Incomplete Contracts in Dynamic Games

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Abstract

I develop a dynamic model of costly private provision of public goods where agents can also invest in cost-reducing technologies. Despite the n+1 stocks in the model, the analysis is tractable and the (Markov perfect) equilibrium unique. The framework is used to derive optimal incomplete contracts in a dynamic setting. If the agents can contract on provision levels, but not on investments, they invest suboptimally little, particularly if the contract is short-term or close to its expiration date. To encourage sufficient investments, the optimal and equilibrium contract is more ambitious if it is short-lasting, and it is tougher to satisfy close to its expiration date. If renegotiation is possible, such a contract implements the first best. The results have important implications for how to design a climate treaty.

Key words: Dynamic private provision of public goods, dynamic common-pool problems, dynamic hold-up problems, incomplete contracts, renegotiation design, climate change and climate agreements

JEL: D86, H87, Q54, F53

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1. Introduction

This paper develops a dynamic model of private provision of public goods. The agents can also invest in cost-reducing technologies, leading to $n + 1$ stocks, but the analysis is nevertheless tractable. I derive and characterize a unique Markov perfect equilibrium for the noncooperative game as well as for situations where the agents can negotiate and contract on contribution levels. In particular, the optimal and equilibrium contract is described.

The model is general and could fit various contexts. The leading example is climate change, and the results have clear implications for how to design an efficient treaty. Consistent with the model's assumptions, a country can reduce its emission in multiple ways: a short-term solution is to simply consume less fossil fuel today, while a more long-term solution might be to invest in new technologies, such as renewable energy sources or abatement technology. The Kyoto Protocol is a bargaining outcome limiting the countries' emission levels, but it does not specify the extent to which countries should invest or simply reduce its short-term consumption. This distinction would, in any case, be difficult to verify. At the same time, the Protocol is relatively short-lasting, since the commitments expire in 2012. This may reflect the difficulties or costs of committing to the distant future.

All these aspects are in line with the model. To fix ideas, I will refer to the players as "countries" and their contributions as "emissions." The public good, or rather its negative: the public bad, can be interpreted as greenhouse gases. The technology provides a private substitute for polluting, and can be interpreted as renewable energy or abatement technology. The model abstracts from heterogeneities across and within countries as well as the difficulties of motivating participation and compliance. I thus describe an idealized benchmark case that isolates the interactions between negotiated quotas and incentives to invest in technologies.

The real investment cost function may be convex or concave (if there are increasing returns to scale). By assuming it is linear, I prove that the continuation value must be linear in *all* the $n+1$ stocks. Thus, the payoff-relevant history is represented by a weighted sum of the stocks. Only one MPE satisfies these conditions, so the MPE is unique. This MPE is stationary and coincides with the unique subgame perfect equilibrium if time were finite but approached infinity. These attractive equilibrium properties hold for every scenario studied in the paper.

First, the noncooperative outcome is characterized. Although the technology is private and investments are selfish, each country's technology stock is, in effect, a public good, since its role is to substitute for the country's contribution to the public bad. If one country happens to pollute a lot, the other countries are, in the future, induced to pollute less since the problem is then more severe. They will also invest more in technology to be able to afford the necessary cuts in emissions. On the other hand, if a country invests a lot in abatement technology, it can be expected to pollute less in the future. This induces the other countries to increase their emissions and reduce their own investments. Anticipating these effects, each country pollutes more and invests less than it would in an otherwise similar static model. This dynamic common-pool problem is thus particularly severe.

Since the MPE is unique, agreements enforced by trigger strategies are not feasible. Instead, I derive the equilibrium outcome assuming the agents can contract on emission levels. For climate agreements, for example, countries may be able to commit at least to the near future, since domestic stakeholders can hold the government accountable if it has ratified an international agreement. Instead of taking a stand on the countries' ability to commit, I derive the equilibrium contract as a function of this ability.

To begin, suppose the time horizon of a contract is represented by the length of "a period" in the model. If there were only one period, contracting on emission levels would be first best since investments in technology are selfish (one country's investment has no spillover effect on the other countries' technologies). With multiple periods, however, the technology stock that survives to the next period is, in effect, a public good. The reason for this is that a hold-up problem arises when the countries negotiate emission levels: if one country has better technology and can cut its emissions fairly cheaply, then its opponents may ask it to bear the lion's share of the burden when collective emissions are reduced. Anticipating this, countries invest less when negotiations are coming up. Thus, the countries underinvest, particularly if the period is short while the technology is long-lasting. With smaller investments, it is ex post optimal to allow for larger emission levels. On the other hand, since the countries are underinvesting, they would like to encourage more investments and they can do this by negotiating a contract that is tough and allows few emissions. Thus, the best (and equilibrium) contract is tougher and stipulates *lower* emissions compared to the optimum ex post, particularly if the length of the contract is relatively short and the technology long-lasting. Surprisingly, the equilibrium pollution level is identical to the level that would have been first best if investments had been efficient.

If the countries can negotiate and contract on the emission level for several periods, then investments are suboptimally low only at the end of the agreement, since the technology that then remains is, in effect, a public good, thanks to the hold-up problem. Thus, investments decline toward the end of the contract. Anticipating this, and to further motivate investments, the optimal and equilibrium contract becomes tougher to satisfy over time.

However, these contracts are not renegotiation-proof. Once the investments are sunk, countries have an incentive to negotiate ex-post optimal emission levels rather than sticking to an overambitious contract. When renegotiation is possible and cannot be prevented, an investing country understands that it does not, in the end, have to comply with overambitious contracts. Nevertheless, with renegotiation, all investments and emissions are *first best*. Intuitively, emission levels are renegotiated to ex-post optimal levels. Countries with poor technology find it particularly costly to comply with an initial ambitious agreement and will be quite desperate to renegotiate it. This gives them a weak bargaining position and a bad outcome. To avoid this fate, countries invest more in technology, particularly if the initial contract is very ambitious. Taking advantage of this effect, the contract should be tougher if it has a relatively short duration, or if it is close to its expiration date, just as in the case without renegotiation.

Observationally, the outcome of these (re)negotiations is equivalent to a time-inconsistency problem. Repeatedly, the countries make very ambitious promises for future actions. But when the future arrives, they relax these promises while, at the same time, they make ambitious promises for the future - once again. However, rather than being evidence of a time-inconsistency problem, this behavior implements the first best in this model.

The results have important implications for the optimal design of a climate treaty. First, even if countries can commit to emission quantities and investments are selfish, countries tend to invest too little, particularly for short-term agreements. Second, the optimal treaty should be tougher if it is short-term and, third, it should be tougher close to its expiration date. Finally, efficiency is achieved by long-term agreements that are renegotiated over time. In other words, when negotiating a new treaty, it is better if the default outcome is some existing treaty rather than the noncooperative outcome. This suggest that climate negotiators have something to learn from international trade policy negotiators, since trade agreements are typically long-lasting, although they can expand or be renegotiated over time.

While this paper is more general and emphasizes the benefits of renegotiation, my companion paper, Harstad (2010), assumes quadratic utilities and goes further when studying short-term agreements, whether such an agreement is valuable, and what the optimal agreement length should be. Furthermore, that paper shows that domestic holdup problems interact with the international one, and that the optimal climate treaty design depends on existing R&D policies, and vice versa.

The next section clarifies the paper's contribution to the literature on dynamic games and incomplete contracts. The model is presented in Section 3. When solving the model in Section 4, I gradually increase the possibilities for negotiations and contracts by analyzing (i) no cooperation, (ii) one-period contracts, (iii) multi-period contracts, and (iii) contracts permitting renegotiation. Section 5 allows for technological spillovers and Section 6 discusses other extensions and generalizations. Section 7 concludes, while the appendix contains all proofs.

2. Contributions to the Literature

By developing a dynamic (difference) game permitting incomplete contracts, the paper contributes to the literature on both of these fields.

2.1. Dynamic Games

The private provision of public goods is often studied in differential games (or a difference game, if time is discrete) where each player's action influences the future stock or state parameter.[1] Given the emphasis on stocks, the natural equilibrium concept is Markov perfect equilibrium.[2] As in this paper, the typical conclusion is that public goods (bads) are underprovided (overprovided).[3]

Differential games are, however, often difficult to analyze. This has several implications. First, many authors restrict attention to linear-quadratic functional forms.[4] Second, while some papers arbitrarily select the linear MPE (e.g., Fershtman and Nitzan, 1991), typically there are multiple equilibria (Wirl, 1996; Tutsui and Mino, 1990). Consequently, many scholars, like Dutta and Radner (2009), manage to construct more efficient nonlinear MPEs.[5] Third, few bother complicating their model further by adding investments in technologies. One exception is Dutta and Radner (2004), who do explicitly add investments in technology. But since the cost of pollution (as well as the cost of R&D) is assumed to be linear, the equilibrium is "bang-bang" where countries invest either zero or maximally in the first period, and never thereafter.

The first contribution of this paper is the development of a tractable model that can be used to analyze investments as well as emissions. By assuming that technology has a linear cost and an additive impact, I find that the continuation values must be linear in all the $n+1$ stocks, permitting only a single MPE. This trick sharpens the predictions and simplifies the model tremendously. Potentially, this trick can also be applied when studying other economic problems. In the literature on industry dynamics, for example, analytical solutions are rare and numerical simulations necessary.[6]

[1] Thus, such games are subclasses of stochastic games. For overviews, see Başar and Olsder (1999) or Dockner et al. (2000).

[2] In experiments, players tend toward Markov perfect strategies rather than supporting the best subgame perfect equilibrium (Battaglini et al., 2010).

[3] This follows if private provisions are strategic substitutes (as in Fershtman and Nitzan, 1991, and Levhari and Mirman, 1980). If they were complements, e.g., due to a discrete public project, efficiency is more easily obtained (Marx and Matthews, 2000).

[4] For a comprehensive overview, see Engwerda (2005).

[5] See also Dockner and Long (1993), Dockner and Sorger (1996), and Sorger (1998).

[6] See the survey by Doraszelski and Pakes (2007). A firm typically overinvests in capacity to get a competitive advantage. While Reynolds (1987) restricts attention to the linear MPE in a linear-quadratic model, simple two-stage games are used by d'Aspremont and Jacquemin (1988) to discuss the benefits of

My second contribution, made possible by the first, is to incorporate incomplete contracts in dynamic games. Few papers allow for policies or negotiation in stochastic games.[7] In Battaglini and Coate (2007), legislators negotiate spending on "pork" and a long-lasting public good. The equilibrium public-good level is suboptimally but strategically low to discourage future coalitions from wasting money on pork. This mechanism relies on majority rule, however, and the contract incompleteness is related to future policies rather than current investments.

2.2. Contract Theory

By permitting contracts on emissions but not on investments, this paper is in line with the literature on incomplete contracts (e.g., Hart and Moore, 1988). Since I assume investments are selfish in that they affect only the investor's technology stock, contracting on quantity would implement efficiency if there were only one period, or if the contract lasted forever. However, if the countries cannot commit to the end of time, I find that investments are lower if the contract length is short, and that investments decrease toward the end of a contract. To encourage more investments, the optimal and equilibrium contract is tougher to comply with if the contract is short-term or close to its expiration date, particularly if the technology is long-lasting compared to the length of the agreement. These results have not been detected earlier, to the best of my knowledge.

In other dynamic settings, hold-up problems may be solved if the parties can invest while negotiating and agreements can be made only once (Che and Sakovics, 2004), or if there are multiple equilibria in the continuation game (Evans, 2008). Neither requirement is met in this paper, however.

The results hold also if renegotiation is permitted. When renegotiation is possible, moral hazard problems are often expected to worsen (Fudenberg and Tirole, 1990). But Chung (1991) and Aghion et al. (1994) have shown how the initial contract can provide

cooperation and by Gatsios and Karp (1992) to show that firms may invest *more* if they anticipate future merger negotiations. When allowing negotiations on price, but not on investments, in a more general setting, Fershtman and Pakes (2000) use numerical analysis.

[7] For example, Hoel (1993) studies a differential game with an emission tax, Yanase (2006) derives the optimal contribution subsidy, Houba et al. (2000) analyze negotiations over (fish) quotas lasting forever, while Sorger (2006) studies one-period agreements. Although Ploeg and de Zeeuw (1992) even allow for R&D, contracts are complete or absent in all these papers.

incentives by affecting the bargaining position associated with particular investments.[8] While these models have only one period, Guriev and Kvasov (2005) present a dynamic moral hazard problem emphasizing the termination time. Their contract is renegotiated at every point in time, to keep the remaining time horizon constant. Contribution levels are not negotiated, but contracting on time is quite similar to contracting on quantity, as studied by Edlin and Reichelstein (1996): to increase investments, Guriev and Kvasov let the contract length increase, while Edlin and Reichelstein let the contracted quantity increase. In this paper, agents can contract on quantity (of emissions) as well as on time, which permits the study of how the two interact. I also allow an arbitrary number of agents, in contrast to the buyer-seller situations in these papers.

3. The Model

3.1. Stocks and Preferences

This section presents a game where a set of $N \equiv \{1, ..., n\}$ agents contribute over time to a public bad while they also invest in technology. The public bad is represented by the stock G. Allowing for a more or less long-lasting stock, let $1 - q_G \in [0, 1]$ measure the fraction of G that "depreciates" from one period to the next. The stock G may nevertheless increase, depending on the contribution or "emission" level g_i from agent $i \in N$:

$$G = q_G G_- + \sum_N g_i. \tag{3.1}$$

Parameter G_- represents the level of the public bad left from the previous period; subscripts for periods are thus skipped.

Each agent $i \in N$ benefits privately from emitting g_i. For example, if G measures the level of greenhouse gases, g_i is fossil-fuel consumption by country i. As an alternative to fossil fuel, i may consume renewable energy. Let the technology stock R_i measure how much energy i can produce using its renewable energy sources. Thus, R_i can be interpreted as the capacity of the "windmill park" in country i. The stock R_i might also

[8] Segal and Whinston (2002) generalize many related models.

depreciate over time, at the rate $1 - q_R \in [0, 1]$. Each "windmill" costs K units, and r_i measures how much i invests in its technology stock. Thus, if $R_{i,-}$ measures i's technology stock in the previous period, its current technology is given by:

$$R_i = q_R R_{i,-} + r_i. \tag{3.2}$$

Since the technology can generate R_i units of energy, the total amount consumed by i is given by

$$y_i = g_i + R_i. \tag{3.3}$$

As an alternative interpretation, R_i may measure i's "abatement technology," i.e., the amount by which i can at no cost reduce (or clean) its potential emissions. If energy production, measured by y_i, is otherwise polluting, the actual emission level of country i is given by $g_i = y_i - R_i$, which again implies equality (3.3). For either interpretation, i's technology provides a private substitute to contributing to the public bad.

The investment stages and the pollution stages alternate over time. Define "a period" to be such that the countries first simultaneously invest in technology, after which they simultaneously decide how much to emit (see Figure 1).

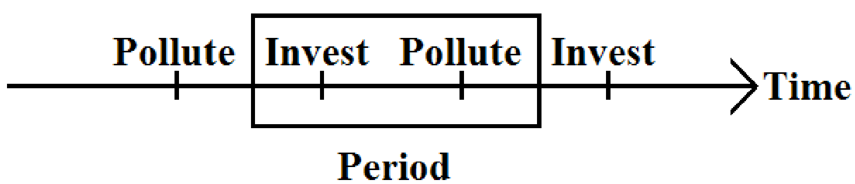

Figure 1: The definition of a period

Let the benefit of consumption be given by the increasing and concave function $B(y_i)$. If $C(G)$ is an increasing convex function representing each country's cost of the public bad, i's utility in a period is:

$$u_i = B(y_i) - C(G) - Kr_i.$$

Country i's objective is to maximize the present-discounted value of its future utilities,

$$U_{i,t} = \sum_{\tau=t}^{\infty} u_{i,\tau} \delta^{\tau - t},$$

where δ is the common discount factor and $U_{i,t}$ is i's continuation value as measured at the start of period t. As mentioned, subscripts denoting period t are typically skipped when this is not confusing.

For alternative applications, one could interpret $-G$ as a public *good* and $-g_i$ as i's contribution. The marginal benefit of the public good is then $C' > 0$, but the private marginal cost of contributing to the public good is $B'(R_i - (-g_i)) > 0$. Naturally, this marginal cost increases in the contribution level $-g_i$, but declines in the (cost-reducing) technology R_i. Sections 5 and 6 discuss how the model can be extended, and the results survive, if we allow for technological spillovers, uncertainty, and heterogeneity.

3.2. The Equilibrium Concept

As in most stochastic games, attention is restricted to Markov perfect equilibria (MPEs) where strategies are conditioned on the physical stocks only. As in Maskin and Tirole (2001), I look for the coarsest set of such strategies. Maskin and Tirole (2001: 192-3) defend MPEs since they are "often quite successful in eliminating or reducing a large multiplicity of equilibria," and they "prescribe the simplest form of behavior that is consistent with rationality" while capturing the fact that "bygones are bygones more completely than does the concept of subgame-perfect equilibrium." In this model, the MPE turns out to be unique and coinciding with the unique subgame-perfect equilibrium if time were finite and approaching infinity. This result is desirable; in fact, Fudenberg and Tirole (1991: 533) have suggested that "one might require infinite-horizon MPE to be limits of finite-horizon MPE."

If the agents are negotiating a contract, I assume the outcome is efficient and symmetric *if* the payoff-relevant variables are symmetric across agents. These assumptions are weak and satisfied in several situations. For example, we could rely on cooperative solution concepts, such as the Nash Bargaining Solution (with or without side transfers). Alternatively, consider a noncooperative bargaining game where one agent can make a take-it-or-leave-it offer to the others, and side transfers are feasible. If every agent has the same chance of being recognized as the proposal-maker, the equilibrium contract is exactly as described below.

All countries participate in the contract in equilibrium, since there is no stage at which they can commit to not negotiating with the others.

4. Analysis

For future reference, the first-best emission level g_i^* ex post (taking the stocks $R_1, ..., R_n$ and G_- as given) equalizes the private marginal benefit of consumption to the social cost of pollution:

$$B' = n\left(C' - \delta U_G\right) > 0, \text{ where} \tag{4.1}$$
$$B' \equiv \partial B\left(g_i^* + R_i\right)/\partial g_i,\ C' \equiv \partial C\left(G\right)/\partial G,\ U_G = -q_G\left(1 - \delta q_R\right)K/n.$$

Implicitly, the g_i^*s are functions of G_- and $\{R_1, ...R_n\}$. The first-best investment level equalizes the marginal benefit to the marginal cost, recognizing that more investments today reduce the need to invest in the next period:

$$B'\left(g_i + R_i^*\right) = \left(1 - \delta q_R\right)K. \tag{4.2}$$

By substituting (4.2) in (4.1), we find the first-best public bad level:

$$C'\left(G\right) = \left(1 - \delta q_G\right)\left(1 - \delta q_R\right)K. \tag{4.3}$$

Combined with (3.1), equation (4.3) pins down $\sum_N g_i$. Since (4.2) implies that y_i is the same across the is, then, when investments are efficient, we can write the first-best emission level as:

$$\begin{aligned} g_i^{fb} &= \frac{1}{n}\left[C'^{-1}\left(\left[1 - \delta q_G\right]\left[1 - \delta q_R\right]K\right) - q_G G_- + R^*\right] - R_i^*, \text{ where} \\ R^* &\equiv \sum_N R_i^*. \end{aligned} \tag{4.4}$$

Given the g_is, (4.2) determines the first-best R_i^*s which, with (3.2), determine the first-best r_is. Throughout the analysis, I assume that $g_i \geq 0$ and $r_i \geq 0$ never bind.[9]

[9] In every equilibrium considered below, $g_i > 0$ and $r_i > 0$ always hold. Thus, it can be verified in retrospect that the constraints will never bind.

