Integration and Information: Firms' Governance in Rational Expectations Equilibrium

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Abstract

We analyze a rational-expectations model of price formation in an intermediate-good market under uncertainty. There is a continuum of firms, each consisting of a party who can reduce cost and a party who can discover information about consumers. Both parties can make specific investments at private cost, and there is a machine that either party can control. As in incomplete-contracting models, different ownership structures create different incentives for the parties' investments. As in rational-expectations models, some parties may invest in acquiring information, which is then incorporated into the market-clearing price by the parties' trading behaviors. The informativeness of the price mechanism affects the returns to specific investments and hence the optimal governance structure for individual firms; meanwhile, the governance choices by individual firms affect the informativeness of the price mechanism. In equilibrium the informativeness of the price mechanism can induce *ex ante* homogenous firms to choose heterogeneous governance structures. (JEL D20, D23)

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1 Introduction

Both practitioners and students of strategy and organizational design have long espoused two approaches: producing existing products efficiently through process control and continuous improvement, on the one hand, and developing innovative new products through R&D and market research, on the other. But most observers quickly emphasize the difficulty of simultaneously pursuing "exploration" and "exploitation" (March (1991)). For example, "Cost leadership usually implies tight control systems, overhead minimization, pursuit of scale economies, and dedication to the learning curve; these could be counterproductive for a firm attempting to differentiate itself through a constant stream of creative new products" (Porter, 1985: 23). Furthermore, as Chandler (1962) famously argued, a firm's strategy and its organizational structure are inextricably linked. In short, "Exploration and exploitation are quite different tasks, calling on different organizational capabilities and typically requiring different organizational designs to effect them" (Roberts, 2004: 255).

In quite a different tradition, economists have long celebrated the market's price mechanism for its ability to aggregate and transmit information (Hayek, 1945; Grossman, 1976). The informativeness of the price mechanism thus raises the possibility that the market can substitute for certain information-gathering activities within the firm, and thus affect the optimal organizational structure. But as Grossman and Stiglitz (1976, 1980) pointed out, when information is costly to acquire, equilibrium prices cannot be fully informative, otherwise no party would have an incentive to acquire the information in the first place.

In this paper we view firms and the market as institutions that shape each other: in an industry equilibrium, each firm takes the informativeness of the price mechanism as an important parameter in its choice of organizational design, but these design decisions in turn affect the informativeness of the price mechanism. We thus complement the large and growing literature on how internal structures and processes affect incentives to acquire and communicate information.[1] In particular, our analysis shows how one firm's optimal

[1] See Milgrom & Roberts (1988), Holmstrom & Tirole (1991), and Aghion and Tirole (1997) for early work

organizational design depends not only on the uncertainty it faces but also on the designs other firms choose. For example, if market prices are very informative, then firms will choose governance structures to improve incentives for other activities (say, cost reduction), effectively free-riding on the informativeness of the price mechanism. But the Grossman-Stiglitz paradox implies that not all firms can free-ride, lest the price mechanism contain no information.

In this paper we explore how the informativeness of the price mechanism and firms' governance structure choices interact. To do so we analyze an economic environment that includes uncertainty. Formally, the uncertainty concerns consumers' valuation of final goods, but we discuss other interpretations below. Parties can resolve this uncertainty at a cost. As in other rational-expectations models, the price mechanism both clears the market and conveys information from informed to uninformed parties. The fact that the price is not perfectly informative provides the requisite incentive for some parties to pay the cost to resolve the uncertainty.

As one specific example, consider a U.S. steel producer deciding how much steel to produce. Demand for steel might depend on future construction in China, which is uncertain. Travelling to China and conducting interviews could give a steel producer a better signal of Chinese construction and hence demand for steel. But other steel producers, who choose not to conduct such research, could look at the market price for an upstream good (such as iron ore) and infer some of the information gathered by the firms that did conduct research. This example parallels our model, in that it is the market-clearing price of an upstream good (iron ore) that conveys information about the uncertain value of the downstream good (steel).

Many other applications of our approach arise if we consider alternative sources of uncertainty, other than the value of goods such as steel. For example, the uncertainty might concern whether tariff barriers will change or whether a new technology will fulfill its promise.

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and Alonso *et al.* (2008) and Rantakari (2008) for a sample of recent work; see Bolton and Dewatripont (2011) for a survey.)

Interestingly, however, not all sources of uncertainty will do: our rational-expectations model applies to common (or partially correlated)- rather than private-value uncertainty. As Grossman (1981: 555) puts it, in non-stochastic economies (and certain economies with private-value uncertainty), "No one tries to learn anything from prices [because] there is nothing for any individual to learn." Often, however, there is something to learn from prices, such as when there is common-value uncertainty.

To pursue these issues, we develop a rational-expectations model similar to Grossman and Stiglitz (1976, 1980) but designed to apply to a market for an intermediate good (prices and net supply are non-negative and the players are risk-neutral). In Gibbons, Holden, and Powell (2009; hereafter GHP), we developed such a model but for the Grossman-Stiglitz case of individual investors. The novel aspect of the present paper is the analysis of alternative governance structures. Specifically, we allow for production by firms (where control can be allocated to either party) rather than by individual investors. Relative to rational-expectations pricing models such as Grossman-Stiglitz (and GHP), the novel component in this paper is the analysis of alternative governance structures. Specifically, we enrich GHP to allow for production by firms rather than only individual investors.

To model these firms, we develop a simplified version of the classic incomplete-contracting approach initiated by Grossman and Hart (1986), but applied to the choice of internal organization (as in Aghion & Tirole (1997)). To keep things simple, our incomplete-contracts model involves only a single control right and hence two governance structures. Regardless of who controls the asset, each party can make a specific investment, but the incentives to make these investments depend on who controls the asset. Following the incomplete-contracts approach (i.e., analyzing one firm in isolation) reveals that the optimal ownership structure is determined by the marginal returns to these investments. In our model all firms are homogeneous *ex ante*, so an incomplete-contracts analysis of a single firm would prescribe that all firms choose the same governance structure. Relative to the incomplete-contracts approach, the novel component of our model is the informativeness of the price mechanism, which endogenizes the returns to the parties' specific investments and hence creates an industry-level determinant of an individual firm's choice of governance structure.

In summary, our model integrates two familiar approaches: rational expectations (where an imperfectly informative price mechanism both permits rational inferences by some parties and induces costly information acquisition by others) and incomplete-contracts (where equilibrium investments depend on the parties' governance structure and control rights are chosen to induce second-best investments). Our main results are that: (1) when firms choose control rights they affect incentives to gather information, and (2) the informativeness of the price mechanism is a central determinant of whether firms control rights to strengthen these incentives (or, instead, make decisions to strengthen other incentives, such as for cost reduction), and this informativeness is endogenously determined by the aggregation of firms' governance-structure choices in market equilibrium. In fact, in our model, certain governance structures may be sustained in market equilibrium *only* because the price system allows some firms to benefit from the information-acquisition investments of others.

Grossman & Helpman (2002), Legros & Newman (2008) and Legros & Newman (2009) analyze other interactions between firms' integration decisions and the market, focusing on interactions that do not involve the *informativeness* of the price mechanism. In these models, supply and demand determine prices, which in turn determine the returns to the parties' actions and hence the parties' optimal governance structures; meanwhile, the parties' actions in turn determine supply and demand, so governance and pricing interact. As Grossman (1981: 555) notes, such Walrasian equilibria are not useful "as a tool for thinking about how goods are allocated... when...information about the future...affects current prices." In contrast to the aforementioned papers, our model focuses of the informative role of prices: transferring information from informed to (otherwise) uninformed parties. We see these two approaches as complementary. Indeed, in economies with uncertainty the price mechanism plays two roles: it clears the market and communicates information.

The remainder of the paper proceeds as follows. In Section 2 we specify and discuss the model. Section 3 analyzes the governance structure choice of a single firm in isolation, and Section 4 analyzes the informativeness of the price mechanism, taking firms' governance structure choices as given. Section 5 then combines the incomplete-contracts and rational-expectations aspects of the previous two sections, analyzing the equilibrium choices of governance structures for all the firms in the industry and hence deriving our main results. Section 6 offers a reinterpretation of our model as a model of firm boundaries, and discusses how it relates to well-known theories of firm boundaries. Section 7 concludes.

