Demand for Slant:
How Abstention Shapes Voters' Choice of News Media*

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Abstract

Political commentators warn that the fragmentation of the modern media landscape induces voters to withdraw into "information cocoons" and segregate along ideological lines. We show that the option to abstain breaks ideological segregation and generates "cross-over" in news consumption: voters with considerable leanings toward a candidate demand information that is *less* biased toward that candidate than voters who are more centrist. This non-monotonicity in the demand for slant makes voters' ideologies non-recoverable from their choice of news media and generates disproportionate demand for media outlets that are centrist or only moderately biased. It also implies that polarization of the electorate may lead to ideological moderation in news consumption. Thus, among rational voters, ideological segregation and information cocoons are perhaps less of a problem than commonly believed.

1 Introduction

News outlets are in a period of wrenching change. With the negligible costs of self-publishing on the web, the proliferation of camera phones, and the ability to broadcast in real time via Twitter, nearly anyone can produce and disseminate news and opinion. At the same time, consumers increasingly have the power to customize the news and opinion they consume, tailoring it to fit their preferences.

Proliferation of news outlets and the ability to customize the news consumed is, no doubt, a boon to individuals. But at a societal level, these features are potentially baneful. In *On Liberty*, John Stuart Mill writes that "it is only by the collision of adverse opinions that the ... truth has any

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chance of being supplied." Voters' ability to tailor news offerings to hear what they want to hear effectively shuts down this "marketplace of ideas" and invites individuals to crawl into cosseted information cocoons, undisturbed by differing viewpoints or even facts contrary to their preferred world view. (See, e.g., Sunstein (2002), Sunstein (2009), Jamieson and Cappella (2010) and Brooks (2010).)

Relying on binary decision models, formal theory by and large confirms this "echo chamber" argument: for purely instrumental reasons, rational voters prefer information sources whose biases conform to their own, both in direction and degree (see Calvert (1985) and Suen (2004)). To see why, consider a liberal and a conservative candidate competing in a majoritarian election. The candidates differ sharply in their policy positions. The "correct" policies to address the polity's problems are determined by a draw from nature. When the state is liberal, the policies espoused by the liberal candidate are more effective, while the reverse is true when the state is conservative. All voters prefer superior outcomes, but they differ in their ideologies—i.e., their prior beliefs that the policies proposed by a given candidate are the most effective in addressing the challenges at hand. Prior to voting, voters consult a news outlet, customized to their taste, to determine which candidate offers the better policies. News outlets are biased, characterized by Type I and Type II error with a centrist outlet having equal errors of each type.

What motivates rational voters to choose a biased outlet is the hurdle in overcoming their own ideology. Under compulsory voting, a voter with an ideology favoring the conservative candidate chooses a news outlet more likely to favor the conservative candidate in either state. The reason is that, should this outlet unexpectedly favor the liberal candidate, it offers compelling evidence, sufficient to overcome the voter's ideology, that the liberal candidate is the correct choice. Thus, in these models, voters exhibit a strong "demand for slant." Indeed, as the population of voters becomes ideologically polarized, relatively centrist media outlets see customers desert in droves.

Despite this prediction, and the greater polarization of the US electorate manifesting itself in the Tea Party and Occupy movements, centrist news outlets have not fallen by the wayside. For instance, the *New York Times* now enjoys more readers than at any time in its history.[1] New centrist outlets, such as Yahoo, have successfully entered the scene. Thus, there is a disconnect between the dark predictions of the existing theory—which suggests that Mill's vision of democracy is in jeopardy—and the rather more reassuring reality.

In this paper, we show that the key to reconciling this apparent disconnect is to enrich the standard model with the possibility of abstention—the option to stay home from the polls. To see how this amendment to the model fundamentally changes the demand for slant, consider the

[1] Subscriptions to the weekday print edition have fallen from around 1.2 million in 1993 to around 800 thousand in 2012. However, this has been more than compensated by over 800 thousand digital subscriptions (A.B.C. (2012)). Including non-subscribers, *nytimes.com* attracts as many as 45 million unique visitors a month. Each month, visitors can read up to 10 articles for free.

situation of a moderately polarized voter. In the standard model with compulsory voting, such a voter chooses a relatively extreme news outlet and follows its advice at the polls. In the model with abstention, such a voter turns out only when the signal favors his ex ante preferred candidate. If the signal favors the candidate of the opposite ideology, he prefers to stay home. As we show, the crux is that, for this modified plan, a centrist or near centrist outlet is in fact better suited than a more extreme outlet. Enriching the model to account for voters' option to abstain implies that, as the population becomes more ideologically polarized, demand for centrist and near centrist outlets can *increase.*

From the perspective of a researcher seeking to infer a voter's ideology from his choice of news outlet, our model presents an important challenge. No longer is it the case that the more a voter's ideology favors a given side, the more biased the news outlet chosen. Instead, the selected outlet offers a jumble of possibilities. Less ideological voters may select more ideological outlets than their more ideological brethren and vice-versa.

The intuition for the abstention-induced non-monotonicity is as follows. First, centrist and mildly polarized voters continue to behave under voluntary voting as they do under compulsory voting. That is, the former consult and follow a centrist outlet, while the latter follow a paper with a mild, conforming bias. Next, consider a liberal voter (say) whose prior beliefs are more pronounced. For such a voter to follow his news outlet when it tells him to vote for the conservative candidate, he needs to be consulting an outlet with a strong liberal bias. This comes at the cost of virtually always voting for the liberal candidate, even when the state is conservative, as an outlet with a strong liberal bias rarely comes out in favor of the conservative candidate. Nonetheless, if the voter had to choose between the two candidates, this would be his best option. Suppose, however, that the voter has the option of supporting *neither* candidate and abstaining. For the usual reasons related to the Swing Voter's Curse (Feddersen and Pesendorfer (1996)), upon receiving the signal that the state is conservative, abstaining rather than voting for the conservative candidate is indeed an attractive alternative for such a liberal-leaning voter.

What does abstaining do to this voter's optimal choice of news outlet? That is, how biased does he want his outlet to be if he only follows its recommendation when the advice is to vote for the liberal candidate, but abstains when the advice is to vote for the conservative candidate? As he no longer plans on voting for the conservative candidate upon receiving the signal that the state is conservative, his original rationale for consulting an outlet with a strong liberal bias has significantly diminished. In relative terms, he now worries more about voting for the liberal candidate when the state is in fact conservative. Hence, he wants to raise the level of certainty conveyed by the "liberal" signal and, therefore, prefers to consult a strictly more centrist outlet. In other words, the outlet of choice of this liberal-leaning voter who sometimes abstains is less biased than that of some intrinsically more centrist voter who follows his news outlet's recommendation in *both* directions. Notice that, as a result, he also votes for the liberal candidate *less* often.

To summarize, the option to abstain makes the demand for slant a non-monotone function of ideology. The implications are as follows: First, abstention induces "cross-over" in news consumption and breaks strict ideological segregation. This constitutes a challenge to the received wisdom about information cocoons and echo chambers. Second, it creates disproportionate demand for relatively centrist news outlets. These outlets are able to serve multiple constituencies who use the same information in different ways. Third, when voters can abstain, polarization of the electorate may lead to ideological moderation in news consumption, potentially further benefiting centrist news outlets. Hence, the oft-prophesied, imminent demise of balanced political discourse is perhaps exaggerated.

The remainder of the paper is organized as follows. In Section 2, we present a simple example illustrating the main point of the paper. Section 3 introduces the general model, which we solve in Section 4. In Section 5 we analyze the demand for slant. Section 6 discusses some extensions and limitations of the model, while Section 7 reviews the related literature. Finally, Section 8 concludes. Formal proofs are relegated to the Appendix.

2 An example

The results in this paper are driven by the observation that the option to abstain makes the demand for slant non-monotone in ideology. That is, certain voters with considerable leanings toward a candidate demand information that is *less* biased toward that candidate than voters who are more centrist. The intuition is nicely illustrated in the following example. Consider a voter with the following preferences: he gains $u = 1$ if he votes for the "right" candidate, but loses $c = 2 > u$ if he votes for the "wrong" candidate. In addition, he has the option to abstain, which gives him a utility of 0. From the perspective of the voter, the "right" candidate is determined by the state of nature: in state d, the right candidate is D; in state r, the right candidate is R. The voter's prior belief that the state is r is equal to $\theta \in [0, 1]$. We say that the higher is θ, the more "biased" is the voter toward candidate R.

The voter can consult one of two news outlets, labeled **r** and **d**. Before the election, the outlets publish editorials supporting one of the candidates. Formally, support for candidate d is expressed by sending a signal s_d, while support for R is expressed by sending a signal s_r. News outlet **d** supports candidate D in state d 75% of the time, while it supports candidate R in state r 65% of the time. Conversely, news outlet **r** supports candidate R in state r 75% of the time, while it supports candidate D in state d 65% of the time. Hence, in both states, outlet **d** supports candidate D more often than does outlet **r** , while outlet **r** supports candidate R more often. In an intuitive sense, this makes **d** more biased toward D, and **r** more biased toward R.

Let $\rho_\theta^{\mathbf{k}}(s)$ denote the voter's posterior belief that the state is r upon receiving a signal s from outlet $\mathbf{k}$. By Bayes' rule, $\rho_\theta^{\mathbf{d}}(s) > \rho_\theta^{\mathbf{r}}(s)$ for all $s \in \{s_d, s_r\}$. That is, a signal from outlet $\mathbf{r}$ provides weaker evidence that the state is r than the same signal from outlet $\mathbf{d}$.

Which news outlet, if any, does the voter consult, and how does this depend on whether voting is mandatory? When voting is mandatory, the voter must choose between: 1) always voting for D, 2) always voting for R, and 3) voting for D after s_d and voting for R after s_r. We denote these strategies by DD, RR, and DR, respectively. (Notice that the strategy RD, i.e., voting for R after s_d and voting for D after s_r, is strictly dominated and, hence, ignored.) When playing DR, the voter also has to decide which news outlet to consult.

The voter's expected utility from voting for D after receiving the signal s_d from outlet $\mathbf{k}$ is

$$U_\theta^{\mathbf{k}}(D, s_d) = 1 \times \left(1 - \rho_\theta^{\mathbf{k}}(s_d)\right) - 2 \times \rho_\theta^{\mathbf{k}}(s_d) \tag{1}$$

Similarly, his utility of voting for R after receiving the signal s_r from outlet $\mathbf{k}$ is

$$U_\theta^{\mathbf{k}}(R, s_r) = 1 \times \rho_\theta^{\mathbf{k}}(s_r) - 2 \times \left(1 - \rho_\theta^{\mathbf{k}}(s_r)\right) \tag{2}$$

The expected utility from playing the voting strategy DR while consulting outlet $\mathbf{k}$ is then

$$U_\theta^{\mathbf{k}}(DR) = U_\theta^{\mathbf{k}}(D, s_d) \times \Pr(s_d \mid \mathbf{k}, \theta) + U_\theta^{\mathbf{k}}(R, s_r) \times \Pr(s_r \mid \mathbf{k}, \theta) \tag{3}$$

Here, $\Pr(s \mid \mathbf{k}, \theta)$ denotes the probability that the voter assigns to outlet $\mathbf{k}$ sending the signal s. For instance, $\Pr(s_d \mid \mathbf{d}, \theta) = .75 \times \theta + .35 \times (1 - \theta)$. By contrast, the expected utilities from the voting strategies DD and RR do not depend on $\mathbf{k}$. They are equal to

$$\begin{aligned} U_\theta(DD) &= 1 \times (1-\theta) - 2 \times \theta \\ U_\theta(RR) &= 1 \times \theta - 2 \times (1-\theta) \end{aligned} \tag{4}$$

Comparing (3) with (4), we find that under compulsory voting: 1) for $\theta \leq \frac{5}{18}$, the voter does not consult a news outlet and always supports D; 2) for $\frac{5}{18} < \theta \leq \frac{1}{2}$, he consults outlet $\mathbf{d}$ and follows its advice at the polls; 3) for $\frac{1}{2} < \theta \leq \frac{13}{18}$, he consults and follows outlet $\mathbf{r}$; 4) for $\frac{13}{18} \leq \theta$, he once again consults neither outlet and always votes for R. Hence, under compulsory voting, the voter only consults news outlets whose biases conform to his own. This is consistent with Calvert (1985) and Suen (2004).

Why does the voter switch news outlet at $\theta = \frac{1}{2}$? Notice that the two outlets provide different services. While outlet $\mathbf{d}$ provides a high level of certainty that the state is r when it sends the signal s_r, outlet $\mathbf{r}$ provides a high level of certainty that the state is d when it sends s_d. Depending on his prior, a voter attaches different values to these certainties. A voter with $\theta < \frac{1}{2}$ is predisposed to vote for D. Hence, in relative terms, he cares less about the *additional* certainty that the state is d provided by signal s_d. By contrast, if he plans to vote against his prior and for R after s_r, he cares a lot about the additional certainty that the state is r provided by signal s_r. He achieves this certainty by consulting outlet $\mathbf{d}$. An analogous argument explains why a voter with $\theta > \frac{1}{2}$ consults outlet $\mathbf{r}$, if any.

When voting is voluntary, the voter's optimal strategy is more complicated—and interesting. First, abstention expands the set of (non-dominated) voting strategies with $D\Phi$, ΦR, and $\Phi\Phi$, which correspond to: 1) voting for D after s_d and abstaining after s_r, 2) abstaining after s_d and voting for R after s_r, and 3) abstaining after both signals. It is easily verified that, for every $\theta \in [0,1]$, there exists at least one news outlet $\mathbf{k} \in \{\mathbf{d}, \mathbf{r}\}$ such that either $U_\theta^{\mathbf{k}}(D, s_d) > 0$ or $U_\theta^{\mathbf{k}}(R, s_r) > 0$. Hence, always abstaining is a dominated strategy for all prior beliefs. In addition, for a liberal-leaning voter (i.e., $\theta < \frac{1}{2}$), DR dominates RR and ΦR, while for a conservative-leaning voter (i.e., $\theta > \frac{1}{2}$), DR dominates DD and $D\Phi$. We may conclude that, for a liberal voter, only the voting strategies DR, $D\Phi$, and DD are relevant, while for a conservative voter the relevant strategies are DR, ΦR, and RR. When using a responsive voting strategy—i.e., $D\Phi$, DR, or ΦR—a voter also needs to decide which news outlet to consult. Because of symmetry, we may limit attention to, say, the liberal side of the ideological spectrum, i.e., $\theta \leq \frac{1}{2}$. Optimal behavior on the conservative side is analogous.

Conditional on consulting outlet $\mathbf{k}$, the expected utility from voting $D\Phi$ is $U_\theta^{\mathbf{k}}(D, s_d) \times \Pr(s_d \mid \mathbf{k}, \theta)$. Using (1) and the relevant expressions for $\Pr(s_d \mid \mathbf{k}, \theta)$, we find that, conditional on $D\Phi$, a voter with beliefs $\theta \leq \frac{1}{3}$ prefers to consult outlet $\mathbf{d}$, while a voter with $\theta > \frac{1}{3}$ prefers to consult outlet $\mathbf{r}$. Comparing the resulting expected payoff with that from the unresponsive voting strategy DD given in (4), we find that a voter with prior beliefs $\theta > \frac{5}{31}$ prefers $D\Phi$ (in combination with the optimal news outlet) to DD, while a voter with $\theta \leq \frac{5}{31}$ prefers DD to $D\Phi$.

It remains to determine which voters prefer DR over DD and $D\Phi$. Conditional on playing DR, a liberal voter always prefers consulting $\mathbf{d}$ over consulting $\mathbf{r}$. The intuition for this rational "confirmation bias" is exactly the same as under compulsory voting. Comparing the expected payoff from DR with $\mathbf{d}$ to the payoff from $D\Phi$ with either $\mathbf{d}$ or $\mathbf{r}$, we find that a liberal voter prefers DR if and only if $\theta > \frac{8}{17}$. (A fortiori, he also prefers DR over DD in that region.) Hence, a liberal voter's optimal strategy as a function of his prior is

$$\begin{array}{lll} DD & \text{for} & \theta \leq \frac{5}{31} \\ D\Phi\ \mathbf{d} & \text{for} & \frac{5}{31} < \theta \leq \frac{1}{3} \\ D\Phi\ \mathbf{r} & \text{for} & \frac{1}{3} < \theta \leq \frac{8}{17} \\ DR\ \mathbf{d} & \text{for} & \frac{8}{17} < \theta \leq \frac{1}{2} \end{array}$$

A conservative voter's optimal strategy is the mirror-image analogue. Figure 1 provides a graphical

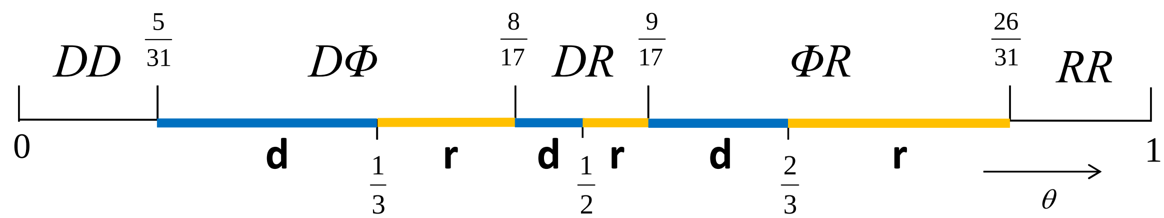

Figure 1: Optimal choice of news outlet and voting strategy as a function of prior belief θ

representation.

The voting behavior is rather intuitive. An extremely liberal voter ($\theta \leq \frac{5}{31}$) always votes for D. A less extreme liberal ($\frac{5}{31} < \theta \leq \frac{8}{17}$) votes for D upon receiving s_d and abstains upon receiving s_r. Finally, a moderate liberal ($\frac{8}{17} < \theta \leq \frac{1}{2}$) votes for D upon receiving s_d and votes for R upon receiving s_r. The optimal choice of news outlet, by contrast, is rather curious: while voters $\frac{5}{31} < \theta \leq \frac{1}{3}$ and $\frac{8}{17} < \theta \leq \frac{1}{2}$ exhibit the familiar rational confirmation bias, voters $\frac{1}{3} < \theta \leq \frac{8}{17}$ exhibit a rational *anti*-confirmation bias. These liberal voters choose to "cross-over" and consult the conservative outlet **r**, rather than the liberal outlet **d**. In other words, in addition to the switch of news outlet at $\theta = \frac{1}{2}$ which also occurs under compulsory voting, on each side of the political spectrum, we now have two more switches.

