Forecasting the Yield Curve in a Data-Rich Environment: A No-Arbitrage Factor-Augmented VAR Approach*

Emanuel Mönch

Humboldt University Berlin

Job Market Paper

November 2006

Abstract

This paper suggests a term structure model which parsimoniously exploits a broad macroeconomic information set. The model uses the short rate and the common components of a large number of macroeconomic variables as factors. Precisely, the dynamics of the short rate are modeled with a Factor-Augmented Vector Autoregression and the term structure is derived using parameter restrictions implied by no-arbitrage. The model has economic appeal and provides better out-of-sample yield forecasts than previously suggested approaches. The reduction of root mean squared forecast errors relative to the competitor models is highly significant and particularly pronounced for short and medium-term maturities.

Keywords: Yield Curve, Factor-Augmented VAR, Affine term structure models, Dynamic factor models, Forecasting

JEL codes: C13, C32, E43, E44, E52

---

*I would like to thank two anonymous referees, Fernando Alvarez, Filippo Altissimo, Theofanis Archontakis, Claus Brand, Sandra Eickmeier, Lars Hansen, Philipp Hartmann, Peter Hördahl, Manfred Kremer, Monika Piazzesi, Diego Rodriguez Palenzuela, Oreste Tristani, Harald Uhlig, David Vestin, Thomas Werner and seminar participants at Deutsche Bundesbank, Humboldt University Berlin, the ECB, the University of Chicago, the Econometric Society World Congress, the European Finance Association Annual Meeting, and the European Economic Association Annual Meeting for helpful comments. I am further grateful to Lucrezia Reichlin and Robert Bliss for sharing their data with me. Parts of this work were done when the author was visiting the ECB. Financial support by the German National Academic Foundation and the Deutsche Forschungsgemeinschaft through the "SFB 649" is gratefully acknowledged. All remaining errors are the authors' responsibility. Address: Emanuel Mönch, Humboldt University, School of Business and Economics, Institute of Economic Policy, Spandauer Str. 1, 10178 Berlin, GERMANY. Email: moench@wiwi.hu-berlin.de, FON: +49 (30) 20931670, FAX: +49 (30) 20935934.

1 Introduction

Traditional models of the term structure decompose yields into a set of latent factors. These models commonly provide a good in-sample fit to the data (e.g. Nelson and Siegel 1987, Knez, Litterman, and Scheinkman 1994, Dai and Singleton 2000) and can also be used to predict interest rates out-of-sample (e.g. Duffee 2002, Diebold and Li 2006). While providing a good statistical fit, however, the economic meaning of such models is limited since they disregard the relationships between macroeconomic variables and interest rates. In this paper, I suggest a model which has both economic appeal and superior predictive ability for yields as compared to traditional approaches.

In a widely recognized paper, Ang and Piazzesi (2003) augment a standard three-factor affine term structure model with two macroeconomic variables which enter the model through some Taylor-rule type of short rate equation. They find that the macro variables account for a large share of the variation in interest rates and also improve yield forecasts. Inspired by this finding, a vivid literature has emerged lately that explores different approaches to jointly model the term structure and the macroeconomy. Examples for such models are Hördahl, Tristani, and Vestin (2006), Diebold, Rudebusch, and Aruoba (2006), and Dewachter and Lyrio (2006). While these studies consistently find that macroeconomic variables are useful for explaining and/or forecasting government bond yields, they only exploit very small macroeconomic information sets. Yet, by limiting the analysis to only a few variables, other potentially useful macroeconomic information is being neglected.

This is particularly important for term structure modeling as a recent literature argues that the central bank acts in a "data-rich environment" (Bernanke and Boivin 2003). This means that the monetary policy authority bases its decisions upon a broad set of conditioning information rather than only a few key aggregates. Consistent with this argument, a number of studies have found that factors which by construction summarize the comovement in a large number of macroeconomic time series help to explain and forecast the evolution of short-term interest rates (e.g. Bernanke and Boivin 2003, Giannone, Reichlin, and Sala 2004, Favero, Marcellino, and Neglia 2005). In a recent paper, Bernanke, Boivin, and Eliasz (2005) suggest to combine the advantages of factor modeling and structural VAR analysis by estimating a joint vector-autoregression of the short-term interest rate and factors extracted from a large cross-section of macro time series. They label this approach a "Factor-Augmented VAR" (FAVAR) and use it to analyze the dynamics of the short rate and the effects of monetary policy on macroeconomic variables.

In this paper, I take the approach of Bernanke et al. (2005) a step further and employ the FAVAR model to study the dynamics of the entire yield curve within an arbitrage-free model. Precisely, I suggest a model that has the following structure. A Factor-Augmented VAR is used as the state equation of an affine term structure model. This delivers a dynamic characterization of the short-term interest rate conditional on a large macroeconomic information set. Given the dynamics of the short rate, the term structure of interest rates is derived using parameter-restrictions implied by no-arbitrage. In sum, my model is an affine term structure model that has the short rate and the common components of a large number of macro time series as factors. My approach can thus be characterized as a No-Arbitrage Factor-Augmented VAR.

Estimation of my model is in two steps. First, I extract common factors from a large macroeconomic dataset using the method suggested by Stock and Watson (2002a,b) and estimate the parameters governing their joint dynamics with the monetary policy instrument in a VAR. Second, I estimate a no-arbitrage vector autoregression of yields on the exogenous pricing factors. Specifically, I obtain the price of risk parameters by minimizing the sum of squared fitting errors of the model following the nonlinear least squares approach of Ang, Piazzesi, and Wei (2006).

The results of the paper can be summarized as follows. The No-Arbitrage FAVAR model based on four macro factors and the short rate fits US yields well in-sample. Compared to a model which incorporates the short rate and four individual measures of output and inflation as factors, there is a clear advantage in using the larger macroeconomic information set. The results from out-of-sample forecasts of yields underpin this finding. The term structure model based on common factors clearly outperforms a model based on individual variables. More importantly, the No-Arbitrage FAVAR model shows a striking superiority with respect to a number of benchmark models in out-of-sample yield forecasts. Except for extremely short forecast horizons and very long maturities, the model significantly outperforms the random walk, a standard three-factor affine model and the model recently suggested by Diebold and Li (2006) which has been documented to be particularly useful for interest rate predictions. A subsample analysis reveals that the No-Arbitrage Factor-Augmented VAR model performs particularly well in periods when interest rates vary a lot.

The paper is structured as follows. In Section 2, the No-Arbitrage Factor-Augmented VAR model is presented and its parametrization discussed. Section 3 describes the estimation of the model. In Section 4, I document the in-sample fit of the model and then discuss the results of the out-of-sample forecasts in Section 5. Section 6 concludes.

2 The Model

Economists typically think of the economy as being affected by monetary policy through the short term interest rate. On the other hand, the central bank is assumed to set interest rates in response to the overall state of the economy, characterized e.g. by the deviations of inflation and output from their desired levels. In a recent paper, Bernanke et al. (2005) point out that theoretical macroeconomic aggregates as output and inflation might not be perfectly observable neither to the policy-maker nor to the econometrician. More realistically, the observed macroeconomic time series will in general be noisy measures of broad economic concepts such as output and inflation. Accordingly, these variables should be treated as unobservable in empirical work so as to avoid confounding measurement error or idiosyncratic dynamics with fundamental economic shocks.

Bernanke et al. (2005) therefore suggest to extract a few common factors from a large number of macroeconomic time series variables and to study the mutual dynamics of monetary policy and the key economic aggregates by estimating a joint VAR of the factors and the policy instrument, an approach which they label "Factor-Augmented VAR" (FAVAR). This approach can be summarized by the following equations:

$$X_t = \Lambda_F F_t + \Lambda_r r_t + e_t \tag{1}$$

$$\begin{pmatrix} F_t \\ r_t \end{pmatrix} = \mu + \Phi(L) \begin{pmatrix} F_{t-1} \\ r_{t-1} \end{pmatrix} + \omega_t. \tag{2}$$

X_t denotes a $M \times 1$ vector of period-t observations of the observed macroeconomic variables, Λ_F and Λ_r are the $M \times k$ and $M \times 1$ matrices of factor loadings, r_t denotes the short-term interest rate, F_t is the $k \times 1$ vector of period-t observations of the common factors, and e_t is an $M \times 1$ vector of idiosyncratic components. Moreover, $\mu = (\mu_f', \mu_r)'$ is a $(k+1) \times 1$ vector of constants, $\Phi(L)$ denotes the $(k+1) \times (k+1)$ matrix of order-p lag polynomials and ω_t is a $(k+1) \times 1$ vector of reduced form shocks with variance covariance matrix Ω. Since affine term structure models are commonly formulated in state-space from, I rewrite the FAVAR in equation (2) as

$$Z_t = \boldsymbol{\mu} + \boldsymbol{\Phi} Z_{t-1} + \boldsymbol{\omega}_t, \tag{3}$$

where $Z_t = (F_t', r_t, F_{t-1}', r_{t-1}, \ldots, F_{t-p+1}', r_{t-p+1})'$, and where $\boldsymbol{\mu}, \boldsymbol{\Phi}$,and $\boldsymbol{\Omega}$ denote the companion form equivalents of μ, Φ, and Ω, respectively. Accordingly, the short rate r_t can be expressed in terms of Z_t as $r_t = \delta' Z_t$ where $\delta' = (0_{1\times k}, 1, 0_{1\times(k+1)(p-1)})$.

Adding the Term Structure

The term structure model which I suggest is built upon the assumption that yields are driven by movements of short term interest rate as well as the main shocks hitting the economy. The latter are proxied by the factors which capture the bulk of common variation in a large number of macroeconomic time series variables. The joint dynamics of these factors and the monetary policy instrument are modeled in a vector autoregression. I thus employ the FAVAR in equation (3) as the state equation of my term structure model. To make the model consistent with the assumption of no-arbitrage, I further impose restrictions on the parameters governing the impact of the state variables on the yields of different maturity. More precisely, I model the nominal pricing kernel as

$$\begin{aligned} M_{t+1} &= \exp(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\omega_{t+1}), \\ &= \exp(-\delta' Z_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\omega_{t+1}), \end{aligned} \tag{4}$$

where λ_t are the market prices of risk. Following Duffee (2002), these are commonly assumed to be affine in the underlying state variables Z, i.e.

$$\lambda_t = \lambda_0 + \lambda_1 Z_t. \tag{5}$$

In order to keep the model parsimonious, I restrict the prices of risk to depend only on contemporaneous observations of the model factors.[1] In an arbitrage-free market, the price of a n-months to maturity zero-coupon bond in period t must equal the expected discounted value of the price of an (n-1)-months to maturity bond in period $t+1$:

$$P_t^{(n)} = E_t[M_{t+1}\, P_{t+1}^{(n-1)}].$$

Assuming that yields are affine in the state variables, bond prices $P_t^{(n)}$ are exponential linear functions of the state vector:

$$P_t^{(n)} = \exp\left(A_n + B_n' Z_t\right),$$

where the scalar A_n and the coefficient vector B_n depend on the time to maturity n. Following Ang and Piazzesi (2003), I show in Appendix A that no-arbitrage is guaranteed by computing coefficients A_n and B_n according to the following recursive equations:

$$A_n = A_{n-1} + B_{n-1}'\left(\boldsymbol{\mu} - \boldsymbol{\Omega}\lambda_0\right) + \frac{1}{2}B_{n-1}'\boldsymbol{\Omega}B_{n-1}, \tag{6}$$

$$B_n = B_{n-1}'\left(\boldsymbol{\Phi} - \boldsymbol{\Omega}\lambda_1\right) - \delta'. \tag{7}$$

[1] Obviously, there is some arbitrariness in this restriction. In principle, one can also think of theoretical models that give rise to market prices of risk which depend on lagged state variables. However, since the dimensionality of the problem requires to make some identification restrictions, assuming that market prices of risk depend only on current observations of the states seems to be a plausible compromise.

Given the price of an n-months to maturity zero-coupon bond, the corresponding yield is thus obtained as

$$\begin{aligned} y_t^{(n)} &= -\frac{\log P_t^{(n)}}{n} \\ &= a_n + b_n' Z_t, \end{aligned} \tag{8}$$

where $a_n = -A_n/n$ and $b_n' = -B_n'/n$.

Altogether, the suggested model is completely characterized by equations (1), (3), (6), (7) and (8). In a nutshell, it is an essentially affine term structure model that has a FAVAR as the state equation. Accordingly, I will refer to my model as a "No-Arbitrage Factor-Augmented VAR" approach.

