	0.160	0.014	0.135	0.973	0.027
HOURS_AVG	0.019	0.032	0.095	0.816	0.961	0.039

HSTARTS_NE	0.009	0.115	0.016	0.679	0.819	0.181
HSTARTS_MW	0.017	0.193	0.115	0.273	0.598	0.402
HSTARTS_SOU	0.058	0.601	0.059	0.152	0.870	0.130
HSTARTS_WST	0.019	0.328	0.075	0.404	0.826	0.174
SFYGM6	0.090	0.041	0.029	0.642	0.802	0.198
SFYGT1	0.067	0.024	0.054	0.698	0.843	0.157
SFYGT10	0.157	0.006	0.025	0.460	0.648	0.352
SFYBAAC	0.034	0.004	0.082	0.811	0.931	0.069
BUS_LOANS	0.279	0.032	0.230	0.251	0.791	0.209
CONS_CREDIT	0.064	0.212	0.065	0.275	0.616	0.384
P_COM	0.038	0.012	0.011	0.335	0.396	0.604
P_OIL	0.008	0.011	0.007	0.263	0.288	0.712
P_NAPM_COM	0.017	0.017	0.010	0.223	0.267	0.733
DLOG_EXR_US	0.008	0.016	0.039	0.118	0.180	0.820
DLOG_EXR_CHF	0.007	0.013	0.030	0.110	0.160	0.840
DLOG_EXR_YEN	0.011	0.010	0.010	0.116	0.147	0.853
DLOG_EXR_GBP	0.007	0.012	0.016	0.117	0.152	0.848
DLOG_EXR_CAN	0.010	0.029	0.058	0.184	0.280	0.720
DLOG_SP500	0.016	0.010	0.026	0.222	0.274	0.726
DLOG_SP_IND	0.016	0.009	0.024	0.259	0.308	0.692
DLOG_DJIA	0.010	0.010	0.017	0.147	0.183	0.817
UMICH_CONS	0.006	0.311	0.046	0.405	0.767	0.233
NAPMI	0.075	0.050	0.016	0.760	0.900	0.100
NAPM_NEW_ORDRS	0.093	0.047	0.010	0.652	0.802	0.198
NAPM_VENDOR_DEL	0.068	0.053	0.015	0.711	0.846	0.154
NAPM_INVENTORIES	0.047	0.046	0.023	0.804	0.919	0.081
RCONS	0.005	0.032	0.196	0.667	0.901	0.099
RCONS_DUR	0.044	0.319	0.144	0.353	0.859	0.141
RCONS_SERV	0.009	0.237	0.099	0.580	0.925	0.075
RINV_GDP	0.005	0.479	0.069	0.415	0.967	0.033
RNONRESINV_STRUCT	0.339	0.184	0.013	0.327	0.863	0.137
RNONRESINV_BEQUIPT	0.095	0.027	0.008	0.750	0.880	0.120
RRESINV	0.092	0.078	0.092	0.596	0.858	0.142
REXPORTS	0.018	0.093	0.196	0.635	0.942	0.058
RIMPORTS	0.055	0.615	0.025	0.119	0.813	0.186
RGOV	0.006	0.339	0.175	0.437	0.957	0.043
LABOR_PROD	0.033	0.044	0.161	0.602	0.839	0.161
RCOMP_HOUR	0.020	0.026	0.176	0.563	0.784	0.216
ULC	0.090	0.215	0.019	0.526	0.850	0.150
PCED_DUR	0.021	0.044	0.023	0.699	0.788	0.212
PCED_NDUR	0.009	0.023	0.006	0.438	0.474	0.526
PCED_SERV	0.007	0.088	0.005	0.457	0.557	0.443
PINV_GDP	0.015	0.036	0.045	0.544	0.639	0.361
PINV_NRES_STRUCT	0.019	0.048	0.023	0.397	0.486	0.514
PINV_NRES_EQP	0.008	0.118	0.023	0.447	0.596	0.404
PINV_RES	0.028	0.080	0.036	0.270	0.414	0.586
PEXPORTS	0.013	0.022	0.015	0.637	0.687	0.313
PIMPORTS	0.012	0.015	0.012	0.499	0.537	0.463
PGOV	0.009	0.019	0.029	0.177	0.233	0.767

Notes: Structural shocks are GOV – government spending, CHI – money demand, MP – monetary policy and Z – neutral technology. Please see the Appendix C. Data: Description and Transformations for the corresponding mnemonics of data indicators reported here.