Let us continue to focus on the liberal side of the spectrum. At the first switch, which occurs at $\theta = \frac{1}{3}$, only the news outlet changes (from **d** to **r**) while the voting strategy, $D\Phi$, remains unchanged. By contrast, the second switch, which occurs at $\theta = \frac{8}{17}$, involves not only a change in news outlet (from **r** back to **d**), but also a change in voting strategy (from $D\Phi$ to DR). The intuition for these two switches is quite different. While the switch at $\theta = \frac{1}{3}$ is a consequence of the discreteness of the space of available news outlets, the switch at $\theta = \frac{8}{17}$ is a consequence of *the discrete nature of voting itself.* As we shall see, this implies that the first switch disappears when we allow for a continuum of news outlets, while the second switch is a robust feature of any voting environment.

First, let us focus on the switch at $\theta = \frac{1}{3}$, where only the news outlet but not the voting strategy changes. The intuition for this switch is quite similar to the one at $\theta = \frac{1}{2}$. Even though all voters in the interval $\frac{5}{31} < \theta \leq \frac{8}{17}$ play $D\Phi$, depending on the value of θ, they have different default actions. That is, if they had to decide purely on the basis of their prior, voters to the left of $\frac{1}{3}$ would vote for D, while the voters to the right of $\frac{1}{3}$ would abstain. This makes these voters concerned about different attributes of the same voting strategy, $D\Phi$. After any signal, a voter whose θ is close to $\frac{5}{31}$ more strongly believes that the state is d than a voter whose θ is close to $\frac{8}{17}$. Therefore, the former values less than the latter any *additional* certainty that the state is d after a signal s_d. At the same time, in order to abstain, the former needs more reassurance that the state is *not d* after a signal

s_r than the latter. Hence, the voter whose θ is close to $\frac{5}{31}$ prefers an outlet, **d**, that provides strong evidence that the state is *not* d when sending the signal s_r, while the voter whose θ is close to $\frac{8}{17}$ prefers an outlet, **r**, that provides strong evidence that the state *is* d when sending the signal s_d. The actual switch in news outlet occurs where the default action changes from D to Φ.

The second switch, from **r** back to **d**, takes place at $\theta = \frac{8}{17}$, and occurs for a rather different reason. Rather than the change in the default action, it is the change in (responsive) voting strategy—from $D\Phi$ to DR—that induces the voter to change his news outlet. To see this, notice that, by its very nature, the strategy space in voting environments is discrete. Hence, any change in voting strategy entails a discontinuous change in the payoff function which, in turn, implies a discontinuous change in the value of information. How does the value of information change when the voting strategy changes from $D\Phi$ to DR? The jump from abstaining to voting for R after the signal s_r makes the voter considerably more "concerned" about the certainty conveyed by s_r. The reason is that, instead of taking the relatively safe action of abstaining, he now plans to take the much riskier action of voting against his prior. Suddenly, he attaches a lot more value to having additional certainty that the state really is r after observing s_r. In order to achieve this, he is willing to give up some certainty that the state is d after s_d. This explains why the switch in voting strategy from $D\Phi$ to DR induces the voter to switch from news outlet **r** to news outlet **d**.

Our example is special in three respects. First, there are only two news outlets. Second, $u < c$; i.e., the benefit of voting for the right candidate is smaller than the cost of voting for the wrong candidate. Third, preferences are "pseudo-expressive," in the sense that a voter's payoff depends on whether his *vote* matches the state. One may wonder which aspects of the example carry over to more canonical voting environments with multiple news outlets, general cost-benefit ratios, and purely instrumental preferences. As we shall see, the discrete and *non-monotone* switch in news outlet between the $D\Phi$ and DR regions is a robust phenomenon, while the discrete and *monotone* switch within the $D\Phi$ region is not. (Again, in our discussion, we limit attention to the liberal side of the ideological spectrum. Behavior on the conservative side is analogous.)

The switch within the $D\Phi$ region is generated by the fact that, while voters' beliefs are changing smoothly, in the example, voters cannot smoothly adjust their choice of news outlet. Instead, they have to choose between either **d** or **r**. However, as we have argued in the introduction, the modern media landscape is characterized by a wide array of news outlets, catering to every ideological bias and belief. Hence, in today's world, people *can* adjust their choice of news outlet more or less smoothly. When we account for this by allowing for a continuum of outlets spanning the ideological spectrum, the discrete jump within $D\Phi$ disappears. Instead, within this interval, there is a smooth and monotone change from liberal to more centrist outlets, as we move from the ideological left toward the center.

As argued above, the discrete and non-monotone switch at the boundary between $D\Phi$ and DR

is caused not by the discreteness of the media landscape—indeed, it occurs in spite of it—but by the change in (responsive) voting strategy. Note, however, that when the benefit of voting for the right candidate is greater than the cost of voting for the wrong candidate—i.e., $\frac{u}{c} > 1$—no voter ever abstains and the boundary between $D\Phi$ and DR does not exist. In that case, because there is no switch from one responsive voting strategy to another, the demand for slant is a monotonic function of ideology and our example has no bite. This raises the question whether $u < c$ or $u > c$ is the "right" assumption.

When we study a voting model with fully rational voters and instrumental preferences, (the equivalent of) the ratio $\frac{u}{c}$ is endogenous. That is, the probabilities of the pivotal events that give rise to the benefits and costs of casting a vote versus abstaining are determined in equilibrium. Moreover, as explained in detail in Feddersen and Pesendorfer (1996), voters who are (close to) indifferent between D and R suffer from the Swing Voter's curse, which implies that they strictly prefer to abstain rather than vote for either candidate. Hence, while in our simple example, we have to assume that $u < c$ in order to generate abstention, in a more general model with instrumental preferences, abstention is endogenously generated. This explains why the non-monotonicity and discontinuity of the demand for slant are in fact robust phenomena, that extend beyond our simple example to canonical voting environments.

3 The model

In Section 2, we studied a simple decision-theoretic example to illustrate the basic intuition behind cross-over in news consumption. Here, we extend the analysis to a fully-fledged strategic voting environment. Specifically, we study a two -candidate election with a Poisson-distributed number of voters who have the option to abstain. Preferences are purely instrumental, i.e., voters only care about the outcome of the election and not about their own vote. There are two kinds of voters, partisan and non-partisan. Partisan voters always prefer one candidate over the other, while non-partisan voters, who constitute the majority, want the winner of the election to match the state of nature. While all non-partisans prefer the Democratic candidate in the Democratic state and the Republican candidate in the Republican state, they do, however, disagree as to the prior probability that these states pertain. While some believe that the state is more likely to be Democratic, others believe that the state is more likely to be Republican. We shall say that the former are leaning to, or "biased" toward, the Democratic candidate, while the latter are leaning to or biased toward the Republican candidate. Voters who think that the two states are more or less equally likely are called centrists.

Before casting a ballot, voters can costlessly and privately consult one news outlet. News outlets, of which there are a continuum, send conditionally independent binary signals that are correlated with the state. News outlets span the ideological spectrum, from strongly biased toward the Democratic candidate at one end, to strongly biased toward the Republican candidate at the other end. An outlet that is biased toward the Democrat is very likely to send the "Democratic" signal when the state is Democratic. In that sense, it has high accuracy in that state. However, high accuracy in the Democratic state comes at the cost of low accuracy in the Republican state. That is, even when the state is Republican, this news outlet is relatively likely to send the Democratic signal. For an outlet that is biased toward the Republican the reverse holds. Finally, a centrist news outlet is one whose accuracy is (close to) the same in the two states.

Once voters have consulted their preferred news outlet, if any, they simultaneously cast their ballots or abstain. The candidate who receives the most votes wins the election and payoffs are realized. Ties are resolved by a coin flip.

Formally, there are two candidates, $j \in \{D, R\}$, and two states of the world, $\omega \in \{d, r\}$. The number of voters is Poisson distributed with mean υ (see Myerson (1998) and Myerson (2000)). The probability that a voter is partisan is equal to $\zeta \in \left(0, \frac{1}{2}\right)$ and independent across voters.[2] A partisan voter prefers candidate R with probability η and candidate D with the remaining probability, again independently across voters.[3] The remaining voters are non-partisan. Non-partisans receive a payoff of zero when the winner of the election matches the state (i.e., $j = D$ and $\omega = d$, or $j = R$ and $\omega = r$), and -1 otherwise. We denote a non-partisan voter's prior belief that the state is r by $\theta \in [0, 1]$. These beliefs are private information and drawn independently from a distribution with cumulative distribution function (CDF) F on $[0, 1]$ that admits a well-behaved probability density function (PDF) f with no mass points.[4]

Voters can cast a ballot for D, for R, or they can abstain, which we denote by Φ. Before casting a ballot, a voter can collect a signal $s \in \{s_d, s_r\}$ from one news outlet. News outlets are characterized by accuracies $(p_d, p_r) \in [0, 1]^2$ in state d and state r, respectively. Here, p_d is the probability that the news outlet sends the (correct) signal s_d in state d. Likewise, p_r is the probability that the news outlet sends the (correct) signal s_r in state r.

We identify a news outlet with its accuracy in state r and assume that there is a continuum of outlets, one for each $p_r \in [0, 1]$. The relationship between news outlets' accuracies in the two states

[2] All results can be extended to the case where $\xi = 0$, but some extra care is needed to prove existence. Details can be provided upon request.

[3] Because the number of voters is Poisson distributed, we could accommodate "partisans" who always prefer to abstain. This would not change the results.

[4] Because we allow for general belief distributions, F, our assumption about the payoffs of non-partisan voters is less restrictive than it may seem. Specifically, the model can accommodate heterogeneity in payoffs, as long as preferences are co-monotone, i.e., non-partisans agree that candidate D is preferred in state d, while candidate R is preferred in state r. To see this, suppose non-partisan voter i earns α_i when D is elected in state d, β_i when D is elected in state r, γ_i when R is elected in state d, and δ_i when R is elected in state r. When comparing a vote for D, a vote for R, and abstaining, only the payoff differences $l_{d,i} \equiv \alpha_i - \gamma_i$ and $l_{r,i} \equiv \delta_i - \beta_i$ matter. Provided that preferences are co-monotone such that $l_{d,i}, l_{r,i} > 0$ for all i, this implies that we can redefine voters' priors to be $\theta' = \frac{\theta l_{r,i}}{(1-\theta) l_{d,i} + \theta l_{r,i}}$ and proceed as before.

is described by a function G that maps an outlet's accuracy in state r into its accuracy in state d. That is, $p_d = G(p_r)$. We assume that G is strictly decreasing, twice continuously differentiable, and strictly concave. Strict concavity implies that increased accuracy in one state becomes progressively more expensive in terms of reduced accuracy in the other state. It ensures that, for a given voting strategy, a voter's optimal choice of news outlet is "well-behaved" (i.e., continuous) in θ and the ratio of pivotal probabilities. We further assume that G runs from $G(0) = 1$ to $G(1) = 0$. This means that a news outlet can achieve perfect accuracy in state d, but only by always sending the signal s_d. Similarly, perfect accuracy in state r comes at the cost of perfect inaccuracy in state d. Finally, let $0 < -G'(0) < -G'(1) < \infty$. As we show later, this implies that, for non-degenerate priors, posterior beliefs remain bounded away from zero and 1.

Summarizing,

Assumption 1 *The accuracy function* $G : [0,1] \to [0,1]$*, which maps* p_r *into* p_d*, is a strictly decreasing, strictly concave, twice continuously differentiable bijection with* $0 < -G'(0) < -G'(1) < \infty$.

Under these conditions, it is easy to show that there always exists a (unique) news outlet, $\tilde{p} > \frac{1}{2}$, whose accuracy is the same in both states. (See Lemma 3, part 3, in the Appendix.) We call this outlet perfectly centrist, and say that a news outlet is leaning to (or biased toward) R if $p_r > \tilde{p}$. Bias toward D is defined analogously.

For ease of exposition, we impose one additional assumption on G, namely, that it gives rise to "increasing elasticities." Specifically, denote by $\delta_{p_r,1-p_d}$ the elasticity of accuracy p_r in state r with respect to *in*accuracy $1-p_d$ in state d. That is, $\delta_{p_r,1-p_d}$ measures the percentage rise in inaccuracy in state d associated with a one percent rise in accuracy in state r. Formally,

$$\delta_{p_r,1-p_d} = \frac{d(1-p_d)}{dp_r}\frac{p_r}{1-p_d} = \frac{-G'(p_r)\,p_r}{1-G(p_r)}$$

Similarly, $\delta_{p_d,1-p_r}$ is the elasticity of accuracy p_d in state d with respect to inaccuracy $1-p_r$ in state r. I.e.,

$$\delta_{p_d,1-p_r} = \frac{d(1-p_r)}{dp_d}\frac{p_d}{1-p_r} = \frac{G(p_r)}{-G'(p_r)(1-p_r)}$$

When $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ are strictly increasing in p_r and p_d, respectively, then each *percentage* increase in accuracy in one state becomes progressively more expensive in terms of reduced accuracy in the other state, again expressed in percentage terms. It is easy to verify that this property implies strict concavity of G, but that the reverse implication does not hold. Strictly increasing elasticities only play a role in the proof of Proposition 1. They ensure that all payoff functions satisfy single-crossing in θ and, thus, make equilibrium behavior particularly simple to characterize.

Assumption 2 *Elasticities* $\delta_{p_r,1-p_d}$ *and* $\delta_{p_d,1-p_r}$ *are strictly increasing in* p_r *and* p_d*, respectively.*

In Section 6, we relax assumption 2 and show that our results remain essentially unchanged.

In order to better understand the informational implications of choosing a particular news outlet p_r, we now study the posterior beliefs it induces. Denote by $\rho_\theta^{p_r}(s)$ a voter's posterior belief that the state is r upon receiving a signal s from news outlet p_r. By Bayes' rule,

$$\rho_\theta^{p_r}(s_r) = \left(1 + \frac{1 - G(p_r)}{p_r} \frac{1-\theta}{\theta}\right)^{-1} \text{ and } \rho_\theta^{p_r}(s_d) = 1 - \left(1 + \frac{1-p_r}{G(p_r)} \frac{\theta}{1-\theta}\right)^{-1} \tag{5}$$

Differentiating with respect to p_r reveals that $\rho_\theta^{p_r}(s_r)$ and $\rho_\theta^{p_r}(s_d)$ are monotonically decreasing in p_r. (See Lemma 3, part 4, in the Appendix.) This is intuitive. Higher p_r means that a news outlet is more biased toward R, such that a signal s_r conveys less evidence that the state is indeed r. This makes $\rho_\theta^{p_r}(s_r)$, the posterior belief that the state is r after s_r, decreasing in p_r. At the same time, higher p_r also means that the signal s_d conveys more evidence that the state is *not* r. This makes $\rho_\theta^{p_r}(s_d)$, the posterior belief that the state is r after s_d, also decreasing in p_r.

For $p_r \downarrow 0$, $\rho_\theta^{p_r}(s_r)$ and $\rho_\theta^{p_r}(s_d)$ approach their suprema $\frac{\theta}{\theta - G'(0)(1-\theta)} < 1$ and θ, respectively. For $p_r \uparrow 1$, $p_r \downarrow 0$, $\rho_\theta^{p_r}(s_r)$ and $\rho_\theta^{p_r}(s_d)$ approach their infima θ and $\frac{\theta}{\theta - G'(1)(1-\theta)} > 0$. (See Lemma 3, part 4, in the Appendix.) Because $-G'(0) > 0$, upon receiving the signal s_r, a voter achieves less than perfect certainty that the state is r even when $p_r \downarrow 0$. Similarly, because $-G'(1) < \infty$, upon receiving the signal s_d, a voter achieves less than perfect certainty that the state is *not* r (i.e., d) even when $p_r \uparrow 1$.[5]

Voters receive signals that are conditionally independent. This implies that voters who choose media outlets with the same accuracy do not necessarily receive the same signal. This assumption is made purely for technical convenience. Assuming conditional independence reduces the level of correlation across voters' signals. Since our model involves continuous types and continuous media outlets, correcting for this reduction in correlation would require taking care of complex measurability issues. As should be clear from the underlying intuition, this would not affect the main result of our paper—i.e., the non-monotonicity and discontinuity of the demand for slant. However, it would considerably increase the complexity and length of the proofs.

The timing of the game is as follows: 1) Nature selects the state, the number of voters, and the profile of voter types. 2) Each voter observes his own type. 3) Voters privately decide which news outlet to consult and they draw a private and conditionally independent signal from the selected outlet. 4) Ballots are cast, the winner is selected according to majority rule, and payoffs are realized. Ties are resolved by a coin flip.

In this game, the optimal behavior of partisan voters is trivial: they simply vote according to their party affiliation. Indeed, since non-partisans are the only ones who act strategically, in the

[5] In Section 6, we relax the assumption of bounded derivatives and show that our results remain essentially unchanged.

remainder, "voter" refers to non-partisan voter. Non-partisans must choose which news outlet to consult, if any, and how to vote. For them, a strategy consists of a tuple (P, V). Here, P is a measurable function mapping a voter's prior θ into a choice of news outlet, p_r. We refer to P as the *demand for slant* and assume that a voter only consults a news outlet if it leaves him strictly better off. The act of *not* consulting an outlet is denoted by $\emptyset$. Hence, $P : [0,1] \to [0,1] \cup \{\emptyset\}$. The function V, which we refer to as the *voting strategy*, maps signals $\{s_d, s_r\}$ into a vote for D, a vote for R, or abstention. Hence, $V = \{s_d, s_r\} \to \{D, R, \Phi\}$. We denote particular voting strategies as follows: DR refers to voting for D after the signal s_d and voting for R after the signal s_r. All other combinations of D, R,and Φ are similarly defined. For example, DD denotes always voting for D, while ΦR denotes abstaining after s_d and voting for R after s_r.

We study pure-strategy symmetric Bayesian Nash equilibria of this game. A symmetric Bayesian Nash equilibrium is a profile of identical strategies (P^*, V^*), one for each voter, such that no voter has a strictly profitable, unilateral deviation.

Micro-foundation for G For maximum generality, we have simply postulated an accuracy function G and endowed it with certain plausible properties. This naturally raises the question what kind of information technology and behavior on the part of media outlets generates accuracy functions with the assumed properties. As we now show, such accuracy functions naturally arise when media outlets observe a continuous signal that they coarsen into a binary voting recommendation using some threshold rule, τ.

