On The Labor Market Inequalities Across Regions

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October, 2007

Abstract

Wages, unemployment and the labor market size vary considerably across regions in U.S. To explain these differences I develop a general equilibrium model in which each region is characterized by a labor market with search frictions: workers search for a job both when unemployed and when employed and firms post wages that maximize their profits. Firms and workers decide where to locate given labor market conditions in each region. Central to their decisions are the probabilities of finding a productive match. These probabilities are derived from a meeting technology that displays increasing returns of scale and is endogenously given by the number of workers and firms in each market. Workers and firms are assumed to be equally productive, however workers differ in the way that they value each region. Within this framework equilibria with unequal and non-empty labor markets can arise even for equally productive workers and firms. The model predicts a positive relationship between wages and markets size and a negative relationship between unemployment and size for a group of identical workers. Both relationships are supported by empirical literature.

1 Introduction

The empirical literature have widely characterized the fact that wages, unemployment rates, and the number of workers and firms vary considerably across regions in U.S. To explain these inequalities two main classes of theoretical models have emerged. On one hand, the compensanting differentials literature claims that in the presence of a a mobile labor force, labor market outcomes

*I thank Ken Burdett, Jan Eeckhout, Randy Wright, Albert Saiz, Roberto Pinheiro and the participants of the Search & Matching Workshop at University of Pennsylvania, the Midwest Theory/Trade Meeting at University of Minnesota, and North-American Summer Meetings of the Econometric Society at Duke University for their comments and useful discussions. A previous version of this paper circulated before under the name "An Equilibrium Theory of Inequality Across Regions". All erros are mine.

adjust to equalize the utility workers derive from being in each region. Wages and unemployment can compensate for each other as in Harris and Todaro (1970) or to compensate for another factor, as amenities or rental prices as in the case of Roback (1982). On the other hand, an extensive literature has built on the idea of existence of agglomeration economies. Such economies, based on sharing, learning or matching mechanisms, generate reasons for which firms and workers concentrate in specific regions and therefore explain the observed inequality on the size distribution of regions (for a review, see Duranton and Puga (2003)). Although these two theoretical explanations share the purpose of explaining labor market inequalities, their focus has been very different: while the compensating differentials literature concentrates on wage and unemployment inequalities, the studies based on agglomeration economies focalize size and wages dispersions. To my knownledge, there are no studies that consider these three inequalities together.

To explain the sources and the implications of labor market inequalities I propose a general equilibrium model in which regional wage distributions, unemployment rates, and market sizes are endogenously and explicitly determined. Each region is characterized by a labor market with search frictions as in Burdett and Mortensen (1998): workers search for a job both when unemployed and when employed and firms post wages that maximize their profits. Firms and workers decide where to locate given labor market conditions in each region. In particular, central to their decisions are the probabilities of finding a productive match. These probabilities are derived from a meeting technology that displays increasing returns of scale and is endogenously given by the number of workers and firms in each market. This meeting technology determines how often workers get a job offer. Workers and firms are assumed to be equally productive, however workers differ in the way that they value each region.

The model is a blend of the two main existent literatures. As in the compensating differentials models, workers move across regions to maximize their utility. When taking their location decisions workers consider the trade-off between wages, unemployment, and their valuation of each region. As all the studies on the agglomeration economy literature I assume a type of increasing returns of scale. More precisely, in my model the meeting echnology displays increasing returns to scale.

Most of the literature on agglomeration economies focus on sharing and learning. However there has been an increasing literature that center on matching.as a source of agglomeration economies (see Strange and Helsley (1990), Wheeler (2001), Shimer (2001), Teulings and Gautier (2003), Gan and Zhang (2005) just to name a few). Differently of this literature, however,I center on the probability of matching and not in the expected quality of matches. This seems to be a novelty. Focusing on the probality rather than in the quality allows me to explain labor market inequalities for equally productive workers and firms. This is relevant because empirical literature shows that wage, unemployment, and market size are remarkably unequal even for groups of similar workers and firms.

Within this model, the rate at which workers meet a firm increases with the thickness of the market. This is a direct consequence of assuming that the meeting technology display increasing returns of scale. Although the hypothesis of constant returns of scale is more common in the search literature, many studies have been advocating in favor of increasing returns of scale: Diamond (1982), Weitzman(1982), Yashiv(2000), Teuling and Gautier (2003), and Gan and Li (2004). Moreover, as commented by Coles and Smith(1996), although there exists some evidence of constant returns of scale in the matching function, this does not mean that the meeting technology does not displays increasing returns of scale.

Within this model I prove that non-empty equilibria with unequal labor markets can arise. Moreover, the model predicts a positive relationship between market size and wages level and a negative relationship between unemployment and market size. These results are endorsed by the empirical literature.

The main contribution of this paper is the construction of a general framework that endogenously determine wages, unemployment, and market size and that can be used to explain observed labor market inequalities across regions. Moreover this framework can be used to shed some light in the relationship between the three considered types of inequality.

I present my theoretical model in Section 3, and then proceed to the determination of equilibria in Section 4, to their characterization in Section 5. The paper concludes with some remarks on Section 6.

2 Model

Assume the existence of two regions, region A and region B, each of them with a labor market in which firms post wages to maximize their profits and workers search for a job both when unemployed and when employed. Both workers and firms are risk-neutral and take as given the interest rate r that they use to discount the future. The workers' search process is charaterized by a job offer arrival rate $\lambda(n_i, v_i)$, where n_i is the endogenous number of workers living in region i and v_i is the endogenous number of firms established in region i. The arrival rate is definied as being equal the number of meetings per worker in a given market, i.e. $\lambda(n_i, v_i) = \frac{m(n_i,v_i)}{n_i}$, where $m(n_i, v_i)$ is a continuous meeting function that displays increasing returns of scale, is increasing in both the number of workers and the number of firms, satisfies Inada conditions, and is such that $\frac{\delta\lambda(n_i,v_i)}{\delta n_i} < 0$ and $\left|\frac{\delta\lambda(\frac{1}{2},\frac{1}{2})}{\delta n_i}\right| < \left|\frac{\delta\lambda(\frac{1}{2},\frac{1}{2})}{\delta v_i}\right|$[1]. The labor market in each region therefore resembles the labor market presented by Burdett and Mortensen(1998) except for the fact that in my paper the arrival rates are determined endogenously by n_i and v_i.

There is a measure one of identical firms and a measure one of equally productive workers that value each region differently. Workers' evaluation of region A is given by $\theta_A \sim U[-\overline{\theta}, +\overline{\theta}]$, where $\overline{\theta} \longrightarrow \infty$, and the value attached

[1]The importance and the tenability of these assumptions will be assessed later on the paper.

to B is $\theta_B = 0$. Therefore θ_A displays the relative value of A in comparison to B. Observe that the distribution of these relative values is symmetric and has zero mean.

