Learning by Employing: The Value of Commitment Under Uncertainty

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Abstract

We analyze a dynamic principal–agent problem where an infinitely–lived principal who can employ at most one agent faces a sequence of finitely–lived agents who differ in their ability to produce output. The ability of an agent is initially unknown to both him and the principal and agents can exert effort when employed, which makes their performance more informative about ability. We characterize equilibrium contracts and show that they display short–term commitment to employment when the effect of effort on output is persistent but delayed. By providing insurance against early termination, commitment encourages agents to exert effort, and thus improves on the principal's ability to identify their type. This helps explain the use of probationary appointments in environments in which there is uncertainty about the talent of workers.

Keywords: dynamic principal–agent model, learning, commitment.

JEL Classification: C73, D21, D83, J41, M12, M51, M54.

1 Introduction

New hires are often offered probationary appointments. These are temporary contracts that promise employment for a pre–specified period of time.[1] The use of such contracts is pervasive in occupations such as academia and the legal profession, where output depends critically on a worker's skill, but the qualities that make an individual successful are usually difficult to assess. Why, then, commit to

[1]The terminology is not uniform. Probationary periods are also understood as the stage at the beginning of an employment relationship during which an employer has greater discretion to dismiss workers. This type of probationary appointment is common in unionized industries.

employ a worker of *uncertain* talent for a *certain* period of time rather than decide on employment as the relationship unfolds? If job performance provides information about talent, and so is an indicator of future performance, having the flexibility to terminate workers when their performance is unsatisfactory seems preferable. In this paper we show, on the contrary, that when the talent of new hires is uncertain, it may be beneficial for a firm to restrict its ability to dismiss workers early in their career. The value of probation is that it induces workers to invest in generating information about their talent, thus improving on the firm's ability to sort workers.

It has long been recognized that skilled individuals can only be identified through careful selection. Firms usually rely on a range of methods to evaluate job applicants and assess their suitability for being hired. Some of the standard practices include the review of resumes, reference checks, various forms of tests, and interviewing. The reason for this multifaceted approach to hiring is the difficulty in identifying talent.[2] As part of their hiring process, many firms also rely on probationary appointments to determine whether new workers are able to handle the duties and challenges associated with their job. For example, in the legal profession, successful probation in team with senior partners is essential for a young associate's retention and promotion to partner. In academia, young PhD's in the social sciences are hired as assistant professors for a predetermined number of years. At the end of this period, promotion and tenure are decided based primarily on the quality of their research.[3,4]

Whenever the talent of workers is uncertain, but the extent to which performance reveals ability cannot be influenced by their behavior, a firm cannot benefit from offering probation. Intuitively,

[2] This difficulty is illustrated by the following quote, which discusses the limitations of interviews: "Despite their advantages, behavioral interviews really only establish a candidate's minimum qualifications; they don't identify star talent. A candidate's experience, for example, is obviously an important hiring factor, but we all know seasoned executives who aren't stars. Similarly, being likable doesn't mean you have the intellectual horsepower to be a stellar leader. In short, behavioral interviews measure knowledge, not intelligence. Knowledge is information acquired through experience or formal training. Intelligence is the skill with which someone uses knowledge to solve a problem." [Justin Menkes, managing director of the *Executive Intelligence Group*, a New York–based consulting firm focused on the assessment of executive talent, as quoted in "*Hiring for Smart*", Harvard Business Review, Vol. 83, No. 11, November 2005.]

[3] The use of probationary appointments is not exclusive to firms in the professional service sector in anglo–saxon countries. For instance, it has been documented that an increasing number of Japanese firms in industries such as manufacturing and trade now offer probationary appointments. These typically last from one to two years.

[4] Even when probationary employment is meant for a candidate worker to develop the skills necessary for future assignments, an important component of this trial period is the evaluation of his potential. The importance of identifying talent through probation is documented, for example, on the website of the Office of Industrial Relations in the New South Wales Department of Commerce (Australia): "A probationary period at the commencement of an employment relationship can help an employee develop skills needed for the specific position. It also allows the employer to assess the employees potential performance. [...]"

in this case, the only effect of commitment to employment is to prevent the firm from dismissing a worker once he is perceived to be unsuited for his job.

In many circumstances, however, not only performance reveals information about ability, but the precision of this information is affected by a worker's behavior. Whether a restructuring project is successful in addressing the needs of a client firm depends not only on the talent of the management consultants involved, but also on their dedication. Likewise, a talented lawyer is more likely to develop a successful legal strategy for a client if he is fully engaged in his work. In academia, researchers are free to choose projects and how much intellectual energy to devote to them. In this case, the prospect of early dismissal might discourage a worker from dedicating himself to his job even if exerting effort helps reveal talent. Offering probation may then be beneficial to a firm despite the loss of flexibility in employment decisions that it entails.

The model we propose features an infinitely–lived firm in a market in which there is a constant inflow of finitely–lived workers who differ in their ability to produce output. The firm can employ at most one worker at a time.[5] The ability of any worker is initially unknown to both him and the firm. When employed by the firm, a worker can exert effort, which makes his output more informative about his ability. Every worker in the market has an outside option that increases with his reputation, the posterior belief by the firm that he is of high ability. Nevertheless, workers of higher reputation are more valuable. Hence, the firm faces an opportunity cost if it retains a worker whose initial performance is poor, since he is more likely to be less talented than a worker who is new to the market.

The question we address is whether probation is effective in inducing new hires to exert effort, thus allowing the firm to better sort the workers it employs. When effort only affects output in the period a worker exerts it, which is our benchmark case, commitment has no value. In this case, the prospect of being retained regardless of performance encourages workers to shirk. Hence, probation is detrimental to the firm: it has an adverse effect on workers' incentives and it limits the firm's ability to replace workers who perform poorly.

Suppose, instead, that effort has a persistent but delayed effect on output, i.e., by exerting effort a worker affects both his current and future performance, but the effect is mostly on the latter.[6] In this case, effort shares the characteristics of an investment in information: it *eventually*

[5]The key assumption is that the firm is capacity constrained, and so cannot create vacancies to absorb all the workers in the market.

