BARGAINING, ON-THE-JOB SEARCH AND LABOR MARKET EQUILIBRIUM

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Apart from Monks and Nuns, and possibly lighthouse keepers all at sometime look for a job while employed. Surprisingly, until recently there were few studies on the topic even though a significant percentage of worker job changes in the US involves no interim unemployment. The small number of studies on this topic possibly reflects the fact that the standard competitive labor market model implies there is no reason for a worker to look for another job while employed. In recent years, however, models of labor markets have been developed where frictions are suitably taken into account. These imply a worker faces only a limited number of job opportunities at any moment in time, and these opportunities change through time. In such a framework it is not unreasonable for a worker to accept a job but continue to look for another one while employed. In this paper we present two versions of a labor market model where workers (both employed and unemployed) can search but only at a cost. It will be shown that with either of the two variants analyzed equilibrium is characterized by on-the-job search if the cost is small enough.

The work on labor market models with frictions can be usefully partitioned into two - those where firms post wages, and those that assume a worker and firm bargain over the wage to be paid. Within the context of wage posting models, on-the-job search by employees plays a central role (see, for example, Burdett and Mortensen (1998)). In these models employees are assumed to receive new job offers from time to time. as well as when unemployed. The worker does not face the problem of whether to search or not - its just part of life's rich process. The vast majority of studies that assume workers and firms bargain to establish the wage paid do not consider on-the-job search (Pissarides (1994) provides a counter example). [1]

When analyzing a model where employees can pay to search, the expected return to search needs to be specified. Here we use the approach taken by Postel-Vinay and Robin (2002). With this approach if an employee contacts another firm, this firm

[1] Recently, Shimer (2005) and Cahuc, Postel-Vinay and Robin (2005) have analyzed an models where employees search within a strategic bargaining environment. In these studies there is again no on-the-job search choice, - employees receive new offers from time to time, by assumption.

and the worker's current employer are assumed to bid for the worker's services. The outcome of this bidding process is binding on the firm that then employs the worker. As workers and firms are assumed to be homogeneous, after the bidding process any employee is always paid the same wage, z. It is simple to show that this is the highest wage that a worker can receive and therefore anybody receiving wage z will never search on-the-job. All this seems reasonable and natural in the situation envisaged.

Consider now the bargaining process between an unemployed worker and a firm. Given the above result it can be shown that if the wage bargained is less than some critical number Q $(Q < z)$ the worker will search while employed, whereas any wage negotiated at least as great as Q implies the employee will not search. Further, as an employee's search behavior cannot be observed, any bargain reached cannot be conditioned on search behavior. It is shown that this implies the bargaining set is not convex when the cost of search is small enough. Given the bargaining set set is not convex, what should be done?

In the vast majority of studies where an unemployed worker and a firm bargain to establish a wage it is assumed they reach a Nash bargain. There are, however, problems with this approach when the bargaining set is not convex. Using an axiomatic approach, Nash demonstrated that if the bargaining set is convex, maximizing the "Nash product" yields the unique solution that guarantees four reasonable axioms are satisfied; one of which is Pareto optimality. Why they maximize the "Nash product" is not discussed as there is no analysis of the bargaining process. Given the bargaining set is not convex Nash's result cannot be used directly. Nevertheless, it has become standard in bargaining theory to use a lottery to convexify the bargaining set so the Nash approach can be used. Some labor economists, however, do not feel comfortable with this approach as they claim lotteries are not observed in practice and therefore doubt the usefulness of including lotteries even though they can Pareto improve the outcome. To take into account this point of view, here we not only analyze the situation where lotteries are allowed, but also the situation where lotteries are ruled out. As there is no particular justification of using Nash bargaining when the bargaining set is not convex, an offer/counter offer game is used in this case.

Following Rubenstein (1982), the offer/counter offer approach to bargaining is now firmly established. The object of this research is to analyze the bargaining process directly. There are several versions of such games. In most versions it is possible show that when the bargaining set is convex there is a unique subgame perfect equilibrium and this equilibrium converges to a "Generalized Nash solution" suitably defined as the time between offers and counter-offers becomes small. Indeed, in the present paper we show that the offer/counter offer model used yields the same equilibrium as the Nash Bargaining when the bargaining set is convex and the time between offers becomes small.

The paper perhaps closest to one presented here is by Pissarides (1994). He investigated equilibrium in the context of a labor market with heterogeneous firms where employees can choose to search at a cost. By assumption, workers and firms cannot negotiate long-term contracts. Hence, what is negotiated is not binding when the outside options faced by the parties change. This restriction motivates Pissarides to assume that a firm and worker always utilize a split the surplus bargain. The present study differs from Pissarides study in three ways. First, a simpler labor market is considered where both workers and firms are homogeneous.[2] Second, we assume that what a firm and worker negotiate when bargaining is binding on the firm. To illustrate, suppose an employee contacts another firm and offers a higher wage. In response the worker's current employer increases the wage paid to its employee and then the other firm withdraws forever. In the present study the firm's increased wage offer, unlike Pissarides' study, is binding on the firm. As many of our academics know, the approach used here is not unreasonable. Finally, although Pissarides assumes unemployed workers and firms reach a split the surplus bargain, we shall explore the consequence of two different bargaining processes - Nash bargaining when lotteries are allowed, and strategic bargaining when lotteries are not allowed. These differences both lead to very different results from those presented by Pissarides. In particular, they imply that if search costs are small, then on-the-job search exists in equilibrium

[2] Of course, the generalization to heterogeneous firms is reasonably straightforward.

even when workers and firms are homogeneous.

There are other studies that have considered costly on-the-job search. First, Burdett, Imai and Wright (2004) have analyzed the search decision within the context of a marriage model where utility is not transferable.[3] In this setting the decision to look for a new partner while married depends, among other things, on whether the individual's partner is also looking around, or not. Note, in the marriage market setting either party can choose to search while matched. In the labor market context, by assumption, only workers can select to search while matched. Second, Nagypal (2005) has analyzed an on-the-job search model where a worker's utility from employment at a particular firm depends on a idiosyncratic element as well as the wage offered. As the idiosyncractic element is private information to the worker, even if all firms offer the same wage some employees in a bad match may select to search while employed.[4]

1 The Model

We assume time is continuous and there is a large fixed number (a continuum) of both workers and employers We normalize the number of both to one. A worker is either employed or unemployed. Any employed worker generates revenue p per unit of time. An unemployed worker obtains b per unit of time. Each firm employs at most one worker.

