

**A UNIFIED FRAMEWORK FOR MEASURING PREFERENCES
FOR SCHOOLS AND NEIGHBORHOODS**

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A Unified Framework for Estimating Preferences for Schools and Neighborhoods

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Abstract

This paper sets out a framework for estimating household preferences over a broad range of housing and neighborhood characteristics, some of which are determined by the way that households sort in the housing market. This framework brings together the treatment of heterogeneity and selection that has been the focus of the traditional discrete choice literature with a clear strategy for dealing with the correlation of unobserved neighborhood quality with both school quality and neighborhood sociodemographics. We estimate the model using rich data on a large metropolitan area, drawn from a restricted version of the Census. The estimates indicate that, on average, households are willing to pay an additional one percent in house prices - substantially lower than in prior work - when the average performance of the local school is increased by 5 percent. There is also evidence of considerable preference heterogeneity. We also show that the full capitalization of school quality into housing prices is typically 70-75 percent greater than the direct effect as the result of a social multiplier, neglected in the prior literature, whereby increases in school quality also raise prices by attracting households with more education and income to the corresponding neighborhood.

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1 INTRODUCTION

Economists have long been interested in estimating the demand for non-marketed goods such as school quality, and for good reason. From a policy perspective, recovering an accurate household valuation of school quality allows the direct benefit of education reforms to be quantified. Estimates of a wider range of underlying preferences parameters also play an important role in understanding the way that households sort in the housing market, which in turn determines the pattern of residential segregation and the matching of households to schools. Building on theoretical work studying the effects of household sorting on equilibrium in the housing market,¹ researchers have used equilibrium models more recently to simulate policy changes, notably choice-based education reforms, identifying important general equilibrium effects as households re-sort that do not appear in partial equilibrium.² However, parameters in this body of research have typically been chosen through calibration or more arbitrarily. Based on direct estimation of demand parameters, more reliable and coherent sets of preference estimates can be used, with the potential to improve our understanding of policy reforms and the workings of the urban economy more generally.

Despite their usefulness, recovering demand parameters from observed choices in the housing market poses considerable challenges for estimation. Underlying household preferences are likely to be heterogeneous, depending on a wide range of household characteristics; to uncover these heterogeneous preferences requires extensive data on both households and the detailed characteristics of choices in the choice set, typically unavailable in public-use datasets. Even with the richest data, it is not possible to characterize fully the factors that make certain houses and neighborhoods particularly desirable – there will invariably be a component that is unobserved to the econometrician. And given that households sort on a non-random basis, in part influenced by these unobservable choice characteristics, endogeneity problems arise as neighborhood sociodemographic composition and school quality will be correlated with these unobservables.

Of the two most prominent approaches to estimating demand in the literature, *hedonic price regressions* relate house prices to housing and neighborhood attributes including school

¹ Much of the intuition for how household sorting affects the equilibrium in the housing market and consequently the matching of households with schools derives from a long line of theoretical work in local public finance. Important contributions to this literature date back to the work of Tiebout (1956) and include more recent research by Epple and Zelenitz (1981), Epple, Filimon, and Romer (1984, 1993), Benabou (1993, 1996), Fernandez and Rogerson (1996), and Nechyba (1997).

² Direct analyses of choice-based and education finance reforms have been conducted by Epple and Romano (1998), Nechyba (1999, 2000), and Fernandez and Rogerson (2003) among others.

characteristics, while *traditional discrete choice models* estimate preferences by attempting to match the location decisions of the households in the data.^{3,4} An attractive feature of the recent hedonic price regression literature has been its focus on addressing the likely correlation of school quality with unobserved housing and neighborhood quality, making use of a boundary fixed effects strategy.⁵ However, it is difficult to address the additional endogeneity problems that arise within the standard framework due to non-random sorting by households. Further, it is difficult to determine how the estimates of a hedonic price regression relate to fundamental preferences in the population, both because the equilibrium price function need not correspond to mean preferences [see Tinbergen (1956) and Rosen (1974)] and because this approach provides no clear way of estimating heterogeneous preferences.

While dealing with heterogeneity in preferences in a straightforward fashion, the discrete choice literature in urban and public economics has traditionally assumed away any systematic, unobserved differences in the quality of houses and neighborhoods. Thus when housing prices are included in the indirect utility function, the positive correlation of price with unobserved housing and neighborhood quality has been routinely ignored, leading to a severe understatement of price elasticities and to biases in the estimation of other taste parameters.⁶ Moreover, the likely correlation of test scores with unobserved neighborhood quality has not been adequately addressed in that literature.

The current paper makes two primary contributions. First, it sets out a new framework for estimating household preferences over a broad range of housing and neighborhood characteristics, some of which are determined by the way that households sort in the housing market. Here, we bring together in a unified framework the treatment of heterogeneity and selection that has been the focus of the discrete choice literature while also addressing the correlation of school quality with unobserved neighborhood quality that has been the focus of

³ This framework was first introduced for the study of housing markets by McFadden (1978). Examples related to estimating preferences for school quality include Quigley (1985), Nechyba and Strauss (1998), Bayer (1999), and Barrow (1999).

⁴ *Hedonic demand models* studied by Rosen (1974), Epple (1987), Bajari and Benkhard (2002), Ekeland, Heckman, and Nesheim (2002) and Heckman, Matzkin, and Nesheim (2003) among others provide another approach to estimating demand for non-marketed goods and attributes. These models have not been employed to date to estimate demand for schooling, although Nesheim (2001) proposes such an empirical exercise. The fundamental difference between hedonic demand and discrete choice models as applied to location choice problems relates to whether households are assumed to be able to select the level of consumption of each element of the bundle of attributes determined by the location decision in order to satisfy the first order condition associated with that element or whether households are constrained to choose among the set of choices (bundles) that exist in the data.

⁵ See, for example, Black (1999), Clapp and Ross (2002), and Kain, Staiger, and Samms (2003).

recent work in the hedonic price regression literature. To account for the non-random sorting of households, we model the sorting process directly, generalizing the traditional discrete choice approach to incorporate unobserved housing and neighborhood quality. The framework we develop nests both of the main approaches as direct restrictions. It provides an intuitive correction for biases in hedonic price regressions that arise when the marginal household's preferences differ from those of the mean household.

An important feature of our approach is the solution we provide to the endogeneity problem arising when neighborhood unobservables and observed neighborhood sociodemographics are correlated. Here, we adapt an approach already used in the hedonic price literature to control for the correlation of school quality and unobserved neighborhood quality using boundary fixed effects, showing how it can profitably be applied to the problem of estimating consistently the valuation of neighborhood sociodemographic composition. The essential idea is intuitive: important sociodemographics are discontinuous at boundaries due to household sorting - we show clear evidence on this below. This means that boundaries become useful places to learn about the valuation of such variables. Boundary fixed effects serve as an attractive way of absorbing out fixed unobservable components when estimating preferences for these sociodemographics⁷ - our estimates lend support to the idea that this is a promising strategy.

The second contribution is to estimate the model using the richest data available, to obtain a broad range of underlying preference estimates that account the measurement difficulties already referred to. Here, we benefit from using restricted-access Census microdata that provide detailed household and housing information including the precise residential location of nearly a quarter of a million households in the San Francisco Bay Area. With the resulting estimates in hand, we are then able to use our equilibrium framework to explore their general equilibrium implications, drawing attention to important effects neglected in the prior literature. Obtaining consistent estimates of a broad range of preferences is necessary for this task.

Our estimates of preferences for school quality indicate that on average households are willing to pay an additional one percent in house price when the average performance of the local school is increased by 5 percent, substantially lower than in prior work. These estimates control directly for detailed measures of neighborhood sociodemographic composition, which previous researchers often have been unable to include in their analysis. We show that the failure to

⁶ Recent papers by Bayer (1999), Bajari and Kahn (2002), and Bayer, McMillan, and Rueben (2002) include a term that captures the unobserved (to the econometrician) quality of houses in neighborhoods in the utility specification and address the endogeneity of price in this context.

control for neighborhood sociodemographics leads to the overstatement of the mean marginal willingness to pay for school quality by over 200 percent. The estimates also reveal substantial heterogeneity in preferences for school quality – for instance, households with children and more educated households value school quality more than those without. Further, we find evidence of very strong social interactions – highly educated households, for instance, are willing to pay a much higher price to live with other highly educated households than those who are not so well educated. This heterogeneity in preferences implies that simple hedonic price regressions will typically not return average preferences and we demonstrate that price regressions lead to significant biases in the estimation of mean preferences for neighborhood sociodemographic characteristics.

Our framework provides a natural device for exploring the general equilibrium implications of these estimates, offering both a complete characterization of the heterogeneity in preferences for schools and neighborhoods and a way of understanding how these aggregate to determine the equilibrium in the housing market. We show that the estimated heterogeneity gives rise to substantial variation in the capitalization of school quality in housing prices throughout the metropolitan region. Further, the full capitalization of school quality into housing prices is typically 70-75 percent greater than the direct effect, as the result of a social multiplier. Focusing on the direct effect only, the vast prior literature on capitalization has neglected this significant general equilibrium mechanism,⁸ whereby increases in school quality also raise housing prices by attracting households with more education and income to the corresponding neighborhoods.

The rest of the paper is organized as follows: the sorting model is presented in the next section. Section 3 discusses estimation issues. The unique data used to estimate the model are described in Section 4. Estimation results are presented in Section 5, and results of general equilibrium simulations, in Section 6. Section 7 concludes.

2 A MODEL OF RESIDENTIAL SORTING

This section of the paper sets out an equilibrium model of a self-contained, urban housing market in which households sort themselves among the set of housing types and locations available in the market. The model consists of two key elements: the household residential location decision problem and a market-clearing condition. While maintaining this simple

⁷ It is important to stress that our analysis calls into question the narrow use of boundaries in the prior literature, though. As soon as households sort non-randomly, as they certainly do in practice, capitalization of house prices at school attendance boundaries picks up more than just differences in school quality.

structure, the model is quite powerful, allowing households to have heterogeneous preferences defined over housing and neighborhood attributes in a very flexible way; it also allows for housing prices and neighborhood sociodemographic compositions to be determined in equilibrium. Importantly, the exact characterization of the conditions for equilibrium is not necessary for the estimation of the model, i.e., recovering the underlying preference parameters. We characterize the equilibrium conditions here primarily because we use the full model later in the paper to carry out a series of general equilibrium simulations, exploring the implications of the preference estimates for the capitalization of school quality into local house prices.

The Residential Location Decision

We model the residential location decision of each household as a discrete choice of a single residence. The utility function specification is based on the random utility model developed in McFadden (1978) and the specification of Berry, Levinsohn, and Pakes (1995), which includes choice-specific unobservable characteristics. Let X_h represent the observable characteristics of housing choice h including characteristics of the house itself (e.g., size, age, and type), its tenure status (rented vs. owned), and the characteristics of its neighborhood (e.g., sociodemographic composition, school, crime, and topography). Let p_h denote the price of housing choice h . Each household chooses its residence h to maximize its indirect utility function V_h^i :

$$(1) \quad \underset{(h)}{Max} \quad V_h^i = \mathbf{a}_X^i X_h - \mathbf{a}_p^i p_h + \mathbf{x}_h + \mathbf{e}_h^i.$$

The error structure of the indirect utility is divided into a correlated component associated with each house that is valued the same by all households, \mathbf{x}_h , and an individual-specific term, \mathbf{e}_h^i . A useful interpretation of \mathbf{x}_h is that it captures the unobserved quality of each house, including any unobserved quality associated with its neighborhood.⁹

⁸ This has a long history of study in the literature. See, for example, Oates (1969), Kain and Quigley (1975), Hayes and Taylor (1996), Black (1999), Bogart and Cromwell (2000), Figlio and Lucas (2000), Clapp and Ross (2002), and Kane, Staiger, and Samms (2003).

⁹ We employ an indirect utility function that is linear in housing prices primarily because it facilitates comparisons with standard hedonic price regressions, as we discuss in Section 3 below. This structure does not seriously limit the flexibility of the model in terms of income elasticities, however, as we allow for interactions of income with all of the choice characteristics and directly with house price. This specification ensures, for example, that households without much income very rarely choose expensive homes. Alternative specifications of the indirect utility function could certainly be estimated, as the linear form is not essential to the model.

Each household's valuation of choice characteristics is allowed to vary with its own characteristics, Z^i , including education, income, race, employment status, and household composition. Specifically, each parameter associated with housing and neighborhood characteristics and price, \mathbf{a}_j^i , for $j \in \{X, p\}$, varies with a household's own characteristics according to:

$$(2) \quad \mathbf{a}_j^i = \mathbf{a}_{0j} + \sum_{r=1}^R \mathbf{a}_{rj} Z_r^i,$$

and equation (2) describes household i 's preference for choice characteristic j .

Given the household's problem described in equations (1)-(2), household i chooses housing choice h if the utility that it receives from this choice exceeds the utility that it receives from all other possible house choices - that is, when

$$(3) \quad V_h^i > V_k^i \Rightarrow W_h^i + \mathbf{e}_h^i > W_k^i + \mathbf{e}_k^i \Rightarrow \mathbf{e}_h^i - \mathbf{e}_k^i > W_k^i - W_h^i \quad \forall k \neq h$$

where W_h^i includes all of the non-idiosyncratic components of the utility function V_h^i . As the inequalities in (3) imply, the probability that a household chooses any particular choice depends in general on the characteristics of the full set of possible house choices. Thus the probability P_h^i that household i chooses housing choice h can be written as a function of the full vectors of house/neighborhood characteristics (both observed and unobserved) and prices $\{\mathbf{X}, \mathbf{p}, \mathbf{x}\}$:

$$(4) \quad P_h^i = f_h(Z^i, \mathbf{X}, \mathbf{p}, \mathbf{x})$$

as well as the household's own characteristics Z^i .

Equilibrium^{10,11}

We define a sorting equilibrium to be a set of residential location decisions and a vector of housing prices such that the housing market clears and each household makes its optimal

¹⁰ For a much broader discussion of the assumptions, conditions, and properties of the sorting equilibrium defined here, see Bayer, McMillan, and Rueben (2002). The notion of a sorting equilibrium we develop is closely related to that of Brock and Durlauf (2001, 2002).

¹¹ The equilibrium concept developed here treats the supply of housing as fixed. This is done for expositional simplicity. A more generic housing supply function could be incorporated in the analysis.

location decision given the location decisions of all other households. The computational requirements of this equilibrium concept are greatly simplified if we smooth the residential location decision problem. In particular, we assume that each household observed in the sample represents a continuum of households with the same observable characteristics, letting the measure of this continuum be \mathbf{m} . When the set of draws $\{\mathbf{e}_h^i\}$ for each household observed in the data is interpreted as unobserved heterogeneity in preferences for each location, we can then work with the choice probabilities defined in equation (4) when deriving the conditions required for equilibrium. These choice probabilities characterize the distribution of housing choices that would result for the continuum of households with a given set of observed characteristics as each household responds to its particular unobserved preferences.

This assumption concerning the distribution of households requires an analogous assumption about the set of housing choices observed in the sample. In particular, we assume that each house observed in the sample represents a particular type of housing in the observed location, and that the continuum of this housing type also has measure \mathbf{m} . Aggregating the probabilities in equation (4) over all households yields the predicted number of households that choose each housing type in each location h , \hat{N}_h :

$$(5) \quad \hat{N}_h = \mathbf{m} \cdot \sum_i P_h^i .$$

In order for the housing market to clear, the number of households choosing houses of type h must equal the measure of the continuum of houses that each observed house represents:¹²

$$(6) \quad \hat{N}_h = \mathbf{m}, \quad \forall h \quad \Rightarrow \quad \sum_i P_h^i = 1, \quad \forall h .$$

That the probabilities add to one for each house observed in the data simply implies that supply must equal demand for each type of housing in each location.

The implications of this market clearing condition for prices are intuitive, with excess demand for a housing type causing price to be bid up and excess supply leading to a fall in price. In equilibrium, the location decisions that arise as a result of these market-clearing prices must aggregate up to give rise to the neighborhood sociodemographic compositions used in calculating

the market-clearing prices. Bayer, McMillan, and Rueben (2002) establish the existence of a sorting equilibrium as long as (i) the indirect utility function shown in equation (1) is decreasing in housing prices for all households; (ii) indirect utility is a continuous function of neighborhood sociodemographic characteristics; and (iii) \mathbf{e} is drawn from a continuous density function. We describe a method for calculating an equilibrium in Section 6 where we conduct a series of counterfactual general equilibrium simulations.

3 ESTIMATION

Estimation of the model follows a two-step procedure related to that developed in Berry, Levinsohn, and Pakes (1995).¹³ In this section of the paper, after briefly describing this estimation procedure, we discuss several additional issues related to the estimation and identification of the model. We begin that discussion by making clear the relation between our framework and a simple hedonic price regression, showing that the latter results from restricting household preferences to be homogeneous – i.e., ruling out any sorting of households across locations and housing choices on the basis of sociodemographic characteristics. In the presence of heterogeneous preferences, the hedonic price regression does not typically bear any direct relationship to the structural preference parameters. Using the broader sorting model, however, we are able to show that a modified price regression that forms the basis for the second step of our estimation procedure does indeed return mean preferences. In this way, we are able to provide intuition for the likely biases in a hedonic price regression, when such a regression is viewed as returning mean marginal willingness to pay measures, and also to make clear that the same variation in the data that forms the basis for estimating hedonic price regressions is also exploited as part of estimating the broader sorting model. Finally, we describe how our framework can be adapted to deal with the correlation of school quality and neighborhood sociodemographic characteristics with unobservable local characteristics.

The Estimation Procedure

We begin by describing the estimation of the model, and here it is helpful to introduce some notation that simplifies the exposition. In particular, we rewrite the indirect utility function as:

¹² Note that the measure \mathbf{m} drops out of the market-clearing condition in equation (6), and so serves simply as a rhetorical device for understanding the use of the continuous choice probabilities shown in equation (4) rather than the actual discrete choices of the individuals observed in the data in defining equilibrium.

$$(7) \quad V_h^i = \mathbf{d}_h + \mathbf{I}_h + \mathbf{e}_h^i$$

where

$$(8) \quad \mathbf{d}_h = \mathbf{a}_{0X} X_h - \mathbf{a}_{0p} p_h + \mathbf{x}_h$$

and

$$(9) \quad \mathbf{I}_h^i = \left(\sum_{k=1}^K \mathbf{a}_{kX} Z_k^i \right) X_h - \left(\sum_{k=1}^K \mathbf{a}_{kp} Z_k^i \right) p_h.$$

In equation (8), \mathbf{d}_h captures the portion of the utility provided by housing choice h that is common to all households, and in (9), k indexes household characteristics. When the household characteristics included in the model are constructed to have mean zero, \mathbf{d}_h is the *mean indirect utility* provided by housing choice h . The unobservable component of \mathbf{d}_h , \mathbf{x}_h , captures the portion of unobserved preferences for housing choice h that is correlated across households, while \mathbf{e}_h^i represents unobserved preferences over and above this shared component.

