Expropriation Risk and Technology

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Abstract

This paper characterizes the set of self-enforcing contracts in an environment where a firm has access to a profitable investment project but faces a government who can confiscate all investment proceeds without incurring legal sanctions. Investment is only sustainable if the government's production technology is sufficiently inferior. Stationary payoffs to the government are a non-monotonic function of its relative production technology. While the reduced incentive problem associated with technological incompetence increases investment efficiency, the lower threat point limits the share of the surplus obtained by the government. The model can generate backloaded or frontloaded transfers to the government and makes predictions about which dynamic pattern emerges. Markov-type discount rate shocks of the government generate expropriation on the equilibrium path with low technology-intensive sectors at the top of the pecking order. Firms are able to mitigate the government's incentive to expropriate via non-horizontal integration. The model predictions are consistent with observed contracts between sovereign countries and foreign direct investors. Special emphasis is given to production sharing agreements, the most common contract form in the oil industry.

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1 INTRODUCTION

"The Government can promise you whatever it wants - it is not binding."
(Bernard Mommer, Venezuela's deputy oil minister)

The recent history of expropriations reveals that this statement is not inconsequential political rhetoric, but rather an accurate description of reality. Venezuela took over the oil projects of ConocoPhilips in spring 2007, resulting in a $4.5bn profit charge. At the same time, foreign oil groups led by Eni of Italy are facing a clash with the government of Kazakhstan over the original contract terms for the development of the Kashagan field in the Caspian sea. Almost provocatively, Kazakhstan's parliament recently passed a law to allow the government to cancel or change retrospectively contracts perceived to harm the national economic interest. This selection of events reveals a general message. If a government breaches a contract – often officially justified by allegations such as tax fraud or environmental violations – the affected party cannot rely on an external punishment mechanism to enforce the written contract.[1] Due to the mere threat of expropriation, firms may not pursue otherwise profitable investment opportunities ex ante. These investment distortions are difficult to measure, but represent a large fraction of the economic cost of expropriation risk.

The lack of legal enforceability raises two related questions: Why do profit maximizing firms invest? What economic rationale prevents a government from expropriating all the time? This paper argues that the comparative technological advantage of the firm mitigates the risk of expropriation. In my model, expropriation risk emerges from a hold-up problem: The government is able to seize all output after the firm invests. This results in high current period cash flows to the government at the expense of lower future income. Driven by capital constraints or political economy reasons, the government is effectively impatient relative to the firm, thus increasing the short-run incentive to expropriate. Within an infinite-horizon repeated game, mutually beneficial agreements between the firm and the government can be sustained through the threat of autarky. The effectiveness of this threat becomes stronger with decreasing relative technological ability of the government.

If firm investment is feasible, any self-enforcing contract specifying the path of tax payments and investment converges to a unique stationary allocation. In the steady state the competing forces of efficiency and incentives are offset. Efficiency requires the impatient government to receive frontloaded transfers. In contrast, backloaded transfers help mitigate the incentive problem of the government. The initial division of the surplus determines the dynamic pattern to reach the steady state.[2] Backloading occurs if the initial surplus division provides the firm with a higher value than in the steady state. Frontloading occurs if the initial surplus division provides the government with a higher value than in the steady state. In the latter case, the firm effectively lends to

[1] For example, Russia's takeover of the Sakhalin-2 oil and gas project managed by Shell in 2006 was officially attributed to environmental concerns.
[2] In Section 4.5, I consider various mechanisms of how this initial split up is determined.

the government. The maximum upfront payment to the government is determined by the break-even condition of the firm and can be interpreted as an endogenous borrowing constraint of the government. This result is related to the seminal paper on sovereign debt by Bulow and Rogoff (1989) who find that sovereign debt cannot be sustained without sanctions. In my setup, the productivity advantage of the firm provides it with an effective sanction mechanism.

The steady state has interesting comparative statics. The government's stationary payoffs are a non-monotonic function of its relative ability. At low levels of relative ability, an improvement in the government's ability allows it to extract a larger fraction of the surplus. However, at some critical level, the short-run temptation of the government to expropriate becomes so strong that firm participation can only be sustained at the cost of underinvestment. These distortions reduce total surplus and thus transfers to the government. If the government's relative ability is sufficiently high, firm investment is no longer sustainable and the project must be operated with the government's second-best technology.

I explore various economic applications of the model. First, I consider the implications of large upfront cost, such as the cost to build a plant. Specifically, I discuss the nature of the oil industry, which is characterized by a costly exploration phase and an ex post profitable extraction phase. Firms are only willing to incur large upfront cost if they expect to receive a high initial share of the ex post surplus relative to the steady state. Thus, the model predicts that significant upfront cost should be associated with backloaded contracts. The application to the oil industry suggests a rationale for vertical integration of exploration and extraction as a means to relax the incentive problem of the government. The productivity advantage in the extraction phase secures the required surplus for the oil firm to cover the cost of the potentially unsuccessful exploration phase.

Secondly, I analyze the implications of a multi-sector economy in the spirit of Bernheim and Whinston (1990). If firms coordinate across sectors on joint punishment, their effective threat point becomes stronger. The sustainability of such a linkage equilibrium is more likely when formal ties between sectors are prevalent, such as with conglomerate structures. This provides a rationale for firms to engage in non-horizontal integration. Without formal ties, an uncoordinated equilibrium in which each firm punishes separately is more plausible.

Thirdly, the model allows for expropriations on the equilibrium path through discount rate shocks. These shocks can be interpreted as regime changes or liquidity shocks. I show that the contract dynamics are unaffected by expropriation events. Cross-sectional and time-series predictions about expropriations depend on whether firms coordinate in equilibrium. The uncoordinated equilibrium features expropriations of firms according to a pecking order that is determined by the technology intensity of the sector. This pecking order also implies that expropriation and privatization cycles adhere to a "Last-In-First-Out" principle. Variations of the strict pecking order could exist in the linkage equilibrium depending on which sectors are linked.

I find empirical support for my model with regards to contract dynamics and expropriation events. Recent anecdotal evidence from Bolivia also highlights the guiding theme of my model: Relative technological inability reduces the risk of expropriation. The announced expropriation of the gas sector upon Morales' election victory had to be abandoned in 2006 due to lack of local expertise.[3] Observable features of contracts with sovereign countries show the empirical relevance of frontloading and backloading. I analyze the contract dynamics associated with select investments of multinational automobile firms in Eastern Europe and reveal these to be consistent with the predictions of my model. Moreover, I find empirical support in the features of production sharing agreements in the natural resources sector.[4] Kobrin's comprehensive collection of expropriation acts in emerging market countries lists 563 acts between 1960 and 1979 (Kobrin, 1980, 1984). Consistent with the predictions of my model, Kobrin finds that firms in less technology-intensive sectors (such as extraction or utilities), where the private sector's technological advantage is presumably smaller, face higher expropriation risk than manufacturing or trade firms. Using the same dataset, Li (2006) documents that expropriations are positively related to high government turnover. Political economy theory suggests that higher government turnover results in an effective reduction of the discount factor (see Aguiar et al., 2007, and Amador, 2003), an event that can trigger expropriation in my model.

My paper is organized as follows. Section 2 provides a summary of the related literature. Section 3 presents the economic setup of my model. The formal analysis is presented in Section 4. This section derives conditions for sustainable firm investment and describes the short-run and long-run properties of efficient self-enforcing contracts. In Section 5, I reveal that the results are robust to various technical extensions. Section 6 discusses the main applications of my model and provides empirical evidence. Section 7 concludes.

2 LITERATURE

The intuition of my paper is related to Rajan and Zingales (2003) who provide historical evidence that the evolution of property rights is connected to productivity gains in managing private assets. Property rights cannot be imposed simply through the creation of a particular institution (see Acemoglu et al., 2001) or laws (see La Porta et al., 1998), but arise endogenously through the development of specialization skills.

My paper considers a setup similar to Thomas and Worrall (1994). They find that backloading of taxes provides efficient incentives to the government, consistent with the

[3] In general, there is strong empirical support that nationalized companies operate less efficiently due to insufficient technological acumen. A FT Energy Study (2006) documents productivity gaps of national oil companies of up to 50% (for Venezuela). Likewise, Mexico's national oil company PEMEX lacks the expertise to do deep-sea drilling in the Gulf of Mexico. A Worldbank study by Kikeri et al. (1992) reveals how production efficiency increases significantly after privatization of state owned enterprises.

[4] These arrangements represent the most common contract form between multinational oil companies and emerging market countries (see Bindemann, 1999).

backloading results obtained in the labor market literature (see Becker and Stigler, 1974, Harris and Holmstrom, 1982, and Thomas and Worrall, 1988). Schnitzer (1999) amends their paper by allowing for upfront cost while restricting periodical investment to a binary variable. My paper makes two new economic contributions. First, the government has access to a relatively inefficient autarky technology, which is the guiding theme of my paper. Secondly, relative impatience of the government can generate efficient upfront transfers to the government and guarantees the existence of a unique stationary contract.

The assumption of relative impatience relates my paper to the theoretical literature on differential time preferences in repeated games. In an early contribution, Rubinstein (1982) derives the unique bargaining outcome of an alternating offer game in terms of the discount factors. The patient player effectively has a larger bargaining power. Lehrer and Pauzner (1999) analyze the set of feasible payoffs in infinite-horizon repeated games when one player is relatively patient. They find that the Folk Theorem (see Fudenberg and Maskin, 1986) does not generalize to the case of heterogenous discount factors. Even if agents are very patient, not all feasible and individually rational repeated game payoffs can be supported by equilibria. Moreover, they find that the difference in discount factors gives rise to trading opportunities by shifting early payoffs to the relatively impatient agent. As a result, the set of sustainable equilibria in the infinite-horizon game may be larger than the convex hull of the stage game payoffs. In my setup, this trading opportunity gives rise to the possibility of upfront payments to the government. In contrast to the findings of Lehrer and Pauzner (1999) optimal transfers to the government are not necessarily frontloaded. When the government obtains relatively little compared to the steady state, the efficient provision of incentives requires backloading.[5] Haag and Lagunoff (2007) restrict their attention to stationary subgame perfect equilibria to determine the highest average level of cooperation that can be sustained when discount factors are heterogenous. Cooperation is shown to be decreasing in mean preserving spreads of the distribution of the discount factor. Moreover, relative impatience can be related to the literature which considers the game between one long-run (patient) player facing a sequence of short-run (impatient) players such as Fudenberg and Levine (1989) or Fudenberg et al. (1990).

The assumption of heterogenous discount factors has also been considered in applied theory. A recent paper by Aguiar et al. (2007) analyzes the effect of relative impatience of the government on the stationary capital stock and level of sovereign debt in an emerging market economy.[6] As in my setup, efficient investment is not reached in the steady state (see also Aguiar et al., 2006). Krüger and Uhlig (2006) develop dynamic risk sharing contracts with one-sided lack of commitment in which they analyze the implications of different discount rates of the principal and the agent.

The methodology used in this paper relies on set-valued techniques developed by

[5] The results can be reconciled if one considers the "appropriate" limit of both discount factors in my setup. In the limit, the incentive constraint of the government can be ignored.

[6] I interpret their findings as follows. Upfront transfers are implicitly present in their model through the existence of sovereign debt. Joint punishment of outside lenders and the firm provides the necessary sanction mechanism to sustain sovereign debt (see Bulow and Rogoff, 1989).

Abreu, Pearce and Stachetti (1986, 1990). Cronshaw and Luenberger (1990, 1994) extend their analysis to games with perfect public information, such as the game in my setup. I rely on the Inner Hyperplane Algorithm considered by Judd et al. (2003) to implement the algorithm numerically.

My findings can also be interpreted in light of Hart and Moore (1994). In their model an entrepreneur seeks financing from an outside creditor, but cannot commit not to withdraw his human capital from the project. The model is reverse compared to my setup because the principal underlying commitment problem is on the firm side. The incentive problem of the firm turns out to be of second order for most of the results of my paper. The liquidation technology of the creditors is analogous to the autarky technology in my paper. Due to the reverse nature of the problem it is ex ante beneficial if the liquidation technology improves. The papers differ fundamentally with regards to the enforceability of property rights. Hart and Moore allow for the enforceable transfer of control rights to the creditor upon the debtor's default. In my setup, the effective control right over the project always rests with the government and cannot be credibly transferred. Kovrijnykh (2007) develops a model of debt contracts with short-term commitment, in which she finds that social welfare is non-monotonic in the borrower's outside option. This non-monotonicity results from the participation constraints of the lender and the borrower, respectively. In my setup, the non-monotonicity of stationary cash-flows to the government is driven by similar model mechanics.

3 SETUP

3.1 The Environment

A country represented by its government (G) possesses an immovable investment opportunity (such as a gold mine), but it lacks the technical know-how and management skills to run this project in an efficient way relative to a multinational firm (F).[7] The productivity gap between the firm and the government can also be interpreted as the difference between local expertise and international best practice. I assume that current period output Y is solely a function of current period investment I. The production technology of the firm $Y_F(I)$ satifies Inada conditions.[8] All figures in this paper are based on the functional form: $Y_F(I) = I^\alpha$ where $0 < \alpha < 1$. The first-best investment level $\hat{I}$, output $\hat{Y}$, and associated per period profits $\hat{\pi}$ are defined as:

$$\begin{aligned} \hat{I} &\equiv \arg\max\left(Y_F(I) - I\right) \\ \hat{Y} &\equiv Y_F\left(\hat{I}\right) \\ \hat{\pi} &\equiv \hat{Y} - \hat{I} \end{aligned} \tag{1}$$

[7] Whether the firm is actually from another country does not matter in terms of the model. This interpretation should be viewed as a motivation for the most relevant empirical implications.

[8] Output and investment is measured in terms of consumption units.

The second-best government technology delivers a fraction ρ of the efficient per period profits (where $0 < \rho < 1$).

$$\pi_{aut} = \rho\hat{\pi} \tag{2}$$

The parameter ρ is a measure for the relative productivity of the government. I denote the associated profits as π_{aut} because these profits are generated in autarky.[9] In Appendix B, I consider the implications of stochastic output shocks and reveal that my findings are robust.

Both players are risk neutral and maximize the net present value of cash flows $C_t^{(i)}$ over an infinite horizon with respective discount factors β_F and β_G.

$$V^{(i)} = \sum_{t=0}^{\infty} \beta_i^{-t} C_t^{(i)} \tag{3}$$

Assumption 1 $\beta_G < \beta_F \leq 1$.

The firm's discount factor β_F is exogenously determined by the capital market. Relative impatience of the government can be viewed as a reduced form implication of two distinct mechanisms. First, political economy considerations suggest that the chance of government turnover effectively increases the discount rate of the government in place (see Aguiar et al., 2007). Secondly, as in Hart and Moore (1994), higher effective discounting can be caused by a combination of financial constraints and profitable reinvestment opportunities. As a robustness check, I consider the case $\beta_G \geq \beta_F$ in Section 5.3.1.

3.2 The Game

I consider an infinite-horizon repeated game Γ^∞ with perfect public information between the firm and the government. There is two-sided lack of commitment: The government can confiscate the proceeds from the project, the firm can leave the country. The commitment problem of the government could be eliminated if it were able to pledge sufficient outside collateral.[10] The stage game Γ features a standard hold-up problem as illustrated in Figure 1. After the firm has incurred the upfront investment I, the government is free to seize the entire output Y. In the isolated analysis of the stage game, the government is the last mover. It would always choose $\tau = Y$, inducing the firm to lose the upfront investment I. Since the firm can get at least a payoff of 0 by refusing to invest in the project, a Nash equilibrium of the stage game Γ features no firm investment. The associated autarky per period cash flows are π_{aut} for the government and 0 and for the firm.

By unraveling, a finite repetition of the stage game Γ^n does not make firm investment sustainable. In the infinite-horizon setup, autarky is not the only sustainable outcome if the government is sufficiently patient.

9 Appendix A provides a summary of the notation.

10 An example is money in a Swiss bank account, which is transferred to the firm in case of a violation of its property rights.

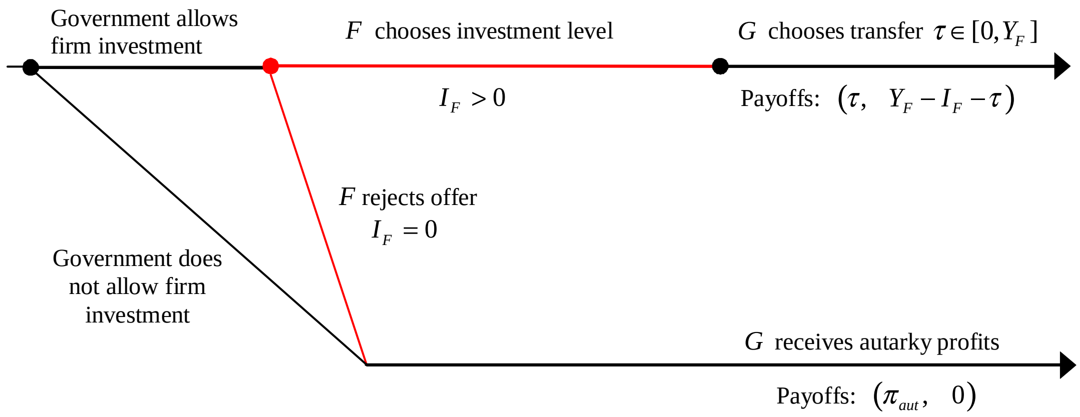

Figure 1: Timeline of Stage Game Γ

Lemma 1 *The autarkic allocation for each player in every period t is subgame perfect and is Pareto dominated by any other subgame perfect equilibrium.*

Proof. Autarky is subgame perfect as repeated play of the static Nash equilibrium is subgame perfect. Moreover, the autarky payoffs are equivalent to the min-max payoffs (Fudenberg and Tirole, 1994). This implies that the static outcome represents the worst possible subgame perfect equilibrium. ■

4 SELF-ENFORCING CONTRACTS

4.1 Feasibility of Firm Investment

Efficiency requires the private firm to operate the project.[11] However, firm investment can only take place if autarky is not the only subgame perfect equilibrium. Following the idea of Abreu, Pearce, and Stachetti (1986, 1990) (henceforth APS) the whole set of subgame perfect equilibria can be enforced with the threat of the worst possible subgame perfect equilibrium (autarky).[12]

[11] With secure property rights, the analysis of Hart and Moore (1990) implies that the firm – the only party that is needed to invest – should be the owner of the assets. This follows directly from proposition 2 of their paper.

[12] As Gale and Hellwig (1989) point out in their model of sovereign debt, the selection of a specific subgame equilibrium is delicate. The choice of the worst subgame perfect equilibrium as the punishment path implicitly suggests that the players could commit ex-ante to the choice of an equilibrium. Nonetheless, it is common in the literature to use the worst subgame perfect equilibrium or autarky as the punishment equilibrium.

Definition 1 *A self-enforcing contract refers to a subgame perfect equilibrium in the infinitely repeated game* Γ^{∞}.

