Efficient Use of Information and Social Value of Information*

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Abstract

This paper analyzes equilibrium and welfare for a tractable class of economies with externalities, strategic complementarity or substitutability, and incomplete information. First, we characterize the *equilibrium use of information*; the key result is that complementarity heightens the sensitivity of equilibrium actions to public noise relative to private, while the converse is true for substitutability. Next, we define and characterize an efficiency benchmark designed to address whether such heightened sensitivity is socially undesirable; the key result is that the *efficient use of information* trades off aggregate volatility for cross-sectional dispersion. Finally, we examine the *social value of information,* that is, the comparative statics of equilibrium welfare with respect to the information structure; the key result is that the latter is determined by the relation between the equilibrium and efficient use of information. We conclude with a few applications, including production externalities, beauty contests, Keynesian frictions, inefficient fluctuations, and large Cournot and Bertrand games.

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1 Introduction

In many environments—including economies with production or network externalities, incomplete financial markets, or monopolistic competition—a coordination motive emerges: the optimal action of an agent depends on his expectation, not only of the underlying fundamentals, but also of other agents' actions. Furthermore, different agents may have different information about the fundamentals and hence different beliefs about what other agents are doing. Although an extensive literature examines the equilibrium properties of such environments, the welfare implications are far less understood. Filling this gap is the goal of this paper.

To fix ideas, consider the following example. A large number of investors are choosing how much to invest in a new sector. The profitability of this sector depends on an uncertain exogenous productivity parameter, as well as on the aggregate investment in that sector. Each investor thus tries to align his investment choice with other investors' choices. Because of this coordination motive, and because public information is a better predictor of what others do relatively to private information, investment choices are highly sensitive to public information. Furthermore, more precise public information, by reducing investors' reliance on private information and increasing their reliance on public information, may dampen the sensitivity of aggregate investment to the true fundamental and amplify its sensitivity to the noise in public information. The economy can thus exhibit high non-fundamental volatility, and the more so the stronger the complementarity in investment decisions.

It is tempting to give a normative connotation to these properties, but their welfare implications are not obvious. Is the heightened sensitivity to public information due to coordination undesirable from a social perspective? And does this mean that public information disseminated, for example, by policy makers or the media can reduce welfare?

To answer the first question, one needs to understand the *efficient use of information*; to answer the second question, one needs to understand the *social value of information.* In this paper we undertake these two tasks in a tractable yet flexible framework that permits us to uncover some general principles, while also capturing a variety of applications.

The environment. A large number of ex-ante identical small agents takes a continuous action. Individual payoffs depend, not only on one's own action, but also on the mean, and possibly the dispersion, of actions in the population—this is the source of external and strategic effects in the model. Agents observe noisy private and public signals about the underlying economic fundamentals—this is the source of dispersed heterogeneous information. We allow for either strategic complementarity or strategic substitutability, but restrict our attention to economies in which the equilibrium is unique. Finally, we assume that payoffs are quadratic and that information is Gaussian, which makes the analysis tractable.

Since our framework allows for various strategic and external effects, the welfare properties of equilibrium are ambiguous in general. For example, there are economies where the heightened sensitivity to public information is socially desirable, and economies where the opposite is true; similarly, there are economies where welfare increases with both private and public information, economies where only one type of information is socially valuable, and economies where neither type is socially valuable. All this is consistent with the folk theorem that in a second-best world anything can happen. Our contribution is to identify a clear structure for *what* happens *when*.

We are able to identify such a structure by comparing the equilibrium with an appropriate constrained efficiency benchmark. We first isolate the inefficiencies that pertain when information is complete from those that emerge only when information is incomplete. The former are manifested in a discrepancy between the complete-information equilibrium and the first-best allocation; the latter in a discrepancy between the *equilibrium degree of coordination*, which summarizes the private value of aligning one's action with those of others, and the *socially optimal degree of coordination*, which summarizes the social value of such alignment. We next show how the social value of information is best understood by classifying economies according to these two forms of inefficiency.

Preview of the results. Consider first the *equilibrium use of information.* Strategic complementarity raises the sensitivity of equilibrium actions to public information; symmetrically, strategic substitutability raises the sensitivity to private information. Noise in public information generates aggregate non-fundamental volatility (that is, common variation in actions due to common noise); noise in private information generates cross-sectional dispersion (that is, idiosyncratic variation in actions due to idiosyncratic noise). It follows that, in equilibrium, complementarity contributes to higher volatility, substitutability to higher dispersion.[1]

We next introduce an efficiency benchmark, aimed at understanding the normative content of the aforementioned positive properties. We consider the strategy that maximizes ex-ante utility under the sole constraint that information cannot be centralized or otherwise communicated among the agents. Our efficiency benchmark thus corresponds to a situation where the "planner" can perfectly control the agents' incentives, and hence can dictate how the agents use their information, but cannot affect the information available to them, for example, by asking them to report their information, aggregating this information, and then sending them informative recommendations (or new signals). We call this benchmark the *efficient use of information.*

We characterize this benchmark and show that it can be represented as the equilibrium of a fictitious game where best responses reflect social incentives. The slope of these fictitious best responses with respect to the mean activity measures the complementarity [or substitutability] that agents must perceive for the equilibrium of the fictitious game to coincide with the efficient

[1]The amplification effects of various sorts of complementarities are the subject of a vast literature. See Cooper (1990) for a review of complete-information applications and Morris and Shin (2002, 2003) for incomplete information.

allocation of the true economy; it defines what we call the *socially optimal degree of coordination.*

The idea behind this concept is the following. When the planner can perfectly control the agents' incentives, he can induce them to play a game with an arbitrary degree of complementarity/substitutability in actions—that is, when choosing the efficient use of information, it is as if the planner controls the degree of coordination perceived by the agents.

A higher degree of coordination leads to a higher sensitivity of actions to public information relative to private information. Higher sensitivity to public information raises aggregate non-fundamental volatility, while lower sensitivity to private information decreases cross-sectional dispersion. It follows that, when choosing the optimal degree of coordination, the planner faces a trade-off between volatility and dispersion.

This explains our first characterization result: *the socially optimal degree of coordination increases with social aversion to dispersion, and decreases with social aversion to volatility.* Equivalently, the efficient use of information requires a higher sensitivity of actions to public information the higher the social aversion to dispersion, or the lower the social aversion to volatility.

But then the question is how social aversion to dispersion and volatility relate to the primitives of the economy, which is what we address next.

Social aversion to both volatility and dispersion originates in the curvature of individual payoffs. When there are no payoff interdependencies across agents, all that matters is the level of noise, not its composition. As a result, the welfare costs of dispersion and volatility are totally symmetric and the socially optimal degree of coordination is zero. Complementarity, on the other hand, reduces social aversion to volatility by alleviating concavity at the aggregate level—if we think of concavity as diminishing returns, this is the familiar property that complementarity alleviates diminishing returns at the aggregate level. As a result, complementarity alone contributes to a lower aversion to volatility and thereby to a positive optimal degree of coordination—equivalently, to a socially desirable heightened sensitivity to public information. Symmetrically, substitutability contributes to a negative optimal degree of coordination—equivalently, to a socially desirable heightened sensitivity to private information.

The impact of complementarity or substitutability on the optimal degree of coordination thus parallels its impact on the equilibrium degree of coordination. However, the optimal degree also depends on second-order external but non-strategic payoff effects, namely external effects that directly affect social preferences over volatility and dispersion without affecting private incentives. Clearly, such external effects can tilt the optimal degree of coordination one way or another, without affecting equilibrium. In the absence of such external effects, the optimal degree of coordination—and hence the sensitivity of efficient allocations to public information—is *higher* than the equilibrium one when agents' actions are strategic complements [and *lower* when they are strategic substitutes]. This is because of the internalization of the externality introduced by the complementarity.

These results highlight the danger in extrapolating normative properties from positive ones: a heightened sensitivity to public noise due to a coordination motive need not be socially undesirable, even if it amplifies volatility. Instead, to evaluate whether such heightened sensitivity is inefficient, one has to understand how the equilibrium degree of coordination compares to the socially optimal one, which in turn requires to evaluate the social cost of dispersion relative to volatility.

We finally characterize the *social value of information* in equilibrium. For this purpose, we find it illuminative (i) to classify economies according to the type of inefficiency—if any—exhibited by the equilibrium, and (ii) to decompose any change in the information structure into a change in *accuracy* and a change in *commonality* . We identify the former with the precision of the agents' forecasts (that is, the reciprocal of total noise), and the latter with the correlation of forecast errors across agents (that is, the extent to which noise is common). While only the accuracy of available information matters in the absence of payoff interdependencies, its commonality plays a special role in the presence of strategic effects because it affects the agents' ability to align their choices.

First, consider economies in which the equilibrium is efficient under both complete and incomplete information—these are economies in which the complete-information equilibrium coincides with the first best and, in addition, the equilibrium degree of coordination coincides with the optimal one. When this is the case, welfare necessarily increases with the accuracy of information. Moreover, welfare increases with the commonality of information if and only if the agents' actions are strategic complements [and decreases if and only if they are substitutes].

Private and public information have symmetric effects on accuracy but opposite effects on commonality. In economies where the equilibrium is efficient, the accuracy effect necessarily dominates, so that welfare increases with either private or public information. This is because the equilibrium coincides with the solution to the planner's problem, in which case an argument analogous to Blackwell's theorem ensures that any source of information is welfare-improving. At the same time, complementarity contributes to a higher social value to public information *relative* to private [and substitutability to a lower]. This is because public information increases commonality, whereas private information decreases it, and because the social value of commonality is positive [negative] in efficient economies with strategic complementarity [substitutability].

Next, consider economies in which inefficiency emerges only due to the incompleteness of information—that is, economies where the equilibrium degree of coordination is different than the socially optimal one, but the complete-information equilibrium coincides with the first best. In this case, welfare continues to increase monotonically with accuracy, as in the case of efficient economies. The welfare effect of commonality, on the other hand, is tilted relative to the efficiency benchmark as a function of the gap between the equilibrium and the optimal degree of coordination. In particular, when agents' actions are strategic complements, the welfare effect of commonality remains positive as long as the socially optimal degree of coordination is higher than the equilibrium one, but turns negative once the equilibrium degree of coordination is excessively high.

To recap, in economies where inefficiency emerges only under incomplete information, more accuracy is necessarily socially valuable, while more commonality can be socially undesirable only to the extent that coordination is socially undesirable. By implication, more precise public or private information can decrease welfare only to the extent that they adversely affect commonality.

Finally, consider economies where inefficiency pertains even under complete information. This is the case when distortions other than incomplete information create a gap between the complete-information equilibrium and the first best. In this case, information affects, not only volatility and dispersion, but also the covariation between the equilibrium and the efficiency gap. This in turn raises the possibility that welfare decreases with accuracy: less noise necessarily brings the incomplete-information equilibrium activity closer to its complete-information counterpart, but now this may mean taking it further away from the first best. As a result, in economies in which the complete-information equilibrium is inefficient, welfare may decrease with the precision of either public or private information.

Applications. We conclude the paper by illustrating how our results can help understand the potential inefficiencies in the equilibrium use of information, and the social value of information, in specific applications.

We start with two examples where the equilibrium is efficient under both complete and incomplete information. The first one is an economy in which agents value being close to each other. The other is an incomplete-market, competitive, production economy. In the first, actions are strategic complements, so that the equilibrium features heightened sensitivity to public information and amplified volatility. In the second, actions are strategic substitutes, so that the equilibrium features heightened sensitivity to private information and high cross-sectional dispersion. But in both cases the equilibrium use of information is efficient, implying that any heightened sensitivity to noise is just right, and ensuring that no source of information can reduce welfare.

We next consider a typical model of production spillovers, like the one outlined at the beginning of the introduction. Complementarities emerge in investment choices, thus amplifying the volatility of aggregate investment, but the equilibrium degree of coordination is actually too low, so that the amplified volatility is anything but excessive. Moreover, welfare unambiguously increases with either the accuracy or the commonality of information, which ensures that welfare necessarily increases with the precision of public information, despite the adverse effect the latter can have on the volatility of investment.

In contrast, the equilibrium degree of coordination is inefficiently high in economies that resemble Keynes' beauty-contest metaphor for financial markets and that are stylized in the example of Morris and Shin (2002). As a result, more precise public information can reduce welfare in these economies—but this is only because coordination, and hence commonality, is socially undesirable.

Keynesian frictions such as monopolistic competition or incomplete markets are at the heart of various macroeconomic complementarities (a.k.a. “multipliers” or “accelerators”). These frictions share with beauty contests the property that complementarity originates in some market imperfection. However, they need *not* share the property that coordination is socially unwarranted. Indeed, new-Keynesian models of the business cycle typically feature a disutility from cross-sectional price dispersion (Woodford, 2001; Hellwig, 2005; Roca, 2005). This effect—which is also the one that micro-founds the social cost of inflation—heightens social aversion to dispersion and thereby raises the social value of coordination. As a result, in these models the optimal degree of coordination is higher than the equilibrium one—exactly the opposite than in beauty-contest economies.

This result appears to provide a case for transparency in central bank communication (if we interpret transparency as dissemination of public information). However, the social value of information—and hence the desirability of central-bank transparency—may critically depend on the source of the business cycle. We highlight this point by constructing an example that features two types of shocks: one that affects equilibrium and first best symmetrically, and another that drives fluctuations in the gap between the two. Whereas information about the former shock increases welfare, information about the latter decreases it. This suggests a case for “constructive ambiguity” in central bank communication, to the extent that the business cycle is driven by shocks to “mark-ups”, “wedges”, or other distortions.

The above examples have a macro flavor. However, our results may also be relevant for micro applications, such as oligopolistic markets with a large number of firms. We show that expected industry profits increase with both the accuracy and the commonality of information in Bertrand-like games (where firms compete in prices), whereas they increase with accuracy but decrease with commonality in Cournot-like games (where firms compete in quantities). As a result, information-sharing among firms, or other improvements in commonly available information, necessarily increases profits in Bertrand games, but not in Cournot games.

Related literature. To the best of our knowledge, this paper is the first one to conduct a complete welfare analysis for the class of economies considered here. The closest ascendants are Cooper and John (1988), who examine economies with complementarities but complete information, and Vives (1988), who examines a class of limit-competitive economies that is a special case of the more general class considered here (see Section 6.2). Related are also Vives (1990) and Raith (1996), who examine the value of information sharing in oligopolies (see Section 6.6).

The social value of information, on the other hand, has been the subject of a vast literature, going back at least to Hirshleifer (1971). More recently, and more closely related to this paper, Morris and Shin (2002) show that public information can reduce welfare in an economy that resembles a “beauty contest” and that features strategic complementarity. Angeletos and Pavan (2004) and Hellwig (2005), on the other hand, provide counterexamples where public information is socially valuable despite strategic complementarity—a real economy with investment complementarities in the first paper, a monetary economy with pricing complementarities in the second. These works illustrate the non-triviality of the welfare effects of information within the context of specific applications, but do not explain the general principles underlying the question of interest. We fill the gap here by showing how the social value of information depends, not only on the form of strategic interaction, but also on other external effects that determine the gap between equilibrium and efficient use of information.

The literature on rational expectations has emphasized how the aggregation of dispersed private information in markets can improve allocative efficiency (e.g., Grossman, 1981). Laffont (1985) and Messner and Vives (2001), on the other hand, highlight how *informational* externalities can generate inefficiency in the private collection and use of information. Although the information structure here is exogenous, the paper provides an input into this line of research by studying how the welfare effects of information depend on *payoff* externalities.

The paper also contributes to the debate about central-bank transparency. While earlier work focused on incentive issues (e.g., Canzoneri, 1985; Atkeson and Kehoe, 2001; Stokey, 2002), recent work emphasizes the role of coordination. Morris and Shin (2002, 2005) and Heinemann and Cornand (2004) argue that central-bank disclosures can reduce welfare if financial markets behave like beauty contests; Svensson (2005) and Woodford (2005) question the practical relevance of this result; Hellwig (2005) and Roca (2005) argue that disclosures improve welfare by reducing price dispersion. While all these papers focus exclusively on whether coordination is inefficiently high or not, we argue that perhaps a more important dimension is the source of the business cycle.

The rest of the paper is organized as follows. We introduce the model in Section 2. We examine the equilibrium use of information in Section 3, the efficient use of information in Section 4, and the social value of information in Section 5. We turn to applications in Section 6 and conclude in Section 7. The Appendix contains proofs omitted in the main text.

2 The model

Actions and payoffs. Consider an economy with a measure-one continuum of agents, each choosing an action $k \in \mathbb{R}$. Let Ψ denote the cumulative distribution function for k in the cross-section of the population, $K \equiv \int k d\Psi(k)$ the mean action, and $\theta = (\theta_1, ..., \theta_N) \in \mathbb{R}^N$ a vector of exogenous payoff-relevant variables (the fundamentals). The analysis is simplest when $N = 1$, but $N > 1$ allows us to capture the possibility that there are fundamentals that are relevant for equilibrium but not for efficient allocations, and vice versa—a possibility that, as shown in Section 5, is important in determining the social value of information.