2 The Model

2.1 Overview of the Model

We begin with an informal description of our model. There is a continuum of firms, each consisting of an "engineer" and a "marketer" in a production process that can transform an intermediate good (a "widget") into a final good. Any firm may purchase a widget from the market. Each firm has an alienable asset (a machine) that can transform one widget into one final good at a cost. Engineers may make investments that reduce the cost of operating the machine; we therefore think of engineers as having human capital that is relevant to the *production* of final goods. Marketers may make investments that deliver information about the value of a final good in the consumer market; we therefore think of marketers as having human capital that is relevant to the *marketing* of final goods.

As is standard in incomplete contracting models, the parties' incentives to make investments depend on the allocation of control. There are two possible governance structures inside the firm: marketing control and engineering control. In particular, in our model, only the party that controls the machine has an incentive to invest. Thus, in firms where the marketer controls the machine, the marketer invests in information about the value of the final good, whereas in firms where the engineer controls the machine, the engineer invests instead in cost reduction and relies solely on the price mechanism for information about the value of the final good. Naturally, if the price mechanism is more informative, the returns to investing in information are lower so firms have a greater incentive to choose engineer ownership and invest instead in cost reduction. As in rational-expectations models, however, when fewer parties invest in gathering information, the price mechanism becomes less informative, thereby making downstream ownership more attractive. An industry equilibrium must balance these two forces. We show that a unique equilibrium exists and is often interior. In this sense, the price mechanism induces heterogenous behavior among homogeneous firms.

2.2 Statement of the Problem

There is a unit mass of risk-neutral firms. Each firm $i \in [0, 1]$ consists of two parties, denoted E_i and M_i, and a machine that is capable of developing one intermediate good (a "widget") into one final good at cost $c_i \sim U[\underline{c}, \bar{c}]$. The machine can be controlled by either party, but it is relationship-specific (i.e., the machine is useless outside the firm). If party E_i controls the machine, we say that the governance structure in firm i is $g_i = E$, whereas if party M_i controls the machine, we say that $g_i = M$.

Final goods have an uncertain value. Party M_i can invest at private cost K_M to learn the value of a final good in the market, $v \sim U[\underline{v}, \bar{v}]$. If M_i incurs this cost, E_i knows that M_i is informed but does not herself observe v. Party E_i can invest at cost K_E in reducing the cost of operating the firm's machine. If she incurs this cost, both parties observe that c_i is reduced to $c_i - \Delta$, where $\Delta \leq \underline{c}$.

We embed these firms in our rational-expectations model of price formation in goods markets from GHP.

Firms may purchase one in the widget market. The supply of widgets, x, is random and inelastic. Assume $x \sim U[\underline{x}, \bar{x}]$.

Equilibrium in the market for widgets occurs at the price p that equates supply and demand (from informed and uninformed firms). In making decisions about widgets, firms that are not directly informed about v make rational inferences about v from the market price for widgets. Firms choose their governance structures (i.e., machine control) taking into account the information that will be inferred from the market price and hence the relative returns from the two parties' investments.

2.3 Timing and Assumptions

We now state the timing and assumptions of the model more precisely. We comment on these assumptions in Section 2.3. There are six periods.

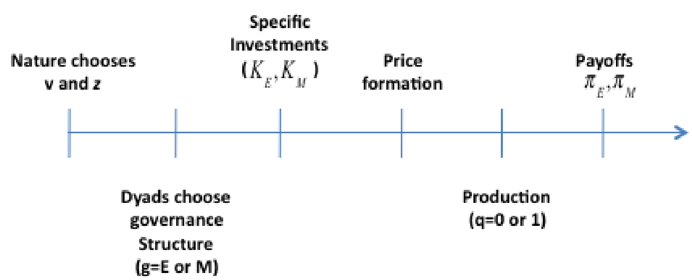

Figure 1: Timeline

In the first period, industry-level uncertainty is resolved: the value of a final good v is drawn from $U[\underline{v}, \bar{v}]$ and the widget supply x is drawn from $U[\underline{x}, \bar{x}]$, but neither of these variables is observed by any parties.

In the second period, each firm negotiates a governance structure $g_i \in \{E, M\}$: under $g_i = E$, party E_i controls the machine that can develop one widget into one final good; under $g_i = M$, party M_i controls this machine. This negotiation of governance structure occurs via Nash bargaining.

In the third period, parties E_i and M_i simultaneously choose whether to make relationship specific investments (or not) at costs K_E and K_M, respectively. In partial accordance with incomplete contracting models, we assume that the acts of making these investments are observable but not verifiable, but we depart from that literature (in a manner that is natural in our setting) by assuming that the outcome of the marketer's investment (namely, learning v) is observable to only M_i, not E_i.

In the fourth period, price formation takes place, in two steps. In period 4a, the parties E_i and M_i commonly observe $c_i \sim U[\underline{c}, \bar{c}]$, the raw cost of running their machine, as well as $\delta_i \in \{0, \Delta\}$, the amount of cost reduction achieved by E_i's specific investment. Also, M_i (but not E_i) observes $\varphi_i \in \{\emptyset, v\}$, a signal about the value v of the final good, where $\varphi_i = \emptyset$ is the uninformative signal that obtains if party M_i has not invested K_M in period 3, and $\varphi_i = v$ is the perfectly informative signal received if K_M has been invested. It is useful to introduce the following notation for the parties' information sets: $s_i^M = (c_i, \delta_i, \varphi_i)$, $s_i^E = (c_i, \delta_i, \emptyset)$, and $s_i = (s_i^M, s_i^E)$. In period 4b, the market for widgets clears at price p. In particular, any firm may buy a widget.

In the fifth period, production occurs: if the party in control of the machine in firm i has a widget, then he or she can run the machine to develop the widget into a final good at cost $c_i - \delta_i$. We denote the decision to produce a final good by $q_i = 1$ and the decision not to do so by $q_i = 0$. Off the equilibrium path, one party might control the machine and the other a widget, in which case the parties bargain over the widget and then the machine controller makes the production decision. We assume that cashflow rights and control rights are linked.

Finally, in the sixth period, final goods sell for v and payoffs are realized. The expected payoffs (before v is realized) are

$$\pi_{E_i}^{g_i} = 1_{\{g_i=E\}} 1_{\{q_i=1\}} \left[E\left[v | s_i^E, p(\cdot,\cdot) = p \right] - p - (c_i - \delta_i) \right], \text{ and}$$

$$\pi_{M_i}^{g_i} = 1_{\{g_i=M\}} 1_{\{q_i=1\}} \left[E\left[v | s_i^M, p(\cdot,\cdot) = p \right] - p - (c_i - \delta_i) \right].$$

2.4 Discussion of the Model

Before proceeding with the analysis, we pause to comment on some of the modeling choices we have made.

First, we assume that the machine is firm-specific. This assumption allows us to focus on the market for widgets by eliminating the market for machines. By allowing both markets to operate, one could analyze whether the informativeness of one affects the other.

Second, we have only one asset, in contrast to the classic incomplete contracts setting. Our choice here is driven purely by parsimony; extending the model to allow more assets (and hence more control/governance structures) could be interesting.

Third, we have binary investments in cost reduction and information acquisition (at costs K_E and K_M, respectively), rather than continuous investment opportunities. It seems straightforward to allow the probability of success (in cost reduction or information acquisition) to be an increasing function of the investment level, which in turn has convex cost.

Fourth, we assume inelastic supply demand x. This uncertain supply plays the role of noise traders, making the market price for widgets only partially informative about v, so that parties may benefit from costly acquisition of information about v.

Fifth, as in GHP, our assumptions that all the random variables are uniform allow us to compute a closed-form (indeed, piece-wise linear) solution for the equilibrium price function at the industry level. This tractability is very useful in the computing the returns to alternative governance structures, at the firm level.