4.1. The Noncooperative Outcome

In principle, the continuation value U_i is a function of the $n+1$ stocks G_- and $\{R_1, ..., R_n\}$. However, note that choosing g_i is equivalent to choosing y_i, once the R_is are sunk. Substituting (3.3) into (3.1), we get:

$$G = q_G G_- + \sum_N y_i - R. \tag{4.5}$$

This way, the R_is are eliminated from the model: they are *payoff-irrelevant* as long as $R \equiv \sum_N R_i$ is given, and i's Markov perfect strategy for y_i is thus not conditioned on them.[10] A country's continuation value U_i is thus a function of G_- and R_-, not $R_{i,-} - R_{j,-}$, and we can therefore write it as $U(G_-, R_-)$, without the subscript i.

Because of the linear investment cost, it turns out that the continuation value U must be linear in *both* payoff-relevant stocks, even though u_i is nonlinear in G. This linearity makes the model tractable and simple to work with. Furthermore, the linearity permits only one equilibrium.

PROPOSITION 1. Equilibrium properties:
(i) *There is a unique symmetric MPE.*
(ii) *The equilibrium is in stationary strategies.*
(iii) *The continuation value* $U_i(G, R_1, ...R_n) = U(G, R)$ *is linear in the stocks, with:*

$$\begin{aligned} U_R &= q_R K/n \text{ and} \\ U_G &= -q_G (1 - \delta q_R) K/n. \end{aligned} \tag{4.6}$$

Proposition 1, along with the other results, is proven in the appendix.[11] The rest of this section describes the equilibrium in more detail.

[10] This follows from the definition by Maskin and Tirole (2001, p. 202), where Markov strategies are measurable with respect to the coarsest partition of histories consistent with rationality.

[11] As the proposition states, this is the unique *symmetric* MPE. Since the investment cost is linear, there also exist asymmetric MPEs in which the countries invest different amounts. Asymmetric equilibria may not be reasonable when countries are homogeneous, and they would cease to exist if the investment cost were convex.

At the emission stage, when the technologies are sunk, i solves

$$\max_{y_i} B(y_i) - C(G) + \delta U(G,R) \text{ s.t. } (4.5) \Rightarrow$$
$$B'(y_i) = C' - \delta U_G. \tag{4.7}$$

First, note that each country pollutes too much compared to the first best (4.1). The marginal benefit of polluting, $B'(g_i + R_i)$, decreases in g_i and it can be interpreted as the shadow value of polluting one more unit, fixing the total level of emission. Thus, B' would be the equilibrium permit price if the emission quotas were tradable across the countries (allowing for such trade would not alter the results). In the noncooperative equilibrium, each country limits its emission (since $B' > 0$), but it internalizes only $1/n$ of the total harm.

Second, (4.7) verifies that each i chooses the same y_i, no matter the R_is. While perhaps surprising at first, the intuition is straightforward. Every country has the same preference for (and marginal benefit from) consuming y_i, and the *marginal* cost, through G, is also the same for every country: one *additional* consumed unit generates one unit of public bad.[12]

Substituting (4.5) in (4.7) implies that a larger R must increase every y_i. This implies that if R_i increases but R_j, $j \neq i$, is constant, then $g_j = y_j - R_j$ must increase. Furthermore, substituting (3.3) in (4.7) implies that if R_i increases, g_i must decrease. In sum, if country i has better technology, i pollutes less but (because of this) other countries pollute more. In addition, in the next period all countries invest less. Clearly, these effects discourage countries from investing.

PROPOSITION 2. Investments:

(i) *Even if past investments differed, every $i \in N$ consumes the same:*

$$y_i^{no} = y_j^{no} \; \forall i, j \in \{1, ...n\} \forall R_i, R_j. \tag{4.8}$$

(ii) *If i invests more, i pollutes less but $j \neq i$ pollutes more, and everyone invests less the*

[12] This follows from (3.3), and would not necessarily be true if I instead had focused on technologies that reduced the emission *content* of *each* produced unit (e.g., $g_i = y_i/R_i$). The additive form (3.3) is chosen - not only because it simplifies the analysis tremendously - but also because the resulting crowding-out effects might be reasonable in reality.

following period:

$$\partial g_i^{no}/\partial R_i = -\frac{C''(n-1)-B''}{nC''-B''} < 0, \tag{4.9}$$

$$\partial g_i^{no}/\partial R_j = \frac{C''}{nC''-B''} > 0 \; \forall j \neq i, \tag{4.10}$$

$$\partial r_i^{no}/\partial R_- = -q_R/n. \tag{4.11}$$

(iii) *Consequently, investments are too low, compared to the first best.*

Results (i)-(ii) mean that a country's technology stock is, in effect, a public good. A larger R_i raises every country's consumption and reduces every investment in the following period. Since i captures only $1/n$ of the benefits, i invests less than optimally.

At the emission stage, as already noted, a country consumes too much since it does not take into account the harm imposed on the other countries. In addition, the appendix shows that, in equilibrium, r_i increases in G_-. Anticipating this, a country may want to pollute a lot in order to induce the other countries to invest more in the next period.[13]

Proposition 3. Emissions and consumption:
(i) *If i pollutes more, every $j \in N$ invests more in the following period:*

$$\partial r_i^{no}/\partial G_- = q_G/n. \tag{4.12}$$

(ii) *Emission levels are too large compared to the first best.*
(iii) *Nevertheless, the equilibrium consumption level y_i is lower than it would be in the first best.*

Part (iii) states that the reduction in r_i always dominates the increase in g_i, such that the consumption level $y_i = g_i + R_i$ is always less in the noncooperative equilibrium than the first-best level of y_i. With these dynamic effects, this common-pool problem is more severe than its static counterpart (or than the open-loop equilibrium).[14]

[13] Adding to the public good $-G$ (by reducing g_i) or to R (by increasing r_i) has somewhat similar effects. However, they are not equivalent since a larger r_i reduces emissions in *every* future period. Increasing r_i thus has a longer-lasting impact than reducing g_i, which is why r_i is referred to as an investment. Moreover, the next sections let g_i be contractible but not r_i.

[14] This is also the case in Ploeg and de Zeeuw (1991), for example, and it is verified in experiments by Battaglini et al. (2010).

4.2. Negotiations and Incomplete Contracts

From now on, I let the countries negotiate and contract on their contributions to the public bad. Whether a country i complies by reducing its current consumption or by investing in a more long-term solution is up to country i. The other countries may, in any case, find it hard to verify which course was chosen.

The model can (and will) be used to analyze agreements of any length. In this subsection, countries negotiate and contract in the beginning of each period. Thus, the period length is defined by the contract length. Obviously, each period and contract can be arbitrarily long, since I have not specified the level of the discount factor, for example.

In each period, the timing is the following. First, the countries negotiate a vector of contribution levels g_i. Thereafter, each country sets r_i and, finally, every country complies with the contract. As mentioned, I assume the bargaining outcome is efficient and symmetric *if* the game itself is symmetric. If negotiations fail, the countries play noncooperatively.

The bargaining game is indeed symmetric, even if $R_{i,-}$ differs across the countries. Just as in Section 4.1, the $R_{i,-}$s are eliminated from the model and the continuation value is a function of only G_- and $R_- \equiv \sum_N R_{i,-}$. Moreover, the linear investment cost implies that U must be linear in both stocks, pinning down a unique equilibrium. In fact, the equilibrium properties simplifying the analysis above continue to hold with incomplete contracts. In particular, there is a unique MPE and the continuation value is linear, with the same slopes as before.

PROPOSITION 4. Equilibrium properties (Proposition 1 continues to hold):
(i) *There is a unique MPE.*
(ii) *The equilibrium is in stationary strategies.*
(iii) *The continuation value* $U_i(G, R_1, ...R_n) = U(G, R)$ *is linear in the stocks, with:*

$$\begin{aligned} U_R &= q_R K/n \text{ and} \\ U_G &= -q_G(1-\delta q_R)K/n. \end{aligned}$$

When investing, $i \in N$ prefers a larger stock of technology if its quota, g_i^{co}, is small, since otherwise its consumption level would be very low. Consequently, r_i decreases in g_i^{co}. For a given g_i^{co}, the investment level r_i increases until R_i satisfies:

$$B'\left(g_i^{co} + R_i^{co}\right) = K - \delta U_R = K\left(1 - \delta q_R / n\right). \tag{4.13}$$

In contrast to the noncooperative game, R_i is no longer a public good: once the emission levels are pinned down, i's investment increases y_i but not y_j, $j \neq i$. However, the technology that survives to the next period, $q_R R_i$, does become a public good, since, for a fixed R, the continuation value at the start of every period is independent of R_i. Intuitively, if the agreement does not last forever, a country anticipates that good technology will worsen its bargaining position in the future, once a new agreement is to be negotiated. At that stage, good technology leads to a lower $g_{i,+}$ since the other countries can hold i up when it is cheap for i to reduce its emissions.[15] In fact, $y_{i,+}$ is going to be the same across the is, no matter what the differences are in the R_is. This discourages i from investing now, particularly if the current agreement is relatively short (δ large), the technology likely to survive (q_R large), and the number of countries n large. Thus, compared to the first best (4.2), countries still underinvest if $\delta q_R > 0$.

PROPOSITION 5. Investments:

(i) *Country $i \in N$ invests more if the contract is tough: $\partial r_i / \partial g_i^{co} = -1$.*

(ii) *Nevertheless, for any given g_i^{co}, i underinvests if the agreement is relatively short-lasting ($\delta > 0$) while the technology long-lasting ($q_R > 0$).*

Thus, if δ and q_R are large, it is important to encourage more investments. On the one hand, this can be achieved by a small g_i^{co}. On the other, the ex-post optimal g_i^{co} is larger when equilibrium investments are low. The optimal g_i^{co}s must trade off these concerns. As shown in the appendix, the equilibrium and optimal g_i^{co}s must satisfy (4.4): the equilibrium quotas are identical to the first-best levels![16]

[15] Or, if no agreement is expected in the future, a large $R_{i,+}$ reduces $g_{i,+}$ and increases $g_{j,+}$, as proven in Section 4.1.

[16] Technically, the reason is that y_i is in equilibrium independent of g_i^{co}, since $\partial R_i / \partial g_i^{co} = -1$. Thus, the marginal costs and benefits of increasing g_i^{co} have the same levels as in the first-best scenario, in which the effect on y_i can be ignored using the envelope theorem.

However, since (4.13) implies that the equilibrium R_i^{co}s are less than optimal, the g_i^{lt}s are suboptimally low *ex post*. Combining (4.3) and (4.13) gives

$$B' = n\left(C' - \delta U_G\right) + \left(1 - 1/n\right) K \delta q_R. \tag{4.14}$$

Not only is the shadow value of polluting, B', larger than in the noncooperative case, but it is even larger than it would be in the first best, (4.1). For a fixed investment level, optimally g_i^{co} should have satisfied $B' = n\left(C' - \delta U_G\right)$ rather than (4.14). Only then would marginal costs and benefits be equalized. Relative to this *ex-post* optimal level, the g_i^{co} satisfying (4.14) must be *lower* since $B' - n\left(C' - \delta U_G\right)$ decreases in g_i^{co}. If n, q_R, and δ are large, the additional term $\left(1 - 1/n\right) K \delta q_R$ is large, and g_i^{co} must decline. This makes the contract more demanding or *tougher* to satisfy at the emission stage, compared to what is ex post optimal. The purpose of committing to such an overambitious agreement is to encourage investments, since these are suboptimally low when n, q_R, and δ are large.

PROPOSITION 6. The equilibrium contract:
(i) *The contracted emission levels are equal to the levels at the first best (4.4).*
(ii) *But the emission levels are lower than what is* ex post *optimal (4.1) if the agreement is short-term ($\delta > 0$) while the technology is long-lasting ($q_R > 0$).*

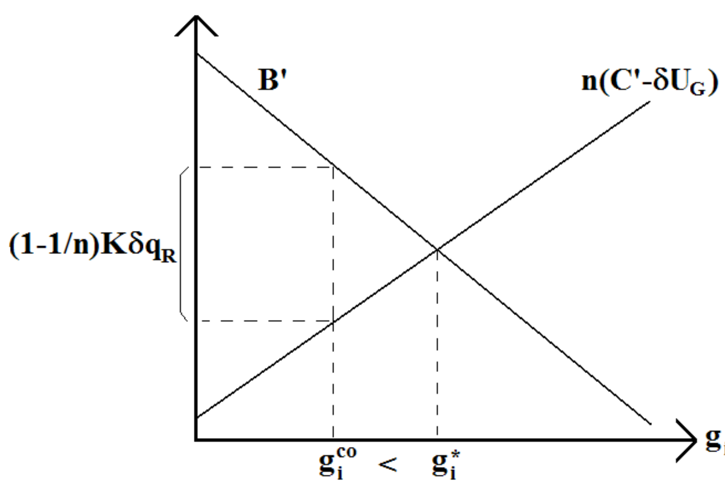

Figure 2: The shorter the agreement, the lower is the contracted emission level relative to the ex post optimum

Figure 2 illustrates the main result: if the length of the agreement is relatively short, δ and q_R are large, and the quotas should be much smaller relative to the emission levels that are ex post optimal. For example, suppose $B(y_i) = -b(\overline{y} - y_i)^2/2$ and $C(G) = cG^2/2$. If g_i^* measures the ex-post optimal pollution level, conditioned on equilibrium investment levels, then:

$$g_i^{co} = g_i^* - \frac{\delta q_R K(1-1/n)}{b+cn^2}.$$

On the other hand, if $\delta q_R = 0$, the right-hand side of (4.14) is zero, meaning that the commitments under the best long-term agreement also maximize the expected utility ex post. In this case, the countries are not concerned with how current technologies affect future bargaining power, either because the existing agreement is lasting forever ($\delta = 0$), or because the technology will not survive the length of the contract ($q_R = 0$). Investments are first best and there is no need to distort the g_i^{co}s downwards.

4.3. Multiperiod Contracts

Assume now that at the beginning of period 1, the countries negotiate the $g_{i,t}$s for every period $t \in \{1, 2, ..., T\}$. The time horizon T may be limited by the countries' ability to commit to future promises.

Just as before, the payoff-relevant stocks at the start of period 1 are G_- and R_- only. Once again, this simplifies the analysis. There is a unique MPE, the continuation value at the start of period 1 (and $T+1$) is linear in the stocks, and has the same slopes as before.

Proposition 7. Equilibrium properties (Proposition 1 continues to hold):

(i) *There is a unique MPE.*

(ii) *The equilibrium is in stationary strategies.*

(iii) *The continuation value* $U_i(G, R_1, ...R_n) = U(G, R)$ *is linear in the stocks, with:*

$$\begin{aligned} U_R &= q_R K/n \text{ and} \\ U_G &= -q_G(1-\delta q_R)K/n. \end{aligned}$$

When investing in period $t \in \{1, 2, ..., T\}$, countries take the $g_{i,t}$s as given, and the continuation value in period $T+1$ is $U(G_T, R_T)$. At the last investment stage, i's problem is the same as in Section 3.2 and i invests until (4.13) holds. Anticipating this, i can invest less in period T by investing more in period $T-1$. The net investment cost is thus $K(1-\delta q_R)$. The same logic applies to every previous period and, in equilibrium,

$$B'(g_{i,t}+R_{i,t}) = K(1-\delta q_R) \text{ for } t < T. \tag{4.15}$$

Thus, the incentives to invest are larger earlier than in the last period, given by (4.13). In fact, investments are equal to the first best (4.2) for every $t < T$.[17]

PROPOSITION 8. Investments:

(i) *Investments decrease toward the end of the agreement.*

(ii) *They are socially optimal for $t < T$, but suboptimally low in the last period.*

Intuitively, the countries invest less when future negotiations are coming up because of the hold-up problem, but they invest more (and optimally) if the emission levels are pinned down for the next period as well. All this is anticipated when the countries negotiate the $g_{i,t}$s.

As shown in the appendix, the optimal and equilibrium $g_{i,t}$s must satisfy (4.3) for every $t \leq T$: the equilibrium pollution level is similar to the first-best level, for every period!

In the beginning of the agreement, when $t < T$, the $g_{i,t}$s are *ex post* optimal as well, since the investments are first best. In the last period, however, investments decline and the contracted emission levels are lower than the ex-post optimal levels. In other words, the optimal contract becomes tougher to satisfy toward its end (and the shadow value of polluting, or the permit price, B', increases).

PROPOSITION 9. The equilibrium contract:

[17] Since $R_T < R_{T-1}$, investments may be negative in period T. The condition for when investments are always positive is:

$$\begin{aligned} \sum_N r_i &= R_T - q_R R_{T-1} = B'^{-1}(K(1-\delta q_R/n)) - g^{co}_{i,T} - q_R B'^{-1}(K(1-\delta q_R)) + g^{co}_{i,T-1} \\ &= B'^{-1}(K(1-\delta q_R/n)) - q_R B'^{-1}(K(1-\delta q_R)) \\ &\quad - (1-q_R)(1-q_G)\frac{1}{n}C'^{-1}[(1-\delta q_G)(1-\delta q_R)K] > 0. \end{aligned}$$

(i) *For every period, the contracted emission levels equal the first-best levels (4.4).*

(ii) *Since investments are suboptimally low in the last period, the contract becomes tougher to satisfy toward the end, and emission levels are then too low, relative to the* ex post *optimum (4.1), if* $\delta q_R > 0$.

4.4. Renegotiation

The contracts above are not renegotiation-proof, since they specify emission levels that are less than what is optimal ex post, after the investments are sunk. The countries may thus be tempted to renegotiate the treaty. This section derives equilibria when renegotiation is costless.

Starting with one-period contracts, the timing in each period is the following. First, the countries negotiate the initial commitments, the g_i^{de}s, referred to as "the default." If these negotiations fail, it is natural to assume that the threat point is no agreement.[18] Thereafter, the countries invest. Before carrying out their commitments, the countries get together and renegotiate the g_i^{de}s. Relative to the threat point g_i^{de}, the bargaining surplus is assumed to be split equally in expectation. As before, this bargaining outcome is implemented by, for example, randomly letting one country make a take-it-or-leave-it offer regarding quantities and transfers.

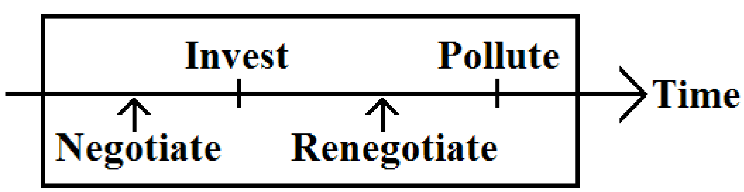

Figure 3: The timing when renegotiation is possible

At the start of each period, any difference in technology is payoff-irrelevant and the continuation value is a function of G_- and R_- only, just as before. Moreover, this continuation value is linear in the stocks, leading to a unique equilibrium. For this game also, the earlier appealing equilibrium properties continue to hold.

[18] However, if the threat point were "short-term" agreements, negotiated after the investment stage, the outcome would be identical.

PROPOSITION 10. Equilibrium properties (Proposition 1 continues to hold):

(i) *There is a unique MPE.*

(ii) *The equilibrium is in stationary strategies.*

(iii) *The continuation value* $U_i(G, R_1, ...R_n) = U(G, R)$ *is linear in the stocks, with:*

$$\begin{aligned} U_R &= q_R K/n \text{ and} \\ U_G &= -q_G(1 - \delta q_R) K/n. \end{aligned}$$

Renegotiation ensures that emission levels are ex post optimal, in contrast to the contracts discussed above. When investing, a country anticipates that it will not, in the end, have to comply with an overambitious contract. Will this jeopardize the incentives to invest?