Suppose that the states of nature $\omega = d$, r, are associated with 0 and 1, respectively. Each news outlet observes a conditionally independent, continuous signal $\sigma \in \mathbb{R}$ equal to the true state plus noise: $\sigma = \omega + \varepsilon$. Noise ε, which is independent of ω, is described by a CDF H on $\mathbb{R}$ that admits a well-behaved PDF h. Specifically, h is strictly positive, differentiable, and single-peaked around zero. A news outlet with threshold rule τ sends signal s_d if and only if $\sigma \leq \tau$, and s_r otherwise. Each threshold, τ, implies a pair of accuracies $(p_d(\tau), p_r(\tau)) \in [0,1]^2$, where $p_d(\tau) = H(\tau)$ and $p_r = 1 - H(\tau - 1)$. There is a continuum of such media outlets, one for each $\tau \in \mathbb{R}$. Hence, the implied accuracy function, G, is

$$G(p_r) = H\left(1 + H^{-1}(1 - p_r)\right) \tag{6}$$

It is easily verified that this function is indeed strictly decreasing in p_r and runs from $G(0) = 1$ to $G(1) = 0$. Strict concavity of G corresponds to strict log-concavity of h. (See Lemma 4 in the Appendix.) Recall that many standard distributions, including the Normal, the Logistic, and the Extreme Value distribution, satisfy this property.[6]

[6] See Bergstrom and Bagnoli (2005) for a list of probability densities satisfying log-concavity.

As shown formally in Lemma 5, strict increasingness of $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ corresponds to h being strictly more log-concave than H and $1-H$. That is, for $x \in \mathbb{R}$,

$$\frac{d^2 \ln h(x)}{(dx)^2} < \frac{d^2 \ln H(x)}{(dx)^2} \text{ and } \frac{d^2 \ln h(x)}{(dx)^2} < \frac{d^2 \ln(1-H(x))}{(dx)^2} \tag{7}$$

When h is symmetric, the two inequalities coincide. This follows from the fact that, under symmetry, $H(x) = 1 - H(-x)$. As proved in Lemma 6, the Normal and the Logistic distributions do satisfy the conditions in (7), while the Extreme Value distribution does not. With the Extreme Value distribution in mind, in Section 6 we discuss what happens when the conditions in (7) fail. While the equilibrium characterization becomes somewhat less elegant, our results remain essentially unchanged.

Let γ_σ denote the likelihood ratio, $\frac{h(\sigma-1)}{h(\sigma)}$, between state r and state d conditional on signal σ. Because $G'(p_r) = -\frac{h\left(1+H^{-1}(1-p_r)\right)}{h(H^{-1}(1-p_r))}$, we have that $G'(0) = -1/\gamma_\infty$ and $G'(1) = -1/\gamma_{-\infty}$. Hence, the assumption that $-\infty < G'(1) < G'(0) < 0$ corresponds to γ_σ remaining bounded when $\sigma \to \infty$, and bounded away from zero when $\sigma \to -\infty$. In other words, there is a limit to how much can be learned from any signal from any news outlet. While the Logistic distribution satisfies this property, the Normal distribution does not.[7] With the Normal distribution in mind, in Section 6 we discuss what happens when $\gamma_\infty = \infty$ and $\gamma_{-\infty} = 0$—i.e., $-G'(1) = \infty$ and $-G'(0) = 0$. As we show, the results remain essentially unchanged.

Finally, as an illustration, we derive the accuracy function G and likelihood ratios γ_∞ and $\gamma_{-\infty}$ implied by Logistic noise.

Example 1 *Suppose that ε is Logistically distributed with precision λ. Then the implied accuracy function G is*

$$G(p_r) = \frac{1}{1 + \frac{1}{e^\lambda}\frac{p_r}{1-p_r}}$$

The likelihood ratios converge to $\gamma_\infty = e^\lambda$ and $\gamma_{-\infty} = e^{-\lambda}$.

4 Solving the Model

To solve the model, we proceed in two steps. First, we derive a voter's optimal choice of news outlet conditional on his voting strategy. This gives rise to a set of "indirect" utility functions, one for each voting strategy. Then we compare these indirect utility functions and, for each θ, identify the voting strategy that generates the highest payoff. Of course, the complication we have to deal with is that voters' payoffs not only depend on their priors, their choice of news outlet, and their voting strategy, but also on the ratios of pivotal probabilities, which are determined in equilibrium.

[7] The Extreme Value distribution constitutes a hybrid: $\gamma_{-\infty} = 0$, while $\gamma_\infty < \infty$.

The remainder of this section is organized as follows. In Section 4.1, we study pivotal events and probabilities. In Section 4.2, we derive the optimal choice of news outlet for each voting strategy. Finally, in Section 4.3, we compare indirect utilities and characterize equilibrium voting behavior.

4.1 Pivotal Events and Probabilities

A rational voter anticipates that his decision at the ballot box only affects his payoff when it changes the outcome of the election. Hence, when deciding how to vote, he conditions on being pivotal. When comparing a vote for D with abstaining, the relevant pivotal events are that R is leading by one vote or that the two candidates are tied. When R is leading by one vote, a vote for D rather than abstaining throws the election into a tie and, hence, raises the probability that D wins by 50%. When the candidates are tied, a vote for D hands the election to D and, therefore, also raises D's probability of winning by 50%. The payoff comparison of voting for R versus abstaining is analogous. In that case, the pivotal events are that D is leading by one vote or that the two candidates are tied. Either way, a vote for R rather than Φ raises R's chances by 50%. Finally, when comparing a vote for D with a vote for R, the pivotal events consist of all of the above, i.e., R is leading by one vote, D and R are tied, or D is leading by one vote. Notice that when a candidate is leading by one vote, the shift in winning probabilities induced by a vote for R rather than D is once more 50%. By contrast, when the two candidates are tied, a vote for R instead of D shifts the election from a sure win for R to a sure win for D.

For a given profile of symmetric strategies (P, V), we now derive expressions for the probabilities of the various pivotal events. Denote by $t_D(\omega)$ the probability that a randomly drawn voter, who may be partisan or non-partisan, casts a vote for D in state ω. Denote by $n_D(\omega)$ the total number of votes for D in state ω. Let $t_R(\omega)$ and $n_R(\omega)$ be similarly defined. As proved by Myerson (2000), $n_D(\omega)$ and $n_R(\omega)$ are independently distributed Poisson random variables with expectation parameters $\upsilon t_D(\omega)$ and $\upsilon t_R(\omega)$. The memorylessness of the Poisson distribution implies that, from the perspective of a given voter i, $n_D(\omega)$ and $n_R(\omega)$ also describe the number of votes for D and R cast by all *other* voters. From i's perspective, the pivotal events in state ω correspond to $|n_D(\omega) - n_R(\omega)| \leq 1$, i.e., situations where, after all others have cast their votes, one candidate leads by at most one vote.

Let $T^x(\omega)$ denote the probability that D leads by $x \in \mathbb{Z}$ votes. The probabilities of the three pivotal events are then $T^{-1}(\omega)$, $T^0(\omega)$, and $T^1(\omega)$. Because $T^x(\omega)$ is Poisson distributed, we have

$$T^{-1}(\omega) = e^{-\upsilon(t_R(\omega)+t_D(\omega))} \sum_{n=0}^{\infty} \frac{(\upsilon t_D(\omega))^n}{n!} \frac{(\upsilon t_R(\omega))^{n+1}}{(n+1)!}$$

The expressions for $T^0(\omega)$, and $T^1(\omega)$ are analogous.

Denote the set of all pivotal probabilities by T, i.e., $T \equiv \left\{ \begin{array}{c} T^{-1}(d), T^{0}(d), T^{1}(d), \\ T^{-1}(r), T^{0}(r), T^{1}(r) \end{array} \right\}$. Partisan voters guarantee that $t_D(\omega)$, $t_R(\omega) > 0$. This implies that every vote count, (n_D, n_R), has strictly positive probability of occurring. As pivotal events are nothing but collections of particular vote counts, T is strictly interior, i.e., $T \in (0,1)^6$. Finally, for future reference, let $\widetilde{T}^{-1}(\omega) \equiv T^{0}(\omega) + T^{-1}(\omega)$ and $\widetilde{T}^{1}(\omega) \equiv T^{0}(\omega) + T^{1}(\omega)$.

4.2 Demand for Slant Conditional on Voting Strategy

In this section, we derive voters' optimal choice of news outlet conditional on their voting strategy. First, notice that only DD, $D\Phi$, DR, $\Phi\Phi$, ΦR, and RR are viable voting strategies. The remaining strategies, RD, ΦD and $R\Phi$, entail the "reversing" of signals. Because signals are informative, such strategies are dominated and, hence, not played in equilibrium. Next, notice that a voter who optimally uses one of the unresponsive voting strategies, DD, $\Phi\Phi$, or RR, is indifferent between all news outlets and, thus, by assumption, chooses not to consult one. Therefore, it only remains to determine the optimal choice of news outlet for voters using $D\Phi$, DR, or ΦR. We denote these optimal outlets by $p_r^{D\Phi}(\theta)$, $p_r^{DR}(\theta)$, and $p_r^{\Phi R}(\theta)$, respectively.

Conditional on playing DR, a voter's expected payoff is

$$\begin{aligned} U_\theta^{p_r}(DR) &= L(\theta) - \left\{ \left(1 - \frac{1}{2}p_r\right) T^{1}(r) + (1-p_r) T^{0}(r) + \frac{1}{2}(1-p_r) T^{-1}(r) \right\} \theta \\ &\quad - \left\{ \frac{1}{2}(1-G(p_r)) T^{1}(d) + (1-G(p_r)) T^{0}(d) + \left(1 - \frac{1}{2}G(p_r)\right) T^{-1}(d) \right\} (1-\theta) \end{aligned} \tag{8}$$

The first term, $L(\theta)$, incorporates the expected payoffs (i.e., losses) associated with all *non*-pivotal events. By definition, these payoffs do not depend on the voter's choice of news media or voting strategy. The remaining terms incorporate the payoffs associated with all *pivotal* events. Consider, for instance, the last term on the first line of (8), i.e., $\frac{1}{2}(1-p_r) T^{-1}(r)\theta$. Its interpretation is as follows: The joint event that the state is r, candidate R leads by one vote, and the voter receives the (wrong) signal s_d occurs with probability $(1-p_r) T^{-1}(r)\theta$. As he is playing DR, the signal s_d makes the voter cast a ballot for D. This throws the election into a tie and, hence, leads to an expected loss of one half. As a result, under DR, the contribution to the expected payoff of this particular event is $\frac{1}{2}(1-p_r) T^{-1}(r)\theta$. All other terms have analogous interpretations.

Differentiating $U_\theta^{p_r}(DR)$ with respect to p_r and rearranging yields the first-order condition

$$-G'\left(p_r^{DR}\right) = \frac{\tilde{T}^{-1}(r) + \tilde{T}^{1}(r)}{\tilde{T}^{-1}(d) + \tilde{T}^{1}(d)} \frac{\theta}{1-\theta} \tag{9}$$

It is easily verified that $U_\theta^{p_r}(DR)$ is strictly concave in p_r. Hence, there exists at most one p_r^{DR}

that solves (9) and, at that p_r^{DR}, the second-order condition for a maximum is satisfied.

The first-order condition can be easily understood in terms of cost-benefit arguments. To see this, rewrite (9) as

$$\frac{1}{2}\left(\tilde{T}^{-1}(d)+\tilde{T}^{1}(d)\right)(1-\theta)\cdot -dG\left(p_r^{DR}\right)=\frac{1}{2}\left(\tilde{T}^{-1}(r)+\tilde{T}^{1}(r)\right)\theta\cdot dp_r^{DR}$$

On the right-hand side, we have the gain in expected payoffs induced by a marginal increase in accuracy in state r. This gain is equal to the joint probability that the state is r and the vote is pivotal times the expected gain from voting for R rather than D. On the left-hand side, we have the loss in expected payoffs induced by a marginal decrease in accuracy in state d, which is equal to the joint probability that the state is d and the vote is pivotal times the expected loss from voting for R rather than D. Of course, when the choice of news outlet is optimal, the expected marginal gains are equal to the expected marginal losses.

Because p_r is a probability, it is bounded between 0 and 1. Define ${}_{DD}\theta_{DR}$ to be the type θ such that the solution to the first-order condition (9) just reaches its lower bound, $p_r^{DR}=0$. Similarly, let ${}_{DR}\theta_{RR}$ be the type θ such that the solution to (9) just reaches its upper bound, $p_r^{DR}=1$. In other words, ${}_{DD}\theta_{DR}$ and ${}_{DR}\theta_{RR}$ are the *transition types* where DR degenerates into DD and RR, respectively. Solving (9) for θ reveals that

$${}_{DD}\theta_{DR}=\frac{-G'(0)}{\frac{\tilde{T}^{-1}(r)+\tilde{T}^{1}(r)}{\tilde{T}^{-1}(d)+\tilde{T}^{1}(d)}+(-G'(0))} \text{ and } {}_{DR}\theta_{RR}=\frac{-G'(1)}{\frac{\tilde{T}^{-1}(r)+\tilde{T}^{1}(r)}{\tilde{T}^{-1}(d)+\tilde{T}^{1}(d)}+(-G'(1))} \tag{10}$$

Hence, the transition types ${}_{DD}\theta_{DR}$ and ${}_{DR}\theta_{RR}$ exist, are unique, and are strictly interior. Moreover, because $-G'(0)<-G'(1)$, we have ${}_{DD}\theta_{DR}<{}_{DR}\theta_{RR}$. Recalling that G and $U_\theta^{p_r}(DR)$ are strictly concave in p_r, we may conclude that

$$p_r^{DR}(\theta)=\begin{cases} 0 & \text{for} \quad \theta\in[0,{}_{DD}\theta_{DR}] \\ (G')^{-1}\left(-\frac{\tilde{T}^{-1}(r)+\tilde{T}^{1}(r)}{\tilde{T}^{-1}(d)+\tilde{T}^{1}(d)}\frac{\theta}{1-\theta}\right) & \text{for} \quad \theta\in({}_{DD}\theta_{DR},{}_{DR}\theta_{RR}) \\ 1 & \text{for} \quad \theta\in[{}_{DR}\theta_{RR},1] \end{cases} \tag{11}$$

When playing DR, a voter with beliefs θ sufficiently close to 0 maximizes his utility by "cornering out" and choosing $p_r^{DR}(\theta)=0$. Hence, such a voter is, in effect, playing DD. Similarly, a voter with θ sufficiently close to 1 maximizes his utility by choosing $p_r^{DR}(\theta)=1$ and, in effect, playing RR. For these voters, there simply do not exist media outlets whose signals are sufficiently convincing to overcome their strong prior beliefs.

Along the same lines, we find that $p_r^{D\Phi}(\theta)$ and $p_r^{\Phi R}(\theta)$ are equal to[8]

[8] Expressions for $U_\theta^{p_r}(D\Phi)$ and $U_\theta^{p_r}(\Phi R)$ can be found in Remark 1 in the Appendix. The corresponding first-order conditions for $p_r^{D\Phi}(\theta)$ and $p_r^{\Phi R}(\theta)$ can be found in Lemma 7.

$$p_r^{D\Phi}(\theta) = \begin{cases} 0 & \text{if} \quad \theta \in [0, {}_{DD}\theta_{D\Phi}] \\ (G')^{-1}\left(-\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}\frac{\theta}{1-\theta}\right) & \text{if} \quad \theta \in ({}_{DD}\theta_{D\Phi}, {}_{D\Phi}\theta_{\Phi\Phi}) \\ 1 & \text{if} \quad \theta \in [{}_{D\Phi}\theta_{\Phi\Phi}, 1] \end{cases} \tag{12}$$

$$p_r^{\Phi R}(\theta) = \begin{cases} 0 & \text{if} \quad \theta \in [0, {}_{\Phi\Phi}\theta_{\Phi R}] \\ (G')^{-1}\left(-\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)}\frac{\theta}{1-\theta}\right) & \text{if} \quad \theta \in ({}_{\Phi\Phi}\theta_{\Phi R}, {}_{\Phi R}\theta_{RR}) \\ 1 & \text{if} \quad \theta \in [{}_{\Phi R}\theta_{RR}, 1] \end{cases} \tag{13}$$

Here, the transition types $\{{}_{DD}\theta_{D\Phi}, {}_{D\Phi}\theta_{\Phi\Phi}\}$ and $\{{}_{\Phi\Phi}\theta_{\Phi R}, {}_{\Phi R}\theta_{RR}\}$ are the analogues of the transition types $\{{}_{DD}\theta_{DR}, {}_{DR}\theta_{RR}\}$. They correspond to the points where the responsive voting strategies $D\Phi$ and ΦR degenerate into the unresponsive voting strategies DD, $\Phi\Phi$, and RR. Closed-form solutions can be found in Lemma 8 in the Appendix.

Implicitly differentiating the first-order conditions with respect to θ reveals that, in the interior, $p_r^{DR}(\theta)$, $p_r^{D\Phi}(\theta)$, and $p_r^{\Phi R}(\theta)$ are strictly increasing in θ. Hence, *for a given voting strategy*, the demand for slant is monotone in ideology. The intuition is essentially the same as in Calvert (1985) and Suen (2004). The more the decision maker leans toward one candidate, the stronger evidence he needs in order to vote for the other candidate (or to abstain). Such strong evidence can only be provided by information sources that share his bias.

Having derived voters' preferred demand for slant for a given voting strategy, we now compare the demand for slant *across* voting strategies. As is evident from (11), (12) and (13), the comparison between $p_r^{DR}(\theta)$, $p_r^{D\Phi}(\theta)$, and $p_r^{\Phi R}(\theta)$ crucially hinges on the equilibrium values of the ratios of pivotal probability, $\frac{\tilde{T}^{-1}(r)+\tilde{T}^{1}(r)}{\tilde{T}^{-1}(d)+\tilde{T}^{1}(d)}$, $\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$, and $\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)}$. In the next lemma we show that, for large electorates, these ratios can be unambiguously ranked.