[6]See Mukoyama and Şahin (2005) for a model of repeated moral hazard with effort persistence.

provides information about ability. Thus, the usual incentive problem is compounded by the time separation of costs and returns typical of investment problems. The worker has an incentive to undertake such an investment only if he is guaranteed to participate in its return, which here is the higher wage that is paid to workers of proven talent. However, even when a worker exerts effort, the firm prefers to replace him if his initial performance is unsatisfactory. Probation then solves a time–consistency problem: in its absence, the firm cannot credibly promise to retain a worker of uncertain talent whose initial performance is poor; this destroys a worker's incentives for effort. If the gain in information about ability when the worker exerts effort is sufficiently large, probation has value to the firm. In addition, since the impact of effort on output is delayed, output–contingent contracts cannot substitute for probation.

This paper is organized as follows. We discuss the related literature in the next section and introduce the model in Section 3. We consider the benchmark case in Section 4 and the case where effort has a persistent effect on output in Section 5. Section 6 concludes the paper. The Appendix contains omitted proofs and a discussion of the robustness of our analysis.

2 Related Literature

Formally, the problem of the firm is a multi–armed bandit, a sequential sampling problem where a decision maker has to choose between a number of alternatives with uncertain rewards.[7] The first application of the multi–armed bandit framework to the analysis of employer learning in labor markets is Jovanovic (1979). Harris and Weiss (1984) extend Jovanovic's analysis to include workers who are finitely–lived and risk–averse. Eeckhout (2006) considers the case where productivity is general instead of match–specific.[8] A fundamental difference between these papers and ours is that in our framework rewards are endogenous: the decisions of the firm affect the behavior of workers.[9]

A related paper on contracting in the presence of uncertainty is Manso (2007), which analyzes the extent to which contracts can motivate innovation: the discovery, through experimentation and learning, of alternatives that are better than the currently known actions. He shows, in a two–period setting, that optimal long–term wage contracts that stimulate innovation can be non–monotone in performance, i.e., they reward early failure and late success.

[7] See Berry and Fristedt (1985) for an exposition of the theory of multi–armed bandits.

[8] Moscarini (2005) embeds Jovanovic's inference problem about match quality in an equilibrium search model.

[9] When effort is persistent, the firm faces an experimentation problem with signal–dependence. See Datta, Mirman, and Schlee (2002) for an analysis of such problems.

Two related papers that also analyze the interplay between explicit and implicit incentives in principal–agent problems are Gibbons and Murphy (1992) and Levin (2003). The first derives the optimal combination of explicit and implicit contracts in a model of career concerns.[10] The second characterizes the optimal stationary self–enforcing contracts in an environment in which explicit contracts can extend for only one period.

In our setting the nature of the relationship between the firm and a worker is akin to that of two parties in a hold–up problem. In our case, despite the value information about ability being general, the fact that investment (exerting effort) may be damaging to workers induces them to underinvest unless protected by a probationary appointment. See Klein, Crawford, and Alchian (1978) for an early reference on the hold–up problem and Che and Sakovics (2004) for an analysis of the role of contracts in a dynamic model of hold–up.

The equilibrium contracts in our setting have the "up–or–out" feature that after probation a worker is either permanently retained or dismissed. Kahn and Huberman (1988) and Waldman (1990) show how up–or–out contracts that specify that an individual should be fired if not promoted within some set time can induce (homogeneous) workers to invest in the acquisition of human capital. O'Flaherty and Siow (1992) analyzes a model of on–the–job screening and shows that the optimal retention decision for the firm has the features of an up–or–out rule. Carmichael (1988) discusses how the institution of tenure can induce academic departments to hire the best available junior researchers.[11] None of these papers, however, consider the optimality of commitment when it involves an opportunity cost to the firm.

3 Basic Setup

Time is discrete and indexed by $t \geq 1$. There is one firm and a countable number of workers.

Workers In each period a new worker enters the market, which initially has no workers. All workers have a strictly increasing and bounded below Bernoulli utility function $v : \mathbb{R}_+ \to \mathbb{R}$ and discount factor $\beta_w \in [0, 1]$. Workers live for $T \geq 3$ periods once they enter the market and can be of one of two types, high (H) or low (L). A worker's type is unknown to both him and the firm. The probability that a worker is of the high type is $\phi_0 \in (0, 1)$.

[10] Holmström (1999) is the seminal reference on career concerns.

[11] See also Bar-Isaac (2004), which considers the problem of selection and retention in partnerships when the contribution of an individual worker cannot be inferred from the total output of the partnership.

Every period he is employed by the firm, a worker either exerts effort ($\overline{e}$), which has cost $c > 0$, or not ($\underline{e}$) and produces either high ($\overline{y}$) or low ($\underline{y}$) output. Output is observable, but effort is not. For simplicity, we assume that output is not verifiable, so that output–contingent contracts are not feasible. We show that our conclusions do not depend on this assumption.

Output depends on effort and ability. We consider two cases, the so–called IID and non–IID cases. In the first case, our benchmark, the output of each type of worker is only affected by his current choice of effort, i.e., conditional on a worker's type, output realizations are independently and identically distributed over time. In the second case, a worker's choice of effort affects his current *and* future output. In both cases we assume that a low type worker cannot produce high output: high output then reveals that a worker is of the high type.

More precisely, let e and e_{-} denote a worker's current and previous choice of effort, respectively. Moreover, let $\Pr\{y|k, e, e_{-}\}$ be the probability, as a function of e and e_{-}, that a worker of type $k \in \{L, H\}$ produces $y \in \{\underline{e}, \overline{e}\}$. In the IID case,

$$\Pr\{\overline{y}|H, e, e_{-}\} = 1 - \Pr\{\underline{y}|H, e, e_{-}\} = \alpha + \eta(e) \quad \text{and} \quad \Pr\{\overline{y}|L, e, e_{-}\} = 0,$$

where $\alpha > 0$, $\eta(\underline{e}) = 0$, and $\eta(\overline{e}) = \eta > 0$. In the non–IID case,

$$\Pr\{\overline{y}|H, e, e_{-}\} = \alpha + \eta(e_{-}, e) \quad \text{and} \quad \Pr\{\overline{y}|L, e, e_{-}\} = 0,$$

where $\alpha > 0$, $\eta(\underline{e}, \overline{e}) \geq \eta(\underline{e}, \underline{e}) = 0$, $\eta(\overline{e}, \overline{e}) \geq \eta(\overline{e}, \underline{e}) > 0$, and $e_{-} = \underline{e}$ for a worker of age 1. We assume that $\eta(\overline{e}, \overline{e}) - \eta(\overline{e}, \underline{e}) \leq \eta(\underline{e}, \overline{e}) - \eta(\underline{e}, \underline{e})$, so that the return to effort exertion in one period decreases with the amount of effort exerted in the previous period.

