# Labour Market Dynamics with Sequential Auctions and 

## Heterogeneous Workers:

On the Dynamics of Wage Distributions and Unemployment Volatility

Jean-Marc Robin*

November 14, 2009

[^0]
#### Abstract

Postel-Vinay and Robin's (2002) sequential auction model is extended to allow for aggregate productivity shocks. Workers exhibit permanent differences in ability and firms are identical. The model first predicts that negative productivity shocks may induce job destruction by driving the surplus of matches with low ability workers to negative values. Endogenous job destruction coupled with worker heterogeneity thus provides a mechanism for amplifying productivity shocks that offers an original solution to the unemployment volatility puzzle (Shimer, 2005). Second, positive or negative shocks may lead employers and employees to renegotiate low wages up and high wages down when agents' individual surpluses become negative. The model thus delivers a rich business cycle dynamics of wage distributions that allows to explain why both low wages and high wages are more procyclical than wages in the middle of the distribution and why wage inequality may be countercyclical, as the data seem to suggest is true.


## 1 Introduction

The initial motivation for this paper is two-fold. First, empirical results in Bonhomme and Robin (2009) and Heathcote, Perri and Violante (2009), among others, suggest that wage and earnings inequality increase in downturns (while earnings mobility decreases) and that low earnings, and low wages to a lesser extent, are more procyclical than high earnings or wages. The reason why it is so is not totally clear by lack of a theory of the business cycle fluctuations of wage distributions. Second, models of individual earnings dynamics in Haider (2001), Baker and Solon (2003), Guvenen (2007), Moffitt and Gottschalk (1995, 2008), and others, consider extensions of the basic permanent-and-transitory-factor model:

$$
y_{i t}=p_{t} \mu_{i}+\lambda_{t} v_{i t}
$$

where $y_{i t}$ is the residual of a regression of log earnings on time dummies, education, etc., and $v_{i t}$ is a stationary ("transitory") process; $p_{t}$ and $\lambda_{t}$ are factor loadings, i.e. timevarying parameters to be estimated. Increasingly more complex structures have been proposed in the literature without strong economic rationale.

The aim of this work is to propose a theory of the interaction of aggregate shocks and worker heterogeneity for understanding business cycle fluctuations of wage distributions. In passing, I will also reconsider the unemployment volatility puzzle (Shimer, 2005) through the lens of this model.

I shall use Postel-Vinay and Robin's (2002) sequential auctions to model wage setting, in a way that is similar to the model in Lise et al. (2009) except than I allow for aggregate shocks to productivity instead of firm-specific shocks. Wage contracts are long term contracts that can be renegotiated by mutual agreement only. Employees search on the job and employers counter outside offers. There is no invisible hand to set wages as in a Walrasian equilibrium. Instead, it is assumed that firms have full monopsony power vis-àvis unemployed workers and hire them at a wage that is only marginally greater than their reservation wage. However, a worker paid less than the competitive wage has a strong incentive to look for an alternative employer in order to trigger Bertrand competition. In
such an environment, at a steady-state equilibrium, with identical workers and identical firms, there are only two wages in the support of the equilibrium distribution: the lower and the upper bounds of the bargaining set; either the firm gets all the surplus, or the worker.

In a very influential paper, Rob Shimer (2005) argues that the search-matching model of Mortensen and Pissarides (1990) cannot reproduce unemployment dynamics well. A long series of papers have tried to offer a remedy, essentially by making wages sticky (Hall, 2005, Hall and Milgrom, 2008, Gertler ad Trigari, 2009, Pissarides, 2007) or by reducing the match surplus to a very small value (Hagedorn and Manovskii, 2008). Mortensen and Nagypal (2007) review this literature and consider other mechanisms. Interestingly, although endogenous job destruction is at the heart of the Mortensen-Pissarides model, this literature has neglected endogenous job destruction as a possible amplifying mechanism when coupled with worker heterogeneity. However, if negative aggregate productivity shocks make the job surplus of low ability workers negative, even a small fraction of workers (around $5 \%$ ) at risk of a negative surplus is enough to amplify the effect of negative productivity shocks on unemployment above and beyond the steady exogenous layoff flows. I will show that realistic unemployment dynamics can be generated with an exogenous layoff rate of $4.3 \%$ and an overall job destruction rate of $4.5 \%$. The $0.2 \%$ difference is the endogenous part. Exogenous job destruction (idiosyncratic) implies a minimum unemployment rate of about $4 \%$ (say frictional unemployment). Endogenous job destruction (macroeconomic) implies additional unemployment between 0 and $5 \%$ (shall we call it classical?).

A few interesting search-matching models with endogenous earnings distribution dynamics have recently been proposed in the literature. Pissarides (2007) suggests a novel approach to solve the unemployment volatility puzzle by assuming that productivity shocks change entry wages in new jobs differently from wages in on-going jobs. Gertler and Trigari (2009) generate wage stickiness using a Calvo-type mechanism such that only a fraction of contracts are renegotiated in every period. Both models generate crosssectional wage dispersion but they do not address the issue of wage inequality dynamics.

In my model, wages in new matches and wages in on-going matches may also be different. However, they reflect state-dependent rent sharing mechanisms with not effect on unemployment dynamics, as unemployment dynamics depends on the level of the rent or surplus (and how it compares to zero), not on how it is split.

Two other recent papers are worth mentioning. Moscarini and Postel-Vinay (2009) study the non-equilibrium dynamics of Burdett and Mortensen's wage posting model. Workers are identical but firms are different. This model yields very interesting insights on the business-cycle dynamics of firm size distributions. Menzio and Shi (2009) also consider a wage posting model but they assume undirected search instead of directed search. Moreover, neither firms nor workers are intrinsically different but a match productivity value is drawn after installing a new partnership.

Here, wage dispersion accrues partly because "starting wages" (upon exiting unemployment) differ from "promotion wages" (from Bertrand competition), partly because of workers' heterogeneous abilities, and partly because the long term nature of wage contracts induces aggregate state dependence (renegotiation occurs after a productivity shock only if it puts the current contract outside the bargaining set). At any point in time, new wage contracts are signed which depend on the current aggregate state. Some of these new wages result from a contact between a worker, employed or unemployed, and an employer. Some of these new wages reflect a contract renegotiation with the same employer due to the recent change in the macroeconomic environment. A low wage may suddenly become lower than the worker's reservation wage and the employer is forced to renegotiate the wage upward. A high wage may suddenly become higher than the employer's reservation value and the worker is forced to accept a wage cut. It is therefore expected that both low and high wages will be more procyclical than wages in the middle of the distribution.