Both firms and workers have to decide where to be located given that they face no costs to move from one region to the other.

I assume that workers are initially unemployed. Those who attach a higher value to A than to B are initially unemployed in A and those that attach a higher value to B are initially unemployed in B.

The workers have to decide either to be in A or in B when unemployed. They don't move when employed[2]. Workers also decide which job offers to accept both when employed and when unemployed.

Unemployed workers have to decide where to locate and do so comparing the value of being unemployed in A with the value of being unemployed in B. In both regions they get an unemployment insurance b and get a job offer from an endogenous wage distribution $F_i(w_i)$ according to the rate $\lambda(n_i, v_i)$. Formally the value of living in A and in B is given by:

$$rU_i = b + \theta_i + \lambda(n_i, v_i) \int_{w_i^*}^{\infty} [E_i(w_i') - U_i] dF_i(w_i') \tag{1}$$

As in a typical search model there exists a wage w_i^* in each region such that unemployed workers accept job offers if and only if they are at least equal to w_i^*.

When workers accept an offer they start to work and to produce p. They keep searching for a job. For matters of simplicity the arrival rate of offers for employed workers is assumed to be equal to the arrival rate when the worker is unemployed: $\lambda(n_i, v_i)$. Employed workers will accept a new offer if this is bigger than their current wage w_i. When employed, workers can become unemployed according to the exogenous rate δ, that is assumed to be a small positive number such that $\delta < \lambda(\frac{1}{2}, \frac{1}{2})$. The value of being employed in each region can be written as:

$$rE_i(w_i) = w_i + \theta_i + \lambda(n_i, v_i) \int_{w_i}^{\infty} [E_i(w_i') - E_i(w_i)] dF_i(w_i') - \delta[E_i(w_i) - U_i] \tag{2}$$

Firms decide where to establish considering that there are no costs to move across regions and then post wages that maximize their profits.

The profit maximization problem can be posed as:

$$\pi_i = \max_w (p - w) l(w | w_i^*, F_i) \tag{3}$$

[2]I am allowing the workers to move just when they are unemployed. I do this for simplicity reasons: the model gets much more difficult to solve when one allow the workers to move when they are employed. Although this does not reflect what happens in the real world, the migration literature shows that individuals are more likely to move when they are young and when they are unemployed. This assumption could be thought to reflect this fact.

The revenue generated by each worker is $(p - w)$ and the endogenous measure of workers employed at this wage is represented by $l(w|w_i^*, F_i)$ that is a increasing function of w[3]. Therefore the firm faces a trade-off: an increase in the wages decreases the profit per worker but increases the number of workers in equilibrium.

Taking n_A, v_A, n_B and v_B as given, one can follow Burdett and Mortensen (1998) and show that in each region there is an unique equilibrium solution to the search and wage posting game, i.e. there is an unique distribution of wages F_i, level of profits π_i, and reservation wage w_i^* in each region such that: i) any wage on the support of F_i generates the same level of profit π_i and any wage that is not on the support of F_i generates profits lower or at most equal to π_i; ii) the level of profits π_i results from a maximization process in which firms choose the optimal wage that solves the following trade-off: higher wages genetare less profits per worker but attract more workers; and iii) the reservation wage w_i^* is such that unemployed workers accept an offer if and only if its is equal or bigger to w_i^*. Moreoverer, again following Burdett and Mortensen(1998), one can show that F_i, π_i and w_i^* are given by:

$$F_i(w_i) = \left[\frac{\delta + \lambda(n_i, v_i)}{\lambda(n_i, v_i)}\right] \left[1 - \left(\frac{p - w_i}{p - b}\right)^{\frac{1}{2}}\right] \tag{4}$$

$$\pi_i = \frac{(p - b)\frac{n_i}{v_i}\lambda(n_i, v_i)\delta}{(1 + \lambda(n_i, v_i))^2} \tag{5}$$

$$w_i^* = b \tag{6}$$

In this type of model there is dispersion in the wages within a region due to on-the-job search. Although workers have the same reservation wage, they move up the wages ladder according to the random arrival of job offers when employed and eventually go back to the position of unemployment according to the exogenous parameter δ. On the other hand, firms post different wages because a higher wage means that the firm is atrracting more workers despite of the fact that the profit per worker is lower. The firm attracts more workers both because it is more likely that a worker accept an offer when she is employed in another firm and because it is less likely that she leaves due to an outside offer.

Furthermore, one can determine the unemployment rates in each region and the total (national) unemployment rate in steady-state.

To do so, observe that in steady-state the inflow of individuals into unemployment is the rate at which the matches are destroyed times the sum of

[3] As $l(w|w_i^*, F_i)$ is the steady-state number of workers available to a firm offering any particular wage w, it can be derived from $l(w|w_i^*, F_i) = \lim_{\varepsilon \to 0} \frac{G(w) - G(w-\varepsilon)}{F(w) - F(w-\varepsilon)}(1 - u)$, where $G(w)$ is the number of employed workers receiving a wage no greater than w and u is the number of unemployed workers. $G(w)$ is got from a steady-state condition equating the number of workers that leave $G(w)$ (those who flow out into unemployment and into higher paying jobs) to the number of workers that flow into firms paying a wage up to w. The number of unemployed, u ,will be derived later in the paper. See Burdett and Mortensen (1988) for details.

the number of employed workers in A and the number of employed workers in B. If u is defined as the total number of unemployed, then $1-u$ is the total number of employed workers. Let the endogenous proportion of employed living in A be represented by γ_e, then $\delta(\gamma_e(1-u)+(1-\gamma_e)(1-u))$ is the steady-state inflow of individuals into unemployment. On the other hand, the outflow of workers from unemployment is the number of unemployed workers in A that get an acceptable wage offer plus the unemployed workers in B that get offers higher than w_B^*. If we define γ_u as the endogenous proportion of unemployed living in A, then the outflow of workers is given by $u\left[\gamma_u\lambda(n_A,v_A)(1-F_A(w_A^*))+(1-\gamma_u)\lambda(n_B,v_B)(1-F_B(w_B^*))\right]$. Observe that $\lambda(n_i,v_i)[1-F_i(w_i^*)]$ is the probability that an unemployed worker receives and accepts a wage offer. In steady-state the inflow and outflow should be equal. Therefore, equating these flows and using the fact that $F_A(w_A^*)=F_B(w_B^*)=0$ (since workers would not accept a wage lower than their reservation wage and since the distribution of wage offers can be shown to be non-degenerate and continuous):

$$u=\frac{[\gamma_e\delta+(1-\gamma_e)\delta]}{\gamma_e\delta+(1-\gamma_e)\delta+\lambda(n_A,v_A)\gamma_u+\lambda(n_B,v_B)(1-\gamma_u)} \tag{7}$$

