Self Control, Risk Aversion, and the Allais Paradox

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1. Introduction

We argue that the choices made in the Allais paradox, reflecting a certain type of violation of the independence axiom in choices over lotteries, can be seen as a consequence of the agent having a self-control problem. In addition, we argue that recent experimental work by Benjamin, Brown and Shapiro [2006] on the effect of cognitive load on small-stakes risk aversion provides further support for the idea that risk preferences and self-control problems are linked phenomena that can and should have a unified explanation.

Our argument extends our work in Fudenberg and Levine [2006] by allowing a more flexible set of consumption options. That paper studied a self-control game between a single long-lived patient self and a sequence of short-term myopic selves, and showed that equilibrium in this game is equivalent to optimization by a single long-run agent with a "cost of self-control." It then specialized the model to the case where the cost of self-control depends only on the short-run utility of the chosen action and the short-run utility of the most tempting alternative. When the cost of self-control is linear in the foregone short-run utility, the model is consistent with Gul and Pesendorfer's [2001] axioms for choice over menus, which includes an analog of the independence axiom. However, we argued that a number of experimental and empirical observations suggest that the cost function is convex.[1] This convexity has a number of important implications, because it implies that the dual-self model fails the independence axiom of expected utility theory.[2]

The convexity of the cost function leads to a particular sort of violation of the independence axiom: Agents should be "more rational" about choices that are likely to be payoff-irrelevant. This is exactly the nature of the violation of the independence axiom in the Allais paradox. Consider the Kahneman and Tversky [1979] version of the

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[1] There were several sets of evidence we discussed. The work of Baumesiter and collaborators (for example, Muraven et al [1998,2000]) argues that self-control is a limited resource. The stylized fact that people often reward themselves in one domain (for example, food) when exerting more self control in another (for example, work) has the same implication. This is backed up by evidence is from Shiv and Fedorikhin [1999] and Ward and Mann [2000] showing that agents under cognitive load exercise less self-control, for example, by eating more deserts. The first two observations fit naturally with the idea that a common "self control function" controls many contemporaneous or nearly-temporaneous choices, and the third does as well on the hypothesis that self-control and some other forms of mental activity draw on related mental systems or resources.

[2] Benahib and Bisin [2005], Bernheim and Rangel [2004], Brocas and Carillo [2005], and Loewenstein and O'Donoghue [2005] present similar dual-self models, but they do not derive them from a game the way we do, and they do not discuss risk aversion, cognitive load, or the possibility of convex costs of self-control.

paradox. In scenario one, the choice is between option A_1, a lottery with probability and respective payoffs $(.01:0,.66:2400,.33:2500)$, and option B_1, which gives \$2400 for certain. In this case option B_1 is chosen. In scenario two, the choices are option $A_2 = (.33:0,.34:2400,.33:2500)$, and option $B_2 = (.32:0,.68:2400)$, and option A_2 is chosen. If we let $A_3 = (1/66:0,16/33:2400,1/2:2500)$, then $A_1 = .66A_3 + .34B_1$ and $B_1 = .66B_1 + .34B_1$. The independence axiom says that the choice between A_1 and B_1 should be the same as the choice between A and B_1 since the other consequence $.34B_1$ is common between the two lotteries. In the second case, $A_2 = .68A_3 + .32*0$ and $B_2 = .68B_1 + .32*0$. Again the independence implies the choice between A_2 and B_2 should be the same as that between A and B since the other consequence $.32*0$ is common between the two lotteries. In other words, the independence axiom means that A is chosen in scenario one if and only if it is chosen in scenario two. The paradox arises from the fact that in experimental data the choices differ in a particular way: many agents choose A_1 over B_1, but between B over between A_2.

From the dual-self perspective, the key aspect of this choice problem is that the chance of winning a prize in scenario two is much less than in scenario one, so the problem of self-control should be correspondingly less. Because the cost of self-control depends on the amount of foregone utility (relative to the most tempting outcome), convex control costs can in principle explain the paradoxical choices in the Allais lotteries. When the cost of self-control is low, the agent chooses the lottery with the highest long-run utility. Because the agent is fairly patient and the lottery is a small share of lifetime wealth, this is equivalent to choosing the lottery with the higher expected value. When the cost of self control is high, the agent chooses the lottery that is most appealing to the short-run player. Since short-run consumption will be essentially the same regardless of whether the agent wins 2400 or 2500, the more tempting lottery is the one with the highest probability of a short-run payout, that is B_1. We should emphasize that our theory does not explain all possible violations of the independence axiom: If the choices in each of the two Allais scenarios were reversed, the independence axiom would still be violated, but our explanation would not apply.

Our goals in this paper are not only to formalize this observation, but to construct a form of the dual-self model that can be calibrated to explain a range of data about choices over lotteries. To do this, we extend the bank/nightclub model we used in

Fudenberg and Levine [2006] by adding an additional choice of "consumption technology" and a corresponding unobserved parameter. This is necessary not only to explain the Allais paradox, but to accommodate reconcile the "Rabin paradox" of substantial risk aversion even to very small stakes. Specifically, while our earlier model can explain the examples in Rabin [2000], those examples (such as rejecting a bet that had equal probability of winning $105 or losing $100) understate the degree of risk aversion in small-stakes experiments where agents are risk averse over much smaller gambles.

The idea of the bank/nightclub model is that agents use cash on hand as a commitment device, so that on the margin they will consume all of any small unexpected winnings. However, when agents win large amounts, they choose to exercise self-control and save some of their winnings. The resulting intertemporal smoothing make the agents less risk averse, so that they are less risk averse to large gambles than to small ones. However, when calibrating the model to aggregate data, we took the underlying utility function to be logarithmic and the same for all consumers. In the present paper we show that this simple specification is not consistent with experimental data on risk aversion and reasonable values of the pocket cash variable against which short-term risk is compared.[3]

For this reason we introduce an extension of the nightclub model in which the choice of venue at which short-term expenditures are made is endogenous. This reflects the idea that over a short period of time, the set of things on which the short-run self can spend money is limited, so the marginal utility of consumption decreases fairly rapidly and risk aversion is quite pronounced. Over a longer time frame there are more possible ways to adjust consumption, and also to learn how to use or enjoy goods that have not been consumed before, so that the long-run utility possibilities are the upper envelope of the family of short-run utilities. With the preference that we specify in this paper, this upper envelope, and thus the agent's preferences over steady state consumption levels, reduces to the logarithmic form we used in our previous paper.

[3] Since this paper was written, Cox et al [2007] have conducted a series of experiments to test various utility theories using relatively high stakes. They also observe that the simple logarithmic model is inconsistent with observed risk aversion, and they argue that the simple linear-logarithmic self-control model does not plausibly explain their data. We will be interested to see whether their data is consistent with the more complex model developed here.

After developing the theory of "endogenous nightclubs," we then calibrate it in an effort to examine three different paradoxes. Specifically, we analyze Rabin paradox data from Holt and Laury [2002], the Kahneman and Tversky [1979] and Allais versions of the Allais paradox, and the experimental results of Benjamin, Brown, and Shapiro [2006], who find that exposing subjects to cognitive load increases their small-stakes risk aversion.[4]

Our procedure is to define a set of sensible values of the key parameters, namely the subjective interest rate, income, the degree of short-term risk aversion, the time-horizon of the short-run self, and the degree of self-control, using a variety of external sources of data. We than ask investigate how well we can explain the paradoxes using the calibrated parameter values and the dual-self model. How broad of set of parameter values in the calibrated range will explain the paradoxes? To what extent can the same set of parameter values simultaneously explain all the paradoxes? Roughly speaking, we can explain all the data if we assume an annual interest rate of 5%, a low degree of risk aversion, and a daily time horizon for the short-run self. In the data on the cognitive load of Chilean students, these are the only calibrated parameters that explain the data. In the case of the Allais paradox, we can find a degree of self-control for every interest rate/income/risk aversion combination in the calibrated range that explains the paradox. Roughly speaking, the Rabin paradox is relatively insensitive to the exact parameters assumed; the Allais paradox is sensitive to choosing a plausible level of risk aversion; and the Chilean cognitive load data is very sensitive to the exact parameter values chosen.

After showing that the base model provides a plausible description of data on attitudes towards risk, gambles, and cognitive load, we examine the robustness of the theory. Specifically, in the calibrations we assume that the opportunities presented in the

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[4] The main focus of Benjamin, Brown and Shapiro [2006], like that of Frederick [2005], is on the correlation between measures of cognitive ability and the phenomena of small-stakes risk aversion and of a preference for immediate rewards. Benjamin, Brown and Shapiro find a significant and substantial correlation between with each of these sorts of preferences and cognitive ability. They also note that the correlation between cognitive ability and time preference vanishes when neither choice results in an immediate payoffs, and that the correlation between small-stakes risk aversion and "present bias" drops to zero once they control for cognitive ability. This evidence is consistent with our explanation of the Rabin paradox, as it suggests that that small-stakes risk aversion results from the same self-control problem that leads to a present bias in the timing of rewards. They also discuss the sizable literature that examines the correlation between cognitive ability and present bias without discussing risk aversion.

experiments are unanticipated, so we consider what happens when gambling opportunities are foreseen. We conclude by discussing a number of modeling choices concerning the long-run and short-run self and how it would impact our analysis.

2. Self-Control, Cash Constraints, and Target Consumption

Fudenberg and Levine [2006] considered a "self control game" between a single long run patient self and a sequence of short-run impulsive selves. The equilibria of this game correspond to the solutions to a "reduced form maximization" by a single long-run agent who acts to maximize the expected present value of per-period utility *u* net of self control costs *C*:

$$U_{RF} = \sum_{t=1}^{\infty} \delta^{t-1} \left(u(a_t, y_y) - C(a_t, y_t) \right), \tag{2.1}$$

where a_t is the action chosen in period t and y_t is a state variable such as wealth whose evolution can be stochastic. That paper, like this one, focuses on the case where preferences satisfy "opportunity-based cost of self control," meaning that the cost *C* depends only on the realized short-run utility and on the highest possible value of short-run utility in the current state. We refer to the latter as the *temptation utility*.

It is important to note that in the dual-self model, there is a single long-run self with time-consistent preferences. The impulsive short-run selves are the source of self-control costs, but the equilibrium of the game between the long run self and the sequence of short run selves is equivalent to the optimization of a reduced-form control problem by the single long-run self. Even though the solution corresponds to that of a control problem, the cost of self-control can lead agents with dual-self preferences to choose actions that correspond to "costly self-commitment" in order to reduce the future cost of self-control. For example, they may pay a premium to invest in illiquid assets, as do the quasi-hyperbolic agents in Laibson [1997]. They may also choose to carry less cash than in the absence of self-control costs, as they do here.

Now we apply the dual self model to an infinite-lived consumer making a savings decision. Each period $t = 1,2,\ldots$ is divided into two sub-periods, the *bank* subperiod and the *nightclub* subperiod. The state $w \in \Re_+$ represents wealth at the beginning of the bank sub-period. During the "bank" subperiod, consumption is not possible, and wealth w_t is divided between savings s_t, which remains in the bank, cash x_t which is carried to the nightclub, and durable consumption c_t^d which is paid for immediately and is consumed in the second sub-period of period t.[5] Consumption is not possible in the bank, so the short-run self is indifferent between all possible choices, and the long-run self incurs no cost of self control. In the nightclub consumption $0 \leq c_t \leq x_t$ is determined, with $x_t - c_t$ returned to the bank at the end of the period. Wealth next period is just $w_{t+1} = R(s_t + x_t - c_t - c_t^d)$. No borrowing is possible, and there is no other source of income other than the return on investment.