Table 1 shows elasticities of three hourly wage inequality measures (D9/D5, D5/D1, D9/D1 where $\mathrm{D} x$ stands for the $x$ th decile). Elasticities are calculated with respect to aggregate unemployment (CPS) and productivity (BLS) using a log-log regression that includes a linear trend. ${ }^{1}$ One can see that the data seem to comply with the model's

[^1]|  | D9/D5 | D5/D1 | D9/D1 |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Hourly Wage |  |  |  |
| Unempl. Rate (linear trend) | $-0.048(0.014)$ | $0.056(0.018)$ | $0.007(0.017)$ |  |
| Unempl. Rate (HP-filtered) | $-0.095(0.003)$ | $0.113(0.011)$ | $0.017(0.009)$ |  |
| Productivity (linear trend) | $0.21(0.14)$ | $-0.53(0.16)$ | $-0.32(0.15)$ |  |
| Productivity (HP-filtered) | $0.44(0.15)$ | $-0.98(0.15)$ | $-0.53(0.03)$ |  |
|  | Annual Earnings |  |  |  |
| Unempl. Rate (linear trend) | $0.010(0.012)$ | $0.25(0.032)$ | $0.26(0.033)$ |  |
| Unempl. Rate (HP-filtered) | $-0.005(.009)$ | $0.36(.024)$ | $0.35(0.025)$ |  |
| Productivity (linear trend) | $-0.10(0.11)$ | $-1.41(0.41)$ | $-1.51(0.43)$ |  |
| Productivity (HP-filtered) | $-0.044(0.16)$ | $-2.46(0.50)$ | $-2.51(0.50)$ |  |

Table 1: Wage and Earnings Inequality. (Source: CPS, 1967-2005. Each case corresponds to the elasticity of the column variable with respect to the row variable. The log of variables has been first detrended using a linear trend or the HP-filter.)
predictions. The table also displays the elasticities of inequality indices of annual earnings. Low wage earners also facing more unemployment risk, low earnings are therefore much more procyclical than high earnings. Moreover, it seems that hours worked are more procyclical than hourly wages. These points were already made by Heathcote et al. (2009). I shall present a calibration of the dynamic sequential auction model that generates more procyclicality in low wages than in high wages and that also produces a swiftier employment response of low ability workers to aggregate productivity.

The paper is organized as follows. I will first start to develop a search-matching, sequential-auction model with heterogeneous workers and identical firms. Then, I will explain how the model's parameters are calibrated or estimated. Lastly, I will interpret the results.

## 2 The Model

We start by laying out the model.
grateful to Gianluca Violante who passed me these data.

### 2.1 Employment and turnover

Aggregate shocks. Time is discrete and indexed by $t \in \mathbb{N}$. The global state of the economy is described by an ergodic Markov chain $y_{t} \in\left\{y_{1}<\ldots<y_{N}\right\}$ with transition probability matrix $\boldsymbol{\Pi}=\left(\pi_{i j}\right)$ (with a slight abuse of notation, $y_{t}$ denotes the stochastic process and $y_{i}$ an element of the support). Aggregate shocks accrue at the beginning of each period.

Workers. There are $M$ types of workers and $\ell_{m}$ workers of each type (with $\sum_{m=1}^{M} \ell_{m}=$ 1). Each type is characterized by a time-invariant ability $x_{m}, m=1, \ldots, M$, with $x_{m}<$ $x_{m+1}$. Workers are paired with identical firms to form productive units. The per-period output of a job, if the worker is of ability $x_{m}$ and aggregate productivity is $y_{i}$, is denoted $y_{i}(m)$, and a natural specification for match productivity is $y_{i}(m)=x_{m} y_{i}$. We assume that firms cannot direct their search to specific worker types. We denote as $S_{i}(m)$ the surplus of a match $\left(x_{m}, y_{i}\right)$, that is, the present value of the match minus the value of unemployment and minus the value of a vacancy (assumed to be nil). Only matches with $S_{i}(m)>0$ are viable.

Turnover. Matches form and break at the beginning of each period, after the aggregate state has been reset for the whole period. Let $u_{t}(m)$ (resp. $1-u_{t}(m)$ ) denote the proportion of unemployed (employed) in the population of workers of ability $x_{m}$ at the end of period $t-1$, and let $u_{t}=\sum_{m=1}^{M} u_{t}(m) \ell_{m}$ denote the aggregate unemployment rate. A the beginning of period $t$, the new aggregate productivity state is revealed to be equal to some $i \in\{1, \ldots, N\}$. A fraction $\mathbf{1}\left\{S_{i}(m) \leq 0\right\}\left[1-u_{t}(m)\right] \ell_{m}$ is then endogenously laid off, and another fraction $\delta \mathbf{1}\left\{S_{i}(m)>0\right\}\left[1-u_{t}(m)\right] \ell_{m}$ is exogenously destroyed.

For simplicity, we assume that workers meet employers at exogenous rates. It is easy to work out an extension of the model with a standard matching function if necessary. Thus, a fraction $\lambda_{0} \mathbf{1}\left\{S_{i}(m)>0\right\} u_{t}(m) \ell_{m}$ of unemployed workers meet an employer and a fraction $\lambda_{1}(1-\delta) \mathbf{1}\left\{S_{i}(m)>0\right\}\left[1-u_{t}(m)\right] \ell_{m}$ of employed workers meet an alternative employer, where $\lambda_{0}$ and $\lambda_{1}$ are the respective search intensities of unemployed and employed
workers. We assume that employers have full monopsony power with respect to workers. Hence, unemployed workers are offered their reservation wage. However, employees benefit from the competition between the incumbent and the poacher. Because firms are identical and there is no mobility cost, Bertrand competition transfers the whole surplus to the worker and the worker is indifferent between staying with the incumbent employer or moving to the poacher. We assume that the tie is broken in favour of the poacher with probability $\tau .{ }^{2}$

The following turnover rates can then be computed:

- Exit rate from unemployment:

$$
f_{0 t}=\lambda_{0} \frac{\sum_{m} \mathbf{1}\left\{S_{i}(m)>0\right\} u_{t}(m) \ell_{m}}{u_{t}} ;
$$

- Quit rate (job-to-job mobility):

$$
f_{1 t}=\tau \lambda_{1}(1-\delta) \frac{\sum_{m} \mathbf{1}\left\{S_{i}(m)>0\right\}\left[1-u_{t}(m)\right] \ell_{m}}{1-u_{t}}
$$

- Lay-off rate:

$$
s_{t}=\delta+(1-\delta) \frac{\sum_{m} \mathbf{1}\left\{S_{i}(m) \leq 0\right\}\left(1-u_{t}(m)\right) \ell_{m}}{1-u_{t}}
$$

The value of unemployment. Let $U_{i}(m)$ denote the present value of remaining unemployed for the rest of period $t$ for a worker of type $m$ if the economy is in state $y_{i}$. An unemployed worker receives a flow-payment $z_{i}(m)$ for the period. At the beginning of the next period, the state of the economy changes to $y_{j}$ with probability $\pi_{i j}$ and the worker receives a job offer with some probability. However, because the employer has full monopsony power, the present value of a new job to the worker is only marginally better than the value of unemployment. Consequently, the value of unemployment solves the

[^2]following linear Bellman equation:
$$
U_{i}(m)=z_{i}(m)+\frac{1}{1+r} \sum_{j} \pi_{i j} U_{j}(m)
$$

The match surplus. After a productivity shock from $i$ to $j$ all matches yielding negative surplus are destroyed. Otherwise, if the worker is poached, Bertrand competition transfers the whole surplus to the worker whether s/he moves or not. Everything that the worker or the firm will earn in the future is included in the definition of the current surplus. It follows that the surplus of a match $\left(x_{m}, y_{i}\right)$ solves the following (nearly linear) Bellman equation:

$$
S_{i}(m)=y_{i}(m)-z_{i}(m)+\frac{1-\delta}{1+r} \sum_{j} \pi_{i j} S_{j}(m)^{+}
$$

where we denote $x^{+}=\max (x, 0)$. This nearly-linear system of equations can be numerically solved by value function iteration.