3 Estimation of the Model

In principle, the Factor-Augmented VAR model can be estimated using the Kalman filter and maximum likelihood. However, this approach becomes computationally infeasible when the number of macro variables stacked in the vector X is large. Bernanke et al. (2005) therefore discuss two alternative estimation methods: a single-step approach using Markov Chain Monte Carlo (MCMC) methods, and a two-step approach in which first principal components techniques are used to estimate the common factors F and then the parameters governing the dynamics of the state equation are obtained via standard classical methods for VARs. Comparing both methods in the context of an analysis of the effects of monetary policy shocks, Bernanke et al. (2005) find that the two-step approach yields more plausible results. Another advantage of this method is its computational simplicity. Since recursive out-of-sample yield forecasts are the main focus of this paper, I therefore employ the principal components approach in my application of the FAVAR model.

Accordingly, the common factors have to be extracted from the panel of macro data prior to estimating the term structure model. As in Bernanke et al., this is achieved using standard static principal components following the approach suggested by Stock and Watson (2002a,b). Precisely, let V denote the eigenvectors corresponding to the k largest eigenvalues of the $T \times T$ cross-sectional variance-covariance matrix XX' of the data. Then, subject to the normalization $F'F/T = I_k$, estimates $\hat{F}$ of the factors and $\hat{\Lambda}$ the factor loadings are given by

$$\begin{aligned} \hat{F} &= \sqrt{T}\, V \quad \text{and} \\ \hat{\Lambda} &= \sqrt{T}\, X'V, \end{aligned}$$

i.e. the common factors are estimated as the k largest eigenvalues of the variance-covariance matrix XX'.[2] In practice, the true number of common factors which capture the common variation in the panel X is not known. Bai and Ng (2002) have provided some panel information criteria which allow to consistently estimate the number of factors. In the application of the FAVAR approach suggested here, the number of factors that can feasibly be included in the model is limited due to computational constraints imposed by the market prices of risk parameters. I therefore fix the number of factors instead of applying formal model selection criteria.

Given the factor estimates, estimation of the term structure model is performed using the consistent two-step approach of Ang et al. (2006). First, estimates of the parameters $(\boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Omega})$ governing the dynamics of the model factors are obtained by running a VAR(p) on the estimated factors and the short term interest rate. Second, given the estimates from the first step, the parameters λ_0 and λ_1 which drive the evolution of the state prices of risk, are estimated by minimizing the sum of squared fitting errors of the model. That is, for a given set of parameter estimates $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Phi}}, \hat{\boldsymbol{\Omega}})$, the model-implied yields $\hat{y}_t^{(n)} = \hat{a}_n + \hat{b}_n' Z_t$ are computed and the sum S is minimized with respect to λ_0 and λ_1 where S is given by[3]

$$S = \sum_{t=1}^{T} \sum_{n=1}^{N} (\hat{y}_t^{(n)} - y_t^{(n)})^2. \tag{9}$$

Due to the recursive formulation of the bond pricing parameters, S is highly nonlinear in the underlying model parameters. It is thus helpful to find good starting values so as to achieve fast convergence. This is done as follows. I first estimate the parameters λ_0 assuming that risk premia are constant but nonzero, i.e. I set to zero all elements of the matrix λ_1 which governs the time-varying component of the market prices of risk. I then take these estimates of λ_0 as starting values in a second step that allows for time-varying market prices of risk, i.e. I let all elements of λ_0 and λ_1 be estimated freely. Standard errors of the prices of risk parameters are obtained by numerically computing the outer product of gradients estimate of their variance-covariance matrix. The standard errors of the state equation parameters are obtained from OLS.

[2] To account for the fact that r is an observed factor which is assumed unconditionally orthogonal to the unobserved factors F in the model (1), its effect on the variables in X has to be concentrated out prior to estimating F. This is achieved by regressing all variables in X onto r and extracting principal components from the residuals of these regressions.

[3] Note that the assumption that only contemporaneous factor observations affect the market prices of risk implies a set of zero restrictions on the parameters λ_0 and λ_1. In particular, $\lambda_0 = (\tilde{\lambda}_0', 0_{1\times(k+1)(p-1)})'$ and $\lambda_1 = \begin{pmatrix} \tilde{\lambda}_1 & 0_{(k+1)\times(k+1)(p-1)} \\ 0_{(k+1)(p-1)\times(k+1)} & 0_{(k+1)(p-1)\times(k+1)(p-1)} \end{pmatrix}$ where $\tilde{\lambda}_0$ is of dimension $(k+1)$ and $\tilde{\lambda}_1$ is a $(k+1)\times(k+1)$ matrix. Hence, in practice only $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ need to be estimated.

4 Empirical Results

4.1 Data

I estimate the model using the following data. The macroeconomic factors are extracted from a dataset which contains about 160 monthly time series of various economic categories for the US. Among others, it includes a large number of time series related to industrial production, more than 30 employment-related variables, around 30 price indices and various monetary aggregates. It further contains different kinds of survey data, stock indices, exchange rates etc. This dataset has been compiled by Giannone et al. (2004) to forecast US output, inflation, and short term interest rates. Notice that I exclude all interest rate related series from the original panel used by Giannone et al. The reason is that if the factors of my arbitrage-free model were extracted from a dataset containing yields, restrictions would have to be imposed on the factor loading parameters in (1) so as to make them consistent with the assumption of no-arbitrage. This would imply a non-trivial complication of the estimation process. Accordingly, I exclude the interest rate related series and thus implicitly assume that the central bank does not take into account the information contained in yields when setting the short term rate.

The principal components estimation of the common factors in large panels of time series requires stationarity. I therefore follow Giannone et al. (2004) in applying different preadjustments to the time series in the dataset.[4] Finally, I standardize all series to have mean zero and unit variance.

I use data on zero-coupon bond yields of maturities 1, 3, 6, and 9 months, as well as 1, 2, 3, 4, 5, 7, and 10 years. All interest rates are continuously-compounded unsmoothed Fama-Bliss yields and have been constructed from US treasury bonds using the method outlined in Bliss (1997). I estimate and forecast the model over the post-Volcker disinflation period, i.e. from 1983:01 to the last available observation of the macro dataset, 2003:09.

4.2 Model Specification

In the first step of the estimation procedure, I extract common factors from the large panel of macroeconomic time series using the principal components approach of Stock

[4] Though with a slight difference as regards the treatment of price series: instead of computing first differences of quarterly growth rates as in Giannone et al. (2004), I follow Ang and Piazzesi (2003) and compute annual inflation rates.

and Watson (2002a,b). Together, the first 10 factors explain about 70% of the total variance of all variables in the dataset. The largest contribution is accounted for by the first four factors, however, which together explain about 50% of the total variation in the panel. Table 1 lists the shares of variance explained by the first four factors as well as the time series in the panel that each of them is most strongly correlated with. Note however, that the factors estimated by principal components are only identified up to a non-singular rotation and therefore do not have a structural economic interpretation.

– Table 1 about here –

As already discussed above, the number of factors that can be included in the No-Arbitrage FAVAR model is limited due to parameterization constraints imposed by the market prices of risk specification. Indeed, unless further restrictions are imposed on the market prices of risk, the number of parameters to estimate in the second step of the estimation procedure increases quadratically with the number of factors. For the sake of parsimony, I therefore restrict the number of factors to the first four principal components extracted from the large panel of monthly time series and the short rate. Unreported results with smaller and larger number of factors have shown that this specification seems to provide the best tradeoff between estimability and model fit. A similar choice has to be made regarding the number of lags to include in the Factor-Augmented VAR which represents the state equation of my term structure model. Applying the Hannan-Quinn information criterion with a maximum lag of 12 months indicates an optimal number of four lags for the joint VAR of factors and the short rate. I therefore employ this particular specification for the in-sample estimation of the model. Note that in the recursive out-of-sample forecast exercise documented in Section 5, the lag length of the FAVAR is re-estimated each time a forecast is produced as it would have to be in the context of truly real-time predictions.

4.3 Preliminary Evidence

Before estimating the term structure model subject to no-arbitrage restrictions, I run a set of preliminary regressions to check whether the extracted macro factors are potentially useful explanatory variables in a term structure model. First, I use a simple encompassing test to assess whether a factor-based policy reaction function provides a better explanation of monetary policy decisions than a standard Taylor-rule based on individual measures of output and inflation. I then perform unrestricted regressions of yields on the model factors.

4.3.1 Test of "Excess Policy Response"

The use of the Factor-Augmented VAR pproach has been justified with the argument that central banks react to a large information set rather than to individual measures of output and inflation alone. Whether this conjecture holds true empirically can be tested by comparing the fit of a standard Taylor-rule policy reaction function with that of a policy reaction function based on dynamic factors. Bernanke and Boivin (2003) present evidence for what they call an "excess policy reaction" of the Fed by showing that the fitted value of the federal funds rate from a factor-based policy reaction function is a significant additional regressor in an otherwise standard Taylor-rule equation. Alternatively, one can separately estimate the two competing policy reaction functions and then perform an encompassing test *à* la Davidson and MacKinnon (1993). This is the strategy adopted by Belviso and Milani (2005). I follow these authors and compare a standard Taylor rule with partial adjustment,[5]

$$r_t = \rho r_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y y_t),$$

to a policy reaction function based on the four factors which represent state variables in the No-Arbitrage FAVAR model,

$$r_t = \rho r_{t-1} + (1-\rho)\phi_F' F_t.$$

The results from both regressions are summarized in Tables 2 and 3 in the appendix. As indicated by the regression R^2s of 0.967 and 0.970, the factor-based policy rule fits the data slightly better than the standard Taylor rule. The Davidson-MacKinnon (1993) encompassing test can be used to asses whether this improvement in model fit is statistically significant. I therefore regress the federal funds rate onto the fitted values from both alternative specifications which yields the following result:

$$\begin{aligned} r_t &= \underset{(0.186)}{0.207}\, \hat{r}_t^{Taylor} + \underset{(0.186)}{0.793}\, \hat{r}_t^{Factors} \end{aligned}$$

Hence, the coefficient on the standard Taylor rule is insignificant whereas the coefficient on the factor-based fitted federal funds rate is highly significant.[6] This result can be interpreted as evidence supporting the hypothesis that the Fed reacts to a broad macroeconomic information set.

5 Inflation π is defined as the annual growth rate of the GDP implicit price deflator (GDPDEF). The output gap is measured as the percentage deviation of log GDP (GDPC96) from its trend (computed using the Hodrick-Prescott filter and a smoothing parameter of 14400). Both quarterly series have been obtained from the St. Louis Fed website and interpolated to the monthly frequency using the method described in Mönch and Uhlig (2005). For the interpolation of GDP, I have used industrial production (INDPRO), total civilian employment (CE16OV) and real disposable income (DSPIC96) as related monthly series. CPI and PPI finished goods have been employed as related series for interpolating the GDP deflator.

6 Unreported results have shown that this is robust to alternative specifications of both reaction functions using a larger number of lags of the policy instrument and the macro variables or factors.

4.3.2 Unrestricted Estimation of the Term Structure Model

To get a first impression whether the factors extracted from the panel of macro variables also capture predictive information about interest rates of higher maturity, Table 4 summarizes the correlations between the yields and various lags of the factors of the No-Arbitrage FAVAR model. This table shows that the short rate is most strongly correlated with yields of any other maturity. Yet, the four macro factors extracted from the panel of monthly US time series also exhibit some correlation with yields. While the short rate is contemporaneously most strongly correlated with yields, the correlations between macro factors and yields tend to be higher for longer lags. This indicates that the factors extracted from the panel of macro data might be useful for forecasting interest rates.

– Table 4 about here –

To further explore the question whether the models factors have explanatory power for yields, Table 5 provides estimates of an unrestricted VAR of yields of different maturities onto a constant, the four macro factors and the federal funds rate, i.e. it estimates the pricing equation for yields,

$$Y_t = A + BZ_t + u_t,$$

where no cross-equation restrictions are imposed on the coefficients A and B. The first observation to make is that the R^2 of these regressions are all very high. Together with the short rate, the four factors explain more than 95% of the variation in short yields, and still more than 85% of the variation in longer yields. Not surprisingly, the federal funds rate is the most highly significant explanatory variable for short maturity yields. However, in the presence of the macro factors its impact decreases strongly towards the long end of the maturity spectrum. This shows that the factors extracted from the large panel of macro variables exhibit strong explanatory power for longer yields and thus represent potentially useful state variables in a term structure model.

– Table 5 about here –

4.4 Estimating the Term Structure Model

4.4.1 In-Sample Fit

In this section, I report results obtained from estimating the FAVAR model subject to the cross-equation restrictions (6) and (7) implied by the no-arbitrage assumption as outlined in Section 2. The model fits the data surprisingly well, given that it does not make use of latent yield curve factors. Table 6 reports the first and second moments of observed and model-implied yields and one-year holding period returns, respectively. These numbers indicate that on average the No-Arbitrage FAVAR model fits the yield curve almost exactly. Figure 1 provides a visualization of this result by showing average observed and model-implied yields across the entire maturity spectrum. Notice that the model seems to be missing some of the variation in longer maturities since the standard deviations of fitted interest rates are slightly lower than the standard deviations of the observed yields, especially at the long end of the curve. This can also be seen in Figure 2 which plots the time series for a selection of observed and model-implied yields.