Independent of a worker's employment status, if flow cost c is paid a firm with a vacancy is contacted at Poisson rate α. Vacancies are not contacted if the worker does not pay this cost. Any worker/employer partnership breaks up at an exogenous rate δ. If such an event occurs, the firm costlessly posts a vacancy, whereas the worker becomes unemployed. Any firm with a vacancy contacts a searching worker at Poisson rate α_f. The probability any worker contacted is unemployed is denoted by π. Throughout we assume the market is in a steady-state and therefore the above

[3] As utility is non-transferable, there is no bargaining - what you see is what you get.

[4] There is also a study by Moscarini (2005) who considers the employee's decision to search within a model of learning about a worker's ability..

aggregates stay constant through time.

If a firm with a vacancy contacts an unemployed worker the wage paid is determined by a bargaining process - but more of that later. Any employee may, or may not, search but this is not observable to the firm. This implies the worker and firm cannot condition on the worker's search behavior. Suppose a firm's employee does search on-the-job. Further, assume this worker contacts another firm with a vacancy. In this case the newly contacted firm and the worker's current employer are assumed to enter a wage bidding competition for the worker's services.

1.1 Firms

Given the model described above let V denote a firm's expected payoff when it posts a vacancy. The object here is to specify a firm's expected return when it hires a worker, taking V as given. Let $J(w,0)$ denote a firm's expected payoff when currently employing a worker at wage w, given the employee does not search while employed. It follows

$$rJ(w,0) = p - w + \delta[V - J(w,0)]$$

Now let $J(w,1)$ denote the firm's expected return when employs a worker at wage w and the employee searches on-the-job. In this case

$$rJ(w,1) = p - w + \delta[V - J(w,1)] + \alpha[J_H - J(w,1)]$$

where J_H denotes the firm's expected payoff after its employee contacts another firm and they bid for the worker's services.

Suppose for a moment that a firm's employee contacts another firm with a vacancy. In this case, by assumption, the two firms bid for the worker's services. As firms are homogeneous each is willing to bid up to wage z; a wage which implies a firm indifferent between employing this worker, or posting a vacancy, i.e., $J_H = V$. Without loss of generality, assume the worker stays at his current employer. Further, as no employee will ever pay a higher wage than z, a worker will not search when employed at this wage. Hence, $J_H = V = J(z,0)$.

In what follows it will be found to more useful to express the expected return of a firm in terms of the surplus generated. Let $S_f(w,0) = J(w,0) - V$ and $S_f(w,1) = J(w,1) - V$ denote these surpluses. It follows

$$S_f(w,0) - V = \frac{p - w - rV}{r + \delta} \tag{1}$$

$$S_f(w,1) = \frac{p - w - rV}{\phi} \tag{2}$$

where $\phi = (\alpha + \delta + r)$. Further, $V = J(z,0)$ implies

$$z = p - rV \tag{3}$$

1.2 Workers

Let U denote a unemployed worker's expected discounted lifetime income. Suppose for the moment a worker is employed at wage w and does not search on-the-job. This worker's expected return in this case, $U(w,0)$, can be written as

$$U(w,0) = w + \delta[U - U(w,0)]$$

Let $U(w,1)$ denote a worker's expected payoff when employed at wage w and searching. It follows

$$rU(w,1) = w + \delta[U - U(w,1)] + \alpha[U(z,0) - U(w,1)] - c$$

where $U(z,0)$ denotes the worker's expected return after the worker has contacted another firm and the two firms have bid for his/her services. Manipulation establishes that the surplus going to the worker when wage w is paid and the worker does not search, $S_w(w,0) = U(w,0) - U$, can be written as

$$S_w(w,0) = \frac{w - rU}{r + \delta} \tag{4}$$

The surplus going to the worker when wage w is paid and the worker searches on-the-job, $S_w(w,1) = U(w,1) - U$, can be written as

$$S_w(w,1) = \frac{w(r+\delta) - \phi rU + \alpha z - c(r+\delta)}{\phi(\delta + r)} \tag{5}$$

We are now in a position to define two reservation wages; R_0 and R_1. In particular,

$$S_w(w,0) \gtrless 0 \text{ as } w \gtrless R_0$$

and

$$S_w(w,1) \gtrless 0 \text{ as } w \gtrless R_1$$

At any wage greater than R_0 a worker strictly prefers to work and not search rather than remain unemployed, whereas at any wage greater than R_1 a worker strictly prefers to work and search rather than be unemployed. Without any real loss of generality we assume at any wage that makes a worker indifferent between employment and unemployment, the worker accepts employment. From (4) and (5) it follows that

$$R_0 = rU$$

and

$$R_1 = \frac{\phi rU - \alpha[z - \frac{c(r+\delta)}{\alpha}]}{(r+\delta)}$$

We are now in a position to specify Q - the search wage of a worker. This is the wage that makes the worker indifferent between searching while employed and not searching while employed. For any fixed U, it follows from (4) and (5) that $S_w(w,1) \gtrless S_w(w,0)$ if and only if $w \lesseqgtr Q$, where

$$Q = z - \frac{c(r+\delta)}{\alpha} \tag{6}$$

The relationship between these three objects R_0, R_1, and Q is now established in Claim 1

Claim 1

(a) If $c < c_0$, then $R_1 < R_0 < Q$.

(b) If $c > c_0$, then $Q < R_0 < R_1$.

(c) If $c = c_0$, then $R_0 = R_1 = Q$.

where

$$c_0 = \frac{\alpha[p - r(V+U)]}{(r+\delta)} \tag{7}$$

Proof

A little math establishes that $\partial S_w(w,0)/\partial w > \partial S_w(w,1)/\partial w > 0$. The results now follow and are illustrated in Figures 1 and 2. This completes the proof.

The above Claim leads to a complete description of an unemployed worker's strategy. Suppose $c < c_0$ and wage w is offered to an unemployed worker.

(a) If $w < R_1$,then the worker remains unemployed,

(b) If $Q > w \geq R_1$,then the worker accepts the job and searches on-the-job.

(c) If $w > Q$,then the worker accepts the job but does not search.

Note, if wage $w = Q$ is offered, the worker is indifferent between accepting the offer and searching, and accepting the offer and not searching. To simplify the analysis, and without any real loss of generality, we assume a worker in this situation does as told by the firm. It will be shown later, at $w = Q$, the firm will always prefer to tell the worker not to search

Suppose now $c > c_0$ and wage w is offered to an unemployed worker.