Unemployment process. The law of motion of individual-specific unemployment rates is:

$$
\begin{aligned}
u_{t+1}(m) & =1-\left[(1-\delta)\left(1-u_{t}(m)\right)+\lambda_{0} u_{t}(m)\right] \mathbf{1}\left\{S_{i}(m)>0\right\} \\
& = \begin{cases}1 & \text { if } S_{i}(m) \leq 0 \\
u_{t}(m)+\delta\left(1-u_{t}(m)\right)-\lambda_{0} u_{t}(m) & \text { if } S_{i}(m)>0\end{cases}
\end{aligned}
$$

It is clear that the dynamics of unemployment is completely independent on how the surplus is split between employers and employees.

Steady-state. If the economy remains in state $i$ for ever, the unemployment rate in group $m$ is

$$
u_{i}(m)=\frac{\delta}{\delta+\lambda_{0}} \mathbf{1}\left\{S_{i}(m)>0\right\}+\mathbf{1}\left\{S_{i}(m) \leq 0\right\}
$$

The aggregate unemployment rate is:

$$
u_{i}=\sum_{m=1}^{M} u_{i}(m) \ell_{m}=\frac{\delta}{\delta+\lambda_{0}} L_{i}+1-L_{i}=1-\frac{\lambda_{0}}{\delta+\lambda_{0}} L_{i},
$$

where $L_{i}=\sum_{m=1}^{M} \ell_{m} \mathbf{1}\left\{S_{i}(m)>0\right\}$ is the number of employable workers.

### 2.2 Wages

Let $W_{i}(w, m)$ denote the present value of a wage $w$ in state $i$ to a worker of type $m$. The worker surplus, $W_{i}(w, m)-U_{i}(m)$, satisfies the following Bellman equation:

$$
\begin{aligned}
W_{i}(w, m)- & U_{i}(m)=w-z_{i}(m) \\
& +\frac{1-\delta}{1+r} \sum_{j} \pi_{i j} \mathbf{1}\left\{S_{j}(m)>0\right\}\left[\lambda_{1} S_{j}(m)+\left(1-\lambda_{1}\right)\left(W_{j}^{*}(w, m)-U_{j}(m)\right)\right]
\end{aligned}
$$

with

$$
W_{j}^{*}(w, m)-U_{j}(m)=\min \left\{\max \left\{W_{j}(w, m)-U_{j}(m), 0\right\}, S_{j}(m)\right\} .
$$

The surplus flow for the current period is $w-z_{i}(m)$. In the following period, the worker is laid off with probability $1\left\{S_{j}(m) \leq 0\right\}+\delta 1\left\{S_{j}(m)>0\right\}$, and suffers zero surplus. Alternatively, with probability $\lambda_{1}$, the worker receives an outside offer and enjoys the whole surplus. In the absence of poaching (with probability $1-\lambda_{1}$ ) wage contracts may still be renegotiated if a productivity shock moves the current wage outside the bargaining set. We follow McLeod and Malcomson (1993) and Postel-Vinay and Turon (2007) and assume that the new wage contract is the closest point in the bargaining set from the old, now infeasible wage. That is, if $W_{j}(w, m)-U_{j}(m)<0$, the worker has a creadible threat to quit to unemployment and her employer accepts to renegotiate the wage up to the point where the worker obtains zero surplus. If $W_{j}(w, m)-U_{j}(m)>S_{j}(m)$, now the employer has a credible threat to fire the worker unless she accepts to renegotiate down to the point where she gets the whole surplus and no more.

For all aggregate states $y_{i}$ and all worker types $x_{m}$, there are only two possible starting
wages. Either the worker was offered a job while unemployed, and he can only claim a wage $\underline{w}_{i}(m)$ such that $W_{i}\left(\underline{w}_{i}(m), m\right)=U_{i}(m)$ (his reservation wage); or he was already employed and he benefits from a wage rise to $\bar{w}_{i}(m)$ such that $W_{i}\left(\bar{w}_{i}(m), m\right)=U_{i}(m)+$ $S_{i}(m)$ (the employer's reservation value).

For all $k$, let us denote the worker surpluses when the economy is in state $k$ evaluated at wages $\underline{w}_{i}(m)$ and $\bar{w}_{i}(m)$ as

$$
\begin{aligned}
& \underline{W}_{k, i}(m)=W_{k}\left(\underline{w}_{i}(m), m\right)-U_{k}(m), \\
& \bar{W}_{k, i}(m)=W_{k}\left(\bar{w}_{i}(m), m\right)-U_{k}(m),
\end{aligned}
$$

and let

$$
\begin{aligned}
& \underline{W}_{k, i}^{*}(m)=\min \left\{\max \left\{\underline{W}_{k, i}(m), 0\right\}, S_{i}(m)\right\}, \\
& \bar{W}_{k, i}^{*}(m)=\min \left\{\max \left\{\bar{W}_{k, i}(m), 0\right\}, S_{i}(m)\right\} .
\end{aligned}
$$

Making use of the definitions of wages, $\underline{W}_{i, i}(m)=0$ and $\bar{W}_{i, i}(m)=S_{i}(m)$, these worker surpluses therefore satisfy the following modified Bellman equations:

$$
\begin{aligned}
& \underline{W}_{k, i}(m)=\underline{W}_{k, i}(m)-\underline{W}_{i, i}(m) \\
& \quad=z_{i}(m)-z_{k}(m)+\frac{1-\delta}{1+r} \sum_{j}\left(\pi_{k j}-\pi_{i j}\right) \mathbf{1}\left\{S_{j}(m)>0\right\}\left[\lambda_{1} S_{j}(m)+\left(1-\lambda_{1}\right) \underline{W}_{j, i}^{*}(m)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
& \bar{W}_{k, i}(m)-S_{i}(m)=\bar{W}_{k, i}(m)-\bar{W}_{i, i}(m) \\
& =z_{i}(m)-z_{k}(m)+\frac{1-\delta}{1+r} \sum_{j}\left(\pi_{k j}-\pi_{i j}\right) \mathbf{1}\left\{S_{j}(m)>0\right\}\left[\lambda_{1} S_{j}(m)+\left(1-\lambda_{1}\right) \bar{W}_{j, i}^{*}(m)\right] .
\end{aligned}
$$

Again, value function iteration delivers a simple numerical solution algorithm.

Having determined $\underline{W}_{k, i}(m)$ and $\bar{W}_{k, i}(m)$ for all $k, i$ and $m$, wages then follow as

$$
\underline{w}_{i}(m)=z_{i}(m)-\frac{1-\delta}{1+r} \sum_{j} \pi_{i j} \mathbf{1}\left\{S_{j}(m)>0\right\}\left[\lambda_{1} S_{j}(m)+\left(1-\lambda_{1}\right) \underline{W}_{j, i}^{*}(m)\right]
$$

and

$$
\bar{w}_{i}(m)=S_{i}(m)+z_{i}(m)-\frac{1-\delta}{1+r} \sum_{j} \pi_{i j} \mathbf{1}\left\{S_{j}(m)>0\right\}\left[\lambda_{1} S_{j}(m)+\left(1-\lambda_{1}\right) \bar{W}_{j, i}^{*}(m)\right] .
$$

### 2.3 Wage distributions

The support of the wage distribution is the union of all sets $\Omega_{m}=\left\{\underline{w}_{i}(m), \bar{w}_{i}(m), \forall i\right\}$. Let $g_{t}(w, m)$ denote the measure of workers of ability $m$ employed at wage $w \in \Omega$ at the end of period $t-1$.