Sixth, as in Grossman-Stiglitz and the ensuing rational-expectations literature, our model of price formation is not an extensive-form model of strategic decision-making (including information transmission during the price-formation process), but rather a reduced-form model of price-taking behavior. See GHP for an extended discussion.

3 Individual Firm Behavior

As a building block for our ultimate analysis, we first analyze the behavior of a single firm taking the market price p as given. To begin, define the expected gross surplus (at the start of period six) for firm i as

$$\begin{aligned} GS_i &= \pi_{E_i}^{g_i} + \pi_{M_i}^{g_i} \\ &= 1_{\{q_i=1\}} \left[E\left[v | s_i^{g_i}, p\left(\cdot,\cdot\right) = p \right] - p - \left(c_i - \delta_i \right) \right]. \end{aligned}$$

Therefore, in period 5 (production), the efficient production decision is $q_i^* = 1$ if $E_{x,v}\left[v | \, s_i^{g_i}, p \right] \geq p + c_i - \delta_i$. The maximized expected gross surplus in period 5 is then

$$GS_i^*\left(g_i, s_i\right) = E_{x,v}\left[\left(v - c_i + \delta_i - p \right) q_i^*\left(g_i, s_i, p\right) | \, s_i^{g_i}, p \right].$$

Working backwards, in period 4, whoever controls the machine receives GS_i^* and the non-owner receives a constant. These payoffs determine the parties' investment incentives in period 3, as follows.

Let the subscript pair $(I, 0)$ denote the situation in which M_i invested and hence is informed about v but E_i did not invest in cost reduction, (U, Δ) the situation in which M_i did not invest but E_i did, hence reducing production costs by Δ, and $(U, 0)$ the situation in which neither invested. Now define the following:

$$\begin{aligned} \pi_{I,0} &= E_{c_i}\left[GS_i^*\left(D, s_i\right) \right] \text{ if } \varphi_i = v, \delta_i = 0, \\ \pi_{U,\Delta} &= E_{c_i}\left[GS_i^*\left(U, s_i\right) \right] \text{ if } \varphi_i = \emptyset, \delta_i = \Delta, \text{ and} \\ \pi_{U,0} &= E_{c_i}\left[GS_i^*\left(g_i, s_i\right) \right] \text{ if } \varphi_i = \emptyset, \delta_i = 0. \end{aligned}$$

Formally, these expectations are triple integrals over (c_i, x, v) space:

$$\pi_{I,0} = \int_{\underline{v}}^{\bar{v}} \int_{\underline{x}}^{\bar{x}} \int_{\underline{c}}^{v - p(x,v)} \left(v - p\left(x, v\right) - c_i \right) dF\left(c_i, x, v\right),$$

$$\pi_{U,\Delta} = \int_{\underline{v}}^{\bar{v}} \int_{\underline{x}}^{\bar{x}} \int_{\underline{c}}^{E[v|p] - p(x,v) + \Delta} \left(v - p\left(x, v\right) + \Delta - c_i \right) dF\left(c_i, x, v\right), \text{ and}$$

$$\pi_{U,0} = \int_{\underline{v}}^{\bar{v}} \int_{\underline{x}}^{\bar{x}} \int_{\underline{c}}^{E[v|p]-p(x,v)} (v - p(x,v) - c_i)\, dF(c_i, x, v),$$

where F is the joint distribution function.

Since one party's expected payoff in period 4 is independent of its investment, at most one party will invest in period 3. If E_i controls the machine ($g_i = E$), she will invest if $\pi_{U,\Delta} - K_E \geq \pi_{E,0}$. Similarly, if M_i controls the machine ($g_i = M$), he will invest if $\pi_{I,0} - K_M \geq \pi_{M,0}$. We assume that K_E and K_M are small relative to the benefits of investment, so at least one party will invest.[2]

To proceed, we need to compute the price function $p(x,v)$. This involves analyzing the behavior of other firms, and it is to this task that we now turn.

4 Rational Expectations in the Market for Intermediate Goods

Recall that there is a unit mass of firms indexed by $i \in [0,1]$. Who buys a widget? Define $c_M(v,p) = v - p$ to be the highest cost at which a marketer that has invested in information (and hence knows v) would be prepared to produce a final good, and similarly let $c_E(p) = E[v|\, p] - p + \Delta$ be the highest cost at which an engineer that has invested in cost reduction (but not information) would be prepared to produce. Suppose (as we will endogenize below) that a fraction λ of firms have M control (and hence know v), whereas fraction $1-\lambda$ have E control (and hence costs reduced by Δ).

Firm i buys a widget if its development costs are sufficiently low (i.e., $E_{x,v}[v|\, s_i, p] - p \geq c_i - \delta_i$). Hence, demand for widgets is given by

[2] This condition can be stated in terms of primitives of the model, but since this is the economic assumption we are making, we state it in this fashion.

$$\begin{aligned}
&\Pr\left[c_i - \delta_i \leq E_{x,v}\left[v|\, s_i, p\left(x,v\right) = p\right] - p\right] \\
&= \left[\lambda \Pr\left[c_i \leq v - p\right] + \left(1-\lambda\right) \Pr\left[c_i - \Delta \leq E\left[v|\, p\left(x,v\right) = p\right] - p\right]\right] \\
&= \left[\lambda \frac{v - p - \underline{c}}{\bar{c} - \underline{c}} + \left(1-\lambda\right) \frac{E\left[v|\, p\left(x,v\right) = p\right] + \Delta - p - \underline{c}}{\bar{c} - \underline{c}}\right]
\end{aligned}$$

The market-clearing price equates demand and supply, which recall is x.

$$p = \left(1-\lambda\right) E\left[v|\, p\left(x,v\right) = p\right] + \lambda v - \left(\bar{c} - \underline{c}\right) x + \left(1-\lambda\right)\Delta - \underline{c}.$$

The conditional expectation of v given p therefore must satisfy

$$E\left[v|\, p\left(\cdot,\cdot\right) = p\right] \equiv \frac{p + \left(\bar{c} - \underline{c}\right) x + \underline{c} - \left(1-\lambda\right)\Delta - \lambda v}{1-\lambda}, \tag{1}$$

where the equivalence relation reminds us that (1) must hold as an identity in x and v.

Definition 1 *Assume fractions $\mu_{I\Delta}, \mu_{I0}, \mu_{U\Delta}, \mu_{U0}$ of firms are, respectively, informed and have cost reduction, informed and do not have cost reduction, uninformed and have cost reduction, and uninformed and do not have cost reduction. Let $\mu = \left(\mu_{I\Delta}, \mu_{I0}, \mu_{U\Delta}, \mu_{U0}\right)$. A* ***rational expectations equilibrium*** *("REE") is a price function $p\left(x,v\right)$ and a production allocation $\{q_i\}_{i\in\{0,1\}}$ such that*

1. *$q_i = q_i^*\left(q_i, s_i, p\right)$ for all i, and*

2. *The market for widgets clears for each $\left(x,v\right) \in [\underline{x}, \bar{x}] \times [\underline{v}, \bar{v}]$.*

The above analysis implies $\mu_{I\Delta} = 0$, and K_E and K_M small implies $\mu_{U0} = 0$. Let $\lambda = \mu_{I0}$ and $1-\lambda = \mu_{U\Delta}$. The problem of finding a rational-expectations price function in this model becomes one of finding a fixed point of (1). In GHP, we solve for this fixed point, finding it to be piecewise-linear over three regions of (x,v) space: a low-price region, a moderate-price region, and a high-price region.

Proposition 1 *Given λ, there exists an REE characterized by a price function*

$$p(x,v) = 1_{\{(x,v)\in R_\lambda^1\}} p^1(x,v) + 1_{\{(x,v)\in R_\lambda^2\}} p^3(x,v) + 1_{\{(x,v)\in R_\lambda^3\}} p^3(x,v),$$

where $p^j(x,v) = \beta_0^j + \beta_1^j x + \beta_2^j v$ for $j = 1,2,3$.