Workers who are in the market have an outside option that pays a wage w that depends on their *reputation*, where a worker's reputation is the firm's posterior belief ϕ that he is of the high type. Notice that since the effort choice of a worker is private, his belief about his type can differ from his reputation. We assume that $w = w(\phi)$ is constant in the interval $[0, \phi_0]$ and strictly increasing in the interval $(\phi_0, 1]$. We also assume that a worker who takes his outside option – which happens when the firm decides not to employ him – leaves the market and so can no longer be hired by the firm. Therefore, the value of a worker's outside option is determined by his reputation at the time he first takes it.

Finally, workers can only observe their own history, so that they cannot condition their behavior on the history of play before they enter the market. This prevents workers from coordinating their behavior in response to deviations by the firm.

Firm The firm is infinitely lived, risk-neutral, and has discount factor $\beta_f \in (0,1)$. It can employ at most one worker at a time. Let Π be the flow payoff to the firm when it does not employ a worker. We assume that $\Pi < \alpha\phi_0\overline{y} + (1-\alpha\phi_0)\underline{y} - \underline{w}$, where $\underline{w} = w(\phi_0)$, so that the firm would rather employ an age 1 worker at his outside option than not employ any worker.

The firm can commit to one–period wage offers, i.e., at the beginning of each period the firm can promise a worker that it will pay him at least a certain wage at the end of the period if he accepts employment. The firm can also promise a worker of age $k \leq T-1$ that it will make one–period wage offers to him in the next $q \in \{1, \ldots, T-k-1\}$ periods. Commitment to wage schedules is not possible, though. In other words, the firm cannot post wage–tenure contracts as in Stevens (2004), for example. We assume, however, that the firm cannot offer a wage that is smaller than $\underline{w}$. This simplifies the exposition. We discuss this assumption in the Appendix. When the firm commits to future wage offers to an age 1 worker, we say that it offers *probation*.

Timing A worker is said to be the incumbent if he was employed by the firm in the previous period and is still alive. Recall that any worker who is not employed by the firm leaves the market. Hence, besides an incumbent, the only other worker that the firm can employ in a given period is the available age 1 worker, i.e., the worker who entered the market in this period.

An offer to a worker is a pair (w,q) consisting of a one–period wage offer w and the number q of subsequent periods in which the firm is committed to make one–period wage offers to him. The sequence of events in a period is as follows. If the firm has no incumbent, then it makes an offer to the available age 1 worker. If the firm has an incumbent to which it is committed to make an offer, then it makes an offer to him. If the firm has an incumbent, but is not committed to make an offer to him, then it either makes an offer to the incumbent or it makes an offer to the available age 1 worker.[12] The worker who receives the offer then decides whether to take it or not. If he accepts the offer, he chooses how much effort to exert, output realizes, and the firm pays him a wage not smaller than the wage it promised at the beginning of the period.[13] A worker who does not receive an offer or rejects his offer collects his outside option, and so does the firm if its offer is rejected.

[12]The firm has also the option of not making an offer – when it has no incumbent or is not committed to make an offer to its incumbent – and collecting its outside option. It turns out that this is never optimal, so we omit this possibility from the very beginning.

[13]An incumbent who accepts an offer by the firm does not excuse it of any previous commitment. For example, if a worker accepts an offer (w,q) with $q > 1$ and in the period after he accepts an offer (w', q') with $q' < q-1$, then the firm is still committed to the initial offer.

Let $\overline{w} = w(1)$. The following restriction is a maintained assumption in the rest of this paper.

(A1) $\alpha\overline{y} + (1-\alpha)\underline{y} - \overline{w} > \phi_0\overline{y} + (1-\phi_0)\underline{y} - \underline{w}$.

Since the right–hand side of the above inequality is an upper bound to the flow payoff the firm can obtain from an age 1 worker, A1 implies that an incumbent known to be of the high type is more desirable to the firm than an age 1 worker or an incumbent who has never produced high output.

Let $\mathcal{H}_w$ be the set of possible histories for a worker. This set is the same for all workers. We can partition $\mathcal{H}_w$ in two subsets, $\mathcal{H}_w^1$ and $\mathcal{H}_w^2$, where $\mathcal{H}_w^1$ is the set of worker histories that end with an offer by the firm and $\mathcal{H}_w^2$ is the set of worker histories that end with an offer being accepted. Denote the set of probability measures over a set S by $\Delta(S)$. A behavior strategy for a worker is a pair $(\widetilde{a}, \widetilde{e})$, where $\widetilde{a} : \mathcal{H}_w^1 \to \Delta(\{\text{accept}, \text{reject}\})$ is his acceptance policy and $\widetilde{e} : \mathcal{H}_w^2 \to \Delta(\{\underline{e}, \overline{e}\})$ is his effort policy. Denote by Σ the set of behavior strategies for a worker. A strategy profile for the workers is a map $\sigma_w : \mathbb{N} \to \Sigma$, where $\sigma_w(t)$ is the behavior strategy of the period t worker, i.e., the worker who enters the market in t.

Now let $\mathcal{H}_f$ be the set of histories for the firm. We can partition $\mathcal{H}_f$ in two subsets, $\mathcal{H}_f^1$ and $\mathcal{H}_f^2$, where $\mathcal{H}_f^1$ is the set of beginning of period histories, i.e., the empty history that starts the game or a history that ends with a payment, and $\mathcal{H}_f^2$ is the set of histories for the firm that end with output being produced by a worker. A behavior strategy σ_f for the firm is a pair $(\widetilde{\nu}, \widetilde{p})$, where $\widetilde{\nu} : \mathcal{H}_f^1 \to \Delta([\underline{w}, +\infty) \times \{0, \ldots, T-1\} \times \mathcal{I})$, with $\mathcal{I} \subseteq \{\text{incumbent, age 1}\}$, is the firm's offer policy and $\widetilde{p} : \mathcal{H}_f^2 \to \Delta([\underline{w}, +\infty))$ is the firm's payment policy. The set $\mathcal{I}$ is the set of possible workers to which the firm can make an offer, and it depends on the history of play: $\mathcal{I} = \{\text{incumbent}\}$ if the firm is committed to make an offer to the incumbent, $\mathcal{I} = \{\text{age 1}\}$ if the worker the firm employed in the previous period was of age T, and $\mathcal{I} = \{\text{incumbent, age 1}\}$ otherwise. Denote the first component of $\widetilde{\nu}$ on the by $\widetilde{\omega}$.