An individual's payoff depends on his own action and on the fundamentals θ; it also depends on the distribution Ψ of others' actions through the mean action K. Payoffs are thus given by

$$u = U(k, K, \theta), \tag{1}$$

where $U : \mathbb{R}^{N+2} \to \mathbb{R}$. For notational convenience, we let $W(K, \theta) \equiv U(K, K, \theta)$ denote utility (also, welfare) when all agents choose the same action.

With complete information, the analysis could easily proceed under a general specification for U. With incomplete information, however, maintaining the analysis tractable requires some sacrifice in generality. We impose that U is quadratic, which ensures linearity of best responses as well as linearity in the structure of the efficient allocations.[2]

We further impose concavity at both the individual and aggregate level in the sense that $U_{kk} < 0$ and $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK} < 0$. Given our assumptions of quadratic payoffs and unbounded k, if U were not concave, best responses would not be well-defined; similarly, if W were not concave, the first-best allocation would not be well-defined. We finally restrict $-U_{kK}/U_{kk} < 1$. Since $-U_{kK}/U_{kk}$ will turn out to be the slope of the best response of an agent with respect to aggregate K, this condition is essentially equivalent to imposing that the equilibrium is unique.[3]

To recap, the key restrictions that our model imposes on the payoff structure are concavity of payoffs and uniqueness of equilibrium. Other than these restrictions, the payoff structure is quite flexible: it allows for either strategic complementarity ($U_{kK} > 0$) or strategic substitutability ($U_{kK} < 0$), as well as for positive or negative externality ($U_K \neq 0$). As for the quadratic specification, this is certainly crucial for the tractability of our analysis, but (we conjecture) not key to our results. It might be viewed as a second-order approximation of a more general class of concave, unique-equilibrium, economies.

Information. Following the pertinent literature, we introduce incomplete information by assuming that agents observe noisy private and public signals about the underlying fundamentals.

[2] Suppose that there are finitely many agents, say $J < \infty$, and that agent i has a payoff $\tilde{U}(k_i, k_{-i}, \theta)$, where $k_{-i} \equiv (k_j)_{j \neq i}$. Also assume that $\tilde{U}$ is symmetric in k_{-i} in the sense that, for any k_{-i} and k'_{-i} such that k'_{-i} is a permutation of k_{-i}, $\tilde{U}(k_i, k_{-i}, \theta) = \tilde{U}(k_i, k'_{-i}, \theta)$. Then, if $\tilde{U}$ is quadratic, it can always be written as $\tilde{U} = U(k_i, K_{-i}, \sigma^2_{-i}, \theta)$, where U is quadratic in (k_i, K_{-i}, θ) and linear in σ^2_{-i}, with $K_{-i} \equiv \frac{1}{J-1}\sum_{j \neq i} k_j$ and $\sigma^2_{-i} \equiv \frac{1}{J-1}\sum_{j \neq i}(k_j - K_{-i})^2$. The continuum-player limit is $U(k, K, \theta, \sigma^2_k)$ where $\sigma^2_k \equiv \int (k-K)^2 d\Psi(k)$ is the second moment of Ψ and U is quadratic in $(k, K, \theta,)$ and linear in σ^2_k. To simplify, we initially drop the dependence of U on σ^2_k, but reintroduce it at the end of Section 4.

[3] Let $\alpha \equiv -U_{kK}/U_{kk}$. To be precise, our model admits a unique equilibrium, under complete information, whenever $\alpha \neq 1$; but for $\alpha > 1$ this uniqueness is an artifact of the simplifying assumption that the action space is unbounded. To see this, consider a variant of our model where k is restricted to $[-M, +M]$ for some $M \in (0, \infty)$. Then, the equilibrium is unique if and only if $\alpha < 1$. This is true no matter M , provided that $M < \infty$. Since taking literally the simplifying assumption that $M = \infty$ would be naive, we view the restriction $\alpha < 1$ as essentially equivalent to uniqueness. See the Supplementary Material for a detailed discussion of this issue, as well as for a discussion of the role that the restriction $\alpha < 1$ plays for comparative statics.

Before agents move, nature draws θ_n, for $n \in \mathbf{N} \equiv \{1, ..., N\}$, from independent Normal distributions with mean μ_n and variance $\sigma_{\theta_n}^2$. The realization of $\theta = (\theta_n)$ is not observed by the agents. Instead, for each n, agents observe private signals $x_n^i = \theta_n + \xi_n^i$ and a public signal $y_n = \theta_n + \varepsilon_n$, where ξ_n^i and ε_n are, respectively, idiosyncratic and common noises, independent of one another as well as of θ, with variances $\sigma_{x_n}^2$ and $\sigma_{y_n}^2$. (Throughout, we use the convenient vector notation $x = (x_n)$, $y = (y_n)$, and similarly for all other variables; we also drop the superscript i whenever it does not create confusion.)[4]

The common posterior for θ_n given public information alone is Normal with mean $z_n \equiv \mathbb{E}[\theta_n|y] = \lambda_n y_n + (1 - \lambda_n)\mu_n$ and variance $\sigma_{z_n}^2$, where $\lambda_n \equiv \sigma_{y_n}^{-2}/\sigma_{z_n}^{-2}$ and $\sigma_{z_n} \equiv (\sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2})^{-1/2}$. In what follows we will often identify public information with z rather than y. Private posteriors, on the other hand, are Normal with mean $\mathbb{E}[\theta_n|x^i, y] = (1 - \delta_n)x_n^i + \delta_n z_n$ and variance $Var[\theta_n|x^i, y] = \sigma_n^2$, where

$$\sigma_n^{-2} \equiv \sigma_{x_n}^{-2} + \sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2} > 0 \quad \text{and} \quad \delta_n \equiv \frac{\sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2}}{\sigma_{x_n}^{-2} + \sigma_{y_n}^{-2} + \sigma_{\theta_n}^{-2}} \in (0, 1). \tag{2}$$

Accuracy and commonality. If we let $\omega_n^i \equiv \theta_n - \mathbb{E}[\theta_n|x^i, y]$ denote agent i's forecast error about θ_n, then we can show that

$$\sigma_n^2 = Var\left(\omega_n^i\right) \qquad \text{and} \qquad \delta_n = Corr(\omega_n^i, \omega_n^j),\ i \neq j.$$

That is, σ_n^2 measures the total noise in agents' forecasts about the fundamentals, while δ_n measures the extent to which noise is correlated across agents. We accordingly identify σ_n^{-2} with the *accuracy* of information and δ_n with its *commonality.*

Note that any given change in the information structure can be decomposed to a change in accuracy and a change in commonality. For example, an increase in the precision of public information $\sigma_{z_n}^{-2}$ (for given private $\sigma_{x_n}^{-2}$) combines an increase in accuracy with an increase in commonality, whereas an increase in the precision of private information (for given public) combines an increase in accuracy with a reduction in commonality. For many applied questions, one is interested in comparative statics with respect to the precision of public and private information—and this is also what we do when we turn to applications in Section 6. However, for our main theoretical analysis, we prefer to parametrize the information structure by (δ_n, σ_n) for two reasons.

First, as we show in Section 3, this is most appropriate for understanding the properties of the equilibrium use of information. When there are no payoff interdependencies across agents, the distinction between private and public information is irrelevant—all that matters for welfare

[4] The assumption that (θ_n) are orthogonal to each other is only a normalization: if (θ_n) were correlated, there would exist a linear one-to-one transformation $(\theta_n) \mapsto (\theta_n')$ such that (θ_n') are orthogonal. The orthogonality in the errors (ξ_n, ε_n) across n, on the other hand, permits us to interpret (x_n, y_n) as signals about θ_n: if the errors were correlated across n, then (x_n, y_n) would include information also for $n' \neq n$.

is the level of noise, not its composition. With strategic interactions, instead, the commonality of information becomes crucial, for it affects the agents' ability to forecast one another's actions—and it is only in this sense that public information is different than private.

Second, as we show in Sections 5 and 6, decomposing the effects of public and private information into those of accuracy and commonality turns out to be illuminative even when one is ultimately interested in the total effect. For example, in economies where inefficiency emerges only due to the incompleteness of information, welfare unambiguously increases with accuracy, and hence more precise private or public information can possibly reduce welfare only through an adverse common effect through the commonality of information.

3 Equilibrium use of information

Definition and characterization. Each agent chooses k so as to maximize his expected utility, $\mathbb{E}[U(k,K,\theta)|x,y]$. The solution to this optimization problem gives the best response for the individual. The fixed point is the equilibrium.

Definition 1 *An equilibrium is any strategy $k:\mathbb{R}^{2N}\to\mathbb{R}$ such that, for all (x,y),*

$$k\left(x,y\right)=\arg\max_{k'}\ \mathbb{E}[\ U(k',K(\theta,y),\theta)\ |\ x,y\ ], \tag{3}$$

where $K(\theta,y)=\mathbb{E}[k\left(x,y\right)|\theta,y]$ for all (θ,y).[5] *A linear equilibrium is any strategy satisfying (3) that is linear in x and y.*

It is useful to consider first the complete-information benchmark. When θ is known, the (unique) equilibrium is $k=\kappa\left(\theta\right)$, where $\kappa\left(\theta\right)$ is the unique solution to $U_k\left(\kappa,\kappa,\theta\right)=0$. Since U is quadratic, κ is linear: $\kappa\left(\theta\right)=\kappa_0+\kappa_1\theta_1+...+\kappa_N\theta_N$, where $\kappa_0\equiv-U_k\left(0,0,0\right)/(U_{kk}+U_{kK})$ and $\kappa_n\equiv-U_{k\theta_n}/(U_{kk}+U_{kK})$, $n\in\mathbf{N}$. It follows that $\kappa_n\neq0$ if and only if $U_{k\theta_n}\neq0$. The incomplete-information equilibrium is then characterized as follows.

Proposition 1 *Let $\kappa\left(\theta\right)=\kappa_0+\kappa_1\theta_1+...+\kappa_N\theta_N$ denote the complete-information equilibrium allocation and*

$$\alpha\equiv\frac{U_{kK}}{|U_{kk}|}. \tag{4}$$

(**i**) *A strategy $k:R^{2N}\to\mathbb{R}$ is an equilibrium if and only if, for all (x,y),*

$$k(x,y)\ \ =\ \ \mathbb{E}[\ (1-\alpha)\cdot\kappa\left(\theta\right)+\alpha\cdot K\left(\theta,y\right)\ \ |\ x,y\ ] \tag{5}$$

where $K(\theta,y)=\mathbb{E}[k\left(x,y\right)|\ \theta,y]$ for all (θ,y).

[5] A state of the world is given by the realizations of θ, y, and $\{x^i\}_{i\in[0,1]}$. However, since ξ^i are i.i.d. across agents, K, as well as any other aggregate variable, is a function of (θ,y) alone.

(**ii**) *A linear equilibrium exists, is unique, and is given by*

$$k(x,y) \;=\; \kappa_0 + \sum_{n\in\mathcal{N}} \kappa_n \left[(1-\gamma_n)x_n + \gamma_n z_n\right], \tag{6}$$

where $\mathcal{N} \equiv \{n \in \mathbf{N} : \kappa_n \neq 0\}$[6] *and*

$$\gamma_n \;=\; \delta_n + \frac{\alpha\delta_n(1-\delta_n)}{1-\alpha(1-\delta_n)} \quad \forall n \in \mathcal{N}. \tag{7}$$

Part (i) states that any equilibrium (linear or not) has to be a fixed point to condition (5). This condition has a simple interpretation: it says that an agent's best response is an affine combination of his expectation of some given "target" and his expectation of aggregate activity. The target is simply the complete-information equilibrium, $\kappa(\theta)$. The slope of the best response with respect to aggregate activity, α, is what we identify with the *equilibrium degree of coordination.*

Part (ii) establishes that there exists a unique linear fixed point to (5), but leaves open the possibility that there exist other non-linear fixed points. Since the best response of an agent is linear in his expectations of θ and K, and since his expectation of θ in turn is linear in x and y (or, equivalently, in x and z), it is natural to conjecture that there do not exist fixed points other than the linear one, so that the equilibrium is unique even outside the linear class. This conjecture can be verified for the case $\alpha \in (-1,1)$, following the same argument as in Morris and Shin (2002).[7,8]

As evident in (7), the sensitivity of the equilibrium to private and public information depends, not only on the relative precision of the two (captured by δ_n), but also on the degree of coordination, α. When $\alpha = 0$, (6) reduces to

$$k(x,y) = \mathbb{E}[\kappa(\theta)\,|x,y].$$

That is, when $\alpha = 0$, the incomplete-information equilibrium strategy is simply the best predictor of the complete-information equilibrium allocation and the weights on signals x_n and z_n are simply the Bayesian weights: $\gamma_n = \delta_n$ if $\alpha = 0$. When, instead, $\alpha \neq 0$, equilibrium behavior is tilted towards public or private information depending on whether agents' actions are strategic complements or substitutes: $\gamma_n > \delta_n$ if $\alpha > 0$, and $\gamma_n < \delta_n$ if $\alpha < 0$.

To understand why this is the case, consider the best response of an agent to a given strategy by the other agents. To simplify, let $N = 1$ and $\kappa(\theta) = \theta$. When other agents follow a linear

[6] In the sequel, we restrict attention to economies in which $\mathcal{N} \neq \varnothing$ which rules out the trivial case where the fundamentals are irrelevant for equilibrium.

[7] To be precise, the argument in Morris and Shin (2002) is incomplete in that it presumes that $\alpha^t \overline{E}^t K \to 0$ as $t \to \infty$, which was not proved. (Here $\overline{E}^t$ denotes the $t-$th order iteration of the average-expectation operator.) With $\alpha \in (-1,1)$, $\alpha^t \to 0$ as $t \to \infty$, but one needs also to ensure that $\overline{E}^t K$ remains bounded. Since K is unbounded in Morris and Shin (2002), this is not obvious; however, this problem is easily bypassed by imposing bounds on the action space.

[8] None of our results relies on $\alpha \leq -1$. Hence, if uniqueness is a concern, one can restrict our analysis to the case $\alpha \in (-1,+1)$.

strategy $k(x,y) = (1-\gamma)x + \gamma z$, then the mean action is $K(\theta,y) = (1-\gamma)\theta + \gamma z$, and hence an agent's best response to this strategy is

$$\begin{aligned} k'(x,y) &= \mathbb{E}\left[(1-\alpha)\theta + \alpha K(\theta,y) \mid x,y\right] \\ &= (1-\alpha\gamma)\mathbb{E}\left[\theta|x,y\right] + \alpha\gamma z \\ &= (1-\gamma')x + \gamma' z \end{aligned}$$

where $\gamma' = \delta + \alpha\gamma(1-\delta)$. Thus, as long as other agents put a positive weight on public information ($\gamma > 0$) and actions are strategic complements ($\alpha > 0$), the best response is to put a weight on z higher than the Bayesian one $(\gamma' > \delta)$, and the more so the higher the other agents' weight or the stronger the complementarity. And, symmetrically, the converse is true for the case of strategic substitutability ($\alpha < 0$). The reason is that public information is a relatively better predictor of others' activity than private information: this leads an agent to adjust upwards his reliance on public information when he wishes to align his choice with other agents' choices (which is the case when $\alpha > 0$), and downwards when he wishes to differentiate his choice from others' (which is the case when $\alpha < 0$).

This property of the best responses is reflected in the equilibrium strategy: when $\alpha > 0$, the term $\frac{\alpha\delta_n(1-\delta_n)}{1-\alpha(1-\delta_n)}$ in condition (7) measures the *excess sensitivity* of equilibrium allocations to public information as compared to the case where there are no strategic effects; and when $\alpha < 0$, it measures the excess sensitivity to private information. This term is increasing in α : stronger complementarity leads to higher relative sensitivity to public information, stronger substitutability to lower.

Volatility and dispersion. If information were complete (i.e., $\sigma_n = 0$ for all $n \in \mathbf{N}$, or at least for all $n \in \mathcal{N}$), then all agents would choose $k = K = \kappa(\theta)$. Incomplete information affects equilibrium behavior in two ways. First, common noise generates *non-fundamental volatility*, that is, variation in aggregate activity around the complete-information level. Second, idiosyncratic noise generates *dispersion*, that is, variation in the cross-section of the population. The first is measured by $Var(K-\kappa)$, the second by $Var(k-K)$. Their dependence on the degree of coordination and the information structure is characterized below.

Proposition 2 ***(i)*** *A higher complementarity α decreases dispersion and increases volatility.*

(ii) *A higher accuracy σ_n^{-2} decreases both dispersion and volatility.*

(iii) *A higher commonality δ_n decreases dispersion if and only if $\alpha > -\frac{1}{1-\delta_n}$, whereas it increases volatility if and only if $\alpha < \frac{1}{1+\delta_n}$.*

Higher complementarity, by leading to a better alignment of individual choices, mitigates dispersion, but amplifies volatility. Higher accuracy necessarily reduces both volatility and dispersion. Higher commonality tends to reduce dispersion, but at the expense of higher volatility. This trade off between dispersion and volatility emerges provided that $-1/(1-\delta_n) \leq \alpha \leq 1/(1+\delta_n)$, which in turn is the case for all δ_n when $\alpha \in [-1, +1/2]$ and for a region of δ_n when $\alpha < -1$ or $\alpha \in (+1/2, 1)$. In what follows, we consider the case that higher commonality decreases dispersion at the expense of volatility as the canonical case for our informal discussion; our formal results, however, cover the other case as well.