Table D8. Regressing Data-Rich DSGE Model Concepts on DFM Factors

Model Concept		R^2
Inflation	PI_t	0.984
Interest Rate	R_t	0.991
Real Consumption	X_t	0.998
Govt Spending shock	GOV_t	0.999
Money Demand shock	CHI_t	0.999
Technology shock	Z_t	0.990

Notes: Each line reports the R^2 from predictive linear regression:

$$S_{i,t}^{(pm)} = \alpha_{0,i} + \boldsymbol{\alpha}'_{1,i} F_t^{(pm)} + v_{i,t} ,$$

where $S_{i,t}^{(pm)}$ is the posterior mean of the i^{th} data-rich DSGE model state variable and $F_t^{(pm)}$ is the posterior mean of the empirical factors extracted by DFM.

Table D9. Regressing DFM Factors on Data-Rich DSGE Model Concepts

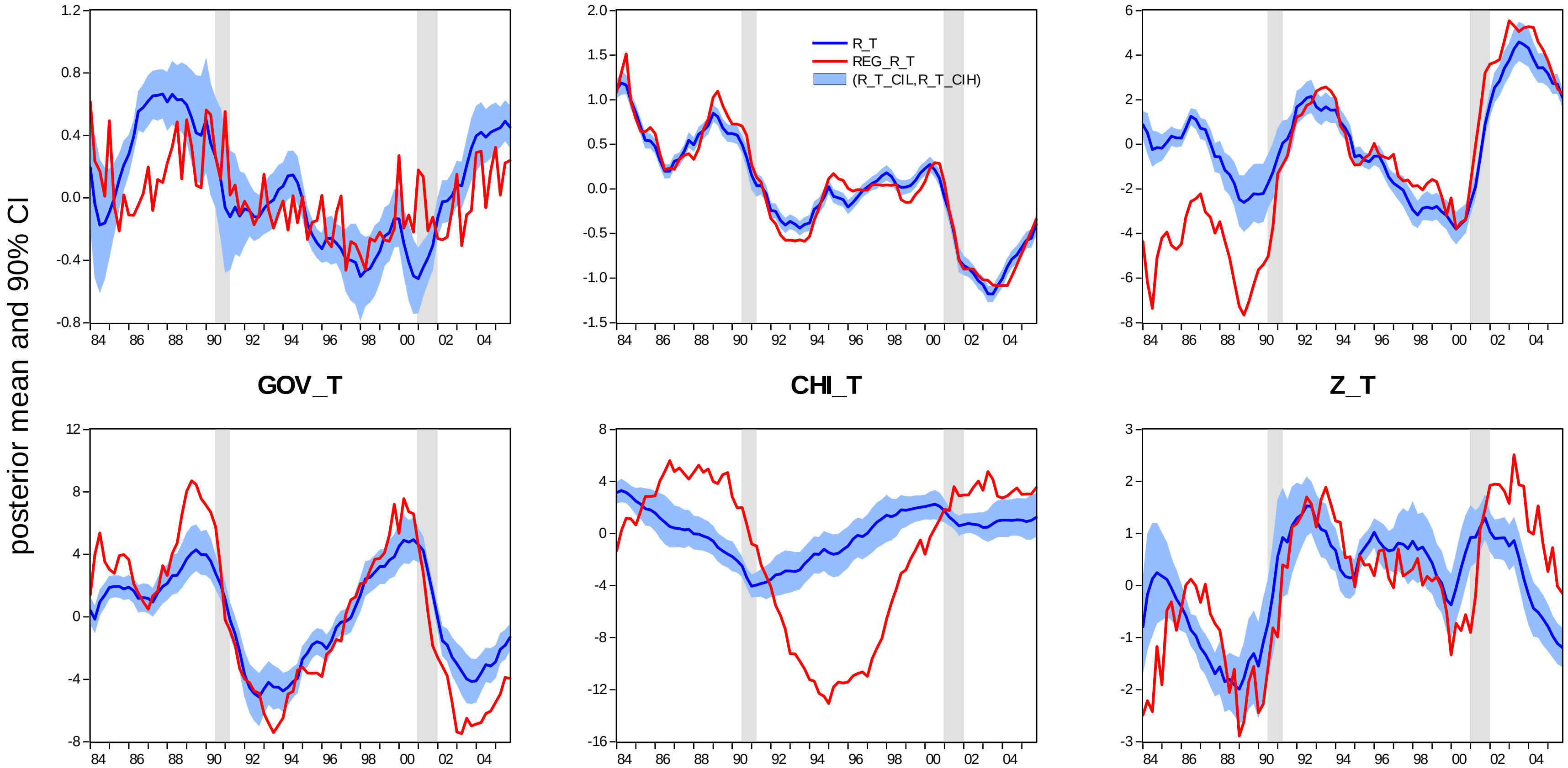
Factors	R^2
Factor 1	0.979
Factor 2	0.924
Factor 3	0.949
Factor 4	0.981
Factor 5	0.989
Factor 6	0.991

Notes: Each line reports the R^2 from predictive linear regression (see (104) in the main text):

$$F_{i,t}^{(pm)} = \beta_{0,i} + \boldsymbol{\beta}'_{1,i} S_t^{(pm)} + u_{i,t} ,$$