A perfect bayesian equilibrium (PBE) is a strategy profile $\sigma = (\sigma_f, \sigma_w)$ where: (i) for all $h \in \mathcal{H}_f$, the strategy of the firm after h is optimal given the behavior of the workers; (ii) for all $t \in \mathbb{N}$ and $h \in \mathcal{H}_w$, the strategy of the period t worker after h is optimal given the behavior of the firm; (iii) each worker uses Bayes rule to update his belief about his type, and so does the firm for every worker it employs.[14] A perfect bayesian equilibrium $\sigma = (\sigma_f, \sigma_w)$ is symmetric if $\sigma_w(t)$ is independent of t, i.e., if all workers follow the same strategy.

[14]Notice that the information sets of workers are singletons and both high and low output are always possible when the firm employs a worker. Hence, the issue of having to define beliefs off the equilibrium path is not present. In particular, the firm updates beliefs as if the workers it employs do not deviate from their prescribed behavior.

We restrict attention to symmetric PBE of this game. Such equilibria are anonymous in the sense that the identity of the workers plays no role. This is a natural condition in our setting.[15] From now on an equilibrium always means a symmetric equilibrium.

Lemma 1. *Consider an equilibrium of this game. The following holds for an incumbent who is known to be of the high type:* (i) $\widetilde{\omega} \leq \overline{w}$ *if he receives an offer;* (ii) *if he accepts an offer, then* $\widetilde{p}$ *is equal to the one–period wage offered, i.e., there are no implicit bonus payments;* (iii) *he never exerts effort when employed.*

An immediate consequence of Lemma 1 is that, in equilibrium, an incumbent who is known to be of the high type never accepts an offer with a wage smaller than $\overline{w}$. Another consequence of Lemma 1 is that, in equilibrium, the firm always pays a worker who produces high output for the first time the one–period wage it offered him. The proof of this is straightforward.

Corollary 1. *In equilibrium, the firm always pays a worker who produces high output for the first time the one–period wage it offered him.*

The key fact for Lemma 1 is that workers have finite lifetimes, and so there is always a last period in which they are employed. A formal proof is somewhat long. For this reason we relegate it to the Appendix and instead present a sketch of the argument. First notice that implicit bonus payments are not possible in a worker's last period of employment, so that no effort is uniquely optimal in this case. But then, implicit bonus payments are not possible when a worker can be employed at most for two periods. Hence, the only incentive he has to exert effort in the first of these two periods is the variation of his future payoff with his output. This variation, however, is zero for a worker who is known to be of the high type, since his reputation does not change with his output. It is now easy to see how this implies that any implicit bonus scheme for a worker who is known to be of the high type unravels and such a worker never exerts effort when employed.

Denote by $V(h|\sigma)$ the firm's normalized present expected lifetime payoff after $h \in \mathcal{H}_f$ when the strategy profile under play is σ.

Lemma 2. *Suppose* σ *is an equilibrium. Then* $V(h|\sigma) = V(h'|\sigma)$ *for any two histories* h *and* h' *for the firm after which it makes an offer to the available age 1 worker.*

Proof: Suppose there exist $h, h' \in \mathcal{H}_f$ after which the firm makes an offer to the available age 1 worker such that $V(h'|\sigma) > V(h|\sigma)$. Consider now the deviation for the firm where it behaves after

[15] One alternative to the assumption that workers follow symmetric strategies is to consider an environment where in each period the identity of the worker who becomes available is random. The approach we follow is simpler.

h as if h' had happened. Since the workers follow symmetric strategies, this deviation increases the firm's payoff after h by $V(h'|\sigma) - V(h|\sigma)$, a contradiction. $\square$

Lemma 3. *In equilibrium, the firm always offers $\overline{w}$ to an incumbent it knows is of the high type, who always accepts the offer.*

Proof: Let σ be an equilibrium. By Lemma 1, an incumbent who is known to be of the high type never exerts effort when employed and always rejects an offer (w, q) with $w < \underline{w}$. Hence, by Assumption A1, $V(h|\sigma) < \alpha\overline{y} + (1-\alpha)\underline{y} - \overline{w}$ when h is the initial history of this game. Therefore, Lemma 2 implies that $V(h|\sigma) < \alpha\overline{y} + (1-\alpha)\underline{y} - \overline{w}$ for all $h \in \mathcal{H}_f$ after which the firm hires the available age 1 worker. Thus, it is strictly optimal for the firm to employ an incumbent it knows is of the high type, so that it should never make an offer to such a worker that is rejected. The desired result now follows from the fact that a worker who is known to be of the high type always rejects a wage smaller than $\overline{w}$ and always accepts a wage greater than $\overline{w}$. $\square$

Now notice that in equilibrium both an age 1 worker and an incumbent of age $T-1$ or less who has never produced high output always take an offer by the firm. Indeed, by accepting employment (at a wage $\underline{w}$) such a worker has the chance of revealing himself to be of the high type before he is of age T, which leads to an outside option that is higher than $\underline{w}$, his current outside option. Also notice that an incumbent of age T who has never produced high output is indifferent between accepting an offer by the firm and taking his outside option. These two facts imply that in equilibrium the firm never offers more that $\underline{w}$ to a worker who has never produced high output, including the age 1 workers. We summarize these results in the following lemma.

Lemma 4. *The following holds in any equilibrium of this game: (i) a worker of age $T-1$ or less who has never produced high output accepts any offer by the firm; (ii) the firm never offers a wage greater than $\underline{w}$ to a worker who has never produced high output.*

Comments We assume that a worker reveals himself to be of the high type if he produces high output. This captures in a simple way the notion that a worker can prove his talent if given enough opportunities. We show in the Appendix that our results survive if there is a small probability that a low type worker can produce high output as well. We also assume that a worker's outside option depends on his reputation. This is a simple way of capturing the idea that markets can reward talent when it is scarce. We are currently working on making outside options endogenous and dependent on a worker's reputation by introducing competition among firms.

We are not concerned with repeated moral hazard in this paper. The role of effort in our model is to make a worker's performance more revealing about ability, so that the firm's employment decisions become better informed. Hence, even though not necessary for our conclusions, a natural assumption to make is that effort is statically inefficient for a high type worker, i.e., the extra output it generates is smaller than its cost.[16] This assumption implies that effort is useful only when there is uncertainty about ability, so that its role is that of an investment in the generation of information about ability.