The measure of unemployed people in A must also be constant in steady-state, therefore the inflow and outflow of people in the group of unemployed in the region A must be equal. The inflow into unemployment is given by $\delta(1-u)\gamma_e$ and the outflow by $u\gamma_u\lambda(n_A,v_A)(1-F_A(w_A^*))$. Again, equating these flows and using the fact that $F_A(w_A^*)=F_B(w_B^*)=0$ one can get:

$$u=\frac{\delta\gamma_e}{\delta\gamma_e+\lambda(n_A,v_A)\gamma_u} \tag{8}$$

From these last two equations:

$$\gamma_e=\frac{\lambda(n_A,v_A)\gamma_u}{\lambda(n_A,v_A)\gamma_u+\lambda(n_B,v_B)(1-\gamma_u)} \tag{9}$$

The national unemployment u can be found through equations (8) and (9).

$$u=\frac{\delta}{\delta+\lambda(n_A,v_A)\gamma_u+\lambda(n_B,v_B)(1-\gamma_u)} \tag{10}$$

The proportion of unemployed people living in A, γ_u ,can be expressed as a function of n_A,v_A,n_B,v_B after one observes that the total number of people living in A is $n_A=\gamma_u u+\gamma_e(1-u)$ and uses the equation for γ_e above to generate:

$$\gamma_u=\frac{n_A(\delta+\lambda(n_B,v_B))}{n_A(\lambda(n_B,v_B)-\lambda(n_A,v_A))+(\delta+\lambda(n_A,v_A))} \tag{11}$$

Using (11) in (10) one can find the value of u as a function of n_A,v_A,n_B and v_B. Observe that I am assuming that the total number of workers is 1, therefore u, the total number of unemployed people, is also the national rate of unemployment.

Proceeding in the same way, the rate of unemployment in A and B can be proven to be:

$$u_A = \frac{\delta}{\delta + \lambda(n_A, v_A)} \tag{12}$$

$$u_B = \frac{\delta}{\delta + \lambda(n_B, v_B)} \tag{13}$$

According to results established in this section, wages, profits, and unemployment rates are completely characterized as a funtion of the parameters of the model and of the number of firms and workers in ech region. In particular, each set (n_A, v_A, n_B, v_B) determines uniquely an equilibrium in each regional labor market. Therefore I proceed to find the values that these variables can assume in equilibrium.

3 Equilibria

When unemployes, workers have to decide to search for a job either in A or in B. Therefore they compare the value of being unemployed in A with the value of being unemployed in B. Workers go to A if $U_A > U_B$ and go to B if $U_A < U_B$. Using equation (1) for each region, letting $r \to 0$, and using integration by parts the comparison between U_A and U_B can be written down as:

$$\begin{aligned} U_A \quad > \quad & U_B \Leftrightarrow \theta_A > \lambda(n_B, v_B) \int_b^\infty \frac{1 - F(w_B')}{\delta + \lambda(n_B, v_B)[1 - F_B(w_B')]} dw_B' \\ & -\lambda(n_A, v_A) \int_b^\infty \frac{1 - F(w_A')}{\delta + \lambda(n_A, v_A)[1 - F_A(w_A')]} dw_A' \end{aligned} \tag{14}$$

Since α_A has a uniform distribution with limits $-\overline{\alpha}$ and $\overline{\alpha}$, where $\overline{\alpha} \to \infty$, we can find a α_A^* such that if $\alpha_A > \alpha_A^*$ workers stay/go to A and if $\alpha_A < \alpha_A^*$ workers stay/go to B. From previous expression α_A^* can be shown to be uniquely given by:

$$\begin{aligned} \theta_A^* \quad = \quad & \lambda(n_B, v_B) \int_b^\infty \frac{1 - F(w_B')}{\delta + \lambda(n_B, v_B)[1 - F_B(w_B')]} dw_B' \\ & -\lambda(n_A, v_A) \int_b^\infty \frac{1 - F(w_A')}{\delta + \lambda(n_A, v_A)[1 - F_A(w_A')]} dw_A' \end{aligned} \tag{15}$$

This expression for α_A^* can be rewritten as a function of the parameters of the model and of the endogenous variables. To do this, one can use (4) to solve the integrals and get:

$$\theta_A^* = (p-b)[\left(\frac{\lambda(n_B, v_B)}{\delta + \lambda(n_B, v_B)}\right)^2 - \left(\frac{\lambda(n_A, v_A)}{\delta + \lambda(n_A, v_A)}\right)^2] \tag{16}$$

The total number of people living in A is given by the measure of workers with θ_A bigger than θ_A^*. Given the assumption that $\overline{\theta} \longrightarrow \infty$, it is guaranteed that always there will exist workers willing to be in each region. Using the assumption that θ_A has a uniform distribution, the total number of people living in A is given by:

$$n_A = \frac{\overline{\theta} - \theta_A^*}{2\overline{\theta}} \tag{17}$$

Using equation (16), this can be rewritten as:

$$n_A = \{\overline{\theta} + (p-b)[\left(\frac{\lambda(n_A, v_A)}{\delta + \lambda(n_A, v_A)}\right)^2 - \left(\frac{\lambda(n_B, v_B)}{\delta + \lambda(n_B, v_B)}\right)^2]\}\frac{1}{2\overline{\theta}} \tag{18}$$

And the total number of people living in B is:

$$n_B = 1 - n_A \tag{19}$$

Given that there will always exist workers in each one of regions, it will never be the case that a region will have no firms. To see that suppose that there is a measure ε_n of workers in a region, say region A, and suppose also that there are no firms in region A. In this case one can use equation (5) to show that profits in region B will be:

$$\pi_B = \frac{(p-b)(1-\varepsilon_n)\lambda(1-\varepsilon_n, 1)\delta}{(1+\lambda(1-\varepsilon_n, 1))^2} \tag{20}$$

Given the specifications of the meeting technology, π_B will be a bounded positive number.

Observe that if a measure of firms, ε_v , decide to go to A their remuneration will be:

$$\pi_A = \frac{(p-b)\frac{\varepsilon_n}{\varepsilon_v}\lambda(\varepsilon_n, \varepsilon_v)\delta}{(1+\lambda(\varepsilon_n, \varepsilon_v))^2} \tag{21}$$

For a sufficiently small ε_v, one can prove that:

$$\lim_{\varepsilon_v \to 0} \pi_A = \frac{\lim_{\varepsilon_v \to 0}[(p-b)\frac{\varepsilon_n}{\varepsilon_v}\lambda(\varepsilon_n, \varepsilon_v)\delta]}{\lim_{\varepsilon_v \to 0}[(1+\lambda(\varepsilon_n, \varepsilon_v))^2]} = \infty \tag{22}$$

Therefore, there will always exist a sufficiently small measure of firms that will take advantage of a moving to A. So it is proved by contradiction that there will not exist an equilibrium with an empty region.