Conditional on the state of the economy changing to $y_{t}=y_{i}$ (maybe equal to $y_{t-1}$ ) at the beginning of period $t$, no worker can be employed if $S_{i}(m) \leq 0$. The inflow into the stock of workers paid the minimum wage $\underline{w}_{i}(m)$ is otherwise made of those unemployed workers drawing an offer $\left(\lambda_{0} u_{t}(m) \ell_{m}\right)$ plus all employees paid a wage $w$ such that $W_{i}(w, m)-U_{i}(m)<0$ who were not laid off but were also not lucky enough to get poached. The outflow is made of those workers previously paid $\underline{w}_{i}(m)$ who are either laid off or poached. That is,

$$
\begin{aligned}
& g_{t+1}\left(\underline{w}_{i}(m), m\right)=\mathbf{1}\left\{S_{i}(m)>0\right\}\left[\lambda_{0} u_{t}(m) \ell_{m}\right. \\
& \left.\quad+(1-\delta)\left(1-\lambda_{1}\right)\left(g_{t}\left(\underline{w}_{i}(m), m\right)+\sum_{w \in \Omega_{m}} \mathbf{1}\left\{W_{i}(w, m)-U_{i}(m)<0\right\} g_{t}(w, m)\right)\right] .
\end{aligned}
$$

The inflow into the stock of workers paid $\bar{w}_{i}(m)$ has two components. First, any employee paid less than $\bar{w}_{i}(m)$ (in present value terms) who is contacted by another employer benefits from a pay rise to $\bar{w}_{i}(m)$. Second, any employee paid more than $\bar{w}_{i}(m)$ (in present value) has to accept a pay cut to $\bar{w}_{i}(m)$ to avoid layoff. The only reason to
flow out is layoff. Hence,

$$
\begin{aligned}
& g_{t+1}\left(\bar{w}_{i}(m), m\right)=\mathbf{1}\left\{S_{i}(m)>0\right\}(1-\delta)\left[\lambda_{1}\left(1-u_{t}(m)\right) \ell_{m}\right. \\
& \left.\quad+\left(1-\lambda_{1}\right)\left(g_{t}\left(\bar{w}_{i}(m), m\right)+\sum_{w \in \Omega_{m}} \mathbf{1}\left\{W_{i}(w, m)-U_{i}(m)>S_{i}(m)\right\} g_{t}(w, m)\right)\right] .
\end{aligned}
$$

Lastly, for all $w \in \Omega \backslash\left\{\underline{w}_{i}(m), \bar{w}_{i}(m)\right\}$, only those workers paid $w$ greater than $\underline{w}_{i}(m)$ and less than $\bar{w}_{i}(m)$ (in value terms), who are not laid off or poached, keep their wage:

$$
\begin{aligned}
g_{t+1}(w, m)=\mathbf{1}\left\{S_{i}(m)>0\right\}(1-\delta)(1- & \left.\lambda_{1}\right) \\
& \times \mathbf{1}\left\{0 \leq W_{i}(w, m)-U_{i}(m) \leq S_{i}(m)\right\} g_{t}(w, m) .
\end{aligned}
$$

Notice that summing up $g_{t+1}(w, m)$ over all wages and dividing by $\ell_{m}$ yields the law of motion for the unemployment rates $u_{t}(m)$ :

$$
1-u_{t+1}(m)=1\left\{S_{i}(m)>0\right\}\left[\lambda_{0} u_{t}(m)+(1-\delta)\left(1-u_{t}(m)\right)\right] .
$$

## 3 Parameterization and Calibration

### 3.1 Aggregate shocks

I use the BLS quarterly series of seasonally adjusted real output per person in the nonfarm business sector (BLS series PS85006163) to construct the aggregate productivity process $y_{t}$. The data cover the period 1947q1-2009q1. The raw data are successively log-transformed, HP-filtered, and exponentiated. ${ }^{3}$

I assume that the aggregate productivity process $y_{t}$ is an ergodic Markov chain. For each value of $y_{t}$ I define the state of the economy as the rank of $y_{t}$ in its marginal/ergodic distribution, say $F$. The joint distribution of two consecutive ranks $F\left(y_{t}\right)$ and $F\left(y_{t+1}\right)$ is

[^3]a copula $C$ (i.e. the cdf of the distribution of two random variables with uniform margins). For example, the usual Gaussian $\mathrm{AR}(1)$ process used in the literature has Gaussian margins and a Gaussian copula. It is commonplace to obtain a discrete approximation of the copula by calculating the transition probability matrix across discretized states (quintiles, deciles, etc.) but fitting a parametric copula (archimedean, elliptical) is a much more parsimonious way than fitting all transition probabilities separately.

I use the following two-stage semi-parametric estimation procedure:

1. Estimate the marginal distribution $F$ by kernel smoothing the empirical distribution.
2. Estimate the copula parameters by maximum likelihood on sample $\left\{F\left(y_{t-1}\right), F\left(y_{t}\right)\right\}$. A simple scatterplot gives a good indication regarding to which parametric specification of the copula to choose.

Chen et al. (2009) argue that a more efficient estimation of the marginal distribution can be obtained if the marginal distribution is peaked and the copula displays strong tail dependence. This should be less of a problem here because this two-step procedure is applied to detrended - hence less autocorrelated - data. Figure 1, panel (a), shows the marginal distribution of detrended productivity. The kernel density estimate is of course much less dented than the histogram. It resembles a normal density except for the left tail that is fatter than the normal.

Figure 1, panel (b), provides a graphical display of the copula. The actual scatterplot (left panel) indicates an elliptical distribution with no specific tail-dependence. Hence, I use a $t$-copula with parameters $\rho$ (linear correlation coefficient) and $\nu$ (the number of degrees of freedom; a large $\nu \geq 30$, indicates Gaussianity). I estimate $\rho=0.89$ and $\nu=$ 13.11. Parameter $\nu$ is large, indicating a close-to-Gaussian copula. The right panel shows a simulation of the $t$-copula with estimated parameters $\rho$ and $\nu$. No apparent discrepancy with the true one can be easily detected. ${ }^{4}$
${ }^{4}$ The simulation algorithm is very simple: given observation $r_{t-1}$ of the $(t-1)$ th rank, generate $r_{t}$ as

$$
t_{\nu}^{-1}\left(r_{t}\right)=\rho t_{\nu}^{-1}\left(r_{t}\right)+e_{t} \sqrt{\frac{\nu+\left(t_{\nu}^{-1}\left(r_{t}\right)\right)^{2}}{\nu+1}\left(1-\rho^{2}\right)}
$$

(a) Marginal productivity distribution

(b) Scatterplot of $t, t+1$ productivity ranks



Figure 1: Two-step Estimation of the Aggregate Productivity Process
(a) Actual HP-filtered productivity series


Figure 2: Simulation of Productivity Dynamics

Figure 2 displays a simulation of productivity levels. Panel (a) shows the actual series of exponentiated HP-filtered log-productivity. Panel (b) shows two simulations obtained with the same sequence of iid uniform innovations: one uses the semi-parametric estimate and the other one uses a Gaussian $\operatorname{AR}(1)$ model. ${ }^{5}$

Finally, a discrete Markov chain approximation can be obtained as follows. Let $a_{0}=$ $\underline{y}<a_{1}<\ldots<a_{N}=\bar{y}$ delimit a grid on the support of the productivity distribution. I use equal-sized intervals $\left(a_{i}-a_{i-1}=\frac{\bar{y}-\underline{y}}{N}\right)$ and extreme points $\underline{y}$ and $\bar{y}$ are chosen according to the estimated marginal distribution $F$ as $F(\underline{y}) \simeq 0$ and $F(\bar{y}) \simeq 1$. Then,