The first step of the estimation procedure is a Maximum Likelihood estimator, which returns estimates of the heterogeneous parameters in \mathbf{I} and mean indirect utilities, \mathbf{d}_h . The ML estimator is based on maximizing the probability that the model correctly matches each household with its chosen house. In particular, for any combination of the heterogeneous parameters in \mathbf{I} and mean indirect utilities, \mathbf{d}_h , the model predicts the probability that each household i chooses house h . We assume that \mathbf{e}_h^i is drawn from the extreme value distribution; in which case this probability can be written:

$$(10) \quad P_h^i = \frac{\exp(\mathbf{d}_h + \hat{\mathbf{I}}_h^i)}{\sum_k \exp(\mathbf{d}_k + \hat{\mathbf{I}}_k^i)}$$

Maximizing the probability that each household makes its correct housing choice gives rise to the following log-likelihood function:

¹³ We provide a more technical discussion of the estimation procedure: relating it to the BLP procedure, discussing methods for simplifying the computation, and describing the asymptotic properties of the estimator, in a technical appendix.

$$(11) \quad \ell = \sum_i \sum_h I_h^i \ln(P_h^i)$$

where I_h^i is an indicator variable that equals 1 if household i chooses house h in the data and 0 otherwise. The first step of the estimation procedure consists of searching over the parameters in \mathbf{I} and the vector of mean indirect utilities to maximize ℓ . Notice that the likelihood function developed here is based solely on the notion that each household's residential location is optimal given the set of observed prices and the location decisions of other households.

The Mechanics of the First Step of the Estimation

Intuitively, it is easy to see how this first step of the estimation procedure ties down the heterogeneous parameters – those involving an interaction of household characteristics with housing and neighborhood characteristics. If more educated households are more likely to choose houses near better schools in the data for instance, a positive interaction of education and school quality will allow the model to fit the data better than a negative interaction would.

What is less intuitive is how the vector of mean indirect utilities is determined. To better understand the mechanics of the first step of the estimation, it is helpful to write the derivative of the log-likelihood function with respect to \mathbf{d}_h :

$$(12) \quad \frac{\partial \ell}{\partial \mathbf{d}_h} = \sum_{i=h} \frac{\partial \ln(P_h^i)}{\partial \mathbf{d}_h} + \sum_{i \neq h} \frac{\partial \ln(P_h^i)}{\partial \mathbf{d}_h} = \sum_{i=h} (1 - P_h^i) + \sum_{i \neq h} (-P_h^i) = 1 - \sum_i (P_h^i) = 0$$

As this equation shows, the likelihood function is maximized at the vector \mathbf{d} that forces the sum of the probabilities to equal one, $\sum_i (P_h^i) = 1$ for each house. That this condition must hold for all

houses results from a fundamental trade-off in the likelihood function. In particular, an increase in any particular \mathbf{d}_h raises the probability that each household in the sample chooses house h . While this increases the probability that the model correctly predicts the choice of the household that actually resides in house h , it decreases the probability that all of the other households in the sample make the correct choice. In this way, the first step of the estimation consists of choosing the interaction parameters that best match each individual with their chosen house, while ensuring that no house is systematically more attractive than any other house according to the metric $\sum_i (P_h^i)$.

For any set of interaction parameters (those in λ), a simple contraction mapping can be used to calculate the vector \mathbf{d} that solves the set of first order conditions: $\sum_i (P_h^i) = 1 \forall h$. For our application, the contraction mapping is simply:

$$(13) \quad \mathbf{d}_h^{t+1} = \mathbf{d}_h^t - \ln(\sum_i \hat{P}_h^i)$$

where t indexes the iterations of the contraction mapping. Using this contraction mapping, it is possible to solve quickly for an estimate of the full vector $\hat{\mathbf{d}}$ even when it contains a large number of elements, thereby dramatically reducing the computational burden in the first step of the estimation procedure.¹⁴

Notice that while we have not explicitly enforced the market clearing conditions derived above, the conditions that result from maximizing the likelihood with respect to \mathbf{d} are identical to the market-clearing conditions shown in equation (6). Thus, there is a clear duality between the equilibrating role of prices in our characterization of equilibrium in the housing market and the way that the vector of mean indirect utilities is determined as a result of maximizing the likelihood that each household chooses its appropriate house. In the context of the model itself, we provide intuition below for why the level of mean indirect utility varies across houses in equilibrium. For the interested reader, we provide a more extensive discussion of the estimation procedure in a technical appendix.

The Second Stage of the Estimation

Having estimated the vector of mean indirect utilities in the first stage of the estimation, the second stage of the estimation involves decomposing \mathbf{d} into observable and unobservable components according to the regression equation (8). Notice that the set of observed residential choices provides no information that distinguishes the components of \mathbf{d} . That is, however \mathbf{d} is broken into components, the effect on the probabilities shown in equation (10) is identical. In estimating equation (8) important endogeneity problems need to be confronted. Most obviously, to the extent that house prices partly capture house and neighborhood quality unobserved to the econometrician, so the price variable will be endogenous. Estimation via least squares will thus

¹⁴ It is worth emphasizing that a separate vector \mathbf{d} is calculated for each set of interaction parameters – and at the optimum, this procedure returns the ML estimates of the interaction parameters and the vector of mean indirect utilities \mathbf{d} .

lead to price coefficients biased towards zero, producing misleading willingness-to-pay estimates for a whole range of choice characteristics. In Section 5 below, we describe the construction of an instrument for price. When correctly specified, the right hand side of (8) will include a variety of other choice characteristics, including those related to the way that households sort across neighborhoods. Later in this section, we present an appealing strategy for dealing with the correlation between neighborhood sociodemographic characteristics and fixed, unobservable neighborhood quality.

A Restricted Version of the Model

For now, we abstract from these endogeneity issues in order to demonstrate that a hedonic price regression is a direct restriction on the full model. Consider a specification of the utility function in which all households share the same value for each house up to an idiosyncratic error term:

$$(14) \quad U_h^i = \mathbf{a}_{0X} X_h - \mathbf{a}_{0p} P_h + \mathbf{x}_h + \mathbf{e}_h^i$$

where \mathbf{e}_h^i is i.i.d. across households and choices. In this case, because the choice probabilities shown in (10) are identical for all households, the first order conditions, $\sum_i (P_h^i) = 1 \forall h$, imply that the ML estimates of \mathbf{d}_h must be identical (equal to a constant K) for all houses. In this case, then, equation (8) can be re-written:

$$(15) \quad \mathbf{a}_{0X} X_h - \mathbf{a}_{0p} P_h + \mathbf{x}_h = K \quad \mathbf{P} \quad P_h = \mathbf{a}_{0X} / \mathbf{a}_{0p} X_h + 1 / \mathbf{a}_{0p} \mathbf{x}_h$$

Equation (15) is a standard hedonic price regression. This equivalence makes clear that a hedonic price regression properly returns the mean valuation of housing and neighborhood attributes when heterogeneity in preferences is limited to only an idiosyncratic component.¹⁵

Note that equation (8), which forms the basis for the second stage regression in the estimation of the sorting model, bears more than a passing resemblance to the hedonic price regression shown in equation (13). In particular, moving price to the left-hand side of equation (8) yields:

¹⁵ This condition holds no matter what assumption is made concerning the distribution of the idiosyncratic error term and in the absence of such idiosyncratic preferences.

$$(16) \quad p_h + \frac{1}{a_{0p}} \mathbf{d}_h = \frac{a_{0x}}{a_{0p}} X_h + \frac{1}{a_{0p}} \mathbf{x}_h$$

Consequently, in the presence of heterogeneous preferences, the mean indirect utility \mathbf{d}_h estimated in the first stage of the estimation procedure provides an adjustment to the hedonic price equation so that the price regression accurately returns mean preferences.

It is useful to spell out the significance of (16). In the context of our model, it provides intuition as to why the equilibrium price function differs from the mean marginal willingness-to-pay when households have heterogeneous preferences. Figure 1 provides this intuition in a simple example in which households value a single, discrete characteristic of a house, such as a view of the Golden Gate Bridge.¹⁶ If such a view were rare, as represented by H_1 in Figure 1, the difference in price between houses with versus without a view would reflect the marginal willingness-to-pay (MWTP) of a household with a relatively strong taste for a view, as indicated by p_1^* in the figure. Put another way, the equilibrium price of a view is set by the households on the *margin* of purchasing a house with a view rather than by the household with mean MWTP, which in this case is clearly infra-marginal. If, on the other hand, a view were widely available, the price of the view would generally reflect the MWTP of someone much lower in the distribution of tastes for a view, as indicated by p_2^* . In the first case, the price of these houses would be exceedingly high relative to the MWTP of the mean household, which is indicated by p_M^* , while in the second case, the price of a view in equilibrium would more closely resemble mean MWTP.

This example also makes clear that \mathbf{d}_h can vary across houses in equilibrium. In the first case, the mean indirect utility of a house with a view would be less than that of a house without a view, as evidenced by the fact that the mean household prefers the house without a view in this case. In the second case, the mean household would prefer the house with a view and, consequently, the mean indirect utility of a house with a view would be greater. In more general cases, the mean indirect utility that house provides will be a function of its characteristics, the distribution of characteristics across the set of available houses, and the distribution of tastes. In essence, the sorting model controls for which individual in the distribution of tastes sets the price of a given attribute given the supply of that attribute. This provides an adjustment that reflects the difference between this household's valuation and that of the mean household so that the adjusted hedonic price regression accurately reflects mean preferences. In the first case, that the

¹⁶ For this example, we ignore the idiosyncratic preference term for expositional simplicity.

mean indirect utility of a house with a view is less than that without a view effectively reduces the left-hand side of equation (16) for houses with a view so that it reflects the amount the mean household would be willing to pay for a view.¹⁷ Without incorporating the adjustment for the difference in mean utilities, a hedonic price regression would clearly return different estimates in the two scenarios.

Hedonic Price Regressions and Selection Bias

Equation (16) also provides intuition for why a seemingly natural way to learn about heterogeneous preferences does not work. In particular, consider estimating a separate hedonic price regression using only the sample of houses chosen by a well-defined subset of the population – households of a particular race for instance. Such regressions might be referred to as *type-specific hedonic price regressions* and intuitively these regressions would seek to estimate the preferences of this subset of the population by exploiting *within-type price variation* – the variation in price and housing/neighborhood characteristics among the set of houses chosen by this type of household. Using equation (16), however, it is straightforward to show that type-specific hedonic price regressions are subject to an important form of selection bias.

To see the selection bias problem more clearly, consider a simple example with two types of households – type 1 and type 2 - in which \mathbf{d}_h is defined to represent the indirect utility provided by housing choice h to type 1 households.¹⁸ In a bid to recover the preferences of type 1 households, suppose a researcher attempted to estimate a hedonic price regression using only the set of houses chosen by that type (i.e., using only within-type price variation). By revealed preference, the set of houses chosen by type 1 households will provide higher indirect utility to those households, given by \mathbf{d}_h , than those not chosen. But by estimating a simple hedonic price regression without making any correction, the researcher omits the \mathbf{d}_h term from the left-hand side of equation (16). This is essentially a case of sample selection on the dependent variable. The prices of these chosen houses will tend to be low, conditional on the observed choice characteristics X_h , given the omission of the \mathbf{d}_h term, and this will lead to an understating of the willingness-to-pay for these characteristics. In essence, the first stage of the full estimation procedure outlined above provides the adjustment to the hedonic price regression so that it accurately returns the preferences of the baseline household category (type 1 households in our example or mean utility when household characteristics are constructed so as to have mean zero).

¹⁷ Notice that the coefficient on \mathbf{d}_h in equation (14) essentially converts utility to prices for the mean household.

This simple example makes clear two issues related to the estimate of heterogeneous preferences. First, in order to properly estimate heterogeneous preferences it is necessary to use another fundamental source of variation in the data related to heterogeneous preferences - *choice variation* (i.e., variation in the characteristics and price of houses chosen versus not chosen by households of a particular type) in order to properly estimate heterogeneous preferences. The use of type-specific price variation alone leads to biased preference estimates. Second, while the estimation of discrete choice models superficially appears to be based entirely on choice variation, when traditional discrete choice models are augmented to allow for terms that capture the unobserved quality of alternatives, the estimation of the full model does indeed use the same form of price variation that forms the basis for hedonic price regressions, as illustrated by the second-stage regression equation shown in (16).

The Endogeneity of School Quality and Neighborhood Sociodemographic Composition

In attempting to estimate preferences for school quality, an important endogeneity issue has been raised in the literature, starting with Black (1999). (See also Clapp and Ross (2002), and Kane *et al.* (2003) among others.) These authors point out that the quality of local schools is likely to be positively correlated with unobserved housing and neighborhood quality, though they do so in the context of a hedonic price regression. The identification strategy developed in Black (1999) uses a sample of houses near school attendance zone boundaries, estimating a hedonic price regression that includes boundary fixed effects. By including these, this strategy essentially compares the prices of houses in otherwise similar neighborhoods, but that fall on opposite sides of a boundary determining where students will attend school. Any differences in prices not associated with housing characteristics are then interpreted as the marginal willingness-to-pay for school quality.

There are very good reasons to think that households will sort on a non-random basis with respect to boundaries so that other differences will help drive differences in capitalization - at the end of Section 4, we present clear evidence showing that sociodemographics vary across boundaries. However, boundary fixed effects are likely to do a good job of absorbing out fixed factors, including ones that are unobservable. In doing so, they provide an appealing means of obtaining more reliable estimates of the valuation of a variety of neighborhood characteristics (including school quality) that would otherwise be correlated with unobservables. In particular, one of the most challenging endogeneity problems in the literature relates to the correlation of

¹⁸ Under this interpretation, the interaction terms in the utility specification represent the additional preferences of type 2 relative to type 1 households.

neighborhood sociodemographic characteristics and unobserved housing and neighborhood quality, a correlation that is mechanical given the non-random sorting of households across locations. To the extent that sorting with respect to school district boundaries is driven by differences in school quality and neighborhood sociodemographics themselves, the use of boundary fixed effects isolates variation in neighborhood sociodemographics that is uncorrelated with variation in unobserved housing and neighborhood quality. Thus, the use of boundary fixed effects provides an appealing way to deal not only with the correlation of school quality with unobservable neighborhood quality, but also that of neighborhood sociodemographics.

Based on this discussion, it is straightforward to incorporate boundary fixed effects into our sorting model. To that end, we assign each house to a region r . When a house is close to a boundary between two school districts, it will fall into a *boundary region* and when a house is more centrally located within a school district, it will fall into a *central region*. Letting \mathbf{y}_r be a region fixed effect for the region r to which house h belongs, we can re-write the utility function shown in equation (1) as:

$$(17) \quad \underset{(h)}{Max} \quad V_h^i = \mathbf{a}_X^i X_h - \mathbf{a}_p^i p_h + \mathbf{y}_r + \mathbf{x}_h + \mathbf{e}_h^i$$

Having accounted for a region fixed effect, \mathbf{x}_h now represents the unobserved quality associated with the particular housing unit h within region r .

In extending this identification strategy to the broader sorting model, an additional issue concerns the treatment of houses not near a school district boundary. In essence, while we seek to use only the variation in the data at the boundaries to estimate preferences for school quality, the logic of the choice model developed in Section 2 requires the use of all houses in the choice set. Notice, however, that given the specification of equation (17), equations (8)-(9) become

$$(18) \quad \mathbf{d}_h = \mathbf{a}_{0X} X_h - \mathbf{a}_{0p} p_h + \mathbf{y}_r + \mathbf{x}_h$$

and

$$(19) \quad \mathbf{I}_h^i = \left(\sum_{k=1}^K \mathbf{a}_{kX} Z_k^i \right) X_h - \left(\sum_{k=1}^K \mathbf{a}_{kp} Z_k^i \right) p_h.$$

That is, the boundary fixed effect appears only in the mean indirect utility regression. Thus, the first stage of the estimation procedure remains unchanged, returning estimates of the interaction parameters and the vector of mean indirect utilities. In the second stage of the estimation

procedure, (i.e., the estimation of equation (18)), we use only an appropriately weighted sample of houses in boundary versus central regions. Thus the estimation of the interaction (heterogeneity) parameters in the utility function shown in equation (19) is based on the full sample of houses, while the estimation of the mean preference parameters (those in equation (17)) is based only on across-boundary variation in prices.

At first glance it may appear that the indirect utility function specified in (18) and (19) may lead to an overstatement of the interactions between certain household and choice characteristics – e.g., overstating the willingness of high-income households to pay for school quality – as the indirect utility function does not include interactions of household characteristics and unobservable choice characteristics. In fact, this problem is avoided by letting the coefficient on price to vary with household characteristics, which in turn permits households with more income, for example, to have a greater marginal willingness-to-pay for unobserved housing and neighborhood quality. Suppose (19) did not have such interactions with price. Then high income households would not be allowed to place a higher value on unobserved quality (positively correlated with price), and the model specification would force them to demand more school quality, also positively correlated with unobserved quality, leading preferences for school quality to be overstated. The more flexible specification that we adopt thus allows for the proper estimation of heterogeneity in preferences for school quality, even in circumstances where school quality is correlated with unobservable housing and neighborhood attributes valued more strongly by some households versus others.

While the boundary fixed approach methodology provides an attractive way of controlling for much of the correlation between unobserved housing/neighborhood quality and a number of relevant neighborhood characteristics, including school quality, there are important reasons to expect that such boundary fixed effects do not control for all of this correlation. In particular, because housing and school quality are both likely to be normal goods, for both their own consumption and for the future re-sale value of their homes, home-owners on the better school quality side of a boundary are likely more likely to invest in improving the quality of their housing unit. This introduces a positive bias between housing and school quality that is very difficult to address given the fact that many of these improvements are likely to be unobserved in the data.¹⁹ Thus, while the use of boundary fixed effects should control for much of the correlation of school quality with unobserved housing and neighborhood quality, we still expect the estimated preferences for school quality to be slightly overstated.