So far, we have followed Fudenberg and Levine [2006]. Now we consider an extension of the model that we will need to explain the degree of risk aversion we observe in experimental data. Specifically, we suppose that there is a choice of nightclubs to go to in the nightclub sub-period; these choices are indexed by their quality of the nightclub $c^* \in (0,\infty)$. Intuitively, the quality represents a “target” level of consumption expenditure. For example, a low value of c^* may represent a nightclub that serves cheap beer, while a high value of c^* represents a nightclub that serves expensive wine. (Recall that c is the amount that is spent in the chosen nightclub.) In the beer bar c represents expenditure on cheap beer, while at the wine bar it represents the expenditure on expensive wine. Thus people with different income and so different planned consumption levels will choose consumption sites with different characteristics.

In a nightclub of quality c^* we assume that the base preference of the short-run self has the form $u(c \mid c^*)$, where $u(c \mid c) = \log c$; this ensures that in a deterministic and perfectly foreseen environment without self control costs, behavior is the same as with standard logarithmic preferences. We want to interpret c^* as the target consumption level, so we must also assume that $u(c \mid c^*) \leq u(c \mid c)$: This implies that when planning to consume a given amount c it is best to choose the nightclub of the same index if that nightclub is feasible. To avoid uninteresting approximation issues, we assume that there are a continuum of different kinds of nightclubs available, so that there are many intermediate choices between the beer bar and wine bar.

[5] Durable and/or committed consumption is a significant fraction (roughly 50%) of total consumption so we need to account for it in calibrating the model, but consumption commitments are not our focus here. For this reason we use a highly stylized model, with consumption commitments reset at the start of each time period. A more realistic model of durable consumption would have commitments that extend for multiple peiods, as in Grossman and Laroque [1990].

The assumption that $u(c \mid c^{*}) \leq u(c \mid c)$ captures the idea that consuming a great amount at a low quality nightclub results in less utility than consuming the same amount at a high quality nightclub: lots of cheap beer is not a good substitute for a nice bottle of wine. Conversely, consuming a small amount at a high quality night club results in less utility than consuming the same amount at a low quality nightclub: a couple of bottles of cheap beer are better than a thimble-full of nice wine. The level of the nightclub can also be interpreted as a state variable or capital stock that reflects experience with a given level of consumption: a wine lover who unexpectedly wins a large windfall may take a while to both to learn to appreciate differences in *grands crus* and to learn which ones are the best values.[6]

There are a great many possible function forms with this property; our choice of a specification is guided both by analytic convenience and by evidence (examined below) that short-term risk preferences seem more risk averse that consistent with the logarithmic specification even when self-control costs are taken into account. This leads us to adopt the following functional form:

$$u(c \mid c*) = \log c * - \frac{(c / c*)^{1-\rho} - 1}{\rho - 1}.$$

Note that $u(c, c) = \log(c)$, and

$$\frac{\partial u(c \mid c^{*})}{\partial c^{*}} = \frac{1}{c^{*}} - \left(\frac{c^{*}}{c}\right)^{\rho-2} \frac{1}{c}.$$

So the first order condition for maximizing $u(c, c*)$ with respect to $c*$ implies $c* = c$, and the second order condition is

$$\left.\frac{\partial^2 u(c \mid c*)}{\partial c^{*2}}\right|_{c=c^{*}} = \frac{-1}{c^2} - (\rho - 2)\frac{1}{c^2} = \frac{1}{c^2}(1 - \rho),$$

which is negative when $\rho > 1$.

The next step is to specify the agent's preferences for durable versus non-durable consumption. Our goal here is simply to account for the fact that durable consumption

[6] To fully match the model, this state variable needs to reflect only recent experience: a formerly wealthy wine lover who has been drinking vin de table for many years may take a while to reacquire both a discerning palate and up-to-date knowledge of the wine market.

exists, and not to explain it, so we adopt a simple Cobb-Douglas-like specification $u(c,c^d \mid c^*) = \tau u(c \mid c^*) + (1-\tau)\log c^d$; this will lead to a constant share τ of spending on durables.

Now that we have specified the base preferences, the next step is to compute the temptations and the cost of self control. In the nightclub the short-run self cannot borrow, and wishes to spend all of the available cash x_t on consumption. The durable good consumption is purchased in the bank, well in advance of consuming it, so this consumption is not subject to temptation. In general, the cost of self-control can depend on the maximum (temptation) utility attainable for the short-run self, $\overline{u}$, the actual realized utility, u, and the cognitive load due to other activities, d; we denote this cost as $g(d+\overline{u}-u)$ and suppose that the function g is continuously differentiable and convex. For most of the paper we suppose that there is no cognitive load from other activities, and set $d_t = 0$; Section 7 discusses the impact of cognitive load on risk preferences. In our calibrations of the model, we will take the cost function to be quadratic: $g(u) = \gamma_0 u + (1/2)\Gamma u^2$.

The long-run self uses discount factor δ, so the reduced form preferences, including self control costs, for the long-run self are

$$U_{RF} = \sum_{t=1}^{\infty} \delta^{t-1}\left[\tau\left(u(c_t \mid c_t^*) - g(u(x_t \mid c_t^*) - u(c_t \mid c_t^*)\right) + (1-\tau)\log c_t^d\right]. \quad (2.2)$$

Because there is no cost of self-control in the bank, the solution to this problem is to choose $c_t^* = c_t = x_t = (1-\delta)\tau w_t$, and $c_t^d = (1-\tau)(1-\delta)w_t$. In other words, cash x_t is chosen to equal the optimal consumption for an agent without self-control costs, and c_t^* is the nightclub of the same quality. The agent then spends all pocket cash at the nightclub, and so incurs no self-control cost there. The utility of the long-run self is

$$U_1(w_1) = \frac{\log(w_1)}{1-\delta} + K,$$

where

$$K = \frac{1}{1-\delta}\left[\log(1-\delta) + \delta\log(R\delta) + \tau\log\tau + (1-\tau)\log(1-\tau)\right].$$

For details (in the special case of $\tau = 1$) see Fudenberg and Levine [2006].[7]

3. Risky Drinking: Nightclubs and Lotteries

Suppose in period 1 (only) that when the agent arrives at the nightclub of her choice, she has the choice between two lotteries, A and B with returns $\tilde{z}_1^A, \tilde{z}_1^B$.We will consider both the situation in which this choice is completely unanticipated – that is, its prior probability is zero – and the case in which it has prior probability one. In the former case $c_t^* = x_t = (1-\delta)\tau w_t$. In the latter case both the amount of pocket cash x_1 and the choice of nightclub c_1^* will be chosen in anticipation of the availability of the lottery. Regardless, given $x\ ,c^*$, the choice of lottery and amount to spend in the nightclub will have to be chosen optimally. So we start by solving the problem of optimal choice of lottery given $x\ ,c^*$. For simplicity – and without much loss of generality[8] – we assume throughout that following the end of period 1 no further lottery opportunities at night clubs are anticipated.

The lotteries $\tilde{z}^A, \tilde{z}^B$ may involves gains or losses, but we suppose that the largest possible loss is less than the agent's pocket cash. There are number of different ways that the dual-self model can be applied to this setting, depending on the timing and "temptingness" of the choice of lottery and spending of its proceeds. In this section, we restrict attention to the following basic model; we explore some alternative specifications in the concluding section.

In the basic model, the short-run player in the nightclub simultaneously decides which lottery to pick and how to spend for each possible realization of the lottery. Since the highest possible short-run utility comes from consuming the entire outcome of the lottery, the temptation utility is calculated as $\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\}$ where $\tilde{z}_1^j$ is the realization of lottery $j = A, B$. This temptation is compared to the expected short-run utility from the chosen lottery. Thus if we let $\tilde{c}_1^j(z_1^j)$ be the consumption chosen contingent on the realization of lottery *j*, the self-control cost is

$$\overline{g}(x_1, \tilde{c}_1^j, c_1^*) = g\Big(\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\} - Eu(\tilde{c}_1^j, c_1^*)\Big)$$

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[7] Note that equation (1) of that paper contains a typographical error: in place of $(1+\gamma)(\log(1-a) + \log y_0)$ it should read $(1+\gamma)\log(1-a) + \log(y_0)$.

[8] The overall savings and utility decision will not change significantly provided that the probability of getting lottery opportunities – whether anticipated or not – at the nightclub are small.

where this is to be interpreted as meaning that $\overline{g}$ is a function of the *distribution* of $\tilde{c}_1^j$ (as well as on the current nightclub).

Consider then random unanticipated income $\tilde{z}_1^j$ at the nightclub. If z_1 is the realized income, the short-run self is constrained to consume $c_1 \leq x_1 + z_1$. Period 2 wealth is given by

$$w_2 = R(s_1 + x_1 + z_1 - c_1 - c_1^d) = R(w_1 + z_1 - c_1 - c_1^d) \ .$$

The utility of the long-run self starting in period 2 is given by the solution of the problem without self control, that is:

$$U_2(w_2) = \frac{\log(w_2)}{1-\delta} + K.$$

Let $\tilde{c}_1$ be the optimal response to the unanticipated income $\tilde{z}_1$. This is a random variable measurable with respect to $\tilde{z}_1$. The overall objective of the long-run self is to maximize

$$\tau\left(Eu(\tilde{c}_1^j, c_1^*) - \overline{g}(x_1, \tilde{c}_1^j, c_1^*)\right) + \frac{\delta}{(1-\delta)} E \log(w_1 + \tilde{z}_1^j - \tilde{c}_1^j - c_1^d) + K \ . \quad (3.1)$$

Let $\overline{u}(x_1, c_1^*) = \max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\}$ denote the maximum possible utility given c_1^* and the pair of lotteries A,B. We then have that

$$\begin{aligned} &Eu(\tilde{c}_1^j, c_1^*) - \overline{g}(x_1, \tilde{c}_1^j, c_1^*) = \\ &Eu(\tilde{c}_1^j, c_1^*) - g\left(\overline{u}(x_1, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\right), \end{aligned}$$

and since $\overline{u}$ does not depend on $\tilde{c}_1^j$, the optimal level of consumption can be determined for each lottery realization by pointwise maximization of (3.1) with respect to $c_1 = c_1^j(z_1^j)$.

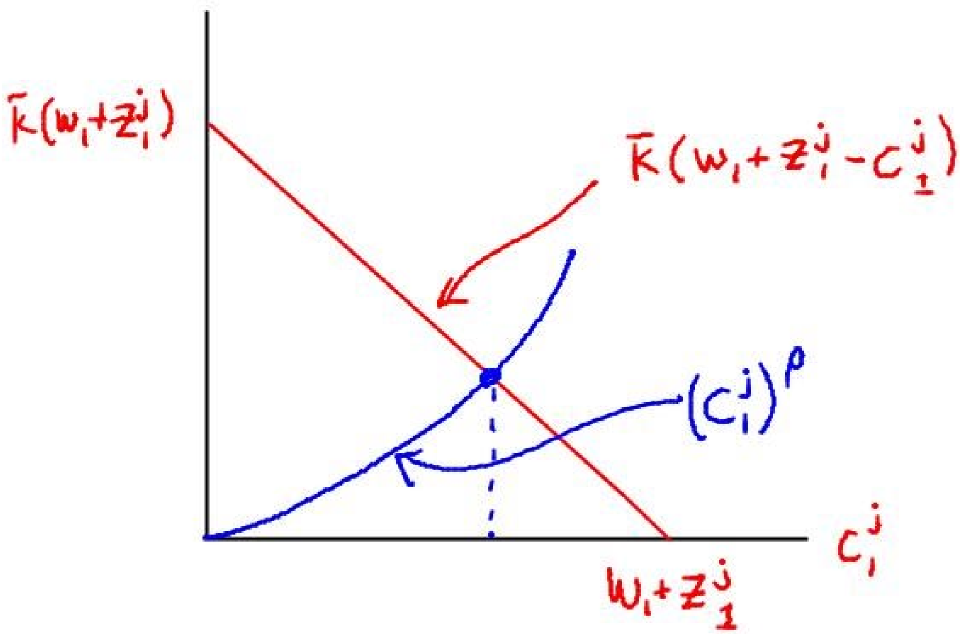
Define

$$\gamma = g'\left(\overline{u}(x_1, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\right) = g'\left(\overline{u}(x_1, c_1^*) - \sum_{z_1^j} \Pr(z_1^j) u(z_1^j, c_1^*)\right). \quad (3.2)$$

In Appendix 1 we show that the objective function is globally concave with respect to c_1^j, and so that the unique maximum is given by the solution to the first order condition, which may be written as

$$\begin{aligned}(c_1^j)^\rho &= \left(c_1^*\right)^{\rho-1}\frac{\tau(1-\delta)(1+\gamma)}{\delta}\left(w_1+z_1^j-c_1^j-c_1^d\right) \\ &= \bar{K}\left(w_1+z_1^j-c_1^j-c_1^d\right)\end{aligned}. \tag{3.3}$$

From Diagram 1 we can see both the uniqueness of the solution of the first order condition, and also see that the solution is increasing in $\bar{K}$, that is, decreasing in δ and increasing in γ, and that the solution is increasing in $w_1 + z_1^j$.