4 The IID Case

Suppose the firm commits to make future offers to a given worker. We say this commitment is binding if there is a positive probability that during the commitment period – the length of time during which the firm is bound to make an offer to this worker – the firm would rather employ the available age 1 worker. If there is an equilibrium of this game in which the firm offers non–binding commitment to a given worker, then there is a payoff equivalent equilibrium in which the firm does not offer this commitment. Hence, if commitment is to be non–trivial, then it must be binding, for otherwise it does not alter the set of equilibrium payoffs.

Proposition 1. *The firm never offers binding commitment to employment in equilibrium.*

We establish this result in three steps. The first step is the crucial one.

Step 1. *Commitment to employment weakens the incentive for effort exertion*

Consider a worker of age $k \in \{1, \dots, T-1\}$ who has never produced high output and let $\pi \leq \phi_0$ be his private belief that he is of the high type. Notice that π is a function of his past effort choices and output realizations. Suppose that the firm employs this worker and let w^y be the wage it pays him at the end of the period if his output is $y \in \{\underline{y}, \overline{y}\}$. By Lemma 1, his IC constraint for effort exertion is given by

$$\begin{aligned}\pi(\alpha+\eta)\{v(w^{\overline{y}}) + \beta_w(1-\beta_w)^{-1}(1-\beta_w^{T-k})v(\overline{w})\} + [1-\pi(\alpha+\eta)]v(w^{\underline{y}}) + \beta_w R(\underline{y}, \overline{e}|\pi, k)\} \\ \geq c + \pi\alpha\{v(w^{\overline{y}}) + \beta_w(1-\beta_w)^{-1}(1-\beta_w^{T-k})v(\overline{w})\} + (1-\pi\alpha)\{v(w^{\underline{y}}) + \beta_w R(\underline{y}, \underline{e}|\pi, k)\}, \quad (1)\end{aligned}$$

where $R(y, e|\pi, k)$ is his expected continuation payoff after he chooses $e \in \{\underline{e}, \overline{e}\}$ and produces y,

[16] This is already the case for low type workers. In the IID case, effort is statically inefficient for a high type worker if $\eta(\overline{y} - \underline{y}) < c$, while in the non-IID case is it statically inefficient for such workers if $[\eta(\underline{e}, \overline{e}) + \eta(\overline{e}, \underline{e})](\overline{y} - \underline{y}) < c$.

which depends on π and k.[17] Notice that $w^{\overline{y}} \leq w^{\underline{y}}$ by Corollary 1. Rewrite (1) as

$$\pi\eta\{\underbrace{v(w^{\overline{y}}) + \beta_w(1-\beta_w)^{-1}(1-\beta_w^{T-k})v(\overline{w}) - [v(w^{\underline{y}}) + \beta_w R(\underline{y}, \overline{e}|\pi, k)]}_{\Delta_1}\}$$
$$+\beta_w(1-\pi\alpha)\underbrace{[R(\underline{y}, \overline{e}|\pi, k) - R(\underline{y}, \underline{e}|\pi, k)]}_{\Delta_2} \geq c. \tag{2}$$

Notice that $R(\underline{y}, e|\pi, k) \geq (1-\beta_w)^{-1}(1-\beta_w^{T-k})v(\underline{w})$ and that equality holds if, and only if, this worker is not employed after (producing) low output. The 'if' part is obvious. The 'only if' part is true because this worker has the chance of revealing himself to be of the high type if he is employed after low output. Hence, for any pair $\{w^{\overline{y}}, w^{\underline{y}}\}$, Δ_1 is maximized when this worker does not receive an offer after low output.

Now observe that when a worker produces $\underline{y}$ his updated private belief is higher in case he did not exert effort. Hence, $R(\underline{y}, \underline{e}|\pi, k) > R(\underline{y}, \overline{e}|\pi, k)$ if there is a chance this worker receives an offer by the firm after producing $\underline{y}$. Indeed this worker can behave after choosing $\underline{e}$ and producing $\underline{y}$ in the same way as he behaves after choosing $\overline{e}$ and producing the same output. The (expected) cost of doing so is the same and the chances of revealing himself to be of the high type are always larger. Therefore, $\Delta_2 \leq 0$, and $\Delta_2 = 0$ if, and only if, this worker is dismissed after low output.

We can then conclude that the left-hand side of (2) is always maximized when low output leads to dismissal. In other words, the incentive for a worker of age $T-1$ or less who has never produced high output to exert effort is highest when he is dismissed after producing low output. To finish this step recall, by Lemma 1, that commitment to employment does not affect the behavior of workers of age $T-1$ or less who are known to be of the high type.

Step 2. *In equilibrium, the firm never offers binding commitment to employment when*

$$\phi_0\eta\beta_w(1-\beta_w^{T-1})[v(\overline{w}) - v(\underline{w})] < (1-\beta_w)c. \tag{3}$$

First notice, by the proof of Lemma 3, that commitment to employment is never binding for a worker who is known to be of the high type. Hence, commitment to employment can only bind if it is offered to a worker who has never produced high output. Now observe, by the previous step, that (2) is never satisfied when (3) holds. Together with Lemma 1 this implies that in equilibrium no worker ever exerts effort when employed. Therefore it is uniquely optimal for the firm to retain

[17] In principle, $R(y, e)$ should also depend on the worker's reputation ϕ, since it affects his outside option, and so what the firm is willing to pay to employ him. However, our assumptions imply that in equilibrium a worker is either paid $\overline{w}$ or $\underline{w}$, so that ϕ does not show up.

an incumbent it knows is of the high type (by A1) and to replace any incumbent who has failed to produce high output at least once with the available age 1 worker. This leads to the desired result.

Step 3. *In equilibrium, the firm never offers binding commitment to employment when*

$$\phi_0\eta\beta_w(1-\beta_w^{T-1})[v(\overline{w})-v(\underline{w})] \geq (1-\beta_w)c.$$

Consider an equilibrium. We know from above that commitment to employment can only bind if it is offered to a worker who has never produced high output. Suppose then, by contradiction, that there is $h \in \mathcal{H}_f$ after which the firm offers $(\underline{w}, q)$, with $q \geq 1$, to a worker of age $T-1$ or less who has never produced high output and this commitment binds with positive probability. Let $\underline{s} \in \{1, \ldots, q\}$ be the smallest number of consecutive low output realizations by this worker after which the commitment binds and let σ_f^* be the optimal continuation strategy for the firm at this point if it is not constrained by its commitment. By construction, σ_f^* is different from the equilibrium continuation strategy for the firm at the same point. Now consider the deviation for the firm where after h it offers $(\underline{w}, 0)$ to the worker under consideration and then behaves as if no deviation has occurred except that after $\underline{s}$ consecutive low output realizations by this worker it plays according to σ_f^*. Step 1 and the assumption that the commitment to employment after h is binding imply that this is a profitable deviation for the firm, a contradiction.