¹⁹ We do, however, show evidence below that the observed housing and neighborhood characteristics are not markedly different on the high versus low side of the school district boundaries used in the analysis.

4 DATA

Having laid out many of the issues concerning the equilibrium model of sorting, estimation, and identification, we now describe the data used in our analysis as well as some empirical issues related to these data. Our analysis is facilitated by access to restricted Census microdata for 1990. These restricted Census data provide the detailed individual, household, and housing variables found in the public-use version of the Census, but unlike the public-use data, also include information on the location of individual residences and workplaces at a very disaggregate level of geography. In particular, while the public-use data specify the PUMA (a Census region with approximately 100,000 individuals) in which a household lives, the restricted data specify the Census block (a Census region with approximately 100 individuals), thereby identifying the local neighborhood that each individual inhabits, as well as the characteristics of each neighborhood, far more accurately than has been previously possible with such a large-scale data set.

We use data from six contiguous counties in the San Francisco Bay Area: Alameda, Contra Costa, Marin, San Mateo, San Francisco, and Santa Clara. We focus on this area for two main reasons. First, it is reasonably self-contained. Examination of Bay Area commuting patterns in 1990 reveals that a very small proportion of commutes originating within these six counties ended up at work locations outside the area; and similarly, a relatively small number of commutes to jobs within the six counties originated outside the area. Second, the area is sizeable along a number of dimensions, including over 1,100 Census tracts, and almost 39,500 Census blocks, the smallest unit of aggregation in the data. The sample consists of about 650,000 people in just under 244,000 households.

The Census provides a wealth of data on the individuals in the sample – race, age, educational attainment, income from various sources, household size and structure, occupation, and employment location.²⁰ The Census data also provide a variety of housing characteristics: whether the unit is owned or rented, the corresponding rent or owner-reported value, property tax payment, number of rooms, number of bedrooms, type of structure, and the age of the building. In constructing neighborhood characteristics, we begin by characterizing the stock of housing in the neighborhood surrounding each house. Using the Census data, we also construct neighborhood racial, education and income distributions based on the households within the same

²⁰ Throughout our analysis, we treat the household as the decision-making agent and characterize each household's race as the race of the 'householder' – typically the household's primary earner. We assign

Census block group, a Census region containing approximately 500 housing units. We merge additional data describing local conditions with each house record, constructing variables related to crime rates, land use, local schools, topography, and urban density. For each of these measures, a detailed description of the process by which the original data were assigned to each house is provided in a Data Appendix. The list of the principal housing and neighborhood variables used in the analysis, along with means and standard deviations, is given in the first column of Table 1.

Refining the House Price Variables Provided in Census

For a variety of reasons, the house price variables reported in the Census are ill-suited for our analysis. House values are self-reported and top-coded, and rents may reflect substantial tenure discounts. Moreover, because we have implicitly defined the model and developed its equilibrium properties in terms of a single price variable for both owner-occupied and rental properties, we must relate house values to rents in some way.²¹ Consequently, we make four adjustments to the housing price variables reported in the Census aiming to get a single measure for each unit that reflects what its monthly rent would be at current market prices. We describe the reasoning behind each adjustment briefly here, leaving a detailed description of the methodology for the Data Appendix.

Because house values are self-reported, it is difficult to ascertain whether these prices represent the current market value of the property, especially if the owner purchased the house many years earlier. Fortunately, the Census also contains other information that helps us to examine this issue and correct house values accordingly. In particular, the Census asks owners to report a continuous measure of their annual property tax payment. The rules associated with Proposition 13 imply that the vast majority of property tax payments in California should represent exactly one percent of the transaction price of the house at the time the current owner bought the property or the value of the house in 1978. Thus, by combining information about property tax payments and the year that the owner bought the house, we are able to construct a measure of the rate of appreciation implied by each household's self-reported house value. We

households to one of four mutually exclusive categories of race/ethnicity: Hispanic, non-Hispanic Asian, non-Hispanic Black, and non-Hispanic White.

²¹ This requirement may seem more restrictive than it actually is. Note that we treat ownership status as a fixed feature of a housing unit in the analysis - whether a household rents or owns is endogenously determined within the model by its house choice. We allow households to have heterogeneous preferences for home-ownership (a positive interaction between household wealth and ownership, for example, will imply that wealthier households are more likely to own their housing unit, as we find below) and other house characteristics. Thus the use of a single house price variable does not impose any serious restrictions on the model.

use this information to modify house values for those individuals who appear to be reporting values much closer to the original transaction price rather than current market value.

A second deficiency of the house values reported in the Census is that they are top-coded at \$500,000, a top-code that is often binding in California. Again, because the property tax payment variable is continuous and only top-coded at \$15,000, it provides information useful in distinguishing the values of the upper tail of the distribution.

The third adjustment that we make concerns rents. While rents are presumably not subject to the same degree of misreporting as house values, it is still the case that renters who have occupied a unit for a long period of time generally receive some form of tenure discount. In some cases, this tenure discount may arise from explicit rent control, but implicit tenure discounts generally occur in rental markets even when the property is not subject to formal rent control. In order to get a more accurate measure of the market rent for each rental unit, we utilize a series of local hedonic price regressions in order to estimate the discount associated with different durations of tenure in each of over 40 sub-regions within the Bay Area.

Finally, we construct a single price vector for all houses, whether rented or owned. In order to make owner- and renter-occupied housing prices as comparable as possible, we seek to determine the implied current annual rent for the owner-occupied housing units in our sample. Because the implied relationship between house values and current rents depends on expectations about the growth rate of future rents in the market, we estimate a series of hedonic price regressions for each of over 40 sub-regions of the Bay Area housing market. These regressions return an estimate of the ratio of house values to rents for each of these sub-regions and we use these ratios to convert house values to a measure of current monthly rent. Again, the procedure is described in detail in the Data Appendix.

School Characteristics

While we have an exact assignment of Census blocks to school districts, we have only been able to attain precise maps that describe the way that city blocks are assigned to schools in 1990 for Alameda County. In the absence of information about within-district school attendance areas, we employ four alternative approaches for linking each house to a school. The crudest procedure assigns average school district characteristics to every house falling in the school district. A refinement on this makes use of distance-weighted averages. For a house in a given Census block group, we calculate the distance between that Census block group and each school in the school district. We then construct weighted averages of each school characteristic, weighting by the reciprocal of the distance-squared as well as enrollment. As a third approach we

simply assign each house to the closest school within the appropriate school district. In our fourth and preferred approach, we adjust this closest school assignment procedure to ensure that the predicted enrollment of each school as calculated by summing over the school-aged children in each Census block group assigned to a school equals the actual enrollment of that school. We describe this procedure in detail in the Data Appendix. In practice, all four methods of defining school characteristics yielded very similar results, with the estimates based on school district averages revealing a small amount of aggregation bias. To keep the exposition of the results manageable below, we simply report results for the fourth method described here.

As our measure of school quality, we use the average test score for each school, averaged over two years. Averaging over two years helps to reduce any year-to-year noise in the measure. When variables that characterize the sociodemographic composition of the school or surrounding neighborhood are included in the analysis, the estimated coefficient on average test score picks up what households are willing to pay for an improvement in average student performance at a school holding the sociodemographic composition constant. While the average test score is an imperfect measure of school quality, it has the advantage of being easily observed by both parents and researchers and consequently has been used in most analyses that attempt to measure demand for school quality.

Boundary Fixed Effects

A number of empirical issues arise in incorporating boundary fixed effects into our analysis, following the description in Section 3. The first issue concerns the choice of jurisdiction for which the boundaries are defined. While Black (1999) uses school attendance zones within a school district, in the analysis presented in this paper we use boundaries between school districts in the Bay Area.²² A central feature of local governance in California helps to eliminate some of the problems that naturally arise with the use of school district boundaries, as Proposition 13 ensures that the vast majority of school districts within California are subject to a uniform effective property tax rate of one percent. A second issue concerns the width of the boundaries. While a narrow band makes the assumption that unobserved neighborhood quality is the same on opposite sides of the boundary more accurate, a wider band allows the use of more data. We experimented with a variety of distances and report the results for 0.25 miles, as these were far more precise due to the larger sample size.²³

²² This difference implies that our results are not directly comparable to Black (1999). It is important to keep this distinction in mind throughout our discussion of the boundary fixed effects.

²³ See data appendix for details about the construction of the distance to the boundaries.

Table 1 displays descriptive statistics for various samples related to the boundaries. The first two columns report means and standard deviations for the full sample; the third column reports means for the sample of houses within 0.25 miles of a school district boundary, the fourth and fifth columns report means on the high versus low average test score side of the school district boundary; the sixth column reports ttests for difference in means of fourth and fifth columns; and the seventh column reports weighted means for the sample of houses within 0.25 miles of a school district boundary (we describe the weight below).

Comparing the first column to the third column of the table, it is immediately obvious that the houses near school district boundaries are not fully representative of those in the Bay Area as a whole. Houses near boundaries tend to be slightly more expensive, more often owner-occupied, and larger in neighborhoods that are less dense and have lower crime rates than the sample as a whole. To address this problem, we create sample weights for the houses near the boundary according to the following procedure: Using a logistic regression, we first regress a dummy variable indicating whether a house is in a boundary region on the vector of housing and neighborhood attributes. Fitted values from this regression provide an estimate of the likelihood that a house is in the boundary region given its attributes. We use the inverse of this fitted value as a sample weight in subsequent regression analysis conducted on the sample of houses near the boundary. Column 7 of Table 1 shows the resulting weighted means. As the numbers clearly demonstrate, using these weights makes the sample near the boundary much more representative of the full sample, column 7 typically being much closer to column 1 than column 3 is.

The principal issue arising from Table 1 concerns differences across school district boundaries, which are displayed in columns 4 and 5. Comparing the average characteristics of houses with 0.25 miles of the boundary on the high versus low school quality side reveals that houses on the high school quality side cost \$53 more per month and are assigned to schools with a 43-point average test score increase. The standard deviation of the test score measure is 74, so this translates into a raw mean marginal willingness to pay for a one standard deviation increase in school quality of approximately \$91 in monthly rent or \$24,100 in house value.²⁴ The remaining differences in average housing characteristics on opposite sides of the school district boundary are fairly small (relative to the overall standard deviation of each variable), but the

²⁴ As described above, we construct a single price vector for all houses, whether rented or owned. Because the implied relationship between house values and current rents depends on expectations about the growth rate of future rents in the market, we estimate a series of hedonic price regressions for each of over 40 sub-regions of the Bay Area housing market. These regressions return an estimate of the ratio of house values to rents for each of these sub-regions and we use the average of these ratios for the Bay Area, 264.1, to convert monthly rent to house value for the purposes of reporting results at the mean.

effect of these differences on housing prices is generally in the direction of increasing prices on the high school quality side of the boundary.

More significantly, houses on the high school quality side of the boundary are more likely to be inhabited by white households and households with more education and income. This pattern is evident when looking at the difference in means test: while percent white, percent high education and average income of the neighborhood have t-stats of 5.14, 9.62 and 10.23 respectively, ownership, elevation and number of rooms have 1.04, 1.14 and 3.13. These types of across-boundary differences in sociodemographic composition are what one would expect if households sort on the basis of preferences for school quality, thereby leading those with stronger tastes or increased ability to pay for school quality to choose the higher school quality side of the boundary

Figures 3 to 6 summarize the points made in Table 1. Figure 1 shows differences in average test scores close to the boundaries. Each point in Figure 1 represents the average test score assigned to all houses located a given distance, within .01 mile ranges, from the boundaries - negative ranges indicate houses located on the lower test score side of the boundary. By construction, there is a clear discontinuity close to the boundaries, and the magnitude of this jump is around 40 points. The same picture is constructed in Figure 4 for house prices. It is evident that the discontinuity is less pronounced close to the boundaries, where cell sizes are small, but it increases as we move further into the high score side. As pointed out by Kane, Staiger and Samms (2003), this effect may be induced by the negative selection of households on the low score side - i.e., those households may have lower income and educational levels. To the extent that the neighborhood composition on the low score side affects prices and neighborhood composition in the high score side, we should not expect to see a dramatic discontinuity in prices at the boundaries. Instead, we observe an increase in prices as we move further from the boundary line.

In order to directly identify the capitalization of school quality into house prices using the measured discontinuities at the boundary, all other relevant variables would need to be continuous at the boundary. Figure 5, however, shows that this is not true for average household income. Families on the higher side of the boundary have incomes that are on average \$3,000 - \$4,000 higher than families on lower side, providing clear evidence of sorting. This is not the case for other variables, such as number of rooms. Figure 6 shows that the discontinuity for the number of rooms is much less pronounced, indicating that the use of boundary fixed effects is likely to control for much of the correlation of school quality with unobserved features of housing and neighborhood quality unrelated to household sorting.

In proceeding to the empirical analysis below, we draw two main conclusions from Table 1 and Figures 3 to 6. First, there is a significant amount of sorting with respect to school district boundaries on the basis of race, education, and income. Consequently, models that do not take residential sorting into consideration are likely to greatly overstate the capitalization of school quality into housing prices. Second, the boundary fixed effects likely absorb much of variation in unobserved neighborhood quality unrelated to school quality and sorting.

5 RESULTS

We begin this section by presenting a series of estimates of the residential sorting model. In so doing, we describe our instrumental variables strategy for dealing with the endogeneity of house prices, before offering a detailed interpretation of the results. We then present a corresponding series of hedonic price regression – i.e., a restricted version of the sorting model, as these can be compared with results in the prior literature. In doing so, we draw attention to and explain important biases that appear in earlier work making use of our more general estimation framework.

Estimates of the Residential Sorting Model

Estimation of the full model proceeds in two stages, as described in Section 3, the first stage recovering interaction parameters and choice-specific constants. Table 2 reports estimates of the interaction parameters for a specification that does not include the variables that characterize neighborhood sociodemographic composition. This specification simultaneously controls for the effect of each of a series of household characteristics (income, education, race, work status, age, and household structure) on a household’s marginal willingness-to-pay for each of a series of housing and neighborhood attributes. While the numbers reported in Table 2 are not directly interpretable in dollar terms (we make this conversion in Table 6 below), the estimates of the coefficients reveal significantly positive interactions of household income, education, and the age of the householder with school quality and significantly negative interactions with school quality if a household has children or is Asian, Black, or Hispanic versus White. The remaining pattern of signs and magnitudes in the table are what one would expect in every important case, with, for example, utility declining rapidly in commuting distance for working individuals and the interaction of income and price revealing a positive income elasticity of demand for housing, especially for home-ownership and larger houses.

Table 3 extends the analysis of Table 2 by including a series of neighborhood sociodemographic characteristics. The inclusion of variables characterizing neighborhood

sociodemographic composition reduces the magnitude of the interaction of income with school quality by 60 percent, of education with school quality by almost 75 percent, and of Black and Hispanic with school quality by 80-90 percent. These reductions are not surprising in the presence of strong sorting social interactions. If, for example, more highly educated households want to live together and have a relatively high taste for school quality, households will stratify on the basis of education in equilibrium, with more highly educated households living in better quality school districts. If one did not account for the fact that part of the corresponding higher prices in these neighborhoods was due to the willingness of these households to pay to live with other highly educated households, one would expect to severely overstate the preference of highly educated households for school quality, as in Table 2. The results for household structure also appear more reasonable in the specification of Table 3 versus Table 2, as households with children now have a significantly positive interaction with school quality rather than a significantly negative one.

Forming an Instrument for Price

Having estimated the vector of mean indirect utilities in the first stage of the estimation procedure, the second stage of the estimation procedure uses these in a decomposition of the mean preferences parameters shown in equation (8), re-written in equation (17) to include boundary fixed effects. For general forms of the utility function, both housing price, p_h , and mean utility, d_h , will be correlated with the unobserved housing/neighborhood quality, \mathbf{x}_h , in equilibrium. In this case, the estimation of equation (8) requires an additional variable that is correlated with p_h but not with unobserved housing/neighborhood quality, \mathbf{x}_h .

We develop a type of instrument that rises naturally out of the sorting model when households value only the features of their own house and attributes of the surrounding neighborhood, where the size of this neighborhood could be potentially quite large. That is, as long as households do not value the features of housing and neighborhoods beyond some threshold distance from their own home when making their residential location decision, the exogenous attributes of houses and neighborhoods that are located beyond this threshold make suitable instruments for housing price. In developing this type of instrument, we exploit an inherent feature of the sorting process – that the overall demand for houses in a particular neighborhood is affected not only by the features of the neighborhood itself, but also by the way these features relate to the broader landscape of houses and neighborhoods in the region. Thus we assume that the exogenous attributes of houses and neighborhoods a sizeable distance away

from a house influence the equilibrium in the housing market, thereby affecting prices, but have no direct effect on utility.

In practice, the precision of the estimation is improved significantly when the logic of this IV strategy is used to construct a single instrument for price that approximates the optimal instrument. The optimal instrument for p_h in the mean indirect utility regression (equation (8)) is given by:

$$(20) \quad E\left(\frac{\partial \mathbf{x}_h}{\partial \mathbf{a}_{0p}}\right) = E(p_h | \Omega)$$

- that is, the expected value of p_h conditional on the information set Ω , which contains the full distribution of *exogenous* choice characteristics (X_h) and individual characteristics (Z^i). Notice that this instrument implicitly incorporates the impact of the full distribution of the set of choices in exogenous characteristic space as well as information on the full distribution of observable household characteristics into a single instrument for price.

For computational purposes, we use a well-defined instrument that maintains the inherent logic of this optimal instrument while being straightforward to compute. This ‘quasi-’ optimal instrument is based on the predicted vector of market-clearing prices calculated for an initial estimate of the parameter values with the vector of unobserved characteristics ξ set identically equal to zero and using only the exogenous features of locations.²⁵ Operationally, the estimation proceeds as follows:

1. Include a full set of variables in the model that account for housing and neighborhood attributes in the region that households value directly when making their location decision – for the analysis conducted below, we assume households care about the land use within five miles of their house.
2. Using a conjecture of the model’s parameters, setting $\mathbf{x}_h=0$ for all h , and including only *exogenous* choice characteristics in X , calculate the vector of housing prices that clears the market, $\hat{p}^*(X_h, Z^i)$. In practice, we make a reasonable conjecture as to the price coefficient and then simply run equation (8) via OLS. In calculating the vector of market clearing prices, we use only variables that describe land use, not those related to neighborhood sociodemographic composition, tests scores, or crime.
3. Using \hat{p}^* as an instrument for p , estimate the mean indirect utility regression.²⁶

²⁵ This condition corresponds to using the prediction at the mean instead of the expected value.