1. Set discrete productivity values as bins' midpoints $y_{i}=\frac{a_{i-1}+a_{i}}{2}$.
2. Estimate marginal state probabilities as $p_{i}=F\left(a_{i}\right)-F\left(a_{i-1}\right)$.
3. Set transition probabilities as:

$$
\begin{aligned}
\pi_{i j}= & \frac{\operatorname{Pr}\left\{\left[a_{i-1}, a_{i}\right] \times\left[a_{j-1}, a_{j}\right]\right\}}{\operatorname{Pr}\left\{\left[a_{i-1}, a_{i}\right]\right\}} \\
= & \frac{1}{p_{i}}\left[C\left(F\left(a_{i}\right), F\left(a_{j}\right)\right)-C\left(F\left(a_{i-1}\right), F\left(a_{j}\right)\right)\right. \\
& \left.\quad-C\left(F\left(a_{i}\right), F\left(a_{j-1}\right)\right)+C\left(F\left(a_{i-1}\right), F\left(a_{j-1}\right)\right)\right] .
\end{aligned}
$$

### 3.2 Parameterization and calibration

I specify match productivity as

$$
y_{i}(m)=y_{i}\left(B x_{m}+C\right)
$$

where $e_{t}=t_{\nu+1}^{-1}(u)$ with $u \sim \operatorname{Uniform}[0,1]$ (or $e_{t} \sim t_{\nu+1}$ ). Then generate $y_{t}=F^{-1}\left(r_{t}\right)$ for any marginal $\operatorname{cdf} F$. Note that for $\nu \rightarrow \infty, t_{\nu} \rightarrow \Phi$ the cdf of the standard normal distribution and the recursive formula for ranks becomes:

$$
\Phi^{-1}\left(r_{t}\right)=\rho \Phi^{-1}\left(r_{t-1}\right)+\sqrt{1-\rho^{2}} e_{t}
$$

where $e_{t} \sim N(0,1)$.
${ }^{5}$ For completeness, the autoregression of detrended log-productivity yields an autocorrelation coefficient $\rho=0.876$ and a standard deviation of residuals $\sigma=0.0097$.
where $B$ and $C$ are two constant and $\left.x_{m} \in\right] 0,1[$. Specifically,

$$
x_{m}=\frac{m-0.5}{M}, \quad m=1, \ldots, M .
$$

The distribution of individual ability is approximately beta-distributed:

$$
\begin{aligned}
\ell_{m} & =\operatorname{betacdf}\left(\frac{m}{M}, \mu, 1\right)-\operatorname{betacdf}\left(\frac{m-1}{M}, \mu, 1\right) \\
& \simeq \frac{1}{M} \operatorname{betapdf}\left(x_{m}, \mu, 1\right) \quad(\text { as } M \rightarrow \infty) .
\end{aligned}
$$

Lastly, the opportunity cost of employment (leisure cost) is specified as:

$$
z_{i}(m)=z_{0}+\alpha\left[y_{i}(m)-z_{0}\right] .
$$

## I also set the unit of time equal to a quarter.

The parameters that have to be estimated are the turnover parameters $\lambda_{0}, \lambda_{1}$ and $\delta$, the probability of moving upon receiving an outside offer $\tau$, the leisure cost parameters $z_{0}$ and $\alpha$, the parameters of the support of worker heterogeneity, $B$ and $C$, and parameter $\mu$ shaping the distribution of heterogeneity. These parameters will be calibrated so as to match a set of moments using the simulated method of moments. I now explain which moments I chose to target.

Shimer $(2005,2007)$ uses CPS data to measure the exit rate from unemployment and the overall separation rate ( $f_{0 t}$ and $s_{t}$ in Section 2.1) assuming that all separations end up in unemployment. To separate quits from layoffs I use the JOLTS data (Job Openings and Labor Turnover Survey) from the BLS that provide information on the number of firm hires per month $(H)$, the number of quits $(Q)$ and involuntary separations (layoffs and discharges), denoted $L$. I also use the total employment series $(E)$ from the Current Employment Statistics (CES), that is supposedly consistent with the JOLTS series. The number of unemployed $(U)$ is extracted from the Current Population Survey (CPS). These are monthly series spanning $2000 \mathrm{~m} 12-2009 \mathrm{~m} 1$.

Assuming that no employee quits her job to become unemployed (the exact opposite

|  | From Unempl. | Job to Job | Job to Unempl. |
| :--- | :---: | :---: | :---: |
| Mean rate, \%/month | $30 \% / \mathrm{mth}$ | $2.1 \%$ | $1.5 \%$ |
| $(\% /$ qtr) | $(66 \% / \mathrm{qtr})$ | $(6.2 \%)$ | $(4.5 \%)$ |
| Elasticity wrt unemployment | -1.01 | -0.61 | 0.33 |
| $\left(R^{2}\right)$ | $(75 \%)$ | $(71 \%)$ | $(33 \%)$ |

Table 2: Turnover (Source: JOLTS)
to assuming that all separations are layoffs) the exit rate from unemployment is $\frac{H-Q}{U}$ (measuring $f_{0 t}$ ), the job-to-job mobility rate is the quit rate $\frac{Q}{E}$ (measuring $f_{1 t}$ ) and the layoff rate is $\frac{L}{E}$ (measuring $s_{t}$ ). The unemployment rate is $\frac{U}{U+E}$. Figure 3, panel (a), displays turnover series, and panel (b) graphs the turnover series as a function of the unemployment rate to emphasize the link with the business cycle. As expected, hiring rates are procyclical and the layoff rate is countercyclical, with elasticities reported in Table 2. Shimer $(2005,2007)$ estimates a separation rate of $3.4 \%$ per month from CPS data, which is roughly the same rate that can be calculated using $\frac{Q+L}{E}$ from JOLTS data, i.e. the sum of the layoff rate and the quit rate. Notice that the elasticity of the layoff rate is not only lower that the other rates (in absolute value), the correlation is also weaker (as indicated by the $R^{2}$ of the log-log regression in brackets). This point was already made by Shimer (2007).

Using results in Jolivet, Postel-Vinay, Robin (2006), who estimate a wage posting, equilibrium search model on PSID data, I estimate the proportion of employees' contacts with alternative employers resulting in actual mobility to $53 \%$. I thus set $\tau=0.5$. Also, because the exit rate of unemployment is so high at the quarterly frequency $-66 \%$ using the JOLTS data $-{ }^{6}$ I arbitrarily set $\lambda_{0}=1$.

I set the number of aggregate states equal to $N=50$, the number of different ability types equal to $M=300$ and I simulate very long series of $T=5000$ observations so as to match the following moments: ${ }^{7}$

- The mean productivity is 1 and the standard deviation of log productivity is equal

[^4](a) Turnover series

(b) Turnover cycle


Figure 3: Turnover (Source: JOLTS)
to 0.02 ; the mean unemployment rate is $5.6 \%$ and the standard deviation of $\log$ unemployment is $0.20 .^{8}$

- The mean layoff rate is $4.5 \%$ per quarter $(1.5 \% /$ month $)$ and the mean job-to-job mobility rate is $6.2 \%$ per quarter (from JOLTS data).
- The standard deviation of $\log$ wages is 0.017 , the elasticity of wages to productivity is 0.53 (from long quarterly BLS series) ${ }^{9}$ and the mean values of D9/D5 and D5/D1 for wages are equal to 2.05 and 2.20 (from CPS).