From Step 2 it is straightforward to see that if $\phi_0\eta\beta_w(1-\beta_w^{T-1})[v(\overline{w})-v(\underline{w})] < (1-\beta_w)c$, then the following is an equilibrium of this game: (i) the firm offers $(\overline{w}, 0)$ to an incumbent it knows is of the high type; (ii) the firm offers $(\underline{w}, 0)$ to the available age 1 worker if it has no incumbent or its incumbent has never produced high output and the firm is not committed to employ him; (iii) the firm offers $(\underline{w}, 0)$ to an incumbent who has never produced high output if it is committed to employ him; (iv) the firm always pays the one–period wage it offers; (v) a worker who has never produced high output accepts any offer by the firm; (vi) a worker who is known to be of the high type accepts an offer (w, q) if, and only if, $w \geq \overline{w}$; (vii) a worker never exerts effort when employed.

Output-Contingent Contracts Suppose now that the firm can offer output–contingent contracts. It is reasonable to assume that it is at least as patient as the workers. In this case, we can restrict attention to one–period output–contingent contracts: one–period wage schedules $\{w^{\underline{y}}, w^{\overline{y}}\}$, where w^y is the wage the firm promises to pay the worker if he produces y.[18] Notice that a one–

[18] If the firm is less patient than the workers, then long–term output–contingent contracts where it backloads wage payments become desirable. Our conclusions do not change, though.

period output–contingent contract reduces to a one–period wage offer when $w^{\underline{y}} = w^{\overline{y}}$. We refer to such a contract as a constant contract.

The same logic used in the proof of Lemma 1 shows that, in equilibrium, if the firm offers a non–constant output–contingent contract to a worker it knows is of the high type, then this worker must be indifferent between exerting effort and not. Hence, in equilibrium, a worker's lifetime payoff after he reveals himself to be of the high type is the same as when output–contingent contracts are not feasible. The logic of the proof of Proposition 1 then implies that among the workers who have not revealed themselves to be of the high type, the ones with the greatest incentive to exert effort are still the age 1 workers who are dismissed after low output. The difference is that now we can have $w^{\overline{y}} > w^{\underline{y}}$ in (2). Thus, it is still the case that the firm can only lose by offering binding commitment to employment. Hence, not surprisingly, the restriction that output contingent–contracts are not feasible does not matter for our conclusion in the IID case.

5 The non–IID Case

We show that there is scope for commitment to probation in the non–IID case when $\eta(\underline{e}, \overline{e})$ is small, so that most of the impact of effort on output takes place in the period after the worker exerts it. For simplicity, we consider the case where $\eta(\underline{e}, \overline{e}) = 0$. It will become clear from our analysis that the results we obtain hold when $\eta(\underline{e}, \overline{e})$ is positive but small. Notice that $\eta(\underline{e}, \overline{e}) = 0$ implies that $\eta(\overline{e}, \overline{e}) = \eta(\overline{e}, \underline{e})$. This also implies that, in equilibrium, an age T worker never exerts effort, even when output–contingent contracts are possible. As in the previous section, we discuss what happens when output–contingent contracts are feasible at the end.

Let $\alpha + \eta(\overline{e}, \underline{e}) = \gamma > \alpha$ and recall that $e_- = \underline{e}$ for an age 1 worker. When $\eta(\underline{e}, \overline{e}) = 0$, the non–IID case is summarized by the following two tables:

$e_- = \underline{e}$	$\overline{y}$	$\underline{y}$
H	α	$1-\alpha$
L	0	1

$e_- = \overline{e}$	$\overline{y}$	$\underline{y}$
H	γ	$1-\gamma$
L	0	1

Now let $\underline{\phi} = (1-\alpha)\phi_0/(1-\phi_0\alpha)$, $\underline{\underline{\phi}}(e) = (1-\alpha)(1-\xi(e))\phi_0/[(1-\alpha)(1-\xi(e))\phi_0 + 1 - \phi_0]$, and $y(\phi, \xi) = \phi\xi\overline{y} + (1-\phi\xi)\underline{y}$, where $\xi(\underline{e}) = \alpha$ and $\xi(\overline{e}) = \gamma$. Notice that: (i) $\underline{\phi}$ is the reputation of an age 2 worker who produces low output in his first period of employment; (ii) $\underline{\underline{\phi}}(e)$ is the reputation of an age 3 worker who chooses $e \in \{\underline{e}, \overline{e}\}$ in his first period of employment and produces low output in his first two periods of employment. We make the following two assumptions:

(A2) $\phi_0(1-\alpha)(\gamma-\alpha)\dfrac{\beta_w^2(1-\beta_w^{T-2})}{1-\beta_w}[v(\overline{w})-v(\underline{w})] > c$;

(A3) $\underline{\phi}(1-\alpha)^2\dfrac{\beta_w^2(1-\beta_w^{T-3})}{1-\beta_w}[v(\overline{w})-v(\underline{w})] < c$.

To understand A2, suppose that an age 1 worker who accepts employment believes that if he produces low output, then the firm offers $(\underline{w}, 0)$ to him in the next period, pays $\underline{w}$ to him regardless of his output, and dismisses him if he produces low output a second time. Suppose also that this worker believes that if he reveals himself to be of the high type, then his flow payoff is $v(\overline{w})$ in every subsequent period, which is what happens in equilibrium. Since $\eta(e,\overline{e}) = \eta(e,\underline{e})$ for $e \in \{\underline{e},\overline{e}\}$, this worker has no incentive to exert effort when he is of age 2 if he produces low output when of age 1. The IC constraint for effort exertion in his first period of employment is then

$$\begin{aligned}
-c + \phi_0\alpha &\left[v(w^{\overline{y}}) + \frac{\beta_w(1-\beta_w^{T-1})}{1-\beta_w}v(\overline{w})\right] \\
&+(1-\phi_0\alpha)\left[v(w^{\underline{y}}) + \beta_w v(\underline{w}) + \underline{\phi}\gamma\frac{\beta_w^2(1-\beta_w^{T-2})}{1-\beta_w}v(\overline{w}) + (1-\underline{\phi}\gamma)\frac{\beta_w^2(1-\beta_w^{T-2})}{1-\beta_w}v(\underline{w})\right] \\
\geq \phi_0\alpha &\left[v(w^{\overline{y}}) + \frac{\beta_w(1-\beta_w^{T-1})}{1-\beta_w}v(\overline{w})\right] \\
&+(1-\phi_0\alpha)\left[v(w^{\underline{y}}) + \beta_w v(\underline{w}) + \underline{\phi}\alpha\frac{\beta_w^2(1-\beta_w^{T-2})}{1-\beta_w}v(\overline{w}) + (1-\underline{\phi}\alpha)\frac{\beta_w^2(1-\beta_w^{T-2})}{1-\beta_w}v(\underline{w})\right],
\end{aligned}$$

where w^y is the wage the firm pays him if he produces $y \in \{\underline{y},\overline{y}\}$ when of age 1. It is straightforward to see that this condition is satisfied by virtue of Assumption A2.