Not necessarily. When renegotiating an *ambitious* agreement, countries that have invested little are desperate to reach a new agreement that would replace the tough initial commitments. Such countries have a poor bargaining position, and so they will, in equilibrium, compensate the others for relaxing the quotas. Fearing this, all countries are induced to invest more, particularly if the default emission levels are small.

PROPOSITION 11. Investments:

Country i's investment level r_i *decreases in the initial quota* g_i^{de}*.*

This is anticipated when negotiating the initial agreement, the g_i^{de}s. The more ambitious this agreement is, the more the countries invest. This is desirable if the countries are otherwise tempted to underinvest. Thus, the agreement should be more ambitious if δ and q_R are large. Since the investments are influenced by the initial agreement, the g_i^{de}s can always be set such that the investments are first best. In any case, the emission levels remain optimal, thanks to renegotiation. In sum, the optimal and equilibrium contract implements the first best.

PROPOSITION 12. The equilibrium contract:

(i) *The initial contract satisfies (4.16), and it is thus tougher if it is relatively short-term (*δ *large) while the technology is long-lasting (*q_R *large).*

(ii) *In equilibrium, all investments and emissions are first best.*

$$
\begin{aligned}
B'\left(g_i^{de}+R_i^*\right) = K &\Rightarrow \qquad (4.16)\\
g_i^{de} &= g_i^* - \left[B'^{-1}\left(K\left[1-\delta q_R\right]\right) - B'^{-1}\left(K\right)\right] < g_i^*.
\end{aligned}
$$

To see the second part of (i), note that (4.16) requires that g_i^{de} decreases in δq_R since R_i^* is increasing in δq_R. Intuitively, if the length of the agreement is short, countries fear that more technology today will hurt their bargaining position in the near future. They thus invest less than what is optimal, unless the agreement is more ambitious.

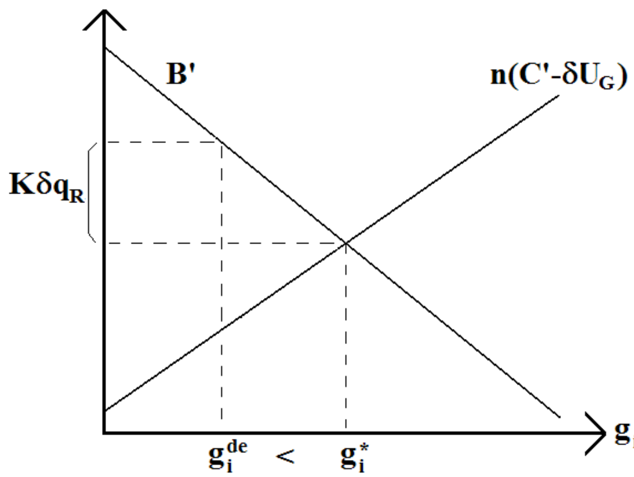

Figure 4: The shorter the agreement, the lower is the contracted emission level relative to the ex post optimum

This result is illustrated in Figure 4, and it confirms the comparative static for the case without renegotiation (Proposition 6).

Compared to the optimal contract without renegotiation, given by (4.14), the initial agreement should be tougher when renegotiation is possible ($g_i^{de} < g_i^{co}$). Intuitively, without renegotiation the contract balances the concern for investments (by reducing g_i^{co}) and for ex-post efficiency (where g_i should be larger). The latter concern is irrelevant when renegotiation ensures ex-post optimality, so the initial contract can be tougher - indeed, so tough that investments are first best.

COROLLARY 1. *The initial contract under renegotiation (4.16) is tougher and specifies lower emission levels than the equilibrium contract when renegotiation is not possible (4.14).*

Implementation:

Note that the equilibrium outcome is observationally equivalent to a time-inconsistency problem where the countries make ambitious plans for the future, while repeatedly backing down from promises made in the past. But rather than reflecting a time-inconsistency problem, this actually leads to the first best.

Corollary 2. *In equilibrium, the countries repeatedly promise to pollute little in the future but when the future arrives, they renege on these promises. This procedure implements the first best.*

Multiple periods:

If the countries can negotiate and commit to a T-period agreement, we know from Section 4.3 that investments (and consumption) are first best in every period - except for the last. Thus, the contracted quantities are also ex-post optimal, and there is no need to renegotiate them. It is only in the last period that the quantities are lower than what is optimal ex post, and only then is there an incentive to renegotiate the contract. When the countries anticipate that the contract will be renegotiated in the last period, they do not need to trade off the concern for ex-post efficiency for the need to encourage investments, and the initial contract can be tougher, and in fact so tough that investments are first best, even in the last period.

Thus, when renegotiation is possible, for every period but the last the optimal and equilibrium initial contract specifies the ex-post optimal quantities, and these are also equal to the first-best quantities since investments are optimal for $t < T$. The initial contract for the last period, $t = T$, is given by (4.16), just as in the one-period contract with renegotiation. As before, the initial contract becomes tougher to satisfy towards its end, since the initial quotas are smaller in the last period than in the earlier periods.[19]

Proposition 13. Multiple periods and renegotiation:

[19] To see this, note that the last-period contract (4.16) can be compared to the earlier (first-best) quantities by writing the latter as

$$B'\left(g_{i,t}^{de} + R_i^*\right) = K\left(1 - \delta q_R\right).$$

(i) *For* $t < T$*, the equilibrium initial contract is given by* $g^{de}_{i,t} = g^*_i$ *but for* $t = T$*, the contract is tougher and given by (4.16).*

(ii) *This contract is renegotiated only after the investment stage in the last period.*

(iii) *The first best is implemented by any* T*-period contract,* $T \geq 1$*, when renegotiation is possible.*

5. Technological Spillovers

In the benchmark case above, investments were selfish. For some applications, however, it might be reasonable that i benefits from j's investments. Coe and Helpman (1995) find that technological spillovers are empirically important, and they let spillovers have an additive impact. Thus, if a larger r_i increases R_i directly by d units, suppose R_j increases by e units, $\forall j \in N \backslash i$. Parameter $e \geq 0$ measures the technological spillover. The total impact of r_i on R is $D \equiv d + e(n-1)$, and we can write:

$$R_i = q_R R_{i,-} + (D - e(n-1)) r_i + \sum_{j \in N \backslash i} e r_j.$$

The appendix solves the model for any e.

In the noncooperative case, the level of e turns out to be irrelevant for investments as well as for consumption. The reason is that R_i is, in any case, a perfect public good, no matter the level of e.

Suppose, next, that countries negotiate emission levels but that renegotiation is not possible. Once the emission levels are pinned down, then a positive e implies that j can consume more if i invests. This externality is a second reason that i invests suboptimally little - in addition to the hold-up problem emphasized so far. The larger e is, the lower the investment levels are, compared to the first-best level. The reduction in investments implies that it is ex post optimal to pollute more. On the other hand, by negotiating smaller emission levels, the countries invest more, and this is beneficial for everyone when the countries invest suboptimally little because of $e > 0$. Balancing these concerns, it turns out that the optimal G and g_i are independent of e, given D. In any case, the equilibrium contract specifies the emission levels given by (4.3)-(4.4), as would have been first best if investments had been efficient.

But since investments are suboptimally low, the negotiated g_is are lower than what is ex post optimal. In the one-period contract analyzed in Section 4.2, the appendix shows that the contracted quotas will satisfy:

$$B' - n\left(C' - \delta U_G\right) = \frac{K}{D}\left(\frac{e(1-\delta q_R)(n-1) + \delta q_R\left(1 - 1/n\right)}{D - e\left(n-1\right)}\right). \qquad (5.1)$$

After the investments are sunk, it would be (ex-post) optimal to pollute more and so much that the left-hand side were equal to zero. Compared to this ex-post optimal level, the agreement should be tougher and more ambitious when e is large. For multiperiod contracts, (5.1) would be satisfied for the last period, when $t = T$. For the earlier periods, $t < T$, the optimal and equilibrium quotas will satisfy:

$$B' - n\left(C' - \delta U_G\right) = \frac{K}{D}\left(\frac{e\left(n-1\right)\left(1-\delta q_R\right)}{D - e\left(n-1\right)}\right). \qquad (5.2)$$

Thus, when the spillover is positive, the optimal agreement is "overambitious" for every period, not only the last.

When renegotiation is possible, the first best is still obtainable by the appropriate initial contract if just $e < D/n$. For one-period contracts, this initial contract should satisfy:

$$B'\left(g_i^{de} + R_i^*\right) = \frac{K}{D - en}. \qquad (5.3)$$

For multiperiod contracts, when $t < T$, the first best is implemented if:

$$B'\left(g_{i,t}^{de} + R_i^*\right) = \frac{K\left(1-\delta q_R\right)}{D - en}. \qquad (5.4)$$

To achieve the first best, note that even the multiperiod contract must be renegotiated in every period when $e > 0$. In that case, a multiperiod contract is overambitious in order to motivate R&D, and for every period it specifies emission levels that are lower than those that are optimal once the investments are sunk. While this encourages investments, it also necessitates renegotiation.

To summarize, for all these cases, a larger spillover implies that the contract should be tougher relative to the level that is ex-post optimal.[20]

[20] For the one-period contract, this finding is also detected by Golombek and Hoel (2005). When renegotiation is possible, a related result is derived for the buyer-seller game in Edlin and Reichelstein (1996). If $e \geq d$, however, the first best can never be implemented, a finding which is in line with Che and Hausch (1999). However, all these contributions limit their attention to one-period models.

Proposition 14. *The larger is the technological spillover* e*, the tougher is the optimal and equilibrium contract relative to the ex post optimum. The* T*-period contract is given by:*

(i) *condition (5.1) for* $t = T$ *if renegotiation is impossible;*

(ii) *condition (5.2) for* $t < T$ *if* $T > 1$ *and renegotiation is impossible;*

(iii) *condition (5.3) for* $t = T$ *if renegotiation is possible;*

(iv) *condition (5.4) for* $t < T$ *if* $T > 1$ *and renegotiation is possible.*

6. Generalizations and Extensions

One purpose of this paper is to present a model that is simple and tractable, despite the complexity of the underlying problem. The results are robust to several generalizations, and the model can fruitfully be expanded in various directions. This section briefly describes some of these extensions.

6.1. Robustness to the Future Regime

When analyzing a particular game, I have so far assumed that the identical game repeats itself after one period or contract has expired. Obviously, the equilibrium in a given period is a function of the future continuation-value function. However, while the derivatives U_R and U_G determine the incentives to invest and to pollute, the *level* of U is irrelevant for these choices. For all the regimes analyzed above, it turned out that U_R and U_G were constant and identical: Proposition 1 continued to hold throughout the analysis. Thus, the equilibrium for a given period or contract is unchanged if, in the next period, the countries play noncooperatively or instead negotiate a contract (of any length, with or without renegotiation).

Corollary 3. *For each period, every equilibrium derived above remains unchanged whether the countries in the next (or any future) period (i) act noncooperatively, (ii) negotiate one-period contracts, (iii) negotiate multiperiod contracts, or (iv) negotiate default contracts that will be renegotiated later.*

6.2. Adding Uncertainty

While the model above is deterministic, certain types of uncertainty leave the results unchanged. Since the continuation values are linear in R, countries are risk-neutral in that it would not matter if, say, the depreciation rate on technology were random, as long as the *expected* depreciation rate is $1-q_R$. The realized depreciation can also be different for every country, as long as the expected depreciation rate is $1-q_R$ for everyone. In addition, the cost of pollution at time t may depend on some state θ_t such that it could be written $C(G_t, \theta_t)$. Again, Propositions 1-14 continue to hold if one simply replaces C' in every expression by the *expected* marginal cost, $\mathrm{E}C'$, where the expectations are taken with respect to the unknown future θ_t. Since $C(.)$ is strictly convex, $\mathrm{E}C'(G_t, \theta_t) > C'(G_t, \mathrm{E}\theta_t)$, and G_t should thus be smaller when θ_t is uncertain. One can thus expect that, for a T-period agreement, the optimal $g_{i,t}$s should decrease in t due to the (increased) uncertainty in θ_t. This strengthens the conclusion that a long-term agreement should become tougher to satisfy over time.

6.3. Endogenizing the Contract Length

Above, I have taken the contract's length, T, to be exogenous. This may be reasonable if T measures the length of time to which the countries can commit. If the countries could choose, they would always prefer T to be as large as possible.

This would no longer be true, however, if uncertainty were added to the model. If θ_t in $C(G_t, \theta_t)$ is stochastic, for example, there is a cost of committing in advance to future emission levels, if these cannot be renegotiated or conditioned on θ_t. In this case, the optimal time horizon may be finite.

Alternatively, one may assume that there is a fixed cost of negotiating every period's emission levels. Since the cost of the hold-up problem is realized only in period T, the present discounted value of this cost is smaller than the fixed negotiation cost if T is large. Thus, such negotiation costs would imply that the optimal T is interior.

Whether T is pinned down by the uncertainty or by the negotiation cost, some comparative static is feasible. For example, if the technology is long-lasting (q_R is large), the hold-up problem in period T is severe and, to delay this cost, the countries may prefer to increase T. In Harstad (2010), I allow for uncertainty and show how the optimal T depends on several parameters.

6.4. Heterogeneity

To provide a benchmark case, it has been assumed that the countries are completely symmetric and there has been no heterogeneity. It did turn out, however, that for a given R_{-}, differences in $R_{i,-}$ (such as $R_{i,-} - R_{j,-}$) were payoff-irrelevant. It is therefore not necessary to assume that all countries start out with the same technology.

Furthermore, some heterogeneity can easily be added to the model. For example, if country i's benefit of consumption is measured relative to some individual bliss point or reference point $\overline{y}_i$, we could write i's benefit as $B(y_i - \overline{y}_i)$. If we define $\widetilde{y}_i \equiv y_i - \overline{y}_i$, every result above holds if y_i is substituted by $\widetilde{y}_i$. While countries with large $\overline{y}_i$ are going to consume more in every equilibrium, i's consumption *relative* to its reference point is going to be constant across the countries.

6.5. Contractible Investments

If the countries negotiated investments but not emission levels, then deriving the best incomplete contract would require an analysis somewhat similar to that above. If both investments and emission levels were contractible, the first best could be implemented trivially, even without renegotiation. One way of implementing the first best is then to subsidize investments across countries. Without subsidies, investments are suboptimally low, particularly if the technology is long-lasting and the contract short-lasting or close to its expiration date. To ensure optimal investments, the subsidy should thus be larger for contracts that have a short time horizon and when they are close to expiring (for details, see Harstad, 2010).

6.6. Participation

In this paper, I have assumed that the agents cannot hide when negotiating a contract. Dixit and Olson (2000), on the other hand, study a two-stage game in which the agents first decide whether to participate in the second cooperative stage. With such a possibility of opting out, many agents prefer to abstain and free-ride. Thus, one might expect less than full participation in the present model as well, if such a stage were added to the model. However, in contrast to Dixit and Olson, the present game has several periods. Thus, if only a few agents decide to participate, they may prefer to contract for fewer periods, hoping that the nonparticipants will turn up later. Since short-term contracts lead to suboptimal investments, this (credible!) threat may discourage agents from considering to free-ride. Thus, participation may be larger than in the two-stage model of Dixit and Olson. Future research should investigate this conjecture - along with many other possible extensions.

7. Conclusions

This paper presents a novel dynamic game in which n agents contribute to a public bad while also investing in substitute technologies. Under the assumption of linear investment costs, the Markov perfect equilibrium (MPE) is unique, the continuation value linear, and the analysis tractable, despite the $n+1$ stocks. While the unique equilibrium rules out self-enforcing agreements, the framework can be employed to analyze incomplete contracts in a dynamic setting.

With only one period, or if the contract lasted forever, contracting on contribution levels would be first best since investments are "selfish" in (most of) the paper. If the agents cannot commit to the end of time, however, investments are suboptimally low, particularly if the contract is short-term or close to expiring. To further motivate investments, the equilibrium and optimal contract is tougher and more ambitious if it is short-lasting or close to the expiration date. If renegotiation is possible, such a contract implements the first best.

While the model and the method are general, the assumptions fit well to the context of climate change, and the results have important consequences for how to design a treaty. First, *even if* the countries can credible commit to emission levels, they will invest too little in renewable energy sources and abatement technology. As a consequence, the climate treaty should be more ambitious compared to what is optimal ex post, after the investments are sunk. In particular, short-term agreements should be more ambitious than long-term agreements, and the agreement should be tougher towards the end.

Currently, the commitments made under the Kyoto Protocol expire in 2012 and the threat point for present negotiations is no agreement at all. This reduces the incentive to invest in new technologies, according to the above results. When the Doha-round trade negotiations broke down, on the other hand, the default outcome was *not* the noncooperative equilibrium but the existing set of long-term trade agreements. Long-lasting agreements permitting renegotiation can implement the first best in the above model. Thus, the procedure used for negotiating trade agreements is more efficient than the one currently used for climate, according to this analysis.

With this application, the paper provides a small step toward a better understanding of how climate treaties interact with the incentives to invest in technologies. The analysis has detected and explored challenges that arise *even if* we abstract from domestic politics, heterogeneity across countries, private information, monitoring, compliance, coalition formation, and the possibility of opting out of the agreement. While the effects discussed in this paper are likely to persist, allowing for such complications will certainly generate several new results and thereby enhance our understanding of the best agreement design. Relaxing these assumptions is thus the natural next step.

8. Appendix

All propositions are here proven allowing for technological spillovers. In Sections 3 and 4, $e = 0$ and $D = d = 1$.

While U_i is the continuation value just before the investment stage, let W_i represent the (interim) continuation value at (or just before) the emission stage. To shorten equations, use $m \equiv -\delta \partial U_i / \partial G_-$, $z \equiv \delta \partial U_i / \partial R_-$, $\widetilde{R} \equiv q_R R_-$ and $\widetilde{G} \equiv q_G G_-$. The proof for the first best (4.1)-(4.3) is omitted since it would follow the same lines as the following proof.

Proofs of Proposition 1-3.

Note that, by substitution,

$$\begin{aligned} G &= q_g G_- + \sum_N y_i - R, \text{ and} \\ u_i &= B(y_i) - C(G) - K r_i. \end{aligned}$$

Thus, all i's are identical w.r.t. y_i and differences in the technology stock do not matter. The game is thus symmetric at the emission stage, no matter differences in R_i.At the investment stage, the game is symmetric, no matter differences in $R_{i,-}$. Analyzing the symmetric equilibrium (where symmetric countries invest identical amounts), I drop the subscript for i on U and W.

At the emission stage, each country's first-order condition for y_i is:

$$\begin{aligned} 0 &= B'(y_i) - C'(G) + \delta U_G(G, R) \\ &= B'(y_i) - C'\left(\widetilde{G} - R + \sum_N y_i\right) + \delta U_G(\widetilde{G} - R + \sum_N y_i, R), \end{aligned} \tag{8.1}$$

implying that all y_is are identical and implicit functions of $\widetilde{G}$ and R only. At the investment stage, i maximizes:

$$W(\widetilde{G}, R) - K r_i = W\left(q_G G_-, \widetilde{R} + \sum_i D r_i\right) - K r_i, \tag{8.2}$$

implying that R is going to be a function of G_-, given implicitly by $\partial W(q_G G_-, R)/\partial R = K/D$ and explicitly by, say, $R(G_-)$. In the symmetric equilibrium, each country invests

$(R(G_-) - q_R R_-)/Dn$. Thus:

$$\begin{aligned} U(G_-, R_-) &= W(q_G G_-, R(G_-)) - K\left(\frac{R(G_-) - q_R R_-}{Dn}\right) \Rightarrow \\ z/\delta &\equiv \frac{\partial U}{\partial R_-} = \frac{q_R K}{Dn} \end{aligned} \tag{8.3}$$

in every period. Hence, $U_{RG} = U_{GR} = 0$, m and U_G cannot be functions of R and (8.1) implies that y_i, G and thus $B(y_i) - C(G) \equiv \gamma(.)$ are functions of $\widetilde{G} - R$ only. Hence, write $G\left(\widetilde{G} - R\right)$. Rewriting (8.2) gives

$$\gamma(q_G G_- - R) + \delta U(G(q_G G_- - R), R) - K r_i$$

and because U_R is a deterministic constant, maximizing this payoff w.r.t. r_i makes $q_G G_- - R$ a constant, say ξ. This gives $\partial r_i / \partial G_- = q_G / Dn$ and U becomes:

$$\begin{aligned} U(G_-, R_-) &= \gamma(\xi) - Kr + \delta U(G(\xi), R) \\ &= \gamma(\xi) - K\left(\frac{q_G G_- - \xi - q_R R_-}{Dn}\right) + \delta U(G(\xi), q_G G_- - \xi) \Rightarrow \\ m/\delta &= \partial U / \partial G_- = -K\left(\frac{q_G}{Dn}\right) + \delta U_R q_G = -\frac{K q_G}{Dn}(1 - \delta q_R), \end{aligned} \tag{8.4}$$

since $G(\xi + \theta)$ and $\gamma(.)$ are *not* functions of G_- when $q_G G_- - R = \xi$.