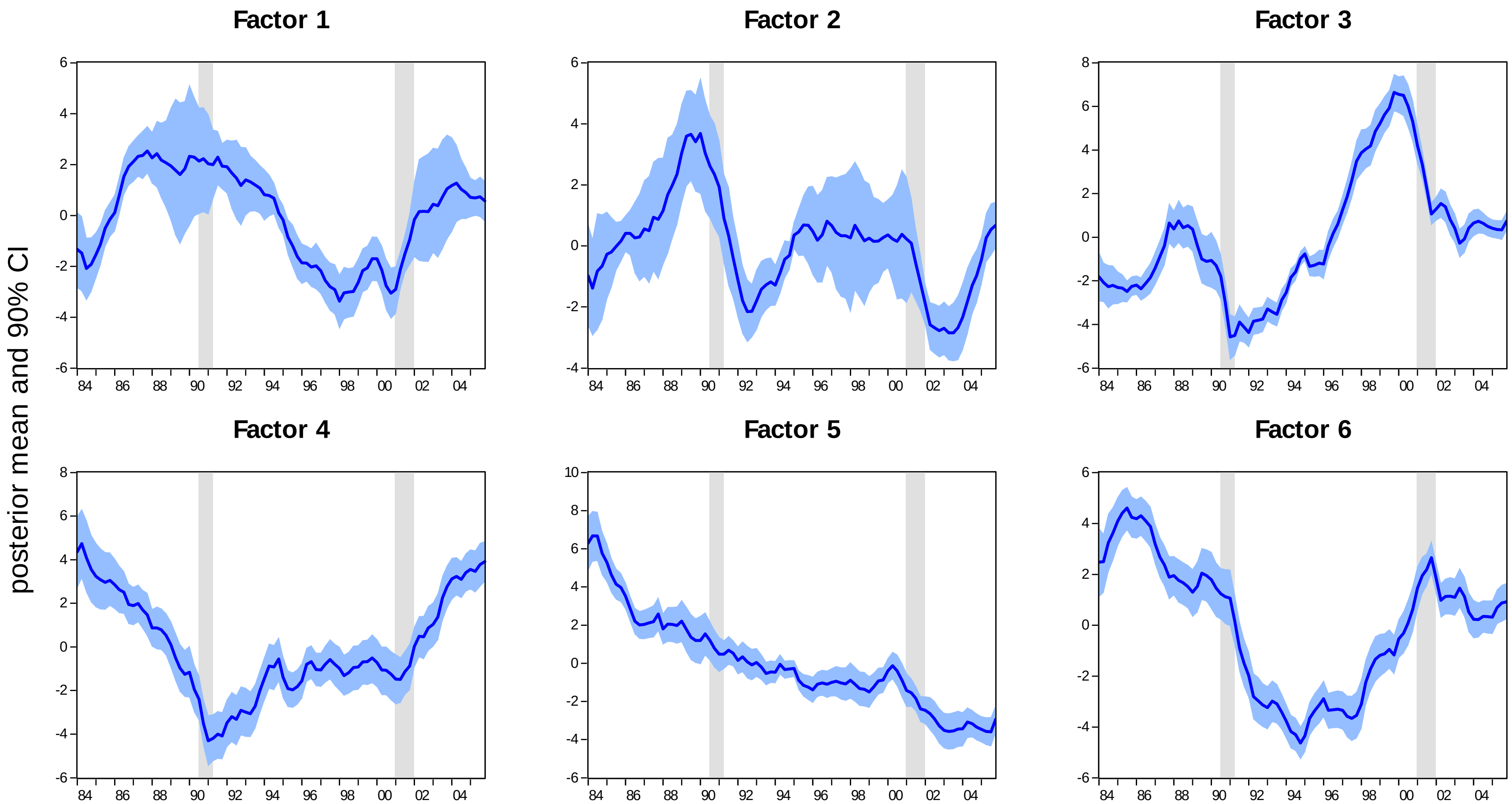
where $F_{i,t}^{(pm)}$ is the posterior mean of the i^{th} empirical factor extracted by DFM and $S_t^{(pm)}$ is the posterior mean of the data-rich DSGE model state variables.

Figure D1. Data-Rich DSGE Model (iid errors): Estimated Model Concepts



Notes: Figure depicts the posterior means and 90% credible intervals of the data-rich DSGE model concepts (blue line and bands): inflation (PI_T, π_t), nominal interest rate (R_T, R_t), real consumption (X_T, x_t), government spending shock (GOV_T, g_t), money demand shock (CHI_T, χ_t), and neutral technology shock (Z_T, Z_t). Red line corresponds to the smoothed versions of the same variables in a *regular* DSGE model estimation derived by Kalman smoother at posterior mean of deep structural parameters (see notes to Table D3 for definition of "regular DSGE estimation").