(a′) If $w < R_0$,then the worker remains unemployment.

(b′) If $w \geq R_0$,then the worker accepts the offer and does not search on-the-job.

2 Feasible Sharing Arrangements

In this section the feasible sharing arrangements are studied by varying the wage paid. Suppose wage w is paid and the worker does not search on-the-job. The total surplus in this case, S_0, can be written as

$$S_0 = S_f(w,0) + S_w(w,0) = \frac{p - r(V+U)}{r+\delta} \tag{8}$$

. At wage $w = z$, where z is defined in (3), $S_f(z,0) = 0$ and the worker receives all the surplus created by the match. On the other hand at wage R_1, $S_w(R_1,0) = 0$,and therefore $S_f(R_1,0) = S_0$. Focussing on the relevant case we assume that V and U are such that $S_0 > 0$,i.e., $p - rV - rU > 0$. It was shown above that if $c > c_0$, then no worker searches while employed and therefore S_0 describes the surplus generated by the match. This is the standard case.

The situation is not so straightforward when $c < c_0$. If $c < c_0$, employees may search while employed. For the moment, assume an employee does search while employed at any given wage. Given this restriction, manipulation establishes

$$S_1 = S_f(w,1) + S_w(w,1) = S_0 - \frac{c}{\phi} \tag{9}$$

Given $c < c_0$, we require $w \geq Q$ for an employee not to search. Suppose an employee is paid wage $w = Q$. At this wage the employee is indifferent between searching, or not searching while employed, As the firm prefers the employee not to search, she is told so and, by assumption, obliges. Manipulation establishes that

$$S_w(Q,0) = S_0 - c/\alpha \text{ and } S_f(Q,0) = c/\alpha \tag{10}$$

is the surplus that goes to the worker and the firm respectively. At any wage less than Q but at least as great as R_1, the worker will search on-the-job and therefore surplus c/ϕ is lost Note, $S_w(Q,0)$ is an strictly increasing function of the given cost of search such that $c \to 0$, $S_w(Q,0) \to 0$ and $S_f(Q,0) \to 0$ The smaller the search cost, the greater the search wage, Q. For any given search cost c, we obtain a unique Q and can therefore determine the benefit of searching on-the-job. As the cost of search becomes smaller the set of wages that do not induce the worker to search $[z, Q]$ becomes smaller. It is simple to see that when $0 < c < c_0$ the feasible bargaining set is not convex. This is illustrated in Figure 3.

As is well known, a lottery can convexify this set. In particular, suppose with probability ρ, wage $w = R_1$ is paid and therefore the employee searches on-the-job. With probability $(1-\rho)$, wage $w = Q$ is paid and the employee is told not to search. Given this lottery is used with mixing probability ρ, the surplus going to the worker can be written as $S_e(\rho) = \rho S_e(R_1, 1) + (1-\rho) S_e(Q,0)$ and the firm's surplus can be written as $S_f(\rho) = \rho S_f(R_1, 1) + (1-\rho) S_f(Q,0)$. The total surplus, $S(\rho)$ can be expressed as

$$S(\rho) = S_0 - \frac{\rho c}{\phi}$$

Using this lottery it is possible to construct the desired convex feasible set. When the lottery option is added, the feasible set is the convex set as illustrated in Figure 4..

2.1 Nash Bargaining with Lotteries

Here we investigate the bargain where the firm and worker maximize the Nash product given lotteries are feasible (and therefore the bargaining set is convex). Suppose for the moment the firm and worker bargain on the assumption that the worker will not search on-the-job. The Nash bargain in this case satisfies the following program

$$\arg\max_{w} S_w(w,0)S_f(w,0)$$

on the feasible set whose frontier was described above. As $\partial S_w(w,0)/\partial w = -\partial S_f(w,0)/\partial w$, it follows that w_{ns} solves the above program if it splits the surplus (STS) in that $S_w(w_{ns},0) = S_f(w_{ns},0)$. This implies

$$w_{ns} = \frac{1}{2}[p - rV + rU]$$

For the moment assume $c \leq c_0$.In what follows we consider the different Nash bargains that result as the given search cost becomes smaller and smaller. As we shall see there will be certain critical costs of search that play a major role in what follows. These are defined and described in the next Claim.

Claim 2

(a) $R_1 \lesseqgtr Q$ as $c \gtrless c_0$, where

$$c_0 = \alpha S_0.$$

(b) $S_0/2 \gtrless S_w(Q,0)$ as $c \gtrless c_1$,where

$$c_1 = \frac{\alpha S_0}{2}.$$

(c) $S_1/2 \gtrless S_w(Q,0)$ as $c \gtrless c_2$, where

$$c_2 = \frac{\alpha S_0}{2}\frac{\phi}{(\phi + \alpha/2)}$$

Note, $c_0 > c_1 > c_2$.

Proof

The results all follow from simple manipulation of the definitions.

Given $c < c_0$, It is simple to show the STS bargain is feasible if and only if $c \geq c_1$. If $c < c_1$,then the STS bargain is not feasible as the employee will search.. There is, however, a "kink" at the boundary of the feasible set when the wage paid is Q. Only two bargains can now maximize the Nash product (suitably defined): (a) The constrained Nash (CN) bargain, where the worker is paid Q and told not to search, or (b) the Lottery Nash (LN) bargain, where with probability $(1-\rho)$ the worker is paid R_1 and searches while employed, and with probability ρ the worker is paid Q(and told not to search).

Without loss of generality define the Nash product as

$$N(\rho) = S_w(\rho)S_f(\rho) = [(1-\rho)S_w(Q,0)][\rho S_1 + (1-\rho)S_f(Q,0)]$$

It is shown in the proof of Claim 3 that maximizing the above with respect to ρ always implies that ρ^* is such that $0 \leq \rho^* \leq 1/2$.The next Claim demonstrates when $\rho^* = 0$

Claim 3

Given $c < c_1$, maximization of $N(.)$, implies $\rho^* = 0$ if and only if $c \geq c_2$.

Proof:

See Appendix.

The above result implies that three Regions can now be described.

Region 1 $(c_0 \geq c > c_1)$: In this Region $S_0/2 \geq S_w(Q,0)$ hence a STS Nash bargain maximizes the Nash surplus and is feasible.