²⁶ In practice, we repeat Steps 2 and 3 of this procedure using the estimated parameters from step 3 to construct a new price instrument in step 2 for the next iteration. While this iterative process is not

Like the optimal instrument, the instrument that we propose provides a measure of the way that the full landscape of possible choices affects the demand for each house/neighborhood. In essence, this instrument extracts additional information from Ω than that which is contained in the vectors of choice characteristics X already appearing in estimating equation (8). In the regressions reported below, we include a full set of controls for the characteristics of the house itself and its neighborhood as well as five variables that described land use²⁷ in each of the 1, 2, 3, 4, and 5 mile rings around the house. In sum, the additional information embedded in our instrument derives from the exogenous features of the land use in a region beyond five miles from the house in question. Importantly, this information is collapsed into a single instrument that uses this information in a concise manner consistent with the logic of the sorting model.

Table 4 reports first stage price regressions analogous to the hedonic price regression reported later in Table 7 but including the optimal price instrument. Table 4 presents the results from six price regressions and these same six specifications form the basis for the analysis that follows. The dependent variable in these regressions is our constructed monthly house price measure, which equals monthly rent for renter-occupied units and an imputed monthly rent for owner-occupied units (see Section 4 for more details). Results are reported for the full sample and for a sample of houses within 0.25 miles of school district boundaries, with and without including fixed effects. For each of these three specifications, results are reported for a specification that does not then does include five neighborhood sociodemographic variables: average income, percent of households with a college degree, percent Asian, percent Black, and percent Hispanic, each measured at the Census block group level. In all cases, when the sample of houses is restricted to those within 0.25 miles of a boundary, sample weights (as described in Section 4) are used in order to make this sample as close to representative of the full sample as possible.

The price instrument, which is derived entirely from the exogenous characteristics of the alternatives and the distribution of household characteristics in the population, adds significantly to the predictive power of these regressions. In each specification, the optimal price instrument is

necessary to ensure consistency, it does ensure that the final estimates are not sensitive to our initial conjecture of the coefficient on price. For this reason, we believe that this iterative procedure is likely to be more efficient than applying the procedure once, but do not have a proof of this.

²⁷ Percent industrial; Percent commercial; Percent residential; Percent open space (lakes and parks); Percent other.

strongly predictive of price, over and above the set of variables included in X , increasing the R^2 of each regression by approximately 2-4 percentage points.²⁸

Estimates of the Mean Indirect Utility Equation

Using the optimal price instrument as an instrument for price, we report in Appendix Table 1 the results of six specifications of the mean indirect utility regression that forms the second stage of the estimation procedure. For these regressions, we use the estimated \mathbf{d} vector from the specification shown in Table 2 when sociodemographic characteristics are excluded and in Table 3 when they are included. Table 5 reports the implied measures of the mean willingness to pay for school quality that result from these six specifications. The estimates are generated by dividing the coefficient associated with each choice characteristic in the \mathbf{d} regressions by the coefficient on price; the six specifications reported in Table 5 are analogous to those reported in Table 4.

No clear changes emerge when the sample is reduced to only those houses near a school district boundary. When neighborhood sociodemographic characteristics are excluded, for example, the implied marginal willingness-to-pay for a one standard deviation increase in school quality moves from \$126 in monthly rent (\$33,300 in house value) to \$123 in monthly rent (\$32,500 in house value), when the sample is restricted to houses near a boundary. The estimated mean MWTP for a one standard deviation in school quality declines to \$82 (\$21,500) when the boundary fixed effects are included in the analysis. This more or less mirrors the raw MWTP of \$91 (\$24,100) calculated using the descriptive statistics in Table 1. Thus, controlling for a host of fixed housing and neighborhood characteristics does very little to this estimate. Including neighborhood sociodemographic characteristics in the analysis dramatically reduces the estimated MWTP for school quality to \$20 in monthly rent (\$5,300 in house value) when boundary fixed effects are not included in the regression and \$26 (\$6,900) in monthly rent when they are included.

The final two specifications of Table 5 also show the impact of including boundary fixed effects on the estimates of mean preferences for neighborhood sociodemographic characteristics. Comparing the coefficients on the neighborhood sociodemographic characteristics with and without the inclusion of boundary fixed effects (columns 5 and 6) yields the pattern of results one would expect if the boundary fixed effects control for unobserved components neighborhood

²⁸ As a side note, it is also important to point out that the coefficients on the other characteristics do not have much meaning as they represent the effect of these characteristics on price controlling for the estimated market-clearing price given only the exogenous attributes of the set of alternatives.

quality unrelated to the sorting of households across the boundary. In particular controlling for fixed effects reduces the coefficient on percent Black from $-\$319$ to $-\$267$; increases the coefficient on percent Hispanic from $\$18$ to $\$139$; changes the sign of the coefficient on percent Asian from $\$96$ to $\$155$; reduces the coefficient on the percent of households with a college degree from $\$206$ to $\$138$; and reduces the coefficient on average neighborhood income ($/\$10,000$) from $\$96$ to $\$88$ per month. Thus, while the boundary fixed effects do not seem to be effective in controlling for differences in the sociodemographic composition across neighborhoods, they do seem to be effective in controlling for more fixed aspects of unobserved neighborhood quality, and thus provide a way of properly estimating preferences for neighborhood sociodemographic characteristics in the presence of this important endogeneity problem.

That the estimated mean MWTP for a one standard deviation increase in school quality of $\$26$ in monthly rent or $\$6,900$ in house value is relatively small may relate to the fundamental informational problem that households face in attempting to distinguish the quality of a school. The ability of households to glean from average test score data the quality of a school as opposed to increased performance due directly to its sociodemographic composition may be very difficult indeed; empirical researchers know that to do this well requires an immense amount of data. Consequently, to the extent that households have difficulty measuring school quality, one would not expect them to value it much when making their residential location decision. Put another way, to the extent that households instead use the posted average test score or the sociodemographic composition of the neighborhood as proxies for school quality when making their location decision, the location decision will be driven much more heavily by neighborhood sociodemographic variables rather than by school quality itself.²⁹

One concern is that the average test score used in the analysis may be a noisy measure of student performance and consequently that neighborhood sociodemographic differences partially capture unobserved differences in school quality as well. To address this issue, we compare the results presented above, which average a school's average test score over two years to results based on a test score for only the first year of these two years of data. We find no difference in the estimate when the test score is based on a one- versus two-year average.³⁰

²⁹ Rothstein (2002) tries to disentangle parental choice of school quality in two components: school effectiveness and peer groups. Instead of modeling residential location and schooling decisions, he uses variation across school districts applied to a set of 1994 SAT-takers, finding little evidence that parents choose schools on the basis of school effectiveness.

³⁰ Kane, Staiger and Samms (2003) point out that the *change* in average test scores from year to year exhibits a great deal of noise unrelated to actual school quality. Not surprisingly, the level exhibits far less noise.

Heterogeneity in Willingness-to-Pay

Table 6 reports the implied estimates of the heterogeneity in MWTP for school quality and neighborhood sociodemographic characteristics across households with different characteristics for our preferred specification, which includes both neighborhood sociodemographic characteristics and boundary fixed effects. Focusing first on the heterogeneity in tastes of school quality, a household's willingness-to-pay increases with income, the presence of children, education, employment, and age. Black households have a significantly lower willingness to pay for school quality relative to White households, although this may result in part from unobservable difference in, for example, wealth that are not included in this analysis.

The presence of children increases demand for house size and school quality, but decreases demand for owner-occupied and newer housing, both of which might proxy in part for housing quality. That the presence of children generally decreases demand for housing quality is most likely due to the fact that disposable income declines as a result of having children. The increased demand for house size and school quality is especially noteworthy given this decline in disposable income. More income and education increases demand for all aspects of housing and school quality as well as for more educated and higher income neighbors. College-educated households in particular have a strong preference to live near other college-educated households.

Finally, Table 6 reveals strong segregating racial interactions, with households of each race preferring to live near others of the same race. The \$97 estimate listed in the fourth row and fifth column of the table, for example, implies that Black (versus White) households are willing to pay \$97 more per month to live in a neighborhood that has 10 percent more Black versus White households. It is important to point out that this is a difference between the *positive* MWTP of Black households for this change and the *negative* MWTP of White households. It is also important to point out that these interactions pick up any direct preferences for living near others of the same race (e.g., a recent immigrant from China may want to interact with neighbors who also have immigrated from China) as well as any unobservable neighborhood or housing amenities valued more strongly by households of this group (e.g., recent immigrants from China may have similar tastes for shops, restaurants, and other neighborhood amenities). As discussed below, it is these strong segregating racial interactions that cause the large difference between the estimates of the neighborhood racial characteristics in the hedonic price regressions and the mean MWTP estimates derived from the broader choice model.

Hedonic Price Regressions

Having set out our preferred estimates, it is instructive to compare these with a series of hedonic price regressions that correspond to estimating a model using the naïve assumption that households have homogeneous preferences and there is no sorting across boundaries. These hedonic price regressions allow us to explore a number of alternative assumptions about the use of boundary fixed effects, the sample, and the effect of including various additional controls in a setting where it is easy to compare results across models. We also show additional evidence that the inclusion of neighborhood sorting variables changes the results dramatically while simply controlling for boundary fixed effects does not.

Table 7 presents the results from six hedonic price regressions analogous to those reported in Table 5 for the full sorting model. As the first row of Table 7 makes clear, the estimated mean MWTP for a one standard deviation increase in school quality as estimated by the hedonic price regressions is generally slightly more than the estimate obtained from the full sorting model. For the specification that excludes both boundary fixed effects and neighborhood sociodemographic characteristics, the mean MTWP for a one standard deviation increase in school quality estimated using the sorting model is \$123 in monthly rent as compared to the \$145 estimate obtained using the standard hedonic price regression. When only boundary fixed effects are included, the estimated mean MWTP is \$82 as compared with \$101 in the hedonic price regression. And, finally, when neighborhood sociodemographic characteristics and boundary fixed effects are included in the model, the estimated mean MWTP rises from \$26 (\$6,900 in house value) to \$28 in monthly rent (\$7,400).

Because of the unusually rich nature of our data, the estimates reported for our preferred specification of the hedonic price regressions in Table 7 (the final column) cannot be compared directly with the results in the prior literature. Black (1999), for example, was never able to include neighborhood sociodemographic variables in her preferred specification because the public use Census data do not match school attendance areas. In comparing our estimates without sociodemographic variables with the Black (1999) results, the corresponding numbers are very similar, although as the final three columns of Tables 5 and 7 make clear, these can be quite misleading as to the true extent of any capitalization. Thus the restricted access Census data allow us both to control properly for neighborhood sociodemographic characteristics as well as model the sorting of households across locations, which is made possible by the information matching households with their housing units in the restricted version of the Census.

The Bias in the Hedonic Price Regression

It is important to point out that we can sign the direction of the bias in the hedonic price regression simply under the assumption that price enters indirect utility negatively ($\mathbf{a}_{0p} > 0$). Re-writing equation (16) as:

$$(21) \quad p_h = \frac{a_{0x}}{a_{0p}} X_h - \frac{1}{a_{0p}} \mathbf{d}_h + \frac{1}{a_{0p}} \mathbf{x}_h,$$

it is immediately obvious that the hedonic price regression includes a negative function of the mean indirect utility that a house provides in the error term. Thus, to the extent that an included regressor is positively correlated with the vector of mean indirect utilities, the hedonic price coefficient is biased downwards and consequently understates mean preferences for the attribute in question. Table 8 shows the partial correlations of key choice attributes with \mathbf{d} for various specifications of the sample and inclusion of boundary fixed effects. In each case, the reported partial correlation conditions on the full set of covariates used in the hedonic price regression including neighborhood sociodemographic characteristics. Focusing on the final column, which includes the boundary fixed effects, the negative correlation between \mathbf{d} and most of the variables, especially the percentage of a block group that is Black and the average income of a block group, implies that a hedonic price regression will tend to *overstate* mean preferences for these characteristics. The correlation of \mathbf{d} with average test is only slightly negative and, consequently, we expect the mean MWTP for average test score that we estimate using the heterogeneous preferences model to be slightly smaller than that estimated via the hedonic price regression. The one partial correlation that is positive in the final column of Table 8 is that with the percentage of the block group that has a college degree or more. Consequently, this coefficient will increase in moving from the hedonic price regression to the estimated mean preferences obtained from the equilibrium choice model.

Comparing the hedonic price regressions reported in Table 7 to the mean MWTP estimates derived from the sorting model in Table 5 reveals the expected pattern of biases given the correlations reported in Table 8. It is worth exploring why a difference arises between these specifications for certain housing and neighborhood attributes and not others. Comparing results of our preferred specification (column 6) in both tables, which includes both boundary fixed effects and neighborhood sociodemographic characteristics, reveals that the estimates related to housing characteristics, school quality, and crime tend to be slightly overstated in the hedonic price regression, while those related to neighborhood sociodemographic composition and race in particular change dramatically. Here, the analysis of Figure 1 is helpful in understanding why

this is the case. Consider, for example, the estimated mean coefficient on Percent Black, which is -\$267 in the full sorting model as opposed to only -\$41 for the hedonic price regression. For simplicity, assume that neighborhoods are completely segregated, so that the equilibrium price of a Black neighborhood is driven by the MWTP of the Black household with the least MWTP for a Black neighborhood (or, alternatively, the White household with the greatest MWTP). Here, the hedonic price regression returns the MWTP of the household on the *margin* between choosing a Black versus White household, which in this case is substantially greater than the MWTP of the *mean* household, which is estimated in the more general sorting model. Put another way, a much lower differential in price between Black and White neighborhoods is required to equilibrate the housing market than would be required to make the mean household indifferent between these neighborhoods.

The case of school quality is also worth discussing in some detail. Because the Bay Area contains over 700 schools, the equilibrium difference in housing prices between each of the neighborhoods associated with each school is more appropriately characterized by Figure 2, which again simplifies the problem to one dimension of the choice characteristic space. In this case, the equilibrium difference in price between each pair of schools ranked according to quality is the MWTP of the household on the corresponding boundary between schools. These equilibrium prices are represented by the p_j^* terms on the vertical axis. If there are roughly an equal number of students in each school, averaging the equilibrium price over all of the houses in the sample corresponds roughly to the mean MWTP and consequently there is only a slight difference between the estimates returned from the model with heterogeneous preferences and the hedonic price regression.

In general, when we can view the choice problem as single-dimensional, one would expect the hedonic price regression to diverge from mean preferences for choice characteristics (especially those in fixed or limited supply) for which the preferences of the marginal household differ systematically from those of the mean household in the population. Which household is on the margin depends explicitly on the set of alternatives and the attributes available in the market as well as on the distribution of households and their preferences. Consequently, the valuation of attributes returned by the hedonic price regression will depend on these distributions, especially for certain types of attributes. It is important to stress that the broader sorting model explicitly accounts for the distribution of characteristics in the population as well as in the set of alternatives.

The intuition that derives from viewing the choice problem as single-dimensional abstracts from the fact that in making their location choices, households choose over discrete

bundles of goods (e.g., housing attributes, neighborhood attributes, commuting distance). In fact, the main argument for using a discrete choice versus hedonic model of demand is precisely that the set of available bundles may not span important subspaces of the multi-dimensional attribute space. This bundling issue is relevant when examining the change from the hedonic regression for the percentage of neighbors with a college degree. In this case, the estimated mean MWTP is greater than the corresponding coefficient in the hedonic price regression. This suggests that the mean household would generally want to increase its consumption of college-educated neighbors at the equilibrium price. In practice, however, because college-educated households themselves demand more housing and neighborhood attributes, this may be difficult to do without also increasing the consumption of these other goods. Consequently, because the single residential location decision determines a full bundle of goods, households may not be able to perfectly satisfy their preferences for any particular element of this bundle even if they are willing to pay the implied marginal price for that element, especially when the elements of the bundle are correlated. In this case, increasing one's consumption of *percent college educated* will tend to imply increases in the consumption of other housing and neighborhood attributes.

6 SIMULATIONS

With estimates of the distribution of preferences for school quality and neighborhood sociodemographic composition in hand, our equilibrium framework can then be used as a tool for exploring a series of economic and policy questions related to equilibrium in the housing market.

In this section, we illustrate the power of the equilibrium framework by exploring the capitalization of an exogenous change in the average test score for each school throughout the metropolitan region. For each school, we calculate both a 'partial equilibrium' increase in house values that accompanies a rise in school quality, holding neighborhood sociodemographic measures constant, and a 'general equilibrium' increase that accounts for the way that neighborhood sociodemographic characteristics would change in moving to a new sorting equilibrium.³¹ The differences between the partial and general equilibrium estimates inform us as to the importance of a social multiplier in the overall elasticity of demand faced by each school. Such a social multiplier arises because the exogenous change in school quality induces higher income and more educated to sort into the corresponding neighborhood, thereby leading to a

³¹ We do not model the dynamics of household mobility directly and, consequently, the household mobility that corresponds to the movement to a new equilibrium may be expected to take place over a number of years. As long as these movements are anticipated following the exogenous increase in school quality, however, house values should move close to their new equilibrium levels almost immediately.

further increase in house prices. Our preference estimates alone indicate that this type of multiplier may be powerful.

In Section 5, we drew attention to a number of problems with estimating household preferences that had not been adequately addressed in prior work. Here, it is important to stress that, even if these problems had been solved and reliable preference estimates had been obtained, these estimates would not provide a useful indication as to the full capitalization effect. As we show in this section, the partial equilibrium effect is significantly smaller than the general equilibrium effect that allows neighborhood sociodemographics to adjust. Thus the full sorting model is needed not only as a way of properly estimating preferences but also for recovering a reliable estimate of the capitalization of an exogenous change in school quality into housing prices, an exercise for which a hedonic price regression naively seems well suited.

Simulation Details

Each of the simulations that we conduct begins by raising the average test score of a given school by one standard deviation (74 points). In this counterfactual environment, we calculate a new equilibrium for the model. Here, an equilibrium consists of a set of location decisions for each household and a set of housing prices such that (i) each household's decision is optimal given the decisions of all other households, and (ii) the set of housing prices clears the market.