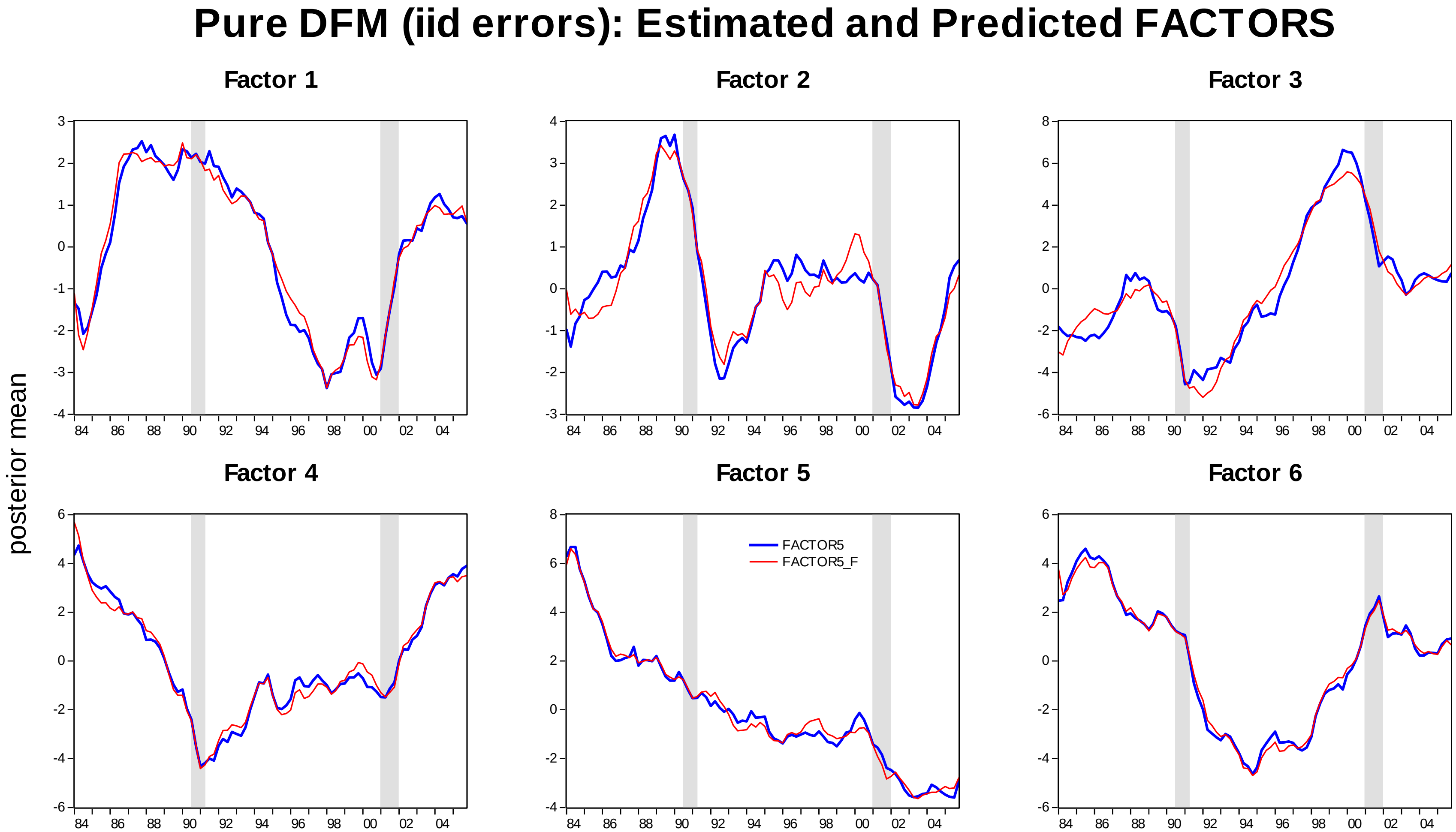
Figure D2. Pure DFM (iid errors): Estimated Factors



Notes: The figure plots the posterior means and 90% credible intervals of the latent empirical factors extracted by the empirical DFM (83)-(85).