The solution to the first order condition (3.3) defines the optimum provided the constraint $c_1^j \leq x_1 + z_1^j$ is satisfied, otherwise the optimum is to spend all available cash $c_1^j = x_1 + z_1^j$. Substituting this equality into the derivative (3.3) and equating to zero, we find that there is a unique value $\hat{z}$ where spending all cash satisfies the first order condition

$$
\begin{aligned}
&(x_1 + z_1^j)^\rho = \\
&\qquad (c_1^j)^\rho = \left(c_1^*\right)^{\rho-1} \frac{\tau(1-\delta)(1+\gamma)}{\delta}\left(w_1 + z_1^j - c_1^j - c_1^d\right) \\
&\qquad\qquad = \left(c_1^*\right)^{\rho-1} \frac{\tau(1-\delta)(1+\gamma)}{\delta}\left(w_1 - x_1 - c_1^d\right)
\end{aligned}
$$

Notice also that no self control is used below $\hat{z}$ so the relevant marginal cost of self-control at the cutpoint is just the coefficient γ_0 on the linear part of the self-control cost: $g(u) = \gamma_0 u + (1/2)\Gamma u^2$. We conclude that

$$
\hat{z} = \left(c_1^*\right)^{\frac{\rho-1}{\rho}} \left[\frac{1-\delta}{\delta}\tau(1+\gamma_0)\left[w_1 - x_1 - c_1^d\right]\right]^{1/\rho} - x_1. \tag{3.3}
$$

Note that for arbitrary x_1, c_1^* we may have $\hat{z}$ negative.

To conclude our analysis of the optimal choice of c_1^j, we observe that when $z_1^j < \hat{z}$ it is optimal to spend all cash (so the cash constraint is binding) while when $z_1^j > \hat{z}$ the optimum is given by (3.3). To see this, it suffices to show that at the left end point when $x_1^j + z_1^j = 0$ the constraint binds. But from the diagram, we see that the solution to (3.3) is always strictly positive – when $x_1^j + z_1^j = 0$ this violates feasibility, so the constraint binds.

Finally, in the first order condition (3.3) the marginal cost of self-control γ is endogenous. To solve the first order condition numerically, we define $\hat{c}_1^j(\gamma)(z_1^j)$ to be the unique solution of (3.3) for a given value of γ. Then define

$$
\begin{aligned}
&\hat{\gamma}^j(\gamma) = \\
&g'(\max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\} - Eu(\min\{\hat{c}_1^j(\gamma)(\tilde{z}_1^j), x_1 + \tilde{z}_1^j\}, c_1^*))
\end{aligned} \tag{3.4}
$$

We show in the Appendix that we can characterize the optimum as follows:

Theorem 1: For given (x_1, c_1^*) and each $j \in \{A, B\}$ there is a unique solution to

$$
\gamma^j = \hat{\gamma}^j(\gamma^j)
$$

and this solution together with $\tilde{c}_1^j = \min(\hat{c}_1^j(\gamma^j)(z_1^J), x_1 + z_1^j\}$ *and the choice of* j *that maximizes (3.1) is necessary and sufficient for an optimal solution to the consumer's choice between lotteries A and B.*

The "consumption function" is $\tilde{c}_1^j = \min(\hat{c}_1^j(\gamma^j)(z_1^j), x_1 + z_1^j\}$. This is sketched in Diagram 2. For $z_1^j < \hat{z}$ no self-control is used, and all winnings are spent. Above this level self control is used, with only a fraction of winnings consumed, and the rest going to savings.

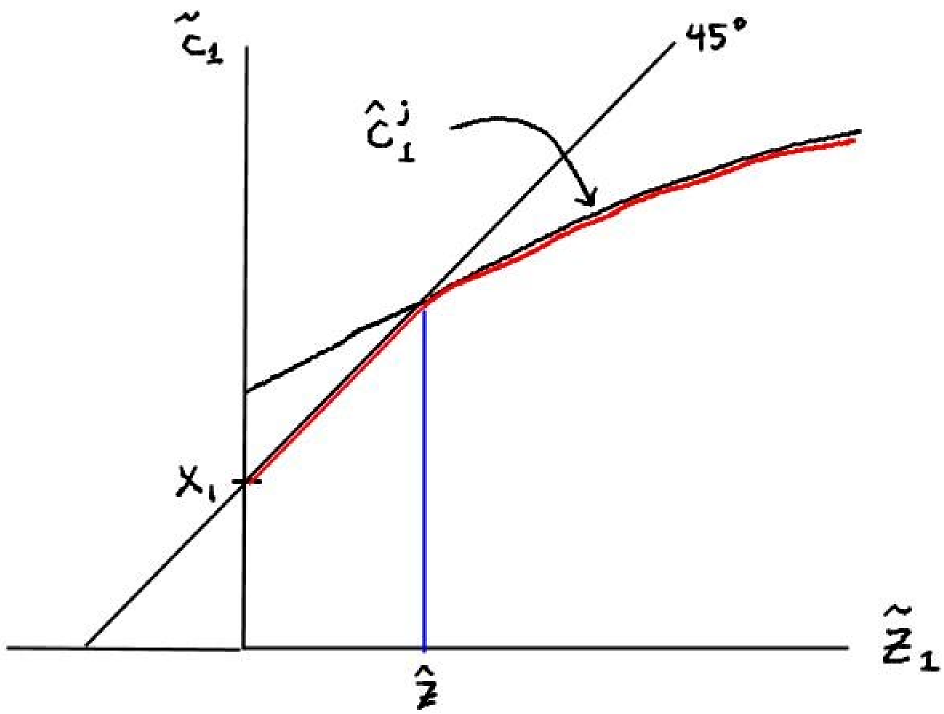


Diagram 2

When the agent exercises self-control and saves, some of the variation in income is being spread over future time periods, which results in a smaller change in marginal utility. The slope of (3.3) is

$$\frac{dc_1^j}{dz_1^j} = \frac{\bar{K}}{\rho(c_1^j)^{\rho-1} + \bar{K}},$$

so its solution in a neighborhood of $z = \hat{z}$ can be approximated by the line

$$
\begin{aligned}
c_1^j &= x_1 + \hat{z} + \frac{\overline{K}}{\rho(x_1 + \hat{z})^{\rho-1} + \overline{K}}(z_1^j - \hat{z}) \\
&= x_1 + \hat{z} + \frac{\tau(1-\delta)(1+\gamma)}{\delta\rho\left(\dfrac{x_1 + \hat{z}}{c_1^*}\right)^{\rho-1} + \tau(1-\delta)(1+\gamma)}(z_1^j - \hat{z})
\end{aligned}
$$

This approximation is generally quite good, since wealth is generally very large relative to pocket cash. For example, in one of our typical calibrations, $\rho = 1.3, \delta = 0.99992, \gamma = 17.66, w_1 = 859{,}000$ we have $x_1 = 40.2, c_1^* = 40.2, \hat{z} = 342$, and for $z_1^j \leq \hat{z} + 10x_1$ the approximation error in computing c_1^j is zero to five significant digits.[9] If the time periods are short, so that $1-\delta$ is very small, the line is very flat, so that only a tiny fraction of the winnings are consumed immediately when receipts exceed the critical level. Thus when the agent is patient he is almost risk neutral with respect to large gambles. However the agent is still risk averse to small gambles, as these will not be smoothed but will lead to a one for one change in current consumption.

4. Basic Calibration

The first step in our calibration of the model is to pin down as many parameters as possible using estimates from external sources of data. We will subsequently use data from laboratory experiments to calibrate risk aversion parameters and to determine the cost of self control.

From the Department of Commerce Bureau of Economic Analysis, real per capita disposable personal income in December 2005 was $27,640. To consider a range of income classes, we will use three levels of income $14,000, $28,000, and $56,000.

To figure consumption from the data in a way minimally consistent with the model, we do not use the currently exceptionally low savings rates, but the higher historical rate of 8% (see FSRB [2002]). This enables us to determine consumption from income. Wealth is simply income y divided by our estimate of the subjective interest rate r.

In determining pocket cash, we need to adjust the model to take into account the fact that there are expenditures that that are not subject to temptation: housing, durables,

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[9] For numerical robustness we used this approximation in all the simulations.

and medical expense. At the nightclub, the rent or mortgage was already paid for at the bank, and it is not generally feasible to sell one's car or refrigerator to pay for one's impulsive consumption. As noted by Grossman and Laroque [1990], such consumption commitments increase risk aversion for cash gambles.[10]

Next we examine durable consumption c^d. Turning to the data, we use the National Income and Product Accounts from the fourth quarter of 2005. In billions of current dollars, personal consumption expenditure was $8,927.8. Of this $1,019.6 was spent on durables, $1,326.6 on housing, and $1,534.0 on medical care, which are the non-tempting categories. This means that $\tau = 0.57$.

The subjective interest rate, r, should be the real market rate, less the growth rate of per capita consumption. From Shiller [1989], we see that over a more than 100 year period the average growth rate of per capita consumption has been 1.8%, the average real rate of returns on bonds 1.9%, and the real rate of return on equity 7.5%. Depending on whether we use the rate of return on bonds or on stocks, this gives a range of 0.1% to 5.7% for the subjective interest rate, although the bottom of the range seems implausible. In our assessments we will use a range of three values: 1%, 3%, and 5%. We should note that while we do not focus on the equity premium here, our model of commitment to "nightclubs" is similar to existing explanations of the equity premium puzzle. In particular, if we assume that once a nightclub is chosen it is locked in for a period of roughly a year, then because self-control does not matter for long-run portfolio balancing in a deterministic environment, our model as applied to the problem of allocating a portfolio between stocks and bonds is essentially the same as that of Gabaix and Laibson [2001], which is a simplified version of Grossman and Laroque [1990].[11] They are able to fit equity premium data using a subjective interest rate of 1%, the bottom of our range.

Finally, we must determine the time horizon Δ of the short-run self. This is hard to pin down accurately, in part because it seems to vary both within and across subjects,

[10] Chetty and Szeidl [2006] extend Grossman and Laroque to allow for varying sizes of gambles and costly revision of the commitment consumption. Postelwaite, Samuelson and Silberman [2006] investigate the implications of consumption commitments for optimal incentive contracts.

[11] They assume that once the nightclub is chosen, no other level of consumption is possible. We allow deviations from the nightclub level of consumption – but with very sharp curvature, so in practice consumers are "nearly locked in" to their choice of nightclub. Chetty and Szeidl [2006] show that these models of sticky consumption lead to the same observational results as the habit formation models used by Constantinides [1990] and Boldrin, Christiano and Fisher [2001].

but the most plausible period seems to be about a day. However, since pocket cash is set equal to the per-period desired consumption, which is $(1 - r / \Delta)\tau y / r$, using a one-day horizon implies implausibly low levels of pocket cash, about \$84 for a person with \$56K of income. This is very low compared to the daily limit on teller machines, and in addition most people do not go to the bank every day. So we will analyze both a daily horizon and a weekly one.