As there will be a positive measure of firms in each region and considerating that they are free to move costlessly across regions, firms will alocate themselves in a way such that $\pi_A = \pi_B$:

$$\pi_A = \pi_B \Longleftrightarrow \frac{(p-b)\frac{n_A}{v_A}\lambda(n_A, v_A)\delta}{(1+\lambda(n_A, v_A))^2} = \frac{(p-b)\frac{n_B}{v_B}\lambda(n_B, v_B)\delta}{(1+\lambda(n_B, v_B))^2} \tag{23}$$

$$v_B = 1 - v_A \tag{24}$$

In this framework an equilibrium is given by a set of values n_A, n_B, v_A and v_B that satisfies equations (21), (22), (23) and (24). Given this set of values, F_i, π_i and w_i^* are determined through equations (4), (5) and (6) and the equilibrium for the wage post - search game in each region is guaranteed as stated above.

Plugging (22) and (24) in (21) and (23) and rearranging them the equilibrium conditions can be characterized by:

$$n_A - \{\overline{\theta} + (p-b)[\left(\frac{\lambda(n_A, v_A)}{\delta + \lambda(n_A, v_A)}\right)^2 - \left(\frac{\lambda(1-n_A, 1-v_A)}{\delta + \lambda(1-n_A, 1-v_A)}\right)^2]\}\frac{1}{2\overline{\theta}} = 0 \tag{25}$$

$$\frac{\frac{n_A}{v_A}\lambda(n_A, v_A)}{(1+\lambda(n_A, v_A))^2} - \frac{\frac{1-n_A}{1-v_A}\lambda(1-n_A, 1-v_A)}{(1+\lambda(1-n_A, 1-v_A))^2} = 0 \tag{26}$$

Equation (25) is the workers' equilibrium condition and equation (26) is the firms' equilibrium condition.

The existence of an equilibrium is trivially given by the solution:

$$n_A = n_B = \frac{n}{2} = \frac{1}{2} \text{ and } v_A = v_B = \frac{v}{2} = \frac{1}{2} \tag{27}$$

Observe that when both the number of workers living in each region and the number of firms in each region is equal to $\frac{1}{2}$, the equations (25) and (26) are satisfied. Moreover, $\lambda(n_A, v_A)$ equals $\lambda(n_B, v_B)$ and therefore the two regions are identical: they have identical levels of unemployment, identical distribution of wages, and firms have the same amount of profits.

Guaranteed the existence of equilibrium I will proceed to investigate the existence of other equilibria.

To check whether unequal equilibria will arise within this model, first observe that both equilibrium conditions are represented by continuous functions that pass throughout point $(\frac{1}{2}, \frac{1}{2})$. Moreover, both curves can be proven to be increasing in a (n_A, v_A) diagram and such that $v_A \longrightarrow 0$ when $n_A \longrightarrow 0$ and $v_A \longrightarrow 1$ when $n_A \longrightarrow 1$[4].

After tedious algebra, one can show that the slope of firms' equation and the slope of workers' equation around the point $(\frac{1}{2}, \frac{1}{2})$ are respectively given by:

[4]Refer to the Appendix to check how these properties are proved to hold.

$$\left.\frac{dn_A}{dv_A}\right|_W = \frac{\overline{\lambda}\frac{\delta\overline{\lambda}}{\delta v}}{(\delta+\overline{\lambda})^3}[\frac{\overline{\theta}}{(p-b)\delta} - \frac{\overline{\lambda}\frac{\delta\overline{\lambda}}{\delta n}}{(\delta+\overline{\lambda})^3}]^{-1} \tag{28}$$

$$\left.\frac{dn_A}{dv_A}\right|_F = \frac{2\overline{\lambda} - \frac{\delta\overline{\lambda}}{\delta v}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}}{2\overline{\lambda} + \frac{\delta\overline{\lambda}}{\delta n}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}} \tag{29}$$

Where $\overline{\lambda} = \lambda(\frac{1}{2}, \frac{1}{2})$ and by assumption $(\delta - \overline{\lambda}) < 0$.

Observe that equations (26) and (29) do not depend on p and b while equations (25) and (28) does. In particular, when productivity is small compared to unemployment insurance (i.e. when $p \longrightarrow b$) it is guaranteed that $\left.\frac{dn_A}{dv_A}\right|_F > \left.\frac{dn_A}{dv_A}\right|_W$ at point $(\frac{1}{2}, \frac{1}{2})$. As the difference $(p-b)$ increases and assumes bigger positive values, the slope of workers' equation changes while the slope of firms' condition remain constant and eventually $\left.\frac{dn_A}{dv_A}\right|_F < \left.\frac{dn_A}{dv_A}\right|_W$[5]. As both equilibrium conditions are continuous in the interval $(\frac{1}{2}, 1)$, a version of the Intermediate Value Theorem can be used to be prove the existence of an equilibrium in this interval.

The same type of argument could be developed to prove the existence of another equilibrium in the interval $(0, \frac{1}{2})$.

As the system of equations given by (25) and (26) exhibits a type of symmetry due the fact that (n_A, v_A) and $(1 - n_A, 1 - v_A)$ can be interchanged without modifying the value that the funtions assumes, for a given equilibrium in the interval $(0, \frac{1}{2})$, say $(n'(v'), v')$, there will exist a correspondent equilibrium $(1 - n'(1 - v'), 1 - v')$ in the interval $(\frac{1}{2}, 1)$.

Therefore the existence of additional equilibria apart from the identical equilibrium has been proven for some combination of parameters. This means that equilibria with unequal regions can arise within this framework, i.e. labor market inequalities across regions can be seen as an equilibrium feature. Therefore, even in a situation in which cities are ex-ante identical, and firms and workers are equally productive, one can observe equilibria in which there are different number of workers and firms across cities what implies that there are differences in the distribution of wages, unemployent and firms' profits across cities. The source of these inequalities are the search frictions derived from a meeting technology that displays increasing returns of scale.

[5] At this point it is used the assuption that $\left|\frac{\delta\lambda(n_i,v_i)}{\delta n_i}\right| < \left|\frac{\delta\lambda(n_i,v_i)}{\delta v_i}\right|$. This inequality means that a small equal increase in the number of workers and firms has a positive impact in the number of offers received by the workers. Observe that for the case of a general Cobb-Douglas meeting technology that displays increasing returns of scale: $m(n_i, v_i) = (n_i^\alpha v_i^{1-\alpha})^\beta$, where $0 < \alpha < 1$ and $\beta > 1$ this is always verified. Check the Appendix for the proof of the relation between the curves' slopes.