Region 2 .$(c_1 > c \geq c_2)$: In this Region the worker is paid Q and told not to search

Region 3 $(0 < c \leq c_2)$: In this Region the LN bargain is used. In this case with probability ρ a worker is paid $R_1$and then searches on-the-job. Assuming the match is not destroyed first, this worker obtains wage z after contacting another firm. With probability $(1-\rho)$ the worker is paid Q and does not search.

Table 1 species the wages paid in Regions 1-3. and Figures 5-7 illustrates the Regions.

Table 1

Region	**Search Costs**	**Nash Wage**	**Wage after Search**	**Type**
1	$c_0 \geq c \geq c_1$	$\frac{1}{2}[p - rV + rU]$	NS	STS
2	$c_1 > c \geq c_2$	$z - c\frac{(\delta+r)}{\alpha}$	NS	CN
3	$c_2 > c > 0$	$Q = \frac{1}{2}[p - rV + rU]$ **or** $R_1 = \frac{\phi rU - \alpha[p - rV - \frac{c(r+\delta)}{\alpha}]}{(r+\delta)}$	$p - rV$	LN

Finally, suppose $c > c_0$.In this case the bargaining set is convex and therefore the standard results follow immediately. In particular, the STS Nash bargain maximizes the Nash product and the worker is paid $w = (1/2)(p - rV + rU)$.and does not search while employed.

2.2 A Strategic Bargaining Game.

The object here is to specify and analyze a relatively standard bargaining game within the context of the market specified above. As lotteries are not allowed the feasible set is not convex when $c \leq c_0$.Indeed, until the end of this subsection it is assumed $c < c_0$.

Suppose a firm and unemployed worker meet, The firm makes an offer. If this is accepted by the worker, it is immediately implemented. If the worker rejects the offer, they wait small time period Δ. Then, with probability $1/2$ the worker (firm) makes a take-it-ot-leave-it offer.

First, consider the take-it-or-leave-it offers made if the worker rejects the firm's first offer. The firm's final offer depends on the search cost faced by the worker, Given $c \leq c_0$, it follows from Claim 2 that $S_f(R_1, 1) \gtrless S_f(Q, 0)$ as $c \gtrless c_1$. Hence, if the given $c_0 \geq c > c_1$, the firm will offer Q, whereas if $c < c_1$, the firm's final offers is R_1. The worker's take-it-or-leave-it offer is always the same, z.

To simplify the analysis, we assume in what follows that $\Delta \rightarrow 0$. This implies the worker's expected surplus if the first offer is rejected, ES, can be written as

$$ES = \begin{cases} (1/2)\{S_0 + 0\} = S_0/2, \text{ if } c < c_1 \\ (1/2)\{S_0 + S_w(Q, 0)\} = S_0 - \frac{c}{2\alpha}, \text{ if } c > c_1 \end{cases}$$

The firm's first offer, w^0, is the wage that maximizes its surplus subject to the expected surplus going to the worker is at least as great as ES.

Given $c \leq c_0$, it is shown below that there are four different regions - depending on the worker's search cost. To help specify these regions we need to define two more critical costs of search.

Claim 4

(a) $S_f(R_1, 1) \gtrless S_f(Q, 0)$ as $c \gtrless c_A$, where

$$c_A = \frac{\phi}{\phi + \alpha} \alpha S_0$$

(b) $S_w(Q - c, 1) \gtrless S_0/2$ as $c \gtrless c_B$, where

$$c_B = \frac{\phi}{\phi + \alpha} \frac{\alpha S_0}{2}$$

Proof

The two claims follow from simple manipulation of definitions.

The regions can now be specified.

Region A $(c_0 > c > c_A)$: Here the firm's take-it-or-leave-it offer is Q, Hence, the firm's first offer is $ES = S_0 - c/(2\alpha) = S_w(w^0, 0)$.The worker accepts this and does not search while employed.

Region B $(c_A \geq c > c_1)$: For $c < c_A$, the firm's take-it-or-leave-it offer is R_1. As $S_0 - c_A/(2\alpha) > S_0/2$, there is a "jump down" in ES at c_A. Nevertheless, $S_0/2 > S_w(Q, 0)$ at $c = c_A$, and therefore in this Region the firm's first offer is w^0 such that $S_w(w^0, 0) = S_0/2$.

At $c = c_1$, $S_w(Q, 0) = S_0/2$. If $c < c_1$, the firm could offer a wage less than Q that is acceptable.. This, however, would induce the worker to search on-the-job. When will the firm offer a wage that implies the worker will search on-the-job rather than Q? To answer this question it is useful to define $w(Q)$ by $S_f(w(Q), 1) = S_f(w, 0)$.. It follows that

$$w(Q) = z - \frac{c\phi}{\alpha} = Q - c$$

We are now in a position to define the last two Regions.

Region C $(c_1 \geq c > c_B)$: For $c > c_B$, the firm's first offer is Q. This the worker accepts and does not search while employed. Note, the expected return to the worker in this region $S_w(Q, 0) > ES$.

Region D $(c_B \geq c > 0)$: In this region c is such that $S_f(w(Q), 1) > S_0/2$ and therefore the firm's first offer, w^0 is such that $S_w(w^0, 1) = ES$. The worker accepts this offer and searches while employed.

Table 2 shows the first wage offered by the firm and Figure 8 illustrates the four regions..

Table 2.

Region	**Search Costs**	w^0
A	$c_0 \geq c > c_A$	$p - rV - \frac{(\delta+r)c}{2\alpha}$
B	$c_A \geq c > c_1$	$\frac{z+rU}{2}$
C	$c_1 \geq c > c_B$	$z - \frac{c(\delta+r)}{\alpha}$
D	$c_B \geq c > 0$	$\frac{(\delta+r)(2c+p+r(U-V))-\alpha(p-r(U+V))}{2(\delta+r)}$

Finally, we consider the above bargaining game when $c \geq c_0$. In this case $ES = S_0/2$. Therefore the firms first offer, w^o is such that $S_w(w^0, 0) = S_0/2$ This implies

$$w^0 = \frac{p - rV + rU}{2}$$

The strategic bargaining game yields the same wage as the one that maximizes the Nash product given $c \geq c_0$.