Putting together all these cases, we find for subjective interest rates, wealth and pocket cash[12]

Percent interest			14K Income			28K Income			56K Income		
r	Day	Wk	Wlth	Day	Wk	Wlth	Day	Wk	Wlth	Day	Wk
1	.003	.020	1.3M	20	141	2.6M	40	282	5.2M	80	563
3	.008	.058	.43M			.86M			1.7M		
5	.014	.098	.30M			.61M			1.2M		

To determine a reasonable range of self control costs, we need to find how the marginal propensity to consume "tempting" goods changes with unanticipated income. The easiest way to parameterize this is with the "self-control threshold" – the level of consumption at which self-control kicks in. From (3.3) the consumption cutoff between the high MPC of 1.0 and the low MPC of order $\tau(1 - \delta)$ is given by

$$\begin{aligned}\hat{c} &= (x_1)^{\frac{\rho-1}{\rho}}\left[\frac{\tau(1-\delta)}{\delta}(1+\gamma_0)[w_2]\right]^{1/\rho} \\ &\approx x_1(1+\gamma_0)^{1/\rho}\end{aligned}$$

where we use the facts that $w_2 \approx w_1 = x_1 / \tau(1 - \delta)$, and that $\delta \approx 1$.

Let y be annual income, and define $\mu(\gamma) = (x_1 / y)(1 + \gamma)^{1/\rho}$.

Then

$$\hat{c} / y \approx (x_1 / y)(1 + \gamma_0)^{1/\rho} = \mu(\gamma_0).$$

[12] The daily interest rate is defined as the annual rate divided by 365; the weekly rate is the daily rate times 7. Wealth (Wlth) is the marginal propensity to consume out of income of 92% times annual income divided by the annual interest rate. Daily (Day) and weekly (Wk) pocket cash are wealth times the temptation factor of 0.57 times the daily or weekly interest rate.

Because γ is measured in units of utility, its numerical value is hard to interpret. For this reason we will report $\mu(\gamma)$ rather than γ.

We can also relate μ to consumption data. Abdel-Ghany et al [1983] examined the marginal propensity to consume semi- and non-durables out of windfalls in 1972-3 CES data.[13] In the CES, the relevant category is defined as "inheritances and occasional large gifts of money from persons outside the family...and net receipts from the settlement of fire and accident policies," which they argue are unanticipated. For windfalls that are less than 10% of total income, they find an MPC of 0.94. For windfalls that are more than 10% of total income they find and MPC of 0.02. Since the reason for the 10% cutoff is not completely clear from the paper, we will take this as a general indication of the cutoff, rather than as an absolutely reliable figure; according to Abdel-Ghany et al this should be about 10%.

5. Small Stakes Risk Aversion

To demonstrate how the model works and calibrate the basic underlying model of risk preference, we start with the "Rabin Paradox": the small-stakes risk aversion observed in experiments implies implausibly large risk aversion for large gambles[14]. Rabin proposes that a gamble with equal probability of losing \$4,000 winning \$635,670 should be accepted. Such a gamble could not possibly be for pocket cash and if is for income at the bank it would certainly be accepted.

The more central issue is the case of small gambles. Following Rabin's proposal let option A be $(.5:-100,.5:105)$, while option B is to get nothing for sure. Here the optimum is to choose option B as Rabin predicts.

Since the combination of pocket cash and the maximum winning is well below our estimates of $\hat{c}$, (even after allowing for commitment to durable consumption) this means that all income is spent, and the consumer simply behaves as a risk-averse individual with wealth equal to pocket cash and a coefficient of relative risk aversion of

[13] The Imbens, Rubin and Sacerdote [2001] study of consumption response to unanticipated lottery winnings shows that big winners earn less after they win, which is useful for evaluating the impact of winnings on labor supply. Their data is hard to use for assessing μ, because lottery winnings are paid as an annuity and are not lump sum, so that winning reduces the need to hold other financial assets. It also appears as though the lottery winners are drawn from a different pool than the non-winners since they earn a lot less before the lottery.

[14] Rabin thus expands on an earlier observation of Samuelson [1963] .

ρ. Let us treat pocket cash as an unknown for the moment, and ask how large could pocket cash be given that a logarithmic consumer is willing to reject such a gamble. That is, we solve $.5\log(x_1 - 100) + .5(x_1 + 105) = \ln(x_1)$ for pocket cash; for larger values of x_1 the consumer will accept the gamble, and for smaller ones he will reject. The indifference point is x_1=\$2100, which is considerably more than any plausible estimate of pocket cash. In this sense, as Fudenberg and Levine [2006] argue, short-run logarithmic preferences are consistent with the Rabin paradox.[15]

The problem with this analysis is that the gamble $(.5 : -100, .5 : 105)$ has comparatively large stakes. Laboratory evidence shows that subjects will reject considerably smaller gambles, which is harder to explain with short-run logarithmic preferences. We use data from Holt and Laury [2002], who did a careful laboratory study of risk aversion. Their subjects were given a list of ten choices between an A and a B lottery. The specific lotteries are shown below, where the first four columns show the probabilities of the rewards.

Option A		Option B		Fraction Choosing A			
\$2.00	\$1.60	\$3.85	\$0.10	1X	20X	50X	90X
0.1	0.9	0.1	0.9	1.0	1.0	1.0	1.0
0.2	0.8	0.2	0.8	1.0	1.0	1.0	1.0
0.3	0.7	0.3	0.7	.95	.95	1.0	1.0
0.4	0.6	0.4	0.6	.85	.90	1.0	1.0
0.5	0.5	0.5	0.5	.70	.85	1.0	.90
0.6	0.4	0.6	0.4	.45	.65	.85	.85
0.7	0.3	0.7	0.3	.20	.40	.60	.65
0.8	0.2	0.8	0.2	.05	.20	.25	.45
0.9	0.1	0.9	0.1	.02	.05	.15	.40
1.0	0.0	1.0	0.0	.00	.00	.00	.00

[15] Note that this theory predicts that if payoffs are delayed sufficiently, risk aversion will be much lower. Experiments reported in Barberis, Huang and Thaler [2003] suggest that there is appreciable risk aversion for gambles where the resolution of the uncertainty is delayed as well as the payoffs themselves. However, delayed gambles are subject to exactly the same self-control problem as regular ones, so this is consistent with our theory. In fact the number of subjects accepting the risky choice in the delayed gamble was in fact considerably higher than the non-delayed gamble, rising from 10% to 22%.

Initially subjects were told that one of the ten rows would be picked at random and they would be paid the amount shown. Then they were given the option of renouncing their payment and participating in a high stakes lottery – depending on the treatment, for either 20X, 50X or 90X of the original stakes. The high stakes lottery was otherwise the same as the original: a choice was made for each of the ten rows, and one picked at random for the actual payment. Everyone in fact renounced their winnings from the first round to participate in the second. The choices made by subjects are shown in the table above.

In the table we have highlighted (in yellow and turquoise respectively) the decision problems where roughly half and 85% of the subjects chose A. We will take these as characterizing median and high risk aversion respectively. The bottom 15th percentile exhibits little risk aversion, suggesting that perhaps they do not face much in the way of a self-control problem.

Since the stakes plus pocket cash remain well below our estimate of $\hat{c}$, we can fit a CES with respect to our pocket cash estimates of \$21, \$42, \$84, \$155, \$310 and \$620, in each case estimating the value of ρ that would leave a consumer indifferent to the given gamble – assuming the chosen nightclub is equal to pocket cash. Taking the CES functional form measured in units of marginal utility of income, we have for utility

$$-x_1 \frac{(c/x_1)^{1-\rho} - 1}{\rho - 1}.$$

We can then compute the utility gain from option A for each of the highlighted gambles for each value of x_1. The theory says this should be zero. We estimate the ρ's corresponding to the median and 85th percentile choices by minimizing the squared sum of these utility gains pooled across all of the gambles in the relevant cells. The results are shown in the next table.

		Pocket Cash x_1					
		\$20	\$40	\$80	\$141	\$282	\$563
ρ median		1.06	1.3	1.8	2.4	3.8	6.5
ρ 85th		2.1	2.8	4.3	6.3	12	22

For each of the estimated risk preferences and each scale of gamble (1X, 20X, 50X, 90X) there is a unique probability of reward that makes the individual indifferent between option A and option B. In the Table below, in the "actual" column we report the probability from the data. For example, in the 1X treatment, when the probability of reward is .60, 45% of the individual choose A and 55% choose B (which we took as an indication that the median individual is roughly indifferent.) Along with these calibrated indifference probabilities, we report for each estimated risk aversion parameter, the theoretical probability that would make an individual indifferent between option A and option B.

		Pocket Cash					
	Actual	\$20	\$40	\$80	\$141	\$282	\$563
ρ median		1.06	1.3	1.8	2.4	3.8	6.5
1X	.60	.47	.46	.46	.46	.45	.45
20X	.70	.65	.62	.60	.59	.57	.56
50X	.70	.72	.71	.71	.71	.70	.69
90X	.80	.79	.79	.81	.82	.83	.83
ρ 85th		2.1	2.8	4.3	6.3	12	22
1X	.70	.50	.48	.47	.47	.47	.47
20X	.80	.81	.79	.78	.78	.79	.78
50X	.90	.90	.91	.94	.94	.96	.96

Notice that even with the very lowest estimates of pocket cash it is impossible to explain even the median degree of risk aversion with logarithmic short-run utility. Notice also that the CES tends to slightly understate the degree of risk aversion in the 1X treatments.[16]

[16] There is some issue over whether the size of the choices might have been confounded with the order in which the choices were given. Harrison, Johnson, McInnes and Rutstrom [2005] find that corrected for order the impact of the size of the gamble is somewhat less than Holt and Laurie found, a point which Holt and Laurie [2005] concedes is correct. However, for us the scale data does not help us in the estimation, it is simply an additional fact that we must explain, and in fact our model predicts less scaling than in the original data. The follow on studies which focus on the order effects do not contain sufficient data for us to get the risk aversion estimates we need.

Note finally that our data does not let us separately identify pocket cash and risk aversion; various combinations of these two are observationally equivalent

Note that consumption commitments alone are not sufficient to explain this data with logarithmic utility. One response would be to replace the logarithmic specification with another more risk averse functions for both long-run and unanticipated short run consumption, but this would cause difficulties in explaining long-term savings. Instead, we will use our expanded model, which allows short term risk preferences to be CES even when long-term preferences are logarithmic.

6. The Allais Paradox

We proceed next to examine the Allais paradox in the calibrated model. We assume that the choice in this (thought) experiment is completely unanticipated. In this case the solution is simple: there is no self-control problem at the bank, so the choices is $c_1^* = x_1$ and spend all the pocket cash in the nightclub of choice. Given this, the problem is purely logarithmic, so the solution is to choose $x_1 = (1-\delta)w_1$. (We discuss the qualitative features of anticipating the choice in section 8.

In the Kahneman and Tversky [1979] version of the Allais Paradox option A_1 is $(.01:0,.66:2400,.33:2500)$, B_1 is 2400 for certain, and many people choose option B_1. Next we consider the pair of choices where the choices are $A_2 = (.33:0,.34:2400,.33:2500)$ and option $B_2 = (.32:0,.68:2400)$.[17] Here many people choose A_1.