– Table 6 about here –

Overall, the No-Arbitrage FAVAR model is able to capture the cross-sectional variation of government bond yields quite well, with a slightly better in-sample fit at the short end of the curve. As we will see further below, this has an impact also on the forecast results obtained from the model. Indeed, the improvement over latent-factor based term structure models is more pronounced at the short than at the long end of the yield curve. Yet, as has been discussed above, estimating a TSM without latent yield factors considerably facilitates estimation of the model and thus makes recursive out-of-sample forecasts feasible.

– Figure 1 about here –

4.4.2 Parameter Estimates

Table 7 in the appendix reports the parameter estimates and associated standard errors of the No-Arbitrage FAVAR model. The upper panel shows parameter estimates of the Factor-Augmented VAR that represents the state equation of the model, the second panel provides the estimates of the state prices of risk which constitute the remaining components of the recursive bond pricing parameters A and B. A noticeable feature of the FAVAR estimates is that most of the off-diagonal elements of the lags of the coefficient matrix Φ are insignificant. Hence, in addition to the unconditional orthogonality of the model factors that is due to the estimation by principal components, there is also little conditional correlation between the factors of the FAVAR model.

As the second panel of Table 7 shows, all elements of the vector $\tilde{\lambda}_0$ governing the unconditional mean of the market prices of risk are large in absolute terms and highly significant. This suggests that risk premia are characterized by a large constant component. As indicated by the size and significance of the estimates $\tilde{\lambda}_1$, there is also some significant amount of time variation in risk premia over the sample period considered. It is difficult to interpret individual elements in the estimated prices of risk matrix, however. Indeed, unreported results from alternative model specifications varying e.g. the number of factors, the number of lags in the state equation or the sample period, have shown that the price of risk estimates are quite sensitive to changes in model specification. Hence, economic reasoning based on the significance of individual parameters governing the state prices of risk is unwarranted. Instead, in order to visualize the relation between risk premia and the model factors, Figure 3 provides a plot of model-implied term premia for the 1-year and 5-year yield. As indicated by these plots, term premia at the short end of the yield curve are inversely related to the first macro factor which is itself highly correlated with output variables. By contrast, premia for longer yields are more closely related to the second factor which is strongly correlated with inflation indicators.

– Figure 3 about here –

Figure 4 shows a plot of the loadings b_n of the yields onto the contemporaneous observations of the model factors. The signs of these loadings are consistent with those obtained from regressing yields onto the model factors without imposing no-arbitrage restrictions, summarized in Table 5. By construction of my arbitrage-free model, the loading of the 1-month yield onto the short rate factor equals unity and those for the macro factors are zero. However, the impact of the short rate on longer yields sharply decreases with maturity. Hence, movements in the short-term interest rate only have a relatively small impact on long-term interest rates. Instead, these are more strongly driven by the macroeconomic factors. Most importantly, the first factor has an equally strong impact on yields of medium and longer maturities. Interestingly, shocks to the third macro factor appear to have a negative effect on yields of very short maturity and an increasingly strong positive impact on medium-term and long-term rates. This indicates that negative shocks to the third macro factor imply a flattening of the yield curve that is commonly associated with an upcoming recession.

– Table 5 about here –

4.5 How are the Macro Factors Related to the Components of the Yield Curve?

In traditional term structure analysis, the yield curve is often decomposed into three factors which together explain almost all of the cross-sectional variation of interest rates. According to their impact on the shape of the term structure, these components are commonly labeled level, slope, and curvature. Since the No-Arbitrage FAVAR model has been shown to explain yields relatively well in-sample, it is interesting to relate the macro factors used in the model to the level, slope, and curvature components of the yield curve. In this section, I thus regress estimates of the latent yield factors onto the macro factors and the short rate. The yield factors are computed as the first three principal components of the yields used to estimate the term structure model. Consistent with results reported in previous studies, the first three principal components explain about 90.8%, 6.4% and 1.6% of the total variance of all yields.

– Table 8 about here –

Table 8 summarizes the results of these regressions. The four macro factors and the short-term interest rate explain almost all of the variation in the yield level which captures the most important source of common variation of interest rates. The main contribution comes from the short rate and the first and third macro factor which are correlated with output and inflation-related variables, respectively. Almost 80% of the variation in the slope of the yield curve is explained by the factors of the FAVAR model. Again, the short rate as well as the first and third macro factors are most strongly correlated with the slope. The short rate has a strongly significant negative coefficient in the slope equation which is consistent with the common view that rises in the short rate lead to a decreasing yield curve slope. Finally note that only about 35% of the variation in the curvature of the yield curve are explained by the macro factors. Hence, variations in the relative size of short, medium and long-term yields seem to be the least related to macroeconomic news.

5 Out-of-Sample Forecasts

The term structure model suggested in this paper is based on the idea that the Federal Reserve uses a large set of conditioning information when setting short-term interest rates and that the FAVAR approach suggested by Bernanke et al. (2005) represents a useful way of capturing this information. Although economically appealing, the model is not structural and should therefore not be used for a qualitative analysis of the economic driving forces behind the yield curve. Accordingly, this paper focuses on the usefulness of the No-Arbitrage FAVAR model for predicting the term structure of interest rates.

In the previous section, it has been shown that the model provides a reasonably good in-sample fit to US yield data. In this section, I study the forecast performance of the No-Arbitrage FAVAR model in a recursive out-of-sample prediction exercise.

Model-implied forecasts are obtained using the following formula:

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}_{t+h|t}, \tag{10}$$

where Z contains the contemporaneous and lagged observations of the short-term interest rate and the four factors explaining the bulk of variation in the panel of monthly time series for the US. The coefficients $\hat{a}_n$ and $\hat{b}_n$ are computed according to equations (6) and (7), using as input the estimates $\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Phi}}$, and $\hat{\boldsymbol{\Sigma}}$ obtained by running a VAR on the states, as well as the estimates $\hat{\lambda}_0$ and $\hat{\lambda}_1$ that result from minimizing the sum of squared fitting errors of the model. Forecasts $\hat{Z}_{t+h|t}$ are obtained from the FAVAR according to

$$\hat{Z}_{t+h|t} = \hat{\boldsymbol{\Phi}}^h Z_t + \sum_{i=0}^{h-1} \hat{\boldsymbol{\Phi}}^i \hat{\boldsymbol{\mu}}. \tag{11}$$

5.1 The Competitor Models

I compare the model's forecast performance to that of several competitor models. In particular, these are a No-Arbitrage Macro VAR model, an unrestricted VAR on yield levels, two different specifications of the Nelson-Siegel (1987) three-factor model recently suggested by Diebold and Li (2006), an essentially affine latent yield factor model $A_0(3)$, and the random walk. The latter three models are expected to be the most challenging competitors. Diebold and Li have shown the Nelson-Siegel model to outperform a variety of alternative yield forecasting models. Duffee (2002) has documented strong out-of-sample forecast performance for the essentially affine latent yield factor model. Finally, the random walk is often reported to be difficult to beat in out-of-sample forecasts of interest rates. In the following, I briefly sketch the individual competitor forecasting models.

5.1.1 No-Arbitrage Macro VAR Model

In order to analyze whether the good forecast performance of the No-Arbitrage FAVAR model can be attributed to the large set of conditioning information incorporated by the model, I compare it to a model that uses individual macroeconomic indicators instead of factors extracted form a large data panel as state variables. In particular, I compare it to a model that has a VAR in the short rate and four measures of output and inflation as the state equation but that is otherwise identically specified. Precisely, I obtain yield forecasts according to

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}^{VAR}_{t+h|t}$$

where Z^{VAR} contains the quarterly growth rate of IP, the help-wanted index, the annual growth rates of CPI and PPI, and the 1-month yield. The coefficients $\hat{a}_n$ and $\hat{b}_n$ are obtained from equations (6) and (7) and guarantee the absence of arbitrage opportunities. Forecasts $\hat{Z}_{t+h|t}^{VAR}$ are computed as in (11). The No-Arbitrage Macro VAR model is denoted "VAR" in the tables below.

5.1.2 VAR(1) on Yield Levels

In this model, forecasts of yields are obtained according to

$$\hat{y}_{t+h|t} = \hat{c} + \hat{\Gamma} y_t,$$

where $\hat{c}$ and $\hat{\Gamma}$ are estimated by regressing the vector y_t onto a constant and its h-months lag. This model is referred to as "VARylds" in the results below.

5.1.3 Diebold-Li Specification of the Nelson-Siegel Model

In a recent paper, Diebold and Li (2006) have suggested a dynamic version of the traditional Nelson-Siegel(1987) decomposition of yields and have shown that this model provides superior yield forecasts with respect to a number of benchmark approaches. According to this model, yields are decomposed into three factors with loadings given by exponential functions of the time to maturity n and some shape parameter τ. Precisely, Diebold and Li suggest to obtain yield forecasts according to

$$\hat{y}_{t+h|t}^{(n)} = \hat{\beta}_{1,t+h|t} + \hat{\beta}_{2,t+h|t} \left(\frac{1 - e^{-\tau n}}{\tau n} \right) + \hat{\beta}_{3,t+h|t} \left(\frac{1 - e^{-\tau n}}{\tau n} - e^{-\tau n} \right)$$

where

$$\hat{\beta}_{t+h|t} = \hat{c} + \hat{\Gamma} \hat{\beta}_t$$

Diebold and Li (2006) obtain initial estimates of the factors β by regressing yields onto the loadings $\left(1, (\frac{1-e^{-\tau n}}{\tau n}), (\frac{1-e^{-\tau n}}{\tau n} - e^{-\tau n})\right)$ for a fixed value of τ. They set $\tau = 0.0609$ which is the value that maximizes the curvature loading at the maturity of 30 months. Diebold and Li consider two different specifications of their model, one where the factor dynamics are estimated by fitting AR(1) processes and another where the factors follow a VAR(1). In my application of their model, I report results for both specifications. These are denoted as "NS(VAR)" and "NS(AR)", respectively.

5.1.4 Essentially Affine Latent Yield Factor Model $A_0(3)$

This is a traditional affine model where all the factors are latent and have to be estimated from the yield data. I implement the preferred essentially affine $A_0(3)$ specification of Duffee (2002) who has shown that this model provides superior out-of-sample forecast results with respect to various other affine specifications. The specification of the market prices of risk is therefore similar to the No-Arbitrage FAVAR model. Within the $A_0(3)$ model, yield forecasts are obtained as

$$\hat{y}^{(n)}_{t+h|t} = \hat{a}_n + \hat{b}_n \hat{Z}^{A_0(3)}_{t+h|t}$$

where $Z^{A_0(3)}$ is composed of three latent yield factors, backed out from the yields using the method by Chen and Scott (1993). In particular, I assume that the 1-month, 1-year and 10-year yield are observed without error. Moreover, the transition matrix Φ in the state equation is assumed to be lower-triangular and the variance-covariance matrix Ω to be an identity matrix so as to ensure exact identification of the model (see Dai and Singleton 2000). Notice that since the latent factors need to be backed out from the yields, estimation of the model takes considerably longer than estimation of the No-Arbitrage FAVAR and VAR models where the parameters of the state equation are estimated in a first stage of the estimation via OLS.

5.1.5 Random Walk

Assuming a random walk model for interest rates implies a simple "no-change" forecast of individual yields. Hence, in this model the h-months ahead prediction of an n-maturity bond yield in period t is simply given by its time t observation:

$$\hat{y}^{(n)}_{t+h|t} = y^{(n)}_t$$

5.2 Out-of-Sample Forecasts

The out-of-sample forecasts are carried out over the time interval 1994:01-2003:09. The forecast period therefore covers a period of almost ten years. The affine models are first estimated over the period 1983:01-1993:12 to obtain starting values for the parameters. All models are then estimated recursively using data from 1983:01 to the time that the forecast is made, beginning in 1994:01.

Table 9 summarizes the root mean squared errors obtained from these forecasts. Three main observations can be made. First, the No-Arbitrage FAVAR model clearly outperforms the No-Arbitrage Macro VAR model except for very short maturities at the 1-month ahead horizon. This implies strong support for the use of a broad macroeconomic information set when forecasting the yield curve based on macroeconomic variables. Second, at the 1-month ahead horizon, the VAR(1) in yield levels and the random walk outperform the macro-based FAVAR and VAR models for yields of all maturities, with the random walk being slightly superior for long yields and the *VARylds* model performing best for short and medium-term maturities. Third and most importantly, however, the No-Arbitrage FAVAR model outperforms all considered benchmark models in yield forecasts 6-months and 12-months ahead. As the first column of panels B and C of Table 9 documents, the FAVAR model implies smaller out-of-sample root mean squared forecast errors than the benchmark models across except for very long maturities for which the essentially affine latent yield factor model $A_0(3)$ performs best.