4 Characterization of equilibria

As commented before, each set of equilibrium values (n_A, v_A, n_B, v_B) fully characterizes regional economies, therefore, given the demonstration of the existence of identical and unequal equilibrium with non-empty regions, I will proceed to the characterization of these equilibria. In particular, I will focus on unequal equilibria.

The fact that both equilibrium conditions are increasing in the plan (n_A, v_A) implies that the number of workers in a certain region is positively related to the number of firms: the region with more workers is also the region with more firms. Moreover from equation (8) it is possible to conclude that $\frac{\partial \gamma_e}{\partial \gamma_u} > 0$, it means that the more populated region will display both a higher number of employed workers and a higher number of unemployed workers than the other region.

The largest region necessarily exhibits a wage distribution that first-order stochastically dominates the distribution of the smaller region. Roughly speaking, the bigger region has higher wages. To verify this, equation (25) can be used. Assume A is the largest region. Thus equation (25) implies that the term $\left(\frac{\lambda(n_A,v_A)}{\delta+\lambda(n_A,v_A)}\right)^2 - \left(\frac{\lambda(1-n_A,1-v_A)}{\delta+\lambda(1-n_A,1-v_A)}\right)^2$ has to be bigger than 0. This guarantees that $\lambda(n_A, v_A) > \lambda(1 - n_A, 1 - v_A)$. Now, using equation (4), the equation that fully characterizes the distribution of wages, one can verify that when $\lambda(n_A, v_A) > \lambda(1-n_A, 1-v_A)$, $F_A(w) < F_B(w)$ for any w in the interval $[b, \overline{w}_A]$. Where b is the lowest wage offered by a firm in both regions and $\overline{w}_A$ is the highest wage offered in region A and is such that $\overline{w}_A > \overline{w}_B$[6].

Furthermore, equations (12) and (13) imply that largest region, say A, will have a lower unemployment rate due to the fact that $\lambda(n_A, v_A) > \lambda(1-n_A, 1-v_A)$.

Thereupon, the model predicts that a region that displays a higher population, will pay higher wages and present lower unemployment rates.

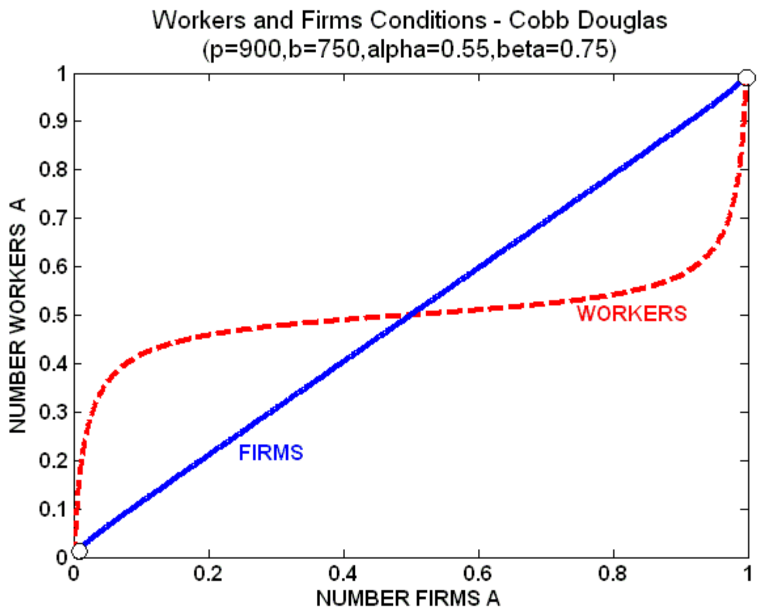
From the demonstration of existence of unequal regions equilibria it is evident that both the productivity level and the unemployment insurance will have a considerable impact in the equilibrium features of both regions. To better access this impact I will assume that meeting technology assumes a Cobb-Douglas of the following form:

$$m(n_i, v_i) = n_i^{\alpha} v_i^{\beta} \tag{30}$$

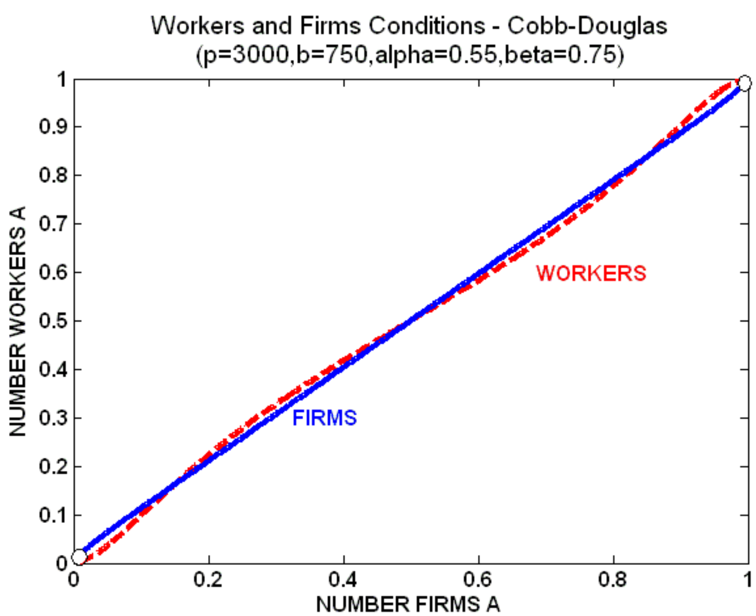
Moreover, I assume that $\theta = 50, \alpha = 0.55, \beta = 0.75, \delta = 0.01$.

Under these assumptions, for small values of $(p - b)$, the workers' and firms' are such that there is an unique equilibrium:

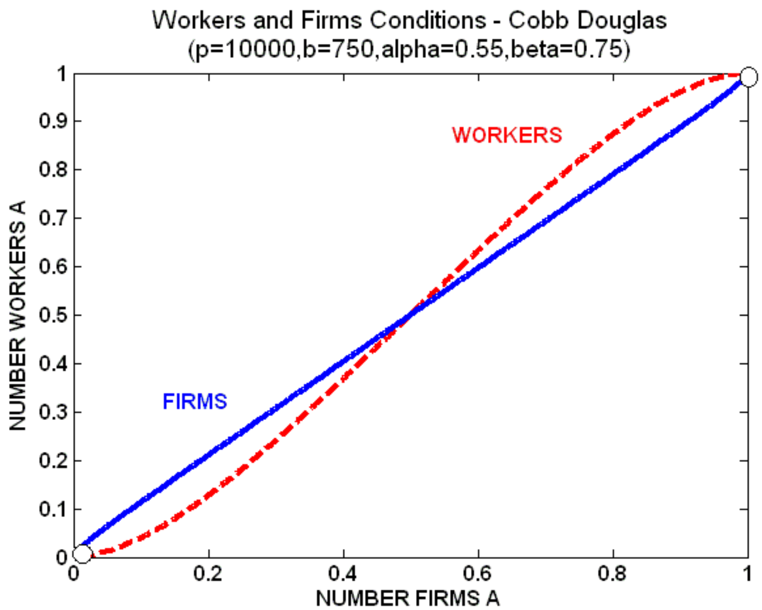
[6]Following Burdett and Mortensen (1998) one can find $\overline{w}_i = \frac{[(\delta+\lambda(n_i,v_i))^2-\delta^2]p+b}{(\delta+\lambda(n_i,v_i))^2}$ and carefully check that $\lambda(n_A, v_A) > \lambda(n_B, v_B)$ implies $\overline{w}_A > \overline{w}_B$.