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[17] These were thought experiments; we are unaware of data from real experiments where subjects are paid over $2000, though experiments with similar "real stakes" are sometimes conducted poor countries. There is experimental data on the Allais paradox with real, but much smaller, stakes, most notably Battalio, Kagel and Jiranyakul [1990]. Even for these very small stakes, subjects did exhibit the Allais paradox, and even the reverse Allais paradox. The theory here cannot explain the Allais paradox over such small amounts, as to exhibit the paradox, the prizes must be in the region of the threshold $\mu(\gamma^*)$, while the prizes in these experiments ranged from $0.12 to $18.00, far out of this range. However, indifference or near indifference may be a key factor in the reported results. In set 1 and set 2 the two lotteries have exactly the same expected value, and the difference between the large and small prize is at most $8.00, and there was only one chance in fifteen that the decision would actually be implemented. So it is easy to imagine that subjects did not invest too much time and effort into these decisions. By way of contrast Harrison [1994] found that with various small stakes the Allais paradox was sensitive to using real rather than hypothetical payoffs, and found in the real payoff case only 15% of the population exhibited the paradox. Although Colin Camerer pointed out the drop from 35% when payoffs were hypothetical was not statistically significant, a follow study by Burke, Carter, Gominiak and Ohl [1996] found a statistically significant drop from 36% to 8%. Conlisk [1989] also finds little evidence of an Allais paradox when the stakes are small. He examines payoffs on the order of $10, much less than our threshold values of $\mu(\gamma^*)$ of roughly 1% of annual

To describe the procedure we will use for reporting calibrations concerning choices between pairs of gambles, let us examine in some detail the choice A_1, B_1 in the base case where the annual interest rate $r = 3\%$, annual income is \$28,000, wealth is \$860,000, the short-run self's horizon is a single day, so pocket cash and the chosen nightclub are $x_1 = c_1^* = 40$.

First, with linear cost of self-control, the curvature parameter *b* is equal to 0, and the marginal costs of self control from (3.4) that correspond to the two different choices are $\gamma^A = \gamma^B = \gamma_0$. We can solve for the numerically unique value γ^* ($\mu(\gamma^*) = 1.36$) such that there is indifference between the two gambles A and B.[18] Next, suppose that $\Gamma > 0$. Suppose we have solved the optimization problem as described by Theorem 1. Let $\overline{u}_1$ be the optimal first period utility. As the cost of self control is quadratic, we have

$$\begin{aligned} \gamma^A &= \gamma_0 + \Gamma\left(\overline{u}_1 - Eu(\tilde{c}_1^A(\gamma^A))\right) \\ \gamma^B &= \gamma_0 + \Gamma\left(\overline{u}_1 - Eu(\tilde{c}_1^B(\gamma^B))\right) \end{aligned} \tag{6.1}$$

Our goal is to characterize the values of γ^A, γ^B for which it is optimal to choose A and B respectively. A numerical computation shows that $Eu(\tilde{c}_1^B(\gamma^*)) > Eu(\tilde{c}_1^A(\gamma^*))$, so that when the long-run self is indifferent, the short-run self prefers the sure outcome *B*. From our analysis of the linear case where $\Gamma = 0$ this implies that for small cost of self control $\gamma^A = \gamma^B < \gamma^*$ the optimal choice is *A*, and for $\gamma^A = \gamma^B > \gamma^*$ the optimal choice is *B*. To analyze the quadratic problem, where increasing marginal cost of self control requires $\Gamma > 0$, it is useful to invert (6.1) to find

$$\begin{aligned} \Gamma &= \frac{\gamma^A - \gamma^B}{Eu(\tilde{c}_1^B(\gamma^B)) - Eu(\tilde{c}_1^A(\gamma^A))} \\ \gamma_0 &= \gamma^A - \frac{\gamma^A - \gamma^B}{Eu(\tilde{c}_1^B(\gamma^B)) - Eu(\tilde{c}_1^A(\gamma^A))}\left(\overline{u}_1 - Eu(\tilde{c}_1^A(\gamma^A))\right) \end{aligned} . \tag{6.2}$$

income. These studies suggest that when played for small real stakes there is no Allais paradox, as our theory predicts.

[18] Notice that this value will be the same if we consider the second pair of choices: with linear self-control cost, the independence axiom is satisfied, and A and B are ranked the same way in both cases. Subsequently when we add some curvature the indifference will be broken, and, as we shall see, in opposite ways for the first and second pair of choices.

In other words, for any given values of γ^A, γ^B we can find corresponding values of γ_0, Γ. Of course it must be that the corresponding solution satisfies $\infty > \gamma_0, \Gamma \geq 0$. Let us refer to values of γ^A, γ^B as *feasible marginal costs of self-control*.

It turns out in the data, the constraint $\infty > \gamma_0, \Gamma \geq 0$ on feasible marginal costs is very tight. Since the marginal cost of self-control is increasing ($b \geq 0$) and as we observed above $Eu(\tilde{c}_1^B(\gamma^*)) > Eu(\tilde{c}_1^A(\gamma^*))$ we have $\gamma^B < \gamma^A$. However, numerically, the difference in first period utility between A and B is quite small. This means as the gap between γ^B and γ^{A} increases the corresponding value of Γ from (6.2) blows up to infinity quite rapidly. This is shown in the following table reporting for various values of γ^A and γ^B the value of Γ computed from (6.2).

$\mu(\gamma^A)(\%)$	$\mu(\gamma^B)(\%), \Gamma$				
1.22	1.21, 9.73	1.21, 25.8	1.20, 57.7	1.19, 152	1.18, 23,200
1.29	1.28, 9.51,	1.28, 24.3	1.27, 50.9	1.26, 112	1.26, 425
1.36	1.36, 9.33	1.35, 23.24	1.34, 46.3	1.34, 92.4	1.33, 231
1.44	1.43, 9.18	1.42, 22.36	1.42, 42.9	1.40, 79.8	1.40, 593
1.51	1.50, 9.06	1.50, 21.6	1.49, 40.4	1.48, 71.4	1.46, 16,000

In the green highlighted cells, where the marginal cost of self-control is low, the optimal choice is A, in the white cells, the optimal choice is B. To a good numerical approximation, the choice between A and B is determined by γ and is the same for any feasible specification of γ^B.[19] Roughly speaking, whatever is the value of Γ, if $\gamma^A > \gamma^*$ the optimal choice will be B, whereas when $\gamma^A < \gamma^*$ it will be A.

We next examine the relationship between the two pairs of gambles, pair 1 and pair 2. As we noted above, both share the same value of γ^*. Let T^{j_k} denote the temptation for decision problem k when the choice is j, that is $T^{j_k} = \overline{u}(x_1, x_1) - Eu(\tilde{c}_1^{j_k}, x_1)$, and let $\overline{\gamma}$ be a parameter explained below. The next table reports what happens when the cost of self-control is linear and the marginal cost of self-control is γ^*.

[19] For γ^A close enough to the indifference point $\mu(\gamma^*) = 1.36$, that is for $1.39 > \mu(\gamma^A) > 1.36$ this is not the case.

r	income	w	$x_1 = c_1^*$	ρ		$\mu(\gamma^*)(\%)$	T^{B_1}	T^{A_2}	$\mu(\overline{\gamma}) - \mu(\gamma^*)$ (%)
3%	28K	.86M	40	1.3	Day	1.37	.72	.51	0.41

The key to the Allais paradox lies in the fact that $T^{B_1} > T^{A_2}$. In other words, even when the less tempting alternative is chosen in problem 1, and the more tempting alternative in problem 2, the temptation is still greater in problem 1. This means that if we hold fixed the marginal utility $\gamma *$ of self control for alternative A in problem 2 while increasing b from zero (and so preserving indifference in that problem), in problem 1 the marginal cost of self-control must now be greater than $\gamma *$ due to the curvature. As we observed above, unless this increase in marginal cost is numerically trivial, it forces a switch to alternative B in problem 1. This reversal is of course exactly the Allais paradox.

More specifically, we can calculate a marginal utility for problem 1 of $\gamma[1] = \gamma_0 + \Gamma T^{B_1}$ and $\gamma[2] = \gamma_0 + \Gamma T^{A_2}$. There is an Allais paradox if and only if $\gamma[1] > \gamma^* > \gamma[2]$: The fact that temptations change as we vary $\gamma[i]$ does not matter, because they move in the correct direction. Evidently then, we must have $\gamma[2] = \gamma_0 + \Gamma T^{A_2} < \gamma *$ for an Allais paradox to occur. The larger is b the smaller must be γ_0, but of course $\gamma_0 \geq 0$. So we must have $\Gamma \leq \gamma * / T^{A_2}$. Conversely, if this inequality is satisfied, there are many values of γ_0 that yields the Allais paradox.

Since, like γ, the parameter Γ is not measured in particularly interesting units, we report this maximal value of Γ by reporting the implied value of marginal self-control for the high temptation T^{B_1}: this represent the greatest marginal cost of self-control in the range of the data. That is, we take $\Gamma = \gamma * / T^{A_2}$, and choose γ_0 so that $\gamma_0 + \Gamma T^{A_2} = \gamma *$, then compute $\gamma_0 + \Gamma T$. We call this parameter

$$\overline{\gamma} = \gamma * -(\gamma * / T^{A_2})T^{A_2} + (\gamma * / T^{A_2})T \quad = \gamma * (T \quad / T^{A_2}).$$

There are 36 different combinations of parameter values corresponding to our different calibrations. Appendix 2 reports a table of values similar to that above for each of the 36 cases. A useful way to summarize the information in that table is by using a regression to report the correlations between the parameters and the values of $\mu(\gamma^*)$. To give an idea of how close the relationship is to being linear, we observe that the $R^2 = 0.63$. Note that as far as the calibration is concerned, the fit is perfect: Because the preference reversals is consistent with convex costs of self control, we can find values of the unobserved parameter γ that give rise to the Allais paradox (or not) for each specification of the other parameters. Our focus therefore is not on whether we can fit the data, but rather whether the values of the unobserved parameter that explain it are sensible and stable across experiments.

	Coefficient
Constant	8.84
r	-0.0029
$\log(income)$	-0.74
ρ	-0.04
week dummy	0.94

To interpret the comparative statics implied by the regression, first note that we do not include pocket cash or wealth as independent variables, as they are computed from income and the interest rate. To a good approximation, our results are not sensitive to the interest rate, the coefficient for all intents being zero. The coefficient of relative risk aversion also player surprisingly little role: increase ρ by 10, which is a very large increase, would decrease $\mu(\gamma*)$ by only 0.4%. The week dummy has a greater impact of roughly 1% with a higher cost of self-control if we assume a weekly horizon than a daily horizon. Most interesting is income. Recall that μ is measured relative to income. Hence if income increases by 10%, the relative percent cutoff declines by 7.4%, but the absolute dollar cutoff increases by 2.6%. Hence if we fit the date with higher incomes, then higher marginal costs of self-control are consistent with the Allais paradox.

One important point to observe: the values of the self-control parameter $\mu(\gamma*)$ are in the range 0.52-3.2% – this is considerably smaller than the 10% figure that we found from consumption survey studies.

The original Allais paradox involved substantially higher stakes – and would be difficult for that reason to implement other than as a thought experiment. In the original paradox where option A_1 was $(.01 : 0, .89 : 1{,}000{,}000, .1 : 5{,}000{,}000)$ and B_1 was 1,000,000 for certain, the paradoxical choice is B_1. The second scenario was $A_2 = (.90 : 0, .10 : 5{,}000{,}000)$ and $B_2 = (.89 : 0, .11 : 1{,}000{,}000)$ with the paradoxical choice being A_1. With logarithmic long-run preferences, regardless of self-control costs, B_1 will never be chosen, so we cannot explain the original paradox with the parameters here. However, the assumption of logarithmic preferences with respect to prizes vastly in excess of wealth implausible. If we modify the utility function so that $u(5,000,000) = \log Y \approx 1{,}149{,}500$ then optimal choices are B_1 and A for similar self-control parameters to those explaining the lower stakes paradox

r	income	w	$x_1 = c_1^*$	ρ		$\mu(\gamma^*)(\%)$	T^{B_1}	T^{A_2}	$\mu(\overline{\gamma}) - \mu(\gamma^*)$ (%)
3%	28K	.86M	40	1.3	Day	1.58	1.06	.13	5.93

It should be emphasized that our explanation of the paradox requires near indifference in both scenarios. Modest changes in the utility function, that is Y, will result in the same choice in both scenarios, so will not exhibit the paradox. Note however, that the required "indifference" is likely to be easier to achieve for thought experiments than for actual ones.