The wage moments aim at identifying parameter $\alpha$, as for any value of $\alpha$ there is an observationally equivalent value of $\left(z_{0}, B, C\right)$ yielding the same unemployment values and surpluses, and also at identifying the range of worker heterogeneity $[C, B+C]$.

I estimate $\alpha=0.5, z_{0}=0.6115, B=0.935, C=0.5935, \eta=1.33, \lambda_{1}=0.13$ and $\delta=0.043$.

The mean leisure cost $z_{t}(m)$ averaged over worker types and time is 0.80 , somewhere between Hagedorn and Manovskii (2008), 0.95, and Hall and Milgrom (2008), 0.70. The estimate of the on-the-job offer arrival rate, $\lambda_{1}=0.13$, is consistent with estimates from micro studies (see e.g. Jolivet et al., 2006). Lastly, note that the exogenous job destruction rate $\delta$ is estimated $4.3 \%$ which is very close to the targeted value for the overall separation rate. This means that, on average, endogenous job destruction contributes little to the overall separation flows.

Figure 4 shows the distribution of worker heterogeneity and how it affects individual productivity given the state of the economy. Every thin line in the top figure corresponds to a different ability type. The thick line in the middle is the aggregate productivity level $y_{i}$. The other thick line at the bottom indicates the viability threshold. For a given

[^5]

Figure 4: Heterogeneous productivity
aggregate state $i$ all individual types $m$ such that $S_{i}(m) \leq 0$ have their productivity below the threshold. Only very few lines are below the threshold (namely 11 low productivity types, $4.85 \%$ of all workers, bear a risk of endogenous layoff). The bottom panel displays the distributions of workers' expected productivity in the whole population, and in the sub-populations of employed and unemployed workers. The distribution is more concentrated in the region of low ability workers. Moreover, as expected low ability workers are over-represented amongst the unemployed.

| mean | $\bar{y}_{t}$ | $u_{t}$ | $f_{0 t}$ | $s_{t}$ | $f_{1 t}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Actual moments (Shimer, 2005) |  |  |  |  |
|  | 1 | 0.056 | 0.83 | 0.045 |  |
| std | 0.020 | 0.19 | 0.118 | 0.075 |  |
| corr wrt $\ln \bar{y}_{t}$ | 1 | -0.41 | 0.40 | -0.52 |  |
| reg on $\ln \bar{y}_{t}$ | 1 | -4.08 | 4.56 | -1.95 |  |
|  | Simulated moments |  |  |  |  |
| mean | 1 | 0.056 | 0.77 | 0.045 | 0.062 |
| std | 0.019 | 0.19 | 0.19 | 0.070 | 0.0035 |
| corr wrt $\ln \bar{y}_{t}$ | 1 | -0.95 | 0.90 | -0.32 | 0.28 |
| reg on $\ln \bar{y}_{t}$ | 1 | -9.49 | 9.16 | -1.18 | 0.048 |
| reg on $\ln u_{t}$ |  | 1 | -0.98 | 0.097 | -0.005 |

Table 3: Fit of employment and turnover moments. (Rows labelled "mean" refer to the mean of levels while the other rows refer to the log of the variable in each column.)

### 3.3 Employment and turnover

Table 3 compares various moments calculated on the actual quarterly series as in Simer (2005) and on the simulated series (as many as 5000 observations were simulated to reduced sampling errors). ${ }^{10}$ The model also predicts an exit rate of unemployment in the right interval albeit with a slightly higher volatility. The moments of the overall separation rate are well reproduced. The model tends to overestimate the correlation with productivity, which induces too much elasticity given that the volatility is well fitted. However, I believe that this is to be expected if there is only one exogenous source of business cycle fuctuations. Figure 5 shows a simulation of the dynamics of unemployment and turnover resulting from an particular simulated history of aggregate productivity shocks (the number of observations is $T=249$ which is the number of quarters in the raw series, between 1947q1-2009q1). The ranges of productivity indices and unemployment rates is as in the actual series. The simulated unemployment series is somewhat less smooth that the true series (see Figure 6).

The elasticity of the exit rate of unemployment with respect to unemployment is correctly reproduced (close to -1 ) but the elasticities of the separation rate and, even more so, the job-to-job mobility rate are underestimated with respect to the values that were

[^6]

Figure 5: Simulation of employment and turnover dynamics


Figure 6: Unemployment series, raw and HP-filtered. (The cycle in the right panel is obtained for a smoothing parameter of $10^{5}$. The left panel shows that the usual smoothing parameter value of 1600 generates a very cyclical trend!)


Figure 7: Unemployment rate as a function of the aggregate shock. (Each step down indicates that a new group of worker becomes employable as aggregate productivity rises. There are 12 steps because only 11 out of $M=300$ workers types, $4.85 \%$ of all workers, face endogenous unemployment risk. When the aggregate productivity index reaches about 1.02 all workers have positive surplus.)
calculated with the JOLTS series (Table 2). Yet it is remarkable that, although unemployment volatility is entirely driven by job destruction dynamics, the model predicts that the elasticity of the exit rate of unemployment is bigger than the elasticity of job separations. This is because the whole volatility of unemployment results from the behaviour of a small fraction of workers (about $5 \%$ ). The distribution of heterogeneity in the unemployment group is also different from its distribution in the population of employees. However, the distorsion is due to such a small fraction of very low ability workers that it may not be easily detected, especially if this heterogeneity is mostly unobserved.

The mechanism by which productivity shocks are amplified is simple to understand. In a boom unemployment is steady, all separations follow from exogenous shocks (there is no endogenous layoff for $i \geq 34$ out of $N=50$ states). When aggregate productivity falls more workers lose their jobs as more match surpluses become negative (see Figure 3.3).

About $4 \%$ unemployment accrues because of the $4.3 \%$ exogenous layoff rate. One may call this minimum unemployment level frictional unemployment. Classical unemployment, due to business cycle conditions, ranges between about 0 and $5 \%$ depending on the severity
of the recession. Notice that although job destruction obviously plays a leading role in determining unemployment, the duration of unemployment however increasing with the depth of the trough, the elasticity of the exit rate of unemployment with respect to productivity is much bigger than the elasticity of the job separation rate. This happens because the elasticity is the correlation divided by the volatility (std). Now the volatility of a rate depends on the volatility of the stock used in the denominator. For the exit rate from unemployment, the denominator is the unemployment stock itself, which is highly volatile. For the job-to-job transition rate and the job destruction rate it is the employment stock which is less volatile because it is much bigger in level than the unemployment stock.

### 3.4 Wages

Table 4 shows that the model can replicate the dynamics of the first and second order wage moments well. In particular, the dynamics of wage inequality in the upper part of the distribution is procyclical and it is counter-cyclical in the bottom part. Overall, countercyclicality dominates. I also compare annual earnings with present values. Given that the model does not have worked hours, it makes sense to consider present values as a way of mixing wages and labour supply. We obtain countercyclicality in both the upper and the bottom parts of the present value distributions. One difference between actual and simulated data is that the volatility of the inequality indices is much lower, equal to about a fourth of what it is in actual data.