Lemma 1 *For ν sufficiently large, in equilibrium,*

$$\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)} < \frac{\tilde{T}^{1}(r)+\tilde{T}^{-1}(r)}{\tilde{T}^{1}(d)+\tilde{T}^{-1}(d)} < \frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$$

Lemma 1 is intuitive. To see this, notice that $\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)} < \frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$ is equivalent to $\frac{\tilde{T}^{1}(r)}{\tilde{T}^{-1}(r)} < \frac{\tilde{T}^{1}(d)}{\tilde{T}^{-1}(d)}$. In essence, the latter inequality merely states that, comparing across states, the relative likelihood of D leading by one versus R leading by one is greater in state d than in state r. In turn, this is a consequence of the positive correlation between votes and signals and between signals and states. Finally, the fact that $\frac{\tilde{T}^{1}(r)+\tilde{T}^{-1}(r)}{\tilde{T}^{1}(d)+\tilde{T}^{-1}(d)}$ lies in between $\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)}$ and $\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$ is simply an arithmetic necessity.[9]

[9] As shown by Feddersen and Pesendorfer (1996, 1999), voters with posterior beliefs ρ such that $\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)} < \frac{\rho}{1-\rho} <$

In combination with (11), (12) and (13), Lemma 1 implies:

Lemma 2 *Suppose* $p_r^{D\Phi}(\theta)$*,* $p_r^{DR}(\theta)$ *and* $p_r^{\Phi R}(\theta)$ *are strictly interior. For* ν *sufficiently large, in equilibrium,*

$$p_r^{D\Phi}(\theta) > p_r^{DR}(\theta) > p_r^{\Phi R}(\theta)$$

The result is immediate and a formal proof is omitted. Intuitively, a voter is relatively more concerned about certainty after the signal s_d when he is playing $D\Phi$ than when he is playing ΦR. Conversely, he is relatively more concerned about certainty after the signal s_r when he is playing ΦR than when he is playing $D\Phi$. As a result, the voter consults a more R-leaning news outlet under $D\Phi$ than under ΦR. When playing DR, his concerns about certainty after s_d and s_r are more balanced, which translates in consulting a paper with intermediate bias.

4.3 Equilibrium Voting Behavior

In the previous section, we have derived voters' optimal choice of news outlet for a given voting strategy. It remains to compare the induced indirect utilities across voting strategies and, for each θ, determine which voting strategy yields the highest payoff. In addition, we must prove that an equilibrium, (P^*, V^*), indeed exists.

As we show below, equilibrium voting strategies can take on one of two forms, which only differ in whether DR is played. When DR *is* played, voting behavior is the same as in the example in Section 2. It moves from DD for beliefs θ close to zero, to $D\Phi$, to DR, to ΦR and, finally, to RR for beliefs θ close to 1. With some abuse of notation, we write $V^* = DD; D\Phi; DR; \Phi R; RR$ to denote an equilibrium voting strategy of this form. When DR is *not* played, $V^* = DD; D\Phi; \Phi R; RR$. Hence, the voting strategy $\Phi\Phi$ is never played in equilibrium.

More formally, $DD; D\Phi; DR; \Phi R; RR$ and $DD; D\Phi; \Phi R; RR$ are defined as follows. Denote by Θ_{DD}^* the set of voters θ for whom, in equilibrium, the voting strategy DD is a best response, and let the sets $\Theta_{\Phi\Phi}^*$ and Θ_{RR}^* be analogously defined. Also, denote by Θ_{DR}^* the set of voters for whom DR is a best response while consulting a *strictly interior* news outlet $0 < p_r^{DR}(\theta) < 1$, and let $\Theta_{D\Phi}^*$ and $\Theta_{\Phi R}^*$ be defined along the same lines.[10] We say that a set is strictly smaller than another set ($\prec$) if almost all elements in the former are strictly smaller than almost all elements in the latter. We say that a set is strictly smaller than a scalar (also using "$\prec$") if almost all elements in the set are strictly smaller than the scalar. "Strictly greater" is analogously defined.

$\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$ suffer from the Swing Voters' Curse. That is, whichever candidate they vote for, these voters are more likely to push the election in the wrong direction than in the right direction. Hence, they are strictly better off abstaining.

[10] We insist on strict interiority of $p_r^{DR}(\theta)$ in the definition Θ_{DR}^* to avoid overlap of Θ_{DR}^* with Θ_{DD}^* and Θ_{RR}^*. Without this additional requirement, significant but spurious overlap would occur because DR reduces to DD whenever $p_r^{DR}(\theta) = 0$, while it reduces to RR when $p_r^{DR}(\theta) = 1$. Similar issues would arise with $\Theta_{D\Phi}^*$ and $\Theta_{\Phi R}^*$.

$V^* = DD; D\Phi; DR; \Phi R; RR$ then means that Θ^*_{DD}, $\Theta^*_{D\Phi}$, Θ^*_{DR}, $\Theta^*_{\Phi R}$, and Θ^*_{RR} are ordered intervals with non-empty interiors that partition the type space $[0, 1]$. That is,

$$0 \prec \Theta^*_{DD} \prec \Theta^*_{D\Phi} \prec \Theta^*_{DR} \prec \Theta^*_{\Phi R} \prec \Theta^*_{RR} \prec 1$$

and

$$\Theta^*_{DD} \cup \Theta^*_{D\Phi} \cup \Theta^*_{DR} \cup \Theta^*_{\Phi R} \cup \Theta^*_{RR} = [0, 1]$$

The only difference with $V^* = DD; D\Phi; \Phi R; RR$ is that, in this case, Θ^*_{DR} has an empty interior.

In the next proposition, we prove existence of equilibrium and formally characterize equilibrium voting strategies.

Proposition 1 *For ν sufficiently large, there exists a pure-strategy symmetric Bayesian Nash equilibrium, (P^*, V^*). The equilibrium voting strategy, V^*, takes on one of two forms:*

$$\text{1) } DD; D\Phi; DR; \Phi R; RR \qquad \text{2) } DD; D\Phi; \Phi R; RR.$$

To understand why $\Phi\Phi$ is never played in equilibrium, notice that abstention is a best response if and only if a voter's posterior belief, ρ, satisfies $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} \leq \frac{\rho}{1-\rho} \leq \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$.[11] For $\Phi\Phi$ to be optimal, the voter's posterior must lie within these bounds after *both* signals. This is only possible if the prior, θ, also lies within these bounds. In other words, the voter must 1) have a centrist prior and 2) read a centrist paper that conveys little information after both signals. Notice, however, that such a voter can profitably deviate by consulting a strongly biased news outlet and play $D\Phi$ or ΦR. Most of the time, he will receive a signal that conforms to the outlet's bias. In that case, he continues to abstain. Sometimes, however, he will receive a signal that goes against the outlet's bias. Provided the bias is sufficiently strong, a contrarian signal will push the voter's posterior belief outside the narrow band where abstention is optimal, such that the voter now wants to follow the signal. As this deviation allows the voter to use his information at least some of the time, he is strictly better off than under the strategy of always abstaining.

The proof of existence follows along the lines of Oliveros (2011). Rather than directly searching for a fixed point in the space of best responses, we search for a fixed point in the space of pivotal probabilities. The idea is that any collection of pivotal probabilities gives rise to a profile of best responses which, in turn, gives rise to a collection of pivotal probabilities. We then show that best responses are continuous functions of pivotal probabilities and vice versa. Hence, the implied mapping of pivotal probabilities into itself is also continuous and we can apply Brouwer's fixed-point theorem. This gives us a fixed point in pivotal probabilities, which corresponds to a fixed point in the space of best responses, i.e., an equilibrium.

[11] See Lemma 9 in the Appendix for a formal proof of this claim.

5 Demand for Slant

Having characterized equilibrium voting behavior, we now study the implications for the demand for slant. Technically, the primary result of this section is to show that the demand for slant is a non-monotone and discontinuous function of ideology, θ. The economically relevant implication is that abstention induces ideological moderation in news consumption and breaks ideological segregation. Specifically:

1) Voters with considerable leanings toward a candidate consult a news outlet that is *less* biased toward that candidate than other, intrinsically more centrist voters.

2) Certain voters with almost identical beliefs consult news outlets with very different ideologies, while voters with very different beliefs consult outlets with the same ideology. As a result, the electorate does not segregate along ideological lines and voters' beliefs and voting behavior are non-recoverable from their choice of news outlet.

3) Centrist or moderately biased news outlets serve multiple constituencies who use the same information in different ways. As a result, these outlets benefit from disproportionate demand.

4) When the electorate becomes more polarized, its choice of news outlets may become ideologically more moderate.

First, we establish the non-monotonicity and discontinuity of the demand for slant. When $V^* = DD; D\Phi; DR; \Phi R; RR$, there exists a unique type, ${}_{D\Phi}\theta_{DR}$, where V^* switches from $D\Phi$ to DR. From Lemma 2, we know that $p_r^{D\Phi}(\theta) > p_r^{DR}(\theta)$. Hence, at ${}_{D\Phi}\theta_{DR}$, the demand for slant is non-monotone and discontinuous: a voter with beliefs slightly to the right of ${}_{D\Phi}\theta_{DR}$ optimally consults a news outlet that is discontinuously more left-leaning than a voter slightly to the left of ${}_{D\Phi}\theta_{DR}$. The intuition is that, upon receiving a signal that contradicts his prior, the former plans on voting against his prior, while the latter merely abstains. This makes the more right-leaning voter significantly more concerned about certainty after signal s_r than the more left-leaning voter. As a result, the right-leaning voter consults a more left-leaning news outlet than the left-leaning voter. At the other side of the ideological spectrum, the analogous phenomenon occurs: a voter with beliefs slightly to the left of the unique crossing point ${}_{DR}\theta_{\Phi R}$ between DR and ΦR optimally consults a news outlet that is significantly more right-leaning than a voter slightly to the right of that point.

When $V^* = DD; D\Phi; \Phi R; RR$, there exists a unique type, ${}_{D\Phi}\theta_{\Phi R}$, where V^* switches from $D\Phi$ to ΦR. Lemma 2 implies that $p_r^{D\Phi}({}_{D\Phi}\theta_{\Phi R}) > p_r^{\Phi R}({}_{D\Phi}\theta_{\Phi R})$. Hence, also at ${}_{D\Phi}\theta_{\Phi R}$, the demand for slant is non-monotone and discontinuous.

We summarize these observations in the following proposition.

Proposition 2 *When voters have the option to abstain, the demand for slant is a non-monotone and discontinuous function of prior beliefs θ. Non-monotone discontinuities occur at the transition*

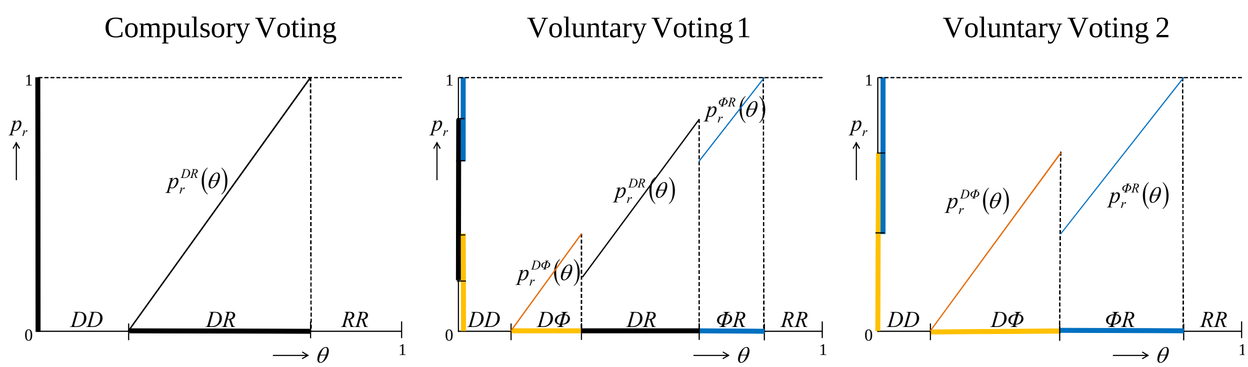

Figure 2: Demand for slant as a function of θ (schematic), under compulsory and voluntary voting.

points between responsive voting strategies, i.e., ${}_{D\Phi}\theta_{DR}$*,* ${}_{DR}\theta_{\Phi R}$*, and* ${}_{D\Phi}\theta_{\Phi R}$*.*

Schematically, the demand for slant as a function of θ is illustrated in the center and right panels of Figure 2. In the interior of $\Theta^*_{D\Phi}$, Θ^*_{DR}, and $\Theta^*_{\Phi R}$, the demand for slant is continuous and monotone in θ. This follows from the continuity and monotonicity of $p_r^{DR}(\theta)$, $p_r^{D\Phi}(\theta)$, and $p_r^{\Phi R}(\theta)$. To the left of ${}_{DD}\theta_{D\Phi}$, voters do not consult a news outlet, as no outlet can dissuade them from voting according to their prior. Just to right of ${}_{DD}\theta_{D\Phi}$, voters do consult a news outlet and, nominally, follow the responsive voting strategy $D\Phi$. However, their preferred outlet has a p_r so close to zero that they almost always receive the signal s_d. In practice, this makes their voting behavior almost indistinguishable from that of voters to left of ${}_{DD}\theta_{D\Phi}$, who do not consult a news outlet at all. Moving farther to the right, voting becomes more responsive, as voters following the voting strategy $D\Phi$ begin to consult more centrist outlets. This continues until we reach either ${}_{D\Phi}\theta_{DR}$ or ${}_{D\Phi}\theta_{\Phi R}$. At ${}_{D\Phi}\theta_{DR}$, the optimal p_r jumps down from $p_r^{D\Phi}({}_{D\Phi}\theta_{DR})$ to $p_r^{DR}({}_{D\Phi}\theta_{DR})$ (center panel). At ${}_{D\Phi}\theta_{\Phi R}$, the optimal p_r jumps down from $p_r^{D\Phi}({}_{D\Phi}\theta_{\Phi R})$ to $p_r^{\Phi R}({}_{D\Phi}\theta_{\Phi R})$ (right panel). On the right side of the political spectrum, behavior is analogous.

What are the implications of voters' demand for slant for ideological segregation in news consumption and for the demand faced by news outlets across the ideological spectrum? As illustrated in Figure 2 (left panel), under compulsory voting, each outlet has a single constituency that, roughly speaking, shares the outlet's ideology. As a result, voters are perfectly ideologically segregated in their news consumption, and the demand for a particular news outlet is roughly proportional to the number (density) of voters with the same ideology as the outlet. Under voluntary voting, by contrast, some news outlets serve two constituencies. When $V^* = DD; D\Phi; DR; \Phi R; RR$, moderately biased outlets have readers who follow the outlet's signal in both directions, as well as readers who follow the signal only when it confirms their prior and abstain otherwise (Figure 2, center panel). The former more or less share the outlet's ideology, while the latter are ideologically more extreme. The situation for moderately right-leaning news outlets is analogous. When V^* is of the form $DD; D\Phi; \Phi R; RR$, it is centrist outlets that serve multiple constituencies, each of whom follows the outlet's signal in one direction only (right panel).

Finally, notice that polarization of the electorate may cause ideological moderation in news consumption. Consider an equilibrium voting strategy of the form $DD; D\Phi; DR; \Phi R; RR$ and transfer probability mass from the interval $({}_{D\Phi}\theta_{DR},\ {}_{D\Phi}\theta_{DR} + \delta)$ to $({}_{D\Phi}\theta_{DR} - \delta,\ {}_{D\Phi}\theta_{DR})$, and from $({}_{DD}\theta_{\Phi R} - \delta,\ {}_{DD}\theta_{\Phi R})$ to $({}_{DD}\theta_{\Phi R},\ {}_{DD}\theta_{\Phi R} + \delta)$ for some $\delta > 0$. Clearly, this makes the electorate more polarized. However, news consumption becomes more moderate, as voters the left of ${}_{D\Phi}\theta_{DR}$ consult more centrist outlets than voters to the right of ${}_{D\Phi}\theta_{DR}$, and voters to the right of ${}_{DD}\theta_{\Phi R}$ consult more centrist outlets than voters to the left of ${}_{DD}\theta_{\Phi R}$. When V^* is of the form $DD; D\Phi; \Phi R; RR$, a similar effect occurs when we transfer probability mass from $({}_{D\Phi}\theta_{\Phi R} - \delta,\ {}_{D\Phi}\theta_{\Phi R})$ to $({}_{D\Phi}\theta_{\Phi R} - 2\delta,\ {}_{D\Phi}\theta_{\Phi R} - \delta)$ and from $({}_{D\Phi}\theta_{\Phi R},\ {}_{D\Phi}\theta_{\Phi R} + \delta)$ to $({}_{D\Phi}\theta_{\Phi R} + \delta,\ {}_{D\Phi}\theta_{\Phi R} + 2\delta)$.[12]

We may conclude,

Proposition 3 *When voters have the option to abstain, strict ideological segregation in news consumption breaks down. As a result, centrist or moderately biased news outlets benefit from disproportionate demand.*

Polarization of the electorate may cause ideological moderation in news consumption.

Political commentators fear that the polarization of the electorate, in combination with voters' new-found ability to customize their news, leads to ideological segregation, and puts John Stuart Mill's vision of democracy in peril (Sunstein (2002), Sunstein (2009), and Brooks (2010).). Proposition 3 suggests that, at least for rational voters, this fear is overblown.

6 Extensions and Limitations

In this section, we discuss some extensions and limitations of our model.

Unbounded Likelihood Ratios So far, we have assumed that $-\infty < G'(1) < G'(0) < 1$. In the micro foundation of G, we saw that this corresponds to likelihood ratios, γ_σ, that remain finite and bounded away from zero when $|\sigma| \to \infty$. In other words, there is a limit to how much can

[12] Somewhat paradoxically, the electorate is better informed under voluntary than under compulsory voting, in the sense that more voters consult a news outlet. To see this, notice that types $\theta \in ({}_{DD}\theta_{D\Phi},\ {}_{DD}\theta_{DR})$ only consult an outlet when they have the option to abstain, but remain uninformed when they must cast a ballot. Intuitively, the option to abstain allows for more "nuanced" voting behavior, which increases the value of information. On a more technical level, recall that the swing voter's curse implies that a voter who is indifferent between D and R strictly prefers to abstain. This implies that, at ${}_{DD}\theta_{DR}$, the voting strategy $D\Phi$ does strictly better than both DD and DR. As the payoff difference between DD and $D\Phi$ is decreasing in θ, the indifference point between $D\Phi$ and DD must lie strictly to the left of ${}_{DD}\theta_{DR}$. For the same reason, ${}_{\Phi R}\theta_{RR} > {}_{DR}\theta_{RR}$.

be learned from any signal from any news outlet. While the Logistic distribution exhibits bounded likelihood ratios, the Normal distribution does not. Hence, one may wonder what happens when $\gamma_\infty = \infty$ and $\gamma_{-\infty} = 0$ or, equivalently, when $G'(1) = -\infty$ and $G'(0) = 0$.