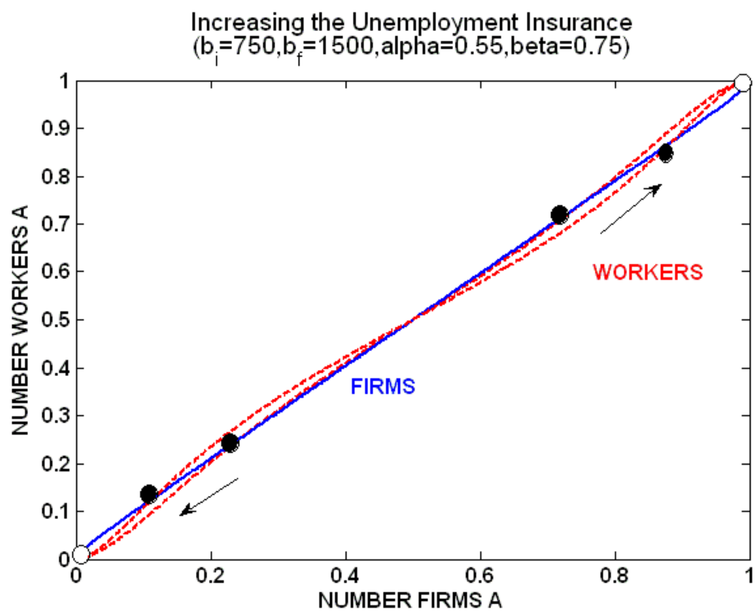
When productivity starts to increases, the difference $(p - b)$ increases, and eventually multiple equilibria will arise. As was observed mentioned before, firms' equilibrium condition does not react to changes on p and b. On the other hand, workers' curve rotate leftwards.



Further increases in p can take the economy back to a situation with an unique equilibrium:



Consider combinations of parameters such that multiple equilibria are observed. In this case, a small increase in b will increase the inequality across regions: the largest region will offer even higher wages, and lower unemployment rate. Graph 4 depicts this phenomenun. This result is consistent with Lkhagvasuren (2006): he has shown that an increase in the UI generates a higher dispersion in the regional unemployment rates in U.S.. As he observes, this increase in the inequality across regions due to an increase in the UI seems to be new in the literature.



5 Final Remarks

I have developed a steady-state, on-the-job search model that endogenously and explicitly determines the wage distribution, the unemployment rates, and the number of firms and workers in each region. To do so I assume the existence of a meeting technology with increasing returns of scale as an agglomeration economy. Within this framework I show that unequal equilibria can arise even when firms and workers are equally productive. This is in accordance with empirical literature. The model predicts a positive relationship between wages and markets size and a negative relationship between unemployment and size for a group of identical workers. Empirical evidence support both predictions.

Although I have focused on the case of equally productive workers and firms, and ex-ante identical regions, this model could be extended to allow differences across regions, workers, and firms. In particular one could investigate what would be the consequences over labor market outcomes of the introduction of productivity and amenities differences across cities. Doing so would create a more general version of models a la Roback (1982) and Ciccone and Hall (1996), for example. Moreover, the model can be adjusted to approach other issues as the creation of an empirical measure for amenities level and international immigration.

6 Appendix

6.1 Appendix A

In this appendix I demonstrate some of the basic features of equations (25) and (26), respectively workers' equilibrium condition and firms' equilibrium condition, that are rewritten and relabeled below:

$$n_A - \{\overline{\alpha} + (p-b)[\left(\frac{\lambda(n_A, v_A)}{\delta + \lambda(n_A, v_A)}\right)^2 - \left(\frac{\lambda(1-n_A, 1-v_A)}{\delta + \lambda(1-n_A, 1-v_A)}\right)^2]\}\frac{1}{2\overline{\alpha}} = 0 \quad \text{(A.1)}$$

$$\frac{\frac{n_A}{v_A}\lambda(n_A, v_A)}{(1+\lambda(n_A, v_A))^2} - \frac{\frac{1-n_A}{1-v_A}\lambda(1-n_A, 1-v_A)}{(1+\lambda(1-n_A, 1-v_A))^2} = 0 \quad \text{(A.2)}$$

Both curves are continuous due to the fact that the $m(n_i, v_i)$ and therefore $\lambda(n_i, v_i)$ are continuous.

To observe that workers' curve is positive, observe that if equation (A.1) is represented by $W(n_A, v_A) = 0$, then its slope can be given by:

$$\left.\frac{dn_A}{dv_A}\right|_W = -[\frac{\delta W(n_A, v_A)}{\delta v_A}][\frac{\delta W(n_A, v_A)}{\delta n_A}]^{-1} \quad \text{(A.3)}$$

The two terms in the the right-hand side of this expression are calculated.

$$\begin{aligned} \frac{\delta W(n_A, v_A)}{\delta v_A} &= -\frac{\lambda(n_A, v_A)\delta\lambda(n_A, v_A)/\delta v_A}{(\delta + \lambda(n_A, v_A))^3} + \\ & \frac{\lambda(1-n_A, 1-v_A)\delta\lambda(1-n_A 1-, v_A)/\delta v_A}{(\delta + \lambda(1-n_A 1-, v_A))^3} \end{aligned} \quad \text{(A.4)}$$

Given the properties of the meeting function, $\frac{\delta\lambda(n_A, v_A)}{\delta v_A} > 0$ and $\frac{\delta\lambda(1-n_A 1-, v_A)}{\delta v_A} < 0$ and therefore expression above is negative. The other term is given by:

$$\begin{aligned} \frac{\delta W(n_A, v_A)}{\delta n_A} &= \frac{\overline{\alpha}}{(p-b)\delta} - \frac{\lambda(n_A, v_A)\delta\lambda(n_A, v_A)/\delta n_A}{(\delta + \lambda(n_A, v_A))^3} \\ & + \frac{\lambda(1-n_A, 1-v_A)\delta\lambda(1-n_A 1-, v_A)/\delta n_A}{(\delta + \lambda(1-n_A 1-, v_A))^3} \end{aligned} \quad \text{(A.5)}$$

$\frac{\delta\lambda(n_A, v_A)}{\delta n_A} < 0$ and $\frac{\delta\lambda(1-n_A 1-, v_A)}{\delta n_A} > 0$ guarantee that expression (1) is positive and therefore the slope of the workers' equilibrium condition is positive.