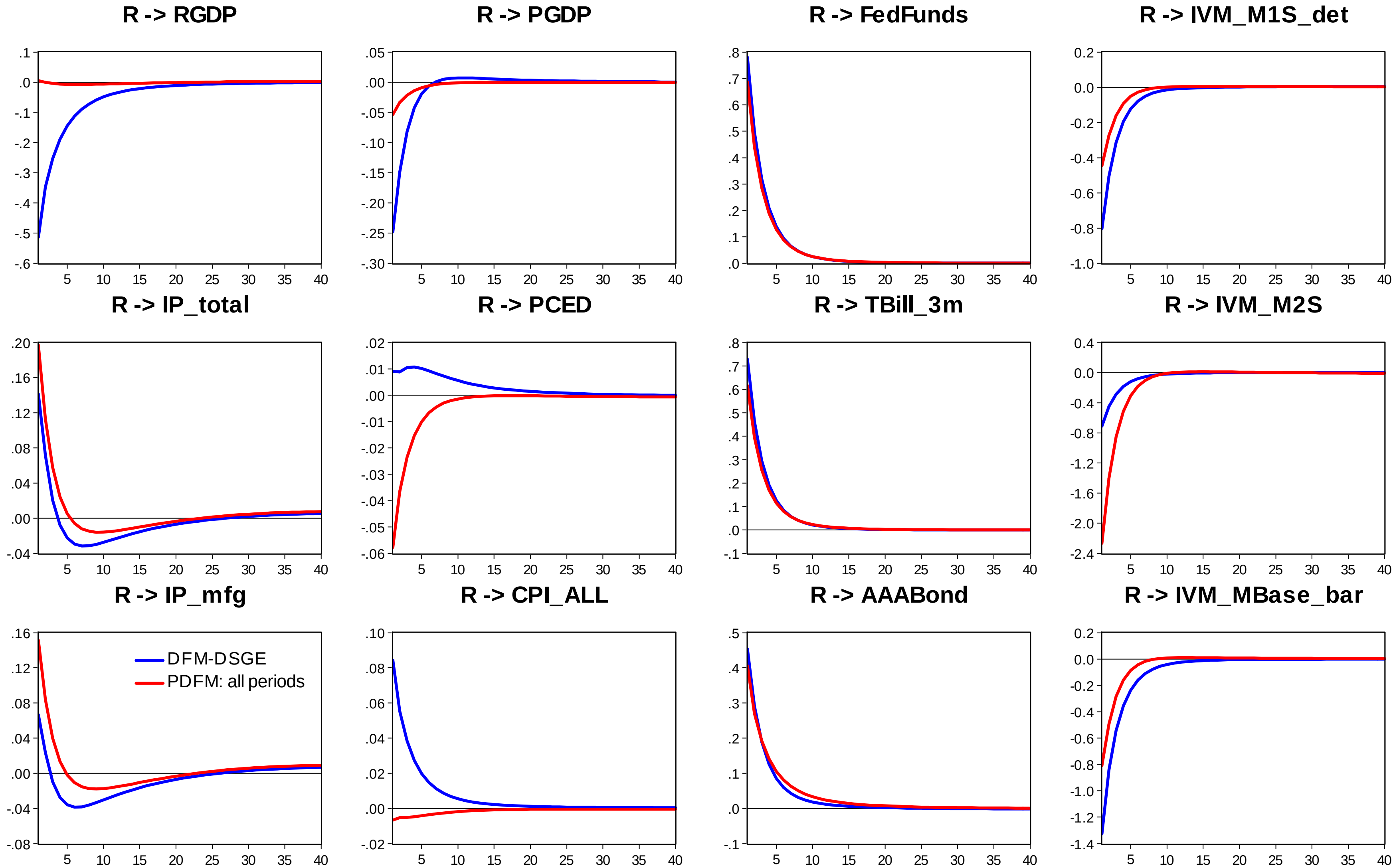
Normalization: block diagonal. Algorithm: Jungbacker-Koopman (2008).

Figure D3. Do Empirical Factors and DSGE Model Concepts Span the Same Space?



Notes: The figure plots the actual empirical factors extracted by the DFM (83)-(85) (blue line) and the empirical factors predicted by the data-rich DSGE model state variables using (105) in the main text (red line).