With unbounded likelihood ratios, DD and RR are dominated by DR and, hence, no longer played. The reason is simple: no matter how extreme a voter's (non-degenerate) prior beliefs, unbounded likelihood ratios imply that there always exists a sufficiently extreme news outlet whose "non-conforming" signal contains so much information that it overcomes these priors. Even though an extremist news outlet can be expected to send such a non-conforming signal only extremely rarely, the equally extremist voter is strictly better off consulting this news outlet and voting DR, rather than always voting according to his prior. As a result, only responsive voting strategies, i.e., $D\Phi$, DR, and ΦR, are played in equilibrium. Indeed, with unbounded likelihood ratios, equilibrium voting strategies are of the form $D\Phi$; DR; ΦR or $D\Phi$; ΦR. Hence, the demand for slant remains a non-monotone and discontinuous function of prior beliefs.

Non-monotone Elasticities Another assumption underlying our characterization of equilibrium is that the elasticities $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ are strictly increasing in p_r and p_d, respectively. Recall that in terms of our micro foundation for G, increasing elasticities correspond to a noise density, h, that is strictly more log-concave than the associated CDF H and the DCDF $1 - H$. While standard distributions such as the Normal and the Logistic satisfy this condition, others, such as the Extreme Value distribution, do not. Hence, we now study what happens when we relax the assumption of strictly increasing elasticities and merely assume that G is strictly concave. Recall that strict concavity of G corresponds to a noise density, h, that is strictly log-concave.

In the proof of Proposition 1, the increasing elasticities served to establish single-crossing of the indirect utility functions $U_\theta(DR)$ and $U_\theta(D\Phi)$, as well as $U_\theta(DR)$ and $U_\theta(\Phi R)$. This guaranteed that $\Theta^*_{D\Phi}$, Θ^*_{DR}, and $\Theta^*_{\Phi R}$ were connected intervals. Under mere concavity of G, Θ^*_{DD} and Θ^*_{RR} remain connected. However, in equilibria where DR is played, connectedness of $\Theta^*_{D\Phi}$, Θ^*_{DR}, and $\Theta^*_{\Phi R}$ may fail. That is, while the ordering of the infima and suprema of $\Theta^*_{D\Phi}$, Θ^*_{DR}, and $\Theta^*_{\Phi R}$ remains unchanged, the sets themselves can no longer be fully ordered. As a result, as we increase θ, the equilibrium voting strategy may jump back and forth multiple times between $D\Phi$ and DR on one side, and DR and ΦR on the other.

Obviously, this loss of connectedness makes the characterization of equilibrium less elegant. For the rest, our results remain essentially unchanged: equilibrium is still guaranteed to exist and $\Phi\Phi$ is never played.[13] More importantly, the demand for slant continues to be a non-monotone and

[13] The proof of existence is somewhat complicated by the fact that we can no longer invoke Brouwer's fixed-point theorem. The reason is that the mapping, Γ, from the set of pivotal probabilities into itself may be a correspondence. (For the construction of Γ, see Step 2 of the proof of Proposition 1.) As Γ remains upper hemicontinuous, convex-valued, and has a closed graph, we can, however, invoke Kakutani's generalization. In fact, Kakutani's fixed-point theorem also implies existence of equilibrium for arbitrary ν (cf. Proposition 1). Details available upon request.

discontinuous function of prior beliefs. Indeed, in the absence of connectedness, discontinuities in the demand for slant occur at each and every one of the now potentially many transition points between $D\Phi$ and DR, and between DR and ΦR. Of these transitions, the ones from $D\Phi$ to DR and from DR to ΦR are non-monotone. By contrast, "reverse" transitions, i.e., from DR to $D\Phi$ and from ΦR to DR, are monotone. Finally, notice that when DR is not played, it does not matter whether $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ are increasing.

Multiple News Outlets and Non-binary Signals In our model, voters may consult at most one news outlet. By contrast, Gentzkow and Shapiro (2011) find that some voters choose to consult multiple outlets. This raises the question whether a rational voter consulting multiple outlets would ever cross over, or whether cross-over is an artifact of voters receiving a single binary signal. While a full analysis is complicated and beyond the scope of this paper, the general principle underlying cross-over does generalize to multiple signals. To see this, recall that the discontinuities in the demand for slant are caused by jumps from one responsive voting strategy to another. In the presence of multiple signals, these jumps continue to exist and, in fact, multiply in number.

Suppose voters collect two conditionally independent signals. For simplicity, assume that both signals have the same accuracy, p_r. Denote by $D\Phi R$ the strategy of voting for D upon receiving two s_d signals, abstaining upon receiving one s_d and one s_r signal, and voting for R upon receiving two s_r signals. Other letter combinations are defined analogously. Now consider the pair of voting strategies $D\Phi\Phi$ and $D\Phi R$, and let ${}_{D\Phi\Phi}\theta_{D\Phi R}$ denote the voter who, conditional on consulting optimal news outlets, is indifferent between $D\Phi\Phi$ and $D\Phi R$. After signals $\{s_r, s_r\}$, a voter just to the right of ${}_{D\Phi\Phi}\theta_{D\Phi R}$ votes for R , while a voter just to the left of ${}_{D\Phi\Phi}\theta_{D\Phi R}$ merely abstains. In that case, the former is relatively more concerned about certainty that the state is R than the latter. As this is the only respect in which the two strategies differ, a voter to the left of ${}_{D\Phi\Phi}\theta_{D\Phi R}$ playing $D\Phi\Phi$ optimally consults a more right-leaning news outlet than a voter to the right of ${}_{D\Phi\Phi}\theta_{D\Phi R}$ playing $D\Phi R$.

An analogous argument reveals that cross-over in news consumption does not crucially depend on the binariness of signals either. Suppose that voters are limited to consulting a single news outlet, but that signals can take on three values, s_d, s_ϕ, or s_r. In state r, news outlet p_r sends signal s_d with probability $(1-p_r)^2$, s_ϕ with probability $2p_r(1-p_r)$, and s_r with probability p_r^2. In state d, the chances of s_d, s_ϕ, and s_r are $G(p_r)^2$, $2G(p_r)(1-G(p_r))$, and $(1-G(p_r))^2$, respectively. In this set-up, $D\Phi R$ denotes the strategy of voting for D upon receiving the signal s_d, abstaining upon receiving s_ϕ, and voting for R upon receiving s_r. Other letter combinations are similarly defined. As this model is isomorphic to the two–signal model discussed above, a voter to the left of ${}_{D\Phi\Phi}\theta_{D\Phi R}$ optimally consults a more right-leaning news outlet than a voter to the right of ${}_{D\Phi\Phi}\theta_{D\Phi R}$.

Consumption Value We have assumed that voters' utility from information is purely instrumental. However, for many people, news also provides consumption value. To construct a model where voters enjoy news as a consumption good, we have to take a stand as to how it enters the utility function. We have to determine whether the consumption value derives from the content of the news—i.e., from receiving a particular signal—or whether it derives from the bias of the outlet that provides it. For example, a voter may derive pleasure from hearing that the Republican candidate should be elected, or he may derive pleasure from listening to a news outlet whose reporting tends to coincides with what he believes. In the first case, it does not matter whether a particular signal is reported by CNN or Fox News. In the second case, listening to CNN provides different utility from listening to Fox News, independently of what is reported. In addition, we would have to decide how the consumption value interacts with the informational value of news.

Suppose that the consumption value of news is separable from its informational value and derives from receiving a signal that conforms to one's biases. Then it is easy to show that, in large elections, voters only consult the most extreme media outlets. The reason is simple and similar to Morgan and Várdy (2012). In large elections, the probability of being pivotal is negligible. Hence, the instrumental value of information disappears. Voters now face the simple task of selecting the news outlet that most often provides their preferred signal. Of course, this outlet lies at one of the extremes of the ideological spectrum. Clearly, this is an unsatisfactory prediction. We leave a more thorough analysis for future research.

Unknown Biases Finally, we have also assumed that voters perfectly know the biases of all news media. While, in practice, voters are roughly aware of the slants of the various media outlets, they may not know how strong these biases are exactly. Hence, one may wonder whether our results carry over to an environment with incomplete information about the precise slants of media outlets.

While we have not undertaken a formal analysis, uncertainty about the bias of media outlets should not affect our main result, i.e., the non-monotonicity and discontinuity of the demand for slant. The reason is that the fundamental cause of the discontinuity—namely, the discrete change in the shape of a voter's payoff function upon a jump from one responsive voting strategy to another—remains present. Also, the direction of the jump should remain unaffected, as it remains true that a voter who switches from $D\Phi$ to DR (say) suddenly cares relatively more about certainty after the signal s_r. As a result, he now prefers to consult a news outlet that, in expectation, is more biased toward D.

7 Literature Review

We now put our findings in the context of the extant literature. In recent years, there has been considerable work analyzing the demand for news and its impact on political outcomes (see, e.g., DellaVigna and Gentzkow (2010) and Prat and Stromberg (2010)). In a decision theoretic environment, Calvert (1985) and Suen (2004) show that rational agents optimally demand information that is biased toward their priors. In Oliveros and Vardy (2011), we show that this result extends to the strategic environment of elections with compulsory voting. Gentzkow and Shapiro (2006) provide an additional rationale for voters' apparent confirmatory bias. In their model, voters are not only uncertain about the state of the world, but also about the quality of the various media outlets. As a consequence, outlets that provide information conforming to voters' prior beliefs are thought to be of higher quality. This gives news media an incentive to pander to the biases and prior beliefs of voters. Besley and Prat (2006) study a different source of media bias. They argue that the government's desire to control the flow of information can induce media bias by capture.

The assumption of rational confirmation bias has been used extensively in reduced-form modelling of voters' demand for information. Examples are Mullainathan and Shleifer (2005) and Baron (2006), who study competition in the market for news via differentiation in price and slant; Chan and Suen (2008), who look at political competition; Krasa et al. (2008), who consider the impact of media bias on election outcomes; and Duggan and Martinelli (2010), who analyze economic policy selection and optimal slant.

Papers such as Strömberg (2004), DellaVigna and Kaplan (2007), Chiang and Knight (2008), Gerber et al. (2007), Gerber et al. (2009), Enikolopov et al. (2011), and Durante and Knight (2011) study whether news media affect voting behavior. They find some evidence to that effect. However, the ideological leanings of media outlets and their customers are not as closely correlated as one might expect. Indeed, Gentzkow and Shapiro (2011) show that news consumption is less ideologically segregated than would be predicted by existing theories. Similar results were found in surveys by Pew (2008) and Pew (2009). The contribution of our paper is to uncover a fundamental rationale for this imperfect segregation in news consumption.

Our paper is closely related to the literatures on abstention and information acquisition in voting. Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999) show that voters who are (close to) indifferent between supporting either one of two candidates suffer from a "Swing Voter's Curse." This gives these voters a strict incentive to abstain. Oliveros (2011) studies the incentives to abstain when information acquisition is endogenous and costly. Martinelli (2006) and Martinelli (2007) also allow for endogenous information acquisition and study the information aggregation properties of elections.

Finally, our paper is related to the literature on learning from coarse information. The aforementioned articles by Calvert (1985) and Suen (2004) are important references. Bøg (2008) studies a decision maker who can observe the binary choices of other experimenters, but not the actual outcomes of their experiments. He discusses how priors, preferences and coarseness of observation interact, such that the decision maker sometimes chooses to observe experimenters with different preferences. Gill and Sgroi (2011) study a company about to launch a new product. It can publicly test the product and condition its price on whether the product passes or fails. The ability to adjust the price in response to the outcome of the test reinforces the positive effect of passing, and mitigates the negative effect of failing. This convexifies the firm's profits and induces the firm to select either the softest or the hardest test. Closer to our paper, Meyer (1991) studies how an employer can optimally generate and use coarse information in promotion decisions that are governed by a sequence of rank-order contests. If the employer can choose not to promote anyone, Meyer (1991) shows that, for purely informational reasons, it may be optimal to handicap the early leader in the final-period contest. This handicapping can be interpreted as a kind of "cross-over" on the part of the employer: despite being positively disposed toward the early leader, the employer chooses to stack the final contest against him. The early leader is promoted only if he wins the final contest. Otherwise, no one is promoted.

8 Conclusions

In many elections, voters have the option to stay away from the polls. As we have shown, this seemingly innocuous fact has interesting and perhaps surprising implications for the kind of news media that rational voters choose to consult. In particular, voters with relatively pronounced leanings toward either side of the political spectrum optimally consult more centrist news outlets than other, intrinsically more centrist voters. As a result, relatively centrist outlets benefit from disproportionate demand for their services, serving multiple constituencies who use the same information in different ways. Moreover, polarization of the electorate may lead to ideological moderation in news consumption.

What do our findings imply for the risk that voters self-segregate along strict ideological lines and end up living in information cocoons and echo chambers? We have shown that rigid ideological segregation is an artifact of compulsory voting, which breaks down when voters have the option to abstain. Because centrist voters naturally demand "balanced" reporting, while more moderately biased voters tend to consult news media whose ideological positions are more centrist than their own, rigid ideological segregation and associated echo chambers should largely be confined to the fringes of the political spectrum. Indeed, our theoretical results are roughly consistent with recent empirical evidence, which suggests that there is far more "cross-over" in news consumption than commonly believed (see Gentzkow and Shapiro (2011)).

While voluntary voting reduces ideological segregation and induces ideological moderation in news consumption, it does induce more polarization at the ballot box. That is, under voluntary voting, people who show up at the polls hold more extreme (posterior) beliefs than the population at large. To see why, recall that voters who receive signals that conform to their bias will go and vote. By contrast, voters who receive signals that go against their bias are more likely to stay home and abstain. As a consequence, abstainers tend to hold more moderate beliefs than active voters and the population at large.

In order to study the demand for slant, in this paper, we have assumed an exogenous *supply* of slant. We believe that this is an important step in developing a sensible model of the market for political news. Informed by our finding that centrist or moderately biased media benefit from disproportionate demand, it would be interesting to endogenize the supply of slant by allowing media outlets to position themselves strategically. Media might be concerned with their circulation, with influencing the outcome of the election, or with both. We leave this for future research.

References

A.B.C. (2012). Top US newspapers for march 2012. Technical report, Audit Bureau of Circulations. http://accessabc.wordpress.com/2012/05/01/the-top-u-s-newspapers-for-march-2012/.

Baron, D. (2006). Persistent media bias. *Journal of Public Economics 90*, 1–36.

Bergstrom, T. and M. Bagnoli (2005). Log-concave probability and its applications. *Economic Theory 26*(2), 445–469.

Besley, T. and A. Prat (2006, June). Handcuffs for the grabbing hand? media capture and government accountability. *American Economic Review 96*(3), 720–736.

Bøg, M. (2008). Whom to observe? Working Paper.

Brooks, D. (2010). Getting obama right. *New York Times*. March 12, 2010.

Calvert, R. L. (1985, June). The value of biased information: A rational choice model of political advice. *The Journal of Politics 47*(2), 530–555.

Chan, J. and W. Suen (2008). A spatial theory of news consumption and electoral competition. *Review of Economic Studies 75*, 699–728.

Chiang, C. F. and B. Knight (2008). Media bias and influence: Evidence from newspaper endorsements. NBER Working Paper 14445.

DellaVigna, S. and M. Gentzkow (2010). Persuasion: Empirical evidence. *Annual Review of Economics 2*, 643–669.

DellaVigna, S. and E. Kaplan (2007, August). The fox news effect: Media bias and voting. *The Quarterly Journal of Economics 122*(3), 1187–1234.

Duggan, J. and C. Martinelli (2010). A spatial theory of media slant and voter choice. *Review of Economic Studies*. forthcoming.

Durante, R. and B. Knight (2011). Partisan control, media bias and viewers's responses: Evidence from berlusconi's italy. *Journal of The European Economic Association*. forthcoming.

Enikolopov, R., M. Petrova, and E. Zhuravskaya (2011, December). Media and political persuasion: Evidence from russia. *American Economic Review 101*(7), 3253–3285. forthcoming.

Feddersen, T. and W. Pesendorfer (1996). The swing voter's curse. *The American Economic Review 86*(3), 408–24.

Feddersen, T. and W. Pesendorfer (1999). Abstention in elections with asymmetric information and diverse preferences. *The American Political Science Review 93*(2), 381–98.

Gentzkow, M. and J. M. Shapiro (2006). Media bias and reputation. *Journal of Political Economy 114*(2), 280–316.

Gentzkow, M. and J. M. Shapiro (2011). Ideological segregation online and offline. *Quarterly Journal of Economics*. forthcoming.

Gerber, A., J. G. Gimpel, D. P. Green, and D. R. Shaw (2007). The influence of television and radio advertising on candidate evaluations: Results from a large scale randomized experiment. Working Paper.

Gerber, A. S., D. Karlan, and D. Bergan (2009, April). Does the media matter? a field experiment measuring the effect of newspapers on voting behavior and political opinions. *American Economic Journal: Applied Economics 1*(2), 35–52.

Gill, D. and D. Sgroi (2011). The optimal choice of pre-launch reviewer. *Journal of Economic Theory*. forthcoming.

Jamieson, K. H. and J. N. Cappella (2010). *Echo Chamber: Rush Limbaugh and the Conservative Media Establishment*. New York: Oxford University Press.

Krasa, S., D. Bernhardt, and M. Polborn (2008, June). Political polarization and the electoral effects of media bias,. *Journal of Public Economics 92*(5-6), 1092–1104.

Martinelli, C. (2006). Would rational voters acquire costly information? *Journal of Economic Theory 129*(1), 225–251.

Martinelli, C. (2007). Rational ignorance and voting behavior. *International Journal of Game Theory 35*(3), 315–394.

Meyer, M. A. (1991). Learning from coarse information: Biased contests and career concerns. *Review of Economic Studies 58*, 15–41.

Mill, J. S. (2002). *On Liberty*. Dover Publications.

Morgan, J. and F. Várdy (2012). Mixed motives and the optimal size of voting bodies. Working Paper.

Mullainathan, S. and A. Shleifer (2005, September). The market for news. *American Economic Review 95*(4), 1031–1053.

Myerson, R. B. (1998). Population uncertainty and poisson games. *International Journal of Game Theory 27*(3), 375–392.

Myerson, R. B. (2000). Large poisson games. *Journal of Economic Theory 94*(1), 7–45.

Oliveros, S. (2011). Abstention, ideology and information acquisition. UC Berkeley, mimeo.

Oliveros, S. and F. Vardy (2011). Unfair and unbalanced: In praise of media bias. Mimeo.

Pew (2008). The state of the news media: 2008. an annual report on american journalism. Technical report, Pew Research Center for the People and the Press.

Pew (2009, October 30). Partisanship and cable news audiences. Technical report, Pew Research Center for the People and the Press.

Prat, A. and D. Stromberg (2010). The political economy of mass media. Manuscript.

Strömberg, D. (2004, February). Radio's impact on public spending. *The Quarterly Journal of Economics 119*(1), 189–221.