3 Market Equilibrium

Above it has been shown (in both bargaining models considered) that if U and V (and the market parameters) are taken as given, the wages paid can be specified as the given cost of search is varied. The object now is to embed these results into a steady-state market equilibrium. To achieve this goal we first specify the steady-state values of the relevant market parameters. Second, within such a steady-state framework we show that the expected return of an unemployed worker and a firm with a vacancy can be written as a function of the stead-state parameters and the wages offered in the bargaining game. A market equilibrium given a particular bargaining game is defined as the situation where the given U, and V in the bargaining game turn out to be the actual U and V generated in a steady-state.

To make progress we first need to specify a steady-state in the market. To achieve this goal we first define an encounter function. This specifies the number of encounters (e) per unit of time as a function of the number of searching workers (s) and firms with a vacancy (v), i.e., $e = e(s, v)$.

Keeping things as simple as possible, we assume $e = sv$. The simplicity generated by using a quadratic encounter function can now be stated.[5] The matching function implies $\alpha s = \alpha_f v = sv$, i.e., the number of searchers who make contact with a firm per unit of time equals the number of encounters between workers and firms, which, in turn, equals the number of firms that contact a worker. Hence, under all circumstances $\alpha = v$ and $\alpha_f = s$. By construction, however, the number of vacancies must equal the number of unemployed workers and therefore $\alpha = u = v$. Firms with vacancies always hire unemployed workers on contact. Therefore, steady-state implies $u\alpha = (1 - u)\delta$. Hence the steady-state number of unemployed workers is always $u = \delta/(\delta + r)$. As $\alpha = u$, we can write

$$\alpha = v = u = \frac{\sqrt{\delta}\sqrt{(\delta + 4)} - \delta}{2} \tag{11}$$

Hence, given the market is in a steady-state, the number of unemployed workers, the number of vacancies, and the arrival rate of offers faced by workers who search are the same and can be written as a function of the job destruction rate.

Assume a fraction γ of employees search on-the-job. This implies $s = u+\gamma(1-u)$ is the number of workers who search. Hence, $\pi = u/(u+\gamma(1-u))$ denotes the fraction of searchers who are unemployed. The rate at which firms contact unemployed workers is therefore $\alpha_f \pi = s\pi = u = \alpha$.

[5] Essentially the same results follow from assuming the encounter function has constant returns to scale. The math, however, is more complicated.

3.1 Market Equilibrium and Nash Bargaining

In the Section on Nash bargaining it was established that given U and V, there are three possible bargaining outcomes depending on the given cost of search. In Regions 1 and 2, the Nash Bargain the wage paid (as shown in Table 1) can be written as a function of U and V, as well as the other market parameters. Further, employed workers do not search. In Region 3, the Nash Lottery bargain implied there are two possible wages paid to newly employed workers and some workers search while employed.

The expected return to an unemployed worker in these three cases can be written as

$$rU = \begin{cases} b - c + \alpha S_w(w,0), \text{ in Region 1} \\ b - c + \alpha S_w(w,0), \text{ in Region 2} \\ b - c + \alpha[\rho S_w(R_1,1) + (1-\rho)S_w(Q,0)], \text{ in Region 3} \end{cases}$$

The expected return to a firm posting a vacancy in the three regions are

$$rV = \begin{cases} \alpha_f S_f(w_{ns},0), \text{ in Region 1} \\ \alpha_f S_f(Q,0), \text{ in Region 2} \\ \alpha_f \pi[\rho S_f(R_1,1) + (1-\rho)S_f(Q,0)], \text{ in Region 3.} \end{cases}$$

To illustrate manipulation establishes a candidate STS Nash bargaining equilibrium, implies

$$U = \frac{\alpha p + (b-c)(\phi + \delta + r)}{2r\phi}, \; V = \frac{\alpha(p+b-c)}{2r\phi}, \; w = \frac{p+b-c}{2}$$

Further, the critical costs c_0 and c_1 can be written as a function of the parameters of the model:

$$c_0 = \frac{\alpha(p-b)}{(\delta + r)} \text{ and } c_1 = \frac{\alpha(p-b)}{(\delta + r + \phi)}$$

We also require $U > 0$ to guarantee workers participate in the market. It is straightforward to show this is always satisfied if c is such that $c_0 > c > c_1$. Thus, the conditions required for a market equilibrium where a STS Nash bargain are specified above..

In an Appendix we establish conditions required for a market equilibrium in the other two regions The results are presented in Table 3.

Table 3

Region	Search Cost	Nash Wage	z
1	$\frac{\alpha(p-b)}{(\delta+r+\phi)}< c <\frac{\alpha(p-b)}{(\delta+r)}$	$\frac{(p+b-c)}{2}$	NS
2	$\frac{\alpha(p-b)}{2\phi}< c <\frac{\alpha(p-b)}{(r+\delta+\phi)}$	$p-\frac{\phi c}{\alpha}$	NS
3	$0 < c <\frac{\alpha(p-b)}{2\phi}$	$R_1=\frac{\phi[(\phi+\alpha\rho)b-\alpha\rho p]-[\phi(\phi-\rho(r+\delta))+\alpha^2\rho^2]c}{\phi^2}$ $Q=\frac{\alpha\phi[(\phi-\rho\alpha)p+\alpha\rho b]-[\phi^2(\phi-\rho\alpha)+\alpha^3\rho^2]c}{\alpha\phi^2}$	$\frac{\phi[(\phi-\rho\alpha)p+\alpha\rho b]-[\phi^2(1-\rho)+\alpha^2\rho^2]c}{\phi^2}$

Finally, the situation where $c > c_0$. This is essentially the same as in Region 1. Here the Nash wage is $w = (p + b - c)/2$ and the worker does not search while employed. The only new element is that we require the cost of search to be no greater than $\bar{c}$, where

$$\bar{c} = b + \frac{\alpha p}{(\phi + \delta + r)} \tag{12}$$

If $c > \bar{c}$,any worker' expected to return when unemployed is negative and therefore workers prefer not to participate in the market..