Figure D4. Impact of Monetary Policy Innovation on Core Macro Series

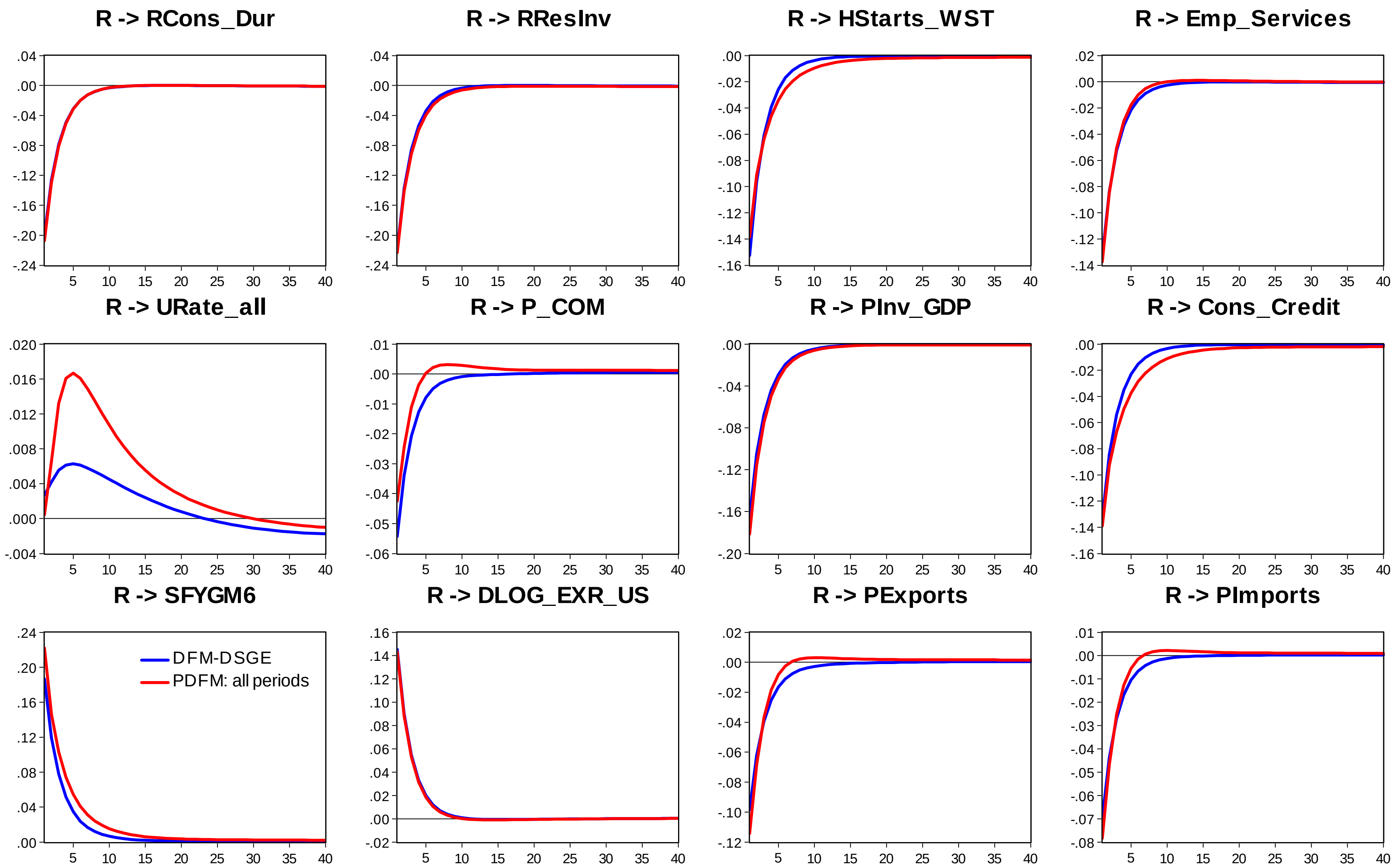


Notes: The figure plots the impulse responses of data indicators to a 1-standard-deviation **monetary policy** innovation $(\varepsilon_{R,t})$ computed in the data-rich DSGE model (blue line, "DFM-DSGE") and in empirical pure DFM (red line, "PDFM: all periods") according to (106) and (107), respectively.

The impact of structural shock is mapped from data-rich DSGE model into empirical DFM every period.

Data indicators are real GDP (RGDP), industrial production: total (IP_total), industrial production: manufacturing (IP_mfg), GDP deflator inflation (PGDP), PCE deflator inflation (PCED), CPI inflation (CPI_ALL), Federal Funds rate (FedFunds), 3-month T-Bill rate (TBill_3m), yield on AAA rated corporate bonds (AAABond), real money balances based on M1S aggregate (IVM_M1S_det), on M2S aggregate (IVM_M2S), and on adjusted monetary base (IVM_MBase_bar). See the corresponding mnemonics in Appendix C. Data: Description and Transformations.