We will examine in more detail the welfare effects of information—and the special role that volatility and dispersion play in this respect—in Section 5. In the next section, we first turn to the characterization of the efficient use of information, and its comparison to equilibrium.

4 Efficient use of information

Our efficiency concept. The property that complementarity amplifies volatility by heightening the sensitivity of equilibrium to common noise [or, symmetrically, that substitutability raises dispersion by heightening the sensitivity to idiosyncratic noise] is interesting on its own. But this is only a *positive* property. To address the *normative* question of whether these effects are socially undesirable, one needs to compare the equilibrium use of information to some efficiency benchmark. The one we propose here is the allocation that maximizes ex-ante utility subject to the sole constraint that information cannot be communicated across agents.

Definition 2 *An efficient allocation is a strategy $k : \mathbb{R}^{2N} \to \mathbb{R}$ that maximizes ex-ante utility,*

$$\mathbb{E}u = \int_{(\theta,y)} \int_x U(k(x,y), K(\theta,y), \theta) dP(x|\theta,y) dP(\theta,y),$$

with

$$K(\theta,y) = \int_x k(x,y)\, dP(x|\theta,y), \text{ for all } (\theta,y),$$

where $P(\theta,y)$ stands for the c.d.f. of the joint distribution of (θ,y) and $P(x|\theta,y)$ for the conditional distribution of x given θ and y.

The allocation defined above can be understood as the solution to a "team problem", where agents get together before they receive their information, choose cooperatively a strategy for how to use the information they will receive, and then follow this strategy. Equivalently, this allocation is the solution to a "planner's problem", where the planner can not make an agent's action depend on other agents' private information, but can otherwise perfectly control how an agent's action depend on his own information.[9]

[9] It is as if the planner can send a supervisor to each agent, with the instruction to perfectly monitor how the latter uses his information, but where the supervisors cannot exchange information.

Our efficiency concept thus answers exactly the question of interest here, namely what is the best a society could do if agents were to internalize their payoff interdependencies and appropriately adjust their use of available information, but without aggregating information beyond what is already done in the available public information. We believe that this notion of efficiency is appropriate for the purposes of this paper. An alternative could have been to allow the planner to solicit information from the agents and possibly communicate some (or all) of it to them. If the planner could also perfectly control the agents' actions, he could then obtain the first best.[10] However, comparing the first best to the incomplete-information equilibrium does not permit us to separate the inefficiency that originates in the incompleteness of information from the one that originates in the way equilibrium processes available information. Since our goal here is precisely to isolate the latter, we find our notion of efficiency more useful.

In certain environments, our efficiency benchmark may also admit an appealing implementation. In particular, suppose that we extend agents' preferences to $\tilde{U}(k, K, \theta, t) = U(k, K, \theta) + t$, where t is a monetary transfer from the government. Suppose further that the government can observe individual choices ex post and can commit ex ante (before agents receive any information) to a budget-balanced transfer scheme, according to which an agent will receive a net transfer $t = T(k, K, \theta)$. As we show after Proposition 4 below, the transfer scheme that maximizes ex-ante welfare implements our efficiency benchmark. What is more, the optimal scheme turns out to be linear in k; that is, our efficiency benchmark can be implemented with the combination of a linear tax/subsidy and a lump-sum transfer. Our efficiency concept may thus provide guidance also for policy. However, this is beyond the scope of this paper—here we only use it to identify the sources of inefficiency in the decentralized use of information, which, as we show in the next section, help us understand the social value of information in equilibrium.

Once again, the emphasis here is on the decentralization of information rather than on moral hazard. Our efficiency concept is thus different from more standard constrained-efficiency concepts that assume costless communication and instead focus on incentive constraints (e.g., Mirrlees, 1971; Holmstrom and Myerson, 1983). Instead, it shares with Hayek (1945) and Radner (1962) the idea that information is dispersed and can not be communicated to a "center".

Characterization. We now turn to the characterization of the efficient use of information.

Lemma 1 *An allocation $k : \mathbb{R}^{2N} \to \mathbb{R}$ is efficient if and only if, for almost all (x, y),*

$$\mathbb{E}[\ U_k(k(x,y), K(\theta,y), \theta) + U_K(K(\theta,y), K(\theta,y), \theta) \mid x, y\ ] \ = \ 0, \tag{8}$$

where $K(\theta, y) = \mathbb{E}[k(x, y) | \theta, y]$ for all (θ, y).

[10] To see this, note that the planner could simply ask each agent to report his private signal x, learn the true state θ by aggregating the private signals, and then dictate to each agent to take the first-best action $\kappa^*(\theta)$. Since, with a continuum of agents, the allocation $\kappa^*(\theta)$ does not depend on any particular agent's own report, truthful revelation is a weakly dominant strategy.

This result has a simple interpretation. Recall that the first-best allocation $\kappa^*(\theta)$ maximizes $W(K,\theta) \equiv U(K,K,\theta)$. It thus solves the first-order condition $W_K(K,\theta) = 0$, or equivalently $U_k(K,K,\theta) + U_K(K,K,\theta) = 0$.[11] The incomplete-information counterpart of this condition is (8).

We can then expand this condition to characterize the efficient allocation under incomplete information in a similar fashion as with equilibrium.

Proposition 3 *Let $\kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta_1 + ... + \kappa_N^*\theta_N$ denote the first-best allocation and*

$$\alpha^* \equiv \frac{2U_{kK} + U_{KK}}{|U_{kk}|} = 2\alpha + \frac{U_{KK}}{|U_{kk}|}. \tag{9}$$

(**i**) *An allocation $k : \mathbb{R}^{2N} \to \mathbb{R}$ is efficient if and only if, for almost all (x, y),*

$$k(x,y) \ = \ \mathbb{E}[\ (1-\alpha^*)\kappa^*(\theta) + \alpha^* K(\theta,y) \mid x,y\ ] \tag{10}$$

where $K(\theta,y) = \mathbb{E}[k(x,y)|\theta,y]$ for all (θ,y).

(**ii**) *The efficient allocation exists, is unique,*[12] *and is given by*

$$k(x,y) = \kappa_0^* + \sum_{n\in\mathcal{N}^*} \kappa_n^* \left[(1-\gamma_n^*)x_n + \gamma_n^* z_n\right], \tag{11}$$

where $\mathcal{N}^* \equiv \{n \in \mathbf{N} : \kappa_n^* \neq 0\}$ and

$$\gamma_n^* = \delta_n + \frac{\alpha^*\delta_n(1-\delta_n)}{1-\alpha^*(1-\delta_n)} \qquad \textit{for all } n \in \mathcal{N}^*. \tag{12}$$

This result characterizes the efficient allocation among all possible strategies, not only linear ones; that the efficient strategy turns out to be linear is because of the combination of quadratic payoffs and Gaussian information structure.

In equilibrium, each agent's action was an affine combination of his expectation of κ, the complete-information equilibrium, and of his expectation of aggregate activity. The same is true for the efficient allocation if we replace κ with κ^* and α with α^*. In this sense, condition (10) is the analogue for efficiency of what the best response is for equilibrium. This idea is formalized by the following.

Proposition 4 *Given an economy $\mathbf{e} = (U;\sigma,\delta,\mu,\sigma_\theta)$, let $\mathcal{U}(\mathbf{e})$ be the set of payoffs U' such that the economy $\mathbf{e}' = (U';\sigma,\delta,\mu,\sigma_\theta)$ admits an equilibrium that coincides with the efficient allocation for $\mathbf{e}$.*

(**i**) *For every $\mathbf{e}$, $\mathcal{U}(\mathbf{e})$ is non-empty.*

(**ii**) *For every $\mathbf{e}$, $U' \in \mathcal{U}(\mathbf{e})$ only if $\alpha' \equiv -U'_{kK}/U'_{kk} = \alpha^*$.*

[11] Since U and hence W is quadratic, $\kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta_1 + ... + \kappa_N^*\theta_N$, where $\kappa_0^* = -W_K(0,0)/W_{KK}$ and $\kappa_n^* = -W_{K\theta_n}/W_{KK}$, $n \in \mathbf{N}$. It follows that $\kappa_n^* \neq 0$ if and only if $W_{K\theta_n} \equiv U_{k\theta_n} + U_{K\theta_n} \neq 0$.

[12] To be precise, the efficient allocation is uniquely determined for *almost* every (x,y).

Part (i) says that the efficient allocation of any given economy $\mathbf{e}$ can be implemented as a linear equilibrium of a fictitious game $\mathbf{e}'$ in which the information structure is the same as in $\mathbf{e}$ but where individual incentives are manipulated so as to coincide with the social incentives of the actual economy. Indeed, since our efficiency concept allows the planner to perfectly control the incentives of the agents, it is as if the planner (whose objective is the true U) can design the payoffs U' perceived by the agents. Part (ii) then explains why we identify α^* with the *optimal degree of coordination*: α^* describes the level of complementarity (if $\alpha^* > 0$) or substitutability (if $\alpha^* < 0$) that the planner would like the agents to perceive for the equilibrium of the fictitious game to coincide with the efficient allocation of the true economy.[13]

In this paper we use Proposition 4 only to give a precise *meaning* to our notion of the socially optimal degree of coordination. However, this proposition also provides an *implementation* result for environments where preferences can be extended to $\tilde{U}(k, K, \theta, t) = U(k, K, \theta) + t$, with t being a monetary transfer, and where the planner can control the incentives of the agents through a (contingent) transfer scheme $t = T(k, K, \theta)$.[14] The payoffs perceived by the agents are then $U'(k, K, \theta) \equiv U(k, K, \theta) + T(k, K, \theta)$ and hence the degree of coordination can be controlled by appropriately choosing T_{kK}, the cross-partial of the transfer with respect to (k, K). Consider then the transfer scheme defined by $T^*(k, K, \theta) \equiv \tau^*(K, \theta)\, k - \tau^*(K, \theta)\, K$, where $\tau^*(K, \theta) \equiv U_K(K, K, \theta)$. By construction, T^* balances the budget and is linear in k. Moreover, the payoff function U^* defined by $U^*(k, K, \theta) \equiv U(k, K, \theta) + T^*(k, K, \theta)$ belongs to $\mathcal{U}(\mathbf{e})$, which means that T^* implements the efficient allocation (see the Appendix for details). This transfer scheme is thus the analogue for incomplete-information economies of what a Pigou scheme is for complete-information economies.

The counterpart of the optimal degree of coordination is the efficient sensitivity to public information: by condition (12), the higher the optimal degree of coordination, the higher the sensitivity of efficient allocations to public information relative to private. Comparing the equilibrium to the efficient use of information then gives the following result.

Corollary 1 *The sensitivity of the equilibrium allocation to public noise is inefficiently high if and only if the equilibrium degree of coordination is higher than the optimal one, which in turn is true if and only if the complementarity is low enough relative to second-order non-strategic effects: for all $n \in \mathcal{N} \cap \mathcal{N}^*$,*

$$\gamma_n \geq \gamma_n^* \iff \alpha \geq \alpha^* \iff U_{kK} \leq -U_{KK}. \tag{13}$$

[13] Note that only the linear equilibrium of the fictitious game $\mathbf{e}'$ can coincide with the efficient allocation of the true economy $\mathbf{e}$ This is because the (unique) efficient allocation of $\mathbf{e}$ is linear. Part (ii) of Proposition 4 thus also implies that, whenever $\alpha^* \in (-1, +1)$, for any $U' \in \mathcal{U}(\mathbf{e})$, the equilibrium of $\mathbf{e}'$ is unique.

[14] Think of this transfer taking place at the very end of the game, after agents' decisions have been committed and uncertainty resolved.

Proposition 3 and Corollary 1 show how the efficient use of information depends on the primitives of the environment, and how it compares to the equilibrium one. As with equilibrium, the optimal degree of coordination is increasing in U_{kK}, the level of complementarity. But unlike equilibrium, the optimal degree of coordination depends also on U_{KK}, a second-order external effect that does not affect private incentives. In the absence of such an effect, $\alpha^* = 2\alpha$: the optimal degree of coordination is higher (in absolute value) than the equilibrium one, reflecting the internalization of the externality associated with the complementarity.

The volatility-dispersion trade-off, and the optimal degree of coordination. To understand better the forces behind the determination of the optimal degree of coordination, an alternative representation is useful. Welfare (ex-ante utility) at the efficient allocation can be expressed as $\mathbb{E}u = \mathbb{E}W(\kappa^*, \theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \frac{|W_{KK}|}{2} Var(K - \kappa^*) + \frac{|U_{kk}|}{2} Var(k - K). \tag{14}$$

Note that $\mathbb{E}W(\kappa^*, \theta)$ is expected welfare at the first-best allocation, whereas $\mathcal{L}^*$ captures the welfare losses associated with incomplete information, namely those due to aggregate volatility and cross-sectional dispersion.[15]

That volatility and dispersion generate welfare losses follows directly from concavity of preferences. Naturally, the weight on volatility is given by W_{KK}, the curvature of welfare with respect to aggregate activity, while the weight on dispersion is given by U_{kk}, the curvature of utility with respect to individual activity. Note that $W_{KK} = U_{kk} + 2U_{kK} + U_{KK}$. When there are no strategic and second-order external effects (in the sense that $U_{kK} = U_{KK} = 0$), aggregate welfare inherits the curvature of individual utility ($W_{KK} = U_{kk}$), so that volatility and dispersion contribute equally to welfare losses. Complementarity ($U_{kK} > 0$) alleviates aggregate concavity by offsetting the diminishing returns faced at the individual level, and therefore lowers social aversion to volatility. The converse is true for substitutability ($U_{kK} < 0$) or external concavity ($U_{KK} < 0$).

When the planner controls how agents use information, it is as if he controls the degree of coordination perceived by the agents. In choosing the optimal degree of coordination, the planner then faces a trade off between dispersion and volatility. Indeed, as shown in Proposition 3, a higher degree of coordination means a higher sensitivity to public information and a lower sensitivity to private information; and since noise in public information generates volatility whereas noise in private information generates dispersion, a higher degree of coordination trades off higher volatility for lower dispersion. It is then not surprising that the optimal degree of coordination reflects social preferences over volatility and dispersion.

[15] Condition (14) follows from a Taylor expansion around $k = K = \kappa^*(\theta)$; see the Appendix.

Corollary 2 *The optimal degree of coordination increases with the social aversion to dispersion and decreases with the social aversion to volatility:*

$$\alpha^* = 1 - \frac{W_{KK}}{U_{kk}}. \tag{15}$$

Extension: external effect from dispersion. In some applications of interest, cross-sectional dispersion has a direct external effect on individual utility. For example, dispersion in prices has a negative effect on individual utility in New-Keynesian monetary models (see the discussion in Section 6.4 below).

We can easily accommodate such an effect—and we do so for the rest of the paper—provided that dispersion enters linearly in the utility function:

$$u = U(k, K, \theta, \sigma_k^2),$$

where $\sigma_k^2 \equiv \int (k-K)^2 d\Psi(k)$ is the cross-sectional dispersion and $U_{\sigma_k^2}$ is a scalar. In analogy to $W_{KK} < 0$, we impose $U_{kk} + 2U_{\sigma_k^2} < 0$; this is necessary and sufficient for the total effect of dispersion on welfare to be negative. Then all our results go through once we replace the welfare weight on dispersion with $U_{kk} + 2U_{\sigma_k^2}$. In particular, the welfare losses due to volatility and dispersion are now given by

$$\mathcal{L}^* = \frac{|W_{KK}|}{2} Var(K - \kappa^*) + \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} Var(k - K). \tag{16}$$

Accordingly, the optimal degree of coordination is

$$\alpha^* = 1 - \frac{W_{KK}}{U_{kk} + 2U_{\sigma_k^2}} = 1 - \frac{U_{kk} + 2U_{kK} + U_{KK}}{U_{kk} + 2U_{\sigma_k^2}}.$$

And finally, condition (13) becomes

$$\gamma_n \geq \gamma_n^* \quad \Longleftrightarrow \quad \alpha \geq \alpha^* \quad \Longleftrightarrow \quad U_{kK} \leq -U_{KK} + 2U_{\sigma_k^2}.$$

Note that α^* is increasing in U_{KK} and decreasing in $U_{\sigma_k^2}$. This is intuitive. A higher U_{KK} decreases the social cost of volatility, while a higher $U_{\sigma_k^2}$ decreases the social cost of dispersion. Both effects are external and non-strategic: they affect the social value of coordination without affecting private incentives. The former contributes to a higher optimal degree of coordination, the latter to a lower.

5 Social value of information

We now examine the comparative statics of equilibrium welfare with respect to the information structure. Throughout this section, when we refer to equilibrium, we mean the linear equilibrium characterized in Proposition 1.[16]

[16] As discussed earlier, uniqueness can be guaranteed for $\alpha \in (-1, +1)$, but we expect this to be the unique equilibrium even when $\alpha \leq -1$.

Definition 3 *Let $\mathbb{E}u$ denote ex-ante utility evaluated at the equilibrium allocation. The social value of commonality [resp. accuracy] is given by the partial derivative $\partial\mathbb{E}u/\partial\delta_n$ [resp., $\partial\mathbb{E}u/\partial\sigma_n^{-2}$]. Similarly, the social value of public [resp., private] information is given by $\partial\mathbb{E}u/\partial\sigma_{z_n}^{-2}$ [resp., $\partial\mathbb{E}u/\partial\sigma_{x_n}^{-2}$].*

We start with economies that are efficient under both complete and incomplete information, continue with economies that are inefficient only when information is incomplete, and conclude with the general case.