I proceed in similar way to show that the slope of firms' curve is always positive. To do so I represent equation (A.2) as $F(n_A, v_A) = 0$ and indicate its slope by:

$$\left.\frac{dn_A}{dv_A}\right|_F = -[\frac{\delta F(n_A, v_A)}{\delta v_A}][\frac{\delta F(n_A, v_A)}{\delta n_A}]^{-1} \quad \text{(A.6)}$$

One can show that:

$$\begin{aligned}\frac{\delta F(n_A, v_A)}{\delta v_A} &= -\frac{n_A}{v_A}[\frac{\lambda(n_A, v_A)}{(\delta + \lambda(n_A, v_A))^2 v_A}] \\ &+\frac{n_A}{v_A}[\frac{(\delta\lambda(n_A, v_A)/\delta v_A)(\delta - \lambda(n_A, v_A))}{(\delta + \lambda(n_A, v_A))^3}] \qquad (1)\\ &-\frac{1-n_A}{1-v_A}[\frac{\lambda(1-n_A, 1-v_A)}{(\delta + \lambda(1-n_A, 1-v_A))^2(1-v_A)} \\ &+\frac{(\delta\lambda(1-n_A, 1-v_A)/\delta v_A)(\delta - \lambda(1-n_A, 1-v_A))}{(\delta + \lambda(1-n_A, 1-v_A))^3}] \qquad (A.7)\end{aligned}$$

By assumption δ is a small number and lower than $\lambda(n_A, v_A)$ for any pair (n_A, v_A) that satisfies the expression above. Moreover $\frac{\delta\lambda(n_A,v_A)}{\delta v_A} > 0$ and $\frac{\delta\lambda(1-n_A 1-,v_A)}{\delta v_A} < 0$ and then the term is negative. On the other hand, the term below can be show to be positive.

$$\begin{aligned}\frac{\delta F(n_A, v_A)}{\delta n_A} &= \frac{n_A}{v_A}[\frac{\lambda(n_A, v_A)}{(\delta + \lambda(n_A, v_A))^2 n_A} - \frac{(\delta\lambda(n_A, v_A)/\delta n_A)(\delta - \lambda(n_A, v_A))}{(\delta + \lambda(n_A, v_A))^3}] \\ &+\frac{1-n_A}{1-v_A}[\frac{\lambda(1-n_A, 1-v_A)}{(\delta + \lambda(1-n_A, 1-v_A))^2(1-n_A)} \\ &+\frac{(\delta\lambda(1-n_A, 1-v_A)/\delta n_A)(\delta - \lambda(1-n_A, 1-v_A))}{(\delta + \lambda(1-n_A, 1-v_A))^3}] \qquad (A.8)\end{aligned}$$

Therefore, after observing expression (A.6), one can verify that the curve that represents firms' equilibrium condition is upward sloping.

Now I will show that each of the equilibrium curves is such that $v_A \longrightarrow 0$ when $n_A \longrightarrow 0$. Rewrite equation (A.1) as:

$$n_A = \{\overline{\alpha} + (p-b)[\left(\frac{\lambda(n_A, v_A)}{\delta + \lambda(n_A, v_A)}\right)^2 - \left(\frac{\lambda(1-n_A, 1-v_A)}{\delta + \lambda(1-n_A, 1-v_A)}\right)^2]\}\frac{1}{2\overline{\alpha}} \qquad (A.9)$$

To prove that $v_A \longrightarrow 0$ when $n_A \longrightarrow 0$ suppose that $v_A \longrightarrow c > 0$ when $n_A \longrightarrow 0$. It clearly implies that left-hand side converges to zero. Analysing the right-hand side, observe that this means that $\lambda(n_A, v_A) \longrightarrow \infty$ due to the assumption that $\frac{\delta\lambda(n_A,v_A)}{\delta n_A} < 0$ and also means that $\lambda(1-n_A, 1-v_A)$ converges to a positive number. These two piece of information imply that $\left(\frac{\lambda(n_A,v_A)}{\delta+\lambda(n_A,v_A)}\right)^2 \longrightarrow \infty$ and $\left(\frac{\lambda(1-n_A,1-v_A)}{\delta+\lambda(1-n_A,1-v_A)}\right)^2 \longrightarrow d > 0$ and therefore left-hand side goes to infinity. This contradicts our initial assumption and therefore proves that workers curve is such that $v_A \longrightarrow 0$ when $n_A \longrightarrow 0$. Clearly the same arguments could be used to show that $v_A \longrightarrow 1$ when $n_A \longrightarrow 1$.

To see that the same feature is displayed by firms' condition I rewrite it here as:

$$\frac{\frac{n_A}{v_A}\lambda(n_A,v_A)}{(1+\lambda(n_A,v_A))^2}=\frac{\frac{1-n_A}{1-v_A}\lambda(1-n_A,1-v_A)}{(1+\lambda(1-n_A,1-v_A))^2} \tag{A.10}$$

Assume that $v_A \longrightarrow c>0$ when $n_A \longrightarrow 0$. This guarantees that the left-hand side converges to zero as $\frac{n_A}{v_A} \longrightarrow 0$ and $\frac{\lambda(n_A,v_A)}{(1+\lambda(n_A,v_A))^2} \longrightarrow 0$ (this last expression is inversely related with $\lambda(n_A,v_A)$ which is going to infinity). The right-hand side converges to a positive number given that $\frac{1-n_A}{1-v_A} \longrightarrow \frac{1}{1-c}$ and $\frac{\lambda(1-n_A,1-v_A)}{(1+\lambda(1-n_A,1-v_A))^2} \longrightarrow d>0$ (due to the fact that $\lambda(1-n_A,1-v_A)$ converges to a positive number. This contradiction proves that the curve that represents firms' equilibrium condition is such that $v_A \longrightarrow 0$ when $n_A \longrightarrow 0$ and $v_A \longrightarrow 1$ when $n_A \longrightarrow 1$.