Suen, W. (2004, April). The self- perpetuation of biased beliefs. *The Economic Journal 114*(495), 377–396.

Sunstein, C. R. (2002). *Republic.Com.* New Jersey: Princeton University Press.

Sunstein, C. R. (2009). *Republic.Com 2.0.* New Jersey: Princeton University Press.

A Proofs

This Appendix contains formal proofs of claims and results presented in the main text of the paper.

A.1 Preliminaries, and Proofs of Claims in Section 3

First, we prove some useful properties of the G function. These properties will be referred to repeatedly in the remainder of the Appendix.

Lemma 3 *The accuracy function, G, has the following properties.*

1. *For all $p_r \in (0,1)$, $p_r + G(p_r) > 1$*
2. $-G'(0) < 1 < -G'(1)$
3. *There exists a unique news outlet, $\tilde{p} > \frac{1}{2}$, such that $G(\tilde{p}) = \tilde{p}$.*
4. *The ratios $\frac{1-G(p_r)}{p_r}$ and $\frac{G(p_r)}{1-p_r}$ are strictly increasing in $p_r \in (0,1)$. For $p_r \downarrow 0$, they take on their infima $-G'(0)$ and 1, respectively. For $p_r \uparrow 1$, they take on their suprema 1 and $-G'(1)$.*

Proof. We prove the claims one by one.

Part 1: Let $K(p_r) = p_r + G(p_r)$ and notice that 1) $K(0) = K(1) = 1$ and 2) K is strictly concave. Hence, for all $p_r \in (0,1)$, it must be that $K(p_r) > 1$. This proves the result.

Part 2: Suppose, by contradiction, that $-G'(0) \geq 1$. In a neighborhood of 0, concavity of G then implies $G(p_r) \leq G(0) - p_r = 1 - p_r$. This contradicts the fact that, by Part 1, $p_r + G(p_r) > 1$ for all $p_r \in (0,1)$.

The proof that $1 < -G'(1)$ is analogous.

Part 3: Let $M(p_r) = G(p_r) - p_r$ and notice that $M(0) = 1 > 0 > M(1) = -1$. By continuity of M, there exists an interior $\tilde{p}$ such that $M(\tilde{p}) = 0$, or, equivalently, $\tilde{p} = G(\tilde{p})$. Because $M'(p_r) < 0$, $\tilde{p}$ is unique. Finally, by Part 1, $\tilde{p} + G(\tilde{p}) = 2\tilde{p} > 1$. Hence, $\tilde{p} > \frac{1}{2}$.

Part 4: Differentiating $\frac{1-G(p_r)}{p_r}$ with respect to p_r yields

$$\frac{d}{dp_r}\left(\frac{1-G(p_r)}{p_r}\right)=\frac{-G'(p_r)\,p_r-(1-G(p_r))}{p_r^2}$$

For $p_r>0$, the sign of $\frac{d}{dp_r}\left(\frac{1-G(p_r)}{p_r}\right)$ turns on the sign of $-p_rG'(p_r)-(1-G(p_r))$ and, for $p_r>0$, the latter expression is strictly positive. To see this, notice that by strict concavity of G

$$-G'(p_r)\,p_r-(1-G(p_r))>\frac{1-G(p_r)}{p_r}p_r-(1-G(p_r))=0$$

Hence, $\frac{1-G(p_r)}{p_r}$ is strictly increasing in p_r, and takes on its infimum for $p_r\downarrow 0$ and its supremum for $p_r=1$. Using L'Hôpital's rule we find that the infimum is equal to $-G'(0)$, while the supremum is equal to 1.

The proof for $\frac{G(p_r)}{1-p_r}$ is analogous. ■

Now we turn our attention to a number of claims made informally in Section 3. The next three lemmas deal with the micro-foundation of the accuracy function, G, in terms of the noise density, h. First, we show that strict log-concavity of h implies strict concavity of G. Then we show that increasing elasticities correspond to PDF h being more log-concave than its CDF H and DCDF $1-H$. Finally, we prove that the Normal and the Logistic distributions satisfy this log-concavity-ranking condition, while the Extreme Value distribution does not.

Lemma 4 *If h is strictly log-concave, then G is strictly concave.*

Proof. Because $G(p_r)=H\left(1+H^{-1}(1-p_r)\right)$, we have

$$G''(p_r)=-\frac{-h'(1+x)+\frac{h(1+x)}{h(x)}h'(x)}{(h(x))^2}$$

where $x=H^{-1}(1-p_r)$. This expression is strictly negative, such that G is strictly concave, iff

$$h(1+x)\,h'(x)>h(x)\,h'(1+x) \tag{14}$$

If $x=0$ then, by differentiability and single-peakedness around zero, $h'(x)=0$ and $h'(1+x)<0$. Hence, (14) holds.

If $x>0$, then $h'(x)<0$ and $h'(1+x)<0$. In that case, (14) is equivalent to

$$\frac{d\ln(h(x))}{dx}>\frac{d\ln(h(x+1))}{dx} \tag{15}$$

This inequality holds if h is log-concave.

Next, suppose that $x<0$, such that $h'(x)>0$. If $h'(1+x)<0$, (14) holds for sure. If $h'(1+x)>0$, (14) is again equivalent to (15), such that strict log-concavity of h suffices once more. ■

Lemma 5 *Let* $(p_d, p_r) \in (0,1)^2$.

$\delta_{p_r,1-p_d}$ *is strictly increasing in* p_r *if* h *is more log-concave than* $1-H$*. I.e.,*

$$\frac{d^2 \ln h(x)}{(dx)^2} < \frac{d^2 \ln (1-H(x))}{(dx)^2}$$

$\delta_{p_d,1-p_r}$ *is strictly increasing in* p_d *if* h *is more log-concave than* H*. I.e.,*

$$\frac{d^2 \ln h(x)}{(dx)^2} < \frac{d^2 \ln H(x)}{(dx)^2}$$

Proof. For $\delta_{p_r,1-p_d}$ to be strictly increasing in $p_r \in (0,1)$, it must be that

$$\frac{-G'(p_r)}{1-G(p_r)} - \frac{G''(p_r)}{G'(p_r)} - \frac{1}{p_r} < 0$$

In terms of H and h, this inequality can be written as

$$\frac{h(1+x)}{1-H(1+x)} - \left(\frac{h'(x)}{h(x)} - \frac{h'(1+x)}{h(1+x)}\right) - \frac{h(x)}{p_r} < 0$$

where $x = H^{-1}(1-p_r)$.

Using that $p_r = 1 - H\left(H^{-1}(1-p_r)\right)$, the condition for $\delta_{p_r,1-p_d}$ to be strictly increasing simplifies to

$$\frac{h(x+1)}{1-H(x+1)} - \frac{h(x)}{1-H(x)} < \frac{h'(x)}{h(x)} - \frac{h'(x+1)}{h(x+1)}$$

Hence, a sufficient condition is that

$$\frac{d}{dx}\left(\frac{h(x)}{1-H(x)}\right) < -\frac{d}{dx}\left(\frac{h'(x)}{h(x)}\right)$$

or, equivalently,

$$\frac{d^2 \ln (1-H(x))}{(dx)^2} > \frac{d^2 \ln h(x)}{(dx)^2}$$

The proof for $\delta_{p_d,1-p_r}$ is analogous. ■

Lemma 6 *For the Normal and the Logistic distributions, the inequalities in* (7) *hold. For the Extreme Value distribution, they do not.*

Proof. Because of symmetry, for the Normal and the Logistic distribution, we only have to check one of the inequalities in (7). Let us focus on the second.

For the Logistic distribution with mean μ and scale parameter σ, the result is immediate:

$$\frac{d^2 \ln (1-H(x))}{(dx)^2} = -\frac{e^{\frac{x+\mu}{\sigma}}}{\left(e^{\frac{x}{\sigma}} + e^{\frac{\mu}{\sigma}}\right)^2 \sigma^2} > -\frac{2e^{\frac{x+\mu}{\sigma}}}{\left(e^{\frac{x}{\sigma}} + e^{\frac{\mu}{\sigma}}\right)^2 \sigma^2} = \frac{d^2 \ln h(x)}{(dx)^2}$$

For the Normal distribution with mean μ and standard deviation σ, we have

$$\frac{d^2 \ln h(x)}{(dx)^2} = -\frac{1}{\sigma^2}$$

while

$$\frac{d^2 \ln(1 - H(x))}{(dx)^2} = -\frac{d}{dx}\frac{h(x)}{1 - H(x)} = -\frac{1}{\sigma}\frac{d}{dx}\frac{\varphi\left(\frac{x-\mu}{\sigma}\right)}{1 - \Phi\left(\frac{x-\mu}{\sigma}\right)} = -\frac{1}{\sigma^2}\lambda'(x)$$

Here, $\varphi(\cdot)$ denotes the PDF of the Standard Normal distribution, $\Phi(\cdot)$ its CDF, and $\lambda(x)$ its hazard rate. The result now follows from the well-known fact that $\lambda'(x) < 1$.

For the Extreme Value distribution with location parameter μ and scale parameter σ,

$$\frac{d^2 \ln h(x)}{(dx)^2} = \frac{e^{\frac{x-\mu}{\sigma}}}{\sigma^2} = \frac{d^2 \ln H(x)}{(dx)^2}$$

Hence, the first inequality in (7) fails. ■

A.2 Payoffs, FOCs, and Transition Types

For future reference, we provide expressions for the expected payoffs of the various voting strategies (Remark 1); list the first-order conditions for the optimal choice of news outlet conditional on voting strategy (Lemma 7); and derive closed-form solutions for the various transition types, ${}_{VW}\theta_{XY}$, where responsive voting strategies degenerate into unresponsive voting strategies (Lemma 8).

Remark 1 *The expected payoffs of the various voting strategies are*

$$\begin{aligned} U_\theta(DD) &= L(\theta) - \left\{\left(T^1(r) + T^0(r) + \frac{1}{2}T^{-1}(r)\right)\theta + \frac{1}{2}T^{-1}(d)(1-\theta)\right\} \\ U_\theta(RR) &= L(\theta) - \left\{\frac{1}{2}T^1(r)\theta + \left(\frac{1}{2}T^1(d) + T^0(d) + T^{-1}(d)\right)(1-\theta)\right\} \\ U_\theta(\Phi\Phi) &= L(\theta) - \left\{\left(T^1(r) + \frac{1}{2}T^0(r)\right)\theta + \left(\frac{1}{2}T^0(d) + T^{-1}(d)\right)(1-\theta)\right\} \end{aligned}$$

$$\begin{aligned} U_\theta^{p_r^{DR}}(DR) = {} & L(\theta) - \left\{\left(1 - \frac{1}{2}p_r^{DR}\right)T^1(r) + \left(1 - p_r^{DR}\right)T^0(r) + \frac{1}{2}\left(1 - p_r^{DR}\right)T^{-1}(r)\right\}\theta \\ & - \left\{\begin{array}{c} \frac{1}{2}\left(1 - G\left(p_r^{DR}\right)\right)T^1(d) + \left(1 - G\left(p_r^{DR}\right)\right)T^0(d) \\ + \left(1 - \frac{1}{2}G\left(p_r^{DR}\right)\right)T^{-1}(d) \end{array}\right\}(1-\theta) \end{aligned}$$

$$
\begin{aligned}
U_\theta^{p_r^{D\Phi}}(D\Phi) &= L(\theta) - \left\{T^1(r) + \left(1 - \frac{1}{2}p_r^{D\Phi}\right)T^0(r) + \frac{1}{2}\left(1 - p_r^{D\Phi}\right)T^{-1}(r)\right\}\theta \\
&\quad - \left\{\frac{1}{2}\left(1 - G\left(p_r^{D\Phi}\right)\right)T^0(d) + \left(1 - \frac{1}{2}G\left(p_r^{D\Phi}\right)\right)T^{-1}(d)\right\}(1-\theta) \\
U_\theta^{p_r^{\Phi R}}(\Phi R) &= L(\theta) - \left\{\left(1 - \frac{1}{2}p_r^{\Phi R}\right)T^1(r) + \frac{1}{2}\left(1 - p_r^{\Phi R}\right)T^0(r)\right\}\theta \\
&\quad - \left\{\frac{1}{2}\left(1 - G\left(p_r^{\Phi R}\right)\right)T^1(d) + \left(1 - \frac{1}{2}G\left(p_r^{\Phi R}\right)\right)T^0(d) + T^{-1}(d)\right\}(1-\theta)
\end{aligned}
$$

Here, $L(\theta)$ incorporates the payoffs associated with all non-pivotal events.

Proof. Trivial. ■

Lemma 7 *The first-order conditions for $p_r^{XY}(\theta)$, $XY \in \{D\Phi, DR, \Phi R\}$, to be optimal are*

$$
-G'\left(p_r^{DR}\right) = \frac{\tilde{T}^{-1}(r) + \tilde{T}^1(r)}{\tilde{T}^{-1}(d) + \tilde{T}^1(d)}\frac{\theta}{1-\theta} \tag{16}
$$

$$
-G'\left(p_r^{D\Phi}\right) = \frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}\frac{\theta}{1-\theta} \tag{17}
$$

$$
-G'\left(p_r^{\Phi R}\right) = \frac{\tilde{T}^1(r)}{\tilde{T}^1(d)}\frac{\theta}{1-\theta} \tag{18}
$$

Proof. The result follows immediately from differentiating with respect to θ the relevant payoff expressions given in Remark 1. ■

Lemma 8 *The unique and strictly interior transition types between responsive and unresponsive voting strategies are*

$$
{}_{DD}\theta_{D\Phi} = \frac{-G'(0)}{\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)} - G'(0)}, \; {}_{DD}\theta_{DR} = \frac{-G'(0)}{\frac{\tilde{T}^{-1}(r)+\tilde{T}^1(r)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)} - G'(0)}, \textit{ and } {}_{\Phi\Phi}\theta_{\Phi R} = \frac{-G'(0)}{\frac{\tilde{T}^1(r)}{\tilde{T}^1(d)} - G'(0)}
$$

$$
{}_{D\Phi}\theta_{\Phi\Phi} = \frac{-G'(1)}{\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)} - G'(1)}, \; {}_{DR}\theta_{RR} = \frac{-G'(1)}{\frac{\tilde{T}^{-1}(r)+\tilde{T}^1(r)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)} - G'(1)}, \textit{ and } {}_{\Phi R}\theta_{RR} = \frac{-G'(1)}{\frac{\tilde{T}^1(r)}{\tilde{T}^1(d)} - G'(1)}
$$

Proof. The responsive strategy $\left(p_r^{DR}(\theta), DR\right)$ degenerates into the unresponsive strategy DD at type $\theta = {}_{DD}\theta_{DR}$ which solves the first-order condition (9) for $p_r^{DR} = 0$. That is,

$$
-G'(0) = \frac{\tilde{T}^{-1}(r) + \tilde{T}^1(r)}{\tilde{T}^{-1}(d) + \tilde{T}^1(d)}\frac{\theta}{1-\theta}
$$

Solving for θ, we find

$$
{}_{DD}\theta_{DR} = \frac{-G'(0)}{\frac{\tilde{T}^{-1}(r)+\tilde{T}^1(r)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)} - G'(0)}
$$

The derivations of the other transition types are analogous. ■

A.3 Proof of Lemma 1

We now turn our attention to Lemma 1, which establishes that $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} < \frac{\widetilde{T}^1(r)+\widetilde{T}^{-1}(r)}{\widetilde{T}^1(d)+\widetilde{T}^{-1}(d)} < \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$.

First, we determine when abstention is a strict best response.

Lemma 9 *A voter with posterior belief ρ that the state is r strictly prefers to abstain if and only if*

$$\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} < \frac{1-\rho}{\rho} < \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$$

Proof. The expected payoff of voting for D is

$$L(\theta) - \left\{ \left(T^1(r) + T^0(r) + \frac{1}{2} T^{-1}(r) \right) \rho + \frac{1}{2} T^{-1}(d)(1-\rho) \right\}$$

while the expected payoff of abstaining is

$$L(\theta) - \left\{ \left(T^1(r) + \frac{1}{2} T^0(r) \right) \rho + \left(\frac{1}{2} T^0(d) + T^{-1}(d) \right)(1-\rho) \right\}$$

Hence, the expected payoff from abstaining is strictly greater than from voting for D if and only if

$$\frac{1}{2}\left(T^{-1}(d) + T^0(d)\right)(1-\rho) - \frac{1}{2}\left(T^{-1}(r) + T^0(r)\right)\rho < 0 \tag{19}$$

The first term corresponds to the chance that the state is d and the voter (rightly) casts the decisive vote in favor of D, while the second term corresponds to the chance that the state is r and the voter (wrongly) casts the decisive vote in favor of D. The inequality in (19) is equivalent to $\frac{1-\rho}{\rho} < \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$. Along the same lines, we find that the voter's payoff from abstaining is strictly greater than from voting for R if and only if $\frac{1-\rho}{\rho} > \frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)}$. Combining these inequalities yields the result. ■

With the help of Lemma 9, we can now prove Lemma 1.

Proof of Lemma 1:

We prove the Lemma by showing that, for ν sufficiently large, $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} < \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$. The fact that $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} < \frac{\widetilde{T}^1(r)+\widetilde{T}^{-1}(r)}{\widetilde{T}^1(d)+\widetilde{T}^{-1}(d)} < \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$ then follows as an arithmetic necessity.