– Table 9 about here –

Interestingly, both specifications of the Nelson-Siegel model considered in Diebold and Li (2006) are outperformed by the No-Arbitrage FAVAR model. This is striking since Diebold and Li have documented their approach to be particularly good at forecasting. This indicates that the combination of a large information set, the rich dynamics of the FAVAR, and the parameter restrictions implied by no-arbitrage together result in a model which is particularly useful for out-of-sample predictions. In the subsample analysis carried out in the next section, I will have a closer look at these results.

Table 10 reports RMSEs of all considered models relative to the random walk forecast. These results show that the improvement in terms of root mean squared forecast errors implied by the FAVAR model is particularly pronounced for short and medium term maturities. At the one-month forecast horizon, all yield-based models outperform the affine models based on macro variables. However, at forecast horizons beyond one month, the No-Arbitrage FAVAR model outperforms all other models for maturities from one month to five years. Relative to the random walk, the No-Arbitrage FAVAR model reduces root mean squared forecast errors up to 30% at the short end of the yield curve and still improves forecast performance of medium-term yields about 15%. Compared to the best performing competitor model, the essentially affine latent factor model $A_0(3)$, the improvement is still remarkable.

– Table 10 about here –

One can formally assess whether the improvement of the FAVAR model over the benchmark models in terms of forecast error is significant by applying White's (2000) "reality check" test. This test uses bootstrap resamples of the forecast error series to derive the empirical distribution of the forecast loss differential of a model with respect to some benchmark model. It can thus be used to evaluate superior predictive ability of a model with respect to one or more competitor models. Here, I test whether the No-Arbitrage FAVAR model has superior predictive accuracy with respect to the five considered competitors. The test statistics are reported in Table 11. Negative numbers indicate that the average squared forecast loss of the No-Arbitrage FAVAR model is smaller than that of the respective competitor model while positive test statistics indicate the opposite. I perform 1,000 block-bootstrap resamples from the prediction error series to compute the significance of the forecast improvement at the 5% level which are indicated by bold figures. As the results in panels B and C of Table 11 show, the documented improvement in terms of root mean squared forecast errors is significant at the 5% level for all but very long maturities at forecast horizons of 6-months and 12-months ahead. This underscores the observation made above that the No-Arbitrage FAVAR model predicts yields considerably better than all studied competitor models, including the Nelson-Siegel model and the $A_0(3)$ model.

– Table 11 about here –

5.3 Subsample Analysis of Forecast Performance

The results documented in the previous section show that the No-Arbitrage FAVAR model exhibits strong relative advantages over a variety of benchmark models which have been documented powerful tools in forecasting the yield curve. This result somewhat challenges the recent findings of Diebold and Li (2006) and therefore a closer look at the forecast performances of the different models is warranted. In this section, I thus perform a subsample analysis of the out-of-sample prediction results. In particular, I analyze the relative performance of the No-Arbitrage FAVAR model with respect to the Nelson-Siegel model over exactly the sample period that has been studied by Diebold and Li (2006).

– Table 12 about here –

Table 12 provides the root mean squared forecast errors of the different models for the out-of-sample prediction period 1994:01-2000:12. At the 1-month ahead horizon, both specifications of the Nelson-Siegel model outperform the other models except for the 5-year yield that is best predicted by the random walk. The absolute size of the RMSEs is very similar to those documented by Diebold and Li (2006). For example, based on the NS(AR) model Diebold and Li report RMSEs of 0.236, 0.292, and 0.260 for the 1-year, 5-year and 10-year yields at the 1-month ahead horizon whereas I find values of 0.249, 0.280, and 0.249, respectively, for the same maturities. The small deviations are likely due to differences in the choice of data and the set of maturities used to estimate the models. Turning to the results for 6-months ahead predictions, the picture becomes less favorable for the Nelson-Siegel model. Only for the 1-month yield, the VAR specification of the Nelson-Siegel model performs best. In contrast, the No-Arbitrage FAVAR model outperforms all other models for the range of maturities between 6-months and 5-years. Again, the absolute size of the RMSEs found here is very similar to those reported by Diebold and Li. For example, while they document

RMSEs of 0.669, 0.777, and 0.721 for the 1-year, 5-year and 10-year yields, I find values of 0.711, 0.764, and 0.694, respectively. The results again change somewhat if one considers 12-months ahead predictions for the sample period studied in Diebold and Li (2006). In this case, there appears to be a clearer advantage of their preferred NS(AR) specification which outperforms all other models except for the 6-months and 10-year maturities.

To visualize these results, Figures 5 to 7 show the actual yields and those predicted by the No-Arbitrage FAVAR, the NS(AR), and the $A_0(3)$ model for some selected maturities. Figure 5 plots the outcomes for the 1-month ahead forecast horizon. According to this, the NS(AR) and the $A_0(3)$ model forecast the persistent movements of yields quite well while the FAVAR model predicts more variation than actual yields exhibit. This confirms the relatively poor predictive ability of the model at very short forecast horizons documented above. Yet, at the 6-months ahead forecast horizon the picture looks strikingly different. In particular, as Figure 6 shows, the No-Arbitrage FAVAR model predicts the surge of yields in 1999 and 2000 quite well. More impressively, it forecasts the strong decline of yields starting towards the end of 2000 very precisely. By contrast, both the NS(AR) and the $A_0(3)$ models miss the particular dynamics in this episode by a few months. Although less pronounced, a similar pattern can be seen for the 12-months ahead forecasts, provided in Figure 7.

– Figures 5 to 7 about here –

Altogether, these results show that the No-Arbitrage FAVAR model performs particularly well compared to yield-based prediction models when interest rates exhibit strong variation. To provide a more quantitative assessment of this finding, Table 13 displays the root mean squared forecast errors of the different models for the subperiod 2000:01-2003:09. As can be seen from the plots above, this period was characterized by an initial surge of yields which was then followed by a sharp and persistent decline of interest rates of all maturities. The results of Table 13 show that over this particular sample period, the No-Arbitrage FAVAR model strongly outperforms all competitor models at forecast horizons 6-months and 12-months ahead. More precisely, the reduction in RMSEs relative to the random walk amounts to a striking 50% for short and medium-term maturities.

– Table 13 about here –

In sum, the results of my subsample analysis show that the strong forecast performance of the Nelson-Siegel model documented by Diebold and Li is partly due to their choice of forecast period. In addition, the superior predictive ability of the model partly vanishes when confronted with the No-Arbitrage FAVAR model. The latter strongly outperforms all benchmark models in periods when interest rates move a lot.

6 Conclusion

This paper presents a model of the term structure based on the idea that the central bank uses a large set of conditioning information when setting the short term interest rate and that this information can be summarized by a few factors extracted from a large panel of macroeconomic time series. Precisely, the Factor-Augmented VAR (FAVAR) approach suggested by Bernanke et al. (2005) is used to model the dynamics of the short-term interest rate. Starting from this characterization of the short rate, the term structure is then built up using restrictions implied by no-arbitrage. This setup is labeled a "No-Arbitrage Factor-Augmented VAR" approach. In contrast to previously proposed macro-finance models of the term structure, the model suggested in this paper does not contain latent yield factors, but is entirely built upon observable macroeconomic information.

Fitting the model to US data, I document that it explains the dynamics of yields quite well. This underlines that no latent yield factors are needed to capture most of the variation of interest rates. Most importantly, I find that the No-Arbitrage FAVAR model exhibits a strikingly good ability to predict the yield curve. In a recursive out-of-sample forecast exercise, the model is shown to outperform various benchmarks including the essentially affine three factor model of Duffee (2002) and the dynamic variant of the Nelson-Siegel model that Diebold and Li (2006) have recently suggested as a prediction model. A subsample analysis of the forecast results documents that the No-Arbitrage FAVAR model performs particularly well in periods when interest rates vary a lot.

Based on the findings of the paper, there are a number of interesting directions for future research. First, while this paper has focused on the predictive ability of the No-Arbitrage FAVAR approach, the model can also be used for structural economic analysis. For example, it would be interesting to identify monetary policy shocks as in Bernanke et al. (2005) and study their impact on the yield curve. Second, based on estimates of term premia, one could use the model to analyze the risk-adjusted expectations of future monetary policy conditional on all macro information available. Finally, estimating the model using a one-step likelihood based Bayesian approach, one could add latent yield factors and assess to what extent these add explanatory and predictive power to the model.

References

ANG, ANDREW AND MONIKA PIAZZESI (2003): "A No-Arbitrage Vector Autoregression of Term Structure Dynamics With Macroeconomic and Latent Variables", *Journal of Monetary Economics*, 50(4), 745–787.

ANG, ANDREW, MONIKA PIAZZESI AND MIN WEI (2006): "What Does the Yield Curve Tell Us About GDP Growth?", *Journal of Econometrics*, 127(1-2), 359–403.

BAI, JUSHAN AND SERENA NG (2002): "Determining the Number of Factors in Approximate Factor Models", *Econometrica*, 70(1), 191–221.

BELVISO, FRANCESCO AND FABIO MILANI (2005): "Structural Factor-Augmented VAR (SFAVAR) and the Effects of Monetary Policy", *mimeo*, Princeton University.

BERNANKE, BEN, JEAN BOIVIN AND PIOTR S. ELIASZ (2005): "Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach", *The Quarterly Journal of Economics*, 120(1), 387–422.

BERNANKE, BEN S. AND JEAN BOIVIN (2003): "Monetary Policy in a Data-Rich Environment", *Journal of Monetary Economics*, 50(3), 525–546.

BLISS, ROBERT R. (1997): "Testing Term Structure Estimation Methods", *Advances in Futures and Options Research*, 9, 197–231.

CHEN, REN-RAW AND LOUIS SCOTT (1993): "Maximum Likelihood Estimation for a Multi-Factor Equilibrium Model of the Term Structure of Interest Rates", *Journal of Fixed Income*, 3, 14–31.

DAI, QIANG AND KENNETH J. SINGLETON (2000): "Specification Analysis of Affine Term Structure Models", *Journal of Finance*, 55(5), 1943–1978.

DAVIDSON, RUSSELL AND JAMES G. MACKINNON (1993): *Estimation and Inference in Econometrics*. Oxford University Press, New York.

DEWACHTER, HANS AND MARCO LYRIO (2006): "Macro Factors and the Term Structure of Interest Rates", *Journal of Money, Credit, and Banking*, 38(1), 119–140.

DIEBOLD, FRANCIS X. AND CANLIN LI (2006): "Forecasting the Term Structure of Government Bond Yields", *Journal of Econometrics*, 127(1-2), 337–364.

DIEBOLD, FRANCIS X., GLENN D. RUDEBUSCH AND BORAGAN S. ARUOBA (2006): "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach", *Journal of Econometrics*, 127(1-2), 309–338.

DUFFEE, GREGORY R. (2002): “Term Premia and Interest Rate Forecasts in Affine Models”, *Journal of Finance*, 57(1), 405–443.

FAVERO, CARLO A., MASSIMILIANO MARCELLINO AND FRANCESCA NEGLIA (2005): “Principal Components at Work: The Empirical Analysis of Monetary Policy with Large Data Sets”, *Journal of Applied Econometrics*, 20(5), 603–620.

GIANNONE, DOMENICO, LUCREZIA REICHLIN AND LUCA SALA (2004): “Monetary Policy in Real Time”, NBER Macroeconomics Annual, 161–225.

HÖRDAHL, PETER, ORESTE TRISTANI AND DAVID VESTIN (2006): “A Joint Econometric Model of Macroeconomic and Term-Structure Dynamics”, *Journal of Econometrics*, 127(1-2), 405–444.

KNEZ, PETER J., ROBERT LITTERMAN AND JOSÉ A. SCHEINKMAN (1994): “Explorations into Factors Explaining Money Market Returns”, *Journal of Finance*, 49(5), 1861–1882.

MÖNCH, EMANUEL AND HARALD UHLIG (2005): “Towards a Monthly Business Cycle Chronology for the Euro Area”, *Journal of Business Cycle Measurement and Analysis*, 2(1), 43–69.

NELSON, CHARLES R. AND ANDREW F. SIEGEL (1987): “Parsimonious Modeling of Yield Curves”, *Journal of Business*, 60(4), 473–489.

STOCK, JAMES H. AND MARK W. WATSON (2002a): “Macroeconomic Forecasting Using Diffusion Indexes”, *Journal of Business & Economic Statistics*, 20(2), 147–162.