Since U_G is a constant, (8.1) implies that if R increases, y_i increases but G must decrease. Thus, $\partial y_i / \partial R \in (0, 1)$, so $\partial g_i / \partial R_j = \partial(y_i - R_i)/\partial R_j > 0$ if $i \neq j$ and < 0 if $i = j$. More precisely, differentiating (8.1) w.r.t. R or R_i gives:

$$\begin{aligned} & B'' \frac{dy_i}{dR} - C'' \frac{n dy_i - dR}{dR} = 0 \Rightarrow \\ \frac{dy_i}{dR} &= \frac{C''}{nC'' - B''} \Rightarrow \\ \frac{dg_i}{dR_i} &= \frac{C''}{nC'' - B''} - 1 = -\frac{C''(n-1) - B''}{nC'' - B''} < 0, \\ \frac{dg_i}{dR_j} &= \frac{C''}{nC'' - B''} > 0, j \neq i. \end{aligned}$$

Since $q_G G_- - R$ is a constant, when investments are symmetric (4.11)-(4.12) follow.

The first-order condition for R_i can be written (using (8.1)):

$$B'(y_i)\frac{dy_i}{dR} - [C'(G) - \delta U_G]\left[n\frac{dy_i}{dR} - 1\right] + \delta U_R = K \Rightarrow$$

$$
\begin{aligned}
B'(y_i)\left[\frac{dy_i}{dR}-n\frac{dy_i}{dR}+1\right] &= K-\delta U_R \Rightarrow \\
B'(y_i)\left[\frac{C''-B''}{nC''-B''}\right] &= (1-\delta q_R/n)\,K. \qquad (8.5)
\end{aligned}
$$

For a given y_i, the left-hand side is smaller and the right-hand side larger than the first best (4.2). Thus, y_i must be smaller than the y_i satisfying (4.2). But since i consumes more than optimally *for a given* R, R must be lower than in the first best. Combined, the pollution level G must be larger than in the first best. By combining (8.5) and (8.1), we can write:

$$
C' = B' + \delta U_G = (1-\delta q_R/n)\,K\frac{nC''-B''}{C''-B''} + \delta U_G,
$$

which is clearly larger than the first best (4.3), since U_G is the same in the two cases.

Proofs of Propositions 4-6.

When g_i is already negotiated, i invests until

$$
\begin{aligned}
K &= dB'(g_i+R_i) + Dz \Rightarrow \qquad (8.6)\\
y_i &= B'^{-1}(K/d - Dz/d)\,,\ R_i = B'^{-1}(K/d-Dz/d) - g_i,\\
dr_i &= B'^{-1}(K/d-Dz/d) - g_i - q_R R_{i,-} - \sum_{j\in N\setminus i} er_j. \qquad (8.7)
\end{aligned}
$$

Anticipating this, the utility before investing is:

$$
U_i = B\left(B'^{-1}(K/d-Dz/d)\right) - C(G) - Kr_i + \delta U(G,R)\,.
$$

If the negotiations fail, the default outcome is the noncooperative outcome, giving everyone the same utility. Since the r_is follow from the g_is in (8.7), everyone understands that negotiating the g_is is equivalent to negotiating the r_is. Since all countries have identical preferences w.r.t. the r_is (and their default utility is the same) the r_is are going to be equal for every i. Symmetry requires that r_i, and thus $\zeta \equiv [g_i + q_R R_{i,-}]$, is the same for all countries. Then, (8.7) becomes

$$
Dr_i = B'^{-1}(K/d-Dz/d) - \zeta.
$$

Efficiency requires (f.o.c. of U_i w.r.t. ζ recognizing $g_i = \zeta - q_R R_{i,-}$ and $\partial r_i/\partial\zeta = -1/D\forall i$):

$$
\begin{aligned}
-nC'(G) + K/D + n\delta U_G - nD\delta U_R(1/D) &= 0 \Rightarrow \\
C'(G) + m + zD &= K/Dn. \qquad (8.8)
\end{aligned}
$$

Combined with (8.7), neither G nor R can be functions of R_- (R_i in (8.7) and G in (8.8) are not functions of R_-). Thus, $U_{R_-} = q_R K/Dn$, just as before, and U_G cannot be a function of R (since $U_{RG} = 0$). (8.8) then implies that G is a constant and, since we must have $\zeta = (G - q_G G_-)/n + q_R R_-/n$, (8.7) gives $\partial r_i/\partial G_- = (\partial r_i/\partial g_i)(\partial g_i/\partial \zeta)(\partial \zeta/\partial G_-) = q_G/Dn$. Hence, $U_{G_-} = -q_G K/Dn + \delta U_R q_G = -q_G(1-\delta q_R)K/Dn$, giving a unique equilibrium, (8.3) and (8.4), just as before.

Substituted in (8.8):

$$C'(G) = (1-\delta q_G)(1-\delta q_R)K/Dn. \tag{8.9}$$

This is the same pollution level as in the first best (4.3). At the same time, for a given g_i, equilibrium investments (8.6) are less than the first best investments (4.2). Thus, ex post the marginal benefit of polluting is larger than the marginal cost. Anticipating the ex post small g_i, r_i increases: from (8.7), $\partial r_i/\partial g_i = -1$. Combining (8.9) with (8.6),

$$B'(g_i + R_i - \overline{y}_i)/n - C'(G) - m = \frac{K}{n}\left(\frac{1}{d} - \frac{1}{D}\right) + \frac{\delta q_R K}{Dn}\left(1 - \frac{D}{dn}\right) =$$

$$\frac{K}{Dn}\left(\frac{1}{1-(n-1)e/D} - 1 + \delta q_R\left(\frac{(D-en)(n-1)}{Dn - en(n-1)}\right)\right) = \frac{K}{Dn}\left(\frac{e + \delta q_R(D/n - e)}{D/(n-1) - e}\right).$$

For the quadratic functions, $B' = b(g_i + R_i - \overline{y}_i)$ and $C' = c(q_G G_- + \sum_i g_i)$, so

$$\begin{aligned}(B'/n - C')^{lt} - (B'/n - C')^{*} &= b\left(-g_i^{lt} + g_i^{*}\right)/n - cn\left(g_i^{lt} - g_i^{*}\right) \Rightarrow \\ g_i^{*} - g_i^{lt} &= \frac{K/D}{b + cn^2}\left(\frac{e/D + \delta q_R(1/n - e/D)}{1/(n-1) + e/D}\right).\end{aligned}$$

Proofs of Propositions 7-9.

At the start of $t = 1$, countries negotiate emission levels for every period $t \in \{1, ..., T\}$. The investment level in period T is (8.7) for the same reasons as given above.

Anticipating the equilibrium $R_{i,T}$ (and $R_{j,T}$) i can invest q_R less units in period T for each invested unit in period $T-1$. Thus, in period $T-1$, i invests until:

$$K = dB'(g_{i,T-1} + R_{i,T-1}) + \delta q_R K \Rightarrow \tag{8.10}$$

$$R_{i,T-1} = q_R R_{i,T-1} + dr_{i,T-1} + \sum_{j \in N \setminus i} er_{j,T-1} = B'^{-1}(K(1-\delta q_R)/d) - g_{i,T-1}. \tag{8.11}$$

The same argument applies to every period $T-t$, $t\in\{1,...T-1\}$, and the investment level is given by the analogous equation for each period but T.

In equilibrium, all countries enjoy the same y_i and default utilities. Thus, just as before, they will negotiate the $g_{i,t}$s such that they will all face the same cost of investment in equilibrium. Thus, $r_i=r_j=r$ and

$$Dr=B'^{-1}\left(K\left(1-\delta q_R\right)/d\right)-g_{i,t}-q_R R_{i,t-1}.$$

For every $t\in(1,T)$, $R_{i,t-1}$ is given by the $g_{i,t-1}$ in the previous period (in line with (8.11)). Thus,

$$\begin{aligned} Dr &= B'^{-1}\left(K\left(1-\delta q_R\right)\right)-g_{i,t}-q_R\left(B'^{-1}\left(K\left(1-\delta q_R\right)/d\right)-g_{i,t-1}\right) \\ &= \left(1-q_R\right)B'^{-1}\left(K\left(1-\delta q_R\right)\right)-g_{i,t}+q_R g_{i,t-1}. \end{aligned} \tag{8.12}$$

Since $r_i=r_j$, (8.10) implies that the equilibrium $g_{i,t}+q_R R_{i,t-1}$ is the same (say ς_t) for all is:

$$g_{i,t}+q_R^{t-\tau}R_{i,\tau-1}=\varsigma_t,\ t\in\{1,...,T\}.$$

All countries have the same preferences over the ς_ts. Dynamic efficiency requires that the countries are not better off after a change in the ς_ts (and thus the $g_{i,t}$s), given by $(\Delta\varsigma_t,\Delta\varsigma_{t+1})$, such that G is unchanged after two periods, i.e., $\Delta\varsigma_t q_G=-\Delta\varsigma_{t+1}$, $t\in[1,T-1]$. From (8.12), this implies

$$\begin{aligned} -nC'\left(G_t\right)\Delta\varsigma_t+\Delta g_t K/D+\delta\left(\Delta\varsigma_{t+1}-\Delta g_t q_R\right)K/D-\delta^2\Delta g_{t+1}q_R K/D &\le 0\forall\Delta\varsigma_t\Rightarrow \\ \left(-C'n+K/D-\delta\left(q_G+q_R\right)K/D+\delta^2 q_G d_R K/D\right)\Delta\varsigma_t &\le 0\forall\Delta\varsigma_t\Rightarrow \\ -C'n+\left(1-\delta q_R\right)\left(1-\delta q_G\right)K/Dn &= 0. \end{aligned}$$

Using (8.10),

$$\begin{aligned} & B'-C'\left(G\right)n-nm=\left(1-\delta q_R\right)K/d-\left(1-\delta q_R\right)\left(1-\delta q_G\right)K/D-\delta q_G\left(1-\delta q_R\right)K/D \\ = & \frac{K\left(1-\delta q_R\right)}{d}-\left(1-\delta q_R\right)K/D=\left(\frac{K}{d}-\frac{K}{D}\right)\left(1-\delta q_R\right)=\frac{K}{D}\left(\frac{e/D}{1/\left(n-1\right)-e/D}\right)\left(1-\delta q_R\right). \end{aligned}$$

The $g_{i,T}$ satisfies (8.9) for the same reasons as in the previous proof (and since they do not influence any $R_{i,t}$, $t<T$). It is easy to check that U_R and U_G are the same as before.

Proofs of Propositions 10-12.

In the default outcome, a country's (interim) utility is:

$$W_i^{de} = B\left(g_i^{de} + R_i\right) - C\left(\widetilde{G} + \sum_N g_j^{de}\right) + \delta U.$$

Since i gets $1/n$ of the renegotiation-surplus, in addition, i's utility is:

$$W_i^{de} + \frac{1}{n}\sum_N \left(W_j^{re} - W_j^{de}\right) - Kr_i, \tag{8.13}$$

where W_j^{re} is j's utility after renegotiation. Maximizing the expectation of this expression w.r.t. r_i gives the f.o.c.

$$\begin{aligned} K &= \left[dB'\left(g_i^{de} + R_i - \overline{y}_i\right) + Dz\right](1 - 1/n) + \\ &+ \frac{D}{n}\partial\left(\sum_N W_i^{re}\right)/\partial R - \sum_{j \in N \setminus i} \frac{1}{n}\left[eB'\left(g_j^{de} + R_j\right) + Dz\right]. \end{aligned} \tag{8.14}$$

Clearly, R_i must decrease in g_i^{de}. Requiring first-best investments, $\partial\left(\sum W_i^{re}\right)/\partial R = K/D$, and since $B'\left(g_i^{de} + R_i - \overline{y}_i\right)$ must be the same for all is,

$$K = B'\left(g_i^{de} + R_i^*\right)(d - D/n) + K/n \Rightarrow B'\left(g_i^{de} + R_i^*\right) = \frac{K(n-1)}{dn - D}. \tag{8.15}$$

Combined with the optimum, (4.2),

$$\begin{aligned} B'\left(g_i^{de} + R_i^*\right) - B'\left(g_i^* + R_i^*\right) &= \frac{K(n-1)}{dn - D} - \frac{K}{D}(1 - \delta q_R) \\ &= \frac{K}{D}\left(\frac{e/D}{1/n - e/D} + \delta q_R\right). \end{aligned} \tag{8.16}$$

Since y_i^{de} is the same for every i in equilibrium, the bargaining game (when renegotiating the g_i^{de}s) is symmetric and the renegotiated g_i^{re}s become efficient (just as under short-term agreements). Since the first best is implemented, U_R and U_G are unique and as before.

Proof of Proposition 13.

The fact that Proposition 13 describes one equilibrium is easy to check. Uniqueness is not claimed for this case: while there cannot be inefficient equilibria (since all countries strictly prefer to negotiate an efficient contract), multiple contracts can implement the first best. For example, any contract specifying every $g_{i,t}$, $t \in \{1, ..., T\}$, implements the first best as long as $g_{i,1}$ is equal to g_i^* and the contract can be renegotiated already in the second period. The level of $g_{i,t}$, $t > 1$, is then not important. If $g_{i,t} \neq g_i^*$ and $T > t > 1$, the $g_{i,t}$s will be renegotiated (to g_i^*) before period t.

Proof of Proposition 14.

The proposition follows from those above, since technological spillovers are allowed in all proofs.

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The Dynamics of Climate Agreements†

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Abstract

This paper provides a model in which countries over time pollute as well as invest in technologies (renewable energy sources or abatement technologies). Without a climate treaty, the countries pollute too much and invest too little, partly to induce the others to pollute less and invest more in the future. Nevertheless, short-term agreements on emission levels can reduce welfare, since countries invest less when they anticipate future negotiations. The optimal agreement is tougher and more long-term if intellectual property rights are weak. If the climate agreement happens to be short-term or absent, intellectual property rights should be strengthened, tariffs should decrease, and investments should be subsidized. Thus, subsidizing or liberalizing technological trade is a strategic substitute for tougher climate treaties.

Key words: Climate agreements, green technology, dynamic common pool problems, dynamic hold-up problems, incomplete contracts, contract-length
JEL: Q54, Q55, F55, H87, D86

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1. Introduction

As demonstrated in Copenhagen, December 2009, implementing efficient climate change policies is going to be tremendously difficult. Not only do independent countries benefit privately from contributing to the public bad, but a lasting solution may require investments in new technology. Recent agreements have two distinct characteristics.[1] First, they have focused on emissions but ignored investments, perhaps because investment levels would be hard to verify by third parties. Second, the commitments are relatively short-term, since committing to the far future may be neither feasible nor desirable. How valuable is such an agreement? How does it affect the incentive to invest in technology, and what characterizes the best agreement?

While these questions are central and important, we do not yet have clear answers, or a good framework for deriving them. This paper attempts to make some progress, and it addresses the questions above head-on by isolating the interaction among negotiations, emissions, and investments. I develop a dynamic framework in which countries pollute as well as invest in technology. The technology reduces the need to pollute, and it can be interpreted as either renewable energy sources or abatement technology. While there is a large number of subgame-perfect equilibria, the Markov perfect equilibria (MPEs) are selected since they are simple and robust. With this refinement, the equilibrium turns out to be unique and the analysis tractable, despite the large number of stocks in the model.

Since the MPE is unique, tacit agreements enforced by trigger strategies are not feasible. But in reality, even domestic stakeholders might act as enforcers if the agreement must be ratified by each country. Although I abstract from domestic politics, I vary the countries' possibilities of negotiating, contracting, and committing, and derive the equilibrium outcome for each situation. Since the equilibrium agreement is also the constrained optimum, the results can be interpreted normatively.

To begin with, countries act noncooperatively at all stages. If one country happens to pollute a lot, the other countries are induced to pollute less in the future since the

[1] The two features characterize the current Kyoto Protocol as well as the recent Copenhagen Accord. The Kyoto Protocol specifies emisson reductions for the five-year period 2008-12, while the Copenhagen Accord lists quantified targets for 2020.

problem is then more severe. They will also invest more in technology to be able to afford the necessary cuts in emissions. If a country invests a lot in abatement technology, it can be expected to pollute less in the future. This induces the other countries to increase their emissions and reduce their own investments. Anticipating these effects, each country pollutes more and invests less than it would in an otherwise similar static model. This dynamic common-pool problem is thus particularly severe.

Short-term agreements on immediate emission levels can nevertheless be worse. A hold-up problem arises when the countries negotiate emissions: if one country has better technology and can cut its emissions fairly cheaply, then its opponents may ask it to bear the lion's share of the burden when collective emissions are reduced.[2] Anticipating this, countries invest less when negotiations are coming up. Consequently, everyone is worse off, particularly if the length of the agreement is short and the number of countries large. This dismal result should provoke us to think hard before recommending a particular climate treaty.

Long-term agreements are better at mitigating the hold-up problem. If commitments are negotiated before a country invests, it cannot be held up by the other countries - at least not before the agreement expires. Thus, countries invest more when the agreement is long-term. Nevertheless, countries underinvest compared to the optimum if the agreement does not last forever. To encourage more investments, the best (and equilibrium) agreement is tougher (in that it stipulates lower emissions) than what is optimal ex post, once the investments are sunk. The equilibrium agreement is derived as a function of its length, while the optimal length is shown to depend on the variance of noncontractible shocks, as well as several other parameters.

The comparative statics are important. In most of the paper, investments are assumed to be noncontractible. This generates the international holdup problem, but it may also lead to domestic holdup problems: Since it is difficult to describe the technology in advance, an innovator will have to develop it first, and then hope that the government is

[2] *Financial Times* reports that "Leaders of countries that want concessions say that nations like Denmark have a built-in advantage because they already depend more heavily on renewable energy" (October 17, 2008: A4). Although the Kyoto Protocol aimed for uniform cuts relative to 1990 levels, exceptions were widespread and there is currently no attempt to harmonize cuts.

willing to pay. If the innovator's intellectual property rights are weak, the government will, in equilibrium, pay less. This "domestic" holdup problem interacts with the international holdup problem, and the optimal agreement reflects them both. If intellectual property rights are weak, investments are low, and a further reduction in investments is particularly harmful. The optimal agreement is then tougher and longer-term, while short-term agreements are more likely to be worse than business as usual.

If technology can be traded or subsidized, then high tariffs or low subsidies discourage investments and, to counteract this, the climate treaty should be tougher and more long-term. The optimal climate treaty is thus a function of trade policies, but the reverse is also true: if the climate treaty is relatively short-term, it is more important to strengthen intellectual property rights, reduce tariffs, and increase subsidies on investments. Negotiating such trade policies is thus a strategic substitute for a tough climate agreement.