To build some intuition for this result, consider the figure below, which shows the three regions of (x,v) space, R_λ^j for $j = 1,2,3$. The low-price region R_λ^1 begins from the lowest feasible price, p_L at $(\bar{x}, \underline{v})$, and extends up to the price $\bar{p}$ at $(\bar{x}, \bar{v})$. The moderate-price region R_λ^2 then extends from price $\bar{p}$ up to the price $\underline{p}$ at $(\underline{x}, \underline{v})$, and the high-price region R_λ^3 extends from $\underline{p}$ up to the highest feasible price, p_H at $(\underline{x}, \bar{v})$.

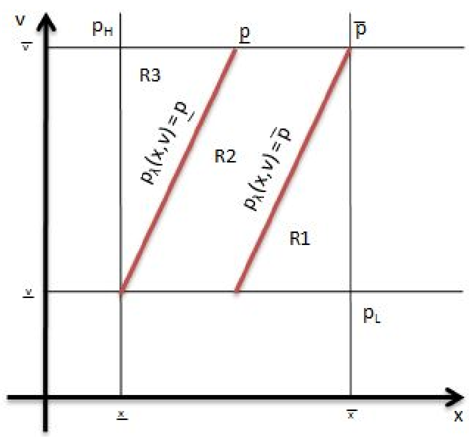

Figure 2: Regions of Piecewise-Linear Pricing Function

Within each region, the iso-price loci are linear. In particular, solving $p^j(x,v) = p$ for v yields

$$v = -\frac{\beta_j^1}{\beta_j^2} x + \frac{p - \beta_j^0}{\beta_j^2}$$

as an iso-price line in (x,v) space. Because x and v are independent and uniform, every (x,v) point on this line is equally likely. Thus, after observing p, an informed party projects this line onto the v-axis and concludes that the conditional distribution of v given p is uniform, with support depending on which region p is in. For example, if $p < \bar{p}$ then the lower bound on v is $\underline{v}$ and the upper bound is some $\bar{v}(p) < \bar{v}$. Alternatively, if $\bar{p} < p < \underline{p}$ then the lower and upper bounds on v are $\underline{v}$ and $\bar{v}$, so p is uninformative. Finally, if $p > \underline{p}$ then the lower bound is some $\underline{v}(p) > \underline{v}$ and the upper bound is $\bar{v}$.

Given this uniform conditional distribution of v given p, the conditional expectation on the left-hand side of (1) is then the average of these upper and lower bounds on v. The coefficients β_0^j, β_1^j, and β_2^j can then be computed by substituting $p^j(x, v)$ for p on both sides of (1) and equating coefficients on like terms so that (1) holds as an identity.

5 Industry Equilibrium

To recapitulate, Section 3 analyzed the production decision, taking $p(\cdot, \cdot)$ as exogenous, and Section 4 endogenized prices. In this section, we endogenize the governance-structure choices of each firm and define an industry equilibrium as follows.

Definition 2 *An* ***industry equilibrium*** *is a set of firms of mass* λ^*, *a price function* $p(x, v)$, *and a production allocation* $\{q_i\}_{i \in \{0,1\}}$ *such that*

1. *Each firm optimally chooses* g_i, *with a fraction* λ^* *choosing* $g_i = M$;

2. *Each party optimally chooses whether or not to invest;*

3. $q_i = q_i^*(q_i, s_i, p)$; *and*

4. *The market for widgets clears for each* $(x, v) \in [\underline{x}, \bar{x}] \times [\underline{v}, \bar{v}]$.

The choice in period 2 is between the two possible governance structures: $g_i = E$ or $g_i = M$. The *ex ante* expected surpluses from choosing the two governance structures are

$$
\begin{aligned}
TS^U(\lambda) &= \pi_{U\Delta}(\lambda) - K_E, \text{ and} \\
TS^D(\lambda) &= \pi_{I0}(\lambda) - K_M.
\end{aligned}
$$

In an interior equilibrium, firms must be indifferent between the two governance structures. Thus our goal is to find λ^* such that $TS^U(\lambda^*) = TS^D(\lambda^*)$ and to characterize how λ^* varies as we change the parameters of the model. For simplicity we assume that $K_E = K_M = K$. (The case where $K_E \neq K_M$ is discussed at the end of this section.) We therefore seek λ^* such that

$$\pi_{I0}\left(\lambda^*\right) = \pi_{U\Delta}\left(\lambda^*\right),$$

or equivalently,

$$\pi_{I,0}\left(\lambda^*\right) - \pi_{U,0}\left(\lambda^*\right) = \pi_{U,\Delta}\left(\lambda^*\right) - \pi_{U,0}\left(\lambda^*\right). \tag{2}$$

To keep notation compact, let $\sigma_v = \frac{1}{\sqrt{12}}\left(\bar{v} - \underline{v}\right)$ and $\sigma_x = \frac{1}{\sqrt{12}}\left(\bar{x} - \underline{x}\right)$. We will make use of the following fact (which is derived in the appendix).

Fact 1 *Assume* $\lambda \leq \left(\bar{c} - \underline{c}\right)\frac{\sigma_x}{\sigma_v}$. *Then*

$$\begin{aligned}
\pi_{I,0}(\lambda) - \pi_{U,0}(\lambda) &= \frac{1}{2}\frac{\sigma_v^2}{\bar{c} - \underline{c}}\left(1 - \frac{1}{2}\frac{\lambda}{\bar{c} - \underline{c}}\frac{\sigma_v}{\sigma_x}\right) \text{ and} \\
\pi_{U,\Delta}(\lambda) - \pi_{U,0}(\lambda) &= \frac{\Delta^2}{\bar{c} - \underline{c}}\lambda - \frac{1}{2}\frac{\Delta^2}{\bar{c} - \underline{c}} + \mu_x\Delta.
\end{aligned}$$

Observe that the first expression is decreasing in λ and the second is increasing in λ. This leads to the following characterization of industry equilibrium.

Proposition 2 *Assume* $\left(\bar{c} - \underline{c}\right)\frac{\sigma_x}{\sigma_v} \geq 1$. *For all* $\bar{c}, \underline{c}, \sigma_x, \sigma_v, \Delta > 0$ *with* $\underline{c} \geq \Delta$, *there exists an industry equilibrium. Further,*

$$\lambda^* = \frac{\sigma_v^2 + \Delta^2 - 2\left(\bar{c} - \underline{c}\right)\mu_x\Delta}{\frac{\sigma_v^2}{2}\frac{\sigma_v/\sigma_x}{\bar{c} - \underline{c}} + 2\Delta^2} \tag{3}$$

if the right-hand side of (3) is in $[0, 1]$. *If the right-hand side of (3) is less than 0, then* $\lambda^* = 0$; *if it is greater than 1, then* $\lambda^* = 1$.

Proof. If $\sigma_v^2 \leq 2\left(\bar{c} - \underline{c}\right)\mu_x\Delta - \Delta^2$, then $\pi_{U,0}(0) \leq \pi_{U,\Delta}(0)$ and thus, since the left hand side is decreasing in λ, it follows that $\lambda^* = 0$. Similarly, if $\sigma_v^2\left(1 - \frac{1}{2}\frac{1}{\bar{c} - \underline{c}}\frac{\sigma_v}{\sigma_x}\right) \geq 2\left(\bar{c} - \underline{c}\right)\mu_x\Delta + \Delta^2$, then $\pi_{U,0}(1) \geq \pi_{U,\Delta}(1)$, and since the right hand side is increasing in λ, we must have that

$\lambda^* = 1$. Otherwise, we want to find λ^* such that

$$
\begin{aligned}
0 &= \left(TS^E - TS^M\right)(\lambda^*) \\
&= \frac{\sigma_v^2 + \Delta^2 - 2\left(\bar{c} - \underline{c}\right)\mu_x \Delta}{2\left(\bar{c} - \underline{c}\right)} - \frac{\lambda^*}{2\left(\bar{c} - \underline{c}\right)}\left(\frac{\sigma_v/\sigma_x}{\bar{c} - \underline{c}}\frac{\sigma_v^2}{2} + 2\Delta^2\right),
\end{aligned}
$$

which yields the expression in the statement of the proposition. ■

Proposition 2 is our main result, establishing that there exists a unique industry equilibrium and providing an explicit expression for the proportion of firms who choose each of the governance structures. As the proposition makes clear, this proportion may well be interior. Recall, however, that our firms are homogeneous *ex ante*, so an incomplete-contracts style analysis (taking each firm in isolation) would prescribe that they all choose the same governance structure. In this sense, the informativeness of the price mechanism can induce heterogeneous behaviors from homogenous firms. To put this point differently, in this model, the price mechanism can be seen as endogenizing the parameters of the incomplete contracts model so that firms are indifferent between governance structures. In a richer model, with heterogeneous investment costs, almost every firm would have strict preferences between governance structures, with only the marginal firm being indifferent.