Three principles emerge through this taxonomy. First, even if one is ultimately interested in the comparative statics of equilibrium welfare with respect to the precisions of private and public information, it seems useful to decompose these comparative statics into their effects through the accuracy and the commonality of information. Second, the social value of accuracy relies crucially on the inefficiency (if any) of the complete-information equilibrium: accuracy can *not* reduce welfare if the complete-information equilibrium is efficient, no matter the equilibrium and optimal degrees of coordination. Third, the impact of commonality relies crucially on the relation between the equilibrium and the socially optimal degree of coordination: when the equilibrium degree of coordination is inefficiently high, commonality can reduce welfare even if the complete-information equilibrium is efficient.

Efficient economies. We first provide necessary and sufficient conditions for an economy to be efficient under incomplete information, that is, for its linear equilibrium to coincide with the efficient allocation.

Proposition 5 *The economy $\mathbf{e} = (U; \sigma, \delta, \mu, \sigma_\theta)$ is efficient if and only if U is such that*

$$\kappa(\theta) = \kappa^*(\theta)\ \forall\theta \quad \text{and} \quad \alpha = \alpha^*.$$

That is, if and only if U satisfies

$$U_{kK} + U_{KK} - 2U_{\sigma_k^2} = 0, \quad U_K(0,0,0) = \frac{U_{kK}}{U_{kk}} U_k(0,0,0), \quad \text{and} \quad U_{K\theta_n} = \frac{U_{kK}}{U_{kk}} U_{k\theta_n}\ \forall n \in \mathbf{N}.$$

The condition $\kappa = \kappa^*$ means that the equilibrium is efficient under *complete* information. But efficiency under complete information alone does not guarantee efficiency under *incomplete* information. What is also needed is efficiency in the equilibrium degree of coordination, namely $\alpha = \alpha^*$. Note that whether these two conditions are satisfied for any given economy depends on the payoff structure—indeed, κ, κ^*, α, and α^* are all functions of U—but not on the information structure. Also note that strategic effects alone do not imply inefficiency: the equilibrium can be efficient despite the fact that $\alpha \neq 0$.

We next show that efficient economies exhibit a clear relation between the form of strategic interaction and the social value of information.

Proposition 6 *Consider the class of economies in which the equilibrium is efficient under both complete and incomplete information (i.e.,* $\kappa = \kappa^*$ *and* $\alpha = \alpha^*$*). The following are true for any* $n \in \mathcal{N} = \mathcal{N}^*$ *:*

(i) *Welfare necessarily increases with accuracy* σ_n^{-2}*;*

(ii) *Welfare increases with commonality* δ_n *if* $\alpha > 0$ *, decreases with* δ_n *if* $\alpha < 0$*, and is independent of* δ_n *if* $\alpha = 0$*.*

As highlighted in the previous section, the impact of information on welfare at the efficient allocation is summarized by the impact of noise on volatility and dispersion (see Condition (16)). An increase in accuracy (for given commonality) reduces both volatility and dispersion and therefore necessarily increases welfare. On the other hand, an increase in commonality (for given accuracy) is equivalent to a reduction in dispersion, at the expense of volatility. Such a substitution is welfare-improving if and only if the social cost of dispersion is higher than that of volatility, which is the case in efficient economies if and only if α $(= \alpha^*)$ is positive.[17]

Consider now the welfare effects of an increase in the precision of private or public information. Both private and public information have symmetric effects on the accuracy of information, but opposing effects on commonality; and while accuracy necessarily increases welfare, the impact of commonality depends on α. Nevertheless, the accuracy effect necessarily dominates, thus ensuring that welfare increases with either private or public information. To see why this is the case, note that, when the equilibrium allocation is efficient, it coincides with the solution to a planner's problem. This planner can never be worse off with an increase in either $\sigma_{z_n}^{-2}$ or $\sigma_{x_n}^{-2}$, for he could always replicate the initial distributions of y and x by simply adding noise to the new distributions.[18] This argument, which is essentially Blackwell's theorem applied to our planner's problem, ensures that *any* source of information is welfare-improving when the equilibrium use of information is efficient, no matter the form of strategic interaction.

At the same time, the form of strategic interaction matters for the *relative* value of different sources of information. Complementarity, by generating a positive value for commonality, raises the value of public information relative to that of private, while the converse holds for substitutability.

Proposition 7 *Consider economies in which the equilibrium is efficient under both complete and incomplete information. The following are true for any* $n \in \mathcal{N} = \mathcal{N}^*$ *:*

(i) *Welfare increases with the precision of either private or public information about* θ_n*, no matter the degree of complementarity or substitutability.*

[17]The informal discussion here refers to the canonical case $\alpha \in (-\frac{1}{1-\delta_n}, \frac{1}{1+\delta_n})$. From Proposition 2, both volatility and dispersion increase with δ_n when $\alpha < -\frac{1}{1-\delta_n}$, whereas they both decrease when $\alpha > \frac{1}{1+\delta_n}$. Thus welfare necessarily decreases with commonality in the former case and increases in the latter, no matter α^*.

[18]The planner's problem we defined in the previous section did not give the planner the option to add such noise. However, if we were to give the planner such an option, he would never use it, because of concavity of W.

(ii) *The relative value of public information is given by*

$$\frac{\partial \mathbb{E}u/\partial\sigma_{z_n}^{-2}}{\partial \mathbb{E}u/\partial\sigma_{x_n}^{-2}} = \frac{\sigma_{x_n}^{-2}}{(1-\alpha)\,\sigma_{z_n}^{-2}}$$

and is increasing in the complementarity α.

To recap, economies where the equilibrium use of information is efficient provide a very clear benchmark for the welfare effects of information: first, any source of information is welfare improving; second, complementarity favors public sources of information, while substitutability favors private ones.

Economies that are efficient under complete information. Consider next the case $\alpha \neq \alpha^*$ but $\kappa = \kappa^*$. This case is of special interest, for it identifies economies where inefficiency emerges *only* because of the incompleteness of information—these are economies where the equilibrium coincides with the first best on average (in the sense that $\mathbb{E}K = \mathbb{E}\kappa^*$) but it fails to be efficient in its response to noise (in the sense that $\gamma \neq \gamma^*$). As we show below, this type of inefficiency crucially affects the social value of commonality, but not that of accuracy.

In this class of economies, the welfare losses associated with incomplete information continue to be the weighted sum of volatility and dispersion, as in (16).[19] Once again, consider the "canonical" case where a higher commonality reduces dispersion at the expense of higher volatility. For given α, and hence given equilibrium strategies and given volatility and dispersion, a higher α^* means only a lower relative weight on volatility, and hence a lower social cost associated with an increase in δ_n. It follows that, relatively to the case where $\alpha^* = \alpha$, inefficiently low coordination ($\alpha^* > \alpha$) increases the social value of commonality, whereas inefficiently high coordination ($\alpha^* < \alpha$) reduces it. Combined with the result in Proposition 6 that, when $\alpha^* = \alpha$, welfare increases with δ_n if and only if $\alpha > 0$, we have that $\alpha^* \geq \alpha > 0$ suffices for the social value of commonality to be positive, whereas $\alpha^* \leq \alpha < 0$ suffices for it to be negative.

As for the social value of accuracy, since a higher σ_n^{-2} reduces both volatility and dispersion, welfare continues to increase with accuracy. Thus, the possibility that the equilibrium degree of coordination is inefficient critically affects the social value of commonality, but not that of accuracy.

Proposition 8 *Consider the class of economies in which the equilibrium is inefficient only when information is incomplete (i.e., $\alpha \neq \alpha^*$ but $\kappa = \kappa^*$). The following are true for any $n \in \mathcal{N} = \mathcal{N}^*$:*

(i) *Welfare necessarily increases with the accuracy σ_n^{-2}.*

(ii) *Welfare increases with the commonality δ_n if $\alpha^* \geq \alpha > 0$ and decreases if $\alpha^* \leq \alpha < 0$.*

[19] As obvious from the derivation of (14) in the Appendix, (14) and similarly (16) extend to $\alpha \neq \alpha^*$ as long as $\kappa = \kappa^*$. This can also be seen from (17) below noting that $W_K(\kappa, \theta) = 0$ when $\kappa = \kappa^*$.

When the equilibrium sensitivity to different sources of noise is inefficient , it is possible that welfare decreases with an increase in the precision of a specific source of information; the result above sheds light on when this is possible.

Corollary 3 *Consider the class of economies in which the equilibrium is inefficient only when information is incomplete. The following are true for any* $n \in \mathcal{N} = \mathcal{N}^*$:

(i) *Welfare can decrease with the precision of public [private] information about* θ_n *only if it decreases [increases] with its commonality.*

(ii) $\alpha^* \geq \alpha \geq 0$ *suffices for welfare to increase with the precision of public information, whereas* $\alpha^* \leq \alpha \leq 0$ *suffices for it to increase with the precision of private information.*

General case. Finally, consider the case in which $\kappa \neq \kappa^*$. This identifies economies in which the equilibrium remains inefficient even under complete information—that is, economies in which the equilibrium is inefficient not only in its sensitivity to different sources of noise but also on its average response to the true fundamentals. As we show next, the latter crucially affects the social value of accuracy.

Equilibrium welfare can now be expressed as $\mathbb{E}u = \mathbb{E}W(\kappa, \theta) - \mathcal{L}$, where

$$\mathcal{L} = -Cov\left(K - \kappa, W_K(\kappa, \theta)\right) + \frac{|W_{KK}|}{2} \cdot Var(K - \kappa) + \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \cdot Var(k - K) \tag{17}$$

are the losses due to incomplete information (relative to the complete-information equilibrium).[20] The last two terms in $\mathcal{L}$ are the familiar losses associated with volatility and dispersion (second-order effects). The covariance term, on the other hand, captures a novel first-order effect. When the complete-information equilibrium is efficient ($\kappa = \kappa^*$ and hence $W_K(\kappa, \theta) = 0$), the covariance term is zero; this is merely an implication of the fact that small deviations around a maximum have zero first-order effects. But when the complete-information equilibrium is inefficient due to distortions other than incomplete information ($W_K(\kappa, \theta) \neq 0$), the covariance term contributes to a welfare gain [or loss]; this is because a positive [negative] correlation between $K - \kappa$, the "error" in aggregate activity due to incomplete information, and $W_K(\kappa, \theta)$, the social return to activity, mitigates [exacerbates] the first-order losses associated with these distortions.

As shown in the Appendix (see the proof of Propositions 9 and 10), this covariance term can be expressed as

$$Cov\left(K - \kappa, W_K(\kappa, \theta)\right) \;=\; |W_{KK}| Cov\left(K - \kappa, \kappa^* - \kappa\right) \;=\; |W_{KK}| \sum_{n \in \mathcal{N}} \phi_n v_n \tag{18}$$

where, for all $n \in \mathcal{N}$,

$$\begin{aligned} \phi_n \;&\equiv\; \frac{\kappa_n^* - \kappa_n}{\kappa_n} \;=\; \frac{Cov\left(\ \kappa^* - \kappa, \kappa \mid \theta_{-n}\ \right)}{Var\left(\ \kappa \mid \theta_{-n}\ \right)}, \\ v_n \;&\equiv\; -\frac{1}{1 - \alpha + \alpha\delta_n} \kappa_n^2 \sigma_n^2 \;=\; Cov\left(\ K - \kappa, \kappa \mid \theta_{-n}\ \right), \end{aligned}$$

[20] Condition (17) follows from a Taylor expansion around $K = \kappa(\theta)$; see the Appendix.

with θ_{-n} standing for $(\theta_j)_{j\neq n}$. The coefficients v_n capture the covariance between $K-\kappa$, the aggregate "error" due to incomplete information, and κ, the complete-information equilibrium, whereas the coefficients ϕ_n capture the covariance between the latter and $\kappa^*-\kappa$, the efficiency gap under complete information.

Note that the coefficients ϕ_n do not depend on the information structure—they only depend on U—and effectively parameterize the inefficiency of the complete-information equilibrium. To see this more clearly, consider the case $N=1$ (a single fundamental variable). Then $\phi=0$ means either that $\kappa=\kappa^*$, or that the efficiency gap is constant, while $\phi>0$ means that the efficiency gap increases in the same direction with equilibrium, and the opposite for $\phi<0$.

The value of ϕ_n is crucial for the welfare effect of accuracy. To see this, note that a higher σ_n^{-2} always implies a v_n closer to zero, since more accuracy brings the incomplete-information activity K closer to its complete-information counterpart κ for any given θ. But how this affects welfare depends on whether getting K closer to κ also means getting K closer to κ^*; this in turn depends on the correlation between the complete-information equilibrium and the first best. Intuitively, less noise brings K closer to κ^* when $\phi_n>0$, but further away when $\phi_n<0$. As a result, the welfare contribution of a higher σ_n^{-2} through the covariance term in (17) is positive when $\phi_n>0$, but negative when $\phi_n<0$. Combining this with the effect of σ_n^{-2} on volatility and dispersion, we conclude that higher accuracy necessarily increases welfare when $\phi_n>0$ (i.e., when the correlation between complete-information equilibrium and first best is positive), but can reduce welfare when ϕ_n is sufficiently negative.

Proposition 9 (accuracy) *There exist functions $\underline{\phi}', \bar{\phi}' : (-\infty,1)^2 \to \mathbb{R}$, with $\underline{\phi}' \leq \bar{\phi}' < 0$, such that the following is true for any $n\in\mathcal{N}$:*

Welfare increases with accuracy σ_n^{-2} for all (σ_n,δ_n) if $\phi > \bar{\phi}'(\alpha,\alpha^*)$, and decreases if $\phi < \underline{\phi}'(\alpha,\alpha^*)$.

This result opens the door to the possibility that welfare decreases with *any* source of information, private or public—a possibility that contrasts sharply with the Blackwell-like result we discussed above for efficient economies. As the following result shows, this is indeed the case when the covariance of the complete-information equilibrium with the first best is sufficiently low, in which case the first-order effects of accuracy dominate any other effects.

Corollary 4 *For any α, α^* and $n\in\mathcal{N}$, ϕ_n sufficiently high ensures that welfare increases with both the precision of private and public information about θ_n , whereas ϕ_n sufficiently low ensures the converse.*

Consider now the social value of commonality. While the sign of the social value of accuracy relies on ϕ_n, that of commonality relies on the product of ϕ_n with α. Indeed, the impact of δ_n on second-order welfare losses (i.e., volatility and dispersion) remains the same as in Proposition 8; but now this must be combined with the impact of δ_n on first-order losses, which is captured by the product $\phi_n v_n$. The impact of δ_n on v_n depends on the sign of α: higher commonality increases the covariance between K and κ when $\alpha > 0$, but decreases it when $\alpha < 0$. How this in turn affects welfare depends on the sign of ϕ_n, the covariance between κ and the efficiency gap $\kappa^* - \kappa$. It follows that the sign of the first-order effect of δ_n is given by the sign of the product of α and ϕ_n . Combining these observations, and noting that the first-order effect dominates when ϕ_n is sufficiently away from zero, we conclude that ϕ_n sufficiently high [low] suffices for the overall welfare effect of commonality to have the same [opposite] sign as α.

Proposition 10 (commonality) *There exist functions* $\underline{\phi}, \bar{\phi} : (-\infty, 1) \times (-\infty, 1) \to \mathbb{R}$, *with* $\underline{\phi} \leq \bar{\phi}$, *such that the following are true for any* $n \in \mathcal{N}$*:*

Strategic Independence. *When* $\alpha = 0$, *welfare increases with* δ_n *for all* (σ_n, δ_n) *if and only if* $\alpha^* > 0$, *and decreases if and only if* $\alpha^* < 0$.

Strategic Complementarity. *When* $\alpha \in (0, 1)$, *welfare increases with* δ_n *for all* (σ_n, δ_n) *if and only if* $\phi_n > \bar{\phi}(\alpha, \alpha^*)$, *and decreases if and only if* $\phi_n < \underline{\phi}(\alpha, \alpha^*)$.

Strategic Substitutability. *When* $\alpha \in (-\infty, 0)$, *welfare increases with* δ_n *for all* (σ_n, δ_n) *if and only if* $\phi_n < \underline{\phi}(\alpha, \alpha^*)$, *and decreases if and only if* $\phi_n > \bar{\phi}(\alpha, \alpha^*)$.

The functions $\underline{\phi}, \bar{\phi}$ *satisfy the following properties: (i)* $\underline{\phi} = \bar{\phi} = -1/2$ *when* $\alpha = \alpha^*$; *(ii) for* $\alpha \in (0, 1)$, $\underline{\phi} < 0$ *if and only if* $\alpha > 1/2$ *or* $\alpha^* > -\alpha^2/(1 - 2\alpha)$, *whereas* $\bar{\phi} < 0$ *if and only if* $\alpha^* > \alpha^2$; *(iii) for* $\alpha \in (-\infty, 0)$, $\underline{\phi} < 0$ *if and only if* $\alpha^* < \alpha^2$, *whereas* $\bar{\phi} < 0$ *if and only if* $\alpha^* < -\alpha^2/(1 - 2\alpha)$.