The basic structure of the simulations consists of a loop within a loop. The outer loop calculates the sociodemographic composition of each neighborhood, given a set of prices and an initial sociodemographic composition of each neighborhood. The inner loop calculates the unique set of prices that clears the housing market given an initial sociodemographic composition for each neighborhood. Thus for any change in the primitives of the model, we first calculate a new set of prices that clears the market; as discussed in Section 2, Berry (1994) ensures that there is a unique set of market clearing prices. Using these new prices and the initial sociodemographic composition of each neighborhood, we then calculate the probability that each household makes each housing choice, and aggregating these choices to the neighborhood level, calculate the predicted sociodemographic composition of each neighborhood. We then replace the initial neighborhood sociodemographic measures with these new measures and start the loop again – i.e., calculate a new set of market clearing prices with these updated neighborhood sociodemographic measures. We continue this process until the neighborhood sociodemographic measures converge. The set of household location decisions corresponding to these new

measures along with the vector of housing prices that clears the market then represents the new equilibrium.³²

It is important to point out that because the model itself does not perfectly predict the housing choices that individuals make, the neighborhood sociodemographic measures initially predicted by model, $\bar{Z}_n^{PREDICT}$, will not match the actual sociodemographic characteristics of each neighborhood, \bar{Z}_n^{ACTUAL} . Consequently, before calculating the new equilibrium for any simulation we first solve for the initial prediction error associated with each neighborhood n :

$$(22) \quad \mathbf{w}_n = \bar{Z}_n^{ACTUAL} - \bar{Z}_n^{PREDICT}$$

In solving for the new equilibrium, we add this initial prediction error \mathbf{w}_n to the sociodemographic measures calculated in each iteration before substituting these measures back into the utility function.

Adjusting Crime Rates and Average Test Scores

Because some neighborhood attributes, such as crime rates and average test scores, depend in part on the sociodemographic composition of the neighborhood, it is natural to expect these neighborhood characteristics to adjust as part of the movement to a new sorting equilibrium. Getting precise measures of the impact of neighborhood sociodemographic characteristics on crime rates and test scores is, of course, an exceedingly difficult exercise, as selection problems abound. For example, an OLS regression of crime rates on neighborhood sociodemographic characteristics almost certainly overstates the role of these characteristics in producing crime as it ignores the fact that households sort non-randomly across neighborhoods. As a result, we take an approach that seeks to provide bounds for the characteristics of the new equilibrium that results for each of our simulations. For one bound, we calculate a new equilibrium without allowing crime rates and average test scores to adjust with the changing neighborhood and school sociodemographic compositions. For the other bound, we calculate a new equilibrium, adjusting crime rates and average test scores according the adjustments implied by an OLS regression of the crime rate and average test score on neighborhood and school sociodemographic composition.

³² While this procedure always converges to an equilibrium, the model does not guarantee that this equilibrium is generically unique. In all of the calculations presented in this paper, we report results that start from the initial equilibrium and follow the procedure summarized here. In general, the counterfactual simulations conducted do not change the overall economic environment very much at all, and consequently, we believe that this procedure yields reasonable results.

These simple production functions are shown in Appendix Table 2, with all of the variables constructed to have mean zero and standard deviation one. The first bound will understate the impact of sociodemographic shifts on the implied crime rate and average test score in each neighborhood, while the second bound will tend to overstate the impact of these sociodemographic shifts. As the results below indicate, these bounds provide a tight range for the predictions from our simulations.

Partial and General Equilibrium Capitalization

Table 9 reports the distribution of average house price changes in the neighborhood corresponding to each school that result from increasing the average test score of that school by 74 points (1 standard deviation).³³ The partial equilibrium results reveal a mean capitalization estimate of \$27.10 per month (\$7,200 in house value) and a median estimate of \$26.40 per month. Not surprisingly, these numbers closely resemble the estimated mean MWTP in our preferred specification of \$26 per month and in the hedonic price regression of \$28 per month. Thus, the hedonic price regression comes close to measuring the mean partial equilibrium capitalization of a marginal change in school quality. The general equilibrium results reveal a mean capitalization estimate of \$46-48 per month (\$12,100-\$12,600 in house value) and a similar median estimate. Thus, the full general equilibrium capitalization of school quality is 70-75 percent greater than the direct (partial equilibrium) effect of school quality on housing prices.

The table also shows changes in sociodemographics for the school neighborhoods where test scores have been increased. Increasing test scores leads to an increase in the proportion of high-income households in the relevant neighborhood and a reduction in the proportion of poor households. The change is magnified comparing the general equilibrium change to that in partial equilibrium, where the sociodemographics are not allowed to adjust fully. For instance, while the change in the proportion of highly educated households rises by 0.7 percentage points in partial equilibrium, it rises by 1.6-1.7 percentage points in general equilibrium. In line with this change, a one standard deviation increase in its test score would raise average incomes in a school's vicinity by an average of \$1050 in partial equilibrium, and \$1650-\$1770 in general equilibrium. And in terms of race, the proportion of white households rises while the proportion of black households falls, the former by over one percentage point, the latter by 0.7-0.8 percentage points in general equilibrium.

³³ In each simulation, average house prices in the Bay Area as a whole are constrained to equal the pre-simulation level.

The other significant feature of these simulation results is the heterogeneity in capitalization across the metropolitan area. In general equilibrium, the price increase accompanying a one standard deviation increase in school quality is roughly twice as large at the 90th percentile (\$56.80-\$59.10) as at the 10th percentile (\$28.80-\$29.60). These numbers, which reflect both the underlying heterogeneity in the population in preferences for neighborhood sociodemographic characteristics as well as school quality, again emphasize the importance of using the broader heterogeneous preferences model when exploring questions related to the demand for school quality. Partial equilibrium estimates are unlikely to be reliable.

7 CONCLUSION

This paper has developed a comprehensive framework for recovering heterogeneous preferences for attributes of schools and neighborhoods, drawing attention to two important and related considerations in empirical work: the notion that households sort on a non-random basis across neighborhoods, and the notion of equilibrium.

The idea that households sort non-randomly is far from new, but it has been given insufficient attention in empirical applications. Using very rich data relating to household choices, we have presented direct evidence that significant sorting does occur, and in a manner that one would expect. For example, at the boundary between high and low quality school districts, higher income and better educated households sort onto the ‘high’ side of the boundary.

Sorting gives rise to difficult challenges for estimation. To address these, we have set out a new approach to estimating preferences for characteristics that are dependent on the way that households sort, including house prices, neighborhood sociodemographics and school quality. Here, we build on the discrete choice literature, modeling the household sorting process directly, also drawing on recent developments in the IO literature that allow households to have preferences over unobservable choice characteristics. This is important: in any data set, researchers are unlikely to observe neighborhood quality entirely, and ignoring the correlation of house prices and neighborhood unobservables, for example, is likely to lead to significant biases in estimates in willingness-to-pay.

Estimating our sorting model makes significant data demands. In this regard, we have been fortunate to have access to a vast data set on a large metropolitan area, providing detailed information on the characteristics and actual housing and neighborhood choices of a very large sample of households. Our estimation results make three things very clear. First, it is essential to control directly for detailed neighborhood sociodemographics when estimating household valuations of important choice characteristics. In prior work, researchers have often been unable

to control for these directly, but the failure to do so involves a fundamental misspecification, leading to more than a 200 percent overstatement of the mean marginal willingness to pay for school quality in our data.

Second, heterogeneity in preferences is an important phenomenon. We would not expect households with children to have the same preferences for school quality as households without, and they do not. Further, we find evidence of very strong social interactions – highly educated households, for instance, are willing to pay a much higher price to live with other highly educated households than those who are not so well educated. Such heterogeneity in preferences implies that simple hedonic price regressions will typically not return average preferences, for reasons discussed at some length in Section 3 – and in fact we find that the use of simple hedonic price regressions leads to significant biases in the estimation of mean preferences for neighborhood sociodemographic characteristics. Preference heterogeneity also has very important implications for the interpretation of the results, to be discussed shortly, as it gives rise to strong multiplier effects in general equilibrium.

Third, given the importance of heterogeneous preferences in driving sorting and the likelihood that households sort, in part, on the basis of unobservables, it is incumbent on the researcher to address an important endogeneity problem - namely the correlation of neighborhood unobservables and observed neighborhood sociodemographics. Here, we have adapted an approach already used in the hedonic price literature to control for the likely correlation of school quality and unobserved neighborhood quality using boundary fixed effects, showing how it can profitably be applied to the problem of estimating consistently the valuation of neighborhood sociodemographic composition. The essential idea is intuitive: having shown that important sociodemographics are discontinuous at boundaries due to household sorting, boundaries become useful places to learn about the valuation of such variables; particularly, boundary fixed effects provide an attractive way of absorbing out fixed unobservable components when estimating preferences for these sociodemographics. Our estimates lend support to the idea that this is a promising strategy. It is important to stress that our analysis calls into question the narrow use of boundaries in the prior literature, though. As soon as households sort non-randomly, as they certainly do in practice, capitalization of house prices at school attendance boundaries picks up more than just differences in school quality. This indicates that prior estimates will overstate the valuation of school quality. This is something else that we find.

Having summarized the main results from the estimation of preferences, drawing attention to the notion that households sorting cannot be ignored, we now turn to the second key idea in the research: the notion of equilibrium. We have dwelt on the difficulties that researchers

face in trying to obtain reliable estimates of a broad range of preference parameters, difficulties that have shaped our own estimation approach. Even if researchers were able to accurately estimate preferences for a variety of choice characteristics, it is not clear that those estimates alone would provide a useful guide when gauging the economic significance of the results, given the size of the interactions we have found. This is because there is likely to be a strong social multiplier that drives a significant wedge between any partial versus general equilibrium effects.

We demonstrate this point very clearly in the context of school capitalization – the impact of an exogenous increase in school quality on house prices. The partial equilibrium estimate we obtain, reliable in its own terms, significantly understates the full impact of a school quality increase on house prices. When school quality at a given school goes up, reinforcing changes occur as certain types of household move, and these effects compound as more highly educated and higher income households move into the neighborhood. In order to explore these general equilibrium effects, it is necessary to specify a model that can accommodate the relevant feedback channels. We have set out a simple equilibrium model for that purpose that allows us to explore the general equilibrium implications of our structural estimates in a clear way.

We believe the approach we have developed and the estimates obtained using it have two key implications for empirical work. First, in a wide variety of settings, researchers should be wary of assuming that sorting can be abstracted from. In our application, doing so produces estimates that are very wide of the mark. While the challenges are non-trivial, we have set out a new approach that addresses a number of these challenges and which can be applied elsewhere. Second, partial equilibrium intuitions can be very misleading as to likely effects in practice, once plausible general equilibrium feedbacks are allowed. This point has been made eloquently using calibrated general equilibrium analysis by Nechyba, among others (see, for example, Nechyba (2000)). We make an analogous point in an equilibrium framework based on a broad set of econometric estimates derived from very detailed household-level data.

In future work, we plan to use the equilibrium model of sorting and the rich dataset we have assembled to analyze a number of related applications. The estimated model provides a well-defined characterization of the relative importance of schooling versus other housing, neighborhood, and geographic factors in driving the location decisions of the heterogeneous households of a major metropolitan area. This combination is extremely powerful for conducting economic and policy research involving the interplay of household mobility/stratification and schools, making it possible, for example, to calculate the elasticity of neighborhood house prices and rents as well as the sociodemographic composition of the local neighborhood and school with respect to school quality for each school in the metropolitan region. These ‘demand’ elasticities

provide a series of measure of the competitiveness of a school's local environment and can be used to explore many aspects of the relations between household mobility and school competition. Moreover, a slightly extended version of the model provides a way of calculating the strength of preferences for school quality (on the basis of both observed and unobserved characteristics) among the households that select into a particular school. This permits the researcher to control directly for the non-random sorting of households across schools and school districts which leads to a form of selection bias (often referred to as Tiebout bias in the local public finance literature) in the estimation of education production functions, voting models, or other models that condition implicitly on the set of households in a particular school or jurisdiction.

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Figure 1: Demand for a View of the Golden Gate Bridge