We are also able to perform some comparative statics. First, when the *ex ante* level of fundamental uncertainty increases (i.e., σ_v is higher), the return to investing in acquiring information increases, so λ increases to the point where the price mechanism has become sufficiently informative to counteract the increase in σ_v. Second, an increase in noise (i.e., σ_x is higher) has an identical effect. Finally, an increase in Δ has two effects. The first is the partial-equilibrium channel through which an increase in the benefits of choosing engineer ownership (and hence investing in cost reduction) makes engineer control relatively more appealing, reducing λ. In an industry equilibrium, however, there is also a price effect. For a fixed fraction $1 - \lambda$ of parties that invest in cost reduction, an increase in Δ makes widgets more valuable, which in turn increases demand and hence average prices. Since firms with engineer control purchase widgets over a larger region of the c_i space than do firms with marketing control, the former face this increase in average price level relatively more than do firms with marketer control, so the price effect militates towards an increase in λ. Which of these two effects dominates depends on the parameters of the model.

Proposition 3 *Assume* $(\bar{c} - \underline{c}) \frac{\sigma_x}{\sigma_v} \geq 1$. *For all* $\bar{c}, \underline{c}, \sigma_x, \sigma_v, \Delta > 0$ *with* $\underline{c} \geq \Delta$ *and* $\lambda^* \in (0, 1)$, *we have that: (i)* λ^* *is increasing in* σ_v, *(ii)* λ^* *is increasing in* σ_x, *(iii)* λ^* *is decreasing in* μ_x, *and (iv) if* $\Delta < (\bar{c} - \underline{c}) \mu_x$, *then* λ^* *is decreasing in* Δ, *otherwise there exists a* $\hat{\sigma}_v$ *satisfying* $0 \leq \hat{\sigma}_v \leq \frac{2\Delta(\bar{c}-\underline{c})\mu_x}{3\Delta+(\bar{c}-\underline{c})\mu_x}$ *such that* λ^* *is decreasing in* Δ *whenever* $\sigma_v > \hat{\sigma}_v$ *and increasing in* Δ *whenever* $\sigma_v < \hat{\sigma}_v$.

Proof. *See appendix.* ■

5.1 REE meets incomplete contracts

A final observation is that the theory of the firm sheds new light on the functioning of the price mechanism. Partially-revealing REE models compare the benefits of acquiring information to the exogenously specified costs of acquiring information. As our model shows, what matters is not only these exogenous costs K_M, but also the opportunity cost of choosing a governance structure that provides incentives to invest in information (namely, the foregone opportunity for cost reduction). To analyze this issue, consider the expression for λ^* when $K_E \neq K_M$:

$$\lambda^* = \frac{\sigma_v^2 + \Delta^2 - 2(\bar{c} - \underline{c})(\mu_x \Delta + K_M - K_E)}{\frac{\sigma_v^2}{2} \frac{\sigma_v / \sigma_x}{\bar{c} - \underline{c}} + 2\Delta^2}.$$

Note the presence of production parameters, such as Δ and K_E, which have nothing *per se* to do with market clearing or price formation. More importantly, note that comparative statics regarding the informativeness of the price mechanism, such as $\partial \lambda^* / \partial K_M$, can depend on production parameters such as Δ.

In addition to comparative statics that illustrate the potential effects of production parameters on rational-expectations equilibrium, we can also say something about how the production environment affects markets. For example, in GHP we showed that (as in Grossman and Stiglitz, 1980) market thickness depends on λ^*, with concomitant implications for economic efficiency and welfare. In this paper, therefore, market thickness depends on production parameters such as Δ and K_E.

6 Markets and Hierarchies Revisited

While our main focus is on the interaction between the choice of internal organization structures by individual firms and the informativeness of the market's price mechanism, a straightforward reinterpretation of our model also sheds light on the interaction between the choice of individual firms' boundaries and the informativeness of the price mechanism. In this section we explore this reinterpretation, thereby taking seriously Coase's (1937: 359) exhortation that "it is surely important to enquire why co-ordination is the work of the price mechanism in one case and of the entrepreneur in the other" (emphasis added). That is, while Coase was explicit that the "price mechanism" is the chief alternative to internal organization, and Williamson's (1975) title famously emphasized "Markets" as the alternative to hierarchy, over the next 35 years the market disappeared from the literature on firms' boundaries. Instead, the literature focused on non-integration versus integration at the transaction level, rather than the functioning of the price mechanism at the market level. Like our analysis of internal organization structures, this section shows that omitting the price mechanism from the analysis of firms' boundaries can be problematic. In particular, we find that incentives to make specific investments affect the informativeness of the price mechanism and vice versa.

To reinterpret our model, consider a vertical production process with three stages (1, 2, and 3) and a different asset used at each stage (A_1, A_2, and A_3). There are again two parties, now denoted upstream (formerly E) and downstream (formerly M). The conditions of production are such that the upstream party (U) owns A_1 and downstream (D) A_3, so there are only two governance structures of interest (namely, E owns A_2 or D owns it), so A_2 is the machine from our original model. In this intermediate-good setting (where upstream necessarily owns A_1 and downstream A_3), we interpret U ownership of A_2 as forward vertical integration and D ownership as backward. Beyond this reinterpretation of governance structures in terms of firms' boundaries, all the formal aspects of the model are unchanged.

Under this reinterpretation, analogs of Propositions 1 through 3 continue to hold.[3] In particular, our characterizations of the rational-expectations equilibrium and the industry equilibrium continue to hold, as do the comparative-statics results. Given this reinterpretation, the next two sub-sections explore the implications of the informativeness of the price mechanism for two leading theories of firms' boundaries: the property-rights theory (PRT) of Grossman & Hart (1986) and Hart & Moore (1990), and the transaction-cost economics (TCE) theory of Williamson (1971, 1975, 1979).Williamson (1971)

6.1 PRT Meets REE

Property-rights theory emphasizes the importance of specific investments for the choice of governance structure: whichever party's investment is more important should own the relevant asset. We can mimic the PRT by eliminating the role of the price mechanism in our model, by supposing that a firm believes $p(x,v) \equiv p$ for all λ, x, v and hence does not recognize that prices are informative.

Fact 2 *If $p(x,v) \equiv p$ for all λ, x, v then the benefits from choosing $g_i = U$ are given by*

$$\pi_{U,0} - \pi_{U,0} = \frac{1}{2} \frac{\sigma_v^2}{\bar{c} - \underline{c}},$$

[3]For formal statements and proofs see an earlier working paper version: Gibbons, Holden and Powell (2009). Available at www.nber.org.

and the benefits from choosing $g_i = D$ are

$$\pi_{U,\Delta} - \pi_{U,0} = \frac{1}{2}\frac{\Delta^2 + 2\left(\mu_v - p - \underline{c}\right)\Delta}{\bar{c} - \underline{c}}.$$

The dyad therefore chooses downstream ownership if $\sigma_v^2 > \Delta^2 + 2\left(\mu_v - p - \underline{c}\right)\Delta$, chooses engineer ownership if this inequality is reversed, and is indifferent if the inequality is replaced with an equality. Generically, one of these two inequalities must hold, so the PRT prescription will be either that all firms are integrated or that all firms are non-integrated (because the firms are identical *ex ante*).

In our model, however, the informativeness of the price mechanism endogenizes the returns to specific investments. In particular, firms that would have chosen to invest in information acquisition (by choosing downstream ownership of the machine) under the assumptions of Fact 2 may now free-ride on the information contained in the market price and choose instead to invest in cost reduction (by choosing to have engineer ownership of the machine). In fact, in our model, certain governance structures may be sustained in equilibrium *only* because the price system allows some firms to benefit from the information-acquisition investments of others.