The slope of firms' curve and of the workers' curve around point $(\frac{1}{2},\frac{1}{2})$ are given respectively by equations (28) and (29) that are now rewritten:

$$\left.\frac{dn_A}{dv_A}\right|_F=\frac{2\overline{\lambda}-\frac{\delta\overline{\lambda}}{\delta v}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}}{2\overline{\lambda}+\frac{\delta\overline{\lambda}}{\delta n}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}} \tag{A.11}$$

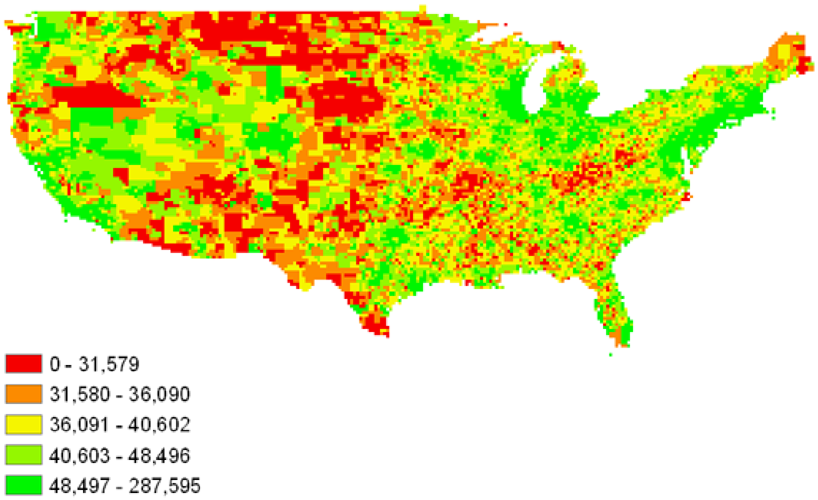
$$\left.\frac{dn_A}{dv_A}\right|_W=\frac{\overline{\lambda}\frac{\delta\overline{\lambda}}{\delta v}}{(\delta+\overline{\lambda})^3}\left[\frac{\overline{\alpha}}{(p-b)\delta}-\frac{\overline{\lambda}\frac{\delta\overline{\lambda}}{\delta n}}{(\delta+\overline{\lambda})^3}\right]^{-1} \tag{A.12}$$

Clearly equation (A.11) does not depend on p. Observe what happens with equation (A.12) when $p \longrightarrow b$. In this case $\frac{\overline{\alpha}}{(p-b)\delta} \longrightarrow \infty$ and $[\frac{\overline{\alpha}}{(p-b)\delta}-\frac{\overline{\lambda}\frac{\delta\overline{\lambda}}{\delta n}}{(\delta+\overline{\lambda})^3}] \longrightarrow \infty$. Therefore $\left.\frac{dn_A}{dv_A}\right|_W \longrightarrow 0$ and $\left.\frac{dn_A}{dv_A}\right|_F > \left.\frac{dn_A}{dv_A}\right|_W$.

When $p \longrightarrow \infty$, $\frac{\overline{\alpha}}{(p-b)\delta} \longrightarrow 0$ and $\left.\frac{dn_A}{dv_A}\right|_W \longrightarrow -\frac{\delta\overline{\lambda}}{\delta v}[\frac{\delta\overline{\lambda}}{\delta n}]^{-1} > 1$, by assumption. Multiply the numerator and denominator by $\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}$ to get $-\frac{\frac{\delta\overline{\lambda}}{\delta v}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}}{\frac{\delta\overline{\lambda}}{\delta n}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}} > 1$. Then add $2\overline{\lambda}$ to the numerator and to the denominator and get $\left.\frac{dn_A}{dv_A}\right|_F = \frac{2\overline{\lambda}-\frac{\delta\overline{\lambda}}{\delta v}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}}{2\overline{\lambda}+\frac{\delta\overline{\lambda}}{\delta n}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}} < -\frac{\frac{\delta\overline{\lambda}}{\delta v}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}}{\frac{\delta\overline{\lambda}}{\delta n}\frac{(\delta-\overline{\lambda})}{(\delta+\overline{\lambda})}} = \left.\frac{dn_A}{dv_A}\right|_W$.

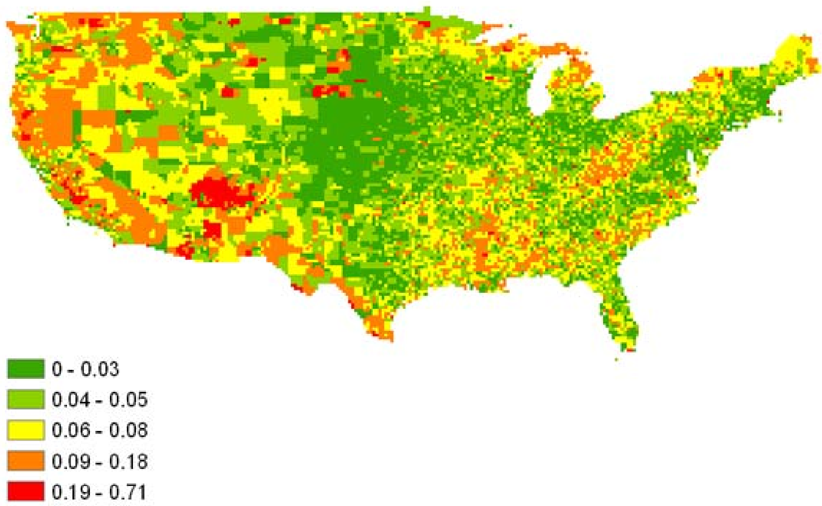
6.2 Appendix B

Wages Inequalities



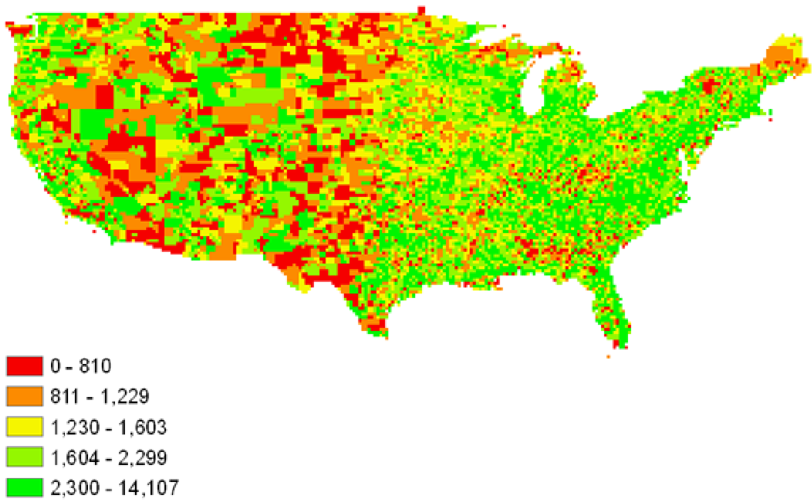
Yearly nominal wages, household average, Census 2000, in US$

Unemployment Inequalities



Unemployment: Census 2000 - proportion of unemployed in the labor force

Labor Force Size Inequalities



Number of workers in the labor force, Census 2000

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