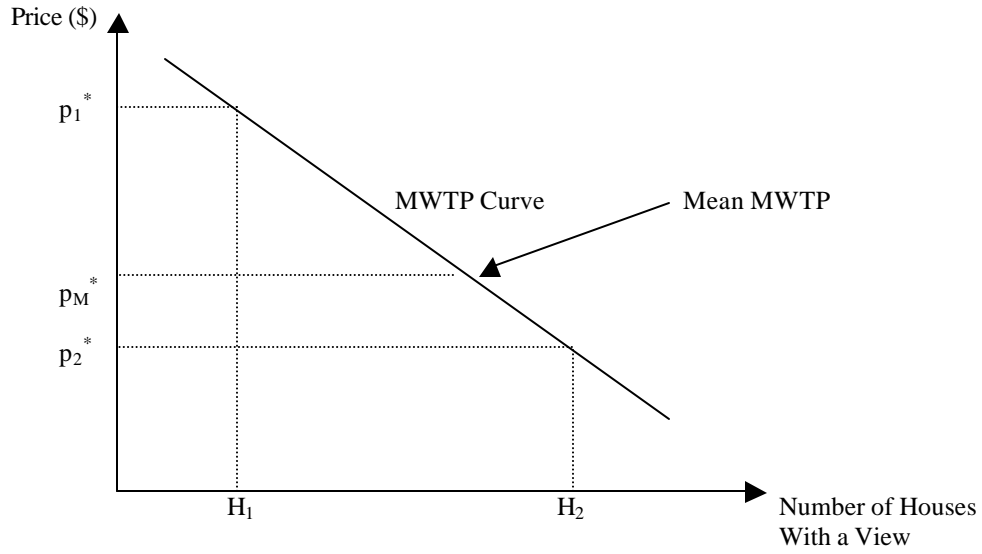


Figure 2: Demand for School Quality

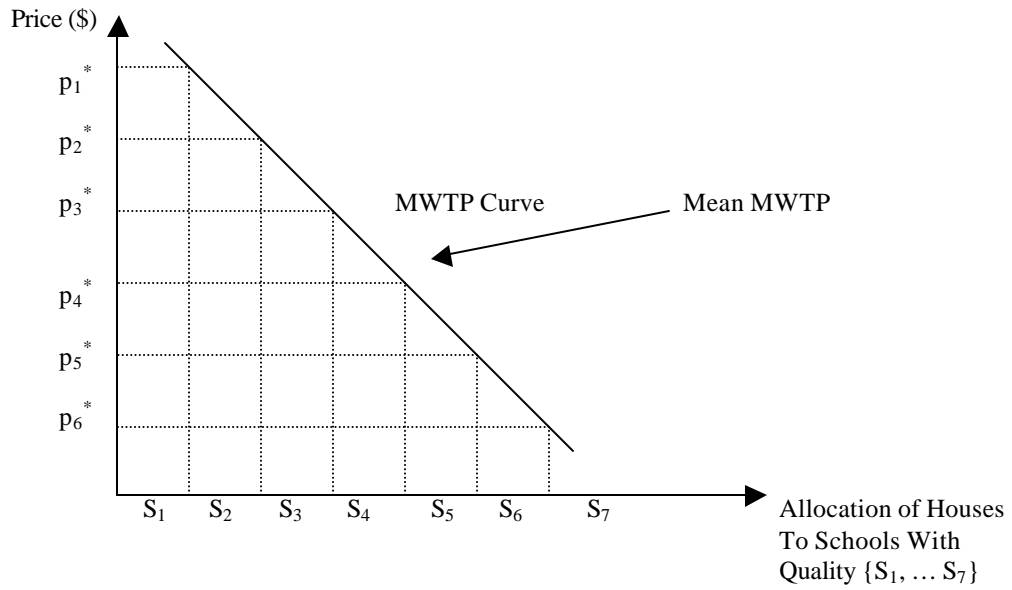


Fig3.Average Test Scores by Distance to the Boundaries

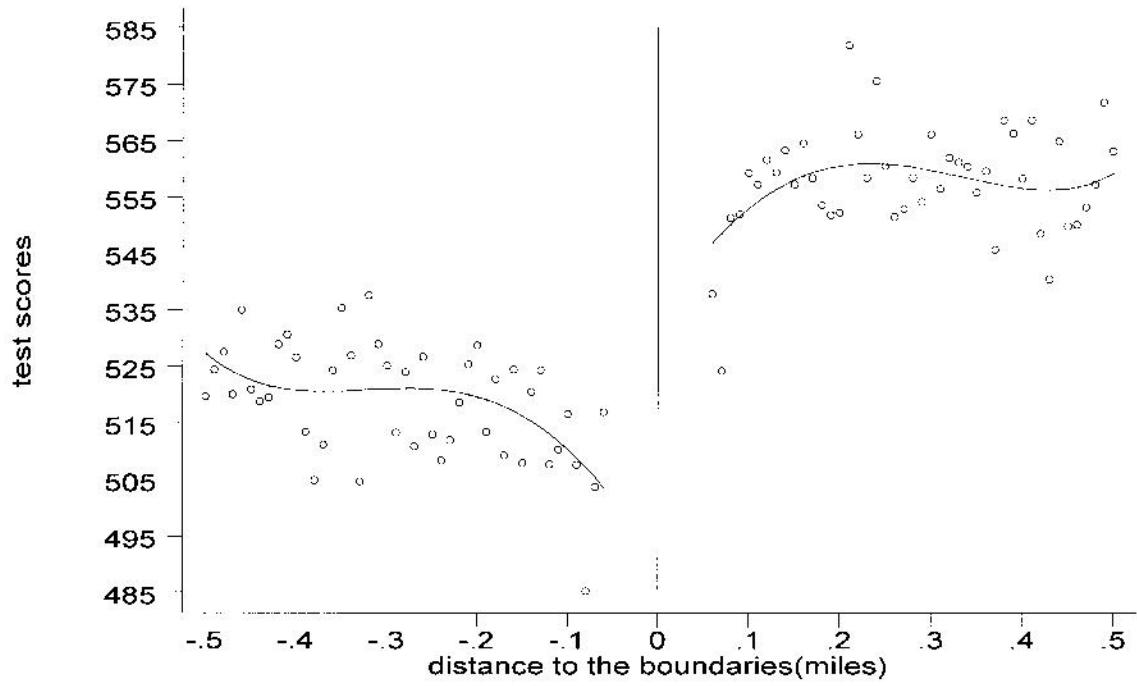


Fig4.Average House Prices by Distance to the Boundaries

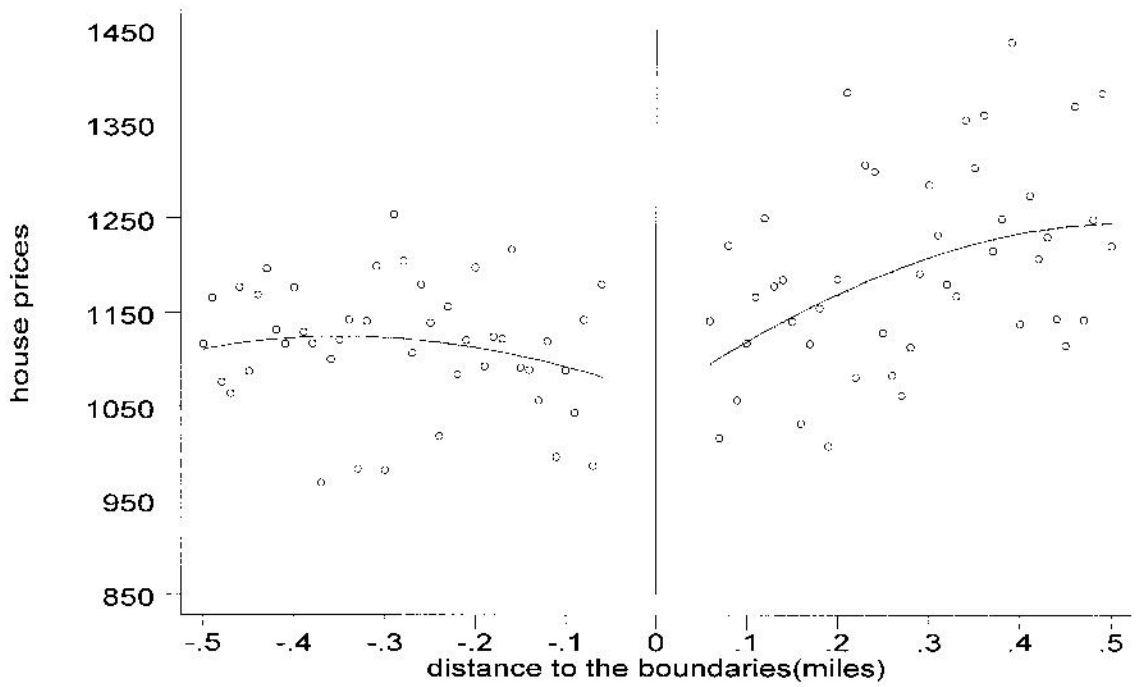


Fig5.Average Household Income by Distance to the Boundaries

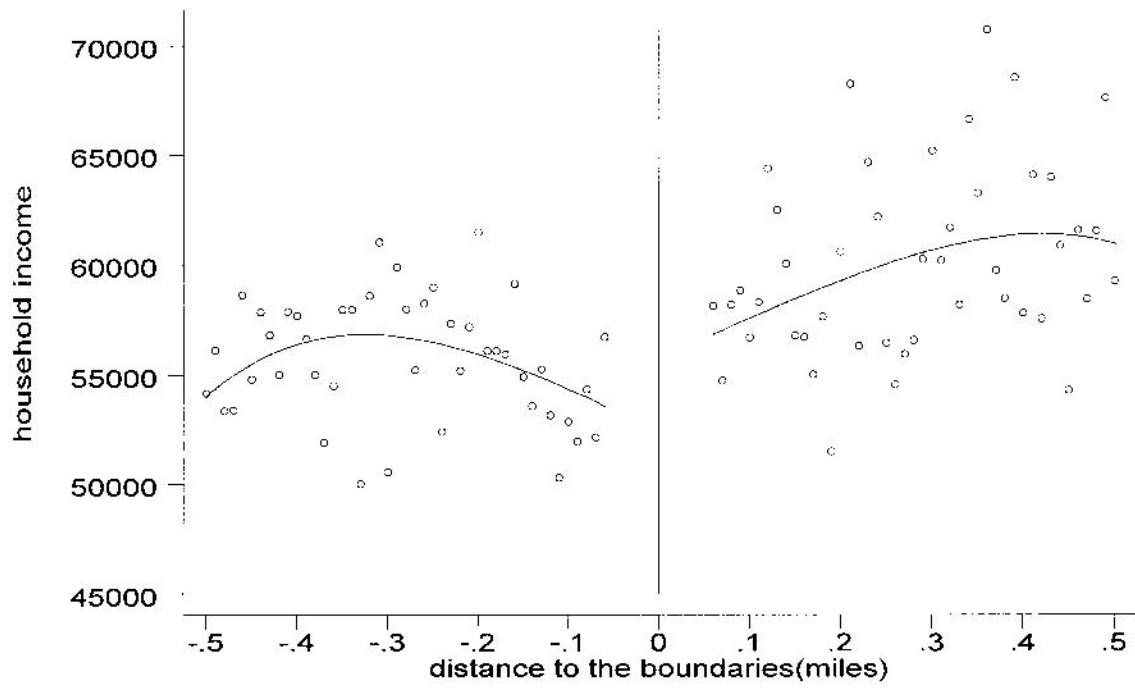


Fig6.Average Number of Rooms by Distance to the Boundaries

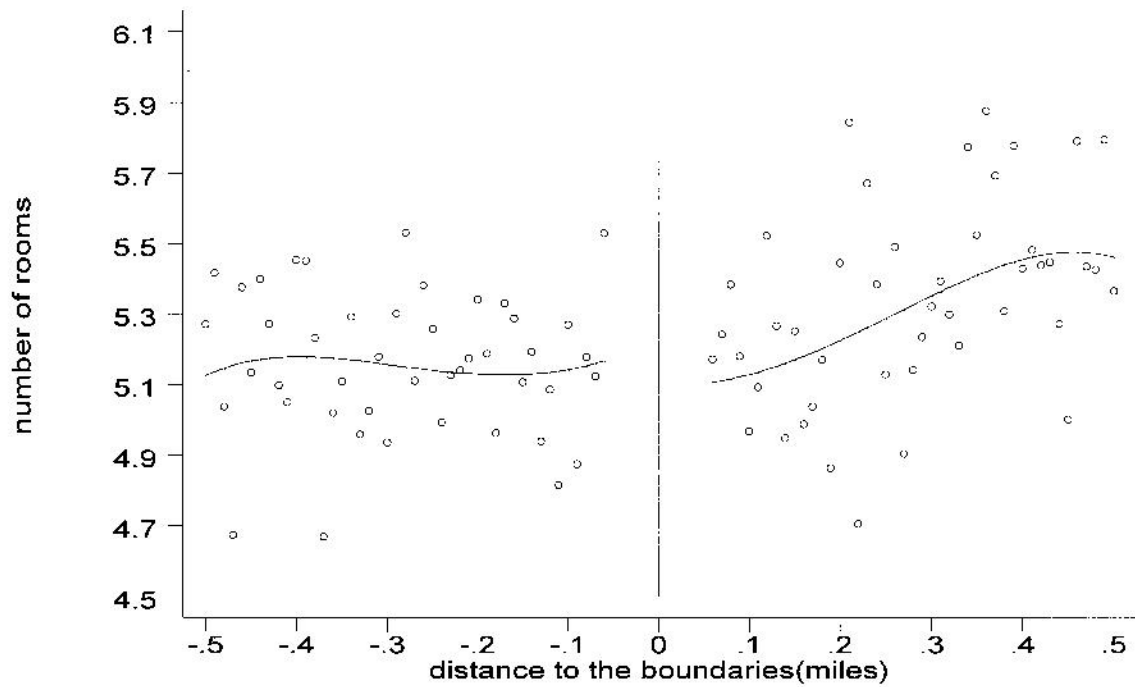


Table 1. Comparing the Sample Near School District Boundaries

Sample Boundary/Weights Observations	full sample		within 0.25 miles of boundaries				t-test for difference in means ((4) versus (5))	weighted sample 27,958 (6) Mean
	242,100		actual sample 27,958	high test score side* 13,348	low test score side* 14,610			
	(1) Mean	(2) S.D.	(3) Mean	(4) Mean	(5) Mean			
<u>Housing/Neighborhood Characteristics</u>								
monthly house price	1,087	755	1,130	1,158	1,105	5.71	1,098	
average test score	527	74	536	558	515	50.96	529	
1 if unit owned	0.597	0.491	0.629	0.632	0.626	1.04	0.616	
number of rooms	5.114	1.992	5.170	5.207	5.134	3.13	5.180	
1 if built in 1980s	0.143	0.350	0.108	0.118	0.099	5.09	0.148	
1 if built in 1960s or 1970s	0.391	0.488	0.424	0.412	0.437	4.22	0.406	
elevation	210	179	193	194	192	1.14	212	
population density	0.434	0.497	0.352	0.349	0.355	2.08	0.374	
crime index	8.184	10.777	6.100	6.000	6.192	2.36	7.000	
% Census block group white	0.681	0.232	0.704	0.712	0.686	9.62	0.676	
% Census block group black	0.081	0.159	0.071	0.065	0.076	6.21	0.080	
% Census block group Hispanic	0.110	0.114	0.113	0.107	0.119	8.62	0.117	
% Census block group Asian	0.122	0.120	0.112	0.110	0.113	2.50	0.121	
% block group college degree or more	0.438	0.196	0.457	0.463	0.451	5.14	0.433	
average block group income	54,744	26,075	57,039	58,771	55,457	10.23	55,262	
<u>Household Characteristics</u>								
household income	54,103	50,719	56,663	58,041	55,405	4.20	55,498	
1 if children under 18 in household	0.333	0.471	0.324	0.322	0.325	0.54	0.336	
1 if black	0.076	0.264	0.066	0.062	0.070	2.69	0.076	
1 if Hispanic	0.109	0.312	0.111	0.102	0.119	4.54	0.115	
1 if Asian	0.124	0.329	0.112	0.114	0.110	1.06	0.121	
1 if white	0.686	0.464	0.706	0.717	0.696	3.86	0.682	
1 if less than high school	0.154	0.361	0.141	0.134	0.147	3.12	0.152	
1 if high school	0.184	0.388	0.176	0.177	0.175	0.44	0.183	
1 if some college	0.223	0.417	0.222	0.222	0.223	0.20	0.225	
1 if college degree	0.291	0.454	0.294	0.295	0.294	0.18	0.286	
1 if more than college	0.147	0.354	0.166	0.172	0.161	2.46	0.155	
age (years)	47.607	16.619	47.890	48.104	47.699	1.99	47.660	
1 if working	0.698	0.459	0.705	0.702	0.709	1.28	0.701	
distance to work (miles)	8.843	8.597	8.450	8.412	8.492	0.82	8.490	

*Note: in constructing columns (4) and (5), we assign each house in the full sample to the nearest school district boundary, noting whether its local school has a higher test score than the school that the closest Census block on the other side of the boundary is assigned to. Hence we determine whether it is on the 'high' versus 'low' side of the boundary.

Table 2. Interaction Parameter Estimates - Model Without Neighborhood Sociodemographics

Household Characteristics	Average Test Score (+1 s.d.)	House Characteristics					Neighborhood Attributes			Distance to Work
		Monthly House Price (/1000)	Owner Occupied	Number of Rooms	Built in 1980s	Built in 1960-1979	Elevation (/100)	Population Density	Crime Index	
household income (/10,000)	0.050 (0.004)	0.121 (0.003)	0.305 (0.010)	0.074 (0.002)	0.142 (0.011)	0.038 (0.009)	0.016 (0.001)	0.028 (0.013)	-0.001 (0.001)	-0.004 (0.001)
1 if children under 18 in household	-0.190 (0.047)	0.063 (0.065)	-0.102 (0.094)	0.544 (0.025)	-0.316 (0.112)	0.146 (0.083)	0.010 (0.022)	-0.740 (0.101)	0.015 (0.005)	0.036 (0.005)
1 if black	-1.395 (0.080)	-0.941 (0.127)	-0.510 (0.167)	0.152 (0.044)	0.004 (0.211)	0.401 (0.144)	-0.062 (0.041)	-1.285 (0.159)	0.110 (0.007)	-0.023 (0.011)
1 if Hispanic	-0.642 (0.072)	0.168 (0.122)	-0.036 (0.130)	-0.268 (0.036)	-0.180 (0.164)	-0.157 (0.115)	-0.104 (0.040)	-0.155 (0.136)	0.050 (0.007)	0.014 (0.007)
1 if Asian	-0.167 (0.062)	0.315 (0.080)	1.765 (0.122)	-0.503 (0.031)	1.037 (0.145)	0.686 (0.108)	-0.015 (0.028)	0.941 (0.095)	0.030 (0.006)	0.003 (0.007)
1 if college degree or more	0.787 (0.053)	0.917 (0.071)	-0.032 (0.108)	-0.012 (0.029)	0.489 (0.135)	-0.045 (0.093)	0.225 (0.024)	-0.007 (0.111)	0.031 (0.006)	-0.006 (0.006)
1 if working	0.007 (0.049)	0.244 (0.067)	0.563 (0.103)	0.032 (0.027)	0.641 (0.125)	0.406 (0.086)	-0.048 (0.025)	-0.437 (0.097)	-0.027 (0.005)	-0.858 (0.008)
age (years)	0.015 (0.001)	0.010 (0.002)	0.090 (0.003)	0.004 (0.001)	-0.034 (0.004)	-0.009 (0.003)	0.003 (0.001)	-0.006 (0.003)	0.001 (0.000)	-0.001 (0.000)

Note: The parameters shown describe the elements of the utility function that interact household characteristics, shown in row headings, with choice characteristics, shown in column headings. Standard errors are in parentheses.

Table 3. Interaction Parameter Estimates - Model With Neighborhood Sociodemographics

	Average Test Score (+1 s.d.)	House Characteristics					Neighborhood Attributes			Neighborhood Sociodemographics					Distance to Work (miles)
		Monthly House Price (/1000)	Owner Occupied	Number of Rooms	Built in 1980s	Built in 1960-1979	Elevation (/100)	Population Density	Crime Index	% Block Group	% Black Group	% Block Hisp Group	% Blk Group Asian	College	
Household Characteristics															
household income (+10,000)	0.020 (0.005)	0.121 (0.004)	0.303 (0.011)	0.076 (0.003)	0.144 (0.012)	0.028 (0.009)	0.010 (0.002)	0.011 (0.017)	-0.001 (0.001)	-0.223 (0.060)	0.113 (0.064)	-0.009 (0.039)	0.385 (0.034)	0.012 (0.002)	-0.004 (0.001)
1 if children under 18 in household	0.102 (0.058)	0.231 (0.075)	-0.238 (0.103)	0.582 (0.028)	-0.399 (0.122)	0.095 (0.092)	0.051 (0.025)	-0.947 (0.127)	0.002 (0.006)	1.594 (0.416)	2.294 (0.527)	1.857 (0.387)	-2.171 (0.331)	0.055 (0.016)	0.027 (0.005)
1 if black	-0.282 (0.116)	0.143 (0.170)	-1.006 (0.205)	0.002 (0.053)	0.027 (0.253)	0.577 (0.184)	-0.068 (0.052)	-1.106 (0.228)	0.045 (0.009)	14.874 (0.560)	7.082 (0.888)	7.371 (0.747)	2.607 (0.680)	-0.023 (0.035)	-0.010 (0.013)
1 if Hispanic	-0.077 (0.089)	0.204 (0.139)	-0.138 (0.147)	-0.246 (0.041)	-0.147 (0.185)	-0.248 (0.131)	-0.067 (0.045)	-0.128 (0.169)	0.005 (0.008)	4.435 (0.568)	12.471 (0.620)	2.757 (0.587)	0.830 (0.492)	0.011 (0.022)	0.012 (0.008)
1 if Asian	0.072 (0.078)	0.558 (0.095)	1.633 (0.138)	-0.571 (0.035)	0.612 (0.166)	0.457 (0.123)	-0.006 (0.033)	-0.053 (0.132)	0.006 (0.007)	4.236 (0.562)	3.330 (0.721)	14.060 (0.429)	-0.016 (0.449)	-0.022 (0.022)	0.012 (0.007)
1 if college degree or more	0.200 (0.065)	0.501 (0.079)	0.428 (0.118)	0.006 (0.032)	0.588 (0.148)	0.106 (0.101)	0.031 (0.027)	0.486 (0.134)	0.022 (0.007)	1.279 (0.504)	-0.638 (0.607)	-1.935 (0.450)	8.986 (0.366)	0.009 (0.020)	0.009 (0.007)
1 if working	0.093 (0.062)	0.272 (0.074)	0.604 (0.113)	0.021 (0.030)	0.897 (0.138)	0.425 (0.096)	0.023 (0.028)	-0.515 (0.125)	-0.019 (0.007)	-0.712 (0.444)	-0.335 (0.563)	-0.434 (0.436)	-1.931 (0.350)	0.033 (0.016)	-0.896 (0.009)
age (years)	0.013 (0.002)	0.011 (0.002)	0.097 (0.003)	0.003 (0.001)	-0.033 (0.004)	-0.010 (0.003)	0.003 (0.001)	-0.011 (0.003)	0.001 (0.000)	-0.022 (0.013)	-0.085 (0.016)	-0.005 (0.013)	-0.018 (0.010)	0.001 (0.001)	-0.001 (0.000)

Note: The parameters shown describe the elements of the utility function that interact household characteristics, shown in row headings, with choice characteristics, shown in column headings. Standard errors are in parentheses.