More specifically, as we began to explain after Proposition 2, the equilibrium fraction of firms choosing downstream ownership in our model, λ^* in (3), is often interior, not zero or one, as is generally true in a PRT analysis.

Figure 4 illustrates the difference between our analysis and PRT by plotting λ^*_{PRT} versus our λ^* from Proposition 2. To plot this figure, we fix $\Delta = 1/4, \bar{c} - \underline{c} = 1$, and $\mu_x = 0.8$, so that a PRT analysis predicts that all firms will choose downstream ownership (i.e., $\lambda^*_{PRT} = 1$) if $\sigma_v^2 > 0.3375$, all firms will choose engineer ownership ($\lambda^*_{PRT} = 0$) when $\sigma_v^2 < 0.3375$, and firms will be indifferent ($\lambda^*_{PRT} \in [0, 1]$) when $\sigma_v^2 = 0.3375$. The figure shows our equilibrium λ^* as a function of σ_v^2 for three different values of σ_x (namely, $1/10, 1$, and 10, with λ^* falling with σ_x for a fixed σ_v^2). Our equilibrium converges to $\lambda^* = 1$ more slowly (and especially slowly for lower values of σ_x).

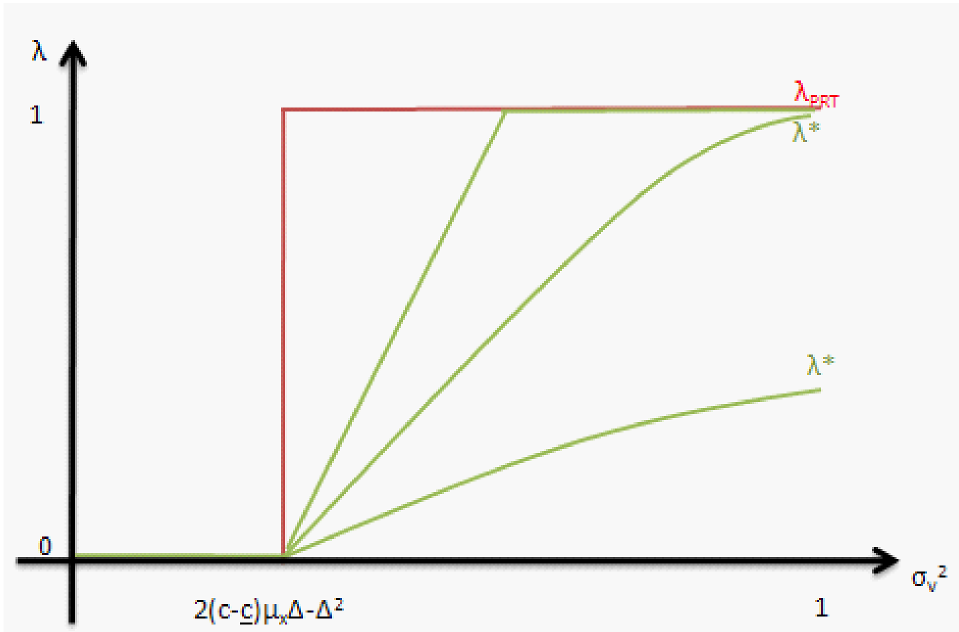

Figure 3: Comparison with PRT

Nonetheless, Figure 4 suggests that empirical tests of PRT that focus solely on the importance of specific investments may be misleading, by failing to consider the role that the price mechanism plays in endogenizing the returns to specific investments. Of course firms may well not be *ex ante* identical, and thus a mixture of these two effects may determine the choice of governance structure.

6.2 TCE Meets REE

Explicitly commenting on Hayek's (1945) discussion of the price mechanism, Williamson (1975: 5) argues that "prices often do not qualify as sufficient statistics and that a substitution of internal organization (hierarchy) for market-mediated exchange often occurs on this account."

Our model allows us to assess those observations, if we can be precise about two things: (i) what it means for prices not to "qualify as sufficient statistics", and (ii) what is meant by "market-mediated exchange." A natural way to think about the first of these is the following.

Definition 3 *The **equilibrium informativeness of the price system** is the expected reduction in variance* $E_{x,v}\left[\sigma_v^2 - \sigma_{v|p}^2\right]$ *that is obtained by conditioning on prices.*

In our model, the informativeness of the price system is given by

$$E\left[\sigma_v^2 - \sigma_{v|p}^2\right] = \lambda \frac{\sigma_v^2}{2} \frac{\sigma_v / \sigma_x}{\bar{c} - \underline{c}}.$$

Naturally, this informativeness is increasing in the fraction of firms that become informed, λ. And in our model "market intermediation" also has a natural interpretation: it means relying on information about v from the price mechanism, rather than acquiring it directly (i.e., engineer ownership rather than downstream). In these terms, Williamson's observation can be stated as: when $E\left[\sigma_v^2 - \sigma_{v|p}^2\right]$ falls, λ^* increases.

In our model, the direct effect of λ runs contrary to Williamson's observation: $E\left[\sigma_v^2 - \sigma_{v|p}^2\right]$ increases with λ. But, of course, λ is endogenous, so it matters what causes λ to increase and what other effects that underlying change has on $E\left[\sigma_v^2 - \sigma_{v|p}^2\right]$. For example, if σ_x increases then it can be shown that informativeness decreases and λ^* increases, as Williamson conjectured. On the other hand, many other changes in exogenous variables can lead simultaneously to an increases in informativeness and an *increase* in λ^*. That is, it is possible for the price system to "work better" and at the same time be used less. For example, it is straightforward to see that an increase in μ_x decreases λ^* and decreases informativeness. And an increase in $\bar{c} - \underline{c}$ can do likewise, as reported in the following result.

Proposition 4 *Assume* $(\bar{c} - \underline{c}) \frac{\sigma_x}{\sigma_v} \geq 1$ *and* $\lambda^* \in (0,1)$. *Define* $\omega = \frac{1}{\bar{c} - \underline{c}}$. *If*

$$\frac{1}{2} \frac{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega}{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega + \Delta^2} \frac{\sigma_v^2 + \Delta^2}{2(\bar{c} - \underline{c}) \Delta} < \mu_x < \frac{\sigma_v^2 + \Delta^2}{2(\bar{c} - \underline{c}) \Delta},$$

then $\frac{\partial E_{x,v}\left[\sigma_v^2 - \sigma_{v|p}^2\right]}{\partial \omega} > 0$ *and* $\frac{\partial \lambda^*}{\partial \omega} > 0$.

Proof. See appendix. ■

7 Conclusion

We view firms and the market not only as alternative ways of organizing economic activity, but also as institutions that interact and shape each other. In particular, by combining features of the incomplete contracting theory of of firms' boundaries and internal governance, and the rational-expectations theory of the price mechanism, we have developed a model that incorporates two, reciprocal considerations. First, firms operate in the context of the market (specifically, the informativeness of the price mechanism affects parties' optimal governance structures). And second, the market for an intermediate good is made up of firms (specifically, parties' governance structures affect how they behave this market and hence the informativeness of the price mechanism).

In the primary interpretation of our model–that of internal organization of firms–we provide a formal explanation for why similar (possibly *ex ante* identical) firms choose different organizational forms. Our analysis also demonstrates that viewing an individual firm, or transaction, as the unit of analysis can be misleading. Because of the interaction between firm-level governance structure choices and the industry-wide informativeness of the price mechanism, equilibrium governance structure choices are determined to some extent by industry-wide factors.

We also showed that our model can be reinterpreted to speak to issues about firm boundaries. There we made similar points, but showed that they relate to the old and ongoing debate about the determinants of firm boundaries. Again, taking the informativeness of prices seriously implies that both property-rights theory and transaction-cost economics abstract from some interesting and important issues by focusing on the transaction as the unit of analysis.

To develop and analyze our model, we have imposed several strong assumptions that might be relaxed in future work. For example, to eliminate the market for machines, we assumed that machines are dyad-specific. Also, as in our paper on price formation (where we analyze individual investors instead of firms), we ignore the possibility of strategic information transmission before or during the price-formation process.