By analyzing environmental agreements in a dynamic game permitting incomplete contracts, I contribute to three strands of literature.

The literature on climate policy and environmental agreements is growing.[3] It usually emphasizes the positive effects of regulation on technological change,[4] and a typical recommendation is decade-long short-term agreements, partly to ensure flexibility (see, for example, Karp and Zhao, 2009). The present paper, in contrast, shows that short-term agreements reduce the incentive to invest in new technology and can be worse than business as usual, while long-term agreements are better at mitigating hold-up problems. This builds on Buchholtz and Konrad (1994), who first noted that R&D might decrease prior to negotiations.[5] Beccherle and Tirole (2010) have recently generalized my one-period model and shown that anticipating negotiations can have adverse effects also if the countries, instead of investing, sell permits on the forward market, allow banking, or set

[3] See Kolstad and Toman (2005) on climate policy and Barrett (2005) on environmental agreements. Aldy et al. (2003) and Aldy and Stavins (2007) discuss alternative climate agreement designs.

[4] See, e.g., Jaffe et al. (2003), Newell et al. (2006), Golombek and Hoel (2005). Even when investments are made prior to negotiations, Muuls (2009) finds that investments increase when the negotiations are anticipated. Hoel and de Zeeuw (2009), in contrast, show that R&D can decrease if countries cooperate because they then reduce pollution even without new technology, although there is no negotiation in their model and their analysis hinges on a "breakthrough technology" and binary abatement levels.

[5] Analogously, Gatsios and Karp (1992) show how firms may overinvest prior to merger negotiations.

production standards. With only one period, however, these models miss dynamic effects and thus the consequences for agreement design.

There is already a large literature on the private provision of public goods in dynamic games.[6] Since the evolving stock of public good influences the incentive to contribute, the natural equilibrium concept is Markov perfect equilibrium. As in this paper, equilibrium provision levels tend to be suboptimally low when private provisions are strategic substitutes (Fershtman and Nitzan, 1991; Levhari and Mirman, 1980). There are often multiple MPEs, however, so Dutta and Radner (2009) investigate whether good equilibria, with little pollution, can be sustained by the threat of reverting to a bad one. Since differential games are often hard to analyze, it is quite standard to assume linear-quadratic functional forms,[7] and few authors complicate the model further by adding technological investments. Dutta and Radner (2004) is an interesting exception, but since their costs of pollution and investment are both linear, the equilibrium is "bang-bang" where countries invest either zero or maximally in the first period, and never thereafter. The contribution of this paper is, first, to provide a tractable model, with a unique MPE, in which agents invest as well as pollute over time. This is achieved by assuming that technology has a linear cost and an additive impact. This trick might also be employed when studying industry dynamics, for example, where analytical solutions are rare and numerical simulations typically necessary (see the survey by Doraszelski and Pakes, 2007). Second, incomplete contracts are added to the model. Incomplete contracts are necessary when the question is how agreements on emissions affect the incentive to invest.[8]

By permitting contracts on emissions but not on investments, this paper is in line with the literature on incomplete contracts (e.g., Hart and Moore, 1988). But the standard model has only two stages, and very few papers derive the optimal contract length.

[6] For treatments of differential games, see Başar and Olsder (1999) or Dockner et al. (2000).

[7] For a comprehensive overview, see Engwerda (2005).

[8] To the best of my knowledge, this is the first paper combining incomplete contracts and difference games. Battaglini and Coate (2007) let legislators negotiate spending on transfers and a long-lasting public good. In equilibrium, the legislators contribute too little to the public good, to induce future coalitions to spend more money on it. While the future coalition is unknown, the contract is complete. Hoel (1993) studies a differential game with an emission tax, Yanase (2006) derives the optimal contribution subsidy, Houba et al. (2000) analyze negotiations over (fish) quotas lasting forever, while Sorger (2006) studies one-period agreements. Although Ploeg and de Zeeuw (1992) even allow for R&D, contracts are either absent or complete in all these papers.

Harris and Holmstrom (1987) discuss the length when contracts are costly to rewrite but uncertainty about the future makes it necessary. To preserve the optimal incentives to invest, Guriev and Kvasov (2005) show that the agents should continuously renegotiate the length. Ellman (2006) studies the optimal probability for continuing the contract and finds that it should be larger if specific investments are important. This is somewhat related to my result on the optimal time horizon, but Ellman has only two agents and one investment period, and uncertainty is not revealed over time. In addition, I find that international contracts should be tougher, and more long-term, to compensate for incomplete domestic contracting. Furthermore, the result that short-term agreements can be worse than no agreement is certainly at odds with the traditional literature that focuses on bilateral trade.

Several of the results in this paper survive, qualitatively, in quite general settings. This is confirmed in Harstad (2010), where I also allow for technological spillovers and renegotiation. However, that paper abstracts from uncertainty, intellectual property rights, and trade policies, crucial in the present paper. Finally, by assuming quadratic utility functions, the following analysis goes further and describes when agreements are beneficial and how long they should last.

The model is presented in the next section. Section 3 compares the noncooperative outcome to short-term agreements, and finds conditions under which climate agreements reduce welfare. Section 4 derives the optimal long-term agreement, describing its length and toughness. The effects of subsidies and trade policies are analyzed in Section 5, while Section 6 discusses other extensions and shows that the results survive if, for example, the permits are tradable. After Section 7 concludes, the proofs follow in the Appendix.

2. The Model

2.1. Pollution and Payoffs

Pollution is a public bad. Let G represent the stock of greenhouse gases, and assume that the environmental cost for every country $i \in N \equiv \{1, ..., n\}$ is given by the quadratic cost function:

$$C(G) = \frac{c}{2}G^2.$$

Parameter $c > 0$ measures the importance of climate change.

The stock of greenhouse gases G is measured relative to its natural level. Since the natural level is thus $G = 0$, G tends to approach zero over time (were it not for emissions), and $1 - q_G \in [0, 1]$ measures the fraction of G that "depreciates" every period. G may nevertheless increase if a country's pollution level g_i is positive:

$$G = q_G G_- + \sum_N g_i + \theta. \tag{2.1}$$

By letting G_- represent the stock of greenhouse gases in the previous period, subscripts for periods can be skipped.

The shock θ is arbitrarily distributed with mean 0 and variance σ^2. It has a minor role in the model and most of the results hold without it (i.e., if $\sigma = 0$). However, it is realistic to let the depreciation and cumulation of greenhouse gases be uncertain. Moreover, the main impact of θ is that it affects the future marginal cost of emissions. In fact, the model would be identical if the level of greenhouse gases were simply $\widehat{G} \equiv q_G G_- + \sum_N g_i$ and the uncertainty were instead in the associated cost function. If the cost could be written as $C\left(\widehat{G} + \Theta\right)$, where $\Theta = q_G \Theta_- + \theta$, then $C' = c\Theta + c\widehat{G}$. In either case, a larger θ increases the marginal cost of emissions. Note that, although θ is i.i.d. across periods, it has a long-lasting impact through its effect on G (or on Θ).

The benefit of polluting g_i units is that country i can consume g_i units of energy. Naturally, country i may also be able to consume alternative or renewable energy, depending on its stock of nuclear power, solar technology, and windmills. Let R_i measure this stock and the amount of energy it can produce. The total amount of energy consumed is thus:

$$y_i = g_i + R_i, \tag{2.2}$$

and the associated benefit for i is:

$$B_i(y_i) = -\frac{b}{2}(\overline{y}_i - y_i)^2. \quad (2.3)$$

The benefit function is thus concave and increasing in y_i up to i's bliss point $\overline{y}_i$, which can vary across countries and be a function of time ($\overline{y}_{i,t}$). The bliss point represents the ideal energy level if there were no concern for pollution: a country would never produce more than $\overline{y}_i$ due to the implicit costs of generating, transporting, and consuming energy. The average $\overline{y}_i$ is denoted $\overline{y}$, which also can be time-dependent. Parameter $b > 0$ measures the importance of energy.

Several other interpretations of R_i are consistent with the model. For example, R_i may measure i's abatement technology, i.e., the amount by which i can at no cost reduce (or clean) its potential emissions. If energy production, measured by y_i, is generally polluting, the actual emission level of country i is given by $g_i = y_i - R_i$, implying (2.2), as before. Alternatively, combine (2.2) and (2.3) to let the cost of *abatement* be given by $-b(\overline{y}_i - R_i - g_i)^2/2$. In this case, the marginal cost of abatement is $b(\overline{y}_i - R_i - g_i)$, increasing in i's abatement level ($-g_i$) but decreasing in its abatement technology (R_i).[9]

2.2. Technology and Time

The technology stock R_i may change over time. On the one hand, the technology might depreciate at the expected rate of $1 - q_R \in [0, 1]$. On the other, if r_i measures country i's investment in the current period, then:

$$R_i = q_R R_{i,-} + r_i.$$

As described by Figure 1, the investment stages and the pollution stages alternate over time.[10] Without loss of generality, define "a period" to start with the investment stage and end with the pollution stage. In between, θ is realized. Information is symmetric at all stages.

[9] If, instead, the model focused on technologies that reduced the emission *content* of *each* produced unit (e.g., $g_i = y_i/R_i$), the analysis would be much harder.

[10] This assumption can be endogenized. Suppose the countries can invest at any time in the interval $[t-1, t]$, where t and $t+1$ denote emission stages, but that the investment must take place at least $\xi < 1$ units (measured as a fraction of the period-length) before time t, for the technology to be effective at time t. Then, all countries will invest at time $t - \xi$, never at time $t - 1$.

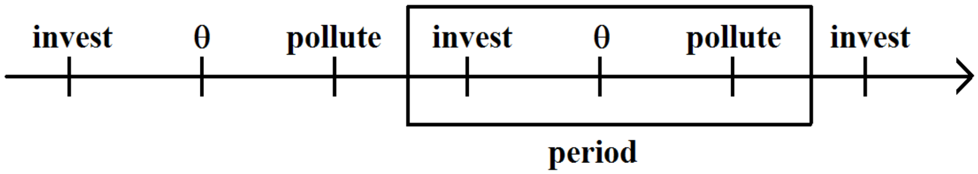

Figure 1: The investment and emission stages alternate over time

Section 5 allows the investments to be verifiable, contractible, and subsidizable. But in most of this paper, the investments are assumed to be observable but not verifiable by third parties. This is in line with the literature on incomplete contracts and may lead to hold-up problems at the international as well as at the national level.

At the international level, it is difficult for countries to negotiate and contract on investment levels. If a country has promised to reduce g_i, it can comply by reducing its short-term consumption or by investing in more long-lasting technology. The difference may be hard to detect by third parties. Therefore, if a country has promised to invest a certain amount, it may be tempted to report other public expenditures as such investments. These problems may explain why the Kyoto Protocol has specified emission quotas, but not investment levels.

At the national level, a government purchasing technology may find it difficult to describe the exact requirements in advance. An innovator or entrepreneur will need to develop the technology first, and then hope the government is willing to pay for it. Let $\mu \in (0,1]$ be the fraction of the government's benefit that the innovator can capture. Thus, μ represents the innovators' intellectual property right. In particular, μ might be the fraction of an investment that is protectable for the innovator, while the fraction $1-\mu$ is available for the government to copy for free.[11] But μ may also decrease if the government is a powerful negotiator.[12] If the cost of developing one unit of technology is given by the constant K, then, with free entry, innovators will earn zero profit and charge

[11] This is a slight modification of Acemoglu, Antras, and Helpman (2007). In their model, μ is the fraction of the tasks for which effort can be specified.

[12] Assuming the innovator can set the price, it will capture a fraction μ of the buyer's value. However, if the innovator and the government were bargaining over the price, the innovator would be able to capture the fraction $\widehat{\mu} \equiv \beta\mu$, where $\beta \in (0,1]$ is the innovator's share of the bargaining surplus (and thus represents its bargaining power). In this case, all the results below continue to hold if only μ is replaced by $\widehat{\mu}$.

the price K.

Consequently, country i's flow utility is:

$$u_i = B_i(y_i) - C(G) - Kr_i,$$

although the equilibrium investment level is given by:

$$\partial \frac{B_i(.) - C(.) + \delta U_{i,+}}{\partial R_i} = \frac{K}{\mu}, \tag{2.4}$$

where $U_{i,+}$ represents country i's continuation value at the start of the next period. The continuation values are given by:

$$U_{i,t} = \sum_{\tau=t}^{\infty} u_{i,\tau} \delta^{\tau-t},$$

where δ is the common discount factor. As mentioned, subscripts denoting period t are often skipped.

This model is more general that might at first appear: For example, if the government is developing technology itself, or if the contract with the developer is complete, then $\mu = 1$, which is a special case of the model, and the results below hold. If the developers of technology are located in foreign countries, the model is unchanged, since such firms earn zero profit in any case. If an innovator can sell (or license) its ideas to several countries at the same time, let K represent the private cost of developing technology that has the potential to raise $\sum_N R_i$ by one unit, and the analysis below needs only small modifications. Tariffs and subsidies on technological trade are discussed in Section 5.

2.3. Definition of an Equilibrium

There is typically a large number of subgame-perfect equilibria in dynamic games, and refinements are necessary. This paper focuses on Markov perfect equilibria (MPEs) where strategies are conditioned only on the pay-off relevant stocks (for further discussion and an exact definition, see Maskin and Tirole, 2001).

There are several reasons for selecting these equilibria. First, experimental evidence suggests that players tend toward Markov perfect strategies rather than supporting the best subgame perfect equilibrium (Battaglini et al., 2010). Second, Markov perfect strategies are simple, since they do not depend on the history in arbitrary ways.[13] This simplifies the analysis as well. Third, focusing on the MPEs is quite standard when studying games with stocks. By doing the same in this paper, its contribution is clarified. Furthermore, in contrast to much of the literature, there is a unique MPE in the present game. This sharpens the predictions and makes institutional comparisons possible. Fifth, the unique MPE coincides with the unique subgame-perfect equilibrium if time were finite but approached infinity. This is particularly important in our context, since the equilibrium is then robust to the introduction of real-world aspects that would make the effective time horizon finite. For example, since fossil fuel is an exhaustible resource, the emission game may indeed have a finite time horizon in the real world. Similarly, politicians' term-limits or short time horizon may force them to view time as expiring.[14] Finally, since the unique MPE makes it impossible to enforce agreements by using trigger strategies, it becomes meaningful to focus instead on settings where countries can negotiate and contract on emission levels - at least for the near future.

I do not explain *why* countries comply with such promises, but one possibility is that the treaty must be ratified domestically and that certain stakeholders have incentives to sue the government unless it complies. By varying the possibilities to negotiate and contract, I derive the outcome for each situation.

At the negotiation stage, I assume the bargaining outcome is efficient and symmetric *if* it should happen that the game and the payoffs are symmetric. This condition is satisfied whether we rely on (i) the Nash Bargaining Solution, with or without side transfers, (ii) the Shapley value, or instead (iii) noncooperative bargaining where one country is randomly selected to make a take-it-or-leave-it offer specifying quotas and transfers. Thus, the condition is quite weak. Note that all countries participate in equilibrium, since there is no stage at which they can close the door to negotiations.

[13] Maskin and Tirole (2001:192-3) defend MPEs since they are "often quite successful in eliminating or reducing a large multiplicity of equilibria," and they "prescribe the simplest form of behavior that is consistent with rationality" while capturing that "bygones are bygones more completely than does the concept of subgame-perfect equilibrium."

[14] More generally, Fudenberg and Tirole (1991:533) suggest that "one might require infinite-horizon MPE to be limits of finite-horizon MPE."

3. Are Agreements Good?

This section discusses the noncooperative outcome as well as the outcome under "short-term" agreements. A comparison reveals that such agreements are not necessarily good.

For future reference, the first-best emission levels are:

$$g_i^*(r) = \overline{y}_i - R_i - \frac{\overline{y}cn^2 + cn\left(q_G G_- + \theta - R\right) + \delta K q_G \left(1 - \delta q_R\right)}{b + cn^2}, \tag{3.1}$$

which is a function of the stocks in the previous period and - as emphasized - of this period's vector of investments $r \equiv (r_1, ..., r_n)$. Given these emission levels, the first-best investments are:

$$r_i^* = \overline{y} - \frac{q_R R_-}{n} + \frac{q_G G_-}{n} - (1 - \delta q_R)\left(\frac{1 - \delta q_G}{cn^2} + \frac{1}{b}\right) K.$$

Combined,

$$G^* = \sum_N g_i^*(r^*) + q_G G_- = \frac{(1 - \delta q_G)(1 - \delta q_R) K}{cn} + \theta \frac{b}{b + cn^2}. \tag{3.2}$$

Throughout the analysis, I assume $g_i \geq 0$ and $r_i \geq 0$ never bind.[15]

3.1. Business as Usual

Solving the model starts with the following two steps. First, at the beginning of every subgame, one can show that the R_is are payoff-irrelevant, given R: substituting (2.2) into (2.1), we get:

$$G = q_G G_- + \theta + \sum_N y_i - R, \text{ where} \tag{3.3}$$

$$R \equiv \sum_N R_i = q_R R_- + \sum_N r_i. \tag{3.4}$$

This way, the R_is are eliminated from the model: they are *payoff-irrelevant* as long as R is given, and i's Markov perfect strategy for y_i is thus not conditioned on them.[16] A

[15] This is satisfied if $g_i < 0$ and $r_i < 0$ are allowed or, alternatively, if q_G and q_R are small while θ has a limited support. A growing $\overline{y}_t$ is also making it necessary with positive investments. The Appendix (proof of Proposition 1) provides the exact conditions.

[16] That is, there is no reason for one player to condition its strategy on R_i, if the other players are not doing it. Thus, ruling out dependence on R_i is in line with the definition by Maskin and Tirole (2001:202), where Markov strategies are measurable with respect to the coarsest partition of histories consistent with rationality.

country's continuation value U_i is thus a function of G_- and R_-, not of $R_{i,-} - R_{j,-}$; we can therefore write it as $U(G_-, R_-)$, without the subscript i.

Second, the linear investment cost is utilized to prove that the continuation value must be linear in R and, it turns out, in G. Naturally, this simplifies the analysis tremendously.

PROPOSITION 0. (i) *There is a unique symmetric Markov perfect equilibrium.*
(ii) *It is in stationary strategies.*
(iii) *The continuation value is linear in the stocks with the slopes:*

$$\begin{aligned} U_R &= \frac{q_R K}{n}, \\ U_G &= -\frac{q_G K}{n}\left(1 - \delta q_R\right). \end{aligned}$$

This result[17] is referred to as Proposition 0 since it is the foundation for the propositions emphasized below. It holds for all scenarios analyzed in the paper, and for any concave $B(.)$ and convex $C(.)$, even if they are not necessarily quadratic. But to get further explicit results, we impose the quadratic forms.

PROPOSITION 1. *With business as usual, countries pollute too much and invest too little:*

$$r_i^{bau} = \overline{y} - \frac{q_R R}{n} + \frac{q_G G_-}{n} - \frac{k\left(b+cn\right)^2}{cb\left(b+c\right)n} + \frac{\delta U_R \left(b+cn\right)^2}{cb\left(b+c\right)n} - \frac{\delta U_G}{cn} < r_i^*, \quad (3.5)$$

$$g_i^{bau}\left(r^{bau}\right) = \overline{y}_i - R_i - \frac{c\left(n\overline{y} + q_G G_- + \theta - R\right) - \delta U_G}{b+cn} > g_i^*\left(r^{bau}\right) > g_i^*\left(r^*\right). \quad (3.6)$$

The first inequality in (3.6) states that each country pollutes too much compared to the first-best levels, conditional on the investments. A country is not internalizing the cost for the others.

Furthermore, note that country i pollutes less if the existing level of pollution is large and if i possesses good technology, but more if the other countries' technology level is large, since they are then expected to pollute less.