Firm investment is not feasible if the government has an incentive to deviate from a stationary contract that gives it the entire firm profits $(Y - I)$. This allocation relaxes the government's incentive problem as much as possible, while allowing the firm to break-even each period. Subgame perfection implies that it is sufficient to check for the best possible one-period deviation (Fudenberg and Tirole, 1994) of the government, which is confiscating the entire output Y. Given firm investment I, the outside option of the government is given by: $Y + \beta_G v_{aut}$. I define v_{aut} as the present value of autarky profits:

$$v_{aut} \equiv \frac{\pi_{aut}}{1 - \beta_G} \tag{4}$$

Formally, the following incentive constraint of the government has to be satisfied for some investment level I:

$$\frac{Y - I}{1 - \beta_G} \geq Y + \beta_G v_{aut} \tag{5}$$

Proposition 1 *If the relative productivity of the government exceeds the threshold level* $\tilde{\rho} = \max_I \frac{Y_F(I) - \beta_G^{-1} I}{\hat{\pi}} < 1$, *autarky is the only subgame perfect equilibrium.*

Proof. The incentive constraint of the government can be simplified by substituting the expression for v_{aut} into equation 5:

$$Y - \beta_G^{-1} I \geq \pi_{aut} \text{ for some } I \tag{6}$$

Since π_{aut} is independent of the firm's investment level, it is sufficient to check whether the following condition holds:

$$\max_I Y_F(I) - \beta_G^{-1} I \geq \pi_{aut} \tag{7}$$

Using the maximized value of the left hand side and the fact that $\pi_{aut} = \rho\left(\hat{Y} - \hat{I}\right)$, we obtain:

$$\rho \leq \max_I \frac{Y_F(I) - \beta_G^{-1} I}{\hat{\pi}} < 1 \tag{8}$$

The threshold level is less than one by the definition of $\hat{\pi} = \max_I Y_F(I) - I$ and since $\beta_G^{-1} > 1$. ■

Using the production function $Y_F(I) = I^{\alpha}$ yields a threshold level of $\beta_G^{\frac{\alpha}{1-\alpha}}$. This proposition has two interpretations. First, for any given discount factor β_G sufficiently low relative ability ρ makes firm investment sustainable. Intuitively, the threat of autarky is more effective if the government is unable to produce on its own. Secondly, for any given level of ρ, there exists a patience level β_G which enables firm investment. Sustainability of firm investment does not depend on the firm's discount rate because the firm obtains zero profits each period in the relevant stationary allocation.

4.2 Recursive Contracting Problem

Following APS the optimal contracting problem is framed recursively. The recursive formulation makes it possible to solve an otherwise intractable problem using standard dynamic programming techniques. The state variable for this problem is given by a promised value v.[13] I define v as the net present value which the firm promises to deliver to the government at the beginning of period t. This promised value is a sufficient statistic for the entire history of investment levels and transfers up to time $t-1$:

$$h^{t-1} \equiv \{(\tau_s, I_s)\}_{s=1}^{t-1} \tag{9}$$

Let $V_F(v)$ denote the net present value of the firm given the promised value v. The current period value to the government can be expressed as the sum of current period transfers τ and the discounted continuation value w. Likewise, the firm value $V_F(v)$ can be expressed as the sum of current period net profits $Y - I - \tau$ and the discounted firm value given next period's promised value w:

$$v = \tau + \beta_G w \tag{10}$$

$$V_F(v) = Y - I - \tau + \beta_F V_F(w) \tag{11}$$

The solution to the following dynamic contracting problem determines the optimal current period investment level I, current period transfers τ, and the promised continuation value to the government w.

$$V_F(v) = \max_{I,\tau,w} Y - I - \tau + \beta_F V_F(w) \text{ s.t.} \tag{12}$$

#	Constraint	Lagrange multiplier
1)	$\tau + \beta_G w \geq v$	λ_{PK}
2)	$\tau + \beta_G w \geq Y_F(I) + \beta_G v_{aut}$	λ_{IC}
3)	$V_F(w) \geq 0$	$\beta_F \lambda_{PC}$
4)	$w \geq v_{aut}$	$\beta_G \lambda_{IR}$
5)	$\tau \geq \tau_{\min} = 0$	$\mu_{\min}$
6)	$\tau \leq \tau_{\max}$	$-\mu_{\max}$

Initial Conditions

$V_F(v_0) = Y - I - \tau + \beta_F V_F(w) \geq 0$

$v_0 \geq v_{aut}$

The problem is stated from the perspective of the firm. The entire set of efficient contracts can be traced out by varying v. The naming of the Lagrange multipliers follows the

[13] The stationary environment does not feature any physical state variables. A more elaborate discussion of the APS machinery is beyond the scope of this paper. The reader is referred to the original essays of Abreu, Pearce and Stachetti (1986, 1990) as well as the book by Ljungqvist and Sargent (2004) which provides many interesting applications.

convention of Sargent and Ljungvist (2004). The first constraint is the promise keeping constraint (PK). It ensures that the current period transfer τ and the promised continuation value w yield at least the promised value of v to the government. The second constraint (IC) ensures that the government finds it incentive compatible to honor the terms of the contract. The value derived from the contract $\tau + \beta_G w$ must be higher than the value implied by the optimal deviation $Y_F(I) + \beta_G v_{aut}$. The third constraint (PC) ensures firm participation in every period. The firm cannot commit to stay in the country, if the net present value becomes negative. The fourth constraint (IR) represents the individual rationality constraint of the government. Next period's promised value must yield a higher value than autarky. The fifth constraint implies that the government is liquidity constrained and cannot provide subsidies.[14] The last constraint puts a restriction on the per period transfers of the firm. Specifically, $\tau_{\max}$ puts an upper bound on the initial sales price. I allow the firm to pay higher transfers than the maximum amount the government can possibly confiscate per period. This is reasonable, unless the multinational firm is financially constrained.[15]

Assumption 2 $\tau_{\max}$ *is greater than the output level* $\hat{Y}$.

The initial condition on the current period firm value $V_F(v_0) \geq 0$ places an *endogenous* upper bound on current period tax payments:

$$\tau \leq Y - I + \beta_F V_F(w) \tag{13}$$

I denote the Lagrange multiplier of this constraint λ_{PC0} since it represents the current period participation constraint. This constraint only binds if the government extracts the highest feasible value from the relationship, denoted as $v_{\max}$:

$$V_F(v_{\max}) = 0 \tag{14}$$

For ease of exposition, I first consider a relaxed problem that ignores this constraint $(\lambda_{PC0} = 0)$ and check in a second step whether the constraint is satisfied.

The initial condition on the promised value to the government has to ensure that the government is at least as well of as in autarky. It will become clear later, that this restriction on v_0 causes the IR constraint of the government to be slack $(\lambda_{IR} = 0)$.[16] Therefore, as long as the initial promised value v_0 satisfes $v_0 \geq v_{aut}$ the IR constraint of the government is slack in any future period.

[14] The mechanics of the contract are essentially unaffected if one allows for the payment of ex-post subsidies. It will become clear later, that subsidies only impact the speed of adjustment to the stationary allocation.

[15] This assumption can be relaxed without changing any of the results presented in the paper.

[16] The reasoning is as follows. If the current period promised value v is small (close to v_{aut}), the optimal dynamics imply backloading of transfers to the government $(w > v)$. Therefore, if $v > v_{aut}$ is satisfied, then $w > v_{aut}$ is also satisfied.

Using these simplifications ($\lambda_{IR} = \lambda_{PC0} = 0$), the Lagrangian can be stated as follows:

$$\begin{aligned} L =& Y_F(I) - I - \tau + \beta_F V_F(w) \\ &+ \lambda_{PK}[\tau + \beta_G w - v] + \lambda_{IC}[\tau + \beta_G w - Y_F(I) - \beta_G v_{aut}] \\ &+ \beta_F \lambda_{PC} V_F(w) + \beta_G \lambda_{IR}[w - v_{aut}] + \mu_{\min}\tau - \mu_{\max}[\tau - \tau_{\max}] \end{aligned} \tag{15}$$

The non-negative Lagrange multipliers are functions of the promised value v. The first-order conditions with respect to I, τ, and w imply:

$$\begin{aligned} I: &\quad Y_F'(I)(1-\lambda_{IC}) - 1 &= 0 \\ \tau: &\quad -1 + \lambda_{PK} + \lambda_{IC} + \mu_{\min} - \mu_{\max} &= 0 \\ w: &\quad \beta_F\left[V_F'(w)(1+\lambda_{PC}) + \tfrac{\beta_G}{\beta_F}(\lambda_{PK} + \lambda_{IC})\right] &= 0 \end{aligned} \tag{16}$$

In addition, the complementary slackness conditions have to hold. The focus of the subsequent analysis lies on the Lagrange multipliers associated with the IC constraint of the government and the PK constraint. The first-order condition with respect to investment reveals that λ_{IC} directly translates into an investment distortion:

$$\lambda_{IC} > 0 \Leftrightarrow I < \hat{I} \tag{17}$$

By the envelope condition, the Lagrange multiplier on the PK constraint λ_{PK} represents the shadow price of promising an additional unit of value to the government (in firm value units):

$$-V_F'(v) = \lambda_{PK} \tag{18}$$

Graphically, the Lagrange multiplier λ_{PK} represents the slope of the value function evaluated at the current period promised value v.

Definition 2 *The Pareto region of v is defined as the compact domain $[v_{\min}, v_{\max}]$ in which $V_F(v)$ is downward sloping.*

Definition 3 *The Pareto frontier is given by: $\{(v, V_F(v)) \text{ s.t. } v \in [v_{\min}, v_{\max}]\}$*

In the Pareto region the value to the firm V_F must be strictly decreasing in the promised value to the government. Otherwise, a Pareto improving allocation would be feasible. An efficient contract is therefore uniquely determined by the promised value v.

Lemma 2 *The value function $V_F(v)$ is concave.*

Proof. See Appendix C.1. ■

Concavity implies that the shadow price of providing an additional unit of value to the government λ_{PK} is increasing in v. This follows from the strict concavity of the production function. Mathematically, these properties ensure that the slope of the value function λ_{PK} is an increasing function of v. Let $\lambda_{PK}^* = -V_F'(w)$ denote the slope of the value function evaluated at next period's promised value w. Then the statement $\lambda_{PK}^{*} > \lambda_{PK}$ implies that the promised continuation value w is higher than the current promised value v. Analogously, if $\lambda_{PK}^{*} < \lambda_{PK}$, the continuation value must be below the current promised value.

Lemma 3 *The Lagrange multiplier λ_{IC} is a strictly decreasing function of the promised value v for all $v \in [v_{\min}, \hat{v}]$ where $\hat{v} = \hat{Y} + \beta_G v_{aut}$. Otherwise, λ_{IC} equals zero.*

Proof. If $\lambda_{IC} > 0$, then $Y_F(I) + \beta_G v_{aut} = v$ (by PK and IC). Therefore, if v increases, so must output $Y_F(I)$ and hence investment I. By the first-order condition on investment, this implies that λ_{IC} becomes smaller. Thus, in the region where the IC constraint is binding, λ_{IC} is strictly decreasing in v. If $v > \hat{Y} + \beta_G v_{aut}$, efficient investment is incentive compatible. Therefore, the Lagrange multiplier λ_{IC} is zero. ■

The more the government obtains from abiding by the contract (v), the smaller the incentive to renege on it (λ_{IC} becomes smaller). This enables the firm to increase its investment level without having to fear expropriation. Hence, Lemma 3 equivalently states that investment is a strictly increasing function of v unless efficient investment is feasible ($v > \hat{v}$) such that $\lambda_{IC} = 0$.

Using the envelope condition (see equation 18) and the first-order condition with respect to the continuation value w, yields a transition law for the slope λ_{PK}:

$$\lambda_{PK}^{*} = \frac{\beta_G}{\beta_F} \frac{\lambda_{PK} + \lambda_{IC}}{1 + \lambda_{PC}} \tag{19}$$

This transition law generates a first-order Markov process for the promised value to the government $w(v)$.

4.3 Efficient Stationary Contracts

Each efficient contract reaches a unique stationary allocation on the Pareto frontier in finite time (a claim yet to be proved). Therefore, the surplus division resulting from an efficient contract can be decomposed into payoffs during the transition phase and payoffs in the steady state. My analysis follows this logical structure. In this section, I characterize the steady state of all efficient contracts, the unique stationary contract on the Pareto frontier.[17] The subsequent section discusses the transition dynamics to the steady state.

A stationary contract on the Pareto frontier associated with a promised value $\bar{v}$ and slope $\bar{\lambda}_{PK}$ satisfies:

$$w = \bar{v} \tag{20}$$

$$\lambda_{PK}^{*} = \bar{\lambda}_{PK} \tag{21}$$

[17] Levin (2003) shows that efficient self-enforcing contracts can often take on a simple stationary form.

At the steady state, the continuation value w and next period's shadow price λ_{PK}^{*} have to coincide with the respective current levels $\bar{v}$ and $\bar{\lambda}_{PK}$.

Lemma 4 *In any stationary contract with positive firm investment, the government receives positive stationary transfers* $(0 < \bar{\tau} < \tau_{\max})$ *that generate a strictly higher value for the government than autarky. This implies:*

$$\bar{\mu}_{\min} = \bar{\mu}_{\max} = \bar{\lambda}_{IR} = 0 \tag{22}$$

Proof. See Appendix C.2 ■

Stationary transfers $\bar{\tau}$ have to be positive ($\mu_{\min} = 0$) to ensure incentive compatibility for the government, but not too large ($\mu_{\max} = 0$) to ensure firm participation. Moreover, the government has to extract more value than in autarky, because its outside option after firm investment is given by $\bar{Y} + \beta_G v_{aut} > v_{aut}$.

I distinguish between two types of stationary contracts: An *interior* stationary contract is defined as a stationary contract with strictly positive firm profits $\left(\bar{\lambda}_{PC} = \bar{\lambda}_{PC0} = 0\right)$. A *corner* stationary contract is referred to as a contract where the firm just breaks even in the steady state $\left(\bar{\lambda}_{PC} = \bar{\lambda}_{PC0} > 0\right)$. Due to stationarity the current period participation constraint $(V_F(\bar{v}) \geq 0)$ coincides with next period's participation constraint such that $\bar{\lambda}_{PC} = \bar{\lambda}_{PC0}$.

Lemma 5 *At the stationary contract, the Lagrange multipliers are related as follows:*

$$\bar{\lambda}_{IC} + \bar{\lambda}_{PK} = 1 + \bar{\lambda}_{PC} \tag{23}$$

Proof. See Appendix C.3. ■

Proposition 2 *There exists a unique stationary contract on the Pareto frontier which yields the government a value of* $\bar{v}$ *such that:*

$$\bar{\lambda}_{PK} = \beta_G/\beta_F \tag{24}$$

Proof. Imposing stationarity on the transition equation (see equation 19 with $\lambda_{PK}^{*} = \lambda_{PK} = \bar{\lambda}_{PK}$) results in the following expression:

$$\bar{\lambda}_{PK} = \frac{\beta_G}{\beta_F} \frac{\bar{\lambda}_{PK} + \lambda_{IC}}{1 + \bar{\lambda}_{PC}} \tag{25}$$

At the steady state, it is possible to apply Lemma 5 such that: $\frac{\bar{\lambda}_{PK} + \lambda_{IC}}{1 + \bar{\lambda}_{PC}} = 1$. Therefore, the stationary slope is given by $\bar{\lambda}_{PK} = \frac{\beta_G}{\beta_F}$. The Lagrange multiplier on the IC constraint binds since $\bar{\lambda}_{IC} = 1 - \beta_G/\beta_F + \bar{\lambda}_{PC} \geq 1 - \beta_G/\beta_F > 0$ by Lemma 5 using $\bar{\lambda}_{PK} = \frac{\beta_G}{\beta_F}$. Due to the Inada property of the production function, the binding IC constraint implies a unique investment level by the first-order condition on investment (see equation 16) and a unique promised value by Lemma 3. ■

Thus, at the steady state, the shadow price for promising an additional unit of value to the government is less than unity ($\bar{\lambda}_{PK} = \beta_G/\beta_F$). The promised value at the steady state can be interpreted as the maximizer of the objective function $V_F(v) + \frac{\beta_G}{\beta_F}v$:

$$\bar{v} = \arg\max V_F(v) + \frac{\beta_G}{\beta_F}v \tag{26}$$

This interpretation shows that the stationary allocation does not maximize total surplus (since $\beta_F \neq \beta_G$). Therefore, the stationary investment level is inefficient. Similar to Rubinstein (1982), the relatively patient firm receives a greater long-run weight.

Proposition 3 ***Interior Stationary Contract***
For $\rho < \bar{\rho} = \frac{\bar{Y} - \beta_G^{-1}\bar{I}}{\hat{\pi}} < \tilde{\rho}$ the stationary contract has the following features:
a) The investment level $\bar{I}$ is inefficient and satisfies: $Y_F'(\bar{I}) = \frac{\beta_F}{\beta_G} > 1$
b) The value to the government $\bar{v}$ is increasing in relative productivity ρ
c) The firm value is strictly positive and decreasing in ρ

Proof. At the interior stationary contract the IC constraint is given by:

$$\bar{\lambda}_{IC} = 1 - \beta_G/\beta_F > 0 \tag{27}$$

Given $\bar{\lambda}_{IC}$ the first-order condition on investment (see equation 16) implies that the marginal product of investment is greater than one:

$$Y_F'(\bar{I}) = \frac{\beta_F}{\beta_G} > 1 \tag{28}$$

The binding IC constraint implies that: $\bar{v} = \bar{Y} + \beta_G v_{aut}$. Using the stationary transfers $\bar{\tau} = (1 - \beta_G)\bar{v}$ the firm profits are given by:

$$V_F(\bar{v}) = \frac{\bar{Y} - \bar{I} - (1 - \beta_G)\bar{v}}{1 - \beta_F} \tag{29}$$

Stationary transfers to the government are increasing in v_{aut} and as such in ρ. In contrast, stationary firm profits are decreasing in ρ. Output and investment are only a function of relative impatience β_G/β_F and independent of ρ. The threshold level $\bar{\rho}$ can be obtained by setting the firm value $V_F(\bar{v})$ to 0:

$$\bar{\rho} = \frac{\bar{Y} - \beta_G^{-1}\bar{I}}{\hat{\pi}} \tag{30}$$

Recall that $\tilde{\rho} = \max_I \frac{Y_F(I) - \beta_G^{-1}I}{\hat{\pi}}$ such that $\bar{\rho} < \tilde{\rho}$. ■

The degree of stationary investment distortions $\left(\bar{\lambda}_{IC} = 1 - \beta_G/\beta_F\right)$ is solely a function of the relative impatience of the government. Since $\bar{I}$ is independent of ρ, relative productivity does not alter the stationary investment efficiency, but it has distributional implications. A higher relative technology level increases transfers to the government and reduces firm net profits. The economic rationale is as follows: The better outside option of the government requires it to obtain larger steady state transfers. Stationary transfers $\bar{\tau}$ are a weighted average of the stationary output level and autarky profits. The weight is given by the discount factor of the government β_G:

$$\bar{\tau} = (1 - \beta_G)\,\bar{Y} + \beta_G \pi_{aut} \tag{31}$$

Higher relative productivity of the government increases the stationary taxes that the firm has to pay while total surplus $\left(\bar{Y} - \bar{I}\right)$ is unaffected. Therefore, firm value is decreasing in ρ.

Proposition 4 ***Corner Stationary Contract***
For $\bar{\rho} \leq \rho \leq \tilde{\rho}$ the stationary contract has the following features:
a) The investment level $\bar{I}$ is inefficient and satisfies: $Y_F'\left(\bar{I}\right) = \frac{\beta_F}{\beta_G}\left(1 - \frac{\beta_F}{\beta_G}\bar{\lambda}_{PC}\right)^{-1}$
b) The government receives the entire surplus, $\bar{v} = v_{\max} = \frac{\bar{Y}-\bar{I}}{1-\beta_G}$
c) The firm value is zero: $V_F(\bar{v}) = 0$

Proof. At the corner contract, the firm participation constraint binds. Therefore, the firm makes zero profits and the government receives the total surplus, i.e. $\bar{v} = v_{\max} = \frac{\bar{Y}-\bar{I}}{1-\beta_G}$. By Lemma 5 and Proposition 2, the IC constraint is given by:

$$\bar{\lambda}_{IC} = 1 - \beta_G/\beta_F + \bar{\lambda}_{PC} \tag{32}$$

By the first order condition on investment (see equation 16) the stationary investment level is uniquely determined as the production function satisfies Inada conditions. ■

Due to the binding participation constraint of the firm, the corner steady state features additional investment distortions relative to the interior stationary contract $\left(\bar{\lambda}_{IC} = 1 - \beta_G/\beta_F + \bar{\lambda}_{PC} > 1 - \beta_G/\beta_F\right)$. Since higher relative productivity translates into a tighter participation constraint, an increase in relative productivity reduces the steady state level of investment and output. Higher relative productivity reduces total surplus and leads to a reduction in the welfare of the government, the bearer of the total surplus. The firm value is unaffected, since the firm is already pushed down to its outside option.