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8 Appendix

8.1 Derivation of Fact 1

$$\begin{aligned} E_{x,v,c_i}\left[\pi_{U,0}\left(\lambda\right)\right] - E_{x,v,c_i}\left[\pi_{U,0}\left(\lambda\right)\right] &= \frac{1}{2}\frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\left(v^2-\mu_{v|p}^2\right)dxdv \\ &= \frac{1}{2}\frac{E_{x,v}\left[\sigma_{v|p}^2\right]}{\bar{c}-\underline{c}} = \frac{1}{2}\frac{\sigma_v^2}{\bar{c}-\underline{c}}\left(1-\frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}-\underline{c}}\right), \end{aligned}$$

which is continuous and strictly decreasing in λ and similarly,

$$
\begin{aligned}
E_{x,v,c_i}\left[\pi_{U,\Delta}(\lambda)\right]-E_{x,v,c_i}\left[\pi_{U,0}(\lambda)\right] &= \frac{\Delta^2}{2\left(\bar{c}-\underline{c}\right)}+\Delta\frac{E_{x,v}\left[\mu_{v|p}(x,v)\right]-\underline{c}-E_{x,v}\left[p_\lambda(x,v)\right]}{\left(\bar{c}-\underline{c}\right)} \\
&= \frac{\Delta^2}{\bar{c}-\underline{c}}\lambda-\frac{\Delta^2}{2\left(\bar{c}-\underline{c}\right)}+\mu_x\Delta,
\end{aligned}
$$

which is continuous and strictly increasing in λ. For the last equalities in these two expressions, we use the following three facts:

$$
\begin{aligned}
E_{x,v}\left[\mu_{v|p}\right] &= \mu_v, \\
E_{x,v}\left[\sigma^2_{v|p}\right] &= \sigma_v^2\left(1-\frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}-\underline{c}}\right), \text{ and} \\
E_{x,v}\left[p_\lambda(x,v)\right] &= \mu_v+(1-\lambda)\Delta-\mu_x\left(\bar{c}-\underline{c}\right)-\underline{c},
\end{aligned}
$$

which we now prove. First note that when $\lambda \leq \left(\bar{c}-\underline{c}\right)\frac{\sigma_x}{\sigma_v}$, $p_\lambda(x,v)=\sum_{j=1}^3 1_{\left\{(x,v)\in R_\lambda^j\right\}}p_\lambda^j(x,v)$, where

$$
\begin{aligned}
p_\lambda^1(x,v) &= (1-\lambda)\frac{\underline{v}+\left(\left(\bar{c}-\underline{c}\right)/\lambda\right)\bar{x}}{2}+(1-\lambda)\Delta-\underline{c}+\frac{1+\lambda}{2}v-\frac{1+\lambda}{2}\frac{\bar{c}-\underline{c}}{\lambda}x \\
p_\lambda^2(x,v) &= (1-\lambda)\frac{\underline{v}+\bar{v}}{2}+(1-\lambda)\Delta-\underline{c}+\lambda v-\left(\bar{c}-\underline{c}\right)x \\
p_\lambda^3(x,v) &= (1-\lambda)\frac{\left(\left(\bar{c}-\underline{c}\right)/\lambda\right)\underline{x}+\bar{v}}{2}+(1-\lambda)\Delta-\underline{c}+\frac{1+\lambda}{2}v-\frac{1+\lambda}{2}\frac{\bar{c}-\underline{c}}{\lambda}x,
\end{aligned}
$$

and

$$
\begin{aligned}
R_\lambda^1 &= \left\{(x,v):p_\lambda^1(x,v)\leq p_\lambda^1\left(\bar{x},\bar{v}\right)\right\} \\
R_\lambda^2 &= \left\{(x,v):p_\lambda^2\left(\bar{x},\bar{v}\right)<p_\lambda^2(x,v)\leq p_\lambda^2\left(\underline{x},\underline{v}\right)\right\} \\
R_\lambda^3 &= \left\{(x,v):p_\lambda^3\left(\underline{x},\underline{v}\right)<p_\lambda^3(x,v)\right\}.
\end{aligned}
$$

We can rewrite the prices as

$$\begin{aligned}
p_\lambda^1(x,v) &= p_\lambda^2(x,v) - \frac{1-\lambda}{2}\left[(\bar{v}-v) - \frac{\bar{c}-\underline{c}}{\lambda}(\bar{x}-x)\right] \\
p_\lambda^2(x,v) &= (1-\lambda)\frac{\underline{v}+\bar{v}}{2} + (1-\lambda)\Delta - \underline{c} + \lambda v - (\bar{c}-\underline{c})x \\
p_\lambda^3(x,v) &= p_\lambda^2(x,v) + \frac{1-\lambda}{2}\left[(v-\underline{v}) - \frac{\bar{c}-\underline{c}}{\lambda}(x-\underline{x})\right].
\end{aligned}$$

For simplicity of notation, define $R_\lambda^j(v) = \left\{x : (x,v) \in R_\lambda^j\right\}$. That is

$$\begin{aligned}
R_\lambda^1(v) &= \left[\bar{x} - \frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v), \bar{x}\right] \\
R_\lambda^2(v) &= \left[\underline{x} + \frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v}), \bar{x} - \frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)\right] \\
R_\lambda^3(v) &= \left[\underline{x}, \underline{x} + \frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})\right].
\end{aligned}$$

Finally, note that

$$\begin{aligned}
\mu_{v|p}^1(x,v) &= \mu_v - \frac{1}{2}\left[(\bar{v}-v) - \frac{\bar{c}-\underline{c}}{\lambda}(\bar{x}-x)\right] \\
\mu_{v|p}^2(x,v) &= \mu_v \\
\mu_{v|p}^3(x,v) &= \mu_v + \frac{1}{2}\left[(v-\underline{v}) - \frac{\bar{c}-\underline{c}}{\lambda}(x-\underline{x})\right].
\end{aligned}$$

Claim 1 $E_{x,v}\left[\mu_{v|p}\right] = \mu_v$

Proof. Follows directly from the Law of Iterated Expectations. ■

Claim 2 $E_{x,v}\left[\sigma_{v|p}^2\right] = \sigma_v^2\left(1 - \frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}-\underline{c}}\right)$

Proof. Here, we want to compute

$$\begin{aligned} E_{x,v}\left[\sigma^2_{v|p}\right] &= \frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)}^{\bar{x}}\left(v^2-\left(\mu^1_{v|p}\right)^2\right)dxdv \\ &+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})}^{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)}\left(v^2-\left(\mu_v\right)^2\right)dxdv \\ &+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})}\left(v^2-\left(\mu^3_{v|p}\right)^2\right)dxdv \end{aligned}$$

If we substitute and rearrange, this becomes

$$\begin{aligned} E_{x,v}\left[\sigma^2_{v|p}\right] &= \frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\left(v^2-\left(\mu_v\right)^2\right)dxdv \\ &+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)}^{\bar{x}}\begin{pmatrix} \mu_v\left[(\bar{v}-v)-\frac{\bar{c}-\underline{c}}{\lambda}(\bar{x}-x)\right] \\ -\frac{1}{4}\left[(\bar{v}-v)-\frac{\bar{c}-\underline{c}}{\lambda}(\bar{x}-x)\right]^2 \end{pmatrix}dxdv \\ &-\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})}\begin{pmatrix} \mu_v\left[(v-\underline{v})-\frac{\bar{c}-\underline{c}}{\lambda}(x-\underline{x})\right] \\ +\frac{1}{4}\left[(v-\underline{v})-\frac{\bar{c}-\underline{c}}{\lambda}(x-\underline{x})\right]^2 \end{pmatrix}dxdv \end{aligned}$$