——— (2002b): “Forecasting Using Principal Components From a Large Number of Predictors”, *Journal of the American Statistical Association*, 97, 1167–1179.

WHITE, HALBERT (2000): “A Reality Check for Data Snooping”, *Econometrica*, 68(5), 1097–1126.

A Derivation of the Bond Pricing Parameters

The absence of arbitrage between bonds of different maturity implies the existence of the stochastic discount factor M such that

$$P_t^{(n)} = E_t[M_{t+1}\ P_{t+1}^{(n-1)}],$$

i.e. the price of a n-months to maturity bond in month t must equal the expected discounted price of an $(n-1)$-months to maturity bond in month $(t+1)$. Following Ang and Piazzesi (2003), the derivation of the recursive bond pricing parameters starts with assuming that the nominal pricing kernel M takes the form

$$M_{t+1} = \exp(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - \lambda_t'\boldsymbol{\omega}_{t+1})$$

and by guessing that bond prices P are exponentially affine in the state variables Z, i.e.

$$P_t^{(n)} = \exp(A_n + B_n' Z_t).$$

Plugging the above expressions for P and M into the first relation, one obtains

$$\begin{aligned} P_t^{(n)} &= E_t[M_{t+1}\ P_{t+1}^{(n-1)}] \\ &= E_t\left[\exp(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - \lambda_t'\boldsymbol{\omega}_{t+1})\exp(A_{n-1} + B_{n-1}'Z_{t+1})\right] \\ &= \exp(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t + A_{n-1})\ E_t\left[\exp(-\lambda_t'\boldsymbol{\omega}_{t+1} + B_{n-1}'(\boldsymbol{\mu} + \mathbf{\Phi}Z_t + \boldsymbol{\omega}_{t+1}))\right] \\ &= \exp(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t + A_{n-1} + B_{n-1}'\boldsymbol{\mu} + B_{n-1}'\mathbf{\Phi}Z_t)E_t\left[\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1})\right] \end{aligned}$$

Since the innovations $\boldsymbol{\omega}$ of the state variable process are assumed Gaussian with variance-covariance matrix $\mathbf{\Omega}$, it is obvious that

$$\begin{aligned} \ln E_t\left[\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1})\right] &= E_t\left[\ln(\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}))\right] + \\ &\quad \frac{1}{2}Var_t\left(\ln(\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1}))\right) \\ &= \frac{1}{2}\left[\lambda_t'\mathbf{\Omega}\lambda_t - 2B_{n-1}'\mathbf{\Omega}\lambda_t + B_{n-1}'\mathbf{\Omega}B_{n-1}\right] \\ &= \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - B_{n-1}'\mathbf{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\mathbf{\Omega}B_{n-1}. \end{aligned}$$

Hence, $E_t\left[\exp((-\lambda_t' + B_{n-1}')\boldsymbol{\omega}_{t+1})\right] = \exp(\frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - B_{n-1}'\mathbf{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\mathbf{\Omega}B_{n-1})$ and thus

$$\begin{aligned} P_t^{(n)} &= \exp(-r_t - \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t + A_{n-1} + B_{n-1}'\boldsymbol{\mu} + B_{n-1}'\mathbf{\Phi}Z_t + \dots \\ &\quad + \frac{1}{2}\lambda_t'\mathbf{\Omega}\lambda_t - B_{n-1}'\mathbf{\Omega}\lambda_t + \frac{1}{2}B_{n-1}'\mathbf{\Omega}B_{n-1}). \end{aligned}$$

Using the relations $r_t = \delta' Z_t$ and $\lambda_t = \lambda_0 + \lambda_1 Z_t$, and matching coefficients finally yields

$$P_t^{(n)} = \exp(A_n + B_n' Z_t),$$

where

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}'(\boldsymbol{\mu} - \boldsymbol{\Omega}\lambda_0) + \frac{1}{2} B_{n-1}' \boldsymbol{\Omega} B_{n-1}, \\ \text{and} \quad B_n &= B_{n-1}'(\boldsymbol{\Phi} - \boldsymbol{\Omega}\lambda_1) - \delta'. \end{aligned}$$

These are the recursive equations of the pricing parameters stated in (6)-(7).

B Tables and Figures

Table 1: **Share of Variance Explained by Factors and Factor Loadings**

This table summarizes R-squares of univariate regressions of the factors extracted from the panel of macro variables on all individual variables. For each factor, I list the five variables that are most highly correlated with it. Notice that the series have been transformed to be stationary prior to extraction of the factors, i.e. for most variables the regressions correspond to regressions on growth rates. The four factors together explain about 50% of the total variation of the time series in the panel.

Factor 1 (25.1% of total variance)	R^2
Index of IP: Total	0.84
Index of IP: Non-energy, total (NAICS)	0.84
Index of IP: Mfg (SIC)	0.84
Capacity Utilization: Total (NAICS)	0.81
Index of IP: Non-energy excl CCS (NAICS)	0.80
Factor 2 (10.9% of total variance)	
CPI: all items less medical care	0.85
CPI: commodities	0.83
CPI: all items (urban)	0.83
CPI: all items less shelter	0.82
CPI: all items ess food	0.79
Factor 3 (7.8% of total variance)	
CPI: medical care	0.66
PCE prices: total excl food and energy	0.48
PCE prices: services	0.45
M1 (in mil of current $)	0.39
Loans and Securities @ all comm banks: Securities, U.S. govt (in mil of $)	0.37
Factor 4 (5.0% of total variance)	
Employment on nonag payrolls: Financial activities	0.27
Employment on nonag payrolls: Other services	0.23
Employment on nonag payrolls: Service-producing	0.19
Employment on nonag payrolls: Mining	0.18
Employment on nonag payrolls: Retail trade	0.17

Table 2: **Policy Rule Based on Individual Variables**

This table reports estimates for a policy rule with partial adjustment based on individual measures of output and inflation, i.e.

$$r_t = c + \rho r_{t-1} + (1-\rho)(\phi_y y_t + \phi_\pi \pi_t),$$

where r denotes the federal funds rate, y the deviation of log GDP from its trend, and π the annual rate of GDP inflation. The sample period is 1983:01 to 2003:09. Standard errors are in parentheses. The R^2 of this regression is 0.967.

c	ρ	ϕ_y	ϕ_π
-0.011	0.955	1.332	2.592
(0.078)	(0.017)	(0.627)	(0.850)

Table 3: **Policy Rule Based on Factors**

This table reports estimates for a policy rule with partial adjustment based on the four factors extracted from a large panel of macroeconomic variables, i.e.

$$r_t = c + \rho r_{t-1} + (1-\rho)(\phi_{F1} F1_t + \phi_{F2} F2_t + \phi_{F3} F3_t + \phi_{F4} F4_t),$$

where r again denotes the federal funds rate and $F1$ to $F4$ the four macro factors extracted from a panel of about 160 monthly time series for the US. The sample period is 1983:01 to 2003:09. Standard errors are in parentheses. The R^2 of this regression is 0.97.

c	ρ	ϕ_{F1}	ϕ_{F2}	ϕ_{F3}	ϕ_{F4}
0.198	0.957	0.115	0.076	-0.008	0.006
(0.088)	(0.016)	(0.025)	(0.031)	(0.025)	(0.026)

Table 4: **Correlation of Model Factors and Yields**

This table summarizes the correlation patterns between the yields and factors used for estimating the term structure model. $F1, F2, F3$ and $F4$ denote the macro factors extracted form the large panel of monthly economic time series for the US, $y^{(1)}$ to $y^{(120)}$ denote the yields of maturities 1-month to 10-years, respectively.

	$y^{(1)}$	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
Panel A: Contemporaneous Correlation of Factors and Yields						
F1	0.243	0.318	0.351	0.382	0.389	0.379
F2	0.597	0.619	0.617	0.570	0.546	0.537
F3	0.150	0.153	0.161	0.270	0.340	0.407
F4	0.315	0.325	0.331	0.354	0.365	0.380
$y^{(1)}$	1.000	0.987	0.975	0.936	0.899	0.833
Panel B: Correlation of 1 month Lagged Factors and Yields						
F1(-1)	0.296	0.365	0.393	0.409	0.409	0.393
F2(-1)	0.600	0.614	0.610	0.564	0.539	0.531
F3(-1)	0.145	0.152	0.161	0.269	0.342	0.411
F4(-1)	0.296	0.309	0.316	0.346	0.358	0.373
$y^{(1)}(-1)$	0.984	0.974	0.960	0.923	0.888	0.822
Panel C: Correlation of 6 Months Lagged Factors and Yields						
F1(-6)	0.445	0.490	0.502	0.473	0.445	0.412
F2(-6)	0.549	0.535	0.521	0.496	0.479	0.470
F3(-6)	0.128	0.151	0.171	0.286	0.364	0.453
F4(-6)	0.285	0.308	0.318	0.343	0.351	0.342
$y^{(1)}(-6)$	0.899	0.880	0.865	0.850	0.829	0.779
Panel D: Correlation of 12 months Lagged Factors and Yields						
F1(-12)	0.548	0.567	0.564	0.502	0.455	0.390
F2(-12)	0.448	0.405	0.385	0.398	0.400	0.408
F3(-12)	0.145	0.186	0.205	0.303	0.378	0.479
F4(-12)	0.275	0.309	0.329	0.349	0.354	0.348
$y^{(1)}(-12)$	0.742	0.712	0.705	0.738	0.745	0.723

Table 5: **Unrestricted Regressions of Yields on Factors**

This table summarizes the results of an unrestricted VAR of yields of different maturities on the four macro factors extracted from the panel of economic time series, and the short rate. The estimation period is 1983:01 to 2003:09. t-values are in brackets.

	$y^{(6)}$	$y^{(12)}$	$y^{(36)}$	$y^{(60)}$	$y^{(120)}$
cst	0.65	1.04	2.29	3.18	4.58
	[3.47]	[3.58]	[7.58]	[10.65]	[12.90]
F1	0.23	0.34	0.45	0.50	0.52
	[5.23]	[4.83]	[6.21]	[6.93]	[7.25]
F2	0.19	0.26	0.26	0.30	0.45
	[3.63]	[2.81]	[1.95]	[2.12]	[2.88]
F3	0.04	0.08	0.37	0.55	0.72
	[1.43]	[1.82]	[4.93]	[6.32]	[6.10]
F4	0.10	0.15	0.26	0.33	0.44
	[3.53]	[3.01]	[2.57]	[2.75]	[2.96]
$y^{(1)}$	0.95	0.93	0.82	0.72	0.52
	[28.64]	[17.59]	[11.71]	[9.07]	[5.57]
$\bar{R}^2$	0.98	0.97	0.93	0.91	0.86

Table 6: **In-sample Fit: Observed and Model-Implied Yields and Returns**

This table summarizes empirical means and standard deviations of observed and fitted yields. Yields are reported in percentage terms. The first and second row in each panel report the respective moment of observed yields and fitted values implied by the No-Arbitrage FAVAR model while in the third row the mean and standard deviation of absolute fitting errors are reported, respectively.