Figure 2 illustrates the steady state welfare comparative statics of relative productivity. The production function is normalized such that the efficient per period profit $\hat{\pi}$ equals 1. The parameter region of ρ is divided into three intervals corresponding to the interior steady state ($0 \leq \rho < \bar{\rho}$), the corner state steady state ($\bar{\rho} \leq \rho \leq \tilde{\rho}$) and autarky ($\tilde{\rho} < \rho \leq 1$). Stationary firm profits are linearly decreasing with slope β_G until the participation constraint becomes binding at $\bar{\rho}$. Stationary profits remain flat at 0 even though the firm is still producing in the region between $\bar{\rho}$ and $\tilde{\rho}$.

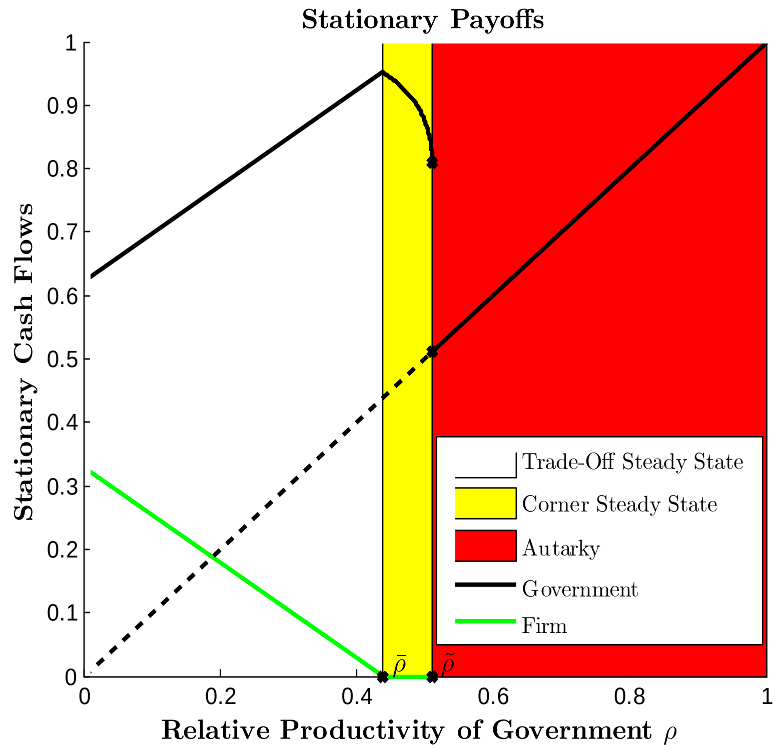

Figure 2: Long-Run Comparative Statics of Relative Productivity ρ
(Parameters: $\beta_G = 0.75$, $\beta_F = 0.85$, $\alpha = 0.7$)

Government cash flows increase linearly in relative productivity with slope β_G as long as the firm participation constraint does not bind ($\rho < \bar{\rho}$). The better outside option allows the government to extract a greater fraction of the total surplus. Once it extracts all the surplus ($\rho \geq \bar{\rho}$), cash flows decrease continuously with slope $\left(\frac{Y'(I_F)-1}{\beta_G^{-1}-Y'(I_F)}\right)$ as the participation constraint of the firm leads to additional investment distortions.[18] At $\tilde{\rho}$ a discrete jump occurs because the efficient production technology of the firm is no longer available and the project is operated with the second-best autarky technology. Once autarky is the only feasible option, cash flows increase one-to-one with relative productivity.

These comparative statics reveal that an increase in relative productivity can have different welfare implications, depending on the current level of technology. Even though technology acquisition is formally not part of the model, one can use these implications to shed light on the incentives to invest in R&D, or more generally in education. Close to the threshold level $\bar{\rho}$ the government has very little incentive to deploy costly resources to

[18] The slope can be determined as follows: Steady state cash flow to the government is given by $C = Y - I$ and investment has to satisfy $Y - \beta_G^{-1} I - \rho = 0$. Using the implicit function theorem one obtains $\frac{dC}{d\rho} = -\frac{Y'(I)-1}{\beta_G^{-1}-Y'(I)}$. Note, that in this region we have $1 < Y'(I) \leq \beta_G^{-1}$. As ρ approaches $\tilde{\rho}$ the slope approaches $-\infty$ as $Y'\left(\tilde{I}\right) = \beta_G^{-1}$.

R&D. A marginal increase in its technology level makes the incentive problem so severe that the firm has to reduce investment. On the other hand, the firm always benefits from a successful upgrade of its relative technology level (lower ρ).

4.4 Efficient Contract Dynamics

Despite the stationary physical environment, the optimal contract has non-trivial dynamic features.

Lemma 6 *The stationary contract is globally stable.*

Proof. See Appendix C.4. ■

Lemma 6 implies that over time promised values are raised to the left of the steady state $(w > v)$ and lowered to the right of the steady state $(w < v)$, where "left" and "right" refer to the cases $v < \bar{v}$ and $v > \bar{v}$, respectively. Thus, if the contract is initialized below the steady state value the government obtains backloaded payoffs. If the contract is initialized above the steady state value it obtains frontloaded payoffs. These distinct dynamics are driven by two competing forces. The efficient provision of intertemporal incentives requires the government to obtain backloaded transfers. The intuition goes back to Becker and Stigler (1974) as well as Harris and Holmstrom (1982). Backloading "works", because deferred rewards provide incentives for current and future periods. Implicitly, the firm acts as a savings bank for the government, where the deposit is used as collateral, a discipline device to induce cooperation. The competing force to the incentive problem is given by the relative impatience of the government (efficiency). The ratio $\frac{\beta_G}{\beta_F}$ can be arbitrarily close to one. Relative impatience suggests that the government should obtain frontloaded payoffs, such that the firm acts as a quasi-lender to the government.

The resulting contract dynamics depend on whether the incentive problem or efficiency dominates. At the stationary contract the effects of incentives and efficiency are offset. To the left of the steady state, the government obtains relatively small current promises v, which implies that the incentive problem dominates. This results in backloaded transfers. To the right of the steady state the incentive problem is second-order, such that relative impatience dominates and the firm acts as a sovereign debt lender. In contrast to the setup of Bulow and Rogoff (1989), the firm can expect to receive a payback on the loan due to its technology advantage on the production side. Without the productivity advantage, the underlying commitment problem would render this loan infeasible.

Proposition 5 ***Optimal Continuation Values***
a) To the left of the steady state ($v < \bar{v}$) promised values are raised each period at the gross interest rate β_G^{-1} until the steady state is reached.
b) To the right of the steady state ($v > \bar{v}$) promised values are lowered each period until the steady state is reached.

$$w(v) = \begin{cases} \min\left(\beta_G^{-1} v, \bar{v}\right) & \text{for} \quad v \leq \bar{v} \\ \max\left((v - \tau_{\max})\beta_G^{-1}, \bar{v}\right) & \text{for} \quad v < \bar{v} \end{cases} \tag{33}$$

Proof. As long as $v \neq v_{\max}$ the PC constraint does not bind, which implies for the transition law:

$$\lambda_{PK}^* = \frac{\beta_G}{\beta_F}\left(\lambda_{PK} + \lambda_{IC}\right) \tag{34}$$

Using the first-order condition on the tax rate: $\lambda_{PK} + \lambda_{IC} = 1 + \mu_{\max} - \mu_{\min}$, we obtain:

$$\lambda_{PK}^* \frac{\beta_F}{\beta_G} = 1 + \mu_{\max} - \mu_{\min} \tag{35}$$

Part a) Suppose the current value is lower than the steady state $(v < \bar{v})$, but the continuation value satisfies $w = \bar{v}$, such that $\lambda_{PK}^* = \bar{\lambda}_{PK} = \frac{\beta_G}{\beta_F}$. Therefore, the left hand side of equation 35 equals 1, such that neither constraint binds: $\mu_{\max} = \mu_{\min} = 0$. The promise keeping constraint $v = \tau + \beta_G \bar{v}$ yields the optimal transfer payment τ. If $w < \bar{v}$ it follows that $\lambda_{PK}^* < \bar{\lambda}_{PK} = \frac{\beta_G}{\beta_F}$ (by strict concavity of the value function to the left of the steady state). Hence, the left hand side of equation 35 is less than 1. This implies that the constraint on the minimum tax binds: $\mu_{\min} > 0$. Therefore, the optimal tax is $\tau = \tau_{\min} = 0$.

Part b) The proof strategy is analogous to part a). If the steady state is reached in the next period neither constraint binds: $\mu_{\max} = \mu_{\min} = 0$. Otherwise, the constraint on the maximum tariff rate binds: $\mu_{\max} > 0$ such that $\tau = \tau_{\max}$. ■

Given the function $w(v)$ the optimal transfer and investment schedule $I(v)$ satisfy :

Proposition 6 ***Optimal Transfer Policy***
If the stationary contract is reached in the next period, taxes are given by $\tau = v - \beta_G \bar{v}$. Otherwise, zero taxes are paid to the left of the steady state and maximum taxes $\tau_{\max}$ are paid to the right of the steady state:

$$\tau(v) = \begin{cases} \tau_{\min} = 0 & \text{for} \quad w(v) < \bar{v} \\ v - \beta_G \bar{v} & \text{for} \quad w(v) = \bar{v} \\ \tau_{\max} & \text{for} \quad w(v) > \bar{v} \end{cases} \tag{36}$$

Proof. See Proof of Proposition 5. ■

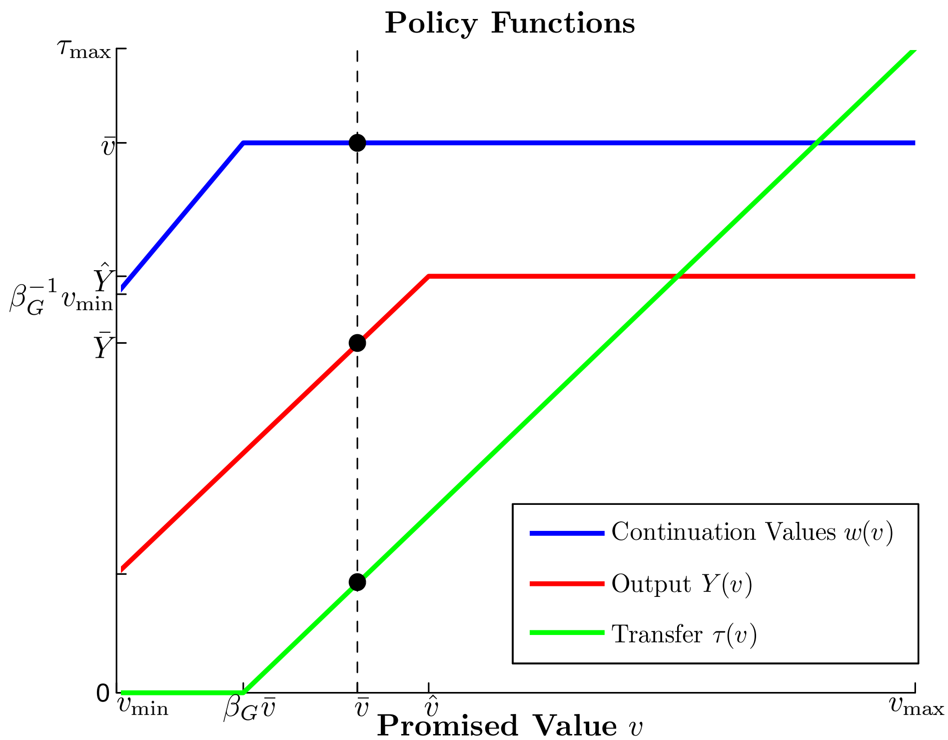

Figure 3: Policy Functions for Continuation Values, Output and Transfers (Parameters: $\beta_G = 0.8$, $\beta_F = 0.9$, $\alpha = 0.6$, $\rho = 0.3$)

Corollary 1 ***Optimal Investment Policy***
a) The optimal investment level is given by:

$$I(v) = \begin{cases} Y_F^{-1}(v - \beta_G v_{aut}) & \textit{for} \quad v < \hat{v} \\ \hat{I} & \textit{for} \quad v \geq \hat{v} \end{cases} \tag{37}$$

where $\hat{v} = \hat{Y} + \beta_G v_{aut} > \bar{v}$
b) For promised values lower than the stationary level $(v < \bar{v})$ *investment is strictly increasing over time.*
c) For promised values greater than the stationary level $(v > \bar{v})$ *investment is decreasing over time.*

Proof. See Appendix C.5. ■

It is instructive to first explore the case when the constraints on the transfer policy $(\tau_{\min} \leq \tau \leq \tau_{\max})$ are lifted or do not bind $(\mu_{\max} = \mu_{\min} = 0)$. Since there are no adjustment cost it is efficient to directly "jump" to the steady state. The optimal transfer τ has to ensure that the promised value $v = \tau + \beta_G \bar{v}$ is delivered. Taking the constraints into account, transfers are chosen such that the contract takes the biggest step towards the steady state contract.

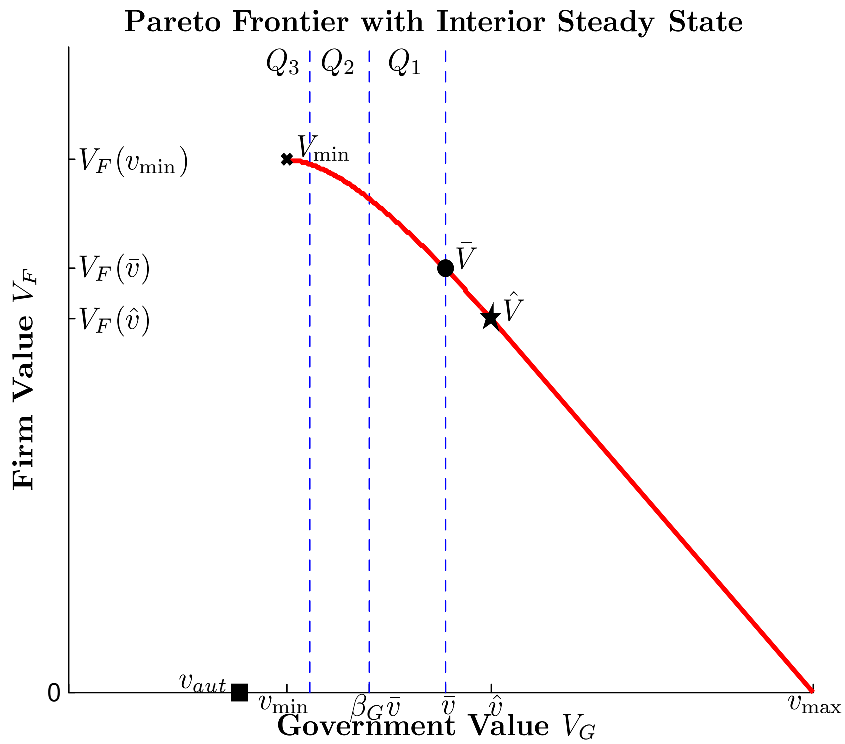

Figure 4: Pareto Frontier with Interior Steady State
(Parameters: $\beta_G = 0.8$, $\beta_F = 0.9$, $\alpha = 0.6$, $\rho = 0.3$)

The optimal policies are plotted in Figure 3. For the purpose of this graph (and the following graphs), I have assumed that the constraint on the maximum transfer $\tau_{\max}$ does not bind. This is equivalent to the assumption that the firm is financially unconstrained. If the current period promised value v is smaller than $\beta_G\bar{v}$, the constraint on the minimum transfer binds, i.e. $\tau = \tau_{\min} = 0$. In this region the firm keeps its promise solely by raising the continuation value to $w = \beta_G^{-1}v$, which is the current value v plus the required interest. For promised values larger than $\beta_G\bar{v}$ the continuation value is given by the steady state value, such that transfers τ have to increase one-to-one with v. This ensures that the firm promise to the government ($\tau + \beta_G\bar{v} = v$) is kept. As long as the incentive constraint of the government binds, output also increases one-to-one with v. This follows from the PK constraint and the IC constraint ($Y + \beta_G v_{aut} = v$). Investment is simply given by $Y_F^{-1}(v - \beta_G v_{aut})$. If the promise is above $\hat{v} = \hat{Y} + \beta_G v_{aut}$, efficient investment and efficient output is feasible.

These constraints on the transfers only affect the short-run dynamics, precisely the number of periods it takes to reach the stationary contract. The stationary contract itself is unaffected. Therefore, as $\tau_{\max}$ is assumed to be sufficiently large, the stationary contract will be reached next period if the current promise is above $\bar{v}$. To the left of the steady state, it is possible to define intervals Q_i such that for each $v \in Q_i$ it takes exactly

i periods to reach the steady state.

$$Q_i \equiv [\beta_G^i \bar{v}, \beta_G^{i-1} \bar{v}] \tag{38}$$

Given the optimal policy functions it is now possible to determine a closed-form solution for the Pareto frontier (see Figure 4). The derivation does not provide additional insight and is moved to Appendix C.6. The dynamics on the frontier over time are uniquely determined by the transition law for the promised value. The relation between v_0 and the steady state value $\bar{v}$ determines the observable dynamics.[19]

4.5 Initial Surplus Division

The model itself does not predict a particular contract starting point v_0 within the Pareto region $[v_{\min}, v_{\max}]$. Therefore, it is necessary to consider mechanisms which remove the ambiguity. The initial surplus division can be interpreted as the outcome of a simple bargaining game of the following structure.[20] Let η denote the probability that the firm is able to make a take-it-or-leave-it offer to the government ($v_0 = v_{\min}$) and $(1-\eta)$ be the probability that the government is able to make a take-it-or-leave-it offer to the firm ($v_0 = v_{\max}$). The probability η can be interpreted as the bargaining power of the firm. Before the realization of the bargaining game, the expected value to the government is given by:

$$v_0 = \eta v_{\min} + (1-\eta)\, v_{\max} \tag{39}$$

Due to the concavity of the Pareto frontier the expected payoffs from this bargaining outcome are not on the Pareto frontier. Similar to Hart and Moore (1998) I therefore augment the bargaining game by allowing the firm to make an offer to the government before the game starts. This adjustment removes the inefficiency such that the firm receives $V_F(v_0)$. It follows directly from equation 39 that high firm bargaining power translates into lower initial promised values to the government (backloading). Analogously, low firm bargaining power results in high initial promised values (frontloading).

An alternative mechanism to generate a starting point on the Pareto frontier is given by an auction process. Suppose there exist multiple firms with different technological capabilities which put in bids to the government v_0 for the exclusive right to operate the project. The technology leader wins the auction. The initial promised value to the government is determined by the break-even condition of the second-best producer. With this type of Bertrand competition among producers, a closer gap between the two leading producers results in higher values to the government (frontloading). In contrast, if the productivity gap between the first- and second-best producer is high, the initial promised

[19] If the stationary contract is a corner contract (such that $v_{\max} = \bar{v}$), backloading is the only feasible dynamic pattern.

[20] For a more rigorous description of bargaining games see Harris and Raviv (1995). Rubinstein (1982) bargaining does not yield closed-form solutions of the bargaining outcome due to the concavity of the Pareto frontier. Depending on the parameters, the bargaining outcome for the government can be below or above the steady state. However, if the government is very impatient, its bargaining position is negatively affected. This makes backloading more likely ($v_0 < \bar{v}$).

value to the government is low (backloading).[21] These ideas can also be framed in terms of uniqueness. If the firm is the essential ingredient providing the government with a unique investment opportunity, it is more likely that the initialization is favorable for the firm. In this case, the firm can exploit competition among countries to extract higher cash flows (backloading). If the investment opportunity is unique to the country, such as through the existence of natural resources or in terms of market access, the government can exploit competition among firms.

In Section 6.1, I extend the basic model and allow for upfront investment cost. I show that large upfront investment cost predict backloading.