Other special cases. Proposition 8 and Corollary 3 applied to economies where $\kappa = \kappa^*$. As a direct implication of Propositions 9 and 10, these results extend to economies where $\kappa^* \neq \kappa$, provided that efficiency gap $\kappa^* - \kappa$ is either invariant or positively correlated with κ. To see this, note that, since $\bar{\phi}' < 0$, $\phi_n \geq 0$ suffices for welfare to increase with accuracy. Furthermore, since $\bar{\phi} < 0$ when either $\alpha^* \geq \alpha > 0$ or $\alpha^* \leq \alpha < 0$, we have that $\phi_n \geq 0$ also suffices for welfare to increase with the commonality of information in the first case and to decrease with it in the latter.

Corollary 5 *The properties established in Proposition 8 and Corollary 3 extend to economies that are inefficient also under complete information* ($\kappa^* \neq \kappa$), *provided that* $\phi_n \geq 0$.

Finally, consider economies where $\kappa \neq \kappa^*$ but $\alpha = \alpha^*$; these are economies where the equilibrium degree of coordination is socially optimal, and hence there is no inefficiency in the relative sensitivity of equilibrium actions to public information, although there is inefficiency in the overall response to the underlying fundamentals. In the Appendix we show that in this case, as in the case of the Blackwell-like result for efficient economies, the accuracy effect of a change in the precision of private or public information necessarily dominates the associated commonality effect. Combining this result with Propositions 9 and 10 and the property that $\underline{\phi} = \bar{\phi} = \underline{\phi}' = \bar{\phi}' = -1/2$ when $\alpha = \alpha^*$, we have the following complete characterization of the social value of information for economies where coordination is optimal.

Corollary 6 *Consider the class of economies in which the equilibrium degree of coordination is optimal ($\alpha = \alpha^*$, but possibly $\kappa \neq \kappa^*$). The following are true for any $n \in \mathcal{N}$:*

(i) *Welfare decreases with accuracy σ_n^{-2} if $\phi_n > -1/2$, and decreases if $\phi_n < -1/2$.*

(ii) *Welfare increases with commonality δ_n if either $(\alpha, \phi_n) > (0, -1/2)$, or $(\alpha, \phi_n) < (0, -1/2)$; and decreases in all other cases.*

(iii) *When $\phi_n > -1/2$, welfare increases with the precision of either private or public information and a higher complementarity raises the relative value of public information.*

(iv) *When $\phi_n < -1/2$, welfare decreases with the precision of either private or public information and a higher complementarity raises the relative cost of public information.*

This result thus extends the properties of efficient economies to economies where $\alpha = \alpha^*$ but $\kappa \neq \kappa^*$, as long as the correlation between the efficiency gap and the complete-information equilibrium is either positive or not too negative ($\phi_n > -1/2$); it also reverses these properties for the alternative case where the correlation is sufficiently negative ($\phi_n < -1/2$).

6 Applications

In the previous section we showed how understanding the inefficiency (if any) in the equilibrium use of information sheds light also on the social value of information. In this section, we show how the results above can guide welfare analysis in specific contexts of interest.

An important applied question is what are the gains (or losses) to society from institutions that aggregate and then publicly disseminate information—these institutions may range from public media and opinion polls, to central banks and financial markets. The answer to this question is not trivial—if the equilibrium use of information is inefficient, the society may prefer not to have such institutions, even if the cost of setting them up is zero.

In what follows, we thus focus primarily on the social value of public information, showing how this can be understood in relation to the inefficiency (if any) of equilibrium, and through our decomposition of its effects on the accuracy and the commonality of information.

6.1 Efficient coordination economies

We start with an example designed to illustrate how efficiency of the equilibrium can be consistent with any arbitrarily high level of complementarity—and hence how the amplification of volatility due to complementarity need not have any negative normative content.

Consider an economy where agents suffer a loss from taking an action away from either some unknown target fundamental θ or from the mean action K in the population:

$$U(k, K, \theta) = -1/2[(1-r)(k-\theta)^2 + r(k-K)^2],$$

where $r > 0$ parameterizes the importance of the distance from the mean action relative to the distance from θ.

It is easy to check that $\kappa^*(\theta) = \kappa(\theta) = \theta$, so that the complete-information equilibrium coincides with the first best, as well as that $U_{kk} = -1$, $U_{kK} = r$, $U_{KK} = -r$, $U_{\sigma_k^2} = 0$, and therefore $\alpha^* = \alpha = r > 0$, so that the equilibrium degree of coordination is optimal. The following is then an immediate implication of Proposition 7.

Corollary 7 *In the coordination economy described above, the equilibrium exhibits heightened sensitivity to public noise, but is efficient. Welfare increases with both private and public information, but public information is more valuable when the two have equal precisions.*

6.2 Efficient competitive economies

We now turn to an incomplete-market competitive economy in which agents' choices are strategic substitutes.

There is a continuum of households, each consisting of a consumer and a producer, and two commodities. Let q_{1i} and q_{2i} denote the respective quantities purchased by consumer i (the consumer living in household i). His preferences are given by

$$u_i = v(q_{1i}, \theta) + q_{2i}, \tag{19}$$

where $v(q, \theta) = \theta q - bq^2/2$, $\theta \in \mathbb{R}$, and $b > 0$. His budget is

$$pq_{1i} + q_{2i} = e + \pi_i, \tag{20}$$

where p is the price of good 1 relative to good 2, e is an exogenous endowment of good 2, and π_i are the profits of producer i (the producer living in household i), which are also denominated in terms of good 2. Profits in turn are given by

$$\pi_i = pk_i - C(k_i) \tag{21}$$

where k_i denotes the quantity of good 1 produced by household i and $C(k)$ the cost in terms of good 2, with $C(k) = k^2/2$.[21]

The random variable θ represents a shock in the relative demand for the two goods. Exchange and consumption take place once θ has become common knowledge. On the contrary, production takes place at an earlier stage, when information is still incomplete.

Consumer i chooses (q_{1i}, q_{2i}) so as to maximize (19) subject to (20), which gives $p = \theta - bq_{1i}$. Clearly, all households consume the same quantity of good 1, which together with market clearing gives $q_{1i} = K$ for all i and $p = \theta - bK$, where $K = \int k d\Psi(k)$. It follows that i's utility can be restated as $u_i = v(K, \theta) - pK + e + \pi_i = bK^2/2 + e + \pi_i$, where $\pi_i = pk_i - C(k_i) = (\theta - bK)k_i - k_i^2/2$. This example is thus nested in our model with

$$U(k, K, \theta, \sigma_k^2) = (\theta - bK)k - k^2/2 + bK^2/2 + e,$$

in which case $\kappa^*(\theta) = \kappa(\theta) = \theta/(1+b)$, $U_{kk} = -1$, $U_{kK} = -b$, $U_{KK} = b$, $U_{\sigma_k^2} = 0$, and therefore $\phi = 0$ and $\alpha^* = \alpha = -b < 0$.

That the complete-information equilibrium is efficient $(\kappa = \kappa^*)$ should not be a surprise. Under complete information, the economy is merely an example of a complete-markets competitive economy in which the first welfare theorem applies. What is interesting is that the equilibrium remains efficient even under incomplete information and despite the absence of ex-ante complete markets. This is because the strategic substitutability perceived by the agents coincides with the one that the planner would like them to perceive $(\alpha^* = \alpha)$. The following is then a direct implication of Proposition 7.

Corollary 8 *In the competitive economy described above, the equilibrium exhibits heightened sensitivity to private noise, but is efficient. Welfare increases with both private and public information, but private information is more valuable when the two have equal precisions.*

The competitive economy considered above is essentially the same as the one in Vives (1988). In particular, Vives considers an incomplete-information quadratic Cournot game with perfect substitutability across firms. He then shows that the maximal expected social surplus (sum of producer and consumer surpluses) is obtained by the equilibrium allocation in the limit as the number of firms goes to infinity. Since the limit case in Vives coincides with the economy analyzed above, the efficiency of equilibrium for this economy follows directly from Vives' analysis. Thus, with respect to this example, our contribution is only to recast this result in relation to the coincidence between the equilibrium and optimal degrees of coordination, and then to spell out the implications for the welfare effects of information.

[21] Implicit behind this cost function is a quadratic production frontier. The resource constraints are therefore given by $\int_i q_{1i} = \int_i k_i$ and $\int_i q_{2i} = e - \frac{1}{2}\int_i k_i^2$ for good 1 and 2, respectively.

6.3 Investment complementarities

The prototypical model of production externalities can be nested in our framework by interpreting k as investment and defining individual payoffs as follows:

$$U(k, K, \theta) = A(K, \theta)k - C(k), \tag{22}$$

where $A(K, \theta) = (1 - a)\theta + aK$ represents the private return to investment, with $a \in (0, 1/2)$ and $\theta \in \mathbb{R}$, whereas $C(k) = k^2/2$ represents the private cost of investment. Variants of this specification appear in Bryant (1983), Romer (1986), Matsuyama (1992), Acemoglu (1993), and Benhabib and Farmer (1994), as well as models of network externalities and spillovers in technology adoption.[22] The important ingredient is that the private return to investment increases with the aggregate level of investment—the source of both complementarity and externality in this class of models.

It is easy to verify that the complete-information equilibrium level of investment is $\kappa(\theta) = \theta$, whereas the first best is $\kappa^*(\theta) = \frac{1-a}{1-2a}\theta$, and hence $\phi = \frac{Cov(\kappa^* - \kappa, \kappa)}{Var(\kappa)} = \frac{a}{1-2a} > 0$. That is, because of the positive spillover, the private return to investment is lower than the social one for all $\theta > 0$, and the more so the higher θ. Furthermore, apart from the complementarity, there are no other second-order external effects: $U_{kK} = a > 0$, but $U_{KK} = U_{\sigma_k^2} = 0$ (and $U_{kk} = -1$). It follows that $\alpha = a > 0$ and $\alpha^* = 2\alpha > \alpha$. That is, in this economy the agents' private incentives to coordinate, and the consequent amplification of volatility featured in equilibrium, are anything but excessive from a normative perspective. The following is then an immediate implication of Corollary 5.

Corollary 9 *In the investment example described above, the complete-information equilibrium is positively correlated with the efficiency gap and the equilibrium degree of coordination is inefficiently low, ensuring that welfare increases with both the accuracy and the commonality of information. By implication, welfare necessarily increases with the precision of public information.*

Economies with frictions in financial markets—where complementarities emerge through collateral constraints, missing assets, or other types of market incompleteness—are often related to economies with investment complementarities like the one considered here. Although this analogy is appropriate for many positive questions, it need not be so for normative purposes. As the examples in the next two sections highlight, the result in Corollary 9 depends on the absence of certain second-order external effects and on positive correlation between equilibrium and first-best activity. Whether these properties are shared by mainstream incomplete-market models is an open question.

[22] This is also the example we examined in Angeletos and Pavan (2004, Section 2), although there we computed welfare conditional on θ, thus omitting the effect of $\phi \neq 0$ on welfare losses.

6.4 "Beauty contests" versus other Keynesian frictions

Keynes contended that financial markets often behave like "beauty contests" in the sense that traders try to forecast and outbid one another's forecasts, but this motive is (presumably) not warranted from a social perspective because it is due to some (unspecified) market imperfection. Capturing this idea with proper micro-foundations is an open question, but one possible shortcut, following Morris and Shin (2002), is to define a *beauty-contest economy* as an economy in which $\alpha > 0 = \alpha^*$ and $\kappa(\cdot) = \kappa^*(\cdot)$. The first condition means that the private motive to coordinate is not warranted from a social perspective; the second means that the inefficiency of equilibrium vanishes as information becomes complete. By Proposition 8 we then have that welfare necessarily increases with the accuracy of information, while by Proposition 10 that it is non-monotonic with its commonality (since $\underline{\phi} < 0 = \phi < \bar{\phi}$).

Corollary 10 *In beauty-contest economies (defined as above), welfare can decrease with the precision of public information, but only when it decreases with the commonality of information—and this is possible only because coordination is socially unwarranted.*

The specific payoff structure assumed by Morris and Shin (2002) is given by

$$u_i = -(1-r) \cdot (k_i - \theta)^2 - r \cdot (L_i - \bar{L})$$

where $\theta \in \mathbb{R}$ is the underlying fundamental, $L_i = L(k_i) \equiv \int (k' - k_i)^2 \, d\Psi(k') = (k_i - K)^2 + \sigma_k^2$ is the mean square-distance of other agents' actions from agent i's action, $\bar{L} = \int L(k) d\Psi(k) = 2\sigma_k^2$ is the cross-sectional mean of L_i, and $r \in (0,1)$.The first term in u_i captures the value of taking an action close to a fundamental "target" θ. The L_i term introduces a private value for taking an action close to other agents' actions, whereas the $\bar{L}$ term ensures that there is no social value in doing so. Indeed, aggregating across agents gives $w = -(1-r) \int (k-\theta)^2 d\Psi(k)$, so that, from a social perspective, it is as if utility were simply $u = -(k-\theta)^2$, in which case there is of course no social value to coordination.

This example is nested in our framework with

$$U(k, K, \theta, \sigma_k^2) = -(1-r) \cdot (k-\theta)^2 - r \cdot (k-K)^2 + r \cdot \sigma_k^2.$$

It follows that $\kappa^*(\theta) = \kappa(\theta) = \theta$, $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{\sigma_k^2} = r$, and hence $\alpha = r > 0 = \alpha^*$. That is, the complete-information equilibrium is efficient, but the equilibrium degree of coordination is inefficiently high.

The point of Morris and Shin (2002) was precisely to highlight how socially unwarranted coordination can lead to a negative welfare effect of public information. However, one should be careful in extrapolating the normative properties highlighted above to environments that resemble beauty contests in that the complementarity emerges through some sort of market friction.

Consider, for example, incomplete-information variants of standard Keynesian business-cycle models, recently examined by Woodford (2002), Hellwig (2005), Lorenzoni, (2005), and Roca (2005). In these models, complementarity emerges in monopolistic competition—a market friction, or a "real rigidity" as some authors have called it. However, imperfect substitutability across goods implies that cross-sectional dispersion in prices creates a negative externality ($U_{\sigma_k^2} < 0$). This in turn contributes to a *higher* optimal degree of coordination—exactly the opposite of what happens in the beauty-contest economy examined above.

Hellwig (2005) provides an excellent analysis of this class of models for the case that the business cycle is driven by monetary shocks. He shows that the efficient sensitivity to public information is higher than the equilibrium one, because of the adverse effect of cross-sectional price dispersion discussed above. This property alone does not guarantee that public information is welfare-improving—but it does so once combined with the property that, in his model, the business cycle is efficient under complete information, in the sense that the gap between first best and equilibrium does not vary with the business cycle. Translating these properties in our framework gives $\alpha^* > \alpha > 0$ and $\phi = 0$, in which case, by Corollary 5, welfare increases with both accuracy and commonality. The same is true for the model of Roca (2005). This helps understand why, unlike in Morris and Shin (2002), public information is necessarily welfare improving in these works.

6.5 Inefficient fluctuations

The focus in the previous section was on how the complementarity and second-order external effects tilt the trade-off between volatility and dispersion. We now focus on first-order effects. In particular, we consider an economy where the efficiency gap $\kappa^* - \kappa$ covaries negatively with κ. This permits us to capture the possibility that recessions are inefficiently deep—a possibility that turns out to have important implications for the social value of information.

To isolate the impact of first-order effects ($\phi \neq 0$), we abstract from strategic and second-order external effects ($U_{kK} = U_{KK} = U_{\sigma_k^2} = 0$), so that $\alpha^* = \alpha = 0$. The latter ensures that welfare is independent of commonality, while $\phi < 0$ opens the door to the possibility that welfare can decrease with accuracy.

There are two types of fundamentals in the economy: one that generates fluctuations in equilibrium without affecting the efficiency gap, and another that generates covariation between equilibrium and the efficiency gap. In particular, agents engage in an investment activity for which private and social returns differ:

$$U(k, K, \theta, \sigma_k^2) = (\theta_1 + \theta_2)\, k - k^2/2 - \lambda\theta_2 K,$$

for some $\lambda \in (0, 1)$. One can interpret the last term as the impact of a "wedge" or "mark-up" that introduces a gap between private and social returns. Indeed, the private return to investment is

$\theta_1 + \theta_2$, the wedge is $\lambda\theta_2$, and the social return is $\theta_1 + (1 - \lambda)\,\theta_2$; the coefficient λ parameterizes the covariation between the private return and the wedge.

The complete-information equilibrium is $\kappa(\theta) = \theta_1 + \theta_2$, whereas the first best is $\kappa^*(\theta) = \theta_1 + (1 - \lambda)\,\theta_2$, and hence $\phi_1 = 0$ but $\phi_2 = -\lambda$. The fact that $\phi_1 = 0$ ensures that any information about θ_1 is welfare-improving; but $\phi_2 < 0$ raises the possibility that information about θ_2 is welfare-damaging. In particular, if $\lambda > 1/2$ and hence $\phi_2 < -1/2$, then welfare *decreases* with the precision of either private or public information about θ_2. The following is then a direct translation of Corollary 6.[23]

Corollary 11 *Consider the economy described above and suppose the correlation between the equilibrium and the wedge is sufficiently negative (i.e. $\phi_2 < -1/2$). Then welfare necessarily increases with either private or public information about the efficient source of the business cycle (θ_1) , whereas it necessarily decreases with private or public information about the inefficient source of the business cycle (θ_2).*

The recent debate on the merits of transparency in central bank communication has focused on the role of complementarities in new-Keynesian models (e.g., Morris and Shin, 2002; Svensson, 2005; Woodford, 2005; Hellwig, 2005; Roca, 2005). The example here suggests that this debate might be missing a critical element—the potential inefficiency of equilibrium fluctuations under complete information.