[17] As the proposition states, this is the unique *symmetric* MPE. Since the investment cost is linear, there also exist asymmetric MPEs in which the countries invest different amounts. Asymmetric equilibria may not be reasonable when countries are homogeneous, and they would cease to exist if the investment cost were convex.

In fact, $\overline{y}_i - R_i - g_i^{bau} = \overline{y}_i - y_i$ is the same across countries, in equilibrium, no matter what the differences in technology are. While perhaps surprising at first, the intuition is straightforward. Every country has the same preference (and marginal utility) when it comes to reducing its consumption level relative to its bliss point, and the *marginal* impact on G is also the same for every country: one *more* energy unit generates one unit of emissions. The technology is already utilized to the fullest possible extent, and producing more energy is going to pollute.

Therefore, a larger R, which reduces G, must increase every y_i. This implies that if R_i increases but R_j, $j \neq i$, is constant, then $g_j = y_j - R_j$ must increase. In words: if country i has a better technology, i pollutes less but (because of this) all other countries pollute more. Clearly, this effect reduces the willingness to pay for technology, and generates another reason why investments are suboptimally low, reinforcing the impact of the domestic hold-up problem when $\mu < 1$. The suboptimal investments make it optimal to pollute more, implying the second inequality in (3.6) and a second reason for why pollution is higher than its first-best level.

In sum, a country may want to invest less in order to induce other countries to pollute less and to invest more in the following period. In addition, countries realize that if G_- is large for a given R, (3.6) implies that the g_is must decrease. Thus, a country may want to pollute more today to induce others to pollute less (and invest more) in the future. These dynamic considerations make this dynamic common-pool problem more severe than its static counterpart.

3.2. Short-term Agreements

If countries can commit to the immediate but not the distant future, they may negotiate a "short-term" agreement. If the agreement is truly short-term, it is difficult to develop new technology during the time-span of the agreement and the relevant technology is given by earlier installations. This interpretation of short-term agreements can be captured by the timing shown in Figure 2.

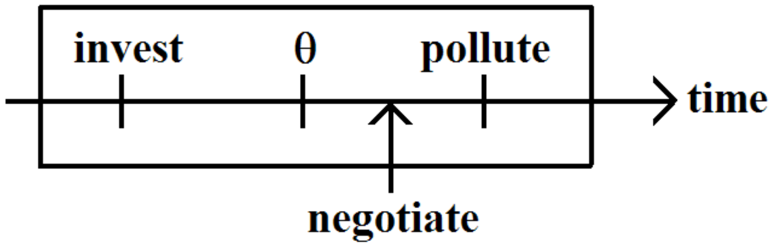

Figure 2: The timing for short-term agreements

Technically, negotiating the g_is is equivalent to negotiating the y_is as long as the R_is are sunk and observable (even if they are not verifiable). Just as in the previous section, (3.3)-(3.4) imply that the R_is are payoff-irrelevant, given R. Even if countries have different R_is, they face the same marginal benefits and costs of reducing y_i relative to $\overline{y}_i$, whether negotiations succeed or not. Symmetry thus implies that y_i is the same for every country in the bargaining outcome. Efficiency requires that the y_is are optimal (all countries agree on this). Consequently, the emission levels are equal to the first-best, conditional on the stocks.

Intuitively, if country i has better technology, its marginal benefit from polluting is smaller, and i is polluting less with business as usual. This gives i a poor bargaining position, and the other countries can offer i a smaller emission quota. At the same time, the other countries negotiate larger quotas for themselves, since the smaller g_i (and the smaller G) reduce the marginal cost of polluting. Anticipating this hold-up problem, every country is discouraged from investing. This international hold-up problem provides a second reason why investments are suboptimally low, in addition to the domestic hold-up problem that arises when $\mu < 1$.

Thus, although emission levels are ex post optimal, actual emissions are larger compared to the first-best levels since the two hold-up problems discourage investments and make it ex post optimal to pollute more.

PROPOSITION 2. *With short-term agreements, countries pollute the optimal amount, given the stocks, but investments are suboptimally low:*

$$\begin{aligned} r_i^{st} &= r_i^* - \left(\frac{n}{\mu} - 1\right)\left(\frac{b + cn^2}{bcn^2}\right) K < r_i^*, \\ g_i^{st}\left(r^{st}\right) &= g_i^*\left(r^{st}\right) > g_i^*\left(r^*\right). \end{aligned}$$

Deriving and describing this outcome is relatively simple because Proposition 0 continues to hold for this case, as proven in the Appendix. In particular, U_G and U_R are exactly the same as in the noncooperative case. This does *not* imply that U itself is identical in the two cases: the levels can be different. But this does imply that when deriving actions and utilities for one period, it is irrelevant whether there will also be a short-term agreement in the next (or any future) period. This makes it convenient to compare short-term agreements to business as usual. For example, such a comparison will be independent of the stocks, since U_G and U_R are identical in the two cases.

3.3. Are Short-Term Agreements Good?

Pollution is less under short-term agreements compared to no agreement. That may not be surprising, since the very motivation for negotiating is to reduce pollution. But what about investments and utilities?

PROPOSITION 3. *Compared to business as usual, short-agreements reduce (i) pollution, (ii) investments, and (iii) utilities if intellectual property rights are weak while the period is short (i.e., if (3.7) holds):*

$$\begin{aligned} \mathrm{E}g^{st}\left(r^{st}\right) &= \mathrm{E}g^{bau}\left(r^{bau}\right) - K\left(\frac{1}{\mu} - \frac{\delta q_R}{n}\right)\frac{n-1}{n\left(b+c\right)}, \\ r_i^{st} &= r_i^{bau} - K\left(\frac{1}{\mu} - \frac{\delta q_R}{n}\right)\frac{\left(n-1\right)^2}{n\left(b+c\right)}, \end{aligned}$$

$$U^{st} < U^{bau} \Leftrightarrow \left(\frac{n}{\mu} - 1\right)^2 - \left(1 - \delta q_R\right)^2 > \sigma^2 \frac{\left(b+c\right)\left(bcn/K\right)^2}{\left(b+cn^2\right)\left(b+cn\right)^2}. \tag{3.7}$$

Rather than being encouraging, short-term agreements impair the motivation to invest. The reason is the following. In anticipating of negotiations, the hold-up problem is exactly as strong as the crowding-out problem in the noncooperative equilibrium; in either case, each country enjoys only $1/n$ of the total benefits generated by its investments. In addition, when an agreement is expected, i understands that pollution will be reduced. A further decline in emissions, made possible by new technology, is then less valuable. Hence, each country is willing to pay less for technology.[18]

Since investments decrease under short-term agreements, utilities can decrease as well. This is the case, in particular, if investments are important because they are already well below the optimal level. Thus, short-term agreements are bad if intellectual property rights are weak (μ small), the number of countries is large, and the period for which the agreement lasts is very short. If the period is short, δ and q_R are large, while the uncertainty from one period to the next, determined by σ, is likely to be small. All changes make (3.7) reasonable, and it always holds when the period is very short ($\sigma \to 0$).

But at the emission stage, once the investments are sunk, all countries benefit from negotiating an agreement. It is the *anticipation* of negotiations which reduces investments and perhaps utility. Thus, if (3.7) holds, the countries would have been better off if they could commit to not negotiating short-term agreements. In particular, it may be better to commit to emission levels before the investments occur.

4. The Optimal Agreement

The hold-up problem under short-term agreements arises because the g_is are negotiated after investments are made. If the time horizon of an agreement is longer, however, it is possible for countries to develop technologies within the time frame of the agreement. The other countries are then unable to hold up the investing country, since the quotas have already been agreed to, at least for the near future.

To analyze such long-term agreements, let the countries negotiate and commit to emission quotas for T periods. The next subsection studies equilibrium investment, as a function of such an agreement. Taking this function into account, the second subsection

[18] A counter-argument is that, if an agreement is expected, it becomes more important to invest to ensure a decent energy consumption level. This force turns out to be smaller, at least for quadratic utility functions.

derives the optimal (and equilibrium) emission quotas, given T. Finally, the optimal T is characterized.

If the agreement is negotiated just before the emission stage in a period, then the quotas and investments for that period are given by Proposition 2, above: the quotas will be first-best, given the stocks, but investments are too low, due to the hold-up problems.

For the subsequent periods, it is irrelevant whether the quotas are negotiated before the first emission stage, or instead at the start of the next period, since no information is revealed, and no strategic decisions are made, in between. To avoid repeating earlier results, I will focus on the subsequent periods, and thus implicitly assume that the T-period agreement is negotiated at the start of period 1, as described by Figure 3.

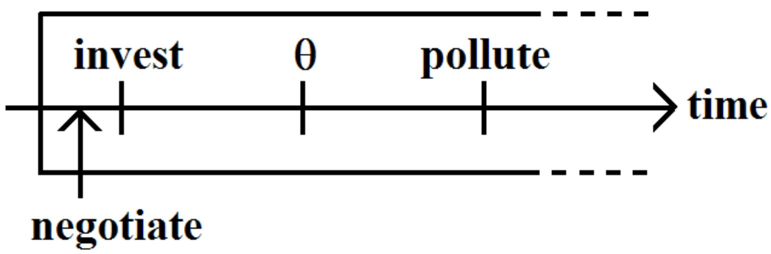

Figure 3: The timing for long-term agreements

4.1. Investments as a Function of the Quotas

When investing in period $t \in \{1, 2, ..., T\}$, countries take the quotas, the $g_{i,t}^{lt}$s, as given, and the continuation value for period $T+1$ is $U(G_T, R_T)$. A country is willing to pay more for innovations and investments if its quota, $g_{i,t}$, is small, since it is going to be very costly to comply if the sum $y_{i,t} = g_{i,t} + R_{i,t}$ is also small. Anticipating this, innovations and investments decrease in $g_{i,t}$.

However, compared to the investments that are first-best conditional on the quotas, $r_{i,t}^{*}(g_{i,t})$, equilibrium investments are likely to be too low. In every period, the innovators fear to be held up, if $\mu < 1$, and thus they invest and innovate only up to the point where the countries' willingness to pay for $1 - \mu$ units equals the cost of developing one unit of technology. Furthermore, a country anticipates that having good technology will worsen its bargaining position in the future, once a new agreement is to be negotiated. At that stage, having good technology leads to a lower $g_{i,t}$ since the other countries can hold up country i when it is cheap for i to reduce its emissions.[19] Anticipating this, countries invest less in the last period, particularly if that period is short (δ large), the technology long-lasting (q_R large), and the number of countries large (n large).

PROPOSITION 4. (i) *Investments increase in μ but decrease in $g_{i,t}$.*
(ii) *Investments are suboptimally low if $\mu < 1$, for any given quota and period.*
(iii) *In the last period, investments are suboptimally low if $\mu < 1$ or $\delta q_R > 0$:*

$$\begin{aligned}
r^*_{i,t}(g_{i,t}) &= \overline{y}_i - q_R R_{i,-} - g_{i,t} - (1-\delta q_R) K/b \\
&\geq \text{ (strict if } \mu < 1) \\
r_{i,t}(g_{i,t}) &= r^*_{i,t}(g_{i,t}) - \left(\frac{1}{\mu} - 1\right)\frac{K}{b} \text{ for } t < T \\
&\geq \text{ (strict if } \delta q_R > 0) \\
r_{i,t}(g_{i,t}) &= r^*_{i,t}(g_{i,t}) - \left(\frac{1}{\mu} - 1 + \delta q_R \left(1 - \frac{1}{n}\right)\right)\frac{K}{b} \text{ for } t = T.
\end{aligned}$$

4.2. The Optimal Emission Quotas

At the emission stage, the (ex post) optimal pollution level is given by $g^*_i\left(r^{lt}\right)$, as before. However, the countries anticipate that the negotiated $g_{i,t}$s are going to influence investments in technology: the smaller the quotas, the larger the investments. Thus, since the investments are suboptimally low, the countries have an incentive to commit to quotas that are actually smaller than the expectation of $g^*_i\left(r^{lt}\right)$, just to encourage investments. The smaller equilibrium investments are compared to the optimal investments, the lower are the negotiated $g^{lt}_{i,t}$s, compared to emission levels that are ex post optimal.

PROPOSITION 5. (i) *If $\mu < 1$, the negotiated quotas are strictly smaller than the ex post optimal levels, in every period.*
(ii) *For the last period, the negotiated quotas are strictly smaller than the ex post optimal quotas if either $\mu < 1$ or $\delta q_R > 0$.*

[19] Or, if no agreement is expected in the future, a large $R_{i,T+1}$ reduces $g_{i,T+1}$ and increases $g_{j,T+1}$, as proven in Section 3.1.

(iii) *The negotiated emission levels are identical to the emission levels that would have been first-best if investments had been first-best*:

$$g_{i,T}^{lt} = \mathrm{E}g_{i,T}^{*}(r^{*}) = \mathrm{E}g_{i,T}^{*}\left(r^{lt}\right) - \frac{1/\mu - 1 + \delta q_R (1 - 1/n)}{b + cn^2} K \text{ for } t = T, \tag{4.1}$$

$$g_{i,t}^{lt} = \mathrm{E}g_{i,t}^{*}(r^{*}) = \mathrm{E}g_{i,T}^{*}\left(r^{lt}\right) - \frac{1/\mu - 1}{b + cn^2} K \text{ for } t < T. \tag{4.2}$$

Parts (i)-(ii) have the following intuition. If μ is small, the last terms of (4.1)-(4.2) are large, and every $g_{i,t}^{lt}$ must decline relative to $g_i^*\left(r^{lt}\right)$. This makes the agreement more demanding or *tougher* to satisfy at the emission stage. The purpose of such a seemingly overambitious agreement is to encourage investments, since these are suboptimally low when μ is small. Encouraging investments is especially important in the last period, since investments are particularly low then, according to Proposition 4. Thus, the optimal agreement is tougher to satisfy over time.[20]

On the other hand, if $\mu = \delta q_R = 0$, the last terms of (4.1)-(4.2) are zero, meaning that the commitments under the best long-term agreement also maximize the expected utility ex post. In this case, there are no underinvestments, and there is no need to distort $g_{i,t}^{lt}$ downwards.

Part (iii) can be explained as follows. While the attempt to mitigate underinvestments reduces $g_{i,t}^{lt}$ compared to $g_i^*\left(r^{lt}\right)$, the fact that investments are low implies that it is ex post optimal to pollute more, so $g_i^*\left(r^{lt}\right)$ *increases* relative to $g_i^*(r^*)$. These two effects turn out to cancel.[21]

Just as in the previous cases, it turns out that the continuation value U is linear in the stocks, making the analysis tractable. Moreover, $U_R^{lt} = q_R K/n$ and $U_G^{lt} = -q_G(1 - \delta q_R) K/n$, as before. The predicted contract and investments are therefore robust to whether there is a long-term agreement, a short-term agreement, or no agreement in the subsequent period.

[20]This conclusion would be strengthened if the quotas were negotiated just before the emission stage in the first period. Then, the first-period quotas would be ex post optimal since these quotas would, in any case, have no impact on investments. It is easy to show that these quotas are expected to be larger than the quotas described by Proposition 5 - whether or not this is conditioned on investment levels.

[21]The reason is that, in this equilibrium as well as in the first-best outcome, y_i is independent of g_i, so a smaller g_i is only reducing G and increasing R_i. Since the marginal cost of increasing R_i is constant, the optimal G is the same in this equilibrium and in the first-best outcome.

4.3. The Optimal Agreement Length

The length of the agreement might be limited by the countries' ability to commit. Otherwise, the optimal and equilibrium T trades off two concerns. On the one hand, investments are particularly low at the end of the agreement, before a new agreement is to be negotiated. This hold-up problem arises less frequently, and is delayed, if T is large. On the other hand, the stochastic shocks cumulate over time, and they affect the future marginal costs of pollution. This makes it hard to estimate the optimal quotas for the future, particularly when T is large.

In general, the optimal length of an agreement depends on the regime that is expected to replace it. This is in contrast to the other contracts studied above, which have been independent of the future regime. When the time horizon is chosen, it is better to commit to a longer-term agreement if everyone expects that, once it expires, the new regime is going to be worse (e.g., business as usual).

On the other hand, if future as well as present negotiators are able to commit to future emissions, then we can anticipate that the next agreement is also going to be optimal. Under this assumption, the optimal agreement is derived and characterized in the Appendix.

PROPOSITION 6. (i) *The agreement's optimal length T^* decreases in μ, b, c, and σ, but increases in n, q_R, and K.*
(ii) *In fact, $T^* = \infty$ if:*

$$q_R K^2 \frac{1-1/n}{b}\left[\frac{1}{\mu} + 3\delta q_R \frac{1-1/n}{2}\right] \geq \frac{c\sigma^2 q_G^2}{2\left(1-q_G^2\right)\left(1-\delta q_G^2\right)}.$$

If θ were known or contractible, the agreement should last forever. Otherwise, the length of the agreement should be shorter if future marginal costs are uncertain (σ large) and important (c large). However, a larger T is preferable if the underinvestment problem is severe. This is the case if the intellectual property rights are weak (μ small), the technology is long-lasting (q_R large), and the number of countries is large. If b is large while K is small, then consuming energy is much more important than the concern for future bargaining power. The hold-up problem is then small, and the optimal T declines.

5. R&D Policies and Climate Agreements

So far, investments in technology have been noncontractible. But since, as a consequence, investments were suboptimally low, the countries have incentives to search for ways by which investments can be subsidized. This section allows for such subsidies and shows that the framework continues to provide important lessons.

Let $\phi \in [0,1)$ be an ad valorem subsidy captured by the innovator or developer of technology. It may denote the share of research expenses borne by the government (as in Grossman and Helpman, 1991:264). As before, K is the cost of increasing R_i by one unit, while $\mu \in (0,1]$ is the fraction of the purchaser's benefit that can be captured by the seller. With free entry of innovators, the equilibrium investments will be given by the following condition (replacing (2.4)):

$$\partial \frac{B_i(.) - C(.) + \delta U_{i,+}}{\partial R_i} = \frac{K(1-\phi)}{\mu}. \tag{5.1}$$

If the typical developer of technology for one country is located abroad, then ϕ can be interpreted as an import subsidy. If $\phi < 0$, then $\tau \equiv -\phi > 0$ may be interpreted as an import tariff.[22]

All the proofs in the Appendix are derived as a function of ϕ. As suggested by (5.1), the effect of ϕ is similar to the effect of μ. If the subsidy is exogenous and low, or the tariff $\tau \equiv -\phi$ is high, then investments decline. A further reduction in investment is then particularly bad, which implies that short-term agreements are worse than business as usual. To encourage more investments, the best climate agreement is tougher and longer-term.

Proposition 7. *If the subsidy ϕ is low or the tariff $\tau \equiv -\phi$ is high, the optimal*

[22] Technically, this requires the tariff, or the import subsidy, to be proportional to the cost of developing technology. If, instead, the import subsidy ϕ', or the tariff $-\phi'$, were proportional to the sales value, (5.1) should be:

$$\partial \frac{B_i(.) - C(.) + \delta U_{i,+}}{\partial R_i} = \frac{K}{\mu(1+\phi')}.$$

Alternatively, with an estate subsidy ϕ'', or tariff $-\phi''$, (5.1) should be:

$$\partial \frac{B_i(.) - C(.) + \delta U_{i,+}}{\partial R_i} = \frac{K}{\mu} - \phi''.$$

In these cases, the effects of ϕ' and ϕ'' would be similar to the effects of ϕ.

agreement is tougher and more long-term, while short-term agreements are likely to be worse than business as usual.

If ϕ, τ, or μ can be specified by international law, one may ask for their ideal levels. In particular, how do the optimal subsidy, tariff, and intellectual property right protection depend on the climate treaty?