Recall that

$$T^{-1}(\omega) = e^{-\upsilon(t_R(\omega)+t_D(\omega))} \sum_{n=0}^{\infty} \frac{(\upsilon t_D(\omega))^n}{n!} \frac{(\upsilon t_R(\omega))^{n+1}}{(n+1)!}$$

Following Feddersen and Pesendorfer (1999), we use the modified Bessel function

$$I_z(x) = \left(\frac{1}{2}x\right)^z \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}x^2\right)^n}{n!\,(n+z)!}$$

to get that

$$T^{-1}(\omega) = e^{-v(t_R(\omega)+t_D(\omega))}\sqrt{\frac{t_R(\omega)}{t_D(\omega)}}I_1\left(2v\sqrt{t_D(\omega)\,t_R(\omega)}\right)$$

For $z \in \{0,1\}$, $\lim\limits_{x\to\infty}\frac{\frac{e^x}{\sqrt{2\pi x}}}{I_z(x)} = 1$. Hence, for large ν,

$$T^{-1}(\omega) \approx \sqrt{\frac{t_R(\omega)}{t_D(\omega)}}\frac{e^{-v\left(\sqrt{t_D(\omega)}-\sqrt{t_R(\omega)}\right)^2}}{\sqrt{4\pi\sqrt{vt_D(\omega)}\sqrt{vt_R(\omega)}}} \tag{20}$$

Analogous calculations for $T^0(\omega)$ and $T^1(\omega)$ yield

$$T^0(\omega) \approx \frac{e^{-v\left(\sqrt{t_D(\omega)}-\sqrt{t_R(\omega)}\right)^2}}{\sqrt{4\pi\sqrt{vt_D(\omega)}\sqrt{vt_R(\omega)}}} \tag{21}$$

$$T^1(\omega) \approx \sqrt{\frac{t_D(\omega)}{t_R(\omega)}}\frac{e^{-v\left(\sqrt{t_D(\omega)}-\sqrt{t_R(\omega)}\right)^2}}{\sqrt{4\pi\sqrt{vt_D(\omega)}\sqrt{vt_R(\omega)}}} \tag{22}$$

Now suppose that, contrary to the claim in the lemma, $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} \geq \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$. Then, for sufficiently large ν, it must be that

$$\frac{1+\sqrt{\frac{t_D(r)}{t_R(r)}}}{1+\sqrt{\frac{t_R(r)}{t_D(r)}}} \geq \frac{1+\sqrt{\frac{t_D(d)}{t_R(d)}}}{1+\sqrt{\frac{t_R(d)}{t_D(d)}}}$$

Because the function $\frac{1+x}{1+\frac{1}{x}}$ is strictly increasing in x, the last inequality is equivalent to

$$\frac{t_D(r)}{t_R(r)} \geq \frac{t_D(d)}{t_R(d)} \tag{23}$$

Denote by Υ_{DR} the set of voters who, in equilibrium, *strictly* prefer $\left(p_r^{DR}(\theta), DR\right)$ over DD and RR. Denote by Υ_{RR} the set of voters who *weakly* prefer RR over $\left(p_r^{DR}(\theta), DR\right)$ and DD. Finally, denote by Υ_{DD} the set of voters who weakly prefer DD over $\left(p_r^{DR}(\theta), DR\right)$ and RR.

Because ${}_{DD}\theta_{DR} < {}_{DR}\theta_{RR}$ by Lemma 8, we have $\Upsilon_{DD} = [0, {}_{DD}\theta_{DR}]$, $\Upsilon_{DR} = ({}_{DD}\theta_{DR}, {}_{DR}\theta_{RR})$, and $\Upsilon_{DD} = [{}_{DR}\theta_{RR}, 1]$. Moreover, from Lemma 9 we know that when $\frac{\widetilde{T}^1(r)}{\widetilde{T}^1(d)} \geq \frac{\widetilde{T}^{-1}(r)}{\widetilde{T}^{-1}(d)}$, nobody abstains. Hence, in state r, the probability that a randomly drawn voter votes for R is

$$t_R(r) = \zeta\eta + (1-\zeta)\left(\int\limits_{\Upsilon_{DR}} p_r^{DR}(\theta)\,f(\theta)\,d\theta + \int\limits_{\Upsilon_{RR}} f(\theta)\,d\theta\right)$$

Similarly, in state d, the probability that a randomly drawn voter votes for R is

$$t_R(d) = \zeta\eta + (1-\zeta)\left(\int\limits_{\Upsilon_{DR}} \left(1 - G\left(p_r^{DR}(\theta)\right)\right) f(\theta)\, d\theta + \int\limits_{\Upsilon_{RR}} f(\theta)\, d\theta \right)$$

Notice that for all $\theta \in \Upsilon_{DR}$, $0 < p_r^{DR}(\theta) < 1$. The first part Lemma 3 then implies that $p_r^{DR}(\theta) > 1 - G\left(p_r^{DR}(\theta)\right)$. Hence, $t_R(r) > t_R(d)$ and $t_D(d) > t_D(r)$, which contradicts the inequality in (23). This completes the proof. ∎

A.4 Proof of Proposition 1

This section contains a proof of Proposition 1; i.e., the claim that equilibrium exists, and that equilibrium voting strategies are of the form DD; $D\Phi$; DR; ΦR; RR or DD; $D\Phi$; ΦR; RR.

First we show that, for any $T \in (0,1)^6$, the payoffs of the various unresponsive voting strategies satisfy single-crossing. We also calculate closed-form solutions for these crossing points. With slight abuse of notation, we use the same notational convention for crossing points as for transition types between responsive and unresponsive voting strategies, i.e., ${}_{VW}\theta_{XY}$.

Lemma 10 *Fix some $T \in (0,1)^6$. The payoffs of unresponsive voting strategies DD, $\Phi\Phi$, and RR satisfy single-crossing in θ.*

Specifically,

1. $U_\theta(DD) \geq U_\theta(\Phi\Phi)$ *iff* $\theta \leq {}_{DD}\theta_{\Phi\Phi} \equiv \frac{1}{\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}+1}$
2. $U_\theta(\Phi\Phi) \geq U_\theta(RR)$ *iff* $\theta \leq {}_{\Phi\Phi}\theta_{RR} \equiv \frac{1}{\frac{\tilde{T}^{1}(r)}{\tilde{T}^{1}(d)}+1}$
3. $U_\theta(DD) \geq U_\theta(RR)$ *iff* $\theta \leq {}_{DD}\theta_{RR} \equiv \frac{1}{\frac{\tilde{T}^{1}(r)+\tilde{T}^{-1}(r)}{\tilde{T}^{1}(d)+\tilde{T}^{-1}(d)}+1}$

Proof. The result trivially follows from the expressions for expected payoffs given in Remark 1. ∎

Together, Lemmas 8 and 10 allow us to derive the following partial ordering of transition types and crossing points.

Lemma 11 *Fix some $T \in (0,1)^6$. Transition types and crossing points ${}_{VW}\theta_{XY}$ satisfy the following partial order:*

$$\begin{array}{ccccccccc}
0 & < & {}_{D\Phi}\theta_{\Phi\Phi} & < & {}_{DR}\theta_{RR} & < & {}_{\Phi R}\theta_{RR} & < & 1 \\
 & & \vee & & \vee & & \vee & & \\
0 & < & {}_{DD}\theta_{\Phi\Phi} & < & {}_{DD}\theta_{RR} & < & {}_{\Phi\Phi}\theta_{RR} & < & 1 \\
 & & \vee & & \vee & & \vee & & \\
0 & < & {}_{DD}\theta_{D\Phi} & < & {}_{DD}\theta_{DR} & < & {}_{\Phi\Phi}\theta_{\Phi R} & < & 1
\end{array}$$

Proof. From Lemma 1, we know that $\frac{\tilde{T}^1(r)}{\tilde{T}^1(d)} < \frac{\tilde{T}^1(r)+\tilde{T}^{-1}(r)}{\tilde{T}^1(d)+\tilde{T}^{-1}(d)} < \frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}$. From the second part of Lemma 3, we know that $0 < -G'(0) < 1 < -G'(1) < \infty$. The ranking then follows from applying these inequalities to the closed-form solutions for the various ${}_{VW}\theta_{XY}$ derived in Lemmas 8 and 10, and the fact that $T \in (0,1)^6$. ■

We now show that, for any $T \in (0,1)^6$, the payoffs of responsive voting strategies also satisfy single-crossing, provided that $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ are strictly increasing.

Lemma 12 *Fix some $T \in (0,1)^6$. If $\delta_{p_r,1-p_d}$ and $\delta_{p_d,1-p_r}$ are strictly increasing in p_r and p_d, respectively, then the indirect utilities of the responsive voting strategies DR, DΦ, and ΦR satisfy single-crossing.*

Specifically,

1. *On $\theta \in [{}_{DD}\theta_{DR},\ {}_{D\Phi}\theta_{\Phi\Phi}]$, $U_\theta(DR) - U_\theta(D\Phi)$ crosses zero at most once, and from below.*
2. *On $\theta \in [{}_{\Phi\Phi}\theta_{\Phi R},\ {}_{DR}\theta_{RR}]$, $U_\theta(DR) - U_\theta(\Phi R)$ crosses zero at most once, and from above.*
3. *On $\theta \in [{}_{\Phi\Phi}\theta_{\Phi R},\ {}_{D\Phi}\theta_{\Phi\Phi}]$, $U_\theta(D\Phi) - U_\theta(\Phi R)$ crosses zero exactly once, and from above.*

Proof. *Part 1:* From the expressions for $U_\theta^{p_r^{DR}}(DR)$ and $U_\theta^{p_r^{D\Phi}}(D\Phi)$ given in Remark 1, we find

$$\begin{aligned}\frac{U_\theta^{p_r^{DR}}(DR) - U_\theta^{p_r^{D\Phi}}(D\Phi)}{1-\theta} &= \left\{\frac{\tilde{T}^{-1}(r)+\tilde{T}^1(r)}{2}p_r^{DR}(\theta) - \frac{\tilde{T}^{-1}(r)}{2}p_r^{D\Phi}(\theta)\right\}\frac{\theta}{1-\theta} \\ &+ \left\{\frac{\tilde{T}^{-1}(d)}{2}\left(1-G\left(p_r^{D\Phi}(\theta)\right)\right) - \frac{\tilde{T}^{-1}(d)+\tilde{T}^1(d)}{2}\left(1-G\left(p_r^{DR}(\theta)\right)\right)\right\}\end{aligned}$$

Using the first-order conditions for $p_r^{DR}(\theta)$ and $p_r^{D\Phi}(\theta)$ given in Lemma 7 and simplifying yields

$$\begin{aligned}&\frac{U_\theta(DR) - U_\theta(D\Phi)}{1-\theta} \\ =\ &\frac{\tilde{T}^{-1}(d)}{2}\left(p_r^{D\Phi}G'\left(p_r^{D\Phi}\right) + \left(1-G\left(p_r^{D\Phi}\right)\right)\right) \\ &-\frac{\tilde{T}^{-1}(d)+\tilde{T}^1(d)}{2}\left(p_r^{DR}G'\left(p_r^{DR}\right) + \left(1-G\left(p_r^{DR}\right)\right)\right)\end{aligned} \tag{24}$$

Hence, $\frac{U_\theta(DR)-U_\theta(D\Phi)}{1-\theta} > 0$ iff

$$\frac{\tilde{T}^{-1}(d)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)} > \frac{p_r^{DR}G'\left(p_r^{DR}\right)+1-G\left(p_r^{DR}\right)}{p_r^{D\Phi}G'\left(p_r^{D\Phi}\right)+1-G\left(p_r^{D\Phi}\right)} \tag{25}$$

To prove the claim, we will show that the RHS of (25) is strictly increasing in θ. Differentiating with respect to θ and simplifying yields that the RHS of (25) is strictly increasing if

$$\begin{aligned} & G''\left(p_r^{DR}(\theta)\right)\frac{dp_r^{DR}(\theta)}{d\theta}p_r^{DR}(\theta)\left(1-G\left(p_r^{D\Phi}(\theta)\right)+G'\left(p_r^{D\Phi}(\theta)\right)p_r^{D\Phi}(\theta)\right) \\ > \; & G''\left(p_r^{D\Phi}(\theta)\right)\frac{dp_r^{D\Phi}(\theta)}{d\theta}p_r^{D\Phi}(\theta)\left(1-G\left(p_r^{DR}(\theta)\right)+G'\left(p_r^{DR}(\theta)\right)p_r^{DR}(\theta)\right) \end{aligned} \tag{26}$$

From the first-order conditions for $p_r^{DR}(\theta)$ and $p_r^{D\Phi}(\theta)$ it follows that

$$\begin{aligned} G''\left(p_r^{DR}\right)\frac{dp_r^{DR}(\theta)}{d\theta} &= G'\left(p_r^{DR}\right)\frac{1}{\theta(1-\theta)} \\ G''\left(p_r^{D\Phi}\right)\frac{dp_r^{D\Phi}(\theta)}{d\theta} &= G'\left(p_r^{D\Phi}\right)\frac{1}{\theta(1-\theta)} \end{aligned}$$

Substituting these expressions for G'' back into (26) and rearranging, we find that the RHS of (25) is strictly increasing in θ if

$$\frac{-G'\left(p_r^{DR}\right)p_r^{DR}}{1-G\left(p_r^{DR}\right)} < \frac{-G'\left(p_r^{D\Phi}\right)p_r^{D\Phi}}{1-G\left(p_r^{D\Phi}\right)} \tag{27}$$

Notice that $\frac{-G'(p_r)p_r}{1-G(p_r)} = \delta_{p_r,1-p_d}$, which is strictly increasing by assumption. Moreover, from Lemma 2, we know that $p_r^{DR}(\theta) < p_r^{D\Phi}(\theta)$. Hence, the inequality in (27) indeed holds.

We may conclude that $\frac{U_\theta(DR)-U_\theta(D\Phi)}{1-\theta}$—and, therefore, $U_\theta(DR)-U_\theta(D\Phi)$—crosses zero at most once, and from below.

Part 2: The proof is analogous to Part 1.

Part 3: The expression for $\frac{U_\theta(DR)-U_\theta(D\Phi)}{1-\theta}$ in (24) and its analogue for $\frac{U_\theta(DR)-U_\theta(\Phi R)}{1-\theta}$ yield

$$\begin{aligned} & \frac{U_\theta(D\Phi)-U_\theta(\Phi R)}{1-\theta} \\ = \; & \frac{U_\theta(DR)-U_\theta(\Phi R)}{1-\theta} - \frac{U_\theta(DR)-U_\theta(D\Phi)}{1-\theta} \\ = \; & \frac{\tilde{T}^1(d)+\tilde{T}^{-1}(d)}{2}\left(G'\left(p_r^{DR}\right)+1\right) - \frac{\tilde{T}^{-1}(d)}{2}\left(p_r^{D\Phi}G'\left(p_r^{D\Phi}\right)+\left(1-G\left(p_r^{D\Phi}\right)\right)\right) \\ & -\frac{\tilde{T}^1(d)}{2}\left(G\left(p_r^{\Phi R}\right)+G'\left(p_r^{\Phi R}\right)\left(1-p_r^{\Phi R}\right)\right) \end{aligned}$$

The first-order conditions (16), (17), and (18) imply that

$$-G'\left(p_r^{DR}\right) = \frac{\tilde{T}^{-1}(d)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)}\left(-G'\left(p_r^{D\Phi}\right)\right) + \frac{\tilde{T}^1(d)}{\tilde{T}^{-1}(d)+\tilde{T}^1(d)}\left(-G'\left(p_r^{\Phi R}\right)\right)$$

Hence,

$$\begin{aligned}
&\frac{U_\theta\left(D\Phi\right)-U_\theta\left(\Phi R\right)}{1-\theta} \qquad (28)\\
=\ &\frac{\tilde{T}^{-1}\left(d\right)}{2}\left(G\left(p_r^{D\Phi}\right)+G'\left(p_r^{D\Phi}\right)\left(1-p_r^{D\Phi}\right)\right)+\frac{\tilde{T}^{1}\left(d\right)}{2}\left(1-G\left(p_r^{\Phi R}\right)+G'\left(p_r^{\Phi R}\right)p_r^{\Phi R}\right)
\end{aligned}$$

To prove the claim, we will show that the RHS of (28) is strictly increasing in θ. Differentiating the RHS with respect to θ gives

$$\frac{\tilde{T}^{-1}\left(d\right)}{2}\left(1-p_r^{D\Phi}\right)G''\left(p_r^{D\Phi}\right)\frac{dp_r^{D\Phi}}{d\theta}+\frac{\tilde{T}^{1}\left(d\right)}{2}p_r^{\Phi R}G''\left(p_r^{\Phi R}\right)\frac{dp_r^{\Phi R}}{d\theta} \qquad (29)$$

From the first-order conditions (17) and (18), it follows that

$$\begin{aligned}
G''\left(p_r^{D\Phi}\right)\frac{dp_r^{D\Phi}}{d\theta} &= -\frac{\tilde{T}^{-1}\left(r\right)}{\tilde{T}^{-1}\left(d\right)}\frac{1}{\left(1-\theta\right)^2}\\
G''\left(p_r^{\Phi R}\right)\frac{dp_r^{\Phi R}}{d\theta} &= -\frac{\tilde{T}^{1}\left(r\right)}{\tilde{T}^{1}\left(d\right)}\frac{1}{\left(1-\theta\right)^2}
\end{aligned}$$

Substituting these expressions back into (29), we find that the derivative of the RHS of (28) is

$$-\frac{1}{2}\left(\left(1-p_r^{D\Phi}\right)\tilde{T}^{-1}\left(r\right)+p_r^{\Phi R}\tilde{T}^{1}\left(r\right)\right)\frac{1}{\left(1-\theta\right)^2}<0$$

We may conclude that $\frac{U_\theta(D\Phi)-U_\theta(\Phi R)}{1-\theta}$ is strictly decreasing in θ on $[{}_{\Phi\Phi}\theta_{\Phi R},\ {}_{D\Phi}\theta_{\Phi\Phi}]$. Hence, $U_\theta\left(D\Phi\right)-U_\theta\left(\Phi R\right)$ crosses zero at most once, and from above.

To see that a crossing indeed takes place, notice that at $\theta={}_{\Phi\Phi}\theta_{\Phi R}$, $p_r^{\Phi R}=0<p_r^{D\Phi}$, where the inequality follows from Lemma 2. Hence, at ${}_{\Phi\Phi}\theta_{\Phi R}$, $U_\theta\left(D\Phi\right)>U_\theta\left(\Phi R\right)=U_\theta\left(\Phi\Phi\right)$. Similarly, at $\theta={}_{D\Phi}\theta_{\Phi\Phi}$, $p_r^{D\Phi}=1>p_r^{\Phi R}$, which implies that $U_\theta\left(D\Phi\right)=U_\theta\left(\Phi\Phi\right)<U_\theta\left(\Phi R\right)$. The intermediate value theorem then proves the result. ■

Next, we prove that, for any $T\in(0,1)^6$, DR and $\Phi\Phi$ never coexist.

Lemma 13 *Fix some* $T\in(0,1)^6$. *If, for this* T, $\Phi\Phi$ *is played by a positive measure of voters* $\theta\in[0,1]$, *then the measure of voters playing* DR *is zero.*

Proof. We prove the lemma by showing that when DR is played for a particular $T\in(0,1)^6$, then $\Phi\Phi$ is not played.

A necessary condition for DR to be played is that, conditional on s_d, D is weakly better than Φ. Similarly, conditional on s_r, R must be weakly better than Φ.