For example, we conjecture that the result in Hellwig (2005) and Roca (2005) that public information has a positive effect on welfare relies on the property that, under complete information, the business cycle is *efficient* in these models. In standard new-Keynesian models, the monopolistic mark-up introduces an efficiency gap. But as long as this mark-up is constant over the business cycle—which is the case in the models of Hellwig and Roca—the efficiency gap also remains constant. If, instead, business cycles are driven primarily by shocks in mark-ups or "labor wedges," it seems possible that providing markets with information that helps predict these shocks can reduce welfare. This is an interesting question that we leave open for future research.

6.6 Cournot versus Bertrand

We now turn to two IO applications, with a large number of firms: a Cournot-like game, where firms compete in quantities and actions are strategic substitutes; and a Bertrand-like game, where firms compete in prices and actions are strategic complements. Efficiency and value of information are now evaluated from the perspective of firms: for this section "welfare" is identified with expected total profits.

[23] A special case of this is when $\lambda = 1$ and $\sigma_{\theta_1} = 0$, so that κ^* is constant and the entire fluctuation in κ is inefficient.

Consider first Cournot. The demand faced by a firm is given by $p = a_0 + a_1\theta - a_2 q - a_3 Q$, where p denotes the price at which the firm sells each unit of its product, q the quantity it produces, Q the average quantity in the market, and θ an exogenous demand shifter ($a_0, a_1, a_2, a_3 > 0$). Individual profits are $u = pq - C(q)$, with a quadratic cost function, $C(q) = c_1 q + c_2 q^2$ ($c_1, c_2 > 0$).

This is nested in our framework with $k \equiv q$, $K \equiv Q$ (actions are quantities), and

$$U\left(k, K, \theta, \sigma_k^2\right) = \left(a_0 - c_1 + a_1\theta - a_3 K\right) k - (a_2 + c_2)k^2,$$

It follows that $U_{kK} = -a_3 < 0$, $U_{kk} = -(a_2 + c_2) < 0$, and $U_{KK} = U_{\sigma_k^2} = 0$, so that the equilibrium degree of coordination is $\alpha = -\frac{a_3}{a_2+c_2} < 0$, while the optimal one is $\alpha^* = 2\alpha < a < 0$. That is, a planner interested in maximizing expected producer surplus would like the firms to increase their reliance on private information, and decrease their reliance on public information. Moreover, under complete information the total profit-maximizing (i.e., monopoly) quantity is $\kappa^*(\theta) = \frac{a_0-c_1+a_1\theta}{2(a_2+c_2+a_3)}$, whereas the equilibrium (i.e., Cournot) quantity is $\kappa(\theta) = \frac{a_0-c_1+a_1\theta}{2(a_2+c_2)+a_3}$, so that $\phi = \frac{\alpha}{2(1-\alpha)} < 0$. That is, both the monopoly and the Cournot quantity increase with the demand intercept, but the monopoly one less so than the Cournot one.

Using $\alpha^* = 2\alpha < \alpha < 0$ with the formulas for the bounds $\bar{\phi}'$ and $\bar{\phi}$ derived in the proof of Propositions 9 and 10 (see Appendix), we have that $\bar{\phi}' = \frac{-(1-\alpha)}{2(1-2\alpha)} > \frac{-2+\alpha}{2(1-2\alpha)} = \bar{\phi}$. Together with $\phi = \frac{\alpha}{2(1-\alpha)}$, this ensures that $\phi > \bar{\phi}'$ and $\phi > \bar{\phi}$. By Proposition 9 and 10, total profits increase with accuracy and decrease with commonality. This in turn ensures that expected profits necessarily increase with the precision of private information, but opens the door to the possibility that they decrease with that of public. In the Appendix we verify that the latter is possible if the degree of strategic substitutability is strong enough, namely if $\alpha < -1$.

Corollary 12 *In the Cournot game described above, firms' actions are strategic substitutes, but less so than what is collectively optimal (i.e., $\alpha^* < \alpha < 0$). Expected total profits necessarily increase with the precision of private information, but can decrease with that of public.*

Next, consider Bertrand. Demand is now given by $q = b_0 + b_1\theta' - b_2 p + b_3 P$, where q denotes the quantity sold by the firm, p the price the firm sets, P the average price in the market, and θ' again an exogenous demand shifter ($b_0, b_1, b_2, b_3 > 0$); we naturally impose $b_3 < b_2$, so that an equal increase in p and P reduces q. Individual profits are $u = pq - C(q)$, with $C(q) = c_1 q + c_2 q^2$ ($c_1, c_2 > 0$).

This is nested in our framework with $k \equiv p - c_1$, $K \equiv P - c_1$ (actions are now prices), and

$$U\left(k, K, \theta, \sigma_k^2\right) = b_2[(\theta - k + bK)\, k - c\,(\theta - k + bK)^2],$$

where $\theta \equiv b_0/b_2 + b_1/b_2\theta' - c_1(1-b)$, $b \equiv b_3/b_2 \in (0,1)$, and $c \equiv c_2 b_2 > 0$; without loss of generality, we let $b_2 = 1$. It follows that $U_{kK} = (1+2c)\, b > 0$, $U_{kk} = -2(1+c) < 0$, $U_{\sigma_k^2} = 0$, and

$U_{KK} = -2cb^2$, so that the equilibrium degree of coordination is $\alpha = \frac{1+2c}{2+2c}b \in (0,1)$ and the optimal one is $\alpha^* = \frac{2+2c(2-b)}{2+2c}b > \alpha$ (as usual, we restrict $\alpha^* < 1$). Moreover, under complete information the monopoly price is $\kappa^*(\theta) = \frac{1+2c(1-b)}{2(1-b)[1+c(1-b)]}\theta$, while the equilibrium price is $\kappa(\theta) = \frac{1+2c}{1+(1+2c)(1-b)}\theta$, so that $\phi > 0$. That is, the Bertrand price reacts too little to θ as compared to the monopoly price. By Corollary 5, the fact that $\alpha^* > \alpha > 0$ together with $\phi > 0$ ensures that welfare increases with both the accuracy and the commonality of information. This immediately implies that more precise public information necessarily increases expected profits. Private information, on the other hand, has a positive effect through accuracy and a negative one through commonality—but the fact that ϕ is sufficiently high turns out to ensure in this example that the positive effect of private information through accuracy dominates.

Corollary 13 *In the Bertrand game described above, firms' actions are strategic complements, but less so than what is collectively optimal (i.e., $\alpha^* > \alpha > 0$). Expected total profits increase with the precision of either public or private information.*

If we interpret information sharing among firms as an increase in the precision of public information, then the above results imply that information-sharing is profit-enhancing under Bertrand competition, but not necessarily under Cournot competition.

This result is closely related to Vives (1990) and Raith (1996), who examine the impact of information-sharing in Cournot and Bertrand oligopolies with a finite number of firms.[24] For example, Proposition 4.2 in Raith's paper shows that expected profits necessarily increase with a uniform increase in the precision of the signals that firms receive about demand (or costs), and that they increase with the correlation of noise across firms in the Bertrand case but decrease in the Cournot case. Since Raith's specification is nested in our framework with $\alpha^* = 2\alpha$ and $\phi > \max\{\bar{\phi}, \bar{\phi}'\}$, this result could be read off our Propositions 9 and 10 if it were not for the difference in the number of players and the information structure. Yet, this coincidence suggests that the logic behind our results is not unduly sensitive to the details of the environment we used.

7 Concluding remarks

This paper examined a class of economies with externalities, strategic complementarity or substitutability, and asymmetric information. For this class, we provided a complete characterization of the equilibrium and the efficient use of information, and a complete taxonomy of the welfare effects of information. We then showed how these results can give guidance for welfare analysis in concrete applications.

[24] The payoff structures in those papers are linear-quadratic, as the ones considered here; a minor difference is that these papers effectively impose $U_{KK} = 0$, and hence $\alpha^* = 2\alpha$, in both the Cournot and the Bertrand case.

Certain modeling choices—namely the quadratic specification for the payoffs and the Gaussian specification for the signals—were dictated by the need for tractability, but do not appear to be essential for the main insights; we expect our analysis to be a good benchmark also for more general environments with a unique equilibrium and concave payoffs. Indeed, an interesting extension is to check whether our results are second-order approximations of this more general class of economies.

On the other hand, the restrictions to unique equilibrium and concave payoffs are clearly essential for our results.

First, when the complementarity is strong enough that multiple equilibria emerge under common knowledge—a possibility that is important for certain applications but ruled out here by the restriction $\alpha < 1$—then the information structure matters not only for the local properties of any given equilibrium but also for the determinacy of equilibria (e.g., Morris and Shin, 2003). In particular, more precise public information contributes towards multiplicity, while more precise private information contributes towards uniqueness. The social value of information may then critically depend on equilibrium selection (e.g., Angeletos and Pavan 2004, Sec. 2).

Second, when aggregate welfare exhibits convexity (a.k.a. increasing aggregate returns) over some region—a possibility ruled out here by the restriction $W_{KK} < 0$—then the planner may prefer a lottery to the complete-information equilibrium.[25] When this is the case, more noise in public information may improve welfare to the extent that aggregate volatility mimics such a lottery.

Therefore, multiple equilibria and payoff convexities introduce effects that our analysis has ruled out. Extending the analysis in these directions is an interesting, but also challenging, next step for future research.

Another promising direction for future research is extending the analysis to environments with endogenous information structures. This is interesting, not only because the endogeneity of information is important per se, but also because inefficiencies in the *use* of information are likely to interact with inefficiencies in the *collection* or *aggregation* of information. For example, in economies with a high social value for coordination, the private collection of information can reduce welfare by decreasing the correlation of expectations across agents and thereby hampering coordination. Symmetrically, in environments where substitutability is important, the aggregation of information through prices or other channels could reduce welfare by increasing correlation in beliefs. [26]

The aforementioned extensions are important for developing a more complete picture of the welfare properties of large economies with dispersed heterogeneous information. The use of the efficiency benchmark identified here as an instrument to assess these welfare properties is the core methodological contribution of this paper.

[25]Indeed, this is necessarily the case when welfare is locally convex around the complete-information equilibrium and the lottery has small variance and expected value equal to the complete-information equilibrium.

[26]The role of prices in coordination environments is also the theme of Angeletos and Werning (2004), but there the focus is on how information aggregation can lead to multiplicity.

Appendix

Proof of Proposition 1. *Part (i).* Take any strategy $k : \mathbb{R}^{2N} \to \mathbb{R}$ (not necessarily a linear one) and let $K(\theta, y) = \mathbb{E}[k(x, y)|\theta, y]$. A best-response is a strategy $k'(x, y)$ that solves, for all (x, y), the first-order condition

$$\mathbb{E}[U_k(k', K, \theta)|x, y] = 0. \tag{23}$$

Since U is quadratic, U_k is linear and hence $U_k(k', K, \theta) = U_k(\kappa, \kappa, \theta) + U_{kk} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa)$, where κ stands for the complete-information equilibrium allocation. By definition, the latter solves $U_k(\kappa, \kappa, \theta) = 0$ for all θ, which implies that (23) reduces to

$$\mathbb{E}[U_{kk} \cdot (k' - \kappa) + U_{kK} \cdot (K - \kappa)|x, y] = 0,$$

or equivalently $k'(x, y) = \mathbb{E}[(1 - \alpha)\kappa + \alpha K|x, y]$. In equilibrium, $k'(x, y) = k(x, y)$ for all x, y, which gives (5).

Part (ii). Since $\mathbb{E}[\theta|x, y]$, and hence $\mathbb{E}[\kappa|x, y]$, is linear in (x, z), it is natural to look for a fixed point that is linear in x and z, where, for any n, $z_n = \lambda_n y_n + (1 - \lambda_n)\mu_n$. Thus suppose[27]

$$k(x, y) = a + b \cdot x + c \cdot z \tag{24}$$

for some $a \in \mathbb{R}$ and $b, c \in \mathbb{R}^N$. Then $K(\theta, y) = a + b \cdot \theta + c \cdot z$ and (5) reduces to

$$k(x, y) = (1 - \alpha)\kappa_0 + \alpha a + ((1 - \alpha)\kappa + \alpha b) \cdot \mathbb{E}[\theta|x, y] + \alpha c \cdot z$$

where $\kappa \equiv (\kappa_1, ..., \kappa_n)$. Substituting $\mathbb{E}[\theta|x, y] = (\mathbf{I} - \mathbf{\Delta})x + \mathbf{\Delta} z$, where $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{\Delta}$ is the $N \times N$ diagonal matrix with n-th element equal to δ_n, we conclude that (24) is a linear equilibrium if and only if a, b and c solve

$$a = (1 - \alpha)\kappa_0 + \alpha a, \quad b = (\mathbf{I} - \mathbf{\Delta})\left[(1 - \alpha)\kappa + \alpha b\right], \quad \text{and} \quad c = \mathbf{\Delta}\left[(1 - \alpha)\kappa + \alpha b\right] + \alpha c.$$

Equivalently $a = \kappa_0$, $b_n = \kappa_n(1 - \alpha)(1 - \delta_n)/[1 - \alpha(1 - \delta_n)]$, and $c_n = \kappa_n\delta_n/[1 - \alpha(1 - \delta_n)]$, $n \in \{1, ..., N\}$. Note that $b_n + c_n = \kappa_n$ always; $b_n = c_n = 0$ whenever $\kappa_n = 0$; and $b_n \in (0, \kappa_n)$ and $c_n \in (0, \kappa_n)$ otherwise. Letting $\gamma_n \equiv c_n/\kappa_n \in (0, 1)$ for any $n \in \mathcal{N}$ gives (6)-(7). ∎

Proof of Proposition 2. From condition (6),

$$\begin{aligned} k(x, y) &= \kappa_0 + \sum_{n \in \mathcal{N}} \kappa_n \left[(1 - \gamma_n)x_n + \gamma_n z_n\right], \\ K(\theta, y) &= \kappa_0 + \sum_{n \in \mathcal{N}} \kappa_n \left[(1 - \gamma_n)\theta_n + \gamma_n z_n\right], \end{aligned}$$

[27] A dot between two vectors denotes inner product, whereas a dot between two scalars denotes standard multiplication.

and therefore $k-K=\sum_{n\in\mathcal{N}}\kappa_n\left[(1-\gamma_n)(x_n-\theta_n)\right]$ and $K-\kappa=\sum_{n\in\mathcal{N}}\kappa_n\gamma_n(z_n-\theta_n)$. Using $Var(x_n-\theta_n)=\sigma_{x_n}^2$, $Var(z_n-\theta_n)=\sigma_{z_n}^2=\left(\sigma_{y_n}^{-2}+\sigma_{\theta_n}^{-2}\right)^{-1}$ and $\delta_n=\sigma_{z_n}^{-2}/\sigma_n^{-2}$, together with (7), we have

$$\begin{aligned} Var(k-K) &= \sum_{n\in\mathcal{N}}\kappa_n^2\left[(1-\gamma_n)^2\sigma_{x_n}^2\right] = \sum_{n\in\mathcal{N}}\kappa_n^2\left[\frac{(1-\alpha)^2(1-\delta_n)}{(1-\alpha+\alpha\delta_n)^2}\sigma_n^2\right], \\ Var(K-\kappa) &= \sum_{n\in\mathcal{N}}\kappa_n^2\gamma_n^2\sigma_{z_n}^2 = \sum_{n\in\mathcal{N}}\kappa_n^2\left[\frac{\delta_n}{(1-\alpha+\alpha\delta_n)^2}\sigma_n^2\right], \end{aligned}$$

from which we get that

$$\frac{\partial Var(k-K)}{\partial\alpha}=-2(1-\alpha)\sum_{n\in\mathcal{N}}\kappa_n^2\frac{(1-\delta_n)\delta_n}{(1-\alpha+\alpha\delta_n)^3}\sigma_n^2$$

$$\frac{\partial Var(K-\kappa)}{\partial\alpha}=2\sum_{n\in\mathcal{N}}\kappa_n^2\frac{(1-\delta_n)\delta_n}{(1-\alpha+\alpha\delta_n)^3}\sigma_n^2$$

$$\frac{\partial Var(k-K)}{\partial\delta_n}=-\frac{(1-\alpha)^2(1+\alpha(1-\delta_n))}{(1-\alpha+\alpha\delta_n)^3}\kappa_n^2\sigma_n^2$$

$$\frac{\partial Var(K-\kappa)}{\partial\delta_n}=\frac{1-\alpha(1+\delta_n)}{(1-\alpha+\alpha\delta_n)^3}\kappa_n^2\sigma_n^2$$

It follows that dispersion decreases whereas volatility increases with α. Furthermore, both dispersion and volatility increase with the noise σ_n (and hence decrease with the accuracy σ_n^{-2}). Finally, dispersion decreases with δ_n if and only if $\alpha>-\frac{1}{1-\delta_n}$, whereas volatility increases with δ_n if and only if $\alpha<\frac{1}{1+\delta_n}$. ■

Proof of Lemma 1. The Lagrangian of the problem in Definition 2 can be written as

$$\begin{aligned}\Lambda = &\int_{(\theta,y)}\int_x U(k(x,y),K(\theta,y),\theta)dP(x|\theta,y)dP(\theta,y)+\\ &+\int_{(\theta,y)}\lambda(\theta,y)\left[K(\theta,y)-\int_x k\left(x,y\right)dP(x|\theta,y)\right]dP(\theta,y).\end{aligned}$$

The first order conditions for $K(\theta,y)$ and $k(x,y)$ are therefore given by

$$\int_x U_K(k(x,y),K(\theta,y),\theta)dP(x|\theta,y)+\lambda(\theta,y)=0 \qquad \text{for almost all } (\theta,y) \tag{25}$$

$$\int_{(\theta,y)}[U_k(k(x,y),K(\theta,y),\theta)-\lambda(\theta,y)]dP(\theta,y|x,y)=0 \qquad \text{for almost all } (x,y) \tag{26}$$

Noting that U_K is linear in its arguments and that $K(\theta,y)=\int_x k\left(x,y\right)dP(x|\theta,y)$, condition (25) can be rewritten as $-\lambda(\theta,y)=U_K(K(\theta,y),K(\theta,y),\theta)$. Replacing this into (26) gives (8). Since U is strictly concave and the constraint is linear, (8) is both necessary and sufficient, which completes the proof. ■

Proof of Proposition 3. *Part* (i). Since U is quadratic, condition (8) can be rewritten as

$$\begin{aligned}&\mathbb{E}[\ U_k(\kappa^*,\kappa^*,\theta)+U_{kk}\cdot(k(x,y)-\kappa^*)+U_{kK}\cdot(K-\kappa^*)+\\ &+U_K(\kappa^*,\kappa^*,\theta)+(U_{kK}+U_{KK})\cdot(K-\kappa^*)\mid x,y\ ]\ =\ 0.\end{aligned}$$

Using $U_k(\kappa^*, \kappa^*, \theta) + U_K(\kappa^*, \kappa^*, \theta) = 0$, by definition of first best, the above reduces to

$$\mathbb{E}[\ U_{kk}\,(k(x,y) - \kappa^*) + (2U_{kK} + U_{KK})(K - \kappa^*) \mid x, y\ ] = 0,$$

which gives (10).