Table 4: First Stage Price Regressions

Sample	Without Neighborhood Socio-demographics			With Neighborhood Socio-demographics		
	full sample	within .25 mile of boundaries		full sample	within .25 mile of boundaries	
Boundary Fixed Effects	No	No	Yes	No	No	Yes
Observations	242,100	27,958	27,958	242,100	27,958	27,958
	(1)	(2)	(3)	(4)	(5)	(6)
average test score (in standard deviations)	25.92 (1.83)	18.57 (5.06)	-9.51 (6.27)	4.96 (1.69)	5.29 (4.68)	11.02 (5.81)
1 if unit owned	44.16 (3.16)	6.62 (8.93)	37.09 (8.77)	37.57 (3.02)	22.32 (8.47)	33.86 (8.47)
number of rooms	37.36 (1.24)	33.20 (3.41)	38.65 (3.41)	31.27 (1.20)	30.20 (3.45)	29.18 (3.41)
1 if built in 1980s	19.24 (3.90)	-34.31 (10.79)	13.81 (11.44)	14.84 (3.72)	5.18 (10.57)	17.91 (11.22)
1 if built in 1960s or 1970s	-0.98 (2.78)	-19.80 (8.05)	-8.97 (8.36)	-4.10 (2.64)	-9.59 (7.58)	-11.29 (7.74)
elevation (/100)	6.72 (0.79)	-18.47 (2.53)	33.57 (4.89)	1.70 (0.75)	-14.55 (2.36)	15.99 (4.77)
population density	-59.53 (4.18)	-113.08 (15.18)	-102.61 (18.67)	10.70 (4.24)	32.79 (15.48)	16.09 (19.12)
crime index	-0.77 (0.18)	-0.11 (0.70)	4.01 (1.74)	0.53 (0.20)	-0.84 (0.80)	2.57 (1.81)
% Census block group Black				-36.28 (6.88)	-50.69 (31.74)	0.47 (37.90)
% Census block group Hispanic				9.14 (10.19)	26.41 (46.21)	138.64 (59.42)
% Census block group Asian				-6.50 (11.55)	-9.46 (37.22)	239.71 (50.47)
% block group college degree or more				163.75 (16.58)	16.06 (30.12)	-47.61 (42.45)
average block group income (/10000)				280.60 (14.24)	29.12 (55.33)	21.17 (60.03)
Optimal price instrument	0.738 (0.006)	0.771 (0.018)	0.663 (0.018)	0.736 (0.008)	0.740 (0.022)	0.731 (0.022)
constant	175.28 (1.90)	158.49 (7.39)	204.63 (17.87)	180.23 (2.17)	187.33 (7.99)	226.98 (17.92)
F-statistic for price instrument	13973	1843	1411	9398	1151	1130
R²	0.40	0.42	0.47	0.46	0.49	0.51

Note: All regressions shown in the table also include controls for land use (% industrial, % residential, % commercial, % open space, % other) in 1, 2, 3, 4, and 5 mile rings around location and six variables that characterize the housing stock in each of these rings. The dependent variable is monthly house price, which equals monthly rent for renter-occupied units and a monthly price for owner-occupied housing calculated as described in the text. Standard errors are in parentheses.

Table 5: Implied Mean MWTP Measures

Sample	Without Neighborhood Sociodemographics			With Neighborhood Sociodemographics		
	full sample	within .25 mile of boundaries		full sample	within .25 mile of boundaries	
Boundary Fixed Effects	No	No	Yes	No	No	Yes
Observations	242,100	27,958	27,958	242,100	27,958	27,958
	(1)	(2)	(3)	(4)	(5)	(6)
average test score (in standard deviations)	126.08	122.89	81.53	20.17	20.19	26.22
	(1.96)	(5.36)	(7.72)	(1.72)	(4.77)	(6.13)
1 if unit owned	209.76	178.37	184.54	165.38	150.77	161.05
	(3.29)	(8.99)	(11.39)	(3.19)	(8.76)	(9.24)
number of rooms	148.98	149.36	138.71	122.03	121.12	118.93
	(1.51)	(4.24)	(5.49)	(1.48)	(4.23)	(4.40)
1 if built in 1980s	129.93	74.74	106.17	99.69	85.50	95.55
	(3.94)	(10.87)	(14.41)	(3.79)	(10.69)	(11.84)
1 if built in 1960s or 1970s	28.48	9.46	15.39	13.79	7.40	4.50
	(2.78)	(8.03)	(10.48)	(2.67)	(7.71)	(8.51)
elevation (/100)	21.09	-4.82	46.46	-1.06	-18.04	12.83
	(0.81)	(2.48)	(6.35)	(0.75)	(2.46)	(5.04)
population density	-100.43	-153.53	-133.08	19.41	41.68	30.33
	(4.23)	(15.64)	(23.85)	(4.30)	(15.76)	(20.09)
crime index	-2.95	-2.30	1.78	0.00	-1.39	1.96
	(0.18)	(0.70)	(2.20)	(0.20)	(0.81)	(1.91)
% Census block group black				-324.67	-318.83	-267.08
				(10.14)	(32.15)	(39.84)
% Census block group Hispanic				-4.42	18.06	138.95
				(14.35)	(46.87)	(63.13)
% Census block group Asian				-97.39	-96.22	155.27
				(11.15)	(37.39)	(55.73)
% block group college degree or more				286.02	206.02	137.71
				(10.50)	(30.58)	(44.53)
average block group income				87.08	96.11	87.61
				(1.25)	(3.86)	(4.00)
F-statistic for boundary fixed effects			5.349			4.162

Note: Specifications shown in the table also include controls for land use (% industrial, % residential, % commercial, % open space, % other) in 1, 2, 3, 4, and 5 mile rings around location and six variables that characterize the housing stock in each of these rings. MWTP measures are reported in terms of a monthly house price. Standard errors are in parentheses.

Table 6. Heterogeneity in Marginal Willingness to Pay for Select Housing/Neighborhood Attributes

	Average Test Score +1 s.d.	House Characteristics			Neighborhood Sociodemographics				
		Own vs. Rent	+1 Room	Built in 1980s vs. pre-1960	+10% Black vs. White	+10% Hisp vs. White	+10% Asian vs. White	+10% College Educated	Blk Group Avg Income + \$10,000
Mean MWTP	26.22 (6.13)	161.05 (9.24)	118.93 (4.40)	95.55 (11.84)	-26.71 (3.98)	13.90 (6.31)	15.53 (5.57)	13.77 (4.45)	87.61 (4.00)
Household Income (+\$10,000)	1.57 (0.35)	21.84 (0.71)	6.12 (0.17)	10.51 (0.76)	-1.53 (0.39)	0.77 (0.41)	-0.05 (0.25)	2.62 (0.22)	1.54 (0.11)
Children Under 18 vs. No Children	7.10 (3.78)	-12.87 (6.67)	40.06 (1.80)	-24.52 (7.94)	10.38 (2.70)	15.03 (3.41)	12.17 (2.51)	-14.18 (2.15)	5.05 (1.06)
Black vs. White	-18.05 (7.50)	-63.55 (13.25)	1.56 (3.40)	2.95 (16.38)	96.82 (3.62)	46.13 (5.75)	48.02 (4.84)	16.99 (4.40)	-0.45 (2.27)
Hispanic vs. White	-4.64 (5.80)	-6.44 (9.53)	-14.14 (2.63)	-8.07 (12.00)	28.89 (3.68)	81.36 (4.01)	18.01 (3.81)	5.43 (3.19)	2.07 (1.41)
Asian vs. White	5.79 (5.08)	113.65 (8.96)	-32.92 (2.27)	43.94 (10.77)	27.74 (3.64)	21.95 (4.67)	92.49 (2.78)	-0.05 (2.91)	1.99 (1.41)
College Degree or More vs. Some College or Less	14.12 (4.24)	33.83 (7.67)	4.50 (2.05)	42.06 (9.57)	8.34 (3.27)	-4.16 (3.94)	-12.70 (2.91)	59.29 (2.37)	3.66 (1.29)
Householder Working vs. Not Working	6.63 (4.02)	42.72 (7.31)	3.69 (1.94)	60.60 (8.92)	-4.71 (2.88)	-2.17 (3.65)	-2.81 (2.82)	-12.62 (2.27)	3.88 (1.04)
Age (+10 years)	0.86 (0.11)	6.49 (0.21)	0.30 (0.06)	-2.07 (0.25)	-0.15 (0.08)	-0.56 (0.10)	-0.03 (0.08)	-0.12 (0.06)	0.11 (0.03)

Note: The first row of the table reports the mean marginal willingness-to-pay for the change reported in the column heading. The remaining rows report the difference in willingness to pay associated with the change listed in the row heading, holding all other factors equal. Standard errors are in parentheses.

Table 7: Hedonic Price Regressions

Sample	Without Neighborhood Sociodemographics			With Neighborhood Sociodemographics		
	full sample	within .25 mile of boundaries		full sample	within .25 mile of boundaries	
Boundary Fixed Effects	No	No	Yes	No	No	Yes
Observations	242,100	27,958	27,958	242,100	27,958	27,958
	(1)	(2)	(3)	(4)	(5)	(6)
average test score (in standard deviations)	145.62	144.96	101.17	28.76	27.63	28.43
	(1.57)	(3.85)	(4.15)	(1.70)	(4.76)	(5.88)
1 if unit owned	151.59	124.67	140.84	125.59	112.21	123.91
	(3.12)	(8.77)	(8.47)	(2.94)	(8.16)	(8.16)
number of rooms	153.00	155.79	143.40	121.88	122.48	121.15
	(0.79)	(2.24)	(2.20)	(0.76)	(2.16)	(2.16)
1 if built in 1980s	129.28	91.28	130.12	89.48	92.79	109.41
	(3.89)	(11.01)	(11.22)	(3.71)	(10.36)	(11.01)
1 if built in 1960s or 1970s	22.98	8.82	93.30	4.13	4.33	6.96
	(2.85)	(8.20)	(8.05)	(2.69)	(7.74)	(8.20)
elevation (/100)	31.80	3.50	48.34	2.53	-17.42	13.29
	(0.79)	(2.53)	(3.25)	(0.76)	(2.43)	(4.85)
population density	-78.26	-155.27	-145.41	46.14	61.17	33.54
	(4.29)	(15.63)	(18.97)	(4.31)	(15.79)	(19.43)
crime index	0.75	2.16	6.84	1.41	0.49	4.97
	(0.19)	(0.71)	(1.79)	(0.20)	(0.81)	(1.85)
% census block group Black				-71.56	-113.70	-40.74
				(10.21)	(32.21)	(38.37)
% census block group Hispanic				128.63	146.56	240.31
				(14.42)	(46.21)	(60.74)
% census block group Asian				-1.38	-76.33	200.60
				(11.25)	(37.85)	(53.62)
% block group college degree or more				286.57	192.17	91.51
				(10.16)	(30.31)	(43.25)
average block group income (/10000)				100.32	110.84	101.26
				(16.14)	(55.33)	(60.03)
F-statistic for boundary fixed effects			23.345			8.754
R ²	0.37	0.38	0.44	0.44	0.47	0.49

Note: All regressions shown in the table also include controls for land use (% industrial, % residential, % commercial, % open space, % other) in 1, 2, 3, 4, and 5 mile rings around each location and six variables that characterize the housing stock in each of these rings. The dependent variable is monthly house price, which equals monthly rent for renter-occupied units and a monthly price for owner-occupied housing, calculated as described in the text. Standard errors are in parentheses.

Table 8. Partial Correlations Between Choice Variables and Choice Specific Constant (d)

Sample	full sample	within .25 miles of boundaries	within .25 miles of boundaries
Boundary Fixed Effects	No	No	Yes
Observations	242,100 (1)	27,958 (2)	27,958 (3)
<hr/> Housing/Neighborhood Variables			
monthly house price	-0.057 (0.001)	-0.057 (0.002)	-0.056 (0.002)
average test score	-0.016 (0.001)	-0.015 (0.002)	-0.003 (0.002)
% Census block group black	-0.078 (0.001)	-0.050 (0.002)	-0.038 (0.002)
% Census block group Hispanic	-0.030 (0.001)	-0.023 (0.002)	-0.012 (0.001)
% Census block group Asian	-0.034 (0.001)	-0.004 (0.002)	-0.004 (0.001)
% block group college degree or more	-0.001 (0.001)	0.003 (0.002)	0.031 (0.001)
average block group income	-0.054 (0.001)	-0.066 (0.002)	-0.045 (0.002)
number of rooms	0.001 (0.001)	-0.009 (0.002)	-0.013 (0.002)

Note: Figures in the table are partial correlations conditional on all other covariates. Other covariates include all housing and neighborhood characteristics shown in Table 2 as well as controls for land use (% industrial, % residential, % commercial, % open space, % other) in 1, 2, 3, 4, and 5 mile rings around location and six variables that characterize the housing stock in each of these rings. Standard errors are in parentheses.

Table 9a. The Capitalization of School Quality: Distribution of Housing Price Changes

Percentile of Simulated Distribution	<u>Partial Equilibrium</u>	<u>General Equilibrium</u>	
	Monthly Price Increase	Unadjusted	Adjusted
		Monthly Price Increase	Monthly Price Increase
90	\$33.95	\$56.83	\$59.05
50	\$26.38	\$45.80	\$47.32
10	\$19.45	\$28.83	\$29.64
Mean	\$27.10	\$46.00	\$47.70

Table 9b. Changes in Sociodemographic Composition of Catchment Areas where Quality is Increased

	<u>Partial Equilibrium</u>	<u>General Equilibrium</u>	
		Unadjusted	Adjusted
Change in Average Income	1,049	1,646	1,771
Percentage Point Change White	0.66%	1.25%	1.35%
Percent. Pt. Chg. Black	-0.53%	-0.75%	-0.83%
Percent. Pt. Chg. Hispanic	-0.43%	-0.86%	-0.90%
Percent. Pt. Chg. Asian	0.31%	0.38%	0.40%
Percent. Pt. Chg. College Degree or More	0.73%	1.61%	1.69%

Note: The figures shown in the upper panel of this table report the mean and distribution of changes in monthly housing prices for a corresponding catchment area following an increase in a school's average test score by 74 points (1 standard deviation). The first column shows partial equilibrium results, which do not account for any subsequent changes to the neighborhood sociodemographic distribution. The second and third columns report general equilibrium results, which account for sociodemographic changes to the neighborhood. In this case, the second column (unadjusted) reports results of simulations that hold crime and the average test score and pre-simulation levels, while the third column adjusts crime and average test scores according to production functions estimated via OLS reported in Appendix Table 2. The lower panel of the table shows the corresponding changes in the sociodemographic composition of the corresponding neighborhood. In calculating the partial equilibrium, these changes in sociodemographic compositions are not accounted for in the utility function.

Appendix Table 1: Choice-Specific Constant Regressions

Sample	Without Neighborhood Sociodemographics			With Neighborhood Sociodemographics		
	full sample	within .25 mile of boundaries		full sample	within .25 mile of boundaries	
Boundary Fixed Effects	No	No	Yes	No	No	Yes
Observations	242,100	27,958	27,958	242,100	27,958	27,958
monthly housing price (/1000)	-10.23 (1.39)	-9.73 (1.13)	-11.34 (1.36)	-15.94 (1.71)	-15.97 (1.56)	-16.19 (1.69)
average test score (in standard deviations)	1.29 (0.02)	1.20 (0.05)	0.92 (0.01)	0.32 (0.03)	0.32 (0.08)	0.42 (0.01)
1 if unit owned	2.15 (0.03)	1.74 (0.09)	2.09 (0.01)	2.64 (0.05)	2.41 (0.14)	2.61 (0.01)
number of rooms	1.52 (0.02)	1.45 (0.04)	1.57 (0.01)	1.95 (0.02)	1.93 (0.07)	1.93 (0.01)
1 if built in 1980s	1.33 (0.04)	0.73 (0.11)	1.20 (0.02)	1.59 (0.06)	1.37 (0.17)	1.55 (0.02)
1 if built in 1960s or 1970s	0.29 (0.03)	0.09 (0.08)	0.17 (0.01)	0.22 (0.04)	0.12 (0.12)	0.07 (0.01)
elevation (/100)	0.22 (0.01)	-0.05 (0.02)	0.53 (0.01)	-0.02 (0.01)	-0.29 (0.04)	0.21 (0.01)
population density	-1.03 (0.04)	-1.49 (0.15)	-1.51 (0.03)	0.31 (0.07)	0.67 (0.25)	0.49 (0.03)
crime index	-0.03 (0.00)	-0.02 (0.01)	0.02 (0.00)	0.00 (0.00)	-0.02 (0.01)	0.03 (0.00)
% Census block group black				-5.18 (0.16)	-5.09 (0.51)	-4.32 (0.06)
% Census block group Hispanic				-0.07 (0.23)	0.29 (0.75)	2.25 (0.10)
% Census block group Asian				-1.55 (0.18)	-1.54 (0.60)	2.51 (0.09)
% block group college degree or more				4.56 (0.17)	3.29 (0.49)	2.23 (0.07)
average block group income				1.39 (0.02)	1.53 (0.06)	1.42 (0.01)
F-statistic for boundary fixed effects			4.545			3.963

Note: Specifications shown in the table also include controls for land use (% industrial, % residential, % commercial, % open space, % other) in 1, 2, 3, 4, and 5 mile rings around location and six variables that characterize the housing stock in each of these

Appendix Table 2: OLS Crime and Education Production Functions

Dependent Variable	Production Function	
	crime index	average test score
Observations	242,100	242,100
R²	0.33	0.41
Percent Black	0.285 (0.005)	-0.188 (0.005)
Percent Hispanic	0.099 (0.004)	-0.074 (0.003)
Percent Asian	0.088 (0.003)	-0.041 (0.003)
Percent College Degree or More	0.017 (0.004)	0.127 (0.004)
Average Income	-0.071 (0.046)	0.311 (0.043)

Note: This table shows the results of the OLS estimation of simple crime and education production functions. These functions are used in the simulations that adjust crime and school quality with changing neighborhood sociodemographic composition. We use these 'adjusted' results to provide a bound on the simulation results. Standard errors are provided below estimated coefficient. All variables are normalized to have mean zero and standard deviation one.

Technical Appendix

Introduction

This appendix supplements the description of the estimation procedure provided in the main text of the paper. To avoid repetition, this document assumes that the reader has read the discussion in the main text. In that discussion, we abstract from many of the issues discussed here, which are of a more technical nature, in order to maintain the focus on issues associated with the identification of preferences for schools and neighbors and on relating our approach to other approaches that appear in the labor and public, rather than the IO, literatures.

Defining the Choice Set

In order to provide a coherent discussion of the properties of the estimator, it is helpful to provide an exact characterization of the choice problem – in particular, whether it should literally be viewed as the choice of a single residence from the full census of residences available in the Bay Area housing market or as the choice of a representative housing type. The assumption that the underlying individual component of the error term \mathbf{e}^i_h is distributed according to the extreme value distribution allows us some flexibility in terms of the interpretation of the underlying choice problem. Given that the IIA property holds for each individual, the econometrician has some flexibility in using a subset of the full set of alternatives in estimating the model. As it turns out, however, it is much more straightforward to develop the equilibrium properties of our model as well as the asymptotic properties of the estimator if we assume that the full census of available houses can be characterized by a smaller set of representative housing types.