The expected payoff of voting for D—respectively, R—conditional on s is

$$\begin{aligned}
U_\theta^{p_r}\left(D\mid s\right) &= L\left(\theta\right)-\left\{\left(T^1\left(r\right)+T^0\left(r\right)+\frac{1}{2}T^{-1}\left(r\right)\right)\rho_\theta^{p_r}\left(s\right)+\frac{1}{2}T^{-1}\left(d\right)\left(1-\rho_\theta^{p_r}\left(s\right)\right)\right\}\\
U_\theta^{p_r}\left(R\mid s\right) &= L\left(\theta\right)-\left\{\frac{1}{2}T^1\left(r\right)\rho_\theta^{p_r}\left(s\right)+\left(\frac{1}{2}T^1\left(d\right)+T^0\left(d\right)+T^{-1}\left(d\right)\right)\left(1-\rho_\theta^{p_r}\left(s\right)\right)\right\}
\end{aligned}$$

while the expected payoff of abstaining is

$$U_{\theta}^{p_r}\left(\Phi \mid s_d\right) = L\left(\theta\right) - \left\{\left(T^1\left(r\right) + \frac{1}{2}T^0\left(r\right)\right)\rho_{\theta}^{p_r}\left(s\right) + \left(\frac{1}{2}T^0\left(d\right) + T^{-1}\left(d\right)\right)\left(1 - \rho_{\theta}^{p_r}\left(s\right)\right)\right\}$$

Hence, $U_{\theta}^{p_r}\left(D \mid s_d\right) \geq U_{\theta}^{p_r}\left(\Phi \mid s_d\right)$ if and only if

$$\frac{\rho_{\theta}^{p_r}\left(s_d\right)}{1 - \rho_{\theta}^{p_r}\left(s_d\right)} \leq \frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)}$$

Using (5), we can rewrite this as

$$\frac{\theta}{1-\theta} \leq \frac{G\left(p_r\right)}{1-p_r}\frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)}$$

By Lemma 3, $\frac{G(p_r)}{1-p_r}$ is strictly increasing in p_r. Hence,

$$\frac{\theta}{1-\theta} \leq \lim_{p_r \uparrow 1}\frac{G\left(p_r\right)}{1-p_r}\frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)} = -G'\left(1\right)\frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)} \tag{30}$$

Analogous calculations show that if $U_{\theta}^{p_r}\left(R \mid s_r\right) \geq U_{\theta}^{p_r}\left(\Phi \mid s_r\right)$, then

$$\frac{\theta}{1-\theta} \geq \lim_{p_r \downarrow 0}\frac{1-G\left(p_r\right)}{p_r}\frac{\tilde{T}^{1}\left(d\right)}{\tilde{T}^{1}\left(r\right)} = -G'\left(0\right)\frac{\tilde{T}^{1}\left(d\right)}{\tilde{T}^{1}\left(r\right)} \tag{31}$$

Together, (30) and (31) imply that

$$-G'\left(0\right)\frac{\tilde{T}^{1}\left(d\right)}{\tilde{T}^{1}\left(r\right)} \leq -G'\left(1\right)\frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)} \tag{32}$$

Next, notice that a necessary condition for $\Phi\Phi$ to be played is that

$${}_{D\Phi}\theta_{\Phi\Phi} < {}_{\Phi\Phi}\theta_{\Phi R}$$

or, equivalently,

$$\frac{{}_{D\Phi}\theta_{\Phi\Phi}}{1 - {}_{D\Phi}\theta_{\Phi\Phi}} = -G'\left(1\right)\frac{\tilde{T}^{-1}\left(d\right)}{\tilde{T}^{-1}\left(r\right)} < -G'\left(0\right)\frac{\tilde{T}^{1}\left(d\right)}{\tilde{T}^{1}\left(r\right)} = \frac{{}_{\Phi\Phi}\theta_{\Phi R}}{1 - {}_{\Phi\Phi}\theta_{\Phi R}} \tag{33}$$

Finally, observe that (33) contradicts (32). This proves the lemma. ■

The following lemma establishes that the voting strategy $\Phi\Phi$ is never played in equilibrium. (Notice that, in contrast to the previous lemmas, Lemma 14 below is only true in equilibrium, and not for arbitrary $T \in (0,1)^6$.)

Lemma 14 *$\Phi\Phi$ is not played in equilibrium.*

Proof. We prove the lemma by showing that, in equilibrium, ${}_{\Phi\Phi}\theta_{\Phi R} < {}_{D\Phi}\theta_{\Phi\Phi}$. This is equivalent to showing that

$$\frac{\tilde{T}^{1}(d)}{\tilde{T}^{1}(r)}\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)} < \frac{-G'(1)}{-G'(0)} \tag{34}$$

The expressions for $T^{x}(\omega)$, $x \in \{-1, 0, 1\}$, in (20), (21), and (22) imply

$$\frac{\tilde{T}^{1}(d)}{\tilde{T}^{1}(r)}\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)} = \frac{t_R(r)}{t_R(d)}\frac{t_D(d)}{t_D(r)} \tag{35}$$

Substituting (35) into (34) gives that ${}_{\Phi\Phi}\theta_{\Phi R} < {}_{D\Phi}\theta_{\Phi\Phi}$ if and only if

$$\frac{t_R(r)}{t_R(d)}\frac{t_D(d)}{t_D(r)} < \frac{-G'(1)}{-G'(0)} \tag{36}$$

Next, notice that

$$\frac{t_R(r)}{t_R(d)} = \frac{\zeta\eta + (1-\zeta)\left(\int_{\Theta^*_{DR}} p^{DR}(\theta)\,dF(\theta) + \int_{\Theta^*_{\Phi R}} p^{\Phi R}(\theta)\,dF(\theta) + \int_{\Theta^*_{RR}} dF(\theta)\right)}{\zeta\eta + (1-\zeta)\left(\int_{\Theta^*_{DR}} (1-G(p^{DR}(\theta)))\,dF(\theta) + \int_{\Theta^*_{\Phi R}} (1-G(p^{\Phi R}(\theta)))\,dF(\theta) + \int_{\Theta^*_{RR}} dF(\theta)\right)}$$

By strict concavity of G, we have

$$G(p) < G(0) + G'(0)\,p$$

which yields

$$1 - G(p) > -G'(0)\,p$$

Therefore,

$$\begin{aligned}
\frac{t_R(r)}{t_R(d)} &< \frac{\zeta\eta + (1-\zeta)\left(\int_{\Theta^*_{DR}} p^{DR}(\theta)\,dF(\theta) + \int_{\Theta^*_{\Phi R}} p^{\Phi R}(\theta)\,dF(\theta) + \int_{\Theta^*_{RR}} dF(\theta)\right)}{\zeta\eta + (1-\zeta)\left(-G'(0)\left(\int_{\Theta^*_{DR}} p^{DR}(\theta)\,dF(\theta) + \int_{\Theta^*_{\Phi R}} (p^{\Phi R}(\theta))\,dF(\theta)\right) + \int_{\Theta^*_{RR}} dF(\theta)\right)} \\
&< \frac{1}{-G'(0)}
\end{aligned} \tag{37}$$

where the last inequality follows from the fact $-G'(0) < 1$.

An analogous argument reveals that

$$\frac{t_D(d)}{t_D(r)} < -G'(1) \tag{38}$$

Combining (37) and (38) implies (36).

This completes the proof. ■

Before proceeding with the actual proof of Proposition 1, we introduce some more notation. Fix some $T \in (0,1)^6$ and let Θ_{DD} be the set of voters $\theta \in [0,1]$ for whom the strategy DD is a best response conditional on T. That is, for given pivotal probabilities, the indirect utility from DD is (weakly) greater than the indirect utility from any other strategy. Let Θ_{RR} and $\Theta_{\Phi\Phi}$ be analogously defined. Define Θ_{DR}, to be the set of voters $\theta \in [0,1]$ for whom the strategy $\left(p_r^{DR}(\theta), DR\right)$ is a best response *and* $0 < p_r^{DR}(\theta) < 1$. Let $\Theta_{D\Phi}$ and $\Theta_{\Phi R}$ be analogously defined to Θ_{DR}.

Proof of Proposition 1:

We prove the proposition in three steps. In Step 1 we show that, for fixed $T \in (0,1)^6$, voting strategies take on one of three forms, namely, 1) $DD; D\Phi; DR; \Phi R; RR$, 2) $DD; D\Phi; \Phi R; RR$, or 3) $DD; D\Phi; \Phi\Phi; \Phi R; RR$. Using the structure this imposes, in Step 2, we prove that an equilibrium always exists. Finally, in Step 3, we invoke Lemma 14 to conclude that, in equilibrium, only voting strategies of forms 1) and 2) can occur.

Step 1: Fix some $T \in (0,1)^6$. Lemma 13 establishes that when $\Phi\Phi$ is played, then DR is *not* played. This implies that we may restrict attention to three possible cases, namely: 1) $\Phi\Phi$ is played and DR is not played; 2) DR is played and $\Phi\Phi$ is not played; 3) Neither $\Phi\Phi$ nor DR are played.

`Case 1:` Suppose that $\Phi\Phi$ is played and DR is not played. In that case, we have to consider the voting strategies DD, $D\Phi$, $\Phi\Phi$, ΦR, and RR.

From Lemma 10, we know that the payoffs of the unresponsive voting strategies DD, $\Phi\Phi$, and RR satisfy single-crossing in θ. Specifically, voter θ prefers DD over $\Phi\Phi$ iff $\theta \leq {}_{DD}\theta_{\Phi\Phi} \equiv \frac{1}{\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}+1}$; he prefers RR over $\Phi\Phi$ iff $\theta \geq {}_{\Phi\Phi}\theta_{RR} \equiv \frac{1}{\frac{\tilde{T}^{-1}(r)}{\tilde{T}^{-1}(d)}+1}$; and he prefers DD over RR iff $\theta \leq {}_{DD}\theta_{RR} \equiv \frac{1}{\frac{\tilde{T}^{1}(r)+\tilde{T}^{-1}(r)}{\tilde{T}^{1}(d)+\tilde{T}^{-1}(d)}+1}$. As to the crossing point between the responsive strategies $D\Phi$ and ΦR, notice the following. Because $\Phi\Phi$ is played, it must be that ${}_{D\Phi}\theta_{\Phi\Phi} < {}_{\Phi\Phi}\theta_{\Phi R}$. Hence, at the point where $D\Phi$ and ΦR generate equal payoffs, both voting strategies have already degenerated into $\Phi\Phi$. Therefore, $D\Phi$ and ΦR do not cross.

From Lemma 11 and the fact that ${}_{D\Phi}\theta_{\Phi\Phi} < {}_{\Phi\Phi}\theta_{\Phi R}$, we know that transition types and crossing points can be ranked as follows:

$$0 < {}_{DD}\theta_{D\Phi} < {}_{DD}\theta_{\Phi\Phi} < {}_{D\Phi}\theta_{\Phi\Phi} < {}_{\Phi\Phi}\theta_{\Phi R} < {}_{\Phi\Phi}\theta_{RR} < {}_{\Phi R}\theta_{RR}$$

$$\text{and } {}_{DD}\theta_{\Phi\Phi} < {}_{DD}\theta_{RR} < {}_{\Phi\Phi}\theta_{RR}$$

This implies that $\Theta_{DD}(T) = [0, {}_{DD}\theta_{D\Phi}]$, $\Theta_{D\Phi}(T) = ({}_{DD}\theta_{D\Phi}, {}_{D\Phi}\theta_{\Phi\Phi})$, $\Theta_{\Phi\Phi}(T) = [{}_{D\Phi}\theta_{\Phi\Phi}, {}_{\Phi\Phi}\theta_{\Phi R}]$, $\Theta_{\Phi R}(T) = ({}_{\Phi\Phi}\theta_{\Phi R}, {}_{\Phi R}\theta_{RR})$, and $\Theta_{RR}(T) = [{}_{\Phi R}\theta_{RR}, 1]$. Hence, if $\Phi\Phi$ is played and DR is not played, $V(T)$ takes on the form $DD; D\Phi; \Phi\Phi; \Phi R; RR$.

`Case 2:` Suppose that DR is played and $\Phi\Phi$ is not played. In that case, we have to consider the voting strategies DD, $D\Phi$, DR, ΦR, and RR.

From Lemma 11, we know that

$$0 < {}_{DD}\theta_{D\Phi} < {}_{DD}\theta_{DR} < {}_{DD}\theta_{RR} < {}_{DR}\theta_{RR} < {}_{\Phi R}\theta_{RR} < 1$$

This implies that $\inf \Theta_{DR} > {}_{DD}\theta_{D\Phi}$ and $\sup \Theta_{DR} < {}_{\Phi R}\theta_{RR}$. In words: Θ_{DR} begins strictly to the right of where $\Theta_{D\Phi}$ begins, and ends strictly to the left of where $\Theta_{\Phi R}$ ends. By Lemma 12, $U_\theta(DR) - U_\theta(D\Phi)$ crosses zero at most once and from above, while $U_\theta(DR) - U_\theta(\Phi R)$ crosses zero at most once, and from below. Because, by assumption, the interior of Θ_{DR} is not empty, the crossing points ${}_{D\Phi}\theta_{DR}$ and ${}_{DR}\theta_{\Phi R}$ must indeed exist and ${}_{D\Phi}\theta_{DR} < {}_{DR}\theta_{\Phi R}$.

This implies that $\Theta_{DD}(T) = [0, {}_{DD}\theta_{D\Phi}]$, $\Theta_{D\Phi}(T) = ({}_{DD}\theta_{D\Phi}, {}_{D\Phi}\theta_{DR}]$, $\Theta_{DR}(T) = [{}_{D\Phi}\theta_{DR}, {}_{DR}\theta_{\Phi R}]$, $\Theta_{\Phi R}(T) = [{}_{DR}\theta_{\Phi R}, {}_{\Phi R}\theta_{RR})$, and $\Theta_{RR}(T) = [{}_{\Phi R}\theta_{RR}, 1]$. Hence, if DR is played and $\Phi\Phi$ is not played, then $V(T)$ takes on the form $DD; D\Phi; DR; \Phi R; RR$.

`Case 3:` Suppose that neither DR nor $\Phi\Phi$ are played. In that case, we have to consider the voting strategies DD, $D\Phi$, ΦR, and RR.

Because neither $\Phi\Phi$ nor DR are played, it must be that ${}_{\Phi\Phi}\theta_{\Phi R} \leq {}_{D\Phi}\theta_{\Phi\Phi}$. (Else, $\Phi\Phi$ would be played by all $\theta \in [{}_{D\Phi}\theta_{\Phi\Phi},\ {}_{\Phi\Phi}\theta_{\Phi R}]$.) The third part of Lemma 12 then implies that the crossing point, ${}_{D\Phi}\theta_{\Phi R}$, between $D\Phi$ and ΦR exists, is unique, and lies between ${}_{\Phi\Phi}\theta_{\Phi R}$ and ${}_{D\Phi}\theta_{\Phi\Phi}$. Combining this with the ranking in Lemma 11, we may conclude that

$$0 < {}_{DD}\theta_{D\Phi} < {}_{D\Phi}\theta_{\Phi R} < {}_{\Phi R}\theta_{RR} < 1$$

This implies that $\Theta_{DD}(T) = [0, {}_{DD}\theta_{D\Phi}]$, $\Theta_{D\Phi}(T) = ({}_{DD}\theta_{D\Phi}, {}_{D\Phi}\theta_{\Phi R})$, $\Theta_{\Phi R}(T) = ({}_{D\Phi}\theta_{\Phi R}, {}_{\Phi R}\theta_{RR})$, and $\Theta_{RR}(T) = [{}_{\Phi R}\theta_{RR}, 1]$. Hence, if neither DR nor $\Phi\Phi$ are played, then $V(T)$ takes on the form $DD; D\Phi; \Phi R; RR$.

Step 2: We prove existence of equilibrium by showing that there exists a fixed point in pivotal probabilities, $T^* \in (0,1)^6$, and that it gives rise to a fixed point in best-responses, i.e., an equilibrium.

From (11), (12), and (13) we know that $p_r^{DR}(\theta;T)$, $p_r^{D\Phi}(\theta;T)$, and $p_r^{\Phi R}(\theta;T)$ are continuous functions of T. As a result, the indirect utility functions, $U_\theta(DR;T)$, $U_\theta(D\Phi;T)$, $U_\theta(\Phi R;T)$, $U_\theta(DD;T)$, $U_\theta(RR;T)$, and $U_\theta(\Phi\Phi;T)$ are also continuous in T.

The probability that a random voter casts a vote for R in state r is

$$t_R(r;T) = \zeta(1-\eta) + \zeta\left(\int_{\Theta_{DR}(T)} p_r^{DR}(\theta) f(\theta)\, d\theta + \int_{\Theta_{\Phi R}(T)} p_r^{\Phi R}(\theta) f(\theta)\, d\theta + \int_{\Theta_{RR}(T)} f(\theta)\, d\theta\right)$$

The expressions for $t_D(d;T)$, $t_D(r;T)$, and $t_R(d;T)$ are analogous. Recall from Step 1 that, for all $T \in (0,1)^6$, the sets $\Theta_{XY}(T)$, $\{X,Y\} \in \{D,\Phi,R\}^2$, partition the interval $[0,1]$. Moreover, the boundaries between them, which consist of transition types and crossing points ${}_{XY}\theta_{VW}$, are continuous in T. This implies that all $t_X(\omega;T)$, where $X \in \{D,R\}$ and $\omega \in \{d,r\}$, are also continuous in T. Finally, notice that the probability of each vote count is a continuous function of $t_X(\omega;T)$. This makes T a continuous function of itself. Denote the implied mapping by Γ.

Since $\{\zeta,\eta\} \in (0,1)^2$, $t_X(\omega;T) > 0$. Hence, there exists a $0 < \alpha < \frac{1}{2}$ such that, for all $T \in (0,1)^6$, $\Gamma(T) \in [\alpha, 1-\alpha]^6$. Applying Brouwer's fixed-point theorem to the mapping $\Gamma : [\alpha,1-\alpha]^6 \to [\alpha,1-\alpha]^6$ implies that Γ admits a fixed point $T^* = \Gamma(T^*) \in [\alpha,1-\alpha]^6$.

Since voters' best responses are pinned down by $p_r^{DR}(\theta;T)$, $p_r^{D\Phi}(\theta;T)$, $p_r^{\Phi R}(\theta;T)$ and $\Theta_{XY}(T)$, $\{X,Y\} \in \{D,\Phi,R\}^2$, a fixed point T^* gives rise to a fixed point in best responses and, hence, an equilibrium of the voting game.

Step 3: Finally, from Lemma 14 we know that, in equilibrium, $\Phi\Phi$ is not played. Hence, at T^*, V^* can only be of the form $DD; D\Phi; DR; \Phi R; RR$ or $DD; D\Phi; \Phi R; RR$.

This completes the proof of Proposition 1. ∎