Part (ii). Uniqueness follows from the fact that the planner's problem in Definition 2 is strictly concave. The characterization follows from the same steps as in the proof of Proposition (1) replacing α with α^* and $\kappa(\cdot)$ with $\kappa^*(\cdot)$. ■

Proof of Proposition 4. Consider first part (ii). Since the (unique) efficient allocation of $\mathbf{e}$ is linear, *only* the linear equilibrium of the economy $\mathbf{e}'$ can coincide with the efficient allocation of the true economy $\mathbf{e}$. Now, take any U' satisfying $\alpha' \equiv -U'_{kK}/U_{kk} < 1$. By Proposition 1, any equilibrium of $\mathbf{e}' = (U'; \sigma, \delta, \mu, \sigma_\theta)$ is a function $k(x, y)$ that solves

$$k(x,y) = \mathbb{E}[\ (1 - \alpha')\kappa' + \alpha' K(\theta, y) \mid x, y\ ]\ \forall (x,y), \tag{27}$$

where $\kappa'(\theta) = \kappa'_0 + \kappa'_1\theta_1 + ... + \kappa'_N\theta_N$ is the unique solution to $U'_k(\kappa', \kappa', \theta) = 0$ and $K(\theta, y) = \mathbb{E}[k\,(x, y) \mid \theta, y]$. The unique linear solution to (27) is the function

$$k(x,y) = \kappa'_0 + \sum_{n \in \mathcal{N}'} \kappa'_n \left[(1 - \gamma'_n)x_n + \gamma'_n z_n\right],$$

where

$$\gamma'_n = \delta_n + \frac{\alpha'\delta_n(1-\delta_n)}{1 - \alpha'(1-\delta_n)} \quad \forall n \in \mathcal{N}' \equiv \{n \in \mathbf{N} : \kappa'_n \neq 0\}.$$

For this function to coincide with the efficient allocation of $\mathbf{e}$ for all $(x, y) \in \mathbb{R}^{2N}$, it is necessary and sufficient that $\kappa'(\cdot) = \kappa^*(\cdot)$ and $\alpha' = \alpha^*$, which proves part (ii).

For part (i), it suffices to let $U'(k, K, \theta) = U(k, K, \theta) + U_K(K, K, \theta)k$, in which case it is immediate that $\kappa'(\cdot) = \kappa^*(\cdot)$ and $\alpha' = \alpha^*$. ■

Proof of Condition (14). Since U is quadratic, a second-order Taylor expansion around $k = K$ is exact:

$$U(k, K, \theta) = U(K, K, \theta) + U_k(K, K, \theta) \cdot (k - K) + \frac{U_{kk}}{2} \cdot (k - K)^2.$$

It follows that ex-ante utility is given by

$$\mathbb{E}u = \mathbb{E}[W(K, \theta)] + \frac{U_{kk}}{2}\mathbb{E}[(k - K)^2],$$

where $k = k(x, y)$ and $K = K(\theta, \varepsilon)$ are shortcuts for the efficient allocation and $W(K, \theta) \equiv U(K, K, \theta)$. A quadratic expansion of $W(K, \theta)$ around κ^*, which is exact since U and thus W are quadratic, gives

$$W(K, \theta) = W(\kappa^*, \theta) + W_K(\kappa^*, \theta) \cdot (K - \kappa^*) + \frac{W_{KK}}{2} \cdot (K - \kappa^*)^2.$$

By definition of κ^*, $W_K(\kappa^*, \theta) = 0$. It follows that

$$\mathbb{E}u = \mathbb{E}W(\kappa^*, \theta) + \frac{W_{KK}}{2} \cdot \mathbb{E}[(K - \kappa^*)^2] + \frac{U_{kk}}{2} \cdot \mathbb{E}[(k - K)^2].$$

At the efficient allocation, $k - \kappa^* = \sum_{n \in \mathcal{N}^*} \kappa_n^* \left[(1 - \gamma_n^*)(x_n - \theta_n) + \gamma_n^*(z_n - \theta_n)\right]$ implying that $\mathbb{E}k = \mathbb{E}K = \mathbb{E}\kappa^*$ and therefore $\mathbb{E}[(K - \kappa^*)^2] = Var(K - \kappa^*)$ and $\mathbb{E}[(k - K)^2] = Var(k - K)$, which gives the result. ■

Proof of Proposition 5. The result follows directly from the proof of Proposition 4 together with the definitions of $\kappa(\cdot)$, $\kappa^*(\cdot)$, α and α^*. ■

Proof of Proposition 6. Suppose $\kappa(\cdot) = \kappa^*(\cdot)$ and $\alpha = \alpha^*$ and consider the set $\mathcal{K}$ of allocations that satisfy

$$k(x, y) = \mathbb{E}[(1 - \alpha')\kappa + \alpha' K | x, y]$$

for some $\alpha' < 1$, or equivalently $k(x, y) = \kappa_0 + \sum_{n \in \mathcal{N}} \kappa_n \left[(1 - \gamma_n')x_n + \gamma_n' z_n\right]$, where

$$\gamma_n' = \delta_n + \frac{\alpha' \delta_n (1 - \delta_n)}{1 - \alpha'(1 - \delta_n)} \quad \text{for all } n \in \mathcal{N}.$$

Clearly, the equilibrium (and efficient) allocation is nested with $\alpha' = \alpha (= \alpha^*)$. Since for any allocation in $\mathcal{K}$ $\mathbb{E}K = \mathbb{E}k = \mathbb{E}\kappa$, ex-ante welfare can be written as $\mathbb{E}u = \mathbb{E}W(\kappa, \theta) - \mathcal{L}$, where

$$\mathcal{L} \;=\; \frac{|W_{KK}|}{2} Var(K - \kappa) + \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} Var(k - K) \;=\; \frac{|U_{kk} + 2U_{\sigma_k^2}|}{2} \Omega,$$

with

$$\Omega \;\equiv\; (1 - \alpha^*) Var(K - \kappa) + Var(k - K).$$

Using

$$\begin{aligned}
Var(K - \kappa) &= \sum_{n \in \mathcal{N}} \kappa_n^2 \gamma_n'^2 \sigma_{z_n}^2 = \sum_{n \in \mathcal{N}} \kappa_n^2 \gamma_n'^2 \left(\sigma_n^2 / \delta_n\right), \\
Var(k - K) &= \sum_{n \in \mathcal{N}} \kappa_n^2 \left[(1 - \gamma_n')^2 \sigma_{x_n}^2\right] = \sum_{n \in \mathcal{N}} \kappa_n^2 \left[(1 - \gamma_n')^2 \left(\sigma_n^2 / (1 - \delta_n)\right)\right],
\end{aligned}$$

we have that

$$\Omega = \sum_{n \in \mathcal{N}} \kappa_n^2 \left\{(1 - \alpha^*) \frac{\gamma_n'^2}{\delta_n} + \frac{(1 - \gamma_n')^2}{1 - \delta_n}\right\} \sigma_n^2.$$

Note that $\mathbb{E}u$ depends on α' and (δ_n, σ_n), for $n \in \mathcal{N}$, only through Ω. Since the efficient allocation is nested with $\alpha' = \alpha^*$, it must be that $\alpha' = \alpha^*$ maximizes $\mathbb{E}u$, or equivalently that $\gamma_n' = \gamma_n^*$ solves $\partial \Omega / \partial \gamma_n' = 0$; that is,

$$(1 - \alpha^*) \frac{\gamma_n^*}{\delta_n} = \frac{1 - \gamma_n^*}{1 - \delta_n}. \tag{28}$$

Next note that Ω increases with σ_n, and hence $\mathbb{E}u$ decreases with σ_n (equivalently, increases with the accuracy σ_n^{-2}). Finally, consider the effect of δ_n. By the envelope theorem,

$$\frac{d\Omega}{d\delta_n} = \left.\frac{\partial\Omega}{\partial\delta_n}\right|_{\gamma_n'=\gamma_n^*} = \kappa_n^2 \left\{ -\frac{(1-\alpha^*)\gamma_n^{*2}}{\delta_n^2} + \frac{(1-\gamma_n^*)^2}{(1-\delta_n)^2} \right\} \sigma_n^2$$

Using (28), we thus have that $d\mathbb{E}u/d\delta_n > [<]0$ if and only if $\gamma_n^*/(1-\gamma_n^*) > [<]\ \delta_n/(1-\delta_n)$, which is the case if and only if $\alpha^* > [<]0$. Using $\alpha = \alpha^*$ (by efficiency) then gives the result. ■

Proof of Proposition 7. Part (i) follows from the Blackwell-like argument in the main text; it can also be obtained by noting that

$$\mathcal{L} = \omega \sum_{n\in\mathcal{N}} \kappa_n^2 \left\{ \frac{(1-\alpha)\,\sigma_{x_n}^2\sigma_{z_n}^2}{\sigma_{x_n}^2 + (1-\alpha)\,\sigma_{z_n}^2} \right\},$$

where $\omega \equiv |U_{kk} + 2U_{\sigma_k^2}|/2$, and hence

$$\begin{aligned} \frac{\partial\mathbb{E}u}{\partial\sigma_{z_n}^{-2}} &= -\frac{\partial\mathcal{L}}{\partial\sigma_{z_n}^2}\left(-\frac{1}{[\sigma_{z_n}^{-2}]^2}\right) = \omega\kappa_n^2 \frac{(1-\alpha)\,\sigma_{x_n}^2\sigma_{z_n}^4}{\left[\sigma_{x_n}^2 + (1-\alpha)\,\sigma_{z_n}^2\right]^2} > 0 \\ \frac{\partial\mathbb{E}u}{\partial\sigma_{x_n}^{-2}} &= -\frac{\partial\mathcal{L}}{\partial\sigma_{x_n}^2}\left(-\frac{1}{[\sigma_{x_n}^{-2}]^2}\right) = \omega\kappa_n^2 \frac{(1-\alpha)^2\,\sigma_{z_n}^2\sigma_{x_n}^4}{\left[\sigma_{x_n}^2 + (1-\alpha)\,\sigma_{z_n}^2\right]^2} > 0 \end{aligned}$$

Part (ii) is then immediate. ■

Proof of Proposition 8. Using $|W_{KK}|/|U_{kk} + 2U_{\sigma_k^2}| = (1-\alpha^*)$, we have that equilibrium welfare is $\mathbb{E}u = \mathbb{E}W(\kappa,\theta) - \mathcal{L}^*$, where

$$\mathcal{L}^* = \tfrac{|U_{kk}+2U_{\sigma_k^2}|}{2}\left\{(1-\alpha^*)\,Var(K-\kappa) + Var(k-K)\right\}. \tag{29}$$

$$= -\tfrac{|U_{kk}+2U_{\sigma_k^2}|}{2}(\alpha^*-\alpha)\,Var(K-\kappa) + \tfrac{|U_{kk}+2U_{\sigma_k^2}|}{2}\left\{(1-\alpha)\,Var(K-\kappa) + Var(k-K)\right\} \tag{30}$$

are the losses due to incomplete information.

(i) Since $Var(K-\kappa)$ and $Var(k-K)$ are both increasing in σ_n, welfare necessarily decreases with σ_n (equivalently, increases in accuracy σ_n^{-2}).

(ii) Consider the "canonical case" in which $Var(k-K)$ is decreasing and $Var(K-\kappa)$ increasing in δ_n. By Proposition 6, the second term in (30) decreases with δ_n if $\alpha > 0$ and increases if $\alpha < 0$. It follows that $\alpha^* \geq \alpha > 0$ suffices for $\mathcal{L}^*$ to decrease (and hence welfare to increase) with δ_n, whereas $\alpha^* \leq \alpha < 0$ suffices for $\mathcal{L}^*$ to increase (and hence welfare decrease) with δ_n. ■

Proof of Condition (17). Since $U\left(k,K,\theta,\sigma_k^2\right)$ is quadratic in k and linear in σ_k^2,

$$U\left(k,K,\theta,\sigma_k^2\right) = U(K,K,\theta,0) + U_k(K,K,\theta,0)(k-K) + \frac{U_{kk}}{2}(k-K)^2 + U_{\sigma_k^2}\sigma_k^2.$$

Using the fact that $\sigma_k^2 = \mathbb{E}[(k-K)^2|\theta,\varepsilon]$ and hence $\mathbb{E}\sigma_k^2 = \mathbb{E}[(k-K)^2]$, we have that ex-ante utility is given

$$\mathbb{E}u = \mathbb{E}W(K,\theta) + \frac{U_{kk} + 2U_{\sigma_k^2}}{2} \cdot \mathbb{E}[(k-K)^2].$$

A Taylor expansion of $W(K,\theta)$ around $K = \kappa$ then gives

$$W(K,\theta) = W(\kappa,\theta) + W_K(\kappa,\theta)(K-\kappa) + \frac{W_{KK}}{2}(K-\kappa)^2$$

and hence

$$\mathbb{E}u = \mathbb{E}W(\kappa,\theta) + \mathbb{E}[W_K(\kappa,\theta)\cdot(K-\kappa)] + \frac{W_{KK}}{2}\cdot\mathbb{E}[(K-\kappa)^2] + \frac{U_{kk}+2U_{\sigma_k^2}}{2}\cdot\mathbb{E}[(k-K)^2].$$

In equilibrium, $\mathbb{E}k = \mathbb{E}K = \mathbb{E}\kappa$ and therefore, $\mathbb{E}[W_K(\kappa,\theta)\cdot(K-\kappa)] = Cov[W_K(\kappa,\theta),(K-\kappa)]$, $\mathbb{E}[(K-\kappa)]^2 = Var(K-\kappa)$ and $\mathbb{E}[(k-K)^2] = Var(k-K)$, which gives the result. ■

Proof of Corollary 4. Using the formula for the $\mathcal{L}$ function given in the proof of Propositions 9 and 10 below, we have that, after some tedious algebra,

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\sigma_{z_n}^2} &= \omega\kappa_n^2\sigma_{x_n}^4\left\{\frac{(1-\alpha^*)\,\sigma_{x_n}^2 + (1-\alpha)\,(1-2\alpha+\alpha^*)\,\sigma_{z_n}^2}{\left[\sigma_{x_n}^2 + (1-\alpha)\,\sigma_{z_n}^2\right]^3} + 2\phi_n\frac{(1-\alpha^*)}{\left[\sigma_{x_n}^2+(1-\alpha)\,\sigma_{z_n}^2\right]^2}\right\} \\
\frac{\partial\mathcal{L}}{\partial\sigma_{x_n}^2} &= \omega\kappa_n^2\sigma_{z_n}^4\,(1-\alpha)\left\{\frac{(1-\alpha-2\alpha^*)\,\sigma_{x_n}^2 + (1-\alpha)^2\,\sigma_{z_n}^2}{\left[\sigma_{x_n}^2+(1-\alpha)\,\sigma_{z_n}^2\right]^3} + 2\phi_n\frac{(1-\alpha^*)}{\left[\sigma_{x_n}^2+(1-\alpha)\,\sigma_{z_n}^2\right]^2}\right\}
\end{aligned}$$

where $\omega \equiv |U_{kk} + 2U_{\sigma_k^2}|/2$. The result then follows immediately. ■

Proof of Propositions 9 and 10. We prove the two result together, in three steps. Step 1 computes the welfare losses due to incomplete information. Step 2 derives the comparative statics. Step 3 characterizes the bounds $\underline{\phi}$, $\bar{\phi}$, $\underline{\phi}'$, $\bar{\phi}'$.