7. Cognitive Load

We turn next to data from an experiment by Benjamin, Brown and Shapiro [2006] that shows the impact of cognitive load on risk preferences. The participants were Chilean high school juniors. These students made choices about uncertain outcomes both under normal circumstances and under the cognitive load of having to remember a seven digit number while responding. The key fact is that the students responded differently to choices involving increased risk when the level of cognitive load was changed.

Two experiments were conducted. In scenario one the choice was between a safe option of 250 pesos and a risky option in which there was a 50% chance of winning a prize of X and a 50% chance of getting nothing. In scenario two the alternative to the "X or 0" gamble was a 50-50 randomization between 200 and 300 pesos. The table below provided to us by the authors summarizes the fraction of subjects who choose the riskier option B as a function of X.

	versus sure alternative		versus 50-50 alternative	
"X"	No load	Cognitive Load	No Load	Cognitive Load
200	7% (1/15)	5%(1/22)	15%(2/13)	14%(3/22)
350	28%4/15)	36%(8/22)	0%(0/15)	9%(2/22)
500	43%(6/14)	41%(9/22)	29%(4/14)	32%(7/22)
650	70%(9/13)	24%(5/21)	73%(11/15)	68%(15/22)
800	77%(10/13)	38%(8/21)	87%(13/15)	86%(19/22)

These were real, and not hypothetical choices, the subjects were paid in cash at the end of the session. To provide some reference for these numbers, 1 $US= 625 pesos; the subjects average weekly allowance was around 10,000 pesos; from this they had to buy themselves lunch twice a week. [20]

The data is obviously noisy: some subjects are choosing the risky option even when its expected value is much less than that of the sure thing, so these subjects are either making a mistake or are risk preferring. Moreover, in the first scenario the number of people choosing the risky option actually drops as the prize increases. It is clear that our model cannot explain these things. Our focus, however, is on the 650 pesos row, where the risky alternative is better in expected value than the safe alternative, but not overwhelmingly so. Here, to a good approximation 70% of the population prefers the risky alternative, except when the safe alternative is completely safe and there is a high cognitive load, in which case only about a quarter of the population wishes to choose the risky alternative. Our goal is to show that this is predicted by our model for parameters consistent with explaining the Allais paradox. That is, our goal is to construct a set of preferences so that the risky alternative is chosen (100% of the time) except when the safe alternative is completely safe and there is a high cognitive load in which case the risky alternative is never chosen.

To calibrate the model, we take pocket cash to be 10000 pesos in the weekly case, or 1/7th that amount in the daily case, or about \$16.00 or \$2.29 respectively. We then work out wealth and income indirectly using the utility-function parameters that we

[20] Many of them buy lunch at McDonald's for 2000 pesos twice a week, leaving an apparent disposable income of 6000 pesos per week.

calibrated in the Allais experiments.[21] The range of calibrated parameters then is the annual interest rate of 1%, 3%, or 5%, the median or high degree of risk aversion, and the time horizon of daily or weekly for the short-run self. Key to any explanation is that the parameters must lead the two choices to have sufficiently similar levels of utility that a reversal is possible due to self-control. Within the calibrated range, the only set of parameters for which this is true is when the annual interest rate is 5%, risk aversion is at the lower median level, and when the horizon of the short-run self is daily. Note that the values of $\mu(\gamma)*$ of 3.543-3.550 needed to create indifference for the Chilean gambles lies in the range $\mu(\gamma^*) = 2.76$ to $\mu(\overline{\gamma}) = 3.82$ from the Allais paradox for the corresponding daily 5% calibration.[22]

The key facts about the relevant calibration in the Chilean case is summarized in the table below, where we report the values of $\mu(\gamma^*)$ that leads to indifference in the first and second scenario respectively.

r	income	w	$x_1 = c_1^*$	ρ		$\mu(\gamma^*)(\%)$ 1	$\mu(\gamma^*)(\%)$ 2
5%	1.6K	29K	2.29	1.06	Day	3.543	3.550

In both scenarios, the risky option has the greater temptation, meaning that it will be chosen only for low marginal cost of self-control or equivalently, low values of $\gamma*$. Recall that in our model the marginal cost of self-control is $\gamma_0 + b(d + \overline{u} - u)$ where d measures the cognitive load. Suppose that $\mu(\gamma_0) < 3.543$ and that b is not too large. Then when cognitive load $d = 0$ is low, marginal cost of self-control is low enough in both scenarios that the risky alternative will be chosen. On the other hand, when cognitive load is high so $d = \overline{d} > 0$, for an appropriate value of $\overline{d}$, we will a greater marginal cost of self-control $3.543 \leq \mu(\gamma) \leq 3.550$. This means that in the sure thing alternative (scenario 1) the marginal cost of self-control is "high" so that the safe alternative will be chosen, while in scenario 2 the marginal cost of self-control is "low" so that the risky alternative will continue to be chosen.

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[21] It is unclear that we should use the same value of τ but the results are not terribly sensitive to this.

8. Making the Evening's Plans: Pocket Cash and Choice of Club

Our base model supposes that that the choice between A and B is completely unanticipated. How does the optimal choice of nightclub c_1^* and pocket cash change if the decision maker realizes that she will face a gamble? Specifically, let π^G denote the probability of getting the gambles. Our assumption has been that $\pi^G = 0$. In this case the solution is simple: there is no self-control problem at the bank, so the choices is $c_1^* = x_1$ and spend all the pocket cash in the nightclub of choice. Given this, the problem is purely logarithmic, so the solution is to choose $x_1 = (1-\delta)w_1$.

To examine the robustness of our results, consider then the polar opposite case in which $\pi^G = 1$, that is, the agent knows for certain she will be offered the choice between A and B. We start by solving the problem conditional on a particular choice of gamble j under the assumption that the temptation will be k, that is

$$Eu(x_1 + \tilde{z}_1^k, c_1^*) = \max\{Eu(x_1 + \tilde{z}_1^A, c_1^*), Eu(x_1 + \tilde{z}_1^B, c_1^*)\}.$$

For each possible combination j,k we can solve for c_1^* and x_1, ignoring the constraint that $Eu(x_1 + \tilde{z}_1^k, c_1^*) \geq Eu(x_1 + \tilde{z}_1^{-k}, c_1^*)$. If for these values it is in fact true that $Eu(x_1 + \tilde{z}_1^k, c_1^*) \geq Eu(x_1 + \tilde{z}_1^{-k}, c_1^*)$, then $\{c_1^*,\ x_1\}$ is the candidate solution for j,k. If $Eu(x_1 + \tilde{z}_1^k, c_1^*) < Eu(x_1 + \tilde{z}_1^{-k}, c_1^*)$ then we need to re-solve for c_1^* and x_1, imposing the "temptation constraint" that $Eu(x_1 + \tilde{z}_1^k, c_1^*) = Eu(x_1 + \tilde{z}_1^{-k}, c_1^*)$; this is then the candidate solution corresponding to j,k. Finally, the solution to the overall maximization is the candidate solution with the highest overall utility. With our specification of the utility function, the temptation constraint simplifies to $E((x_1 + \tilde{z}_1^A)^{1-\rho}) = E((x_1 + \tilde{z}_1^B)^{1-\rho})$, which does not depend on c_1^*. Consequently, the first order condition for c_1^* will be valid regardless of whether or not the constraint is binding; if the constraint is binding, then the first order condition for x_1 must be replaced with the constraint.

Since we will derive qualitative results only, we will simplify to the case $\tau = 1$ for the remainder of the section. The objective function, given the choice of gamble of gamble j, is

$$Eu(\tilde{c}_1^j, c_1^*) - \overline{g}(x_1, \tilde{c}_1^j, c_1^*) + \frac{\delta}{(1-\delta)} E\log(w_1 + \tilde{z}_1^j - \tilde{c}_1^j) + K_2. \tag{8.1}$$

Consider first the optimal choice of nightclub c_1^*. In the Appendix we show that

Proposition 2: *The following first order condition is necessary for an optimum:*

$$c_1^* = \left((1+\gamma)E\left(\tilde{c}_1^j\right)^{1-\rho} - \gamma E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho}\right)^{1/(1-\rho)} \tag{8.2}$$

We can use this condition to examine how the optimal choice of nightclub c_1^* is determined. It is useful to rewrite (8.2) as

$$c_1^* = \left(E\left(\tilde{c}_1^j\right)^{1-\rho} + \gamma\left(E\left(\tilde{c}_1^j\right)^{1-\rho} - E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho}\right)\right)^{1/(1-\rho)}.$$

Suppose first that $\tilde{c}_1^j = \overline{c}$ a constant, that is, consumption has zero variance, and that the marginal cost of self control $\gamma = 0$; then $c_1^* = \overline{c}$. Now suppose that $\tilde{c}_1^j$ has non-zero variance with $\overline{c} = E\tilde{c}_1^j$. Notice that $\rho - 1$ is negative so that $(\bullet)^{1/(\rho-1)}$ is decreasing, and $(\bullet)^{\rho-1}$ is convex, so when the variance of consumption is positive, we have $E(\tilde{c}_1^j)^{1-\rho} > \overline{c}^{1-\rho}$. This implies that higher variance of consumption leads to smaller values of c_1^*. Intuitively, with concave utility, losses are more important than gains. To hedge against increased variance of consumption, it is optimal to lower the cost of losses by going to a slightly less good nightclub.

Next, consider the role of the cost of self-control. Since $\tilde{c}_1^j \le x_1 + \tilde{z}_1^j$ and utility is proportional to the negative of $c^{\rho-1}$

$$\begin{aligned}
&E\left(\tilde{c}_1^j\right)^{1-\rho} - E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho} = \\
&E\left(\tilde{c}_1^j\right)^{1-\rho} - \min\{E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho}, E\left(x_1 + \tilde{z}_1^j\right)^{1-\rho}\} \ge \\
&E\left(\tilde{c}_1^j\right)^{1-\rho} - E\left(x_1 + \tilde{z}_1^j\right)^{1-\rho} \ge 0
\end{aligned}$$

This implies that c_1^* decreases as the marginal cost of self-control increases. This makes sense also: some of the cost of temptation is avoided if a lesser nightclub is selected. Of course, since the solution for c_1^* is a smooth function, “small” variance of consumption and “small” cost of self-control implies that c_1^* while smaller than $\overline{c}$ will be “close” to it.

Now we turn to the optimal choice of x_1. In the Appendix we show that

Proposition 3: *The necessary first order condition determining x_1 is*

$$(1+\gamma)\left(c_1^*\right)^{\rho-1}\sum\nolimits_{z_1^j<\hat{z}}(x_1+z_1^j)^{-\rho}\,pr(z_1^j)-\frac{\delta}{1-\delta}\frac{pr(z_1^j<\hat{z})}{w_1-x_1} \\ -\gamma\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^k)^{-\rho}\leq 0 \tag{8.3}$$

with equality if $x_1>0$.

Notice that while it is necessary that the optimal x_1 must satisfy this first order condition, we cannot prove that it has a unique solution. Since the remaining first order conditions for $c_1^*, \tilde{c}_1^j$ can be uniquely solved for given x_1 computationally we adopt the strategy of searching over x_1 to find the optimal solution.