	$y^{(1)}$	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$	$y^{(24)}$	$y^{(36)}$	$y^{(48)}$	$y^{(60)}$	$y^{(84)}$	$y^{(120)}$
						Mean					
$y^{(n)}$	5.22	5.47	5.62	5.74	5.89	6.27	6.55	6.78	6.90	7.14	7.27
$\hat{y}^{(n)}$	5.22	5.47	5.61	5.76	5.88	6.27	6.56	6.76	6.91	7.14	7.26
$\lvert y_t^{(n)} - \hat{y}_t^{(n)} \rvert$	0.00	0.14	0.19	0.24	0.29	0.41	0.46	0.50	0.51	0.56	0.58
						Standard Deviation					
$y^{(n)}$	2.11	2.20	2.25	2.29	2.32	2.33	2.27	2.24	2.21	2.14	2.06
$\hat{y}^{(n)}$	2.11	2.19	2.25	2.28	2.29	2.27	2.21	2.16	2.12	2.04	1.92
$\lvert y_t^{(n)} - \hat{y}_t^{(n)} \rvert$	0.00	0.19	0.25	0.31	0.37	0.50	0.57	0.63	0.65	0.72	0.73

Table 7: **Parameter Estimates for No-Arbitrage FAVAR Model**

State dynamics : $Z_t = \mu + \Phi_1 Z_{t-1} + \ldots \Phi_4 Z_{t-4} + \omega_t, \quad E[\omega_t \omega_t'] = \Omega$

	Φ_1					Φ_2				
F1	0.977	-0.057	-0.107	-0.103	0.011	0.244	-0.143	0.013	0.143	0.044
	(0.096)	(0.109)	(0.118)	(0.064)	(0.061)	(0.140)	(0.180)	(0.165)	(0.088)	(0.079)
F2	0.196	1.357	0.174	0.038	0.028	-0.055	-0.387	-0.306	0.005	0.086
	(0.064)	(0.073)	(0.079)	(0.043)	(0.041)	(0.094)	(0.121)	(0.111)	(0.059)	(0.053)
F3	-0.160	0.098	0.945	-0.042	-0.043	0.112	-0.340	-0.014	0.072	0.012
	(0.072)	(0.082)	(0.088)	(0.048)	(0.046)	(0.105)	(0.135)	(0.124)	(0.066)	(0.060)
F4	-0.102	-0.172	0.170	1.007	-0.071	-0.068	0.336	0.051	-0.192	-0.044
	(0.123)	(0.140)	(0.151)	(0.082)	(0.079)	(0.179)	(0.231)	(0.212)	(0.112)	(0.102)
$y^{(1)}$	0.140	0.086	-0.045	-0.106	0.860	0.163	-0.057	-0.198	0.147	-0.042
	(0.100)	(0.113)	(0.123)	(0.066)	(0.064)	(0.146)	(0.188)	(0.173)	(0.091)	(0.083)

	Φ_3					Φ_4				
F1	-0.621	0.057	-0.006	-0.089	0.045	0.315	0.079	0.145	0.072	-0.102
	(0.139)	(0.178)	(0.165)	(0.088)	(0.079)	(0.107)	(0.107)	(0.110)	(0.061)	(0.060)
F2	-0.171	-0.049	0.257	-0.028	-0.071	0.084	0.045	-0.117	-0.016	-0.040
	(0.094)	(0.120)	(0.111)	(0.059)	(0.053)	(0.072)	(0.072)	(0.074)	(0.041)	(0.040)
F3	0.013	0.314	-0.350	-0.012	0.039	-0.087	-0.040	0.334	0.041	-0.006
	(0.104)	(0.134)	(0.124)	(0.066)	(0.059)	(0.081)	(0.081)	(0.082)	(0.046)	(0.045)
F4	0.347	-0.358	-0.111	-0.259	0.091	-0.016	0.165	-0.030	0.293	0.040
	(0.178)	(0.228)	(0.212)	(0.113)	(0.101)	(0.138)	(0.138)	(0.141)	(0.078)	(0.077)
$y^{(1)}$	-0.124	0.135	0.293	-0.001	-0.060	-0.022	-0.045	-0.082	-0.005	0.187
	(0.145)	(0.186)	(0.172)	(0.092)	(0.082)	(0.112)	(0.112)	(0.114)	(0.064)	(0.063)

	Ω					μ
F1	0.100					0.013
	(0.009)					(0.084)
F2	-0.020	0.045				-0.003
	(0.005)	(0.004)				(0.057)
F3	0.054	-0.013	0.057			-0.036
	(0.006)	(0.003)	(0.005)			(0.063)
F4	-0.072	-0.016	-0.044	0.165		-0.091
	(0.010)	(0.006)	(0.007)	(0.015)		(0.108)
$y^{(1)}$	0.006	-0.003	-0.005	-0.018	0.109	0.246
	(0.007)	(0.005)	(0.005)	(0.009)	(0.010)	(0.088)

Market prices of risk : $\lambda_t = \lambda_0 + \lambda_1 Z_t$

$\tilde{\lambda}_0$	$\tilde{\lambda}_1$				
-26.970	0.619	0.228	0.306	3.557	-0.120
(0.001)	(0.102)	(0.225)	(0.310)	(0.007)	(0.474)
-15.418	4.486	0.263	0.826	-1.342	0.043
(0.006)	(0.038)	(0.041)	(0.002)	(0.003)	(0.094)
-91.031	-0.770	-1.083	-0.191	-4.263	0.350
(0.007)	(0.078)	(0.232)	(0.001)	(0.010)	(0.460)
-79.287	-1.756	-1.867	0.275	0.554	0.241
(0.017)	(0.042)	(0.007)	(0.418)	(0.007)	(0.176)
-19.460	-0.250	-0.682	-0.040	-0.790	-0.241
(0.009)	(0.103)	(0.011)	(0.773)	(0.075)	(0.060)

Table 8: **Regression of Latent Yield Factors on the Model Factors**

This table summarizes the results obtained from a regression of level, slope, and curvature yield factors onto the factors of the FAVAR model. Level, slope, and curvature are computed as the first three principal components extracted from the yields used to estimate the term structure model. They explain 90.8%, 6.4% and 1.6% of the total variance of all yields, respectively. The sample period is 1983:01-2003:9. *t*-statistics are in brackets.

	Level	Slope	Curvature
cst	0.23	1.65	-0.05
	[10.88]	[9.40]	[-0.11]
F1	0.04	0.13	-0.37
	[7.15]	[4.18]	[-3.91]
F2	0.03	0.10	-0.13
	[2.76]	[1.48]	[-0.96]
F3	0.04	0.30	0.02
	[5.89]	[6.23]	[0.30]
F4	0.02	0.14	-0.09
	[2.92]	[2.44]	[-1.26]
$y^{(1)}$	0.07	-0.29	0.02
	[13.41]	[-7.22]	[0.33]
$\bar{R}^2$	0.95	0.77	0.35

Table 9: **Out-of-sample RMSEs - Forecast Period 1994:01-2003:09**

This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. The models have been estimated using data from 1983:01 until the period when the forecast is made. The forecasting period is 1994:01-2003:09. "FAVAR" refers to the No-Arbitrage Factor-Augmented VAR model; "VAR" denotes an arbitrage-free model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate as factors; "VARylds" refers to a VAR(1) on yield levels; "NS(VAR)" and "NS(AR)" denote the Diebold-Li (2006) version of the three-factor Nelson-Siegel model with VAR and AR dynamics of the latent factors, respectively; "$A_0(3)$" refers to the essentially affine latent yield factor model, and "RW" denotes the random walk forecast.

$y^{(n)}$	FAVAR	VAR	VARylds	NS(VAR)	NS(AR)	$A_0(3)$	RW
			Panel A: 1-month ahead forecasts				
n=1	0.534	0.334	**0.249**	0.262	0.275	0.681	0.305
n=6	0.496	0.347	**0.204**	0.218	0.256	0.216	0.222
n=12	0.517	0.452	**0.250**	0.268	0.293	0.300	0.259
n=36	0.628	0.771	**0.308**	0.313	0.312	0.386	0.309
n=60	0.676	0.935	0.314	0.316	0.316	0.357	**0.307**
n=120	0.711	1.093	0.293	0.289	0.289	0.289	**0.282**
			Panel B: 6-months ahead forecasts				
n=1	**0.601**	0.707	0.779	0.745	0.838	1.189	0.856
n=6	**0.603**	0.898	0.904	0.871	0.931	0.977	0.853
n=12	**0.694**	1.040	1.006	0.958	0.981	1.059	0.876
n=36	**0.753**	1.278	1.021	0.958	0.922	0.962	0.873
n=60	**0.789**	1.377	0.969	0.915	0.870	0.848	0.830
n=120	0.823	1.435	0.872	0.764	0.720	**0.671**	0.696
			Panel C: 12-months ahead forecasts				
n=1	**0.919**	1.307	1.366	1.448	1.357	1.741	1.395
n=6	**0.977**	1.542	1.613	1.569	1.458	1.487	1.417
n=12	**1.053**	1.652	1.728	1.633	1.495	1.506	1.391
n=36	**1.062**	1.769	1.599	1.504	1.349	1.264	1.236
n=60	**1.066**	1.813	1.464	1.359	1.233	1.076	1.138
n=120	1.072	1.806	1.313	1.108	1.022	**0.853**	0.942

Table 10: **RMSEs Relative to Random Walk - Forecast Period 1994:01-2003:09**

This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. The models have been estimated using data from 1983:01 until the period when the forecast is made. The forecasting period is 1994:01-2003:09. "FAVAR" refers to the No-Arbitrage Factor-Augmented VAR model; "VAR" denotes an arbitrage-free model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate as factors; "VARylds" refers to a VAR(1) on yield levels; "NS(VAR)" and "NS(AR)" denote the Diebold-Li (2006) version of the three-factor Nelson-Siegel model with VAR and AR dynamics of the latent factors, respectively; "$A_0(3)$" refers to the essentially affine latent yield factor model, and "RW" denotes the random walk forecast.

$y^{(n)}$	FAVAR	VAR	VARylds	NS(VAR)	NS(AR)	$A_0(3)$
Panel A: 1-month ahead forecasts						
n=1	1.751	1.096	**0.816**	0.859	0.900	2.232
n=6	2.237	1.567	**0.921**	0.984	1.154	0.972
n=12	1.996	1.743	**0.964**	1.034	1.131	1.160
n=36	2.032	2.497	**0.996**	1.013	1.011	1.250
n=60	2.204	3.046	**1.022**	1.029	1.031	1.165
n=120	2.525	3.880	1.039	1.028	**1.025**	1.027
Panel B: 6-months ahead forecasts						
n=1	**0.702**	0.826	0.910	0.870	0.979	1.389
n=6	**0.707**	1.052	1.059	1.022	1.092	1.145
n=12	**0.793**	1.187	1.148	1.094	1.119	1.209
n=36	**0.863**	1.465	1.171	1.098	1.056	1.103
n=60	**0.951**	1.659	1.167	1.102	1.048	1.022
n=120	1.183	2.064	1.254	1.099	1.035	**0.964**
Panel C: 12-months ahead forecasts						
n=1	**0.659**	0.937	0.979	1.038	0.973	1.249
n=6	**0.689**	1.088	1.139	1.107	1.029	1.049
n=12	**0.757**	1.187	1.242	1.174	1.075	1.083
n=36	**0.860**	1.431	1.293	1.217	1.091	1.023
n=60	**0.937**	1.594	1.287	1.194	1.084	0.946
n=120	1.138	1.916	1.393	1.175	1.085	**0.905**

Table 11: **White's Reality Check Test - Forecast Period 1994:01-2003:09**

This table summarizes "White's Reality Check" test statistics based on a squared forecast error loss function. I choose the No-Arbitrage FAVAR model as the benchmark model and compare it bilaterally with the competitor models. Negative test statistics indicate that the average squared forecast loss of the FAVAR model is smaller than that of the respective competitor model. Bold figures indicate significance at the 5% interval. Significance is checked by comparing the average forecast loss differential with the 5% percentile of the empirical distribution of the loss differential series approximated by applying a block bootstrap with 1,000 resamples and a smoothing parameter of 1/12.

$y^{(n)}$	VAR	VARylds	NS(VAR)	NS(AR)	$A_0(3)$	RW
			Panel A: 1-month ahead forecasts			
n=1	1.858	2.390	2.315	2.241	-1.915	2.057
n=6	1.355	2.205	2.143	1.950	2.153	2.126
n=12	0.697	2.222	2.123	1.971	1.924	2.172
n=36	**-2.099**	3.270	3.237	3.237	2.691	3.264
n=60	**-4.387**	3.930	3.915	3.907	3.619	3.975
n=120	**-7.305**	4.599	4.622	4.626	4.622	4.668
			Panel B: 6-months ahead forecasts			
n=1	**-1.456**	**-2.727**	**-2.062**	**-3.581**	**-11.050**	**-3.946**
n=6	**-4.613**	**-4.942**	**-4.327**	**-5.371**	**-6.325**	**-4.003**
n=12	**-6.239**	**-5.813**	**-4.848**	**-5.173**	**-6.917**	**-3.248**
n=36	**-11.142**	**-5.281**	**-3.982**	**-3.119**	**-4.019**	**-2.359**
n=60	**-13.287**	**-3.593**	**-2.517**	**-1.559**	-1.255	**-0.948**
n=120	**-14.444**	**-1.081**	0.727	1.516	2.216	1.832
			Panel C: 12-months ahead forecasts			
n=1	**-8.760**	**-10.630**	**-12.891**	**-10.170**	**-22.505**	**-11.504**
n=6	**-14.341**	**-17.253**	**-15.864**	**-12.136**	**-13.404**	**-11.477**
n=12	**-16.235**	**-19.577**	**-16.546**	**-11.801**	**-12.570**	**-9.299**
n=36	**-20.234**	**-14.953**	**-12.169**	**-7.337**	**-5.467**	**-4.899**
n=60	**-21.841**	**-10.567**	**-7.692**	**-4.137**	-0.707	**-2.151**
n=120	**-21.456**	**-5.984**	**-1.097**	0.939	4.064	2.372

Table 12: **Out-of-sample RMSEs - Forecast Period 1994:01-2000:12**

This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. The models have been estimated using data from 1983:01 until the period when the forecast is made. The forecast period is 1994:01-2000:12 which is exactly the sample considered by Diebold and Li (2006). "FAVAR" refers to the No-Arbitrage Factor-Augmented VAR model; "VAR" denotes an arbitrage-free model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate as factors; "VARylds" refers to a VAR(1) on yield levels; "NS(VAR)" and "NS(AR)" denote the Diebold-Li (2006) version of the three-factor Nelson-Siegel model with VAR and AR dynamics of the latent factors, respectively; "$A_0(3)$" refers to the essentially affine latent yield factor model, and "RW" denotes the random walk forecast.