Figure D5. Impact of Monetary Policy Innovation on Non-Core Macro Series

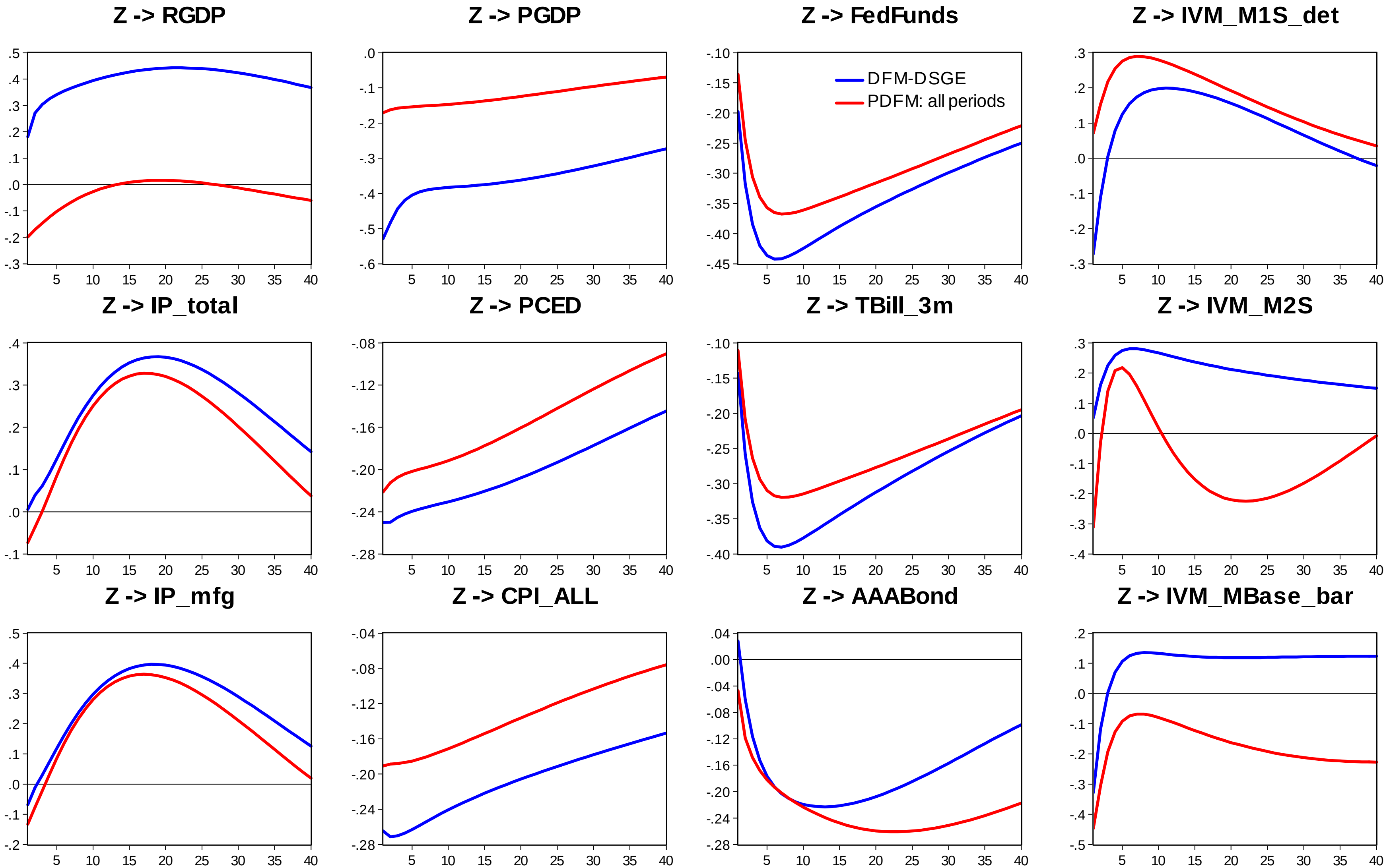


Notes: The figure plots the impulse responses of data indicators to a 1-standard-deviation **monetary policy** innovation $(\varepsilon_{R,t})$ computed in the data-rich DSGE model (blue line, "DFM-DSGE") and in empirical pure DFM (red line, "PDFM: all periods") according to (106) and (107), respectively.

The impact of structural shock is mapped from data-rich DSGE model into empirical DFM every period.

Data indicators are real consumption of durables (RCons_Dur), real residential investment (RResInv), housing starts: West (HStarts_WST), employment in services sector (Emp_Services), unemployment rate (URate_all), commodity price inflation (P_COM), investment deflator inflation (PInv_GDP), consumer credit outstanding (Cons_Credit), 6-month over 3-month T-Bill rate spread (SFYGM6), US effective exchange rate depreciation (DLOG_EXR_US), exports price index (PExports), imports price index (PImports). See the corresponding mnemonics in Appendix C. Data: Description and Transformations.