Consider now an equilibrium of this game. It is straightforward to see that there are no implicit bonus payments to workers of age T and $T-1$. Hence, an age $T-1$ worker never exerts effort when employed. Indeed, a worker's choice of effort when of age $T-1$ only affects his age T output. Thus, a worker of age $T-1$ is only willing to exert effort if his age T wage responds to his age T output, which is not the case.

Suppose then, by induction, that there exists $\underline{k} \geq 3$ such that: (i) the firm always pays a worker of age $k \geq \underline{k}-1$ the one–period wage it offers him; (ii) a worker of age $k \geq \underline{k}$ never exerts effort when employed. Consider now an incumbent of age $\underline{k}-1$ who has never produced high output and suppose that the firm employs him (at a wage $\underline{w}$). If he is to exert effort, then it must be that the firm employs him in the next period after he produces low output. Suppose that this is the case, let ϕ be his reputation, and ξ be the probability that he produces high output in the current period. Notice that $\phi \leq \underline{\phi}$ and that $\xi \geq \alpha$. Moreover, let $R(e)$ be the present lifetime payoff of this worker as a function of his choice of effort and $\phi^{\underline{y}} = (1-\xi)\phi/(1-\xi\phi)$. Then, by the induction hypothesis, an upper bound for $R(\overline{e})$ is

$$-c + \underline{\phi}\xi \left[v(w^{\overline{y}}) + \frac{\beta_w(1-\beta_w^{T-\underline{k}-1})}{1-\beta_w} v(\overline{w}) \right]$$
$$+(1-\underline{\phi}\xi) \left[v(w^{\underline{y}}) + \beta_w v(\underline{w}) + \phi^{\underline{y}} \frac{\beta_w^2(1-\beta_w^{T-\underline{k}-2})}{1-\beta_w} v(\overline{w}) + (1-\phi^{\underline{y}}) \frac{\beta_w^2(1-\beta_w^{T-\underline{k}-2})}{1-\beta_w} v(\underline{w}) \right],$$

which is achieved when his type is revealed in the next period, while a lower bound for $R(\underline{e})$ is

$$\underline{\phi}\xi \left[v(w^{\overline{y}}) + \frac{\beta_w(1-\beta_w^{T-\underline{k}-1})}{1-\beta_w} v(\overline{w}) \right]$$
$$+(1-\underline{\phi}\xi) \left[v(w^{\underline{y}}) + \beta_w v(\underline{w}) + \phi^{\underline{y}}\alpha \frac{\beta_w^2(1-\beta_w^{T-\underline{k}-2})}{1-\beta_w} v(\overline{w}) + (1-\phi^{\underline{y}}\alpha) \frac{\beta_w^2(1-\beta_w^{T-\underline{k}-2})}{1-\beta_w} v(\underline{w}) \right].$$

Hence, $R(\overline{e}) < R(\underline{e})$ by A3, and so an incumbent of age $\underline{k}-1$ who has never produced high output does not exert effort when employed. Consequently, the firm always pays a worker of age $\underline{k}-2$ the one–period wage it offers him.

Thus, by induction, we have the following result as a consequence of Assumption A3.

Lemma 5. *In equilibrium, a worker of age $k \geq 2$ never exerts effort when employed and the firm always pays a worker the one–period wage it offers to him.*

Lemma 6. *There is $\overline{\beta}_f \in (0,1)$ such that if $\beta_f \geq \overline{\beta}_f$, then in any equilibrium of this game an incumbent of age $k \geq 2$ who never produced high output is dismissed unless the firm is committed to make an offer to him.*

Proof: See the Appendix. □

From now on we assume the firm is patient enough for Lemma 6 to hold. Hence, in light of A2, the only way the firm can get an age 1 worker to exert effort in equilibrium is if it offers probation. The question we then need to address is whether the gain to the firm from inducing an age 1 worker to exert effort is larger than the loss due to the lack of flexibility in employment decisions that commitment entails.

When an age 1 worker exerts effort, the firm benefits in two ways. First, it gains additional output when this worker is of age 2. Second, the output of this worker when he is of age 2 is now more informative about his ability. However, the increased informativeness is only important for a worker who fails to produce high output in his first period of employment. Given this, it is intuitive that the informational gain from effort exertion is proportional to the difference between $\phi_0\alpha$ and $\underline{\phi}\gamma$. The first term is the probability that an age 1 worker reveals himself to be of the high type (whether he exerts effort or not). The second is the probability that an age 2 worker who exerted effort in his first period of employment and produced low output reveals himself to be of the high type. We are interested in the case where this informational gain is positive, so that the offer of commitment to probation cannot be attributed solely to its effect on output.