So, with this calibration at least, the median wage is found to be less procyclical than the first and last deciles. Table 3.4 confirms this point and also shows an interesting phenomenon. This non-monotonicity is entirely due to the fact that the bottom and the top of the wage distribution are made of two different types of wages: starting wages $\left(\underline{w}_{i}(m)\right)$ and promotion wages $\left(\bar{w}_{i}(m)\right)$. The first decile is made of starting wages whereas the last decile is made of promotion wages, and for this calibration the median wage is also a promotion wage. Starting wages are considerably more pro-cyclical than promotion wages. Moreover, the procyclicality of starting wages diminishes with the rank in the correspond-

|  | Hourly wages |  |  |  | Annual earnings |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | D9/D5 | D5/D1 | D9/D1 | D9/D5 | D5/D1 | D9/D1 |
|  | Actual series |  |  |  |  |  |  |
| mean |  | 2.05 | 2.20 | 4.55 | 2.08 | 2.88 | 6.00 |
| std ${ }^{(*)}$ | 0.017 | 0.023 | 0.030 | 0.026 | 0.018 | 0.078 | 0.081 |
| corr $\mathrm{w} / \bar{y}_{t}{ }^{(*)}$ | 0.64 | 0.15 | -0.31 | -0.21 | -0.10 | -0.31 | -0.32 |
| reg on $\bar{y}_{t}{ }^{(*)}$ | 0.53 | 0.21 | -0.53 | -0.32 | -0.10 | -1.41 | -1.51 |
| Simulated series |  |  |  |  |  |  |  |
|  | Wages |  |  |  | Present values |  |  |
|  | Mean | D9/D5 | D5/D1 | D9/D1 | D9/D5 | D5/D1 | D9/D1 |
| mean | 0.84 | 1.56 | 2.87 | 4.47 | 1.20 | 1.31 | 1.57 |
| std | 0.012 | 0.0055 | 0.011 | 0.0069 | 0.0061 | 0.0076 | 0.0124 |
| corr w/ $\bar{y}_{t}$ | 0.86 | 0.94 | -0.84 | -0.54 | -0.83 | -0.85 | -0.93 |
| reg on $\bar{y}_{t}$ | 0.56 | 0.28 | -0.47 | -0.20 | -0.26 | -0.34 | -0.61 |

Table 4: Fit of wage moments. (Column "Mean" either refers to the BLS series (deflated per person compensation) or to the cross-section mean in the simulated data. Columns "D9/D5", "D5/D1" and "D9/D1" are the decile ratios of either hourly wages and annual earnings calculated from the CPS (panel "Actual series"), or of wages and present values ( $U_{i}(m)$ or $\left.W_{i}(m)\right)$ for simulated data.)
ing distribution, while the opposite is true for promotion wages: procyclicality increases with rank. Hence, wages in the middle of the distribution are the least procyclical.

Pissarides (2009) builds an argumentation based on initial wages in new jobs being different from wages in on-going spells. He also documents a long list of empirical papers looking at initial wages at the beginning of job spells and wages in on-going spells. This literature usually finds initial wages more procyclical than on-going wages. The opposition between initial and on-going wages that is used in the literature is less clear than the opposition between starting and promotion wages that we make here. First, because ongoing wages are not necessarily different from initial wages, and second, because initial wages are not necessarily wages negotiated with unemployment as only threat point. Therefore, I expect the difference in elasticities that the model predicts between starting wages, one one side, and promotion wages and job-to-job mobility wages, on the other side, to be effectively found in the data.

In order to better understand why starting wages and promotion wages have these distinct cyclicality patterns, I next consider the following variance decompositions. Let

|  | Starting wages | Promotion wages | All wages |
| :---: | :---: | :---: | :---: |
| Mean |  |  |  |
| D9 | 0.57 | 1.37 | 1.33 |
| D5 | 0.36 | 0.98 | 0.85 |
| D1 | 0.08 | 0.68 | 0.30 |
| Cyclicality (elasticity wrt productivity) |  |  |  |
| D9 | 1.08 | 0.70 | 0.67 |
| D5 | 1.31 | 0.57 | 0.38 |
| D1 | 3.38 | 0.36 | 0.91 |

Table 5: Cyclicality of starting vs promotion wages (simulated data)
$w_{i t}$ denote the wage of an individual $i$ of type $z_{i t}$ at time $t$, and $z_{i t}$ some characteristics, then

$$
\operatorname{Var} w_{i t}=\underbrace{\operatorname{Var} \mathbb{E}\left(w_{i t} \mid z_{i t}\right)}_{\text {between }}+\underbrace{\mathbb{E} \operatorname{Var}\left(w_{i t} \mid z_{i t}\right)}_{\text {within }} .
$$

Three conditioning variables $z_{i t}$ can be used that contribute to wage dispersion ( $w \in$ $\left.\left\{\underline{w}_{i}(m), \bar{w}_{i}(m)\right\}\right)$ : worker heterogeneity $(m)$, aggregate state dependence $(i)$ and the type of bargaining ( $\underline{w}$ or $\bar{w}$ ).

Table 3.4 shows the between and within contributions of each of these three sources of wage dispersion. The bargaining type explains $60 \%$ of wage dispersion; aggregate state dependence, $40 \%$; and ability only $15 \%$. Bertrand competition, via the difference between starting wages and promotion wages is the main determinant of the level of inequality.

However, only aggregate state dependence contributes negatively to cyclicality. When productivity increases (in a boom) workers with a very low wage (a starting wage) credibly threaten to quit to unemployment as their reservation wage increases with aggregate productivity, and firms are thus forced to renegotiate wages up. This is the main determinant of the stronger procyclicality of low wages. At the other end of the distribution, when aggregate productivity falls (in a downturn) workers with very high wages are forced by their employer to accept a cut as the firm surplus becomes negative. This is the main determinant of the stronger procyclicality of high wages. Wage renegotiation without alternative offers therefore has an interesting effect on wage inequality dynamics.

| Variance | Total | Between | Within |
| :--- | :---: | :---: | :---: |
|  | Group = ability |  |  |
| Share | 0.14 | $0.022(15 \%)$ | 0.116 |
| Std of $\log$ | 0.027 | 0.055 | 0.022 |
| Elasticity | 1.29 | 2.92 | 1.00 |


|  | Group = aggregate state |  |  |
| :--- | :---: | :---: | :---: |
| Share | 0.14 | $0.056(40 \%)$ | 0.082 |
| Std of $\log$ | 0.027 | 0.21 | 0.12 |
| Elasticity | 1.29 | -3.07 | 3.85 |


|  | Group $=$ starting $/$ promotion |  |  |
| :--- | :---: | :---: | :---: |
| Share | 0.14 | $0.083(60 \%)$ | 0.055 |
| Std of $\log$ | 0.027 | 0.021 | 0.039 |
| Elasticity | 1.29 | 0.89 | 1.90 |

Table 6: Wage variance decomposition

## 4 Conclusion

We have proposed a simple dynamic search-matching model with cross-sectional wage dispersion and worker heterogeneous abilities. Worker heterogeneity interacts with aggregate shocks to match productivity in a way that allows for endogenous job destruction. It suffices that a small fraction of the total workforce be at risk of a shock to productivity that renders match surplus negative to amplify productivity shocks enough to generate the observed unemployment volatility. Moreover, we show that the model can generate inequality dynamics similar to the observed pattern: wages in the middle of the distribution are less procyclical than wages in the bottom and the top. We argue that it may reflect the renegotiation process that is implied by long-term contracts following productivity shocks. Extreme wages are subject to renegotiation as low wages may become lower than workers' reservation wages after a positive productivity shock and high wages may become greater than firms' reservation wages following a negative shock. Wages in the middle of the distribution are more likely to remain in the bargaining set.