PROPOSITION 8. *The optimal ϕ and μ are larger if the agreement is short-term or absent. They are given by:*
(i) *Equation (5.2) for short-term agreements as well as for business as usual;*
(ii) *Equation (5.3) for a long-term agreement's last period;*
(iii) *Equation (5.4) for a long-term agreement, except for its last period:*

$$\phi_{st}^{*} = \phi_{bau}^{*} = 1 - \mu/n > \tag{5.2}$$

$$\phi_{lt,T}^{*} = 1 - \mu\left[1 - \delta q_R \left(1 - 1/n\right)\right] > \tag{5.3}$$

$$\phi_{lt,t}^{*} = 1 - \mu. \tag{5.4}$$

If the climate treaty is short-term, the hold-up problem is larger and it is more important to encourage investments by protecting intellectual property rights, subsidizing technological trade, and reducing tariffs. Such trade agreements are thus strategic substitutes for climate treaties: weakening cooperation in one area makes further cooperation in the other more important. As before, the optimal agreement is also going to be the equilibrium when the countries negotiate, since they are symmetric at the negotiation stage w.r.t. $\overline{y}_{i,t} - y_{i,t}$, no matter what their technological differences are.

As suggested by Proposition 7, the optimal agreement has a shorter length if the subsidy is large. If the subsidy can be freely chosen and set in line with Proposition 8, short-term agreements are actually first-best: while the optimal subsidy induces first-best investments, the negotiated emission levels are also first-best, conditional on the investments. Long-term agreements are never first-best, however, due to the stochastic θ.

PROPOSITION 9. *If ϕ and μ can be set according to Proposition 8, short-term agreements implement the first-best outcome, but long-term agreements do not.*

6. Robustness and Limitations

This paper has focused on the interaction between investments in technology and climate agreements on emissions. To isolate these effects, the model abstracted from a range of real-world complications. While some assumptions have been crucial for the results, others can easily be relaxed.

For example, the discussion has ignored trade in emission allowances, presuming that such trade is prohibited. This assumption is not necessary, however.

PROPOSITION 10. *Suppose the emission allowances are tradable.*
(i) *All results survive, whether or not side payments are available at the negotiation stage.*
(ii) *In every period, the equilibrium permit price is* $b(\overline{y}_i - y_{i,t})$*, which decreases in* μ *but increases toward the end of the agreement.*

The permit price is $b(\overline{y}_i - y_{i,t}) = b(\overline{y}_i - g_{i,t} - R_{i,t})$, i.e., the marginal benefit of polluting one more unit, keeping the stocks constant. The larger $R_{i,t}$ is, the lower the marginal benefit of polluting is, and thus the permit price. It follows that the equilibrium permit price is larger if intellectual property rights are weak, as well as in the last period. Under a short-term agreement, which is negotiated taking technologies as given, the permit price declines in the allocated quotas. This is not true for long-term agreements, however, since a smaller $g_{i,t}^{lt}$ will raise $R_{i,t}$ by the very same amount: $\partial r_{i,t}^{lt}/\partial g_{i,t}^{lt} = -1$.

The quadratic functional forms are necessary when comparing utilities (as in Proposition 3) and when deriving the optimal length of an agreement (as in Proposition 6). Many of the other results, however, continue to hold, qualitatively, for any concave $B_i(.)$ and convex $C(.)$, even if they are not quadratic. This is confirmed in Harstad (2010), which also allows for technological spillovers. In addition, that paper permits the commitments to be renegotiated over time, and shows that this might be beneficial.[23]

The paper has allowed for certain types of heterogeneity. In particular, countries can have different initial technology, and their bliss point for energy consumption may vary.

[23] Renegotiation is nevertheless ignored in the present paper, since the possibility to renege may undermine the ability to commit in the first place. While Harstad (2010) assumes that the threat point under renegotiating is the existing agreement, this requires some discipline. If negotiators can credible threaten to exit the agreement if renegotiation fails, then allowing for renegotiation is equivalent to a sequence of short-term agreements, studied above.

Other types of heterogeneity would be harder to analyze. For example, suppose the cost of developing technology, K, varied across countries. In equilibrium, only countries with a small K would invest. This would also be optimal, but, just as before, the investing countries would invest too little. In a long-term agreement, one could encourage these countries to invest more by reducing g_i^{lt}. Such small g_is would not be necessary (or optimal) for noninvesting countries. Naturally, the investing countries may require some compensation to accept the small quotas. At the same time, a small g_i would not motivate country i to invest if i were allowed to purchase permits from noninvesting countries with more permits. Thus, with heterogeneous investment costs, Proposition 10 would be false. Evaluating political instruments under heterogeneity is thus an important task for future research.

The model above has predicted full participation in a climate treaty. This followed since there was no stage at which countries could opt out of the negotiation process. If such a stage were added to the model, free-riding may emerge. For example, in the one-period model analyzed by Barrett (2005), only three countries participate in equilibrium, when utility functions are quadratic. One may conjecture, however, that the number could be larger in a dynamic model, like the one above. If just a few countries decided to participate, they may find it optimal to negotiate short-term agreements, rather than long-term agreements, in the hope that the nonparticipants will join later. Since the participants invest less under short-term agreements, this credible threat might discourage countries when considering to free-ride.

7. Conclusions

While mitigating climate change will require emission reduction as well as the development of new technology, recent agreements have focused on short-term emissions. What is the value of such an agreement? How does it influence the incentive to invest, and what is the best agreement?

To address these questions, this paper provides a framework where countries over time both pollute and invest in environmentally friendly technologies. The analysis generates a number of important lessons.

First, the noncooperative outcome is particularly bad. With business as usual, countries pollute too much, not only because they fail to internalize the externality, but also because polluting now motivates the other countries to pollute less and invest more in the future. Similarly, countries invest too little in technology, to induce the others to invest more and pollute less themselves.

Second, short-term agreements can, nevertheless, be worse. At the negotiation stage, a country with good technology is going to be held up by the others, requiring it to reduce its pollution a great deal. Anticipating this, countries invest less when negotiations are coming up. This makes the countries worse off relative to business as usual, particularly if the agreement has a short duration and intellectual property rights are poorly enforced.

Third, the optimal agreement described. A tough agreement, if long-term, encourages investments. Thus, if the countries can commit to the far future, the optimal and equilibrium agreement is tougher and more longer-term if, for example, technologies are long-lasting but intellectual property rights weak.

Fourth, the optimal climate treaty is a function of trade policies. If technologies can be traded or subsidized, high tariffs and low subsidies discourage investments and, to counteract this, the climate treaty should be tougher and more long-term. If the climate treaty is absent or relatively short-lasting for exogenous reasons, then tariffs should decrease, intellectual property rights should be strengthened, and investments or trade in technology should be subsidized. Negotiating such policies is thus a strategic substitute to a tough climate treaty: if one fails, the other is more important.

The benchmark model is intentionally simple. In isolating the interaction between negotiations, emissions, and technological investments, it illuminates the challenges that would arise *even if* countries were similar, information complete, participation full, and the countries capable of committing to their promises. Future research should relax these assumptions to deepen our understanding of good climate policies.

8. Appendix

The following proofs allow for the subsidy ϕ, introduced in Section 5 (Propositions 0-6 follows if $\phi = 0$). While U_i is the continuation value for a subgame starting with the investment stage, let W_i represent the (interrim) continuation value at (or just before) the emission stage. To shorten equations, use $m \equiv -\delta \partial U_i / \partial G_-$, $z \equiv \delta \partial U_i / \partial R_-$, $\widetilde{R} \equiv q_R R_-$, $\widetilde{G} \equiv q_G G_- + \theta$ and $\widetilde{y}_i \equiv y_i + \overline{y} - \overline{y}_i$, where $\overline{y} \equiv \sum_N \overline{y}_i / n$. The proof for the first-best is omitted since it would follow the same lines as the following proofs.

Proof of Proposition 0.
Just before the emission stage, θ is known and the payoff-relevant states are R and $\widetilde{G}$.[24] A country's (interrim) continuation value is $W(\widetilde{G}, R)$. Anticipating this, equilibrium investments are given by:

$$\frac{\partial \mathrm{E} W(\widetilde{G}, \sum_N R_i)}{\partial R_i} = \frac{\partial \mathrm{E} W(\widetilde{G}, \sum_N R_i)}{\partial R} = k \equiv \frac{K(1-\phi)}{\mu}, \tag{8.1}$$

where expectations are taken w.r.t. θ. This implies, since the marginal cost of increasing R is constant, that the equilibrium R must be independent of R_-. Thus, when all countries invest the same, a marginally larger R_- implies that R will be unchanged, but r_i can decline by q_R/n units. It follows that:

$$\frac{\partial U}{\partial R_-} = \frac{q_R K}{n}. \tag{8.2}$$

At the emission stage, a country's first-order condition for y_i is:

$$0 = B'(\widetilde{y}_i - \overline{y}) - C'\left(\widetilde{G} - R + \sum_N \widetilde{y}_j\right) + \delta U_G(\widetilde{G} - R + \sum_N \widetilde{y}_j, R), \tag{8.3}$$

implying that all $\widetilde{y}_i$s are identical. From (8.2), we know that $U_{RG} = U_{GR} = 0$, and U_G cannot be a function of R. Therefore, (8.3) implies that $\widetilde{y}_i$, G and thus $B(\widetilde{y}_i - \overline{y}) - C(G) \equiv \gamma(.)$ are functions of $\widetilde{G} - R$ only. Hence, write $G\left(\widetilde{G} - R\right)$. Then, (8.1) becomes:

$$\frac{\partial \mathrm{E}\left[\gamma(q_G G_- + \theta - R) + \delta U\left(G(q_G G_- + \theta - R), R\right)\right]}{\partial R} = k.$$

This requires $q_G G_- - R$ to be a constant, say ξ, which is independent of the stocks. This implies that $\partial r_i / \partial G_- = q_G / n$ and U becomes:

$$\begin{aligned} U(G_-, R_-) &= \mathrm{E}\gamma(\xi + \theta) - Kr + \mathrm{E}\delta U\left(G(\xi + \theta), R\right) \\ &= \mathrm{E}\gamma(\xi + \theta) - K\left(\frac{q_G G_- - \xi - q_R R_-}{n}\right) + \mathrm{E}\delta U\left(G(\xi + \theta), q_G G_- - \xi\right) \Rightarrow \\ \partial U / \partial G_- &= -K\left(\frac{q_G}{n}\right) - \delta U_R q_G = -\frac{K q_G}{n}(1 - \delta q_R). \end{aligned}$$

[24] As explained in the text, there is no reason for one country, or one firm, to condition its strategy on R_i, given R, if the other players are not doing it. Ruling out such dependence is consistent with the definition of Markov and Tirole (2001).

Proof of Proposition 1.
From (8.3),

$$\widetilde{y}_i = \overline{y} - \frac{m+cG}{b} \Rightarrow y_i = \overline{y}_i - \frac{m+cG}{b} \Rightarrow \tag{8.4}$$

$$G = \sum_N (y_i - R_i) + \widetilde{G} = \widetilde{G} - R + n\left(\overline{y} - \frac{m+cG}{b}\right) = \frac{b\overline{y}n - mn + b\left(\widetilde{G} - R\right)}{b+cn} \tag{8.5}$$

$$y_i = \overline{y}_i - \frac{m}{b} - \frac{c}{b}\left(\frac{b\overline{y}n - mn + b\left(\widetilde{G}-R\right)}{b+cn}\right) = \overline{y}_i - \frac{c\overline{y}n + c\left(\widetilde{G}-R\right) + m}{b+cn} \Rightarrow$$

$$g_i = y_i - R_i = \overline{y}_i - \frac{c\overline{y}n + c\left(\widetilde{G}-R\right)+m}{b+cn} - R_i,$$

A comparison to the first-best gives (3.6). Interrim utility (after investments are sunk) can be written as:

$$W_i^{no} \equiv -\frac{c}{2}G^2 - \frac{b}{2}\left(\overline{y}_i - y_i\right) + \delta U(G,R) = -\frac{c}{2}\left(1+\frac{c}{b}\right)G^2 - \frac{Gmc}{b} - \frac{m^2}{2b} + \delta U(G,R).$$

Since $\partial G/\partial R = -b/\left(b+cn\right)$ from (8.5), equilibrium investments are given by:

$$\mathrm{E}\partial W_i^{no}/\partial R = c\left(1+\frac{c}{b}\right)\left(\frac{b}{b+cn}\right)\mathrm{E}G + \frac{bm\left(1+c/b\right)}{b+cn} + z = k. \tag{8.6}$$

Taking expections of G in (8.5), subsituting in (8.6) and solving for R gives:

$$R = \overline{y}n + \mathrm{E}\widetilde{G} - k\frac{\left(b+cn\right)^2}{bc\left(b+c\right)} + \frac{m}{c} + z\frac{\left(b+cn\right)^2}{bc\left(b+c\right)} \Rightarrow \tag{8.7}$$

$$r_i = \frac{R - q_R R_-}{n} = \overline{y} + \frac{q_G G_-}{n} - k\frac{\left(b+cn\right)^2}{bc\left(b+c\right)n} + \frac{m}{c} + z\frac{\left(b+cn\right)^2}{bc\left(b+c\right)n}.$$

A comparison to the first-best gives (3.5).

In steady state,

$$\begin{aligned} G &= \frac{b\overline{y}n - mn}{b+cn} + \frac{b}{b+cn}\left(\theta + k\frac{\left(b+cn\right)^2}{bc\left(b+c\right)} - \frac{m}{c} - z\frac{\left(b+cn\right)^2}{bc\left(b+c\right)} - \overline{y}n\right) \\ &= \frac{b}{b+cn}\theta + \frac{\left(b+cn\right)}{c\left(b+c\right)}\left(k-z\right) - \frac{m}{c}, \\ R &= \overline{y}n - \frac{\left(b+cn\right)^2}{bc\left(b+c\right)}\left(k-z\right) + \frac{m}{c} + q_G\left[\frac{b}{b+cn}\theta_- + \frac{\left(b+cn\right)}{c\left(b+c\right)}\left(k-z\right) - \frac{m}{c}\right] \\ &= \overline{y}n - \left(k-z\right)\frac{\left(b+cn\right)}{c\left(b+c\right)}\left[\frac{b+cn}{b} - q_G\right] + \frac{m}{c}\left(1-q_G\right) + \frac{q_G b\theta_-}{b+cn}. \end{aligned}$$

If the support of θ is $\left[\underline{\theta},\overline{\theta}\right]$, investment levels are *always* positive if:

$$\begin{aligned}0 &\leq \max_{\theta_t,\theta_{t-1}} R-q_R R_- = (1-q_R)\left[-\left(k-\frac{\delta q_R K}{n}\right)\frac{(b+cn)}{c(b+c)}\left[\frac{b+cn}{b}-q_G\right]+\frac{\delta q_G(1-\delta q_R)}{c}(1-q_G)\right]\\&+\frac{q_G b}{b+cn}\left(\underline{\theta}-q_R\overline{\theta}\right)+n\overline{y}_t-q_R n\overline{y}_{t-1},\end{aligned}$$

while the emission level is *always* positive if:

$$0 \leq \min_{\theta_t,\theta_{t-1}} G-q_G G_- -\theta = (1-q_G)\left[\frac{(b+cn)}{c(b+c)}(k-z)-\frac{m}{c}\right]-\frac{cn+q_G b}{b+cn}\overline{\theta}.$$

Proof of Proposition 2.
At the emission stage, the countries negotiate the g_is. g_i determines $\widetilde{y}_i$, and since countries have symmetric preferences over $\widetilde{y}_i$ (in the negotiations as well as in the default outcome), the $\widetilde{y}_i$s must be identical in the bargaining outcome and efficiency requires:

$$0 = B'(\widetilde{y}_i-\overline{y})/n - C'\left(\widetilde{G}-R+\sum\widetilde{y}_i\right)+\delta U_G(\widetilde{G}-R+\sum\widetilde{y}_i, R). \tag{8.8}$$

The rest of the proof of Proposition 0 continues to hold: R will be a function of G_- only, so $U_{R_-}=q_R K/n$. This makes $\mathrm{E}\widetilde{G}-R$ a constant and $U_{G_-}=-q_G(1-\delta q_R)K/n$, just as before. The comparative static becomes the same, but the *levels* of g_i, y_i, r_i, u_i and U_i are obviously different from the previous case.

The first-order condition (8.8) becomes:

$$\begin{aligned}0 &= -ncG+b\overline{y}-b\widetilde{y}_i-nm \Rightarrow y_i=\overline{y}_i-\frac{nm+ncG}{b}.\\G &= \widetilde{G}+\sum_j(y_j-R_j)=\widetilde{G}+n\left(\overline{y}-\frac{nm+ncG}{b}\right)-R\Rightarrow\\G &= \frac{b\overline{y}n-mn^2+b\left(\widetilde{G}-R\right)}{b+cn^2}.\end{aligned} \tag{8.9}$$

Interrim utility is

$$W_i^{st} = -\frac{c}{2}G^2-\frac{b}{2}\left(\frac{nm+ncG}{b}\right)^2+\delta U(G,R).$$

Since (8.9) implies $\partial G/\partial R = -b/(b+cn^2)$, equilibrium investments are given by:

$$\begin{aligned}k &= \mathrm{E}\frac{\partial W_i^{st}}{\partial R} = \mathrm{E}G\left(c+\frac{c^2n^2}{b}\right)\left(\frac{b}{b+cn^2}\right)+\frac{cmn^2}{b}\left(\frac{b}{b+cn^2}\right)+m\left(\frac{b}{b+cn^2}\right)+z\\&= c\mathrm{E}G+m+z.\end{aligned}$$

Subsituted in (8.9), after taking the expecion of it, and solving for R, gives

$$R^{st} = q_G G_- + n\overline{y}+\frac{m}{c}-\left(\frac{b+cn^2}{b}\right)\left(\frac{k}{c}-\frac{z}{c}\right). \tag{8.10}$$

The proof is completed by comparing r_i^* to $r_i^{st} = (R^{st} - q_R R_-)/n$, which is:

$$\begin{aligned} r_i^{st} &= \overline{y} - \frac{q_R R_-}{n} + \frac{q_G G_-}{n} - \left(\frac{b+cn^2}{bcn}\right)(k - \delta U_R) - \frac{\delta U_G}{cn} \\ &= r_i^* - K\left(\frac{b+cn^2}{bcn^2}\right)\left(\frac{nk}{K} - 1\right) < r_i^*. \end{aligned}$$

Proof of Proposition 3.
Part (i) and (ii) follow after some algebra when comparing emissions and investments for business as usual to short-term agreements. Substituted in u_i, which in turn should be substituted in $U = u_i + \delta U_+(.)$, allows us to compare U^{bau} and U^{st}. Then, straightforward algebra gives part (iii).