4.6 Comparative Statics

The previous section highlighted several focal points on the Pareto frontier. I define contract V as the unique contract on the Pareto frontier that provides the government a value of v and the firm a value of $V_F(v)$. Hence, the contract $V_{\min}$ represents the least favorable contract for the government. The level of $v_{\max}$ is determined by the break-even condition of the firm and can be interpreted as an endogenous borrowing constraint. Figure 5 reveals that lower relative productivity causes the Pareto frontier to move outward. Both parties can be made better off at the initialization of the contract. In the following analysis, I compare contracts across Pareto frontiers where the slopes are equal. The slope can be interpreted as the relative weight ψ that the social planner gives the government in the following maximization problem:

$$\max_{v} V_F(v) + \psi v$$

For example, to compare steady state allocations, the appropriate weight is given by $\psi = \frac{\beta_G}{\beta_F}$. The comparative statics for the firm value are unambiguous. Holding fixed ψ, lower comparative advantage of the firm (higher ρ) reduces firm value, particularly for the contracts labelled as $V_{\min}$ and $\bar{V}$. For the government, however, the results are more interesting. The contract $V_{\min}$ becomes strictly more attractive for the government as its relative technology level improves. As shown in Section 4.3 the value derived from the stationary contract is non-monotonic in ρ (see also Figure 2). These stationary comparative statics are indicated by the dashed line. The most favorable contract for the government $V_{\max}$ provides higher payoffs to the government as its relative productivity decreases. In this scenario, the firm makes higher profits in the steady state which enables it to provide more upfront financing to the impatient government. In light of the bargaining game described in Section 4.5, these comparative statics imply that the government prefers to possess low relative ability if its bargaining power is high (contract is close to $V_{\max}$). In the case of low bargaining power, it prefers to possess high relative ability because the contract is close to $V_{\min}$.

[21] An interesting question in the multiple firm setup is whether the government can use the second-best firm technology instead of its autarky technology as a threat point. This can be ruled out if there exists a continuum of firms with the second-best technology. In this case, the second-best producer can always be replaced by a company with the same technology and will never invest.

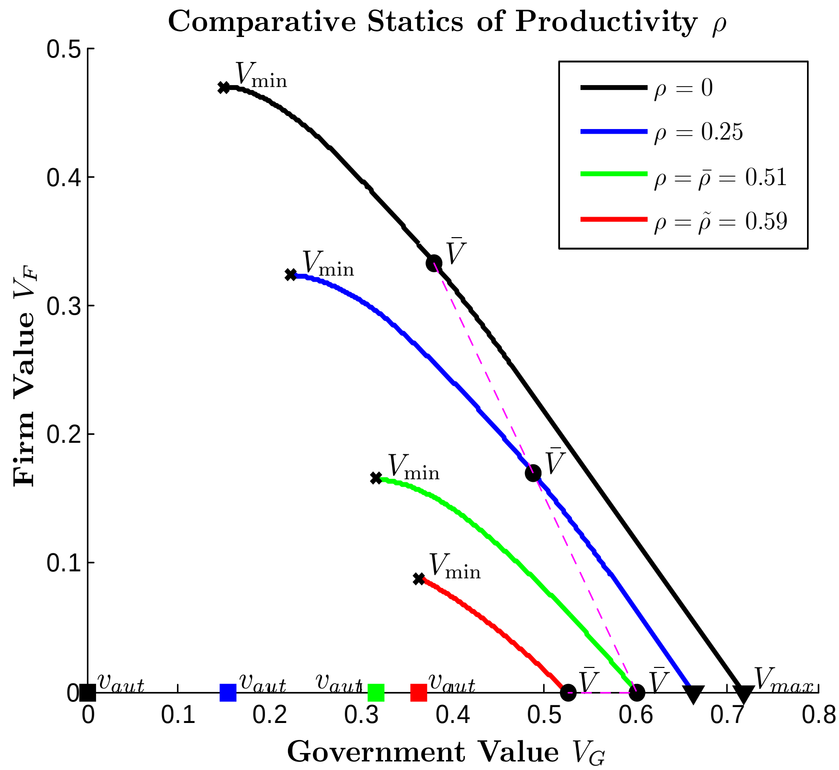

Figure 5: Comparative Statics of Productivity
(Parameters: $\beta_G = 0.7$, $\beta_F = 0.8$ and $\alpha = 0.6$)

5 EXTENSIONS

In this section, I show that the predictions of the basic model are robust to changes in various modeling assumptions. The applications explored in Section 6 do not rely on these extensions. First, I consider the additional implications of stochastic output fluctuations. I find that the optimal transfers and continuation values are positively correlated with output. Provided that the stochastic fluctuations are sufficiently small, the contract converges to a stationary contract as in the deterministic setup. Otherwise, the promised value to the government converges to a unique non-degenerate invariant distribution. In Section 5.2 I consider the implications of renegotiation-proofness in the spirit of van Damme (1991). I also discuss the assumption regarding the relative impatience of the government and the implications of private benefits of control in Section 5.3.

5.1 Stochastic Output

Suppose total factor productivity $\tilde{A}$ follows an i.i.d. process with S possible realizations. Output in state s, which occurs with probability p_s, can be written as:

$$Y_s = A_s Y(I) \tag{40}$$

where $Y(I)$ satisfies Inada conditions. States are ordered according to increasing factor productivity: $A_1 < A_2 < ... < A_S$. I normalize average factor productivity to one: $\sum_s p_s A_s = 1$. Like in the deterministic setup, the problem can be solved recursively and the only state variable of the value function is given by the current period promised value. The latter fact follows from the i.i.d. nature of the stochastic process. In contrast to the deterministic setup, all constraints, except for the promise-keeping constraint, have to hold for each state s. The solution yields the optimal investment level I, state-contingent transfers τ_s, and promised continuation values w_s. The resulting self-enforcing contract is expropriation-proof.

For the purpose of this section, I focus on a parameter constellation that implies the existence of a unique interior stationary contract in the corresponding deterministic setup, i.e. $\rho < \bar{\rho}$. Moreover, I assume that the firm is financially unconstrained such that the constraint on the maximum transfer does not bind. All proofs of this section are moved to Appendix B which covers a self-contained treatment of the stochastic output setup.

Definition 4 *$\bar{v}$ is defined as the promised value v that satisfies $\lambda_{PK}(v) = \frac{\beta_G}{\beta_F}$.*

Thus, the slope of the value function at $\bar{v}$ is determined by the relative impatience level. As in the deterministic setup a financially unconstrained firm only pays taxes to the government if next period's promised value is $\bar{v}$.[22] In addition, the stochastic setup implies that transfers are positively correlated with output. Intuitively, the firm has to make larger payments to the government when the government's outside option from reneging $(A_s Y(I) + \beta_G v_{aut})$ becomes more attractive.

Proposition 7 ***Transfers with i.i.d. Productivity Shocks***
a) Transfers are non-zero, if and only if next period's promised value w_s is given by $\bar{v}$
b) Transfers are positively correlated with output: $Cov(\tau_s, A_s) > 0$

Due to the sorting of the states, the state-contingent transfer τ_s is an increasing function of s. Analogously, the firm must provide the government with strictly higher continuation values when output shocks are more favorable (unless $\bar{v}$ is reached), i.e. the optimal continuation value w_s is also an increasing function of the state.

[22] In the deterministic setup, the transfers of the firm ensure that the promised value reaches the steady state as fast as possible. If the firm is financially unconstrained and $v > \bar{v}$, the continuation value is given by the steady state value $\bar{v}$.

Proposition 8 ***Dynamics with i.i.d. Productivity Shocks***
Continuation values are positively correlated with output: $Cov\left(w_s, A_s\right) \geq 0$.

In contrast to the deterministic setup, the promised value $\bar{v}$ may not be an absorbing state. In this case, continuation values are positively correlated with output, even once the ergodic set is reached.

Proposition 9 ***Unique Stationary Distribution***
The promised value v *converges strongly to a unique invariant distribution with support set between* $\underline{v} \geq v_{\min}$ *and* $\bar{v}$*. Investment is inefficient in the ergodic set.*

It is possible to determine a condition under which the promised value $\bar{v}$ is the deterministic steady state. In this case the unique stationary distribution is degenerate. As a first step, I solve for the stationary transfers in a relaxed problem (indicated with a star) that does not consider the constraint on per period transfers $\tau_{\min}$:

$$\bar{\tau}_s^* = \bar{Y} A_s - \beta_G \left(\bar{Y} - \pi_{aut}\right) \tag{41}$$

where $Y'\left(\bar{I}\right) = \frac{\beta_F}{\beta_G}$. Thus, investment $\bar{I}$ and associated output $\bar{Y}$ are identical to the respective stationary levels in the deterministic setup. Transfers are an increasing affine function of current period output $\bar{Y} A_s$. The expected transfer $E\left(\bar{\tau}_s\right)$ is identical to the transfer in the deterministic setup, such that the stationary promised value to the government is:

$$\bar{v}^* = \frac{E\left(\bar{\tau}_s\right)}{1 - \beta_G} = \bar{Y} + \beta_G v_{aut} \tag{42}$$

Proposition 10 *If* $\bar{\tau}_1^* \geq \tau_{\min} = 0$ *the promised value converges strongly to* $\bar{v}$*. The associated allocation for the stationary promised value* $\bar{v}$ *has the following features:*
a) The investment level $\bar{I}$ *is inefficient and satisfies:* $Y'\left(\bar{I}\right) = \beta_F / \beta_G$
b) State contingent transfers are given by: $\bar{\tau}_s^* = \beta_G \pi_{aut} + \bar{Y}\left(A_s - \beta_G\right)$
c) The stationary value to the government is given by: $\bar{v} = \beta_G v_{aut} + \bar{Y}$

Proof. If $\bar{\tau}_s^* \geq \tau_{\min}\ \forall s$, the relaxed problem also solves the constrained problem. Moreover, as τ_s^* is increasing in s, it is sufficient to check whether $\bar{\tau}_1^* \geq \tau_{\min}$. ■

In this case, the stochastic shocks neither impact total stationary surplus, nor its division relative to the deterministic setup. Figure 6 illustrates the effect of stochastic productivity with two equiprobable states and a mean preserving spread of 10%, 70% and 100%. A mean preserving spread of 10% refers to the case where $A_1 = 0.9$ and $A_2 = 1.1$ occur with probability 0.5. For small volatility (10%), the stationary allocation is deterministic. For a spread of 70% or 100% the condition on $\tau_{\min}$ is not satisfied. An increase in volatility increases the ergodic set of the promised value (indicated by triangles). The respective frontiers are calculated using an extension of the Inner Hyperplane

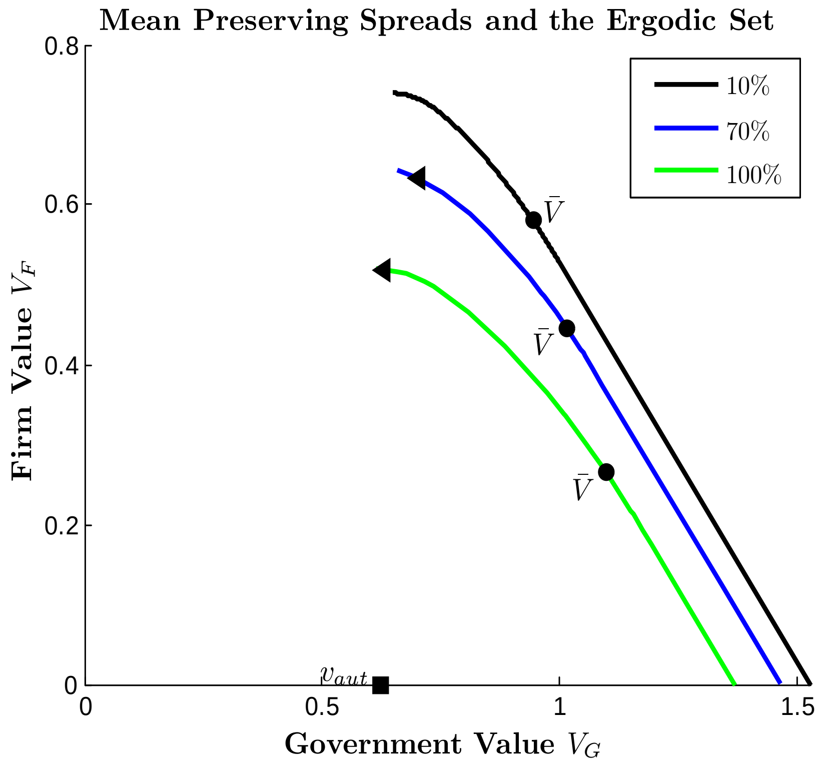

Figure 6: Stochastic Productivity Shocks: Mean Preserving Spreads and the Ergodic Set (Parameters: $\beta_G = 0.8$, $\beta_F = 0.9$, $\alpha = 0.5$, $\rho = 0.5$, $p_1 = p_2 = 0.5$)

Algorithm of Judd et al. (2003) (see Appendix B.2) that allows for stochastic shocks, state dependent actions and different discount factors.

This result can be interpreted in terms of a familiar option pricing analogy. The firm is the underwriter of an American call option which gives the government the right to obtain a payoff equal to the sum of current period output and the discounted autarky value. As the variance of productivity increases, transfers in good states have to increase while the minimum per period transfer is constrained by $\tau_{\min} = 0$. This nonlinearity creates option value which the firm has to deliver.

5.2 Renegotiation-Proofness

I consider the concept of renegotiation-proofness in the spirit of van Damme (1991).[23] The renegotiation-proof set of equilibria can be obtained by using the worst subgame perfect equilibrium on the Pareto frontier as the threat point to induce cooperative behavior. First, consider a relaxed problem in which the constraint on the initial promised value

[23] Another popular concept of renegotiation-proofness in infinite-horizon repeated games has been developed by Farrell and Maskin (1989). Fudenberg and Tirole (1994) compare various concepts in great detail.

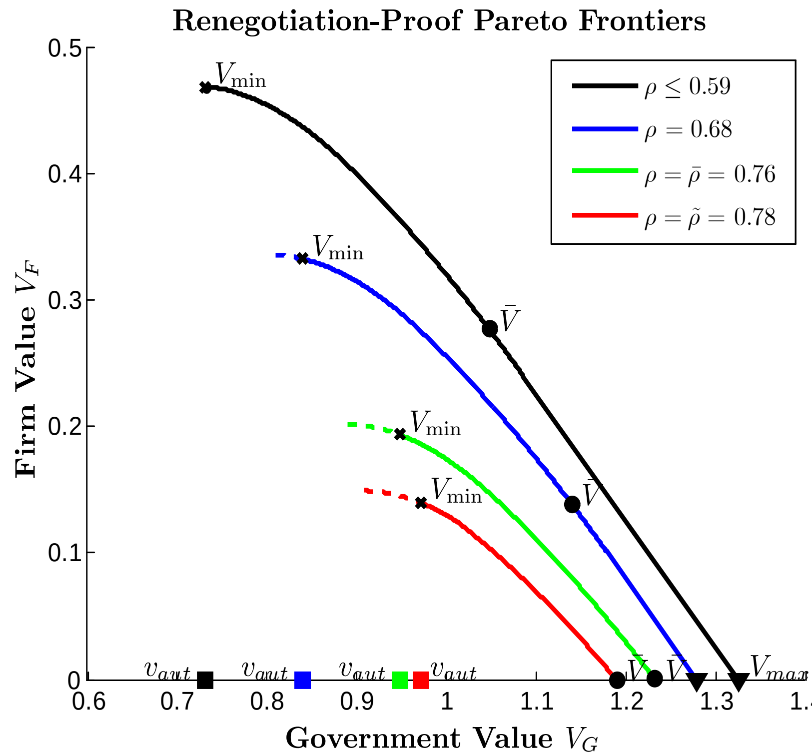

Figure 7: Renegotiation-Proof Comparative Statics of Productivity
(Parameters: $\beta_G = 0.85$, $\beta_F = 0.9$ and $\alpha = 0.6$)

($v_0 \geq v_{aut}$) is ignored. I define the minimum value to the government on the renegotiation-proof frontier $v_F^*(v)$ as $v_{\min}^*$. The problem has the same structure as the original problem (see equation 12), with the difference that $v_{\min}^*$ instead of v_{aut} is used as the threat point to the government. The IC constraint of the government becomes:

$$\tau + \beta_G w \geq Y + \beta_G v_{\min}^* \tag{43}$$

The PC constraint of the firm is unaffected because the worst possible subgame perfect equilibrium on the Pareto frontier from the firm perspective also yields zero profits.[24] The value of $v_{\min}^*$ is given by the fixed point of the problem:

$$v_{\min}^* = \arg\max v_F^*(v) \tag{44}$$

Since the relaxed problem does not consider the autarky constraint, it is necessary to verify whether $v_{\min}^* > v_{aut}$. As long as $v_{\min}^* > v_{aut}$, the renegotiation-proof Pareto frontier is unaffected by the value of v_{aut}. However, if $v_{\min}^* \leq v_{aut}$ the value of v_{aut} is the relevant threat point. Thus, for a sufficiently large autarky value v_{aut}, the original problem (see

[24] The IR constraint of the government: $w > v_{\min}^*$ is irrelevant. Rewrite the IC constraint, such that $w - v_{\min}^* \geq \beta_G^{-1}(Y - \tau) > 0$. Since $Y > \tau$, the IR constraint is fulfilled if the IC constraint is satisfied.

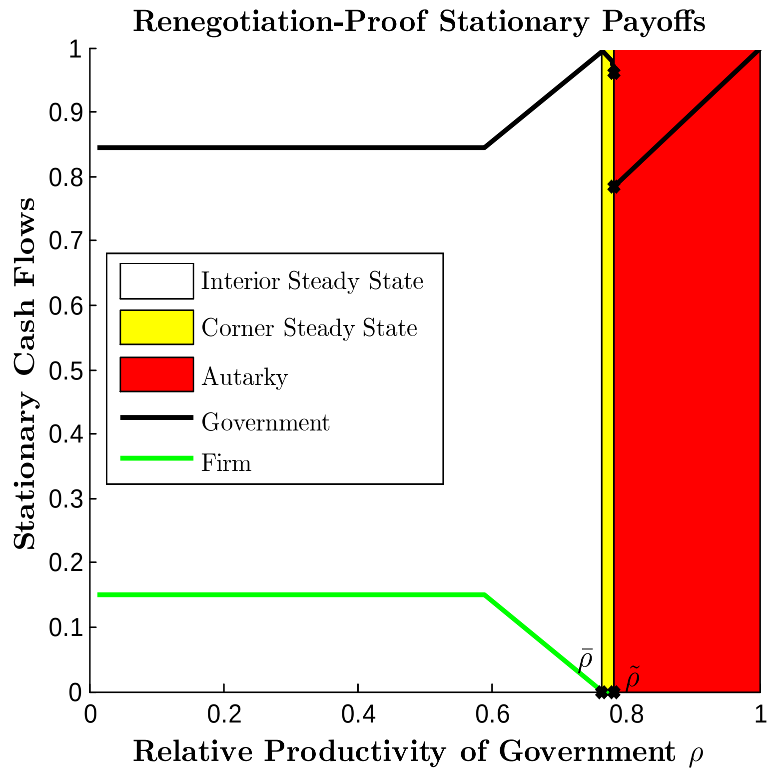

Figure 8: Renegotiation-Proof Long-Run Comparative Statics of Productivity (Parameters: $\beta_G = 0.85$, $\beta_F = 0.9$ and $\alpha = 0.6$)

equation 12) delivers a renegotiation-proof Pareto frontier.[25] These two cases imply that the minimum value on the renegotiation-proof Pareto frontier is given by:

$$v_{\min} = \max\left(v^*_{\min}, v_{aut}\right) \tag{45}$$

The comparative statics of ρ, the determinant of v_{aut}, are illustrated in Figure 7. In contrast to the original setup, the Pareto frontier is identical for all $\rho < 0.59$ in the chosen parametrization. At the threshold value it is true that $v^*_{\min} = v_{aut} = \frac{\rho\hat{\pi}}{1-\beta_G}$. Renegotiation-proofness affects the non-monotonicity result of stationary payoffs in the following way. Using the same parameters as in Figure 7, the autarky productivity has no influence on stationary payoffs if $\rho \leq 0.59$ (see Figure 8). For $\rho > 0.59$ the renegotiation-proof stationary payoffs are enforced with the threat of return to autarky payoffs. As in the basic setup, stationary payoffs to the government are non-monotonic. In contrast to the previous findings, an increase in relative ability does not have positive effects if relative productivity is low.