3.2 Market Equilibrium with Strategic Bargaining.

In this case workers either reach a no search bargain, or a search bargain; depending on the parameters In this case the expected payoff to an unemployed worker can be written as

$$rU = \begin{cases} b - c + \alpha S_w(w, 0), & \text{if No search Bargain} \\ b - c + \alpha S_w(w, 1), & \text{if Search Bargain,} \end{cases}$$

whereas the expected return to a firm posting a vacancy can be written as

$$rV = \begin{cases} \alpha_f \pi S_f(w, 0), & \text{if No Search bargain} \\ \alpha_f \pi S_f(w, 1), & \text{if Search Bargain} \end{cases}$$

In an appendix we utilize the same approach as the Nash bargaining case to establish when a which type of market equilibrium exists as the given cost of search vary

Table 4

Region	**Search Cost**	**wage**	z
A	$\frac{\alpha(p-b)}{\phi} < c \leq \frac{\alpha(p-b)}{\delta+r}$	$\frac{2\alpha p - c(\phi+\alpha)}{2\alpha}$	NS
B	$\frac{\alpha(p-b)}{\phi+r+\delta} < c < \frac{\alpha(p-b)}{\phi}$	$\frac{p+b-c}{2}$	NS
C	$\frac{\alpha(p-b)}{2\phi+\alpha} < c < \frac{\alpha(p-b)}{\phi+r+\delta}$	$p - \frac{\phi c}{\alpha}$	NS
D	$0 < c < \frac{\alpha\phi(p-b)}{(\phi+\alpha+r)(\phi+\alpha)}$	$\frac{\phi(\delta+r)p+\phi(\phi+\alpha)b+(\delta+r)(\phi+\alpha)c}{2\phi^2}$	$\frac{\phi(\phi+\delta+r)p+\alpha\phi b+\alpha(\delta+r)c}{2\phi^2}$

Suppose now that $c > c_0$. It is straightforward to show in this case if a market equilibrium exists the wage offered (and accepted) is $w = (p + b - c)/2$. This wage implies the expected return to an unemployed worker, $U \geq 0$, if and only if $c \leq \overline{c}$,where $\overline{c}$ is defined in (12).

Appendix

Proof of Claim 3

Recalling that $S_f(R_1, 1) = S_1$, the first two derivatives of the Nash product can be written as

$$N'(\rho) = S_e(Q,0)[(1-2\rho)S_1(c) - 2(1-\rho)S_f(Q,0)]$$

and

$$N"(\rho) = -2S_e(Q,0)[S_1(c) - S_f(Q,0)]$$

It is straightforward to show that $c < c_1$ implies $S_1 > S_f(Q,0)$ and therefore $N(.)$ is concave. Further,

$$\begin{aligned} lim_{\rho\to 0}N'(\rho) &= S_e(Q,0)[S_1(c) - 2S_f(Q,0)], \text{ and} \\ lim_{\rho\to 1/2}N'(\rho) &= -S_e(Q,0)S_f(Q,1) \end{aligned}$$

These results imply that $N(.)$ reaches an interior maximum at ρ^* if $S_1/2 > S_f(Q,0)$, where $0 < \rho^* < 1/2$. It is simple to establish $S_1 > 2S_f(Q,0)$ if and only if $c < c_2 < c_1$, where c_2 is defined in the Claim. Further, given ρ^* is interior, i.e., $0 < \rho^* < 1$, it follows

$$\frac{d\rho^*}{dc} = -\frac{[\frac{(1-2\rho^*)}{(r+\delta+\alpha)} + 2(1-\rho^*)/\alpha]}{[S_1 - S_f(Q,0)]2} < 0$$

This completes the proof.

Derivation of Market Equilibrium with Nash Bargaining.

A candidate CN bargaining market equilibrium must imply

$$U = \frac{\alpha p + (\delta + r)b - (\phi + \delta + r)c}{r\phi}, \; V = \frac{c}{r}, \quad w = \frac{\alpha p - c(\delta + \rho + \alpha)}{\alpha}$$

It can now be shown the critical search cost in such an equilibrium are as specified in Table 3. Further, it can be shown that $U > 0$ in this range.

Tedious calculation establishes that a candidate LN bargaining market equilibrium must imply the following:

$$U = \frac{\alpha(1-\rho)\phi p + \phi(\delta + r + \alpha\rho)b - (\alpha^2\rho^2 + (1-\rho)\phi + (\delta + r)(\phi + \alpha\rho)c}{r\phi^2}$$

$$V = \frac{\alpha\rho\phi(p-b) + (\alpha^2\rho^2 + (1-\rho)\phi^2)c}{r\phi^2}$$

and the three wages specified in Table 3, These can be used to establish the desired result.

Market Equilibrium with Strategic Bargaining

Calculation establishes that the expected return to an unemployed worker and a firm with a vacancy can be written as follows in the four regions:

Region	U	V
A	$\frac{\alpha p+(\delta+r)b-(\phi+\delta+r)c}{r\phi}$	$\frac{c}{r}$
B	$\frac{\alpha p+(\phi+\delta+r)(b-c)}{2r\phi}$	$\frac{\alpha(p+c-b)}{2r\phi}$
C	$\frac{2\alpha p+2(\delta+r)b-c(\phi+2(\delta+r))}{2r\phi}$	$\frac{c}{2r}$
D	$\frac{\alpha\phi p+\phi(\phi+2(\delta+r))b-(\delta+r)(2\phi+(\delta+r))c}{2r\phi^2}$	$\frac{\alpha\phi p-\alpha\phi b-\alpha(\delta+r)c}{2r\phi^2}$

It is now possible to establish the critical costs of search and the wages paid in the regions. These are presented in Table 4.