One important case is the one in which the gamble is small in the sense that $z_1^j<\hat{z}$ for all positive probability outcomes, so that all available cash is spent, that is, $c_1^j=x_1+z_1^j$. In this case, the first order condition simplifies to

$$\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^j)^{-\rho}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}=0\,. \tag{8.4}$$

From the formula (8.4) for c_1^* we see there are two subcases of the case $z_1^j<\hat{z}$: If $j=k$, then

$$c_1^*=\left((1+\gamma)E\left(\tilde{c}_1^j\right)^{1-\rho}-\gamma E\left(x_1+\tilde{z}_1^k\right)^{1-\rho}\right)^{1/(1-\rho}=\left(E\left(x_1+\tilde{z}_1^k\right)^{1-\rho}\right)^{1/(1-\rho)}$$

and so (8.4) simplifies to

$$\frac{E(x_1+\tilde{z}_1^j)^{-\rho}}{E(x_1+\tilde{z}_1^j)^{1-\rho}}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}=0\,. \tag{8.5}$$

If $j\neq k$ then

$$c_1^*=\left((1+\gamma)E\left(x_1+\tilde{z}_1^j\right)^{1-\rho}-\gamma E\left(x_1+\tilde{z}_1^k\right)^{1-\rho}\right),$$

and substitution into (8.4) yields

$$\frac{E(x_1+\tilde{z}_1^j)^{-\rho}}{\left((1+\gamma)E\left(x_1+\tilde{z}_1^j\right)^{1-\rho}-\gamma E\left(x_1+\tilde{z}_1^k\right)^{1-\rho}\right)^{1-\rho}}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}=0$$

In one important special case, it is easy to solve for x_1, c_1^*. Suppose that the lotteries $\tilde{z}_1^A, \tilde{z}_1^B$ are both constant and equal to $\overline{z}$. If $\overline{z}\geq\hat{z}$ then (8.4) is negative: the "lottery" will pay out so much that there is no need to carry any pocket cash. Conversely, if $x_1 > 0$ then we must have $\overline{z} < \hat{z}$. This implies immediately that $\tilde{c}_1^j = x_1 + \overline{z}$, so we may plug into (8.3) to find $c_1^* = x_1 + \overline{z}$. In addition, since the temptation is the same for both lotteries, (8.5) applies and may be written as

$$\frac{1}{x_1 + \overline{z}} - \frac{\delta}{1-\delta}\frac{1}{w_1 - x_1} = 0\,.$$

This gives the usual logarithmic solution $x_1 = (1-\delta)w_1 - \delta\overline{z}$. That is, the optimal pocket cash simply is adjusted down by expected earnings.

9. Alternative Models

Our analysis of the Allais and Rabin paradox is based on a particular model of the timing with which decisions are made. An important aspect of the dual-self theory is that timing and the scope of the short-run self are crucial in explaining behavior. It is useful at this point to consider some other models of timing and behavior of the short-run self – and examine how robust the conclusions concerning the Allais and Rabin paradoxes are. For simplicity, we continue to deal only with the case $\tau = 1$, and we simplify by suppressing the choice of nightclub.

Choice and Realization of Lottery Separate: There are two short-run players. The first chooses the lottery A or B, while the second takes over from the first before the lottery is realized. In this case, the first short-run player is indifferent as he does not receive a short-run payoff, so the decision over lotteries is made without self-control cost by the long-run self. If lottery *j* is chosen, then the second short-run player creates a temptation $E\log(x_1 + \tilde{z}_1^j)$. The self-control cost is then

$$g(E\log(x_1 + \tilde{z}_1^j) - E\log\left[\tilde{c}_1^j\right])$$

Because we think of the time period between the choice of lottery and its realization as being fairly short, in most situations it does not make much sense that the division of horizon between the first and the second short-run player should fall in between. The one case where this timing might make sense is if (a) the resolution of the lottery occurs with some delay, say a week, and (b) the potential winnings and their probability are large enough that the agent starts planning how they might be spent before the lottery resolves. We include this primarily to illustrate the range of modeling possibilities.

Lottery and Expenditure Separate: There are two short-run players. The first short-run player chooses the lottery A or B and observes the outcome, the second takes over from the first after the lottery is realized to collect the money and make the consumption decision. In this case the long-run player again chooses the lottery without facing self-control costs. After the lottery realizes z_1^j, the short-run player faces a temptation of $\log(x_1 + z_1^j)$. The expected cost of self-control associated with a given lottery is then simply the expected cost associated with the induced distribution of consumption, which is

$$\overline{g} = Eg\left(\log(x_1 + \tilde{z}_1^j) - \log \tilde{c}_1\right)$$

and the overall objective function is

$$E\left\{\log(\tilde{c}_1) - g(\log\left[x_1 + \tilde{z}_1^j\right] - \log \tilde{c}_1) + \frac{\delta}{(1-\delta)}\log(w_1 + \tilde{z}_1 - \tilde{c}_1)\right\} + K$$

Because $\tilde{c}_1$ depends only on the realization of $\tilde{z}_1^j$, this objective function is linear in the probability distribution over outcomes, and is thus an expected utility theory, so it cannot explain the Allais paradox.

Sophisticated Short-Run Player: This model has the same timing as the previous case, but now the first short-run player cares about the consumption received by the second short-run player. This seems like a plausible model, but it creates the complication that the expectations of the first "short-run" player about what will happen in the second part of the period matters.

In our setting of "infrequent" events that cause pocket cash to change, this is not so much of an issue. But as soon as we move out of this world, such a model becomes badly behaved. For example, the long-run player might be able to coerce the first short-run player by threatening to allow no consumption if the lottery the long-run player prefers is not chosen, and by doing so avoid any self-control costs. In this infinite setting, if similar situations might arise in the future this might well be a subgame perfect equilibrium of the game between long-run and short-run selves, where the threat is made credible by the prospect of increased future self-control costs if it is not carried out. On the other hand, there will generally be equilibria as well in which there are not such threats, and self-control is costly.

Discussion: While the notion that each short-run self cares only about the "moment" and does not overlap with other short-run selves is a simplification that one might want to relax, assuming that the short-run selves think explicitly about the consequences of actions for the future does not do justice to the basic myopic, reactive notion of a short-run impulsive self. An alternative modeling strategy is to have the short-run self care about the future not through explicit forward looking behavior, but rather by rote stimulus-response learning. In this model, the short-run self learns "cues" from the past and has given preferences. The strategic interaction between long-run and short-run self now comes about because the long-run self may manipulate the learning of the short-run self, but individual behavior can still be described as the solution to an optimization problem, not a game with possibly many expectations driven equilibria.

10. Conclusion

We have argued that a simple self-control model with quadratic cost of self-control and logarithmic preferences can account quantitatively for both the Rabin and Allais paradoxes. We have argued also that the same model can account for risky decision making of Chilean high school students faced with differing cognitive loads.

We find it remarkable that we can explain the data on the behavior of Chilean high school students with essentially the same parameters that explain the Allais paradox (for a variety of populations). We should therefore emphasize emphasize that there are indeed possible observations that are not consistent with the theory. For example, cognitive load in the Chilean experiment could have caused preferences to switch in the reverse, "anti-Allais," direction, which we would not be able to explain. Also while we have allowed ourselves some flexibility in the parameters we use to explain the data, it is important that all the parameters we use fall within a "plausible" range. It could easily be, for example, that the self-control costs needed to provide a quantitative explanation of the

Allais paradox led to a 90% propensity to consume out of unanticipated gains of \$1,000,000. The main anomaly we find is with respect to the degree of self-control. The model predicts a threshold level of unanticipated income below which the marginal propensity to consume is 100% and above that is extremely low. There is some permanent consumption data that indicates that this may be true, and that the threshold is about 10% of annual income. We find, however, that to explain the paradoxes and data we consider, the threshold must be in the range of 0.58-3.2% – that is considerably smaller than in household consumption surveys.

The existing model most widely used to explain a variety of paradoxes, including the Allais paradox, is prospect theory, which involves an endogenous reference point that is not explained within the theory.[23] In a sense, the dual-self theory here is similar to prospect theory in that it has a reference point, although in our theory the reference point is a particular value, pocket cash. Pocket cash is in principle is observable, and can be manipulated by experimental design. The theories are also quite different in a number of respects. Prospect theory makes relatively *ad hoc* departure from the axioms of expected utility, while our departure is explained by underlying decision costs. Our theory violates the independence of irrelevant alternatives, with choices dependent on the menu from which choices are made, while prospect theory satisfies independence of irrelevant alternatives. Our theory can address issues such as the role of cognitive load and explains intertemporal paradoxes such as the hyperbolic discounting phenomenon and the Rabin paradox about which prospect theory is silent. Finally, a primary goal of our theory is to have a self-contained theory of intertemporal decision making; by way of contrast, it is not transparent how to embed prospect theory into an intertemporal model.[24]

In the other direction, prospect theory allows for individuals who are simultaneously risk averse in the gain domain and risk loving over losses. This is done in part through the use of different value functions in the gain and loss domains, and in part through its use of a probability weighting function, which can individuals to overweight rare events.[25] Most work on prospect theory has estimated a representative-agent model;

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[23] See Kozegi and Rabin [2006] for one way to make the reference point endogenous, and Gul and Pesendorfer [2007] for a critique.

[24] Kozegi and Rabin [2007] develop but do not calibrate a dynamic model of reference dependent choice.

[25] See Prelec [1998] for an axiomatic characterization of several probability weighting functions, and a discussion of their properties and implications.

Bruhin, Fehr-Duda, and Epper [2007] refine this approach by classifying individuals as expected utility maximizing or as prospect theory types,[26] and find that most individuals are prospect theory types. It is interesting to note that given the functional forms they estimate, individuals with expected utility preferences are assumed to be risk averse throughout the gains domain, while in their data individuals are risk loving for small probabilities of winning, while for higher probability of success they are risk averse. This can be explained within the expected utility paradigm by means of a Savage-style S-shaped utility function that is risk loving for small increases in income and risk averse for larger increases.[27]

While S-shaped utility can explain risk seeking for small chances of gain and risk aversion for larger chances, it does not explain the Allais paradox, while prospect theory can potentially do so. But it appears that the parameters needed to explain individuals who are simultaneously risk averse and risk loving cannot at the same time explain the Allais paradox. Neilson conducts a systematic examination of the parameters needed to fit prospect theory to various empirical facts, and concludes that

> parameterizations based on experimental results tend to be too extreme in their implications. The preference function estimated by Tversky and Kahneman (1992) implies an acceptable amount of risk seeking over unlikely gains and risk aversion over unlikely losses, but can accommodate neither the strongest choice patterns from Battalio, Kagel, and Jiranyakul (1990) nor the Allais paradox, and implies some rather large risk premia. The preference functions estimated by Camerer and Ho (1994) and Wu and Gonzalez (1996) imply virtually no risk seeking over unlikely gains and virtually no risk aversion over unlikely losses, so that individuals will purchase neither lottery tickets nor insurance…. We show that there are no parameter combinations that allow for both the desired

[26] Their estimation procedure tests for and rejects the presence of additional types.

[27] Notice that it is possible to embed such short-run player preferences in our model although we have focused on the risk averse case. Indeed, such preferences are consistent even with long-run risk aversion: the envelope of S-shaped short-term utility functions can be concave provided that there is a kink between gains and losses, with strictly higher marginal utility in the loss domain. There is evidence that this is the case.

> gambling/insurance behavior and a series of choices made by a strong majority of subjects and reasonable risk premia. So, while the proposed functional forms might fit the experimental data well, they have poor out-of-sample performance.

Neilson's survey examines the original Allais paradox holding relative risk aversion constant, which as we have already noted is quite difficult because with expected utility individuals are not near indifference with reasonable degrees of risk aversion. However, if we use the Bruhin, Fehr-Duda, and Epper [2007] estimates from the Zurich 03 gains- domain treatment, the prospect theory types have preferences give by

$$U = \sum_i \frac{.846 p_i^{.414}}{846 p_i^{.414} + (1-p_i)^{.414}} x_i^{1.056}$$

where p_i is the probability of winning the prize x_i.[28] In the Kahnemann and Tversky version of the Allais paradox, recall that A_1 is $(.01:0,.66:2400,.33:2500)$, and B_1 is 2400 for certain. This gives $U(A_1)=3874.58$ and $U(B_1)=3711$. In other words, an individual with these preferences would prefer A_1 to B_1 and so would not exhibit an Allais paradox.