Our prototypical model is extremely simple to simulate outside the steady-state equilibrium and still generates very rich dynamics. This is due to two very strong assumptions: firms have full monopsony power and they are identical. Giving workers some bargaining
power as in Cahuc et al. (2006) and Dey and Flinn (2005) and allowing for firm heterogeneity as in Lise et al. (2009), in a macrodynamic model, are very exciting avenues for further research.

## References

[1] Baker, Michael (1997) "Growth-rate Heterogeneity and the Covariance Structure of Life-Cycle Earnings," Journal of Labor Economics, Vol. 15, pp. 338-375.
[2] Baker, Michael, and Gary Solon. 2003. "Earnings Dynamics and Inequality among Canadian Men, 1976-1992: Evidence from Longitudinal Income Tax Records," Journal of Labor Economics, vol. 21, no. 2
[3] Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin. 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence." Econometrica, 74: 323-364.
[4] Chen, Xiaohong, Wei Biao Wu, and Yanping Yi, 2009. "Efficient Estimation of Copula-Based Semiparametric Markov Models." arXiv:0901.0751v3 [math.ST]
[5] Dey, Matthew S., and Christopher J. Flinn, 2005. "An Equilibrium Model of Health Insurance Provision and Wage Determination," Econometrica, 73(2): 571-627.
[6] Diamond, Paul. 1971. "A model of price adjustment." Journal of Economic Theory, 3: 156-168
[7] Diamond, Paul. 1982. "Aggregate Demand Management in Search Equilibrium." Journal of Political Economy, 90: 881-894.
[8] Gertler, Mark, and Antonella Trigari, 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." Journal of Political Economy, University of Chicago Press, vol. 117(1), 38-86, 02.
[9] Guvenen, Fatih. 2009. "An Empirical Investigation of Labor Income Processes." Review of Economic Dynamics, Vol. 12, No 1, pp. 58-79.
[10] Guvenen, Fatih. 2007. "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?" American Economic Review, Vol. 97, No. 3, pp. 687-712.
[11] Haefke, Christian, Marcus Sonntag, and Thijs van Rens. 2007. "Wage Rigidity and Job Creation." Unpublished.
[12] Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." American Economic Review, 98(4): 16921706.
[13] Haider Steve J. 2001 "Earnings Instability and Earnings Inequality of Males in the United Stated: 1967-1991." Journal of Labor Economics, Vol. 19, pp. 799-836.
[14] Hall, Robert E. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." American Economic Review, 95(1): 50-65.
[15] Hall, Robert E. 2006. "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years." In NBER Macroeconomics Annual 2005, ed. Mark Gertler and Kenneth Rogoff, 103-37. Cambridge, MA: MIT Press.
[16] Hall, Robert E, and Paul R. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." American Economic Review, 98(4): 1653-1674.
[17] Lise J., C. Meghir and J.-M. Robin. 2009. "Matching, Sorting and Wages." Mimeo. University College London.
[18] Meghir, Costas, and Luigi Pistaferri. 2004. "Income Variance Dynamics and Heterogeneity," Econometrica, Vol. 72, pp. 1-32.
[19] Menzio, Guido and Shouyong Shi. 2009. "Efficient Search on the Job and the Business Cycle," NBER Working Papers 14905.
[20] Moffitt, Robert A., and Peter Gottschalk. 1995. "Trends in the Autocovariance Structure of Earnings in the US, 1969-1987," Working Paper.
[21] Mortensen, Dale T., and Éva Nagypál. 2007. "More on Unemployment and Vacancy Fluctuations." Review of Economic Dynamics, 10(3): 327-47.
[22] Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." Review of Economic Studies, 61(3): 397-415.
[23] Osborne, Martin J., and Ariel Rubinstein. 1990. Bargaining and Markets. New York: Academic Press.
[24] Petrongolo, Barbara, and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." Journal of Economic Literature, 39(2): 390-431.
[25] Pissarides, Christopher. 1990. Equilibrium Unemployment Theory. Oxford: Blackwell.
[26] Pissarides, Christopher. 2007. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" Walras-Bowley Lecture, North American Summer Meetings of the Econometric Society, Duke University. Forthcoming in Econometrica.
[27] Postel-Vinay, Fabien, and Jean-Marc Robin. 2002. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity." Econometrica, 70(6): 2295-2350.
[28] Postel-Vinay, F. and H. Turon, 2009. "On-the-job Search, Productivity Shocks, and the Individual Earnings Process." Forthcoming in International Economic Review.
[29] Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." Econometrica, 50(1): 97-109.
[30] Rubinstein, Ariel, and Asher Wolinsky. 1985. "Equilibrium in a Market with Sequential Bargaining." Econometrica, 53(5): 1133-50.
[31] Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." American Economic Review, 95(1): 25-49.
[32] Shimer, Robert. 2007. "Reassessing the Ins and Outs of Unemployment." http://robert.shimer.googlepages.com/reassess-print.pdf.


[^0]:    *The Walras-Bowley lecture, North American Summer Meetings of the Econometric Society, Boston University, June 2009. Various versions of this paper were presented in seminars and conferences and greatly benefited from the comments of participants (seminars at IFS, Stocholm University, Sciences Po, Crest-Insee, Cambridge University, Paris School of Economics; conferences at the Milton Friedman Institute, University of Chicago, Jerusalem (Bank of Israel), Venice (CSEifo)). I greatly benefited from the comments of Rob Shimer, Fabien Postel-Vinay, Francis Kramarz, Etienne Wasmer, Pierre Cahuc, Francis Kramarz.

[^1]:    ${ }^{1}$ The inequality data are obtained from about 20 years of CPS surveys starting in1967. I am immensely

[^2]:    ${ }^{2}$ The randomness in the eventual mobility may explain why employers engage in Bertrand competition in the first place.

[^3]:    ${ }^{3}$ I follow the usual practice since Shimer (2005) and use a smoothing parameter of $10^{5}$ instead of the value of 1,600 that is usually used with quarterly data. The usual smoothing parameter seems to put too much cycle in the trend. This is particularly clear for the nearly non-trended unemployment series (see Figure 6 below).

[^4]:    ${ }^{6}$ Shimer $(05,07)$ estimates an even higher rate of $83 \%\left(45 \%\right.$ per month, hence $1-(1-.45)^{3}=83 \%$ per quarter) using CPS data.
    ${ }^{7}$ A high number of worker types is necessary to smooth the dynamics of unemployment (more on this later) and I simulate a large number of observations to reduce the variance of empirical moments.

[^5]:    ${ }^{8}$ These values moments were calculated using HP-filtered, long (1947q1-2009q1), quarterly series from the BLS as in Shimer (2005).
    ${ }^{9}$ I use hourly compensation (PRS85006103) divided by the implicit output deflator (PRS85006113), readjusted per person by multiplying by hours (PRS85006033) and dividing by employment (PRS85006013). One argument in favor of this series is that when I detrend it using the HP-filter with the same smoothing parameter, I obtain exactly the same trend as for productivity, and regressing wage on productivity gives a coefficient of one. Note that the estimated elasticity is close to that calculated by Gertler and Trigari (2009) from CPS data (series posterior to 1967).

[^6]:    ${ }^{10}$ Aggregate productivity is calculated as $\bar{y}_{t}=\frac{\sum_{m}\left(1-u_{t}(m)\right) \ell_{m} y_{t}(m)}{1-u_{t}}$ with $y_{t}(m)=y_{i}(m)$ if $y_{t}=y_{i}$.