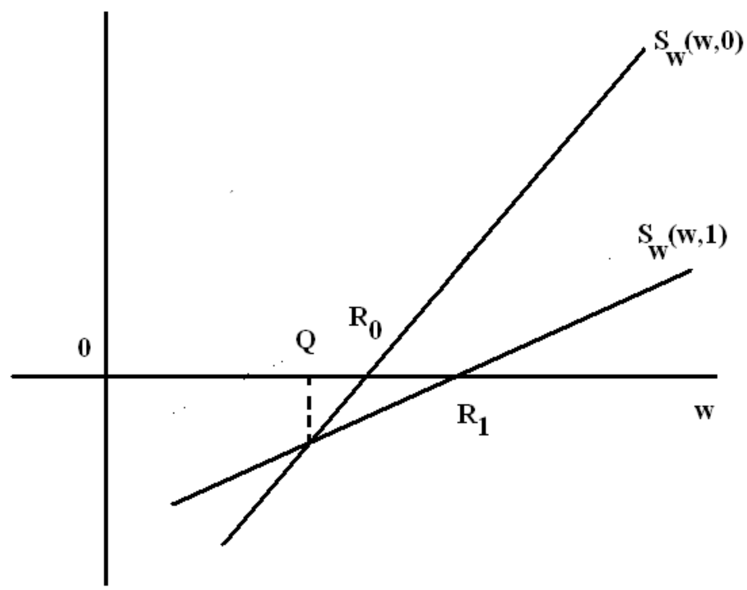

Figure 1: $c > c_0$

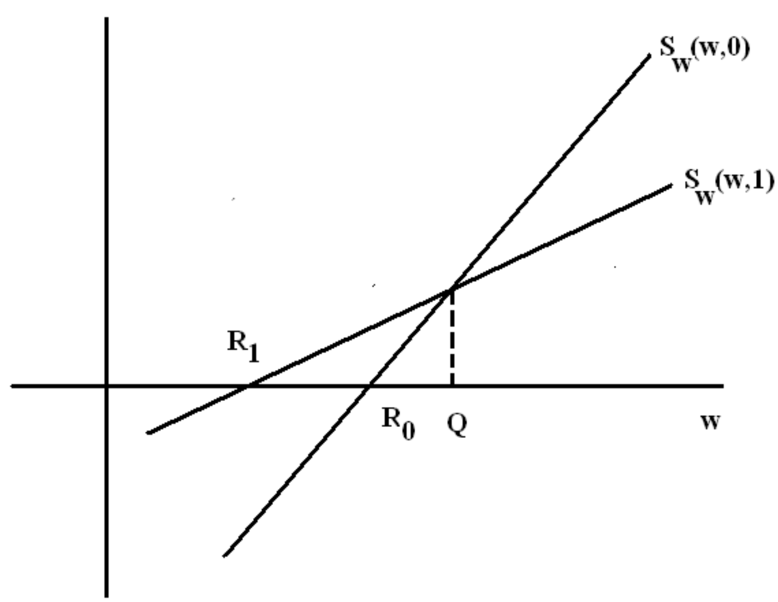

Figure 2: $c \leq c_0$

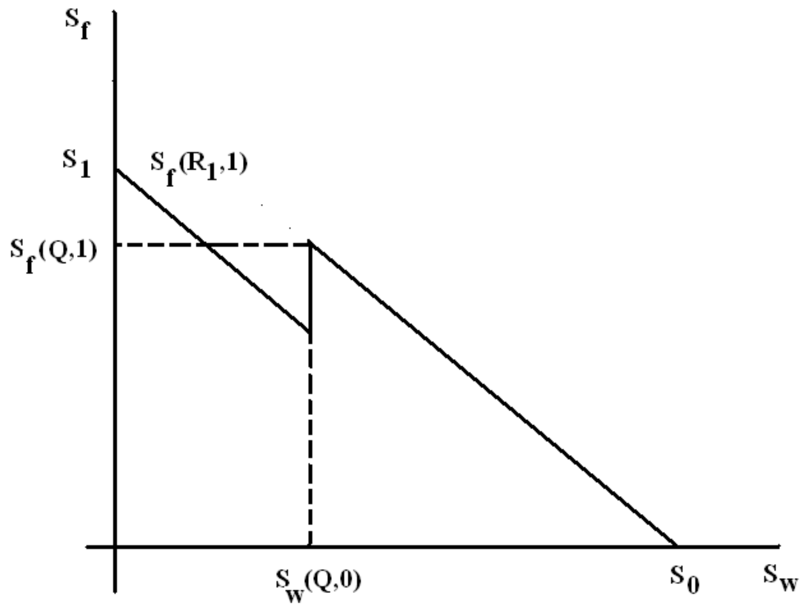

Figure 3: Bargaining Set when $c < c_0$

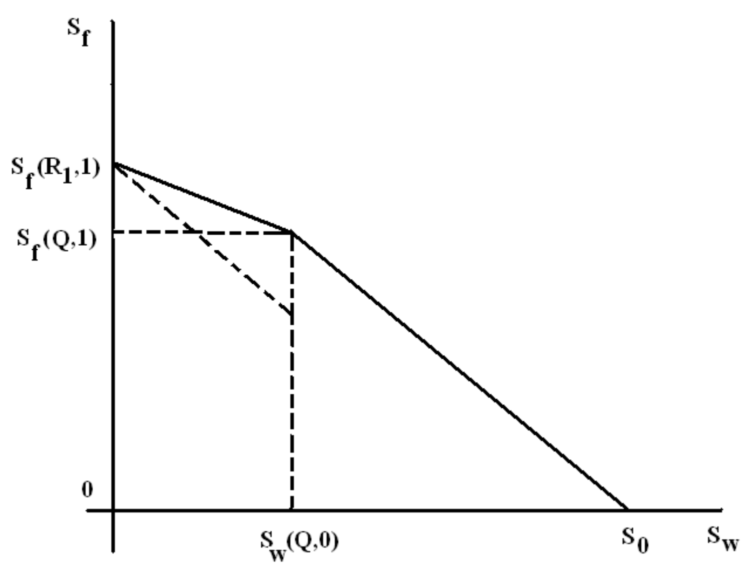

Figure 4: Bargaining Set with Lotteries when $c < c_0$