With business as usual, since $\widetilde{G} = q_G G_- + \theta$, (8.5) gives $G = \text{E}G + \theta b/(b+cn)$. Substituted in (8.4) gives:

$$y_i = \overline{y}_i - \frac{m+cG}{b} = \overline{y}_i - \frac{(k-z)(b+cn)}{b(b+c)} - \frac{\theta c}{b+cn}.$$

This is helpful when calculating u_i^{bau}. It becomes:

$$\begin{aligned} u_i^{bau} &= -\frac{c}{2}\left(\frac{k(b+cn)}{c(b+c)} - \frac{m}{c} - \frac{z(b+cn)}{c(b+c)} + \frac{\theta b}{b+cn}\right)^2 - \frac{b}{2}\left(\frac{(k-z)(b+cn)}{b(b+c)} + \frac{\theta bc}{b(b+cn)}\right)^2 \\ &\quad -\frac{K}{n}\left(-\widetilde{R} + q_G G_- - \frac{k(b+cn)^2}{cb(b+c)} + \overline{y}n + \frac{z(b+cn)^2}{cb(b+c)} + \frac{m}{c}\right) \Rightarrow \\ \text{E}u_i^{bau} &= -\frac{1}{2}(k-z)^2\left(\frac{b+cn}{b+c}\right)^2\left(\frac{1}{c} + \frac{1}{b}\right) - \frac{m^2}{2c} + \frac{m}{c}\left(\frac{b+cn}{b+c}\right)(k-z) \\ &\quad -\frac{K}{n}\left(q_G G_- - \widetilde{R} - \frac{(b+cn)^2}{bc(b+c)}(k-z) + \overline{y}n + \frac{m}{c}\right) - \frac{bc(b+c)\sigma^2}{2(b+cn)^2}. \end{aligned}$$

With short-term agreements,

$$\begin{aligned} y_i &= \overline{y}_i - \frac{nm}{b} - \frac{nc}{b}\left(\frac{b\overline{y}n - mn^2 + b\left(\widetilde{G} - R\right)}{b+cn^2}\right) = \overline{y} + \frac{b\overline{y} - mn - cn\left(\widetilde{G} - R\right)}{b+cn^2} \text{ and} \\ g_i &= \overline{y}_i - \overline{y} + \frac{b\overline{y} - mn - cn\left(\widetilde{G} - R\right)}{b+cn^2} - R_i. \end{aligned}$$

$$
\begin{aligned}
G &= \frac{k}{c} - \frac{m+z}{c} + \frac{b\theta}{b+cn^2} \Rightarrow \qquad (8.11)\\
\overline{y} - \widetilde{y}_i &= \frac{nm}{b} + \frac{nc}{b}\left(\frac{k}{c} - \frac{m+z}{c} + \frac{b\theta}{b+cn^2}\right) = \frac{n}{b}\left(k - z + \frac{bc\theta}{b+cn^2}\right) \Rightarrow \\
u_i^{st} &= -\frac{c}{2}G^2 - \frac{b}{2}\left(\overline{y} - \widetilde{y}_i\right)^2 - Kr \\
&= -\frac{c}{2}\left(\frac{k}{c} - \frac{m+z}{c} + \frac{\theta b}{b+cn^2}\right)^2 - \frac{n^2}{2b}\left(k - z + \frac{\theta bc}{b+cn^2}\right)^2 - Kr \Rightarrow \\
\mathrm{E}u_i^{st} &= -\frac{1}{2}(k-z)^2\left(\frac{1}{c} + \frac{n^2}{b}\right) - \frac{m^2}{2c} + \frac{m(k-z)}{c} \\
&\quad -\frac{K}{n}\left(q_G G_- - q_R R_- + n\overline{y} + \frac{m}{c} - \left(\frac{b+cn^2}{b}\right)\left(\frac{k}{c} - \frac{z}{c}\right)\right) - \frac{\sigma^2 bc}{2\left(b+cn^2\right)}.
\end{aligned}
$$

Comparing (8.7) with (8.10) and (8.5) with (8.11),

$$
\begin{aligned}
R^{no} - R^{st} &= -\frac{k\left(b+nc\right)^2}{bc\left(b+c\right)} + \frac{z\left(b+nc\right)^2}{bc\left(b+c\right)} + \left(\frac{b+cn^2}{b}\right)\left(\frac{k}{c} - \frac{z}{c}\right) \\
&= \frac{k\left(n-1\right)^2}{b+c}\left(1 - \frac{\delta q_R K}{nk}\right) > 0. \\
G^{no} - \mathrm{E}G^{st} &= \left(\frac{k}{c} - \frac{z}{c}\right)\left(\frac{b+nc}{b+c} - 1\right) = k\left(\frac{n-1}{b+c}\right)\left(1 - \frac{\delta q_R K}{nk}\right) = \frac{R^{no} - R^{st}}{n-1} > 0.
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E}u_i^{st} - \mathrm{E}u_i^{no} &= -\frac{1}{2}(k-z)^2\left(\frac{1}{c} + \frac{n^2}{b} - \left(\frac{1}{c} + \frac{1}{b}\right)\left(\frac{b+nc}{b+c}\right)^2\right) + m\frac{k-z}{c}\left(1 - \frac{b+nc}{b+c}\right) \\
&\quad -\frac{K}{n}(k-z)\left(\frac{\left(b+nc\right)^2}{bc\left(b+c\right)} - \frac{b+cn^2}{bc}\right) + \frac{\sigma^2 bc}{2}\left(\frac{b+c}{\left(b+cn\right)^2} - \frac{1}{b+cn^2}\right) \\
&= \left(\frac{\sigma^2 bc}{2\left(b+nc\right)^2\left(b+cn^2\right)} - \frac{\left(k-z\right)^2}{2bc\left(b+c\right)} + \frac{K}{nbc\left(b+c\right)}(k-z)\right) \\
&\quad \bullet \left[(b+c)\left(b+cn^2\right) - \left(b+nc\right)^2\right] - \frac{m\left(k-z\right)}{b+c}(n-1)
\end{aligned}
$$

$$
= \left(\frac{\left(bc\sigma\left[n-1\right]\right)^2}{2\left(b+nc\right)^2\left(b+cn^2\right)} - \frac{\left(k-z\right)\left[n-1\right]^2}{2\left(b+c\right)}\left[k - z - \frac{2K}{n}\right]\right) - \frac{m\left(k-z\right)\left(n-1\right)}{\left(b+c\right)}.
$$

Thus, we get $U^{st} > U^{no}$ if

$$
\mathrm{E}u_i^{st} - \mathrm{E}u_i^{no} + m\frac{\left(k-z\right)\left(n-1\right)}{\left(b+c\right)} - z\frac{k\left(n-1\right)^2}{b+c}\left(1 - \frac{\delta q_R K}{nk}\right) =
$$

$$
\left(\frac{\left(bc\sigma\right)^2\left[n-1\right]^2}{2\left(b+nc\right)^2\left(b+cn^2\right)} - \frac{\left[n-1\right]^2}{2\left(b+c\right)}\left[\left(k-z\right)^2 - \frac{2K}{n}(k-z) + 2zk\left(1 - \frac{\delta q_R K}{nk}\right)\right]\right) > 0
$$

$$\Rightarrow \frac{(bc\sigma)^2 (b+c)}{(b+nc)^2 (b+cn^2)} > \left(\frac{K}{n}\right)^2 \left[\left(n\frac{1-\phi}{\mu}-1\right)^2 - (1-\delta q_R)^2\right].$$

Proof of Proposition 4.
In the last period, investments are given by:

$$\begin{aligned} k &= B'(g_{i,T} + R_{i,T} - \overline{y}_i) + z \Rightarrow \\ \widetilde{y}_i - \overline{y} &= -\frac{k-z}{b},\ R_{i,T} = \overline{y}_i - g_{i,T} - \frac{k-z}{b}, \\ r_i &= \overline{y}_i - g_{i,T} - \frac{k-z}{b} - q_R R_{i,-}. \end{aligned} \tag{8.12}$$

Anticipating the equilibrium $R_{i,T}$, i can invest q_R less units in period T for each invested unit in period $T-1$. Thus, in period $T-1$, equilibrium investments are given by:[25]

$$\begin{aligned} k &= B'(g_{i,T-1} + R_{i,T-1} - \overline{y}_i) + \delta q_R K \Rightarrow \\ R_{i,T-1} &= \overline{y}_i - g_{i,T-1} - \frac{k-\delta q_R K}{b}, \\ r_{i,T-1} &= \overline{y}_i - g_{i,T-1} - \frac{k-\delta q_R K}{b} - q_R R_{i,-}. \end{aligned}$$

The same argument applies to every period $T-t$, $t \in \{1, ...T-1\}$, and the investment level is given by the analogous equation for each period but T. Proposition 4 follows since the optimal R_i and r_i, given g_i, are:

$$\begin{aligned} R_i^* &= \overline{y}_i - g_i - \frac{K(1-\delta q_R)}{b}, \\ r_i^* &= \overline{y}_i - g_i - \frac{K(1-\delta q_R)}{b} - q_R R_{i,-}. \end{aligned}$$

Proof of Proposition 5.
If the negotiations fail, the default outcome is the noncooperative outcome, giving everyone the same utility. Since the r_is follow from the g_is in (8.12), everyone understands that negotiating the g_is is equivalent to negotiating the r_is. All countries have identical preferences w.r.t. the r_is (and their default utility is the same), and symmetry requires that r_i, and thus $\varsigma_t \equiv \overline{y}_i - g_{i,t} - q_R R_{i,t-1}$, is the same for all countries, in equilibrium.

For the last period, (8.12) becomes

$$r_{i,T} = \varsigma_T - \frac{k - \delta q_R K/n}{b}.$$

Anticipating the equilibrium investments, the utility for the last period is:

$$U_i = -\frac{(k-z)^2}{2b} - \mathrm{E}C(G) - K r_{i,T} + \delta U(G,R).$$

[25] This presumes that country i's cost of future technology is K, which is correct since, in equilibrium, country i pays $K(1-\phi)$ plus the subsidy ϕK (or minus the tax $-\phi K$), even if this price is for the remaining fraction μ, after the fraction $1-\mu$ has been expropriated.

Efficiency requires (f.o.c. of U_i w.r.t. ς recognizing $g_i = \overline{y}_i - q_R R_{i,-} - \varsigma$ and $\partial r_i / \partial \varsigma = 1 \forall i$):

$$n\mathrm{E}C'(G) - K - n\delta U_G + n\delta U_R = 0 \Rightarrow \mathrm{E}C'(G) + m + z = K/n. \tag{8.13}$$

For the earlier periods, $t < T$, $r_{i,t} = r_{j,t} = r_t$ and

$$r_t = \varsigma_t - \frac{k - \delta q_R K}{b}.$$

Note that for every $t \in (1, T)$, $R_{i,t-1}$ is given by the g_i in the previous period:

$$\begin{aligned} r_t &= \left(\overline{y}_i - g_{i,t} - q_R\left(\overline{y}_i - g_{i,t-1} - \frac{k - \delta q_R K}{b}\right)\right) - \frac{k - \delta q_R K}{b} \\ &= \overline{y}_i(1 - q_R) - g_{i,t} + q_R g_{i,t-1} - (1 - q_R)\frac{k - \delta q_R K}{b}. \end{aligned} \tag{8.14}$$

All countries have the same preferences over the ς_ts. Dynamic efficiency requires that the countries are not better off after a change in the ς_ts (and thus the $g_{i,t}$s), given by $(\Delta\varsigma_t, \Delta\varsigma_{t+1})$, such that G is unchanged after two periods, i.e., $\Delta\varsigma_{t+1} = -\Delta\varsigma_t q_G$, $t \in [1, T-1]$. From (8.14), this implies

$$\begin{aligned} -n\mathrm{E}C'(G_t)\Delta\varsigma_t + \Delta g_t K + \delta(\Delta\varsigma_{t+1} - \Delta g_t q_R)K - \delta^2 \Delta g_{t+1} q_R K &\leq 0 \forall \Delta\varsigma_t \Rightarrow \\ \left(-\mathrm{E}C' n + K - \delta(q_G + q_R)K + \delta^2 q_G d_R K\right)\Delta\varsigma_t &\leq 0 \forall \Delta\varsigma_t \Rightarrow \\ (1 - \delta q_R)(1 - \delta q_G)\frac{K}{cn} &= \mathrm{E}G = \mathrm{E}G^*. \end{aligned}$$

Thus, neither G_t nor $g_{i,t}$ (and, hence, neither R) can be functions of R_-. At the start of period 1, therefore, $U_{R_-} = q_R K/n$, just as before, and U_G cannot be a function of R (since $U_{RG} = 0$). Since $\mathrm{E}G$ is a constant, we must have $\varsigma_1 = \overline{y} - (\mathrm{E}G^* - q_G G_0)/n - q_R R_0 / n$. (8.12) gives $\partial r_{i,t=1}/\partial G_- = (\partial r_i / \partial g_i)(\partial g_i / \partial \varsigma)(\partial \varsigma / \partial G_-) = q_G/n$. Hence, $U_{G_-} = -q_G K/n + \delta U_R q_G = -q_G(1 - \delta q_R)K/n$, giving a unique equilibrium. Substituted in (8.13), $\mathrm{E}G_T = \mathrm{E}G^*$, just as in the earlier periods. Thus, $g_{i,t}^{lt} = g_i^*(r_i^*)$ in all periods.

Proposition 5 follows since, from (3.1), $\partial g_i^* / \partial r_j = -b/(b + cn^2)$, so $g_{i,t}^{lt} = g_i^*(r_i^*) = g_i^*\left(r_{i,t}^{lt}\right) - \left(r_i^* - r_{i,t}^{lt}\right) b/(b + cn^2)$.

Proof of Proposition 6.

The optimal T balances the cost of underinvestment when T is short and the cost of the uncertain θ, increasing in T. In period T, countries invest suboptimally not only because of the domestic hold-up problem, but also because of the international one. When all countries invest less, u_i declines. The loss in period T, compared to the earlier periods, is:

$$\begin{aligned} H &= B(y_{i,t} - \overline{y}_i) - B(y_{i,T} - \overline{y}_i) - K(r_{i,t} - r_{i,T})(1 - \delta q_R) \\ &= -\frac{b}{2}\left(\frac{k - \delta q_R K}{b}\right)^2 + \frac{b}{2}\left(\frac{k - z}{b}\right)^2 - K\left(\frac{k - z}{b} - \frac{k - \delta q_R K}{b}\right)(1 - \delta q_R) \\ &= \delta q_R K^2 \frac{1 - 1/n}{b}\left[\frac{1 - \phi}{\mu} + 3\delta q_R \frac{1 - 1/n}{2}\right]. \end{aligned}$$

Note that H increases in n, q_R, K, but decreases in μ, ϕ, and b.

The cost of a longer-term agreement is associated with θ. Although $\mathrm{E}C'$ and thus $\mathrm{E}G_t$ is the same for all periods,

$$\begin{aligned} \mathrm{E}\frac{c}{2}(G_t)^2 &= \mathrm{E}\frac{c}{2}\left(\mathrm{E}G_t + \sum_{t'=1}^{t}\theta_{t'}q_G^{t-t'}\right)^2 = \frac{c}{2}(\mathrm{E}G_t)^2 + \mathrm{E}\frac{c}{2}\left(\sum_{t'=1}^{t}\theta_{t'}q_G^{t-t'}\right)^2 \\ &= \frac{c}{2}(\mathrm{E}G_t)^2 + \frac{c}{2}\sigma^2\sum_{t'=1}^{t}q_G^{2(t-t')} = \frac{c}{2}\left(\mathrm{E}G_t^2\right) + \frac{c}{2}\sigma^2\left(\frac{1-q_G^{2t}}{1-q_G^2}\right). \end{aligned}$$

For the T periods, the total present discounted value of this loss is L, given by:

$$\begin{aligned} L(T) &= \sum_{t=1}^{T}\frac{c}{2}\sigma^2\delta^{t-1}\left(\frac{1-q_G^{2t}}{1-q_G^2}\right) = \frac{c\sigma^2}{2\left(1-q_G^2\right)}\sum_{t=1}^{T}\delta^{t-1}\left(1-q_G^{2t}\right) \\ &= \frac{c\sigma^2}{2\left(1-q_G^2\right)}\left[\frac{1-\delta^T}{1-\delta} - q_G^2\left(\frac{1-\delta^T q_G^{2T}}{1-\delta q_G^2}\right)\right] \Rightarrow \qquad (8.15) \\ L'(T) &= \frac{c\sigma^2\left(-\delta^T\ln\delta\right)}{2\left(1-q_G^2\right)}\left[\frac{1}{1-\delta} - \frac{q_G^{2T+2}\left(1+\ln\left(q_G^2\right)/\ln\delta\right)}{1-\delta q_G^2}\right]. \end{aligned}$$

If all future agreements last $\widehat{T}$ periods, the optimal T for this agreement is given by

$$\begin{aligned} &\min_T L(T) + \left(\delta^{T-1}H + \delta^T L\left(\widehat{T}\right)\right)\left(\sum_{\tau=0}^{\infty}\delta^{\tau\widehat{T}'}\right) \Rightarrow \\ &0 = L'(T) + \delta^T\ln\delta\left(H/\delta + L\left(\widehat{T}\right)\right) = L'(T) + \delta^T\ln\delta\left(H/\delta + L\left(\widehat{T}\right)\right) \\ = \; &-\delta^T\ln\delta\left[\frac{c\sigma^2/2}{1-q_G^2}\left(\frac{1}{1-\delta} - \frac{q_G^{2T+2}\left(1+\ln q_G^2/\ln\delta\right)}{1-\delta q_G^2}\right) - \frac{H/\delta + L\left(\widehat{T}\right)}{1-\delta^{\widehat{T}'}}\right], \qquad (8.16) \end{aligned}$$

assuming some T satisfies (8.16). Since $\left(-\delta^T\ln\delta\right) > 0$ and the bracket-parenthesis increases in T, the loss decreases in T for small T but increases for large T, and there is a unique T minimizing the loss (even if the loss function is not necessarily globally concave). Since the history (G_- and R_-) does not enter in (8.16), T satisfying (8.16) equals $\widehat{T}$, assuming also $\widehat{T}$ is optimal. Substituting $\widehat{T} = T$ and (8.15) in (8.16) gives:

$$H/\delta = \frac{c\sigma^2 q_G^2}{2\left(1-q_G^2\right)\left(1-\delta q_G^2\right)}\left(\frac{1-\delta^T q_G^{2T}}{1-\delta^T} - q_G^{2T}\left(1+\frac{\ln\left(q_G^2\right)}{\ln\delta}\right)\right), \qquad (8.17)$$

where the r.h.s. increases in T. $T = \infty$ is optimal if the left-hand side of (8.17) is larger than the right-hand side even when $T \to \infty$:

$$\frac{c\sigma^2 q_G^2}{2\left(1-q_G^2\right)\left(1-\delta q_G^2\right)} \leq H/\delta. \qquad (8.18)$$

If k/K and n are large, but b small, H is large and (8.18) is more likely to hold and if it does not, the T satisfying (8.17) is larger. If c or σ^2 are large, (8.18) is less likely to hold and if it does not, (8.17) requires T to decrease.

Proofs of Propositions 7-9.
Proposition 7 follows since $\phi > 0$ is allowed in the proofs above (k is a function of ϕ).[26] To see Proposition 8: Under short-term agreements (as well as business as usual), if interrim utility is $W\left(\widetilde{G}, R\right)$, investments are given by $\mathrm{E}W_R = k$ while they should optimally be $\mathrm{E}W_R = K/n$, requiring (5.2). For long-term agreements, investments are optimal in the last period if $k - \delta q_R K/n = K(1 - \delta q_R)$, requiring (5.3). For earlier periods, the requirement is $k = K$, giving (5.4). Proposition 9 follows from the text.

Proof of Proposition 10.
First, note that there is never any trade in permits in equilibrium. Hence, *if* country i invests as predicted in Sections 3-4, the marginal benefit of more technology is the same whether permits are tradable or not. Second, if i deviated by investing more (less), it's marginal utility of a higher technology decreases (increases) not only when permit-trade is prohibited, but also when trade is allowed since more (less) technology decreases (increases) the demand for permits and thus the equilibrium price. Hence, such a deviation is not attractive. When permits are tradable, altering their allocation is a form of side transfer, making the feasibility of explicit transfers irrelevant.

[26] Some caution is necessary, however. The proofs of Propositions 4-6 are unchanged only if the innovator receives the subsidy or pays the tariff *before* negotiating the price. With the reverse timing, ϕ would have no impact when the buyer is a government. In that case, the subsidy must be paid by foreign countries (as an international subsidy), and the proofs of Propositions 4-6 would need minor modifications, although the results would continue to hold. The proofs of Propositions 0-3 can stay unchanged in all these cases.

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