$y^{(n)}$	FAVAR	VAR	VARylds	NS(VAR)	NS(AR)	$A_0(3)$	RW
			Panel A: 1-month ahead forecasts				
1	0.380	0.303	0.255	0.265	**0.249**	0.722	0.297
6	0.349	0.345	0.194	**0.186**	0.215	0.209	0.192
12	0.392	0.447	0.242	**0.238**	0.249	0.280	0.239
36	0.563	0.731	0.281	0.286	**0.272**	0.368	0.277
60	0.678	0.913	0.290	0.289	0.280	0.343	**0.275**
120	0.796	1.158	0.270	0.256	**0.249**	0.254	0.253
			Panel B: 6-months ahead forecasts				
1	0.625	0.597	0.696	**0.509**	0.532	1.140	0.635
6	**0.590**	0.769	0.799	0.660	0.648	0.936	0.655
12	**0.659**	0.892	0.898	0.778	0.711	0.999	0.742
36	**0.649**	1.184	0.947	0.877	0.747	0.938	0.834
60	**0.740**	1.348	0.949	0.885	0.764	0.834	0.821
120	0.877	1.516	0.911	0.793	0.694	**0.637**	0.730
			Panel C: 12-months ahead forecasts				
1	0.900	1.006	1.025	0.899	**0.812**	1.654	0.945
6	**0.907**	1.205	1.179	1.002	0.908	1.430	0.977
12	0.955	1.288	1.268	1.078	**0.932**	1.414	1.017
36	0.981	1.560	1.333	1.168	**0.937**	1.188	1.078
60	1.056	1.714	1.331	1.179	**0.979**	1.007	1.072
120	1.160	1.825	1.333	1.089	0.941	**0.775**	0.985

Table 13: **Out-of-sample RMSEs - Forecast Period 2000:01-2003:09**

This table summarizes the root mean squared errors obtained from out-of-sample yield forecasts. The models have been estimated using data from 1983:01 until the period when the forecast is made. The forecast period is 2000:01-2003:09. "FAVAR" refers to the No-Arbitrage Factor-Augmented VAR model; "VAR" denotes an arbitrage-free model with IP growth, the index of help-wanted adds in newspapers, CPI growth, PPI growth and the short rate as factors; "VARylds" refers to a VAR(1) on yield levels; "NS(VAR)" and "NS(AR)" denote the Diebold-Li (2006) version of the three-factor Nelson-Siegel model with VAR and AR dynamics of the latent factors, respectively; "$A_0(3)$" refers to the essentially affine latent yield factor model, and "RW" denotes the random walk forecast.

$y^{(n)}$	FAVAR	VAR	VARylds	NS(VAR)	NS(AR)	$A_0(3)$	RW
			Panel A: 1-month ahead forecasts				
1	0.762	0.417	0.300	**0.296**	0.349	0.648	0.366
6	0.690	0.379	**0.214**	0.256	0.316	0.228	0.257
12	0.677	0.495	**0.250**	0.297	0.348	0.326	0.281
36	0.725	0.884	**0.336**	0.341	0.359	0.396	0.342
60	0.648	1.003	**0.339**	0.344	0.360	0.366	0.342
120	0.489	0.961	**0.310**	0.330	0.336	0.325	0.312
			Panel B: 6-months ahead forecasts				
1	**0.581**	0.904	0.896	1.027	1.231	1.391	1.165
6	**0.611**	1.071	1.038	1.148	1.298	1.085	1.118
12	**0.728**	1.232	1.153	1.213	1.327	1.169	1.078
36	**0.873**	1.420	1.150	1.095	1.158	0.972	0.956
60	**0.822**	1.425	1.019	0.969	1.020	0.841	0.868
120	0.656	1.284	0.798	0.716	0.759	0.709	**0.634**
			Panel C: 12-months ahead forecasts				
1	**0.939**	1.763	1.896	2.191	2.079	1.927	2.052
6	**1.112**	2.062	2.297	2.382	2.221	1.654	2.108
12	**1.246**	2.213	2.459	2.461	2.288	1.749	2.030
36	**1.217**	2.133	2.085	2.084	1.974	1.471	1.601
60	**1.075**	1.995	1.738	1.711	1.660	1.252	1.329
120	**0.845**	1.748	1.275	1.175	1.185	1.022	0.891

Figure 1: **Observed and Model Implied Average Yield Curve**

This figure plots average observed yields against those implied by the No-Arbitrage FAVAR model.

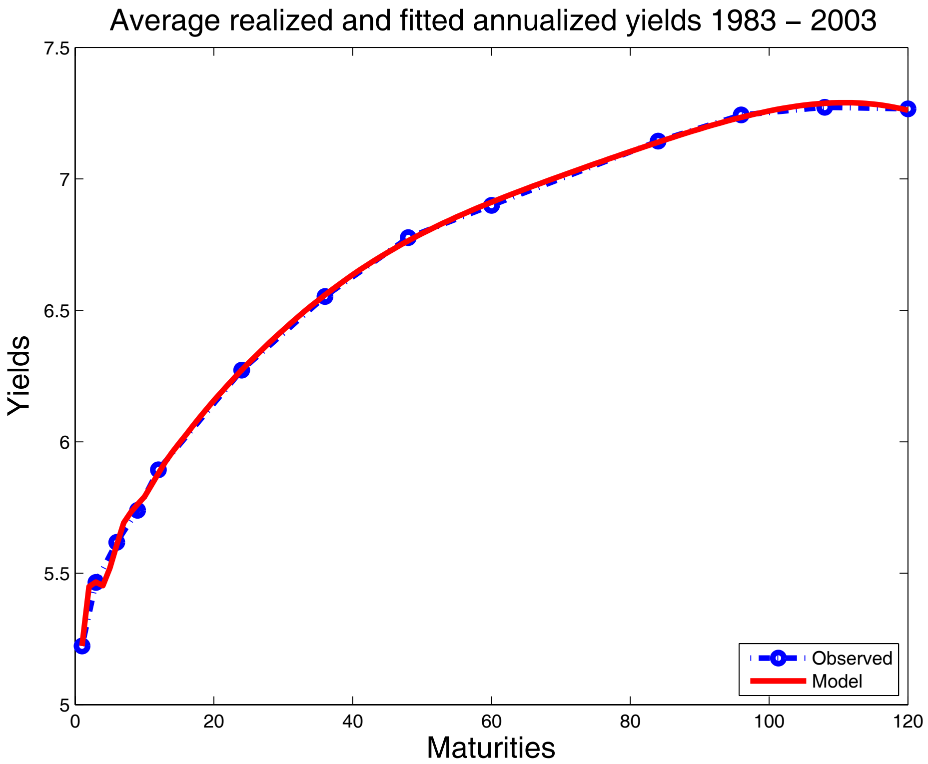

Figure 2: **Observed and Model-Implied Yields**

This figure provides plots of observed and model-implied time series for four selected interest rates, the 6-months yield, the 12-months yield and the 3-and 10-years yields.

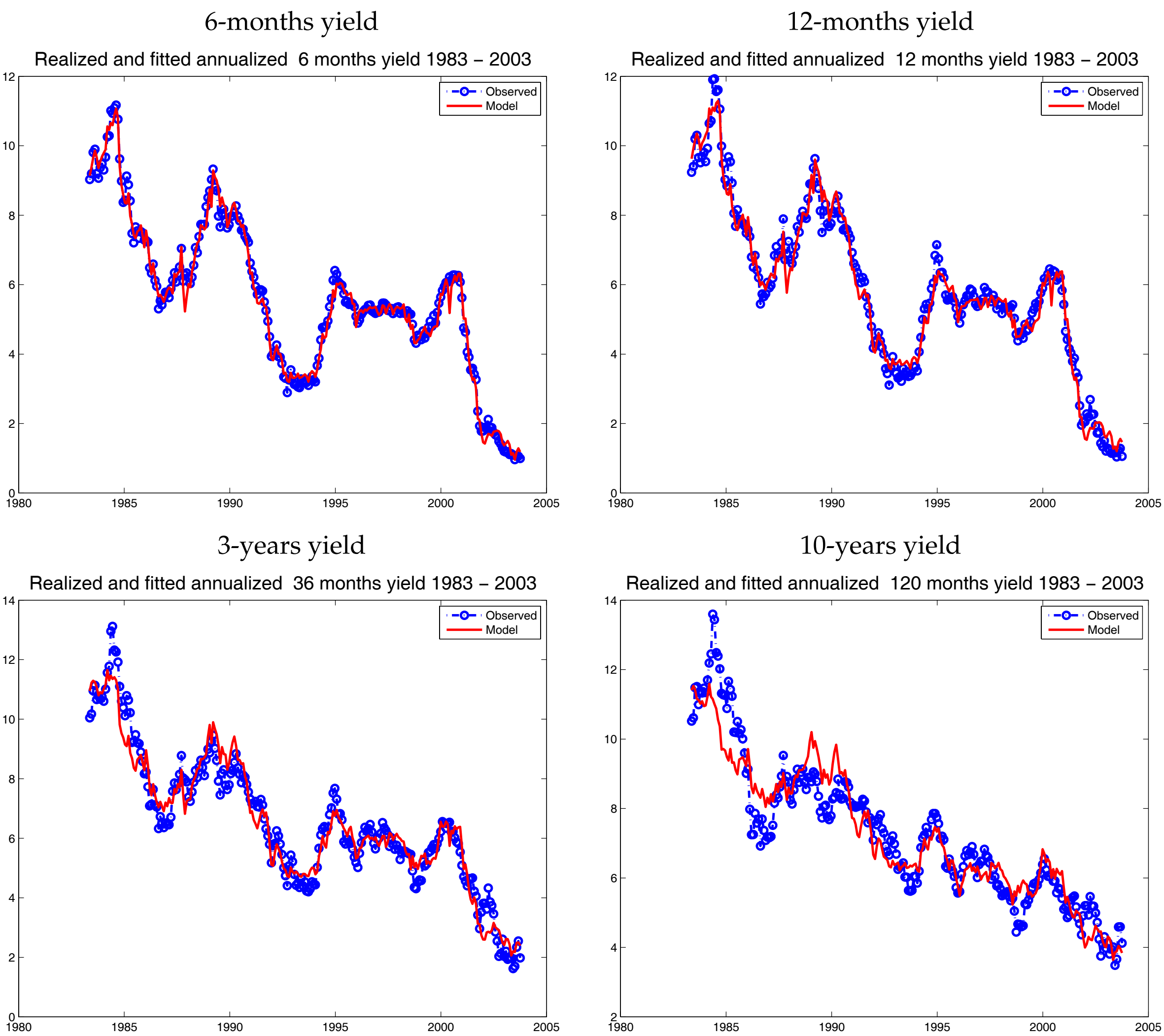

Figure 3: **Risk Premia Dynamics**

This figure provides a plot of the term premia for the 1-year and 5-year yield as implied by the No-Arbitrage FAVAR model. For comparison, they are related to the first and second model factor, respectively.