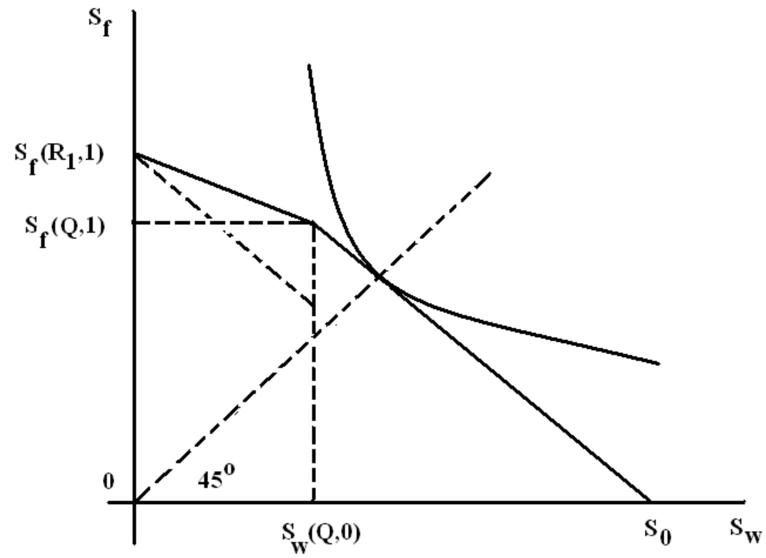

Figure 5: Bargain in Region 1

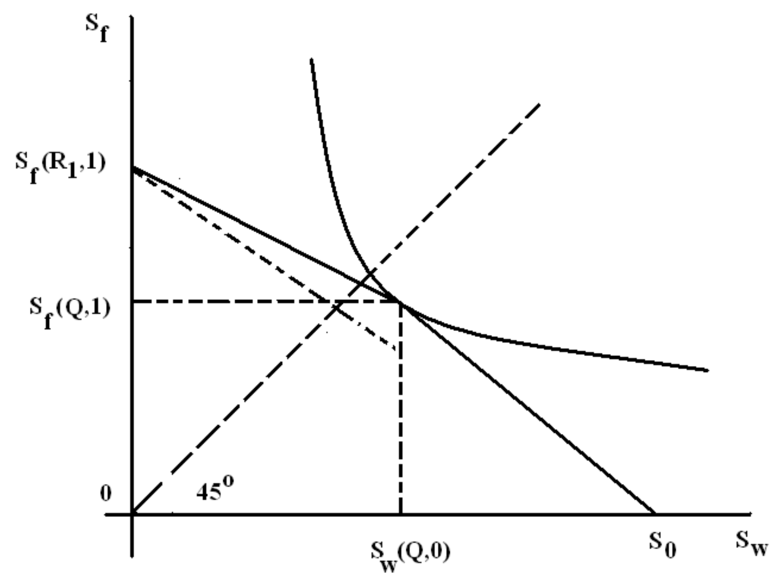

Figure 6: Bargain in Region 2

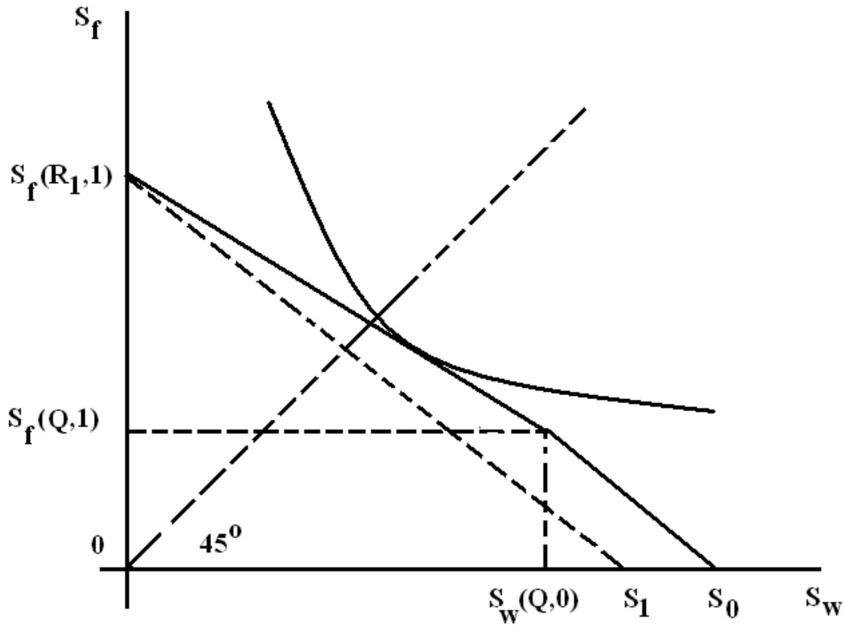

Figure 7: Bargain in Region 3

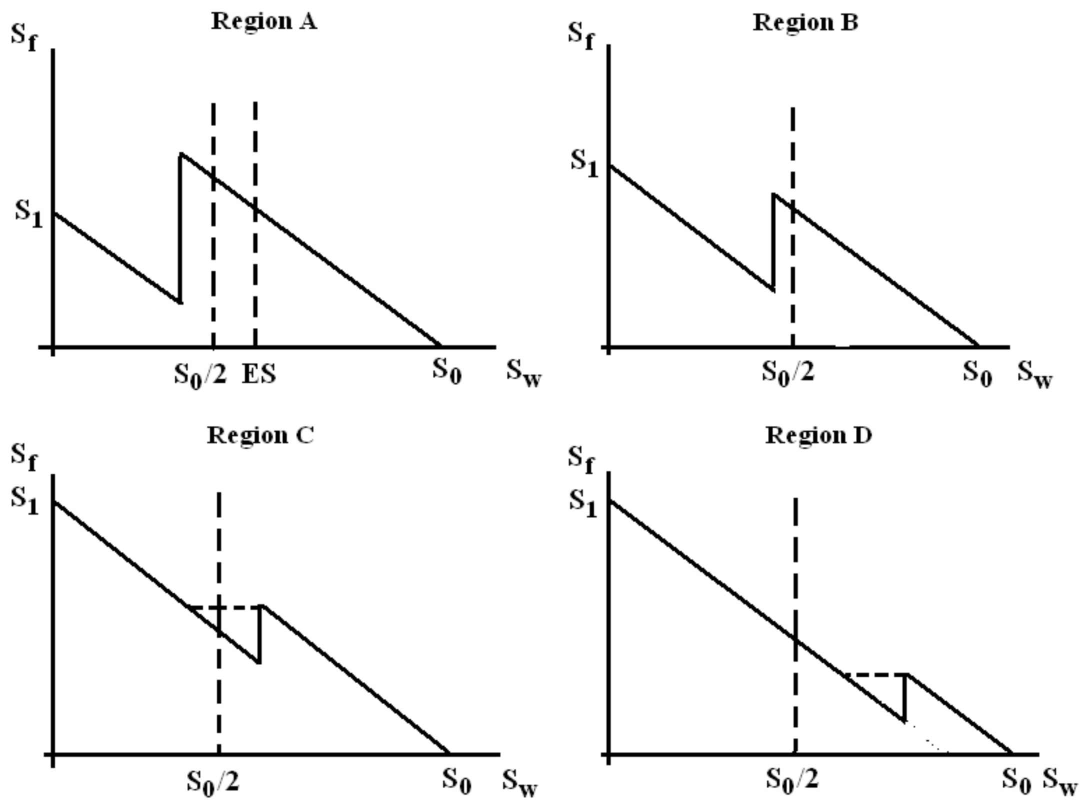

Figure 8: Strategic Bargaining