[25] If the restriction on $\tau_{\min}$ is lifted, the frontier enforced with the threat of autarky is always renegotiation-proof as $v^*_{\min} = 0$.

5.3 Robustness

5.3.1 Assumption on Relative Impatience

First, I consider the case of equal discount factors, i.e. $\beta_G = \beta_F = \beta$. If relative productivity is sufficiently low, i.e. $\rho \leq \bar{\rho}$, efficient investment is reached in the steady state. This result can be obtained by taking the limit of the stationary investment distortions with relative impatience:

$$\lim_{\frac{\beta_G}{\beta_F} \to 1} \bar{\lambda}_{IC} = \lim_{\frac{\beta_G}{\beta_F} \to 1} (1 - \beta_G / \beta_F) = 0 \tag{46}$$

Since the constraint on the IC constraint does not bind in the steady state, the promised value to the government is not uniquely determined. For all promised values that enable efficient investment, i.e. $v > \hat{v} = \hat{Y} + \beta_G v_{aut}$, the dynamics are also indeterminate. For $v < \hat{v}$ promised values are raised each period until the promised value reaches the first-best frontier. If the relative productivity is sufficiently high, i.e. $\bar{\rho} < \rho < \tilde{\rho}$, the stationary contract is a corner contract and backloading is the observed dynamic. Thus, unless the corner stationary contract is reached, common discount factors do neither imply a unique stationary surplus allocation nor the dynamics when the incentive constraint does not bind. It should be noted that my results are valid as long as $\beta_G \leq \beta_F - \varepsilon$ for any $\varepsilon > 0$.

Secondly, I consider the case of a relatively patient government, $\beta_G > \beta_F$. If firm investment is feasible, i.e. $\rho < \tilde{\rho}$, the associated dynamics and the stationary surplus allocation are unique. Backloading occurs until the corner stationary contract is reached. In this case, the effects of efficiency (relative patience of the government) and incentives provision reinforce each other, such that the government obtains all cash flows in the long-run.

Thus, the absence of relative impatience of the government removes the rationale for frontloading. The comparative statics of productivity with respect to feasibility of firm investment and stationary payoffs are qualitatively the same.

5.3.2 Other Cost and Benefits of Expropriation

This paper takes the extreme view that there are no ex post sanctions and/or other associated adjustment cost which would make the return to autarky more costly. On the other hand, the paper also neglects potential private benefits (see Zingales 1994, 1995) from expropriation, such as a control rent in the utility function of the government. Both factors can be incorporated into my model. Let ζ denote the monetarized present value of net benefits (control rent net of cost), one can redefine $v^*_{aut} = v_{aut} + \zeta$ and solve the contracting problem in the described way. A positive value of ζ has the same contracting implications as higher relative productivity.

6 APPLICATIONS

This section shows how the basic workhorse model can be applied in more general settings. In Section 6.1 I show that the model can accommodate upfront cost, such as the cost of building a plant. I choose to tailor this extension to the specifics of the oil industry, which is characterized by a cost-intensive exploration phase followed by a profitable extraction phase. The model provides a rationale for vertical integration of oil companies. In Section 6.2 I allow for multiple sectors and discuss linkage across sectors in the spirit of Bernheim and Whinston (1990). Conglomerate structures across industries may serve as a coordination device to induce joint punishment. In Section 6.3, I show that the contract dynamics are unaffected by expropriation events on the equilibrium path. Expropriations are generated by Markov-type discount rate shocks. In the uncoordinated equilibrium with multiple sectors, the model predicts a pecking order of expropriations. Firms with a low comparative advantage are most susceptible to expropriations.

6.1 Upfront Cost

In many industries where expropriation risk is particularly severe, a large upfront investment is necessary before projects result in positive cash flows. Consider first the simple case where the firm has to incur irreversible upfront cost K, which can be interpreted as the cost of building a plant. Theoretically, the upfront cost represents the "ticket fee" for firm participation in the infinite-horizon game with the government. The ex post Pareto frontier conditional on the sunk investment K and the contract dynamics are described by the basic model. From an ex ante perspective, however, the firm may not choose to invest in the first place. The firm rents from the initial surplus division (see Section 4.5) on the ex post frontier may not be sufficient to cover the upfront investment cost.

Graphically (see Figure 9), the ex ante Pareto frontier is a downward shift of the respective ex post frontier (by K). A necessary condition for firm investment is that the maximum firm value on the ex post frontier $V_F(v_{\min})$ is greater than K.[26] The wedge between the ex ante participation constraint of the firm and the ex post participation constraint has implications for contract dynamics. Higher cost K are more likely to be associated with backloaded contracts as firms only select to invest in projects if the anticipated initial surplus division is favorable.[27]

[26] This condition is necessary and sufficient if the firm possesses full bargaining power, i.e. the contract is initialized at the promised value $v_0 = v_{\min}$.

[27] Upfront investment cost K can also be interpreted as a choice variable for the employed technology. For simplicity, assume there are two ways to set up a plant. The first one uses a highly automated production technology that initially requires high upfront cost K_1, but scales up output in return. The second production technology involves relatively little upfront cost K_2, but requires the presence of (foreign) high-skilled workers. It is reasonable to assume that $\rho_1 > \rho_2$, i.e. the highly automated production technology can be relatively better exploited under autarky than the other one. Even if production technology 1 is better in absolute terms, the comparative advantage of the firm in using production technology 2 may lead the firm to choose the latter one in the presence of expropriation risk.

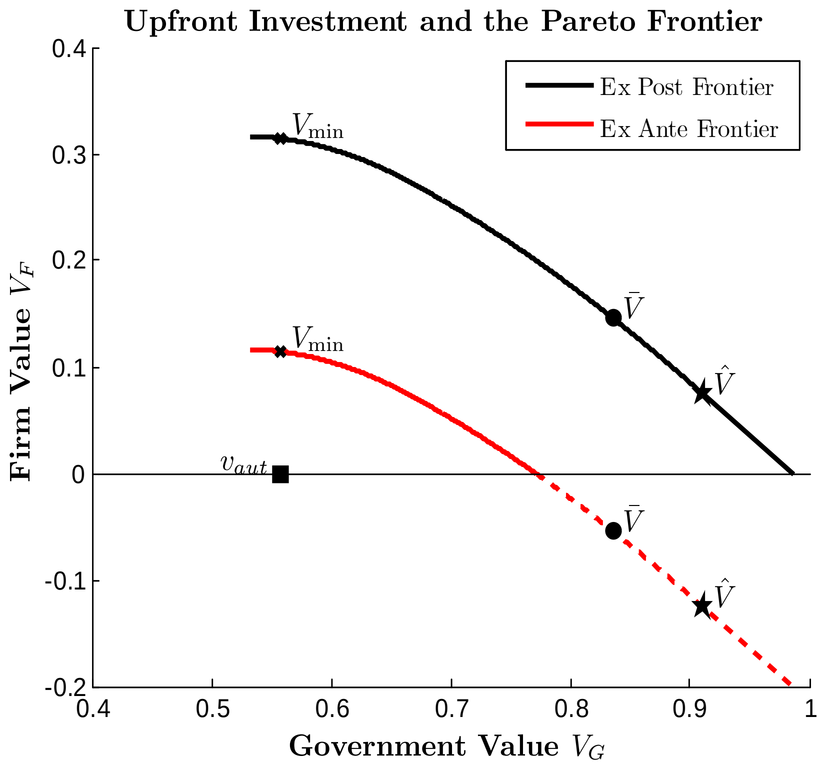

Figure 9: Upfront Investment and the Pareto Frontier
(Parameters: $\beta_G = 0.8$, $\beta_F = 0.9$, $\alpha = 0.6$, $\rho = 0.6$, $K = 0.2$)

To capture the characteristics of the oil industry I introduce a small variation of the analysis.[28] Let K now denote the cost of the exploration phase and q denote the probability of finding oil, the probability of entering the profitable extraction phase. Moreover, let v_0 represent the initial promised value to the government, conditional on a successful exploration phase. Initial investment takes place if and only if:

$$q \cdot V_F(v_0) - K > 0 \tag{47}$$

The probability of success q is a measure of the firm's exploration capability.[29] Condition 47 illustrates that successful firm investment requires simultaneously a high absolute exploration capability (high q) and a high comparative operating advantage in the extraction business (low ρ). A firm that is specialized in the exploration business (say $q = 1$) but only possesses a relative small comparative advantage in developing the field

[28] The oil industry example has some caveats. Within the special oligopoly setting of this industry, nationalization and the associated return to autarky technology may not lead to a large rent destruction for the host country: Technological incompetence of state-owned oil companies acts as a commitment device not to increase production over the agreed upon cartel quotas. This makes it possible to sustain higher prices. From this perspective, the OPEC cartel and the wave of nationalizations in the 1970's are fundamentally connected.

[29] Exploration capability could be equivalently framed in terms of the cost K.

(say $\rho > \tilde{\rho}$) cannot recover the exploration cost.[30] This provides a rationale for vertical integration of exploration and extraction, even in the absence of operating synergies.[31] Consistent with this prediction, all major oil companies ("The Big Five") are engaged in both the exploration as well as the extraction business.[32]

6.2 Multiple Sectors

Suppose that a country consists of J industrial sectors, where each sector j is characterized by relative productivity ρ_j. The sectors are sorted by relative productivity such that $\rho_1 < \rho_2 < .. < \rho_J$.

Definition 5 ***Uncoordinated Equilibrium***
In an uncoordinated equilibrium strategies in any sector j can only depend on the history for the respective sector $h_{t-1,j}$.

If one restricts the analysis to the set of uncoordinated equilibria, the predictions of the basic model are applicable sector by sector. The worst possible threat point is given by return to autarky for each sector. The comparative statics of productivity (see Figure 5) explain the cross section of self-enforcing agreements. Specifically, in any sector j with $\rho_j < \tilde{\rho}$ firm investment is not sustainable.

In light of the findings of Bernheim and Whinston (1990), an ad hoc focus on uncoordinated equilibria is not without loss of generality. Their results suggest that multimarket contacts of duopolists may enhance cooperation to sustain collusive behavior.

Definition 6 ***Linkage Equilibrium***
In a linkage equilibrium strategies in any sector j depend on the history for multiple sectors $h_{t-1,\{i\}}$ where $\{i\}$ is a subset of all sectors.

In this case, the worst possible punishment is given by the threat of autarky in all linked sectors.[33] This punishment path is subgame perfect, as autarky itself is subgame perfect (see Proposition 1). Joint punishment makes expropriation less attractive because a government cannot cherry pick its targets. As a result, technology intensive sectors (low ρ) have a positive externality on other sectors, possibly enabling sustainable firm investment in high ρ sectors. These externalities are stronger if relative productivities vary

[30] The validity of this statement hinges on the assumption that the government cannot provide sufficient upfront financing. However, even if it could, it is questionable whether large upfront financing to the firm would provide the right incentives to the firm.

[31] A more general theory of vertical integration is provided by Hart and Moore (1986).

[32] It is possible that a company with a high comparative advantage in the extraction business uses specialized subcontractors for the exploration phase. This assumes that firms can write legally enforceable contracts with each other. The "Big Five" is an informal expression for the following companies: ExxonMobil, Royal Dutch Shell, BP, Chevron and ConocoPhillips.

[33] The associated threat becomes more effective the greater the number of participating sectors.

considerably across sectors. In contrast to the uncoordinated equilibrium, the relative size of sectors, which is to some degree endogenous in the model, matters. If the mass of high-technology firms is low, linkage only provides a limited effect on the government's incentive to expropriate.[34] From this perspective, the curse of resource-rich countries is that a dominant share of their economies is exogenously concentrated in high ρ sectors which reduces the effectiveness of joint punishment.

Which equilibrium outcome should be expected? I first consider the case where firms in different sectors cannot write legally enforceable contracts on joint punishment and informal ties are absent. In this situation, joint punishment is unlikely to be sustainable. For simplicity, suppose there are only two sectors with $\rho_1 < \tilde{\rho} < \rho_2$ such that a linkage equilibrium enables firm investment in sector 2. By continuity, this is always possible as long as the weight of sector 1 is sufficiently large.[35] Since $\rho_2 > \tilde{\rho}$ the uncoordinated equilibrium features no firm investment in sector 2. Now suppose that both firms have invested in period t in anticipation of a linkage equilibrium and the government expropriates firm 2 while leaving the agreed-upon amount to firm 1. What is the appropriate response of firm 1 in period $t+1$? If firm 1 believes that the government will expropriate it in the subsequent period(s), the prescribed punishment equilibrium (leaving the country) is subgame perfect. However, if the government had wanted to expropriate firm 1, it could have already done so in period t. As firm 1 cannot be punished by firm 2, wouldn't it be reasonable for firm 1 to stay in the country and not abandon the profitable project? This rationale eliminates the linkage equilibrium. In this setup, it is difficult to sustain joint punishment because firm 1 is always less prone to being expropriated than firm 2.[36]

Formal or informal ties across sectors enhance the sustainability of joint punishment. In the most extreme form, firms within a conglomerate may credibly threaten to jointly punish because punishment can be effectively executed under single ownership. From a firm perspective, there exists a strong rationale for non-horizontal integration. Looser connections through alliances or semi-formal associations (such as chambers of commerce) may also help to coordinate joint punishment.[37] From a government perspective, the active encouragement of coordination across sectors would serve as a commitment device.

It is unclear whether the government is better off in the uncoordinated equilibrium or in the linkage equilibrium with the maximum number of participating sectors. This depends on whether the government obtains a sufficient fraction from the greater surplus in the linkage equilibrium to compensate for the reduced threat point. Specifically, if its bargaining power is low, the government is better off in the uncoordinated equilibrium, because it can extract high transfers (albeit from a smaller number of sustainable sec-

[34] The effect of relative sector size can be so extreme, that no firm investment is sustainable in a linkage equilibrium with all sectors, even though some small high-technology sectors would be sustainable on a stand-alone basis. One can eliminate this extreme outcome by allowing for sub-coalitions among higher technology sectors.

[35] For ease of exposition, this weight is assumed to be exogenous.

[36] This result may differ if the relative productivities can change over time in such a way that firm 2 becomes the high-technology firm.

[37] According to Greif et al. (1994) merchant guilds served precisely this purpose in the medieval period.

tors).[38] If the government could choose which sectors to link, it cannot be worse off in a linkage equilibrium.[39] The implications of the two equilibrium concepts for expropriations on the equilibrium path are discussed in the subsequent section.

6.3 Expropriation on the Equilibrium Path

In a non-stationary environment expropriations can occur on the equilibrium path. The term "expropriation" refers to the nationalization of private firms without compensation. The deterministic model suggests that either large productivity shocks or discount rate shocks can cause expropriations. Empirically, expropriations tend to happen almost exclusively after regime changes (see Kobrin 1980, 1984). Since regime changes are presumably unrelated to relative productivity, I use stochastic changes in the discount rate as the identifying shock. For simplicity, I assume that the discount rate shock occurs at the start of the stage game Γ:[40]

Assumption 3 *Each period discount rate shocks are publicly observable before firm investment takes place.*

Otherwise, the timeline is identical to the stationary setup (see Figure 1). I assume that governments only care about cash flows during their tenure, i.e. until a discount rate shock occurs. There are N possible government types characterized by respective discount factors β_1, β_2, ..,β_N. The transition probabilities are summarized in the publicly known time-homogenous matrix $P = [p_{ij}]$, where p_{ij} denotes the probability of moving from state i to state j. This assumption implies that the effective discount factor of government i can be written as:

$$\beta_{Gi} \equiv \beta_i p_{ii} \tag{48}$$

where p_{ii} is the probability that the government stays in power.[41] After a discount rate shock happens, either a new initialization of the contract (renegotiation) or a break-up of the relationship (expropriation) occurs.[42] Analogously to the stationary setup, it is

[38] These ideas about joint punishment are related to the paper of Diamond (2006) who considers the effects of joint punishment in an environment with weak legal protection.

[39] This idea suggests a role for industrial policy. The resulting planning problem solves for the optimum organizational structure of sectors from the perspective of the government.

[40] Since the firm knows the realization of the discount rate shock before its periodical investment, expropriation risk only matters if there are large upfront cost (see Section 6.1). An earlier draft of this paper assumed that the discount rate shock becomes publicly observable after the periodical investment. Under this assumption the analysis becomes more complicated, but delivers no interesting additional insights.

[41] The model implicitly assumes that a government does not take into account a possible return to power after an interim period out of office. Equivalently, one can assume that a future government with discount factor β_i is a different government with the same discount factor. If there is a chance of returning to power later the government becomes effectively more patient. An additional extension is to allow for private information about the respective discount factors. In this setup, learning about the type of the government plays a crucial role.

[42] Any investment adjustment or tax change under a new regime can be interpreted as a renegotiated contract.

possible to solve for the optimal contract dynamics by formulating the problem recursively (see Appendix C.7).

Proposition 11 *The characterization of the optimal dynamic contract is equivalent to the stationary setup.*

Proof. See Appendix C.7. ■

Expropriation on the equilibrium path occurs if the discount factor jumps to a prohibitively low value. In a single sector economy expropriation takes place in all states i where $\tilde{\rho}\left(\beta_{Gi}\right) \equiv \max_I \frac{Y_F(I)-\beta_{Gi}^{-1}I}{\hat{\pi}} < \rho$. In a multiple sector economy (see Section 6.2) the expropriation states and expropriated sectors depend on whether the linkage or the uncoordinated equilibrium is played. In the uncoordinated equilibrium, expropriation in private sector j occurs after a regime change to government type i if and only if:

$$\tilde{\rho}\left(\beta_{Gi}\right) < \rho_j \tag{49}$$

This leads to the following proposition:

Proposition 12 *If the uncoordinated equilibrium is played, expropriations occur according to a pecking order, determined by relative productivity ρ_j. Low-technology sectors are expropriated first.*

This testable proposition (see outline of test in Section 6.4.2) implies that expropriation and privatization follow a "Last-in-First-Out" principle. A sector which has just been expropriated features a higher productivity gap than the sectors expropriated in the previous periods and a lower productivity gap than the sectors which are still private. Once the type of the government changes to a more favorable regime, this sector is the first one that renders private investment feasible again. Sectors with heavy expropriation activity should have a relative productivity low enough to allow for profitable private investments in some states, but high enough to cause expropriation in other states.

In the linkage equilibrium, a strict pecking order may not hold. Consider a simple example with three sectors ($\rho_1 < \rho_2 < \rho_3$) and 2 government types ($\beta_{GL} < \beta_{GH}$). The productivities of the sectors satisfy:

$$\rho_1 < \rho_2 < \tilde{\rho}\left(\beta_{GH}\right) < \rho_3$$
$$\rho_1 < \tilde{\rho}\left(\beta_{GL}\right) < \rho_2 < \rho_3$$

In the uncoordinated equilibrium, firm investment in sector 3 cannot be sustained under either type of government. Firm investment in sector 1 is sustainable for both types of government. If the government switches from β_{GH} to β_{GL}, sector 2 gets nationalized. Vice versa, if the government switches from β_{GL} to β_{GH}, sector 2 gets privatized. This is the pecking order. To highlight the differences of the linkage equilibrium, I assume that sector 2 is relatively large compared to sector 1 while sector 1 is relatively large compared to sector 3. I denote the appropriate weighted average of productivities of linked sectors i and j as $\rho_{i\&j}$ and assume:[43]

$$\begin{aligned} \rho_{1\&3} &< \tilde{\rho}(\beta_{GL}) < \tilde{\rho}(\beta_{GH}) \\ \rho_{1\&2\&3} &> \tilde{\rho}(\beta_{GL}) \end{aligned}$$

Thus, if sector 1 and sector 3 are linked together, both types of government do not expropriate sectors 1 and 3. However, the threat of punishment by sector 1 is not enough to deter the impatient government type (β_{GL}) from expropriating all three sectors if they were linked. This is because sector 2 is relatively large compared to sector 1 and is not sustainable on a stand-alone basis. Thus, linkage across sectors makes firm investment in sector 3 sustainable and causes a violation of the strict pecking order. If the government switches to the impatient type, the intermediate sector 2 is expropriated. This example illustrates how the interaction of size and productivity impacts the pecking order. The pecking order holds if sectors are redefined according to their linkages.