Our overall summary, then, is that the dual-self model explains choices over lotteries about as well as prospect theory, while explaining phenomena such as commitment and cognitive load that prospect theory cannot. Moreover, the dual-self model is a fully dynamic model of intertemporal choice that is consistent with both traditional models of savings (long-run logarithmic preferences) and with the equity premium puzzle.[29]

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[28] Bruhin, Fehr-Duda, and Epper [2007] specify a utility function only for two outcome gambles, this seems the natural extension to the three or more outcomes demanded to explain the Allais paradox. Note also that this utility function has the highly unlikely global property that is we fix the probabilities of the outcomes it exhibits strict risk loving behavior.

[29] The "behavioral life cycle model" of Shefrin and Thaler [1988] can also explain many qualitative features of observed savings behavior, and pocket cash in our model plays a role similar to that of "mental accounts" in theirs. The behavioral life cycle model takes the accounts as completely exogenous, and does not provide an explanation for preferences over lotteries. It does seem plausible to us that some forms of mental accounting do occur as a way of simplifying choice problems. In our view this ought to be derived

In conclusion, there is no reason to think that the dual-self model has yet arrived at its best form, but its success in providing a unified explanation for a wide range of phenomena suggests that it should be viewed as a natural starting point for attempts to explain other sorts of departures from the predictions of the standard model of consumer choice.

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Appendix 1

Here we prove the results from the text.

Theorem 1*:* (a) *For given* (x_1, c_1^*) *and each* $j \in \{A, B\}$ *there is a unique solution to*

$$\gamma^j = \hat{\gamma}^j(\gamma^j).$$

This solution together with $\tilde{c}_1^j = \min(\hat{c}_1^j(\gamma^j)(z_1^J), x_1 + z_1^j\}$ *and the choice of* j *that maximizes (3.1) is necessary and sufficient for an optimal solution.*

Proof:

First we show that the first order conditions corresponding to optimal consumption for a given choice *j* have a unique solution. Observe that

$$\frac{d\gamma}{dc_1^j(z_1^j)} = -\Pr(z_1^j)u'(c_1^j(z_1^j), c_1^*)g''\left(\overline{u}(x_1, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\right) \le 0.$$

Because

$$\frac{\partial u(c_1^j, c_1^*)}{\partial c} = (c_1^j)^{-\rho}\left(c_1^*\right)^{\rho-1},$$

the derivative of (3.1) with respect to $c_1^j = c_1^j(z_1^j)$ evaluated at z_1^j as

$$\tau(1+\gamma)(c_1^*)^{\rho-1}(c_1^j)^{-\rho} - \frac{\delta}{(1-\delta)}\frac{1}{w_1 + z_1^j - c_1^j - c_1^d} \qquad \text{(A.1)}$$

From this we can compute the second derivative

$$\tau(c_1^*)^{\rho-1}(c_1^j)^{-\rho}\frac{d\gamma}{dc_1^j} - \tau\rho(1+\gamma)\left(c_1^*\right)^{\rho-1}(c_1^j)^{-\rho-1} - \frac{\delta}{(1-\delta)}\frac{1}{\left(w_1 + z_1^j - c_1^j - c_1^d\right)^2} < 0$$

implying that the function is globally concave.

We now show that the conditions in the Theorem are necessary and sufficient for an optimum. Examine necessity first. Suppose that an optimum exists. Once we know the choice j, for any given consumption plan in j the marginal cost of self control γ is defined by 3.2, and the optimal consumption plan must satisfy the first order condition with respect to that γ because our conditions preclude a boundary solution. That is, (3.4) must hold.

Next we show sufficiency. Suppose we have j, γ^j satisfy the conditions of the theorem and that this is not the optimum. Since the problem is one of maximizing a continuous function over a compact space, an optimum exists. That optimum must yield more utility in (3.1) than choosing $-j$ and any consumption plan in $-j$, so the unique consumption plan that comes from solving $\gamma^{-j} = \hat{\gamma}^{-j}(\gamma^{-j})$. Given that j is chosen, the optimal consumption is the unique solution of the first order condition. On the other hand, if $-j$ was chosen, we could do no better than the consumption plan defined by $\gamma^{-j} = \hat{\gamma}^{-j}(\gamma^{-j})$, and by assumption this is not as good as choosing j.

☑

Proposition 2: *The following first order condition is necessary for an optimum:*

$$c_1^* = \left((1+\gamma)E\left(\tilde{c}_1^j\right)^{1-\rho} - \gamma E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho}\right)^{1/(1-\rho)} \tag{8.2}$$

Proof: Only the first current utility terms in (8.2) depends on c_1^*, and we can rewrite these terms as follows

$$\begin{aligned}
&Eu(\tilde{c}_1^j, c_1^*) - \overline{g}(x_1, \tilde{c}_1^j, c_1^*) = \\
&Eu(\tilde{c}_1^j, c_1^*) - g\left(\overline{u}(x_1, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\right) = \\
&Eu(\tilde{c}_1^j, c_1^*) - g\left(Eu(x_1 + \tilde{z}_1^k, c_1^*) - Eu(\tilde{c}_1^j, c_1^*)\right)
\end{aligned}$$

Observe that

$$\frac{\partial u(c, c^*)}{\partial c^*} = \frac{1}{c^*} - (c^*)^{\rho-2} c^{1-\rho}.$$

The first order condition for maximizing (8.1) with respect to c_1^* is

$$\begin{aligned}
&\frac{1}{c_1^*} - (c_1^*)^{\rho-2} E\left(\tilde{c}_1^j\right)^{1-\rho} - \gamma\left(-(c_1^*)^{\rho-2} E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho} + (c_1^*)^{\rho-2} E\left(\tilde{c}_1^j\right)^{1-\rho}\right) \\
&= \frac{1}{c_1^*} - (1+\gamma)(c_1^*)^{\rho-2} E\left(\tilde{c}_1^j\right)^{1-\rho} + \gamma(c_1^*)^{\rho-2} E\left(x_1 + \tilde{z}_1^k\right)^{1-\rho} = 0
\end{aligned}$$

which solves to give the result of the Proposition.

☑

Proposition 3: *The necessary first order condition determining* x_1 *is*

$$(1+\gamma)\left(c_1^*\right)^{\rho-1}\sum\nolimits_{z_1^j<\hat{z}}(x_1+z_1^j)^{-\rho}\,pr(z_1^j)-\frac{\delta}{1-\delta}\frac{pr(z_1^j<\hat{z})}{w_1-x_1}$$
$$-\gamma\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^k)^{-\rho}\leq 0 \tag{8.3}$$

with equality if $x_1>0$.

Proof: As we observed above, if the temptation constraint is binding x_1 is determined from $E((x_1+\tilde{z}_1^A)^{1-\rho})=E((x_1+\tilde{z}_1^B)^{1-\rho})$. So we solve the case where the temptation constraint is not binding. When we differentiate the utility function

$$Eu(\tilde{c}_1^j,c_1^*)-g\left(Eu(x_1+\tilde{z}_1^k,c_1^*)-Eu(\tilde{c}_1^j,c_1^*)\right)+\frac{\delta}{(1-\delta)}E\log(w_1+\tilde{z}_1^j-\tilde{c}_1^j)+K$$

with respect to x_1, we get the first order condition

$$(1+\gamma)\left(c_1^*\right)^{\rho-1}E(\tilde{c}_1^j)^{-\rho}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}-\gamma\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^k)^{-\rho}=$$
$$\sum\nolimits_{z_1^j}\left[(1+\gamma)\left(c_1^*\right)^{\rho-1}E(\tilde{c}_1^j)^{-\rho}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}\right]pr(z_1^j)$$
$$-\gamma\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^k)^{-\rho}=$$
$$(1+\gamma)\left(c_1^*\right)^{\rho-1}\sum\nolimits_{z_1^j<\hat{z}}(x_1+z_1^j)^{-\rho}\,pr(z_1^j)-\frac{\delta}{1-\delta}\frac{pr(z_1^j<\hat{z})}{w_1-x_1}$$
$$-\gamma\left(c_1^*\right)^{\rho-1}E(x_1+\tilde{z}_1^k)^{-\rho}\leq 0$$

as stated in the Proposition. Here we make use of the fact that for $z_1^j\geq\overline{z}$

$$(1+\gamma)\left(c_1^*\right)^{\rho-1}E(\tilde{c}_1^j)^{-\rho}-\frac{\delta}{1-\delta}\frac{1}{w_1-x_1}=0$$

by the first order condition defining the optimal choice of c_1^j.

☑

Appendix 2: Allais Paradox Parameters

r	income	w	$x = c*$	ρ	weekly 1=true 0=false	$\mu(\gamma^*)(\%)$	T^{B_1}	T^{A_2}	$\mu(\overline{\gamma}) - \mu(\gamma^*)$ (%)
1%	14000	1288000	20	1.06	0	2.79	1.44	1.01	1.07
1%	14000	1288000	141	2.40	1	3.10	0.14	0.10	0.42
1%	14000	1288000	20	2.10	0	1.03	0.10	0.08	0.14
1%	14000	1288000	141	6.30	1	1.81	0.01	0.01	0.03
3%	14000	429330	20	1.06	0	2.77	1.44	1.01	1.07
3%	14000	429330	141	2.40	1	3.09	0.13	0.10	0.42
3%	14000	429330	20	2.10	0	1.03	0.10	0.08	0.14
3%	14000	429330	141	6.30	1	1.80	0.01	0.01	0.03
5%	14000	257600	20	1.06	0	2.76	1.44	1.01	1.06
5%	14000	257600	141	2.40	1	3.08	0.14	0.10	0.42
5%	14000	257600	20	2.10	0	1.03	0.10	0.08	0.14
5%	14000	257600	141	6.30	1	1.80	0.01	0.01	0.03
1%	28000	2576000	40	1.30	0	1.38	0.72	0.51	0.41
1%	28000	2576000	282	3.80	1	2.00	0.05	0.04	0.15
1%	28000	2576000	40	2.80	0	0.58	0.04	0.04	0.05
1%	28000	2576000	282	12.00	1	1.37	0.00	0.00	0.00
3%	28000	858670	40	1.30	0	1.37	0.72	0.51	0.41
3%	28000	858670	282	3.80	1	2.00	0.05	0.04	0.15
3%	28000	858670	40	2.80	0	0.58	0.04	0.04	0.05
3%	28000	858670	282	12.00	1	1.37	0.00	0.00	0.00
5%	28000	515200	40	1.30	0	1.37	0.72	0.51	0.40
5%	28000	515200	282	3.80	1	2.00	0.05	0.04	0.15
5%	28000	515200	40	2.80	0	0.58	0.04	0.04	0.05
5%	28000	515200	282	12.00	1	1.37	0.00	0.00	0.00
1%	56000	5152000	80	1.80	0	0.69	0.28	0.20	0.13

1%	56000	5152000	563	6.50	1	1.49	0.02	0.02	0.06
1%	56000	5152000	80	4.20	0	0.35	0.02	0.02	0.01
1%	56000	5152000	563	22.00	1	1.19	0.00	0.00	0.00
3%	56000	1717300	80	1.80	0	0.69	0.28	0.20	0.13
3%	56000	1717300	563	6.50	1	1.49	0.02	0.02	0.06
3%	56000	1717300	80	4.20	0	0.35	0.02	0.02	0.01
3%	56000	1717300	563	22.00	1	1.19	0.00	0.00	0.00
5%	56000	1030400	80	1.80	0	0.69	0.28	0.20	0.13
5%	56000	1030400	563	6.50	1	1.49	0.02	0.02	0.06
5%	56000	1030400	80	4.20	0	0.35	0.02	0.02	0.01
5%	56000	1030400	563	22.00	1	1.19	0.00	0.00	0.00