Accordingly, we characterize the economic environment as follows: The model is estimated on data drawn from a single, large metropolitan area. The complete metropolitan area housing market consists of a total of I individuals who must choose from H distinct types of housing, with H assumed to be less than I . Each individual i is characterized by a set of characteristics Z^i and a set of idiosyncratic preferences $\{\mathbf{e}^i_h\}$ defined over the full set of distinct housing alternatives, and identically and independently distributed across choices according to the extreme value distribution. Households are assumed to follow the model's decision rule at the true parameter vector. Each distinct housing type k is characterized by the set of characteristics $\{X_k, \mathbf{x}_k\}$. The $\{X_k, \mathbf{x}_k\}$ vectors are assumed to be exchangeable draws from some larger population of possible house types.

We do not observe the full census of households and houses in the metropolitan area, but instead observe a random sample of households S^I of size N and their corresponding houses S_H . We assume that N is large relative to H so that the market shares for the H distinct types of housing are given by n_H/N ,

where n_H is the number of times that a house of distinct type H is sampled. Thus the relative market share of each house type can be calculated exactly using the observed sample of housing alternatives.

Relationship to the Estimator in BLP and Other IO Applications

Before discussing the equilibrium and asymptotic properties of the model, it is helpful to relate our estimator to that of Berry, Levinsohn, and Pakes (1995). The first step of our estimation procedure is a Maximum Likelihood estimator, which returns estimates of the heterogeneous parameters \mathbf{q}_I and mean indirect utilities, \mathbf{d}_h . As we show in the text, maximizing the log-likelihood function with respect to \mathbf{d} implies that \mathbf{q}_I and \mathbf{d} must be chosen such that the sum of the probabilities over individuals equals one, $\sum_i (P_h^i) = 1$, for each house in the sample of houses S_H . For any \mathbf{q}_I , a simple contraction mapping can be used to solve for the vector \mathbf{d} that forces these conditions to hold exactly. For our application, the contraction mapping is simply:

$$(1) \quad \mathbf{d}_h^{t+1} = \mathbf{d}_h^t - \ln\left(\sum_i \hat{P}_h^i\right)$$

where t indexes the iterations of the contraction mapping. Using this contraction mapping, it is possible to solve quickly for an estimate of the full vector $\hat{\mathbf{d}}$ even when it contains a large number of elements, and consequently the likelihood function can be concentrated as: $\ell(\mathbf{d}, \mathbf{q}_I) = \ell^c(\hat{\mathbf{d}}(\mathbf{q}_I), \mathbf{q}_I)$. This reduces our free parameter search to \mathbf{q}_I , thereby dramatically reducing the computational burden in the first step of the estimation procedure.

Many of the features of the first step of our estimation and the interpretation of the parameters and mean indirect utilities have a clear analogy to those in BLP (1995). It is important to draw a distinction, however, as to how the conditions enforced by the contraction mapping come about in the two applications. In particular, as part of the first step of the estimator developed in BLP, the authors force the model to fit the market share of each product directly. In the context of our estimator, analogous conditions come about directly from the first order conditions of the likelihood function, which imply that the condition $\sum_i (P_h^i) = 1$ holds at the maximum of the likelihood function for each house h in the sample of houses S^H .

One can view these conditions as forcing the ‘market share’ of each house in the sample to be $1/N$, but this interpretation can lead to confusion, especially because it naturally leads one to view the choice set as the full census of houses available in the market. More precisely, given the characterization

of the data generating process above, these conditions force the market share of each distinct house type H to be n_H/N , which is its true market share. In this way, an analogy with IO applications is fairly direct under our preferred interpretation of the choice problem; and the fact the conditions wind up as $\sum_i (P_h^i) = 1$ simply reflects the fact that these sums are taken over the houses in the sample, not distinct house types .

Sampling

An important aspect of the underlying IIA property for each individual is that we can estimate the model using only a sample of the alternatives not selected by the individual, following McFadden (1978). This permits estimation despite having many alternatives – i.e., many distinct house types. The particular procedure that we use is as follows. Using the sample of households S^i and their corresponding houses S_H from the full data set, for each household i observed in this sample, we construct a subset S_H^i that consists of the household’s chosen house and a random sample of the remaining alternatives in S_H . In practice, because we estimate a mean indirect utility for each house observed in the sample, the precision of the estimation procedure increases greatly if we ensure that each alternative appears in the choice set of the same number of households. To this end, we employ the following random sampling procedure: Starting with the assignment of each household’s chosen house, we assign each household a first additional (not chosen) alternative by randomly re-shuffling the full set of houses across households. We then repeat this random re-shuffling of houses as many times as is necessary to generate the desired size of the sample of additional (not chosen) alternatives. In this way, with an additional random draw for each household, we ensure that each alternative is sampled exactly once.

In this way, the probability that household i chooses house h can be written as:

$$(2) \quad P_h^i = \frac{(C+1)}{N} \cdot \frac{\exp(d_h + \hat{m}_h^i)}{\sum_{k \in S_H^i} \exp(d_k + \hat{m}_k^i)}$$

where N is the total number of alternatives in the sample, C is the number of additional (not chosen) alternatives sampled for each household and the sum in the denominator is now taken over only those alternatives in the subset associated with household i . The probability in (2) is used in the log-likelihood function – although notice that the multiplicative component does not affect the first order conditions.

The use of a random sample of the full census of alternatives for each household necessitates a slight adjustment to the calculation of the predicted number of households that choose each house that is

used in the BLP-style contraction mapping that makes up part of the first stage ML estimation. In particular, because the sampling procedure ensures that each household's actual choice is included in the subset of alternatives when calculating the choice probabilities shown in equation (2), the sum of the probabilities for each house must be corrected for this inherent over-sampling. This requires the following straightforward adjustment:

$$(3) \quad \sum_i P_h^i = \sum_{i=h} P_h^i + \frac{(N-1)}{C} \sum_{h \in S_H^i, i \neq h} P_h^i$$

where the notation $i=h$ refers to the household that actually chooses house h . In equation (3), the first term captures the contribution to \hat{N}_h made by the household who actually chose house h , while the second term sums the contributions of the other households in the sample which could have chosen house h (i.e., the house was in the household's randomly drawn choice set) but did not.

Asymptotic Properties of the Full Estimator

Finally, the characterization of the choice problem as a choice of a representative house is also helpful in developing the conditions that ensure the consistency and asymptotic normality of our estimates. Under the assumptions concerning the data generating process described above, our problem fits within a class of models for which the asymptotic distribution theory has been developed. In this section, therefore, we summarize the requirements necessary for the consistency and asymptotic normality of our estimates and provide some intuition for these conditions.

In general, there are three dimensions in which our sample can grow large, namely as H , N , or C grow large. For any set of distinct housing alternatives of size H and any random sampling of these alternatives of size C , the consistency and asymptotic normality of the first-stage estimates (δ, \mathbf{q}_I) follows directly as long as N grows large. This is the central result of McFadden (1978), justifying the use of a random sample of the full census of alternatives. Intuitively, even if each household is assigned only one randomly drawn alternative in addition to its own choice, the number of times that each house is sampled (the dimension in which the choice-specific constants are identified) grows as a fixed fraction of N .

If the true vector \mathbf{d} were used in the second stage of the estimation procedure, the consistency and asymptotic normality of the second-stage estimates \mathbf{q}_d would follow as long as $H \rightarrow \infty$.¹ In practice,

¹ This condition requires certain regularity conditions. See Berry, Linton, and Pakes (2002) for details.

ensuring the consistency and asymptotic normality of the second-stage estimates is complicated by the fact the vector \mathbf{d} is estimated rather than known. Berry, Linton, and Pakes (2002) develop the asymptotic distribution theory for the second stage estimates \mathbf{q}_d for a broad class of models that contains our model as a special case and, consequently, we employ their results. In particular, the consistency of the second-stage estimates follows as long as $H \rightarrow \infty$ and N grows fast enough relative to H such that $H \log H / N$ goes to zero, while asymptotic normality at rate \sqrt{H} follows as long as H^2 / N is bounded. Intuitively, these conditions ensure that the noise in the estimate of \mathbf{d} becomes inconsequential asymptotically and thus that the asymptotic distribution of \mathbf{q}_d is dominated by the randomness in \mathbf{x} as it would be if \mathbf{d} was known.

DATA APPENDIX

1. Introduction

This document details the sources for the data and the construction of the variables used in “A Unified Approach for Measuring Preferences for Schools and Neighborhoods,” by Patrick Bayer, Fernando Ferreira, and Robert McMillan.

2. Census Variables

House Prices

Because house values are self-reported, it is difficult to ascertain whether these prices represent the current market value of the property, especially if the owner purchased the house many years earlier. Fortunately, the Census contains other information that helps us to examine this issue and correct house values accordingly. In particular, the Census asks owners to report a continuous measure of their annual property tax payment. The rules associated with Proposition 13 imply that the vast majority of property tax payments in California should represent exactly 1 percent of the transaction price of the house at the time the current owner bought the property or the value of the house in 1978. Thus, by combining information about property tax payments and the year that the owner bought the house (also provided in the Census in relatively small ranges), we are able to construct a measure of the rate of appreciation implied by each household’s self-reported house value. We use this information to modify house values for those individuals who report values much closer to the original transaction price rather than current market value. In our study most households list the purchase price of their house rather than an estimated market value for their house. Thus if two identical houses were found in the census data but one was last sold in 1989 and one was last sold in 1969 we find on average the listed market price of the more recently sold house is on average 15 percent higher than the other house.

A second deficiency of the house values reported in the Census is that they are top-coded at \$500,000, a top-code that is often binding in California. Again, because the property tax payment variable is continuous and not top-coded, it provides information useful in distinguishing the values of the upper tail of the value distribution. We find that top-coding was fairly predominant in the Bay Area and that higher top-codes may be useful to gain a better understanding of house prices in expensive markets like California or New York.

The exact procedure that we use to adjust self-reported house values is as follows. We first regress the log of self-reported house value on the log of the estimated transaction price (100 times the property tax payment), and a series of dummy variables that characterize the tenure of the current owner:

$$(1) \quad \log(V_j) = \mathbf{a}_1 \log(T_j) + \mathbf{a}_2 y_j + \mathbf{w}_j$$

where V_j represents the self-reported house value, T_j represents the estimated transaction price, and y_j represents a series of dummy variables for the year that the owner bought the house. If owner-estimated house values were indeed current market values and houses were identical except for owner tenure, this regression would return an estimate of 1 for \hat{a}_1 and the estimated \hat{a}_2 coefficients would indicate the appreciation of house values in the Bay Area

over the full period of analysis. If owners tend to underreport house values, especially when they have lived in the house for a long time, the estimated $\hat{\alpha}_2$ parameters will likewise underreport appreciation in the market. In this way, the estimated $\hat{\alpha}_2$ parameters represent a conservative estimate of appreciation. Given the estimates of equation (2), we construct a predicted house value for each house in the sample and replace the owner-reported value with this measure when this predicted measure exceeds the owner-reported value. In practice, in order to allow for different rates of appreciation in different regions of the housing market, we conduct these regressions separately for each of the 45 Census PUMA (areas with at least 100,000 people) in our sample and allow appreciation to vary with a small set of house characteristics within each PUMA. In this way, the first adjustment that we make to house prices is to adjust owner-reported values for likely under-reporting.

The adjustment to top-coded house prices uses the same approach, using the information on property taxes that are continuous and not top-coded. Using estimates of equation (2) based on a sample of houses that does not include the top-coded house values, we construct predicted house values for all top-coded houses. This allows us to assign continuous house values for top-coded measures.

Reported Rental Value

We next examined questions of reported monthly rents. While rents are presumably not subject to the same degree of misreporting as house values, it is still the case that renters who have occupied a unit for a long period of time generally receive some form of tenure discount. In some cases, this tenure discount may arise from explicit rent control, but implicit tenure discounts generally occur in rental markets even when the property is not subject to formal rent control. Thus while, this will not lead to errors in the answering of the listed census question it may lead to an inaccurate comparison of rents faced by households if they needed to move. In order to get a more accurate measure of the market rent for each rental unit, we utilize a series of locally based hedonic price regressions in order to estimate the discount associated with different durations of tenure in each of over 40 sub-regions within the Bay Area.

In order to get a better estimate of market rents for each renter-occupied unit in our sample, we regress the log of reported rent R_j on a series of dummy variables that characterize the tenure of the current renter, y_j , as well as a series of variables that characterize other features of the house and neighborhood X_j :

$$(2) \quad \log(R_j) = \mathbf{b}_1 y_j + \mathbf{b}_2 X_j + \mathbf{u}_j$$

again running these regressions separately for each of the 45 PUMAs in our sample. To the extent that the additional house and neighborhood variables included in equation (3) control for differences between the stock of rental units with long-term vs. short-term tenants, the $\hat{\alpha}_1$ parameters provide an estimate of the tenure discount in

each PUMA.¹ In order to construct estimates of market rents for each rental unit in our sample, then, we inflate rents based on the length of time that the household has occupied the unit using the estimates of $\hat{\alpha}_1$ from equation (2). In this way, these three price adjustments bring the measures for rents and house values reported in the Census reasonably close to market rates.

Calculating Cost Per Unit of Housing Across Tenure Status

Finally, in order to make owner- and renter-occupied housing prices comparable in our analysis we need to calculate a current rental value for housing. Because house prices reflect the expectations about the future rents for the property they incorporate beliefs about future housing appreciation. To appropriately deflate housing values – and especially to control for differences in expectations about appreciation in different segments of the Bay Area housing market – we regress the log of house price (whether monthly rent or house value) $\ln P_j$ on an indicator for whether the housing unit is owner-occupied o_j and a series of additional controls for features of the house including the number of rooms, number of bedrooms, types of structure (single-family detached, unit in various sized buildings, etc.), and age of the housing structure as well as a series of neighborhood controls X_j :

$$(3) \quad \log(\ln P_j) = \mathbf{g}_1 o_j + \mathbf{g}_2 X_j + \mathbf{h}_j$$

We estimate these hedonic price regressions for each of 40 sub-regions (Census Public Use Microdata Areas -PUMAs) of the Bay Area housing market. These regressions return an estimate of the ratio of house values to rents for each of these sub-regions and we use these ratios to convert house values to a measure of current monthly rent.

3. External Data

We next discuss the additional variables we have added to the Census data to provide a more nuanced understanding of the neighborhood characteristics that affect house prices and residential location decisions. These data sets are linked to census blocks and can be used to determine the appropriateness of the questions and sampling techniques used. This additional data includes:

School and School District Data

The Teale data center in California provided a crosswalk that matches all Census blocks in California to the corresponding public school district. We have further matched Census blocks to particular schools using a variety of procedures that takes account of the location (at the block level) of each Census block within a school district and the precise location of schools within the district using information on location from the Department of Education. Other school information in these data include:

¹ Interestingly, while we estimate tenure discounts in all PUMAs, the estimated tenure discounts are substantially greater for rental units in San Francisco and Berkeley, the two largest jurisdictions in the Bay Area that had formal

- 1992-93 CLAS dataset provides detailed information about school performance and peer group measures. The CLAS was a test administered in the early 1990s that will give us information on student performance in math, literature and writing for grades 4, 8 and 10. This dataset presents information on student characteristics and grades for students at each school overall and across different classifications of students, including by race and education of parents.
- 1991-2 CBEDS (California Board of Education data sets) datasets including information from the SIF (school information form) which includes information on the ethnic/racial and gender make-up of students, PAIF – which is a teacher based form that provides detailed information about teacher experience, education and certification backgrounds and information on the classes each teacher teaches, and (LEP census) a language census that provides information on the languages spoken by limited-English speaking students.

Procedures for Assigning School Data:

While we have an exact assignment of Census blocks to school districts, we have only been able to attain precise maps that describe the way that city blocks are assigned to schools in 1990 for Alameda County. In the absence of information about within-district school attendance areas, we employ the alternative approaches for linking each house to a school. The crudest procedure assigns average school district characteristics to every house falling in the school district. A refinement on this makes use of distance-weighted averages. For a house in a given Census block, we calculate the distance between that Census block and each school in the school district. We have detailed information characterizing each school and construct weighted averages of each school characteristic, weighting by the reciprocal of the distance-squared as well as enrollment.

As a third approach we simply assign each house to the closest school within the appropriate school district. Our preferred approach (which we use for the results reported in the paper) refines this closest-school assignment by using information about individual children living in each Census block - their age and whether they are enrolled in public school. In particular, we modify the closest-school assignment technique by attempting to match the observed fourth grade enrollment for every school in every school district in the Bay Area. Adjusting for the sampling implicit in the long form of the Census, the 'true' assignment of houses to schools must give rise to the overall fourth grade enrollments observed in the data.

These aggregate numbers provide the basis for the following intuitive procedure: we begin by calculating the five closest schools to each Census block. As an initial assignment, each Census block and all the fourth graders in it are assigned to the closest school. We then calculate the total predicted enrollment in each school, and compare this with the actual enrollment. If a school has excess demand, we reassign Census blocks out of its catchment area, while if a school has excess supply, we expand the school's catchment area to include more districts.

To carry out this adjustment, we rank schools on the basis of the (absolute value of) their prediction error, dealing with the schools that have the greatest excess demand/supply first. If the school has excess demand, we reassign the Census

block that has the closest second school (recalling that we record the five closest schools to each Census block, in order), as long as that second school has excess supply. If a school has excess supply, we reassign to it the closest school district currently assigned to a school with excess demand. We make gradual adjustments, reassigning one Census block from each school in disequilibrium each iteration. This gradual adjustment of assignments of Census blocks to schools continues until we have 'market clearing' (within a certain tolerance) for each school. Our actual algorithm converges quickly in practice, and produces plausible adjustments to the initial, closest-school assignment.

Land use

Information on land use/land cover digital data is collected by USGS and converted to ARC/INFO by the EPA available at: <http://www.epa.gov/ost/basins/> for 1988. We have calculated for each Census block, the percentage of land in a ¼, ½, 1, 2, 3, 4 and 5 mile radii that is used for commercial, residential, industrial, forest (including parks), water (lakes, beaches, reservoirs), urban (mixed urban or built up), transportation (roads, railroad tracks, utilities) and other uses.

Crime data

Information on crime was drawn from the rankings of zipcodes on a scale of 1-10 on the risk of violent crime (homicide, rape or robbery). A score of 5 is the average risk of violent crime and a score of 1 indicates a risk 1/5 the national average and a 10 is 10 or more times the national average. These ratings are provided by CAP index and were downloaded from APBNews.com.

Geography and Topography

The Teale data center in California provided information on the elevation, latitude and longitude of each Census block.