6.4 Empirical Implications

6.4.1 Contract Evidence

Empirical evidence on contract features between foreign firms and sovereign countries is relatively scarce. Fortunately, production sharing agreements, the most common contractual form for petroleum exploration and development, have been analyzed in great detail by Bindemann (1999). Her dataset consists of 268 production sharing agreements signed by 74 emerging market countries between 1966 and 1998. Production sharing agreements are contracts outside of the general taxation system between foreign oil companies and a government. The foreign oil company assumes the entire exploration risk and is rewarded with participation in the extraction phase. Thus, bundling of exploration and extraction is an essential feature of production sharing agreements. This is well captured by my extended model (see Section 6.1) in which the firm has to incur upfront cost K followed by a profitable extraction phase occurring with probability q. According to my model, vertical integration of oil companies helps mitigate the commitment problem of the government.

Upon the start of the extraction phase, transfers to the government are mainly backloaded.[44] Backloading occurs in the form of tax holidays or high initial allowances for cost oil.[45] The dominance of backloading is consistent with the predictions of my model when upfront costs are large. An inherent feature of these agreements is a positive cor-

[43] It only matters that the weighted average satisfies $\rho_i < \rho_{i\&j} < \rho_j$. It is irrelevant for this argument to the determine the precise functional form of $\rho_{i\&j}$.

[44] Frontloading is rarely observed, but can be implemented via signature or discovery bonuses.

[45] Cost oil refers to the share of oil that does not have to be taxed. Suppose cost oil is specified as 60% of production, then the firm only has to pay taxes on 40% of its output (the so called profit oil).

relation between total transfers and output value.[46] This is consistent with the results obtained in the environment with stochastic productivity shocks (see Section 5.1).

Anecdotal contract evidence is also available for other industries. Backloading in the form of tax holidays is a common tool used by countries in Eastern Europe (see Axarloglou and Meanor, 2006) to attract foreign direct investment. For example, when Audi invested in a production plant in Győr (Hungary), it was granted a tax holiday until 2011. Likewise, Poland granted a 10 year tax holiday in 1996 to the GM subsidiary Adam Opel. As many countries in Eastern Europe were competing for foreign direct investment at the same time, the firms were in a good bargaining position. Moreover, they had to incur considerable upfront cost for the construction of production facilities. Consistent with the model, backloading was the observed dynamic. Volkswagen's investment in Skoda for \$1.4bn in 1991 exhibited frontloaded patterns, as many automobile producers were interested in this particular investment.

My model can also be used to analyze the implicit agreements between a government and domestic firms or citizens. Consistent with a model of no commitment, taxation schemes in virtually all countries require firms with higher profits to pay higher taxes, i.e. non lump-sum taxes.[47] Moreover, the model gives a rationale for tax-loss carry-forwards which can be observed in many tax codes. Tax-loss carryforwards generate positive covariance between current period profits and the discounted value of future tax payments (see Section 5.1). Current period losses imply lower future tax payments. Thus, existing taxation codes can be viewed as a constrained efficient solution to the commitment problem of the government.

6.4.2 Pecking Order of Expropriation

This section provides a recipe for testing the pecking order prediction using within-country variation in technology intensity.[48] Controlling for the existing stock of sectoral foreign direct investment in each country, the model predicts that sectors with low productivity gaps are selected first. To make the problem testable, I assume that the relative productivity of country c in sector s can be separated into a country component and a sector specific component (common across countries) which can be written as a decreasing function g of the technology intensity for that sector, denoted as t_s.

$$\rho_{sc} = f_c \cdot g(t_s)$$

Since the country component f_c is irrelevant for the within-country variation, it is sufficient to find an appropriate measure of the technology intensity t_s. A natural proxy for technology intensity is given by the "typical" share of revenues that is spent on R&D. Analogous to Rajan and Zingales (1998), one can identify technology-intensive sectors

[46] Transfers to the government are a function of numerous parameters (royalty rate, tax rate, cost oil, profit oil, etc., see Bindemann, 1999), which is beyond the scope of this paper. Profit oil represents the share of production that is taxed.

[47] The economic rationale for non-lump sum taxes is not a unique implication of my model.

[48] Discount rates differ across countries, so that it is necessary to have country fixed effects.

as the corresponding sectors in which publicly traded U.S. firms spend a large share of their revenues on R&D.

To my knowledge, this empirical study has not been conducted, though the findings of Kobrin (1980, 1984) are suggestive. He finds that extractive and utilities companies face a significantly higher risk of being expropriated than manufacturing sectors. The average R&D intensity of firms in the extractive sector and the utility sector is 1.1% and 0.1%, respectively, vs. 3.3% in the manufacturing sector.[49]

7 CONCLUSION

This paper studies repeated interactions between a firm and a government in an environment where neither party can commit to a contract. The government can unilaterally seize all firm output while the technologically superior firm can refuse to invest. In this environment, weak relative technological ability of the government effectively reduces its commitment problem and makes ex ante firm investment more likely. The key result of the paper is that the government may not be better off if its incentive problem is reduced. A lower commitment problem increases investment efficiency but lowers the threat point used to extract cash flows from the firm. In the unique stationary allocation, either effect can dominate. When the government is weak, an increase in its relative ability is beneficial. However, when it becomes "too strong", firm participation can only be sustained at the cost of greater investment distortions.

The model makes predictions about the dynamics of optimal self-enforcing contracts. Backloading of taxes and investment occurs if the firm is better off at the time of the initial surplus division than in the steady state. Otherwise, frontloading is optimal. The paper considers various determinants, such as upfront cost and competition among countries or firms, that determine the initial division of surplus and therefore the contract dynamics. When frontloading occurs, the firm acts as a sovereign debt lender. The technology advantage in production gives the firm an effective sanction mechanism without which this loan would not be feasible (see Bulow and Rogoff, 1989).

The paper derives testable implications about expropriations. Expropriations should follow a pecking order, with low-technology intensive sectors at the top. The outlined test (see Section 6.4.2) provides a useful benchmark hypothesis for the empirical analysis of expropriations, which is largely unexplored in the current literature. Moreover, it would be interesting to study empirically whether and how firms coordinate on joint punishment to deter the government from expropriation.

On the theory side, incorporating learning dynamics about technology into the model

[49] The numbers are from Compustat for the year 2005. The extractive sector is represented by firms with SIC-Code: $1000-1499$, the manufacturing sector is represented by SIC-Code: $2000-2999$ and the utility sector is represented by: $4900-4999$. According to a study by Congressional Budget Office (2006), the R&D intensity of the pharmaceuticals and communications equipment sector are 19% and 14%, respectively.

would be an interesting extension. Anecdotal evidence suggests that theft of proprietary technology is an important dimension of expropriation risk. The current analysis implies that learning of technology can only occur on the equilibrium path as long as it is not "too fast". Otherwise the firm does not invest in the first place. International automobile producers manage these learning dynamics by producing outdated models in countries where they perceive the risk of technology expropriation to be high. The future access to firm technology prevents the government from taking over current assets.

A NOTATION

Variable	Formula	Meaning
α		Concavity parameter of the production function in all figures
β_G		Discount factor of government
β_F		Discount factor of firm
$\bar{I}$	$\left(\frac{\beta_G}{\beta_F}\alpha\right)^{\frac{1}{1-\alpha}}$	Stationary investment level
$\hat{I}$	$\alpha^{\frac{1}{1-\alpha}}$	Efficient investment level
π_{aut}	$\rho\hat{\pi}$	Per-period profit under autarky
ρ		Relative productivity of government
$\tilde{\rho}$		Maximum ρ that makes firm investment feasible
$\bar{\rho}$	$\left(\frac{\beta_G}{\beta_F}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-\frac{\alpha}{\beta_F}}{1-\alpha}$	Maximum ρ that guarantees interior steady state
τ		Per period transfer to the government
v		Current period promised value to government
v_{aut}	$\frac{\pi_{aut}}{1-\beta_G}$	Discounted present value under autarky
$\bar{v}$		Value to government in the steady state
$\hat{v}$		Threshold promised value that enables efficient investment
w		Optimal continuation value to the government
$\bar{Y}$	$\bar{I}^\alpha$	Stationary output level
$\hat{Y}$	$\hat{I}^\alpha$	Efficient output level

The formulas refer to the specific production technology $Y_F(I) = I^\alpha$.

B STOCHASTIC PRODUCTIVITY

B.1 Theory

The stochastic setup relaxes several assumptions of the basic model. Output depends on an exogenous productivity shock A_s $(s = 1, .2...S)$ occurring with probability p_s, such that output in state s $(Y_s(I))$ satisfies the following assumptions:

Assumption 4 *$Y_s(I) = A_s Y(I)$ where $Y(I)$ satisfies Inada conditions and $\sum p_s A_s = 1$.*

Assumption 5 *The government technology produces expected autarky profits of π_{aut}, generating an autarky value of $v_{aut} = \pi_{aut}/(1-\beta_G)$.*

Assumption 6 *The firm is financially unconstrained, such that the restriction on $\tau_{\max}$ does not bind.*

Assumption 6 implies that the firm has access to sufficient outside capital to finance any profitable project. I use the following shorthand notation:

$$w_s = w(v, s) \tag{50}$$
$$\tau_s = \tau(v, s) \tag{51}$$

Thus, the state contingent transfers τ_s and continuation values w_s are functions of the current promised value v. Let $V_F(v)$ represent the expected firm profits given a promised expected value of v to the government. As in the static setup, the problem can be written recursively:

$$V_F(v) = \max_{I,\tau_s,w_s} E\left[Y_s(I) - I - \tau_s + \beta_F V_F(w_s)\right] \text{ s.t.} \tag{52}$$

#	Constraint	Lagrange multiplier	
1)	$E[\tau_s + \beta_G w_s] \geq v$	λ_{PK}	
2)	$\tau_s + \beta_G w_s \geq A_s Y(I) + \beta_G v_{aut}$	$p_s \lambda_{ICs}$	$s = 1, 2..S$
3)	$V_F(w_s) \geq 0$	$p_s \beta_F \lambda_{PCs}$	$s = 1, 2..S$
4)	$w_s \geq v_{aut}$	$p_s \beta_G \lambda_{IRs}$	$s = 1, 2..S$
5)	$\tau_s \geq 0$	$p_s \mu_{\min s}$	$s = 1, 2..S$

Lemma 7 *The value function $V_F(v)$ is strictly decreasing and concave in the Pareto region.*

Proof. See Proof of Lemma 2 in Appendix C.1. ■

Lemma 8 *If $v_{\min} > v_{aut}$ then $\lambda_{IRs} = 0$ $\forall s$.*

Proof. If $v_{\min} > v_{aut}$ it follows that $\lambda_{PK}(v_{\min}) = 0$. At $v_{\min}$ the transition equation is equal to:

$$\lambda^*_{PK} = \frac{\beta_G}{\beta_F}\left(\lambda_{ICs} + \lambda_{IRs}\right) \geq 0 \tag{53}$$

Since next period's slope is non-negative (in absolute value), the associated continuation value must be greater or equal to $v_{\min}$, which is greater than v_{aut}, such that $\lambda_{IRs} = 0$. ■

The Lagrangian can be written as:

$$\begin{aligned} L = & \sum_{s=1}^{S} p_s \left[A_s Y(I) - I - \tau_s + \beta_F V_F(w_s)\right] \\ & + \lambda_{PK}\left[\sum_{s=1}^{S} p_s \left[\tau_s + \beta_G w_s - v\right]\right] \\ & + p_s \lambda_{ICs}\left[\tau_s + \beta_G w_s - A_s Y(I) - \beta_G v_{aut}\right] \\ & + p_s \beta_F \lambda_{PCs} V_F(w_s) + p_s \beta_G \lambda_{IRs}\left[w_s - v_{aut}\right] + p_s \mu_{\min s} \tau_s \end{aligned} \tag{54}$$

The first-order conditions are:

$$\begin{array}{llr} I: & E\left[A_s Y'(I)(1-\lambda_{ICs})\right]-1 & =0 \\ \tau_s: & p_s\left[-1+\lambda_{PK}+\lambda_{ICs}+\mu_{\min s}\right] & =0 \\ w_s: & \beta_F p_s\left[V_F'(w_s)(1+\lambda_{PCs})+\frac{\beta_G}{\beta_F}(\lambda_{PK}+\lambda_{ICs}+\lambda_{IRs})\right] & =0 \end{array} \tag{55}$$

The first-order condition on investment can be rewritten as:

$$Y'(I)=\frac{1}{1-E(A_s\lambda_{ICs})} \tag{56}$$

Using the envelope condition $V_F'(w_s)=\lambda_{PK}^*$, the law of motion for the slope of the value function becomes:

$$\lambda_{PK}^*=\frac{\beta_G}{\beta_F}\frac{\lambda_{PK}+\lambda_{ICs}+\lambda_{IRs}}{1+\lambda_{PCs}} \tag{57}$$

Definition 7 *Let $\bar{v}$ denote the value of v, such that $\lambda_{PK}(\bar{v})=\frac{\beta_G}{\beta_F}$.*

Definition 8 *$\hat{v}$ is the smallest v, such that efficient investment possible.*

Whether $\hat{v}$ violates the firm participation constraint does not matter at this point.

Lemma 9 *For all v that satisfy $\hat{v}<v<v_{\max}$:*
a) The current period slope is given by $\lambda_{PK}=1$
b) The continuation value is given by $\bar{v}$ with $\lambda_{PK}(\bar{v})\equiv\bar{\lambda}_{PK}=\frac{\beta_G}{\beta_F}$

Proof. Part a) Since investment is undistorted, value can be exchanged one-to-one. Part b) follows from the fact that feasibility of efficient firm investment means $\lambda_{ICs}=\lambda_{IRs}=\lambda_{PCs}=0\ \forall s$ and $\lambda_{PK}=1$. Using the transition equation 57 we obtain: $\lambda_{PK}^*=\frac{\beta_G}{\beta_F}$. ■

Lemma 10 *The continuation value is given by $\bar{v}$ if and only if the constraint on $\tau_{\min}$ does not bind in state s, i.e. $\mu_{\min s}=0$.*

Proof. Rewrite the transition equation 57 by using the first-order condition on the transfers. In the relevant region ($\lambda_{PCs}=\lambda_{IRs}=0$) the transition equation becomes:

$$\lambda_{PK}^*=\frac{\beta_G}{\beta_F}(1-\mu_{\min s}) \tag{58}$$

Hence, if $\mu_{\min s}=0$, we have: $\lambda_{PK}^*=\frac{\beta_G}{\beta_F}=\bar{\lambda}_{PK}$. ■

Thus, transfers to the government only occur if next period's promised value is given by $\bar{v}$.

Lemma 11 *The IC constraint is given by* $\lambda_{ICs} = 1 - \lambda_{PK}$ *if and only if the constraint on* $\tau_{\min}$ *does not bind in state* s*, i.e.* $\mu_{\min s} = 0$.

Proof. Use the first-order condition on the transfer to obtain:

$$\lambda_{ICs} = 1 - \lambda_{PK} - \mu_{\min s} \tag{59}$$

The result immediately follows. ■

It is helpful to determine the continuation value if the IC constraint does not bind in state s.

Definition 9 $w_{IC0}(v)$ *defines the optimal continuation value for all states* s*, such that* $\lambda_{ICs}(v) = 0$

Lemma 12 $w_{IC0} = \max\left[\lambda_{PK}^{-1}\left(\lambda_{PK}(v)\frac{\beta_G}{\beta_F}\right), v_{aut}\right]$

Proof. Whenever the IC constraint in state s does not bind, the transition law implies that:

$$\lambda_{PK}^{*} = \frac{\beta_G}{\beta_F}\left(\lambda_{PK} + \lambda_{IRs}\right) \tag{60}$$

Thus, the continuation value must be either w, such that $\lambda_{PK}(w) = \frac{\beta_G}{\beta_F}\lambda_{PK}(v)$, or the autarky value (when $\lambda_{IRs} > 0$). ■

Lemma 13 ***Continuation Value Policy***

$$w(v,s) = \begin{cases} \max\left(\min\left(\beta_G^{-1}A_sY(v) + v_{aut}, \bar{v}\right), w_{IC0}\right) & \text{for} \quad v < \hat{v} \\ \bar{v} & \text{for} \quad v \geq \hat{v} \end{cases} \tag{61}$$

Proof. If the IC constraint binds and $\bar{v}$ is not reached, transfers are 0 by Lemma 10. A binding IC constraint implies:

$$\tau_s + \beta_G w_s = A_sY(I) + \beta_G v_{aut} \tag{62}$$

such that: $w_s = \beta_G^{-1}A_sY(v) + v_{aut}$. Unless the transfers are 0, the continuation value is $\bar{v}$. If the IC constraint does not bind, then $w = w_{IC0}$. ■

Thus, in all states where the incentive constraint binds ($\lambda_{ICs} > 0$) the continuation value is given by: $\min\left(\beta_G^{-1}A_sY(v) + v_{aut}, \bar{v}\right)$. Note, that if there is only one state (as in the basic model) the result collapses to $\min\left(\beta_G^{-1}v, \bar{v}\right)$. In the stochastic setup, it is also possible that the IC constraint does not bind, in which case $w_s = w_{IC0}$.

Corollary 2 *Continuation values are positively correlated with output, i.e.* $\forall j \geq 1$

$$\begin{array}{ll} w_s = w_{s-j} & \text{if } \lambda_{ICs} = 0 \text{ or } w_{s-j} = \bar{v} \\ w_s > w_{s-j} & \text{else} \end{array} \tag{63}$$

Proof. This follows directly from Lemma 13. ■

Proposition 13 ***Dynamic Transfer Policy***

$$\tau(v,s) = \begin{cases} \tau_{\min} = 0 & \text{for} \quad w_s < \bar{v} \\ A_s Y(I) - \beta_G(\bar{v} - v_{aut}) & \text{for} \quad w_s = \bar{v} \text{ and } v < \hat{v} \\ \tilde{\tau}_s & \text{for} \quad w_s = \bar{v} \text{ and } v \geq \hat{v} \end{cases} \tag{64}$$

where $\tilde{\tau}_s \geq A_s Y(I) - \beta_G(\bar{v} - v_{aut})$ *and* $\sum p_s \tilde{\tau} + \beta_G \bar{v} = v$.

Proof. If transfers are paid in state s the IC constraint is given by: $\lambda_{ICs} = 1 - \lambda_{PK}$. Thus, if $v < \hat{v}$ and hence $\lambda_{PK} < 1$, the IC constraint binds which implies:

$$\tau_s + \beta_G \bar{v} = A_s Y(I) + \beta_G v_{aut} \tag{65}$$

This yields the optimal transfer. If $v \geq \hat{v}$, the IC constraint does not bind in any state and the exact transfer is indeterminate. There exist many transfer schedules $\tilde{\tau}_s$ that satisfy the IC constraint in each state and the promise-keeping constraint $\sum p_s \tilde{\tau} + \beta_G \bar{v} = v$. ■

Lemma 14 *Investment is strictly increasing in* v *for* $v < \hat{v}$.