Figure D6. Impact of Technology Innovation on Core Macro Series

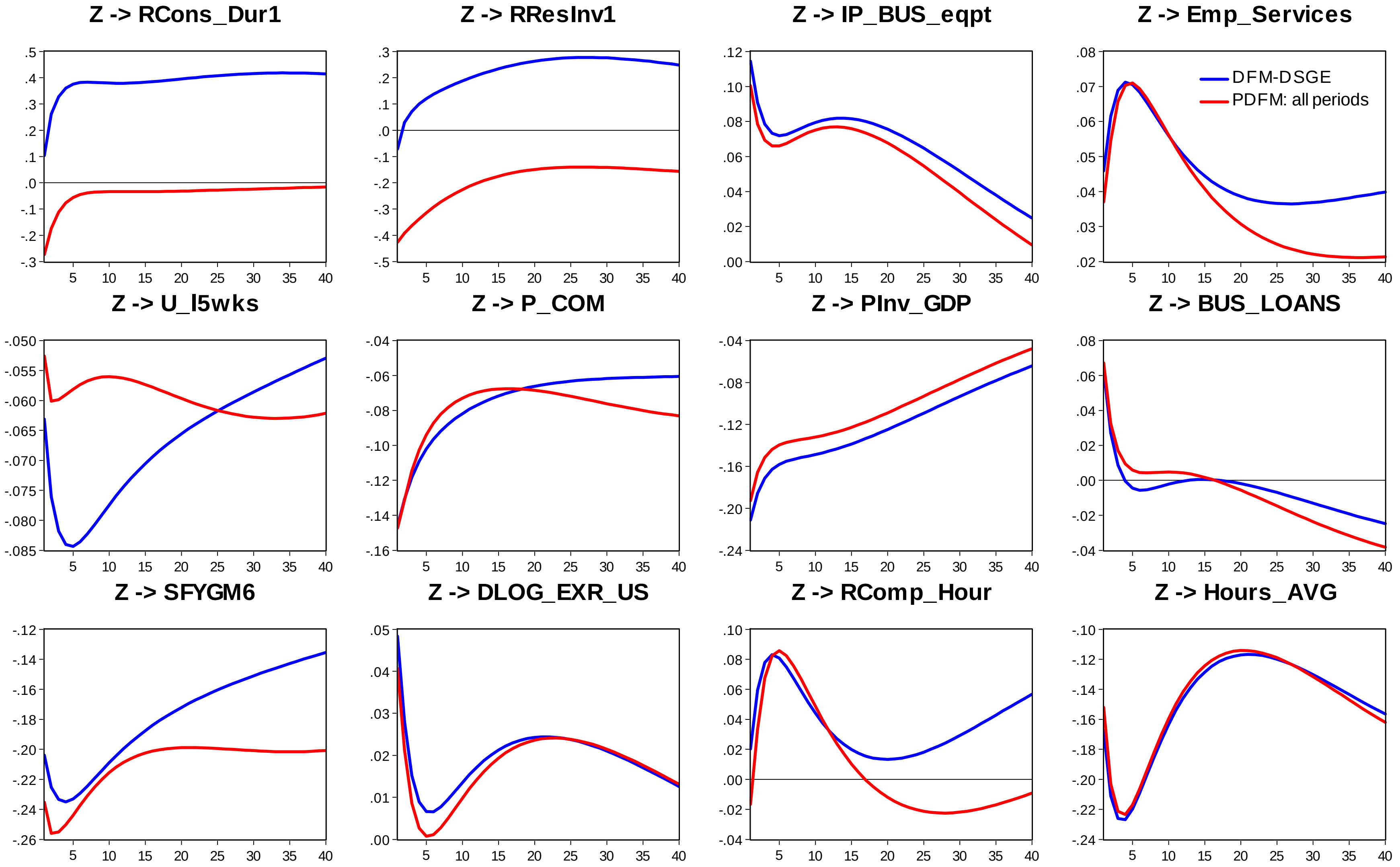


Notes: The figure plots the impulse responses of data indicators to a 1-standard-deviation **technology** innovation $(\varepsilon_{Z,t})$ computed in the data-rich DSGE model (blue line, "DFM-DSGE") and in empirical pure DFM (red line, "PDFM: all periods") according to (106) and (107), respectively.

The impact of structural shock is mapped from data-rich DSGE model into empirical DFM every period.

Data indicators are real GDP (RGDP), industrial production: total (IP_total), industrial production: manufacturing (IP_mfg), GDP deflator inflation (PGDP), PCE deflator inflation (PCED), CPI inflation (CPI_ALL), Federal Funds rate (FedFunds), 3-month T-Bill rate (TBill_3m), yield on AAA rated corporate bonds (AAABond), real money balances based on M1S aggregate (IVM_M1S_det), on M2S aggregate (IVM_M2S), and on adjusted monetary base (IVM_MBase_bar). See the corresponding mnemonics in Appendix C. Data: Description and Transformations.

Figure D7. Impact of Technology Innovation on Non-Core Macro Series



Notes: The figure plots the impulse responses of data indicators to a 1-standard-deviation **technology** innovation $(\varepsilon_{Z,t})$ computed in the data-rich DSGE model (blue line, "DFM-DSGE") and in empirical pure DFM (red line, "PDFM: all periods") according to (106) and (107), respectively.
The impact of structural shock is mapped from data-rich DSGE model into empirical DFM every period.
Data indicators are real consumption of durables (RCons_Dur1), real residential investment (RResInv1), industrial production: business equipment (IP_BUS_eqpt), employment in services sector (Emp_Services), persons unemployed less than 5 weeks (U_l5wks), commodity price inflation (P_COM), investment deflator inflation (PInv_GDP), commercial and industrial loans (BUS_LOANS), 6-month over 3-month T-Bill rate spread (SFYGM6), US effective exchange rate depreciation (DLOG_EXR_US), real compensation per hour (RComp_Hour), average weekly hours worked (Hours_AVG). See the corresponding mnemonics in Appendix C. Data: Description and Transformations.