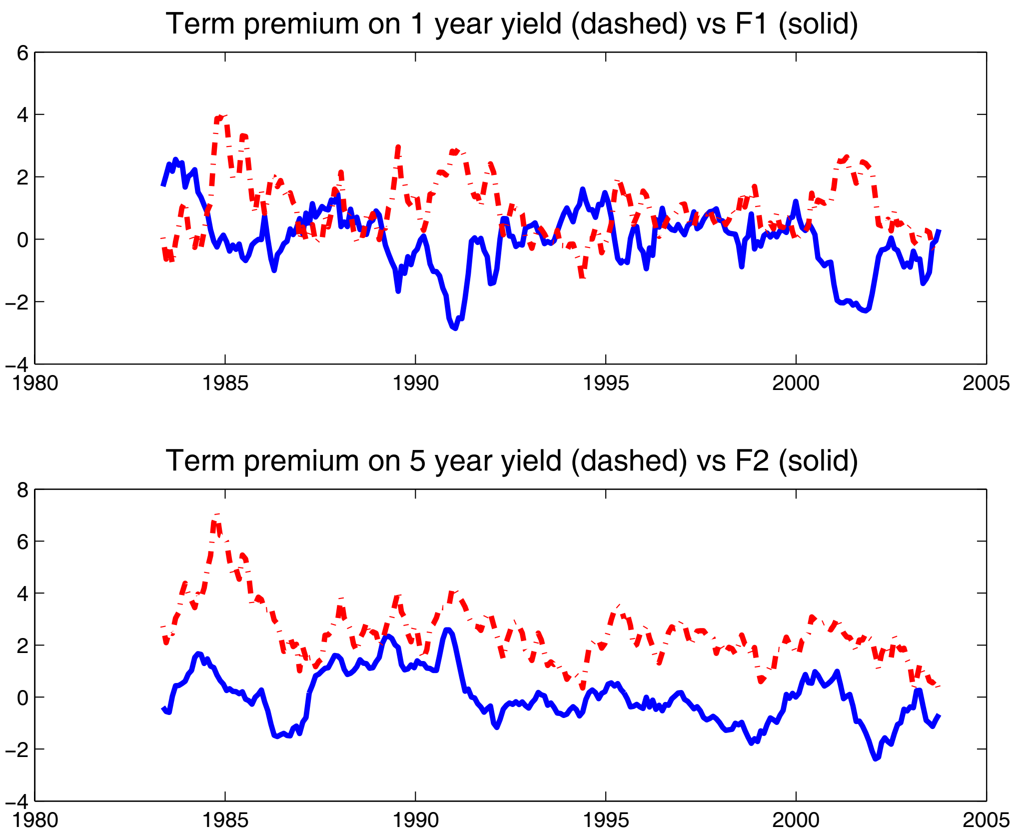

Figure 4: **Implied Yield Loadings**

This figure provides a plot of the yield loadings b_n implied by the No-Arbitrage FAVAR model. The coefficients can be interpreted as the response of the n-month yield to a contemporary shock to the respective factor.

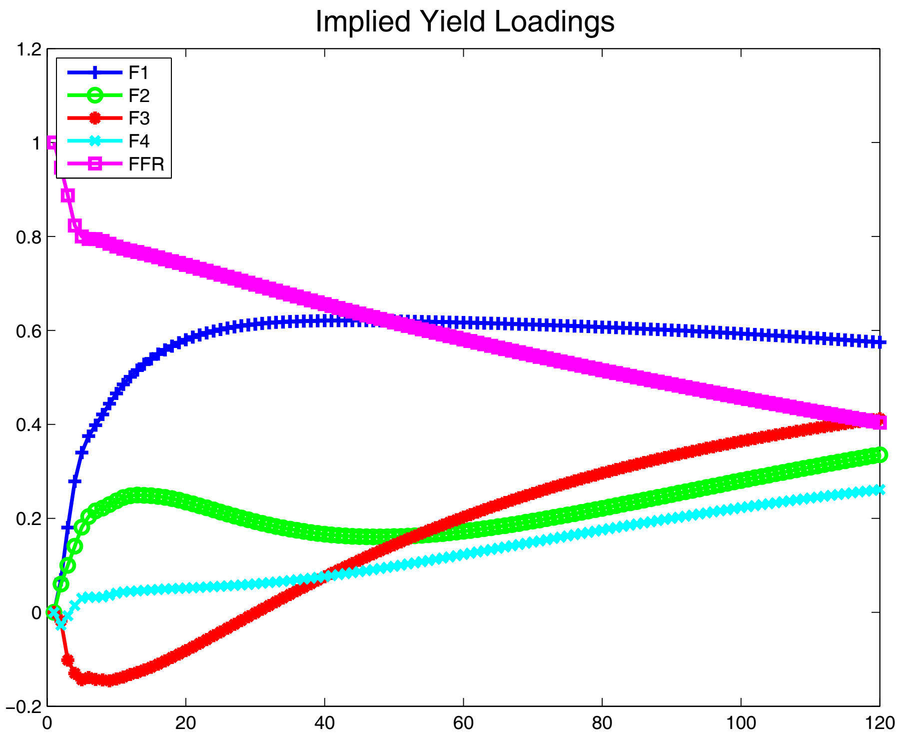

Figure 5: **Observed and Predicted Yields - 1 Month Ahead**

This figure provides plots of the observed and 1-month ahead predicted time series for four the 1-month, the 12-month, the 3- and 10-year maturities. The observed yields are plotted by solid lines, whereas dashed, dash-dotted, and dotted lines correspond to predictions of the No-Arbitrage FAVAR model, the NS(AR) model, and the $A_0(3)$ model.

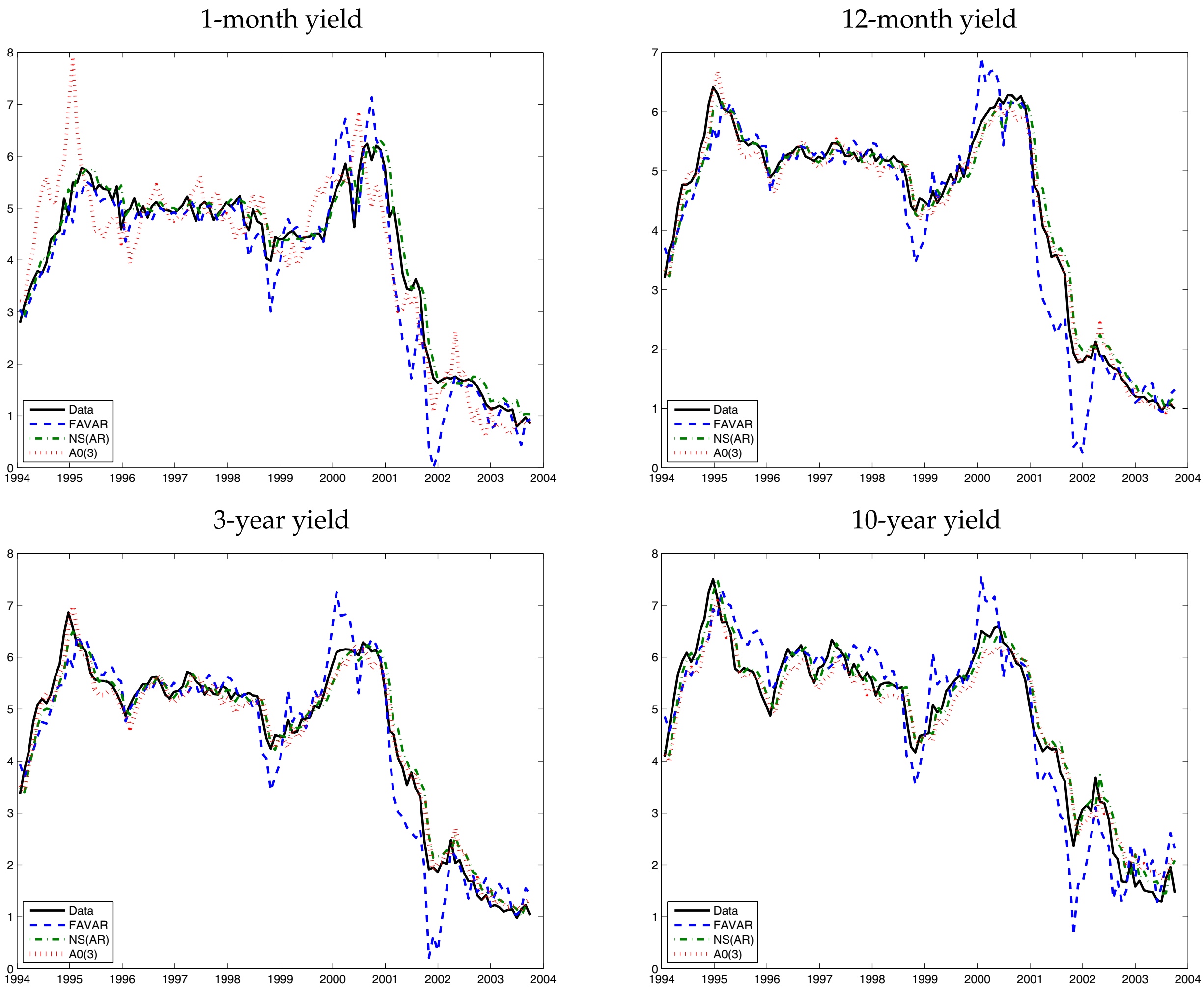

Figure 6: **Observed and Predicted Yields - 6 Months Ahead**

This figure provides plots of the observed and 6-months ahead predicted time series for four the 1-month, the 12-month, the 3- and 10-year maturities. The observed yields are plotted by solid lines, whereas dashed, dash-dotted, and dotted lines correspond to predictions of the No-Arbitrage FAVAR model, the NS(AR) model, and the $A_0(3)$ model.

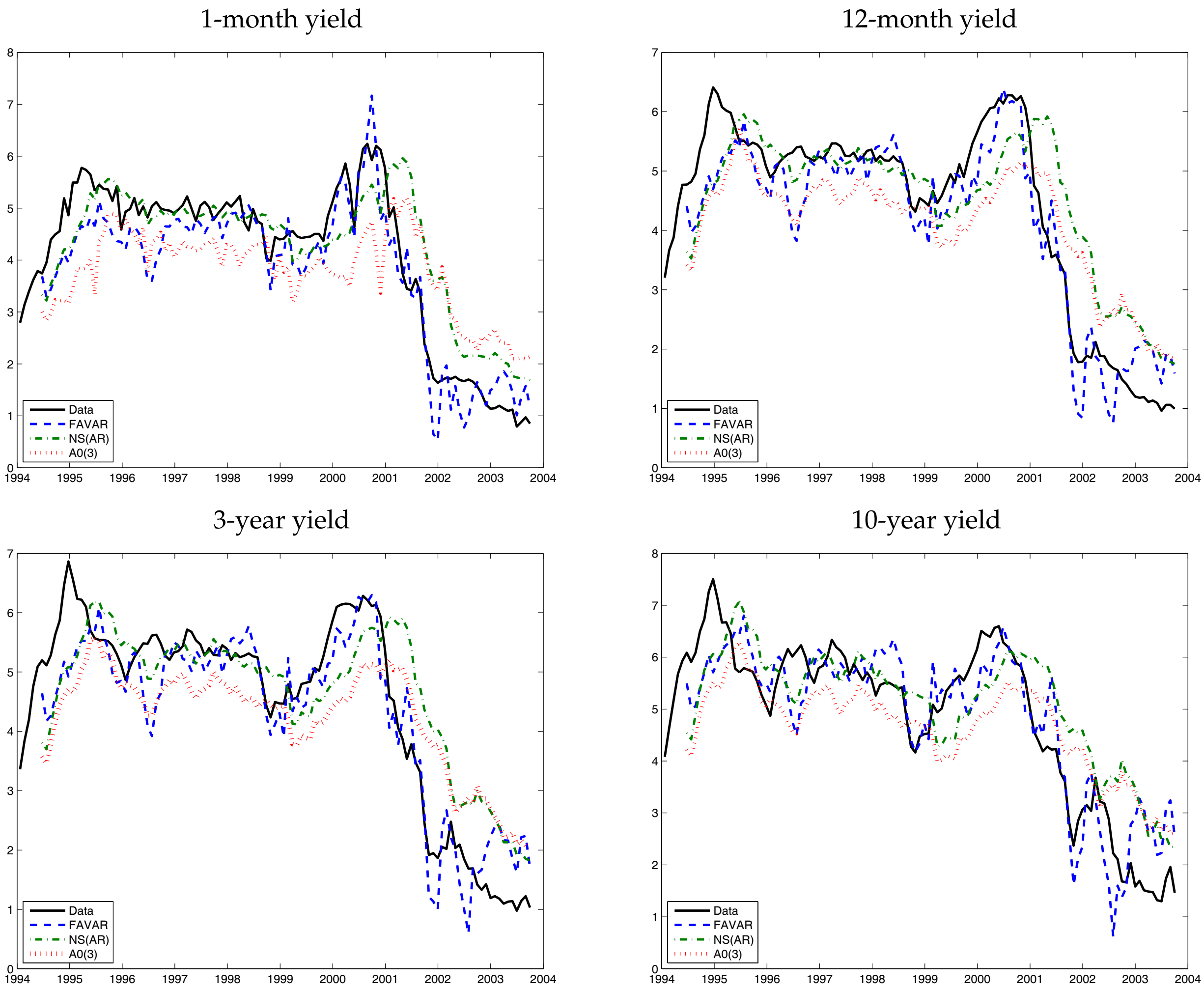

Figure 7: **Observed and Predicted Yields - 12 Months Ahead**

This figure provides plots of the observed and 12-months ahead predicted time series for four the 1-month, the 12-month, the 3- and 10-year maturities. The observed yields are plotted by solid lines, whereas dashed, dash-dotted, and dotted lines correspond to predictions of the No-Arbitrage FAVAR model, the NS(AR) model, and the $A_0(3)$ model.