Proof. Case 1) First consider the case where the IC constraint binds in all states. Substituting the state-by-state IC constraints into the PK constraint yields:

$$\begin{aligned} v &= \sum_{s=1}^{S} p_s \left[A_s Y(I) + \beta_G v_{aut}\right] \\ &= \beta_G v_{aut} + Y(I) \end{aligned} \tag{66}$$

Thus, expected output Y (and as such investment) is strictly increasing in v.

Case 2) Now consider the case where the IC constraint binds in some states, $s^*+1, ...S$. Whenever the IC constraint does not bind, current period transfers are transfers are zero and the continuation value is given by $w_{IC0} = \max\left(\lambda_{PK}^{-1}\left(\lambda_{PK}(v)\frac{\beta_G}{\beta_F}\right), v_{aut}\right)$. In this case, the current promised value is given by:

$$\begin{aligned} v &= \sum_{s=1}^{S} p_s \left[\tau_s + \beta_G w_s\right] \\ &= \beta_G p_{IC0} w_{IC0} + \sum_{s=s^*+1}^{S} p_s \left[A_s Y + \beta_G v_{aut}\right] \end{aligned} \tag{67}$$

where $p_{IC0} \equiv \sum_{s=1}^{s^*} p_s$. If $w_{IC0} = v_{aut}$, the implicit function theorem implies:

$$\frac{dY}{dv} = p_s A_s > 0 \tag{68}$$

If $w_{IC0} = \lambda_{PK}^{-1} \left(\lambda_{PK} (v) \frac{\beta_G}{\beta_F} \right)$, the implicit function theorem implies:

$$\frac{dY}{dv} = \frac{p_s A_s}{1 - \beta_G p_{IC0} \frac{dw_{IC0}}{dv}} > 0 \tag{69}$$

Thus, expected output Y (and as such investment) is strictly increasing in v.

Case 3) If the IC constraint does not bind in any state ($v > \hat{v}$), the first-order condition on investment implies:

$$Y'(I) = 1 \tag{70}$$

In this case, output and investment are independent in v. ■

Definition 10 *$P^N(v, A)$ defines the N-period transition function from v to a measurable set A that is implied by the policy function $w(v, s)$ and the exogenous stochastic process for s.*

Lemma 15 *Independent of the current promised value v, the value $\bar{v}$ will be reached with probability 1, i.e. there exists an integer N and $\varepsilon > 0$ such that $P^N(v, \bar{v}) \geq \varepsilon \; \forall v$.*

Proof. If $\lambda_{PK} = 1$, we will reach $\bar{v}$ in one step (see Proposition 9), so we can restrict ourselves to the case where $\lambda_{PK} < 1$. Consider the path in which the highest state S occurs at each point in time. If $\bar{v}$ is never reached, then transfers must be zero at all times in state S (see Lemma 11). However, then transfers must be zero in all other states. Thus, the government would never obtain transfers. This is impossible (the proof is essentially the same as in Thomas and Worrall, 1994). ■

Definition 11 *Let Υ define the closed set $[v_{\min}, v_{\max}]$.*

Definition 12 ***Condition M***
There exists an $\varepsilon > 0$ and an integer $N \geq 1$ such that for any measurable set A, either $P^N(v, A) \geq \varepsilon$, all $v \in \Upsilon$, or $P^N(v, A^c) \geq \varepsilon$, all $v \in \Upsilon$ (see Lucas and Stokey, 1989).

Lemma 16 *$P(v, A)$ satisfies Condition M.*

Proof. For any set A we either have $\bar{v} \in A$ or $\bar{v} \notin A$. Suppose $\bar{v} \in A$, then by Lemma 15 $P^N(v, A) \geq \varepsilon \; \forall v$. Now, suppose $\bar{v} \notin A$, then by Lemma 15 $P^N(v, A^c) \geq \varepsilon \; \forall v$. ■

Proposition 14 *The promised value v converges strongly to a unique invariant distribution with support set between $\underline{v} \geq v_{\min}$ and $\bar{v}$. Investment is inefficient in the ergodic set.*

Proof. As Condition M is satisfied, Theorem 11.12. of Lucas and Stokey (1989) implies the above statement. The constraint on the lower bound can be obtained as follows. The transition equation in the ergodic set is given by:

$$\lambda_{PK}^{*}=\frac{\beta_G}{\beta_F}\left(\lambda_{PK}+\lambda_{ICs}+\lambda_{IRs}\right) \tag{71}$$

Consider first the case that $v_{\min}>v_{aut}$. By Lemma 8 $\lambda_{IR_s}=0$, such that $\lambda_{PK}^{*}=\frac{\beta_G}{\beta_F}\left(\lambda_{PK}+\lambda_{ICs}\right)\geq 0$. In this case $w\geq v_{\min}$ for any v on the Pareto frontier. Next consider the case that $v_{\min}=v_{aut}$, then the lowest feasible continuation value is given by: $v_{\min}=v_{aut}$. Hence, the lower bound of the ergodic set $\underline{v}$ must be greater or equal than $v_{\min}$.

Now, consider the upper bound. Suppose the current period slope is equal to 1, then by Lemma 9 the next period's slope λ_{PK}^{*} is given by $\bar{\lambda}_{PK}$ for all states s. Suppose the current period slope is given by $\bar{\lambda}_{PK}$ or lower, then next period's slope will be equal to $\bar{\lambda}_{PK}$ or lower. Therefore, by monotonicity for any current period slope between $\lambda_{PK}\left(v_{\min}\right)$ and 1, next period's slope is bounded above by $\bar{\lambda}_{PK}$. Therefore, the upper bound of the ergodic set is given by $\bar{v}$.

Suppose investment was efficient, then the current period slope must be equal to one by Lemma 9. However, as just described, λ_{PK} is bounded above by $\beta_G/\beta_F<1$ in the ergodic set. This is a contradiction. ■

Lemma 17 ***Stationarity Condition***
An interior stationary contract on the Pareto frontier exists if and only if $\beta_G\pi_{aut}+\bar{Y}\left(A_1-\beta_G\right)\geq\tau_{\min}=0$.

Proof. By Lemma 10, it is sufficient to check whether the constraint on the minimum transfer does not bind in all states s. Moreover, by Lemma 11, it must be the case that the IC constraint binds in all states $\left(\lambda_{ICs}=1-\bar{\lambda}_{PK}\right)$ when the current promised value is equal to $\bar{v}$. A binding IC constraint in all states implies:

$$\bar{\tau}_s+\beta_G w_s=A_s Y\left(I\right)+\beta_G v_{aut} \tag{72}$$

where stationary transfers in state s are denoted as $\bar{\tau}_s$. Imposing stationarity yields for the continuation value $w_s\left(\bar{v}\right)=\bar{v}$:

$$\bar{v}=\frac{\sum_{s=1}^{S}p_s\bar{\tau}_s}{1-\beta_G} \tag{73}$$

Thus, we obtain a linear system of S equations for the S stationary transfers $\bar{\tau}_s$. The solution is:

$$\bar{\tau}_s^{*}=\beta_G\pi_{aut}+\bar{Y}\left(A_s-\beta_G\right) \tag{74}$$

It only remains to be checked whether $\bar{\tau}_s^{*}>\tau_{\min}$ for $\forall s$. Since $\bar{\tau}_s^{*}$ is monotonic in s, it is sufficient to confirm that the condition holds in the worst state $s=1$. ■

Proposition 15 *Suppose the stationarity condition holds, then the stationary contract has the following properties:*
a) The investment level $\bar{I}$ *is inefficient and satisfies:* $Y'\left(\bar{I}\right) = \beta_F/\beta_G$
b) State contingent transfers are given by: $\bar{\tau}_s^* = \beta_G \pi_{aut} + \bar{Y}\left(A_s - \beta_G\right)$
c) The stationary value to the government is given by: $\bar{v} = \beta_G v_{aut} + \bar{Y}$

Proof. Part a) Since the contract is stationary, Lemma 11 implies that: $\lambda_{ICs} = 1 - \bar{\lambda}_{PK}$ $\forall s$. The first-order condition on investment becomes:

$$Y'\left(\bar{I}\right) = \frac{1}{1 - E\left(A_s\left(1 - \bar{\lambda}_{PK}\right)\right)} = \frac{1}{\bar{\lambda}_{PK}} = \frac{\beta_F}{\beta_G} \tag{75}$$

Due to the assumed Inada conditions there exists a unique (inefficient) investment level $\bar{I}$ such that $Y'\left(\bar{I}\right) = \frac{\beta_F}{\beta_G}$.

Part b) State contingent transfers ensure that the IC constraint binds in each state (see Proof of Lemma 17).

Part c) Multiplying the stationary transfers $\bar{\tau}_s^*$ with the probability p_s and summing across all states yields an expected transfer of:

$$E\left(\bar{\tau}_s^*\right) = \beta_G \pi_{aut} + \bar{Y}\left(1 - \beta_G\right) \tag{76}$$

The stationary value is given by:

$$\bar{v} = \frac{E\left(\bar{\tau}_s^*\right)}{1 - \beta_G} \tag{77}$$

■

Thus, the stationary contract in the stochastic setup is identical with the one in the deterministic setup (same investment level, same government and firm value) except for the fact that transfers are now state contingent. While expected transfers are identical to the deterministic setup, they are now positively correlated with output.

B.2 Algorithm

I present a variation of the Inner Hyperplane Algorithm of Judd et al. (2003) which requires discretization of the action set for each player. As in Section 5.1 I assume that stochastic productivity affects output by $Y_s(I) = A_s Y(I)$ where $Y(I)$ satisfies Inada conditions. After discretization, the game can be described by the firm choice of an investment level $I \in \{0, I_1, I_2, ..., \hat{I}\}$ and the government choice of a state contingent transfer $\tau_{(s)} \in \{0, \tau_1, \tau_2, ..., \tau_{\max}\}$. Before starting the algorithm let us define a as the collection of state contingent transfers τ_s and the investment level I of the firm:

$$a = \{\tau_1, \tau_2, ..., \tau_S, I\}$$

Given a the current period payoffs of both players can be determined for each state s:

$$\Pi(a,s) \equiv \begin{bmatrix} \Pi_G(a,s) \\ \Pi_F(a,s) \end{bmatrix} = \begin{bmatrix} \tau_s \\ Y_s(I) - I - \tau_s \end{bmatrix}$$

In line with Judd et al. (2003), I normalize the value function $\check{V}^i = V^{(i)}(1-\beta_i)$ such that values can be interpreted as per period payoffs. The IC constraint of the government and the participation constraint of the firm becomes:

$$(1-\beta_G)\Pi_G(a,s) + \beta_G w_{Gs} \geq (1-\beta_G) Y_s(I) + v_{aut} \tag{78}$$

$$w_{Fs} \geq 0 \tag{79}$$

where w_{is} is the continuation value of player i if state s occurs. Since these constraints have to hold state by state, I introduce the following vectors that make it possible to stack the IC constraints:

$$w_s \equiv \begin{bmatrix} w_{Gs} & w_{Fs} \end{bmatrix}' \tag{80}$$

$$w \equiv \begin{bmatrix} w_1' & w_2' & ... & w_S' \end{bmatrix}' \tag{81}$$

$$IC_s(a) \equiv \begin{bmatrix} -\left(\beta_G^{-1} - 1\right)[Y_s(I) - \Pi_G(a,s)] - v_{aut} & 0 \end{bmatrix}' \tag{82}$$

$$IC(a) \equiv \begin{bmatrix} IC_1' & IC_2' & ... & IC_S' \end{bmatrix}' \tag{83}$$

Thus, given a all the $2S$ constraints (see conditions 78 and 79) can be summarized by the following condition on the vector w :

$$-w \leq IC(a) \tag{84}$$

In addition, the break-even condition of the firm at time 0 has to hold, i.e.:

$$(1-\beta_F) E[\Pi_F(a,s)] + \beta_F E[w_{Fs}] \geq 0 \tag{85}$$

where the expectation has been taken with respect to the probability distribution of A_s determined by the vector p :

$$p \equiv \begin{bmatrix} p_1 & p_2 & ... & p_S \end{bmatrix}' \tag{86}$$

It is useful to define a vector $\tilde{p}$:

$$\tilde{p} \equiv p \otimes \begin{bmatrix} 0 & 1 \end{bmatrix}' \tag{87}$$

The break-even condition can also be expressed as a condition on w :

$$-\tilde{p}'w \leq IR(a) \tag{88}$$

where:

$$IR(a) \equiv \left(\beta_F^{-1} - 1\right) E[\Pi_F(a,s)] \tag{89}$$

I define H to be a $D \times 2$ matrix of subgradients with $h_d \in H$, $d = 1, 2, ..., D$ that determine the search direction. For example, the direction $[\xi, 0]$ gives all weight to the first agent (in this case the government). Since it is only of interest to determine the Pareto frontier, one can restrict the subgradient to positive weights on each agent such that h_d can be written as $h_d = [\xi, 1]$ where ξ is increasing in d. The starting set of sustainable values is defined by D vertices z_0 ensuring that the true Pareto frontier lies in the interior of $co(z_0)$. Let z_0 be gathered in a $D \times 2$ matrix. A particular point $v = \begin{bmatrix} v_G & v_F \end{bmatrix}'$ is in the set $W = co(z_0)$ if and only if:

$$Hv \leq k \tag{90}$$

where $k \equiv diag(H * Z_0\prime)$. Since this condition has to hold for all continuation values, this puts the following restriction on w:

$$(\Lambda_S \otimes H) w \leq \iota_S \otimes k \tag{91}$$

Step 1)

For each subgradient $h_d \in H$, $d = 1.., D$.

(a) For each $a \in A$, determine the optimal continuation policy w. As a is fixed for any maximization problem, this maximization problem is simply:

$$\begin{aligned} \tilde{c}_D(a) = \min_w -(p' \otimes h_D B) w \text{ s. t.} \\ \Theta w \leq \kappa \end{aligned}$$

where

$$B = \begin{bmatrix} \beta_G & 0 \\ 0 & \beta_F \end{bmatrix} \text{ and } \Theta = \begin{bmatrix} -\tilde{p}' \\ -\Lambda_{2S} \\ \Lambda_S \otimes H \end{bmatrix} \text{ and } \kappa = \begin{bmatrix} IR(a) \\ IC(a) \\ \iota_S \otimes k \end{bmatrix} \tag{92}$$

Let $w_D(a)$ be the solution to this linear programming problem. Then:

$$c_D(a) = \begin{cases} (p' \otimes h_D B) w_D(a) + (p' \otimes h_D (\Lambda_2 - B)) \Pi(a) & \text{if solution satisfies the constraints} \\ -\infty & \text{else} \end{cases}$$

where $\Pi(a) = \begin{bmatrix} \Pi(a,1)' & \Pi(a,2)' & ... & \Pi(a,S)' \end{bmatrix}'$

(b) Find the best action profile a and corresponding values

$$\begin{aligned} a_d^* &= \arg\max \{ c_d(a) | \, a \in A \} \\ z_d^+ &= (\Lambda_2 - B) E\Pi(a,s) + BE(w_s) \\ &= p' \otimes (\Lambda_2 - B) \Pi(a_d^*) + p' \otimes Bw_d(a_d^*) \end{aligned}$$

Step 2)

Define new set of vertices $Z^+ = \{ z_d^+ | \, d = 1, ..., L \}$ and define $W^+ = co(Z^+)$. The new set is characterized by a new vector $k^+ = diag(H * Z^+\prime)$. Continue step 1 and step 2 until the difference of k^+ and k is "sufficiently small".

C PROOFS

C.1 Proof of Lemma 2

We can write the respective firm value for promised values v_1 and v_2 (where $v_1 < v_2$) as:

$$V_F(v_1) = Y_1 - I_1 - \tau_1 + \beta_F V_F(w_1) \tag{93}$$
$$V_F(v_2) = Y_2 - I_2 - \tau_2 + \beta_F V_F(w_2) \tag{94}$$

The focus lies on the interior of the Pareto region where λ_{PC} and λ_{IR} do not bind. All the other constraints must be satisfied since the values for τ_i, $I(v_i)$, w_i and $Y_i = Y_F(I_i)$ represent the solution to the contracting problem, i.e.:

$$\tau_i + \beta_G w_i \geq Y_i + \beta_G v_{aut} \tag{95}$$
$$\tau_i + \beta_G w_i = v_i \tag{96}$$
$$\tau_{\min} \leq \tau_i \leq \tau_{\max} \tag{97}$$

Consider the value function at the point $\breve{v} = \gamma v_1 + (1-\gamma) v_2$ for some $0 < \gamma < 1$. Since we want to show concavity, the following relationship has to hold:

$$V_F(\breve{v}) \geq \gamma V_F(v_1) + (1-\gamma) V_F(v_2) \equiv V_F^{\gamma} \tag{98}$$

The proof relies on a standard procedure of identifying a feasible policy that delivers the government a value of $\breve{v}$ and the firm at least a value of V_F^{γ}. Consider the policy $\left(\breve{\tau}, \breve{w}, \breve{I}\right)$ which is defined as:

$$\breve{\tau} = \gamma \tau_1 + (1-\gamma) \tau_2 \tag{99}$$
$$\breve{w} = \gamma w_1 + (1-\gamma) w_2 \tag{100}$$
$$\breve{I} = Y_F^{-1}(\gamma Y_1 + (1-\gamma) Y_2) \tag{101}$$

By construction, the policy satisfies all the conditions specified in equations 95, 96 and 97. It has to be confirmed whether the value derived from this feasible policy (denoted as $\breve{V}_F(\breve{v})$) indeed yields a higher firm value than V_F^{γ}, i.e. $\breve{V}_F(\breve{v}) \geq V_F^{\gamma}$. Rewriting yields:

$$-Y_F^{-1}\left(\breve{Y}\right) + \beta_F V_F(\breve{w}) \geq -\left[\gamma Y_F^{-1}(Y_1) + Y_F^{-1}((1-\gamma) Y_2)\right] + \gamma \beta_F V_F(w_1) + (1-\gamma) \beta_F V_F(w_2)$$

which is satisfied due to the strict concavity of the production function (convexity of the inverse function). Note, that only as long as $Y_1 = \hat{Y}$ (i.e. efficient investment at v_1) and thus $Y_2 = \hat{Y}$ (because $v_2 > v_1$) the condition above holds with equality. Thus, the value function is strictly concave as long as efficient investment is not feasible.

C.2 Proof of Lemma 4

Suppose first that $\bar{\mu}_{\min} > 0$, then the stationary contract features zero transfers from the firm to the government. This would imply a stationary value of $\bar{v} = 0$ to the government, strictly less than its outside option $\bar{Y} + \beta_G v_{aut}$. This violates the IC constraint of the government. Now suppose that the constraint on the maximum transfers was binding, then firm profits would be negative as $\tau_{\max} > \hat{Y}$. This violates the participation constraint of the firm. Suppose that the IR constraint of the government was binding, then: $\bar{v} = v_{aut}$. The PK and IC constraint imply that $\bar{v} \geq \bar{Y} + \beta_G v_{aut}$. Hence, $\bar{Y} \leq (1 - \beta_G)\, v_{aut}$ and $\bar{\tau} = (1 - \beta_G)\, \bar{v} = (1 - \beta_G) v_{aut}$. Stationary firm profits are: $\bar{Y} - \bar{I} - \bar{\tau} \leq -\bar{I} < 0$. This violates the participation constraint of the firm.

C.3 Proof of Lemma 5

Using $\bar{\mu}_{\min} = \bar{\mu}_{\max} = 0$ (by Lemma 4) the first-order condition on transfers (see equation 16) in the relaxed problem (setting $\bar{\lambda}_{PC0} = 0$) imply:

$$1 = \bar{\lambda}_{IC} + \bar{\lambda}_{PK} \tag{102}$$

In the interior stationary contract, the assumption of the relaxed problem is satisfied since $\bar{\lambda}_{PC} = \bar{\lambda}_{PC0} = 0$. Therefore, it is trivial to write: $\bar{\lambda}_{IC} + \bar{\lambda}_{PK} = 1 + \bar{\lambda}_{PC}$.