Integrating, we get

$$\begin{aligned} E_{x,v}\left[\sigma^2_{v|p}\right] &= \sigma^2_v+\frac{\sigma_v}{\sigma_x}\frac{\lambda}{\bar{c}-\underline{c}}\left(\mu_v\frac{(\bar{v}-\underline{v})}{6}-\frac{1}{4}\sigma^2_v\right)-\frac{\sigma_v}{\sigma_x}\frac{\lambda}{\bar{c}-\underline{c}}\left(\mu_v\frac{(\bar{v}-\underline{v})}{6}+\frac{1}{4}\sigma^2_v\right) \\ &= \sigma^2_v\left(1-\frac{\lambda}{2}\frac{\sigma_v/\sigma_x}{\bar{c}-\underline{c}}\right), \end{aligned}$$

which was the original claim. ■

Claim 3 $E_{x,v}\left[p_\lambda(x,v)\right]=\mu_v+(1-\lambda)\Delta-\mu_x(\bar{c}-\underline{c})-\underline{c}$

Proof. Similarly as above,

$$\begin{aligned}
E_{x,v}\left[p_\lambda(x,v)\right] &= \frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)}^{\bar{x}} p_\lambda^1(x,v)\,dxdv \\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})}^{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)} p_\lambda^2(x,v)\,dxdv \\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})} p_\lambda^3(x,v)\,dxdv.
\end{aligned}$$

If we substitute and rearrange, we get

$$\begin{aligned}
E_{x,v}\left[p_\lambda(x,v)\right] &= \frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}} p_\lambda^2(x,v)\,dxdv \\
&-\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\bar{x}-\frac{\lambda}{\bar{c}-\underline{c}}(\bar{v}-v)}^{\bar{x}} \frac{1-\lambda}{2}\left[(\bar{v}-v)-\frac{\bar{c}-\underline{c}}{\lambda}(\bar{x}-x)\right]dxdv \\
&+\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\underline{x}+\frac{\lambda}{\bar{c}-\underline{c}}(v-\underline{v})} \frac{1-\lambda}{2}\left[(v-\underline{v})-\frac{\bar{c}-\underline{c}}{\lambda}(x-\underline{x})\right]dxdv
\end{aligned}$$

or since the last two expressions are equal but with opposite signs,

$$E_{x,v}\left[p_\lambda(x,v)\right] = \mu_v + (1-\lambda)\,\Delta - (\bar{c}-\underline{c})\,\mu_x - \underline{c},$$

which is the desired expression ■

8.2 Derivation of Fact 2

Explicit computation yields the following benefit for choosing $g = U$

$$
\begin{aligned}
E\left[\pi_{U,0}\right]-E\left[\pi_{U,0}\right] &= \frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\int_{\underline{c}}^{v-p}\left(v-p-c_i\right)dc_i dx dv \\
&\quad -\frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\int_{\underline{c}}^{\mu_v-p}\left(v-p-c_i\right)dc_i dx dv \\
&= \frac{1}{2}\frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\left(v-\mu_v\right)^2 dx dv \\
&= \frac{1}{2}\frac{\sigma_v^2}{\bar{c}-\underline{c}},
\end{aligned}
$$

and similarly the benefits for choosing $g = D$ are

$$
\begin{aligned}
E\left[\pi_{U,\Delta}\right]-E\left[\pi_{U,0}\right] &= \frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\int_{\underline{c}}^{\mu_v-p+\Delta}\left(v-p+\Delta-c_i\right)dc_i dx dv \\
&\quad -\frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\int_{\underline{c}}^{\mu_v-p}\left(v-p-c_i\right)dc_i dx dv \\
&= \frac{1}{\bar{c}-\underline{c}}\frac{1}{\bar{v}-\underline{v}}\frac{1}{\bar{x}-\underline{x}}\int_{\underline{v}}^{\bar{v}}\int_{\underline{x}}^{\bar{x}}\left(\left(v-p\right)\Delta-\underline{c}\Delta+\frac{\Delta^2}{2}\right)dx dv \\
&= \frac{1}{2}\frac{\Delta^2+2\left(\mu_v-p-\underline{c}\right)\Delta}{\bar{c}-\underline{c}}.
\end{aligned}
$$

8.3 Omitted Proofs

Proof of Proposition 3. To establish that λ^* is increasing in σ_v, note that at $\lambda = 0$, the gains from choosing integration (and hence becoming informed) instead of non-integration (and hence enjoying a cost reduction) are given by

$$
\left(TS^U - TS^D\right)(\lambda = 0) = \frac{\sigma_v^2+\Delta^2-2\left(\bar{c}-\underline{c}\right)\mu_x\Delta}{2\left(\bar{c}-\underline{c}\right)}
$$

and at $\lambda = 1$, the gains from choosing integration over non-integration are

$$
\left(TS^U - TS^D\right)(\lambda = 1) = \frac{\sigma_v^2}{2\left(\bar{c}-\underline{c}\right)}\left(1-\frac{1}{2}\frac{\sigma_v/\sigma_x}{\bar{c}-\underline{c}}\right)-\frac{\Delta^2+2\left(\bar{c}-\underline{c}\right)\mu_x\Delta}{2\left(\bar{c}-\underline{c}\right)}.
$$

Since we are at an interior solution, $\left(TS^U - TS^D\right)(\lambda = 0) > 0$ and $\left(TS^U - TS^D\right)(\lambda = 1) < 0$. Next, note that $\left(TS^U - TS^D\right)(\lambda = 0)$ is increasing in σ_v and $\left(TS^U - TS^D\right)(\lambda = 1)$ is increasing in σ_v if $(\bar{c} - \underline{c}) \frac{\sigma_x}{\sigma_v} > \frac{3}{4}$, which is true since $(\bar{c} - \underline{c}) \frac{\sigma_x}{\sigma_v} > 1$. Since $\left(TS^U - TS^D\right)(\lambda)$ is linear in λ, this then implies that λ^* is increasing in σ_v.

The comparative statics with respect to μ_x and σ_x are straightforward. Finally, note that

$$\frac{\partial \lambda^*}{\partial \Delta} = 2 \frac{\Delta - (\bar{c} - \underline{c}) \mu_x - 2\lambda^* \Delta}{\frac{\sigma_v / \sigma_x}{\bar{c} - \underline{c}} \frac{\sigma_v^2}{2} + 2\Delta^2}.$$

When $\Delta < (\bar{c} - \underline{c}) \mu_x$, this is clearly negative. Otherwise, if , note that at $\sigma_v = 0$, $2\lambda^* \Delta = \Delta - 2(\bar{c} - \underline{c}) \mu_x$, so this expression is positive. For $\sigma_v > \frac{2\Delta(\bar{c} - \underline{c})\mu_x}{3\Delta + (\bar{c} - \underline{c})\mu_x}$, the expression is negative. Since λ^* is increasing in σ_v, this implies that there is a cutoff value $0 \leq \hat{\sigma}_v \leq \frac{2\Delta(\bar{c} - \underline{c})\mu_x}{3\Delta + (\bar{c} - \underline{c})\mu_x}$, a function of the other parameters of the model, for which $\sigma_v < \hat{\sigma}_v$ implies that $\frac{\partial \lambda^*}{\partial \Delta} > 0$ and $\sigma_v > \hat{\sigma}_v$ implies that $\frac{\partial \lambda^*}{\partial \Delta} < 0$. ■

Proof of Proposition 4. Note that

$$\frac{\partial \lambda^*}{\partial \omega} = \frac{2\omega^{-2} \mu_x \Delta - \frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \lambda^*}{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega + 2\Delta^2} > 0$$

whenever

$$\frac{1}{2} \frac{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega}{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega + \Delta^2} \frac{\sigma_v^2 + \Delta^2}{2\omega^{-1} \Delta} < \mu_x < \frac{\sigma_v^2 + \Delta^2}{2\omega^{-1} \Delta},$$

and

$$\frac{\partial E_{x,v}\left[\sigma_v^2 - \sigma_{v|p}^2\right]}{\partial \omega} = \frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \left(\frac{2\Delta^2}{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega + 2\Delta^2} \lambda^* + \frac{2\omega^{-1} \mu_x \Delta}{\frac{\sigma_v^2}{2} \frac{\sigma_v}{\sigma_x} \omega + 2\Delta^2} \right) > 0,$$

so that equilibrium informativeness is always increasing in ω. ■