Step 1. The property that W is quadratic, along with $W_K(\kappa^*,\theta) = 0$ (by definition of the first best), and $W_{KK} < 0$, imply that

$$W_K(\kappa,\theta) = W_K(\kappa^*,\theta) + W_{KK}\cdot(\kappa-\kappa^*) = |W_{KK}|\cdot(\kappa^*-\kappa).$$

It follows that

$$Cov(K-\kappa, W_K(\kappa,\theta)) = |W_{KK}|\cdot Cov\,(K-\kappa,\kappa^*-\kappa). \tag{31}$$

Since $K-\kappa = \sum_{n\in\mathbf{N}}\kappa_n\gamma_n(z_n-\theta_n)$, $z_n - \theta_n = [\lambda_n(\varepsilon_n) + (1-\lambda_n)(\mu_{\theta_n}-\theta_n)]$, and $(\varepsilon_n,\varepsilon_j,\theta_n,\theta_j)$ are mutually orthogonal whenever $n \neq j$, we have

$$\begin{aligned}
Cov\,(K-\kappa,\kappa^*-\kappa) &= Cov\left(\sum \kappa_n\gamma_n(z_n-\theta_n)\ ,\ \sum(\kappa_n^*-\kappa_n)\,\theta_n\right) = \\
&= \sum(\kappa_n^*-\kappa_n)\,\kappa_n\gamma_n Cov\,(\theta_n, z_n-\theta_n) = \\
&= \sum_{n\in\mathcal{N}}\left(\frac{\kappa_n^*-\kappa_n}{\kappa_n}\right)\kappa_n^2\gamma_n\left[-(1-\lambda_n)\,Var\,(\theta_n)\right]
\end{aligned}$$

Using $\phi_n \equiv (\kappa_n^* - \kappa_n)/\kappa_n,\ \gamma_n = \delta_n/(1-\alpha+\alpha\delta_n)$, and $(1-\lambda_n)\,Var\,(\theta_n) = (\sigma_{\theta_n}^{-2}/\sigma_{z_n}^{-2})\sigma_{\theta_n}^2 = \sigma_{z_n}^2 = \sigma_n^2/\delta_n$, we have that

$$Cov\,(K-\kappa, \kappa^* - \kappa) = \sum_{n\in\mathcal{N}} \phi_n \left\{ -\frac{1}{1-\alpha+\alpha\delta_n}\kappa_n^2\sigma_n^2 \right\} \tag{32}$$

while

$$Cov\,(\ K-\kappa, \kappa \mid \theta_{-n}\ ) = \kappa_n^2\gamma_n Cov\,(z_n - \theta_n, \theta_n) = -\frac{1}{1-\alpha+\alpha\delta_n}\kappa_n^2\sigma_n^2.$$

Next, as in the proof of Proposition 2,

$$\begin{aligned} Var(K-\kappa) &= \sum_{n\in\mathcal{N}} \frac{\delta_n}{(1-\alpha+\alpha\delta_n)^2}\kappa_n^2\sigma_n^2 & (33)\\ Var(k-K) &= \sum_{n\in\mathcal{N}} \frac{(1-\alpha)^2(1-\delta_n)}{(1-\alpha+\alpha\delta_n)^2}\kappa_n^2\sigma_n^2. & (34) \end{aligned}$$

Substituting (31)-(34) into (17), using $v = (1-\alpha^*)|U_{kk} + 2U_{\sigma_k^2}|$, and rearranging, we get

$$\mathcal{L} = \omega \sum_{n\in\mathcal{N}} \Lambda\,(\alpha, \alpha^*, \phi_n, \delta_n)\,\kappa_n^2\sigma_n^2 \tag{35}$$

where $\omega \equiv |U_{kk} + 2U_{\sigma_k^2}|/2$ and

$$\Lambda\,(\alpha, \alpha^*, \phi, \delta) \equiv \frac{(1-\alpha^*)\,[2\phi(1-\alpha+\alpha\delta)+\delta] + (1-\alpha)^2(1-\delta)}{(1-\alpha+\alpha\delta)^2}. \tag{36}$$

Step 2. $\mathbb{E}W(\kappa,\theta)$ is independent of (δ_n, σ_n) and hence the comparative statics of welfare with respect to (δ_n, σ_n) coincide with the opposite of those of $\mathcal{L}$. Also note that

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\sigma_n^2} &= \omega\kappa_n^2\Lambda\,(\alpha, \alpha^*, \phi_n, \delta_n)\\ \frac{\partial\mathcal{L}}{\partial\delta_n} &= \omega\kappa_n^2\sigma_n^2\frac{\partial\Lambda\,(\alpha, \alpha^*, \phi_n, \delta_n)}{\partial\delta_n} \end{aligned}$$

where $\omega \equiv |U_{kk} + 2U_{\sigma_k^2}|/2 > 0$. We thus only need to understand the sign of Λ and that of $\partial\Lambda/\partial\delta_n$.

Note that $\Lambda >$ [<] 0 if and only if $\phi >$ [<] $g(\alpha, \alpha^*, \delta)$, where

$$g(\alpha, \alpha^*, \delta) = -\frac{(1-\alpha)^2(1-\delta) + \delta(1-\alpha^*)}{2(1-\alpha+\alpha\delta)(1-\alpha^*)} < 0.$$

Letting

$$\underline{\phi}'(\alpha, \alpha^*) \equiv \min_{\delta\in[0,1]} g(\alpha, \alpha^*, \delta) \quad \text{and} \quad \bar{\phi}'(\alpha, \alpha^*) \equiv \max_{\delta\in[0,1]} g(\alpha, \alpha^*, \delta),$$

we then have that $\partial\mathcal{L}/\partial\sigma_n^2 > 0$ [< 0] for all $\delta_n \in [0,1]$ if $\phi_n > \bar{\phi}'$ [$< \underline{\phi}'$], whereas $\partial\mathcal{L}/\partial\sigma_n^2$ alternates sign as δ_n varies if $\phi_n \in (\underline{\phi}', \bar{\phi}')$.

Next, consider the effect of commonality. By condition (36),

$$\frac{\partial\Lambda}{\partial\delta_n} = \frac{\alpha^2[(1-\delta_n)(1-\alpha) - \delta_n] - \alpha^*(1-\alpha-\alpha\delta_n) - 2\alpha\phi(1-\alpha^*)(1-\alpha+\alpha\delta_n)}{(1-\alpha+\alpha\delta_n)^3}$$

When $\alpha = 0$, this reduces to

$$\frac{\partial \Lambda}{\partial \delta_n} = -\alpha^*$$

and hence, for any $n \in \mathcal{N}$, $\partial \mathcal{L}/\partial \delta_n > [<]0$ if and only if $\alpha^* < [>]0$.

When instead $\alpha \neq 0$,

$$\frac{\partial \Lambda}{\partial \delta_n} = \frac{2(1-\alpha^*)}{[1-\alpha+\alpha\delta_n]^2}\alpha[f(\alpha,\alpha^*,\delta_n) - \phi_n],$$

where

$$f(\alpha,\alpha^*,\delta) \equiv \frac{\alpha^2[(1-\delta)(1-\alpha)-\delta] - \alpha^*(1-\alpha-\alpha\delta)}{2\alpha(1-\alpha+\alpha\delta)(1-\alpha^*)}.$$

Since $\alpha^* < 1$, $sign[\partial \mathcal{L}/\partial \delta_n] = sign[\alpha] \cdot sign[f(\alpha,\alpha^*,\delta_n) - \phi_n]$. Let

$$\underline{\phi}(\alpha,\alpha^*) \equiv \min_{\delta\in[0,1]} f(\alpha,\alpha^*,\delta) \quad \text{and} \quad \bar{\phi}(\alpha,\alpha^*) \equiv \max_{\delta\in[0,1]} f(\alpha,\alpha^*,\delta).$$

If $\phi_n \in (\underline{\phi},\bar{\phi})$, then $\partial \mathcal{L}/\partial \delta_n$ alternates sign as δ_n varies between 0 and 1, no matter whether $\alpha > 0$ or $\alpha < 0$. Hence, $\phi_n < \underline{\phi}$ is necessary and sufficient for $\partial \mathcal{L}/\partial \delta_n > 0\ \forall \delta_n$ when $\alpha > 0$ and for $\partial \mathcal{L}/\partial \delta_n < 0\ \forall \delta_n$ when $\alpha < 0$, whereas $\phi_n > \bar{\phi}$ is necessary and sufficient for $\partial \mathcal{L}/\partial \delta_n < 0\ \forall \delta_n$ when $\alpha > 0$ and for $\partial \mathcal{L}/\partial \delta_n > 0\ \forall \delta_n$ when $\alpha < 0$.

Step 3. Note that both f and g are monotonic in δ, with

$$\frac{\partial f}{\partial \delta} = 2\frac{\partial g}{\partial \delta} = \frac{(1-\alpha)}{(1-\alpha^*)(1-\alpha+\alpha\delta)^2}(\alpha^* - \alpha)$$

When $\alpha^* = \alpha$, both f and g are independent of δ, and

$$\underline{\phi}'(\alpha,\alpha^*) = \underline{\phi}(\alpha,\alpha^*) = \bar{\phi}(\alpha,\alpha^*) = \bar{\phi}'(\alpha,\alpha^*) = -\frac{1}{2} < 0.$$

When instead $\alpha^* > \alpha$, both f and g are strictly increasing in δ, so that

$$\begin{aligned}
\underline{\phi}(\alpha,\alpha^*) &= f(\alpha,\alpha^*,0) < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,1), \\
\underline{\phi}'(\alpha,\alpha^*) &= g(\alpha,\alpha^*,0) < \bar{\phi}'(\alpha,\alpha^*) = g(\alpha,\alpha^*,1),
\end{aligned}$$

and when $\alpha^* < \alpha$, both f and g are strictly decreasing in δ, so that

$$\begin{aligned}
\underline{\phi}(\alpha,\alpha^*) &= f(\alpha,\alpha^*,1) < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,0) \\
\underline{\phi}'(\alpha,\alpha^*) &= g(\alpha,\alpha^*,1) < \bar{\phi}'(\alpha,\alpha^*) = g(\alpha,\alpha^*,0).
\end{aligned}$$

Consider first the case $\alpha \in (0,1)$. If $\alpha^* > \alpha$, then $\alpha^2 + (1-2\alpha)\alpha^* > 0$ (using the fact that $\alpha^* < 1$) and therefore

$$\underline{\phi}(\alpha,\alpha^*) < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,1) = -\frac{\alpha^2 + (1-2\alpha)\alpha^*}{2\alpha(1-\alpha^*)} < 0.$$

If instead $\alpha^* < \alpha$, then

$$\underline{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,1) = -\frac{\alpha^2 + (1-2\alpha)\alpha^*}{2\alpha(1-\alpha^*)} < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,0) = -\frac{\alpha^* - \alpha^2}{2\alpha(1-\alpha^*)}$$

and therefore $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1-2\alpha)$, while $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$. Since $-\alpha^2/(1-2\alpha) < 0$ whenever $\alpha < 1/2$, we conclude that, for $\alpha \in (0,1)$, $\underline{\phi} < 0$ if and only if $\alpha > 1/2$ or $\alpha^* > -\alpha^2/(1-2\alpha)$, and $\bar{\phi} < 0$ if and only if $\alpha^* > \alpha^2$.

Next, consider the case $\alpha \in (-\infty, 0)$. If $\alpha^* > \alpha$, then

$$\underline{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1-\alpha^*)} < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,1) = \frac{\alpha^2 + (1-2\alpha)\alpha^*}{(-2\alpha)(1-\alpha^*)}$$

and hence $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, while $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1-2\alpha)$. If instead $\alpha^* < \alpha$, then $\alpha^* < 0 < \alpha^2$ and hence

$$\underline{\phi}(\alpha,\alpha^*) < \bar{\phi}(\alpha,\alpha^*) = f(\alpha,\alpha^*,0) = \frac{\alpha^* - \alpha^2}{(-2\alpha)(1-\alpha^*)} < 0.$$

We conclude that, for $\alpha \in (-\infty, 0)$, $\underline{\phi} < 0$ if and only if $\alpha^* < \alpha^2$, and $\bar{\phi} < 0$ if and only if $\alpha^* < -\alpha^2/(1-2\alpha)$.

Finally, note that

$$g(\alpha,\alpha^*,0) = -\frac{(1-\alpha)}{2(1-\alpha^*)} < 0 \quad \text{and} \quad g(\alpha,\alpha^*,1) = -\frac{1}{2} < 0.$$

Hence, $\underline{\phi}' = -\frac{(1-\alpha)}{2(1-\alpha^*)} < -1/2 = \bar{\phi}'$ for $\alpha^* > \alpha$, $\underline{\phi}' = \bar{\phi}' = -\frac{1}{2}$ for $\alpha = \alpha^*$, and $\underline{\phi}' = -1/2 < \bar{\phi}' = -\frac{(1-\alpha)}{2(1-\alpha^*)} < 0$ for $\alpha^* < \alpha$. ■

Proof of Corollary 6. Parts (i) and (ii) follow directly from Propositions 9 and 10, noting that $\underline{\phi} = \bar{\phi} = \underline{\phi}' = \bar{\phi}' = -1/2$ when $\alpha = \alpha^*$. For parts (iii) and (iv), note that $\alpha = \alpha^*$ implies that

$$\mathcal{L} = \omega \sum_{n \in \mathcal{N}} \kappa_n^2 \left\{ (1 + 2\phi_n) \frac{(1-\alpha)\,\sigma_{x_n}^2 \sigma_{z_n}^2}{\sigma_{x_n}^2 + (1-\alpha)\,\sigma_{z_n}^2} \right\},$$

where $\omega \equiv |U_{kk} + 2U_{\sigma_k^2}|/2$. The above is identical to the formula for $\mathcal{L}$ in the proof of Proposition 7, except for the multiplication by the term $(1 - 2\phi_n)$. The result then follows immediately from the proof of Proposition 7. ■

Proof of Corollary 12. That welfare increases with private information follows from the property that $\alpha < 0$ and $\phi > \bar{\phi}$ (which ensures that welfare decreases with commonality) and the property that $\phi > \bar{\phi}'$ (which ensures that welfare increases with accuracy). As for the effect of public information, substituting $\alpha^* = 2\alpha$ and $\phi = \frac{\alpha}{2(1-\alpha)}$ in (35)-(36), we have that

$$\mathcal{L} = \frac{\sigma_x^2 \sigma_z^2 \left[(1-2\alpha)\,\sigma_x^2 + \left(1 - 2\alpha + \alpha^3\right) \sigma_z^2 \right]}{(1-\alpha)\left(\sigma_x^2 + \sigma_z^2\right)\left(\sigma_x^2 + (1-\alpha)\,\sigma_z^2\right)}$$

and hence

$$\frac{\partial\mathcal{L}}{\partial\sigma_z^2}=\frac{\sigma_x^2\left[\left(1-\alpha^2\right)(1-\alpha)^2\sigma_z^4+(1-2\alpha)\,\sigma_x^4+2\left(1-2\alpha+\alpha^3\right)\sigma_x^2\sigma_z^2\right]}{(1-\alpha)\left(\sigma_x^2+\sigma_z^2\right)^2\left[\sigma_x^2+(1-\alpha)\,\sigma_z^2\right]^2}$$

Note that the denominator is always positive. When $\alpha\in[-1,0)$, the numerator is also positive for all σ_x and σ_z. When instead $\alpha<-1$, we can find values for σ_x and σ_z such that the numerator is negative. (Indeed, it suffices to take σ_z high enough, for then the term $\left(1-\alpha^2\right)(1-\alpha)^2\sigma_z^4$, which is negative when $\alpha<-1$, necessarily dominates the other two terms in the numerator.) It follows that the social value of public information is necessarily positive when $\alpha\in[-1,0)$, but can be negative when $\alpha<-1$. ■

Proof of Corollary 13. That welfare necessarily increases with public information follows directly from Corollary 5 since $\alpha^*>\alpha>0$ and $\phi>0$. For the social value of private information, after some tedious algebra, it is possible to show that

$$\frac{\partial\mathcal{L}}{\partial\sigma_x^2}=\frac{\sigma_z^4\left[\lambda_1\sigma_x^4+\lambda_2\sigma_x^2\sigma_z^2+\lambda_3\sigma_z^4\right]}{2(1+c)(1+2c)\left(\sigma_x^2+\sigma_z^2\right)^2\left[(b+2bc)\,\sigma_z^2-2(1+c)\left(\sigma_x^2+\sigma_z^2\right)\right]^2}$$

where λ_1,λ_2, and λ_3 are positive functions of b and c. (This result has been obtained with Mathematica; the code and the formulas for the λ's are available upon request.) It follows that welfare also increases with the precision of private information. ■

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