By the definition of the corner stationary contract, the relaxed problem does not solve the original problem because $\bar{\lambda}_{PC} = \bar{\lambda}_{PC0} > 0$. In this case, the initial break-even contraint binds:

$$V_F(\bar{v}) = \bar{Y} - \bar{I} - \bar{\tau} + \beta_F V_F(\bar{v}) \geq 0 \tag{103}$$

Since the participation contraint also binds in the future, it is possible to rewrite this contraint as:

$$\bar{\tau} = \bar{Y} - \bar{I} \tag{104}$$

Using $\bar{\mu}_{\min} = \bar{\mu}_{\max} = \bar{\lambda}_{IR} = \lambda_{IR0} = 0$ (by Lemma 4) and $V_F(w(\bar{v})) = 0$, the problem at the steady state becomes:

$$V_F(\bar{v}) = \max_{I,\tau} Y - I - \tau \text{ s.t.} \tag{105}$$

#	**Constraint**	**Lagrange multiplier**
1)	$\tau \geq (1 - \beta_G)\, \bar{v}$	$\bar{\lambda}_{PK}$
2)	$\tau + \beta_G \bar{v} \geq Y_F(I) + \beta_G v_{aut}$	$\bar{\lambda}_{IC}$
3)	$\bar{\tau} = \bar{Y} - \bar{I}$	$-\bar{\lambda}_{PC0}$

The first-order condition on taxes implies:

$$\bar{\lambda}_{PK} + \bar{\lambda}_{IC} = 1 + \bar{\lambda}_{PC0} \tag{106}$$

Since $\bar{\lambda}_{PC0} = \bar{\lambda}_{PC}$, the result immediately follows.

C.4 Proof of Lemma 6

I define $x \equiv \lambda^*_{PK} - \lambda_{PK}$. If $x > 0$ it follows that $\lambda^*_{PK} > \lambda_{PK}$ and hence $w > v$ by concavity. The transition law (see equation 19) can be written as:

$$\lambda_{PK} + x = \frac{\beta_G}{\beta_F} \frac{\lambda_{PK} + \lambda_{IC}}{1 + \lambda_{PC}} \tag{107}$$

Solving for x in the interior region yields:

$$x = \frac{\beta_G}{\beta_F} \left[\lambda_{IC} - \lambda_{PK} \left(\frac{\beta_F}{\beta_G} - 1 \right) \right] \tag{108}$$

At the steady state, $x = 0$. For $v < \bar{v}$, x is strictly positive, because λ_{IC} is a strictly decreasing function of v by Lemma 3 and λ_{PK} is a strictly increasing function of v which implies for all $v < \bar{v}$ that $\lambda_{IC} > \bar{\lambda}_{IC}$ and $\lambda_{PK} < \bar{\lambda}_{PK}$. The strict monotonicity follows from the binding IC constraint at the steady state. Therefore, if $v < \bar{v}$ continuation values are increasing $w > v$. Analogous arguments yield that if $v > \bar{v}$ continuation values must be decreasing. If the stationary contract is a corner contract, then only the region to the left is relevant.

C.5 Proof of Corollary 1

As long as the IC constraint binds, investment is given by the PK and IC constraint:

$$I(v) = Y_F^{-1}(v - \beta_G v_{aut}) \tag{109}$$

If the IC constraint does not bind, then investment is given by the efficient level $I(v) = \hat{I}$. Since $Y_F^{-1}(v - \beta_G v_{aut})$ is continuous, increasing in v and unbounded, there exists a critical value of $\hat{v}$ such that:

$$I(v) = \begin{cases} Y_F^{-1}(v - \beta_G v_{aut}) & \text{for} \quad v < \hat{v} \\ \hat{I} & \text{for} \quad v \geq \hat{v} \end{cases} \tag{110}$$

where $\hat{v} = \hat{Y} + \beta_G v_{aut} > \bar{v}$ since $\hat{Y} > \bar{Y}$. Part b) and c) follow from the monotonicity of $I(v)$ and the dynamics of the promised value: $w > v$ for $v < \bar{v}$ and $w < v$ for $v > \bar{v}$.

C.6 Derivation of Pareto Frontier

For this section, I specify $\tau_{\max}$ large enough such that the stationary contract is reached from any promised value v to the right of the steady state $v \in [\bar{v}, v_{\max}]$, i.e. $\tau_{\max} > v_{\max} - \beta_G \bar{v}$. This implies that the firm is financially unconstrained. The Pareto frontier is defined separately for the left and the right hand side of the steady state.

The value function to the left of the steady state can be determined as follows: As a first step, I determine for each $v \in [v_{\min}, \bar{v}]$ how many periods it takes to reach the stationary contract. It follows from the optimal continuation value policy (see Proposition 5) that for each $v \in Q_i = [\beta_G^i \bar{v}, \beta_G^{i-1} \bar{v}]$ it will take i periods. Within the first interval $Q_1 = [\beta_G \bar{v}, \bar{v}]$ the transfer payment is given by $v - \beta_G \bar{v}$ such that the firm value in this region $V_F^{(1)}(v)$ can be written as:

$$V_F^{(1)}(v) = Y(v) - I(v) - (v - \beta_G \bar{v}) + \beta_F V_F(\bar{v}) \tag{111}$$

For any region $i \geq 2$ the continuation value satisfies $w = \beta_G^{-1} v$ (by Proposition 5) and transfers are 0. Hence, the value function $V_F^{(i)}(v)$ for each $v \in Q_i = [\beta_G^i \bar{v}, \beta_G^{i-1} \bar{v}]$ can be determined recursively by:

$$V_F^{(i)}(v) = Y(v) - I(v) + \beta_F V_F^{(i-1)}\left(\beta_G^{-1} v\right) \tag{112}$$

Using $Y(v) = v - \beta_G v_{aut}$ one can obtain a closed-form solution after repeated substitution (and using the geometric sum formula):

$$V_F^{(n)}(\mathbf{v}) = \underbrace{\frac{\left(\frac{\beta_F}{\beta_G}\right)^{n-1} - 1}{\frac{\beta_F}{\beta_G} - 1}\mathbf{v} + \frac{\beta_F^{n-1}\beta_G \bar{v}}{1-\beta_F} - \frac{\beta_G v_{aut}}{1-\beta_F}}_{\text{Present Value of Output Net of Taxes}} - \underbrace{\sum_{i=0}^{n-1} \beta_F^i I\left(\beta_G^{-i}\mathbf{v}\right) - \beta_F^n \frac{\bar{I}}{1-\beta_F}}_{\text{Present Value of Investment}} \tag{113}$$

for $v \in Q_n = [\beta_G^n \bar{v}, \beta_G^{n-1} \bar{v}]$

The function $V_F^{(n)}(v)$ is continuously differentiable and strictly concave to the left of the steady state since investment (see Corollary 1) is a continuously differentiable and strictly convex function of v to the left of the steady state. The unique maximum is attained at v^* which satisfies the first-order condition:

$$\lambda_{PK}(v^*) = 0 \tag{114}$$

where $\lambda_{PK}(v) = \frac{dV_F^{(n)}(v)}{dv}$ for $v \in [\beta_G^n \bar{v}, \beta_G^{n-1} \bar{v}]$. The Pareto region cannot include values lower than v^* because both the government and the firm would lose from a further decrease in the promised value to the government. Whether v^* is the lower boundary of the Pareto region $v_{\min}$ depends on the relation between v^* and v_{aut}. Since the government can never be forced to accept a contract which offers a value below autarky, the lower boundary point of the Pareto region is given by:

$$v_{\min} = \max(v_{aut}, v^*) \tag{115}$$

Note, that for relatively low values of ρ the boundary point is given by v^*. High values of ρ imply that the autarky constraint binds (see different values of $v_{\min}$ for $\rho = 0$ and $\rho = 0.25$ vs. $\rho = \bar{\rho}$ and $\rho = \tilde{\rho}$ in Figure 5). Whenever $v_{\min} = v_{aut}$, the derived contract is not only subgame perfect but also renegotiation-proof in the spirit of van Damme (1991).[50] A more detailed discussion of this issue is presented in Section 5.2.

[50] Autarky is not renegotiation-proof because it represents an inefficient subgame perfect equilibrium, off the Pareto frontier.

The right hand side of the frontier only needs to be determined if the stationary contract is interior. By the assumption on $\tau_{\max}$, the stationary contract will be reached in one period such that the value function to the right of the steady state $V_F^{(R)}(v)$ is given by:

$$V_F^{(R)}(v) = \begin{cases} \beta_G \bar{v} + \beta_F \bar{V}_F - \beta_G v_{aut} - I(v) & \text{for} \quad v \leq \min(\hat{v}, v_{\max}) \\ \beta_G \bar{v} + \beta_F \bar{V}_F + \hat{\pi} - v & \text{for} \quad v > \hat{v} \end{cases} \tag{116}$$

The value function is strictly concave up to the point where efficient investment becomes feasible, i.e. $\hat{v} = \hat{Y} + \beta_G v_{aut}$. For $v > \hat{v}$, value is exchanged one-to-one between the government and the firm via the upfront payment. This implies a slope of 1 (see Figure 4). Efficient investment is always feasible if $\beta_G \bar{v} + \beta_F \bar{V}_F - \beta_G v_{aut} > \hat{I}$. Otherwise, the maximum investment level on the Pareto frontier $I_{\max}$ is obtained by the participation constraint of the firm:

$$I_{\max} = \min\left(\beta_G \bar{v} + \beta_F \bar{V}_F - \beta_G v_{aut}, \hat{I}\right) \tag{117}$$

Using the maximum investment level $I_{\max}$, one can determine the maximum value contract to the government ($v_{\max}$). The value of $v_{\max}$ pins down the endogenous borrowing constraint of the government as $v_{\max} = \tau + \beta_G \bar{v}$.

$$v_{\max} = \begin{cases} Y_F(I_{\max}) + \beta_G v_{aut} & \text{for} \quad I_{\max} < \hat{I} \\ \hat{\pi} + \beta_G \bar{v} + \beta_F V_F(\bar{v}) & \text{for} \quad I = \hat{I} \end{cases} \tag{118}$$

This completes the technical characterization of the optimal dynamic contract.

C.7 Proof of Proposition 11

Let $V_{F0}^{(i)} = V_F^{(i)}(v_0^i)$ be the net present value of the firm at the initialization with the government of type i that is promised a value of v_0^i.

$$V_F^{(i)}(v) = \max_{I,\tau,w} \underbrace{(Y - I - \tau) + p_{ii}\beta_F V_F^{(i)}(w)}_{\text{Continued Relationship with Government } i} + \underbrace{\beta_F \sum_{j=1, j\neq i}^{N} p_{ij} V_{F0}^{(j)}}_{\text{New Government}} \tag{119}$$

#	Constraint	Lagrange multiplier
1)	$\tau + p_{ii}\beta_G w \geq v$	λ_{PK}
2)	$\tau + p_{ii}\beta_G w \geq Y_F(I) + p_{ii}\beta_G v_{aut}$	λ_{IC}
3)	$V_F^{(i)}(w) \geq 0$	$p_{ii}\beta_F\lambda_{PC}$
4)	$w \geq v_{aut}$	$p_{ii}\beta_G\lambda_{IR}$
5)	$\tau \geq \tau_{\min} = 0$	$\mu_{\min}$
6)	$\tau \leq \tau_{\max}$	$-\mu_{\max}$

The first-order conditions are the same as in the deterministic setup:

$$\begin{array}{lll} I: & Y_F'(I)(1-\lambda_{IC})-1 & =0 \\ \tau: & -1+\lambda_{PK}+\lambda_{IC}+\mu_{\min}-\mu_{\max} & =0 \\ w: & V_F'(w)(1+\lambda_{PC})+\frac{\beta_G}{\beta_F}(\lambda_{PK}+\lambda_{IC}) & =0 \end{array} \tag{120}$$

Therefore, the characterization is equivalent.

References

[1] **Abreu, Dilip, David G. Pearce and Ennio Stacchetti** (1986): "Optimal cartel equilibria with imperfect monitoring", *Journal of Economic Theory*, 39, 251-269.

[2] **Abreu, Dilip, David G. Pearce and Ennio Stacchetti** (1990): "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", *Econometrica*, 58, 1041-1063.

[3] **Acemoglu, Daron, Simon Johnson, and James A. Robinson** (2001): "The Colonial Origins of Comparative Development: An Empirical Investigation", *American Economic Review*, 91, 1369-1401.

[4] **Axarloglou, Kostas, and Frank G. Meanor, Jr.** (2006): "Taxation and Ownership Structure in Supplying Foreign Markets", *Eastern Economic Journal*, 32, 685-698.

[5] **Aguiar, Mark, Manuel Amador and Gita Gopinath** (2006): "Efficient Expropriation: Sustainable Fiscal Policy in a Small Open Economy", *Boston Fed Working Paper 06-09.*

[6] **Aguiar, Mark, Manuel Amador and Gita Gopinath** (2007): "Investment Cycles and Sovereign Debt Overhang", *unpublished Working Paper.*

[7] **Amador, Manuel** (2003): "A Political Model of Sovereign Debt Repayment", *unpublished Working Paper.*

[8] **Becker, Gary S. and George J. Stigler** (1974): "Law Enforcement, Malfeasance, and Compensation of Enforcers", *The Journal of Legal Studies*, 3, 1-18.

[9] **Bernheim, B. Douglas and Michael D. Whinston** (1990): "Multimarket Contact and Collusive Behavior", *Rand Journal of Economics*, 21, 1-26.

[10] **Bindemann, Kerstin** (1999): "Production-Sharing Agreements: An Economic Analysis", *Oxford Institute for Energy Studies Working Paper WPM 25.*

[11] **Bulow, Jeremy, and Kenneth Rogoff** (1989): "Sovereign Debt: Is to forgive to forget", *American Economic Review*, 79, 43-50.

[12] **Cronshaw, Mark, and David G. Luenberger** (1990): "Subgame Perfect Equilibria in Infinitely Repeated Games with Perfect Monitoring and Discounting", *unpublished Working Paper.*

[13] **Cronshaw, Mark, and David G. Luenberger** (1994): "Strongly Symmetric Subgame Perfect Equilibrium in Infinitely Repeated Games with Perfect Monitoring and Discounting", *Games and Economic Behavior*, 6 220-237.

[14] **Diamond, Douglas W.** (2006): "Delegated Monitoring and Legal Protection", *unpublished working paper.*

[15] **Farrell, Joseph, and Eric Maskin** (1989): "Renegotiation in Repeated Games", *Games and Economic Behavior*, 1, 327-360.

[16] **Fudenberg, Drew and Eric Maskin** (1986): "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information", *Econometrica*, 54, 533-554.

[17] **Fudenberg, Drew, and David K. Levine** (1989): "Reputation and Equilibrium Selection in Games with a Patient Player", *Econometrica*, 57, 759-778. [

[18] **Fudenberg, Drew, David M. Kreps, and Eric S. Maskin** (1990): "Repeated Games with Long-run and Short-run Players", *Review of Economic Studies*, 57, 555-573.

[19] **Fudenberg, Drew and Jean Tirole** (1994): *Game Theory*, The MIT Press, Cambridge, Massachusetts.

[20] **Gale, Douglas, and Martin Hellwig** (1989): "Repudiation and Renegotiation: The Case of Sovereign Debt", *International Economic Review*, 30, 3-31.

[21] **Greif, Avner, Paul Milgrom and Barry R. Weingast** (1994): "Coordination, Commitment, and Enforcement: The Case of the Merchant Guilt", *The Journal of Political Economy*, 102, 745-776.

[22] **Grossman, Sanford J., and Oliver D. Hart** (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration", *The Journal of Political Economy*, 94, 691-791.

[23] **Haag, Matthew and Roger Lagunoff** (2007): "On the size and structure of group cooperation", *Journal of Economic Theory*, 135, 68-89

[24] **Harris, Milton, and Bengt Holmstrom** (1982): "A Theory of Wage Dynamics", *The Review of Economic Studies*, 49, 315-333.

[25] **Harris, Milton, and Artur Raviv** (1995): "The Role of Games for Security Design", Review of Financial Studies, 8, 327-367.

[26] **Hart, Oliver D., and John Moore** (1990): "Property Rights and the Nature of the Firm", *The Journal of Political Economy*, 98, 1119-1158.

[27] **Hart, Oliver D., and John Moore** (1994): "A Theory of Debt Based on the Inalienability of Human Capital", *The Quarterly Journal of Economics*, 109, 841-879.

[28] **Hart, Oliver D., and John Moore** (1998): "Default and Renegotiation: A Dynamic Model of Debt", *The Quarterly Journal of Economics*, 113, 1-41.

[29] **Judd, Kenneth L., Sevin Yeltekin, and James Conklin** (2003): "Computing Supergame Equilibria", *Econometrica*, 71, 1239-1254.

[30] **Kobrin, Stephen J.** (1980): Foreign Enterprise and Forced Divestment in LDCs, *International Organization*, 34, 65-88.

[31] **Kobrin, Stephen J.** (1984): Expropriation as an Attempt to Control Foreign Firms in LDCs: Trends from 1960 to 1979, *International Studies Quarterly*, 28, 329-348.

[32] **Kovrijnykh, Natalia** (2007)**:** "Debt Contracts with Short-Term Commitment", *unpublished working paper.*

[33] **Krüger, Dirk, and Harald Uhlig** (2006): "Competitive risk sharing contracts with one-sided commitment", *Journal of Monetary Economics*, 53, 1661–1691.

[34] **La Porta, Rafael., Florencio Lopez-de-Silanes, Andrei Shleifer, Robert W. Vishny** (1998): "Law and Finance", *The Journal of Political Economy*, 106, 1113-1155.

[35] **Lehrer, Ehud, and Ady Pauzner** (1999): "Repeated Games with Differential Time Preferences", *Econometrica*, 67, 393-412.

[36] **Levin, Jonathan** (2003): "Relational Incentive Contracts", *American Economic Review*, 93, 835-847.

[37] **Li, Quan** (2006): "Democracy, Autocracy, and Expropriation of Foreign Direct Investment", *unpublished Working Paper.*

[38] **Lucas, Robert E., Jr., and Nancy L. Stokey** (1989): *Recursive Methods in Economic Dynamics*, Cambridge, MA: Harvard University Press.

[39] **Kikeri, Sunita, Nellis, John and Shirley, Mary** (1992): "Privatization: The Lessons of Experience", Washington, DC: The World Bank.

[40] **Ljungqvist, Lars and Thomas J. Sargent** (2004): *Recursive Macroeconomic Theory*, Vol 2, The MIT Press, Cambridge, Massachusetts.

[41] **Rajan, Rhaguram G., and Luigi Zingales** (1998): "Financial Dependence and Growth", *American Economic Review*, 88, 559-586.

[42] **Rajan, Rhaguram G., and Luigi Zingales** (2003): *Saving Capitalism from the Capitalists,* Chapter 6, Crown Business, New York, New York.

[43] **Rubinstein, Ariel** (1982): "Perfect Equilibrium in a Bargaining Model", *Econometrica*, 50, 97-109.

[44] **Monika Schnitzer** (1999): "Expropriation and control rights: A dynamic model of foreign direct investment", *International Journal of Industrial Organization*, 17, 1113-1137.

[45] **Thomas, Jonathan, and Tim Worrall** (1988): "Self-Enforcing Wage Contracts", *The Review of Economic Studies*, 55, 541-553.

[46] **Thomas, Jonathan, and Tim Worrall** (1994): "Foreign Direct Investment and the Risk of Expropriation", *The Review of Economic Studies*, 61, 81-108.

[47] **Van Damme, Eric** (1991): Stability and Perfection of Nash Equilibria, Vol 2, Springer, Berlin.

[48] **Zingales, Luigi** (1994): "The Value of the Voting Right: A Study of the Milan Stock Exchange Experience", *Review of Financial Studies*, 7, 125-148.

[49] **Zingales, Luigi** (1995): "What Determines the Value of Corporate Votes?", *Quarterly Journal of Economics*, 110, 1047-1073.