In order to quantify the informational gain to the firm from inducing an age 1 worker to exert effort, consider first the case where it cannot offer commitment to probation. Suppose that σ' is an equilibrium and denote $V(h|\sigma')$ for $h \in \mathcal{H}_f$ after which the firm makes an offer to an age 1 worker by V_1. Then V_1 is the normalized equilibrium payoff to the firm in σ'. By Lemma 6, any age 1 worker who is not offered commitment to probation is dismissed after low output, and so does not exert effort. Since the firm always pays $\underline{w}$ to such an age 1 worker, for otherwise it could increase its payoff by lowering how much it pays after low output, V_1 satisfies the following recursion:

$$V_1 = (1-\beta_f)[y(\phi_0,\alpha) - \underline{w}] + \phi_0\alpha\left\{\beta_f(1-\beta_f^{T-1})[y(1,\alpha) - \overline{w}] + \beta_f^T V_1\right\} + (1-\phi_0\alpha)\beta_f V_1.$$

Solving the above equation for V_1 we obtain that

$$V_1 = \frac{(1-\beta_f)[y(\phi_0,\alpha) - \underline{w}] + \phi_0\alpha\beta_f(1-\beta_f^{T-1})[y(1,\alpha) - \overline{w}]}{1-\beta_f+\phi_0\alpha\beta_f(1-\beta_f^{T-1})}.$$

Notice that $V_1 = \lambda_1[y(\phi_0,\alpha) - \underline{w}] + (1-\lambda_1)[y(1,\alpha) - \overline{w}]$, where

$$\lambda_1 = \frac{1-\beta_f}{1-\beta_f+\phi_0\alpha\beta_f(1-\beta_f^{T-1})}.$$

Consider now the case where commitment is possible, but the extra output a worker generates when he exerts effort is lost, so that the only gain from commitment is informational. Let σ^* be an equilibrium and denote by V^* the payoff to the firm in this equilibrium. Since the firm has the option of always offering $(\overline{w}, 1)$ to an age 1 worker, Lemma 6 and Assumption A2 imply that $V^* \geq V_2'$, where V_2' satisfies the following recursion:

$$\begin{aligned}V_2' = {} & (1-\beta_f)[y(\phi_0,\alpha) - \underline{w}] + \phi_0\alpha\left\{\beta_f(1-\beta_f^{T-1})[y(1,\alpha) - \overline{w}] + \beta_f^T V_2'\right\} + \\ & (1-\phi_0\alpha)\Big\{\beta_f(1-\beta_f)[y(\underline{\phi},\alpha) - \underline{w}] + \underline{\phi}\gamma\Big\{\beta_f^2(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^T V_2'\Big\} + (1-\underline{\phi}\gamma)\beta_f^2 V_2'\Big\}.\end{aligned}$$

The payoff V_2' is obtained when the firm always pays the wages it offers. Solving the above equation for V_2' we obtain that

$$\begin{aligned} V_2' &= \frac{(1-\beta_f)[y(\phi_0,\alpha)-\underline{w}]+\beta_f(1-\beta_f)\{\phi_0\alpha[y(1,\alpha)-\overline{w}]+(1-\phi_0\alpha)[y(\underline{\phi},\alpha)-\underline{w}]\}}{1-\beta_f^2+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})} \\ &\qquad + \frac{\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})[y(1,\alpha)-\overline{w}]}{1-\beta_f^2+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})} \\ &= \frac{(1-\beta_f)[y(\phi_0,\alpha)-\underline{w}]+[\beta_f(1-\beta_f)+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})][y(1,\alpha)-\overline{w}]}{1-\beta_f+\beta_f(1-\beta_f)+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})} \\ &\qquad - \frac{(1-\phi_0\alpha)\beta_f(1-\beta_f)\{[y(1,\alpha)-\overline{w}]-[y(\underline{\phi},\alpha)-\underline{w}]\}}{1-\beta_f^2+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})}. \end{aligned}$$

Notice that $V_2' = \lambda_2[y(\phi_0,\alpha)-\underline{w}]+(1-\lambda_2)[y(1,\alpha)-\overline{w}]-\beta_f\lambda_2(1-\phi_0\alpha)[\Delta+y(\phi_0,\alpha)-y(\underline{\phi},\alpha)]$, where $\Delta=\{[y(1,\alpha)-\overline{w}]-[y(\phi_0,\alpha)-\underline{w}]\}$ and

$$\lambda_2 = \frac{1-\beta_f}{1-\beta_f+\beta_f(1-\beta_f)+\phi_0(\alpha+\gamma(1-\alpha))\beta_f^2(1-\beta_f^{T-2})}.$$

By construction, a sufficient condition for the informational gain of commitment to be positive is then that $V_2' > V_1$. It turns out that this condition is also necessary (prove this). We know from the two previous paragraphs that

$$V_2' - V_1 = [\lambda_1-\lambda_2(1+\beta_f(1-\phi_0\alpha))]\Delta-\beta_f\lambda_2(1-\phi_0\alpha)[y(\phi_0,\alpha)-y(\underline{\phi},\alpha)].$$

Hence, a necessary condition for $V_2' > V_1$ is that $\lambda_1 > \lambda_2(1+\beta_f(1-\phi_0\alpha))$. Simple algebra shows that $\lambda_1-\lambda_2(1+\beta_f(1-\phi_0\alpha))$ is (positively) proportional to

$$\begin{aligned} -\phi_0\gamma(1-\alpha)\beta_f^T+\phi_0\frac{\beta_f^2(1-\beta_f^{T-1})}{1-\beta_f}\left\{\gamma(1-\alpha)+\phi_0\alpha^2-\alpha\right\} &= \\ \phi_0(\beta_f^2+\cdots+\beta_f^{T-1})\{\gamma(1-\alpha)+\phi_0\alpha^2-\alpha\}+\phi_0\beta_f^T(\phi_0\alpha^2-\alpha), \end{aligned}$$

and so $V_2' > V_1$ only if,

$$\phi_0\{\gamma(1-\alpha)+\alpha(\phi_0\alpha-1)\} > \frac{\phi_0\alpha(1-\phi_0\alpha)\beta_f^T}{\beta_f^2+\cdots+\beta_f^{T-1}} \iff \underline{\phi}\gamma > \phi_0\alpha\frac{1-\beta_f^{T-1}}{1-\beta_f^{T-2}}. \tag{4}$$

Notice that the gap between $\underline{\phi}\gamma$ and $\phi_0\alpha$ necessary for (4) to be satisfied decreases with T, the lifetime of the workers. Perhaps surprisingly, this gap increases with β_f. A sufficient condition for (4) is that $\underline{\phi}\gamma > \phi_0\alpha(T-1)/(T-2)$.

We need that $\underline{\phi} > \phi_0\alpha$ if there is to be any informational gain from commitment. This condition is equivalent to $(1-\alpha) > \alpha(1-\phi_0\alpha)$, which is satisfied when $\alpha < 1/2$ no matter the value of ϕ_0. Also notice that $(1-\phi_0\alpha)[y(\phi_0,\alpha)-y(\underline{\phi},\alpha)] = \alpha^2\phi_0(1-\phi_0)(\overline{y}-\underline{y})$, so that (4) gets closer and closer to being sufficient for $V_2' > V_1$ as either α or ϕ_0 becomes small. It is, however, cumbersome to obtain a sufficient condition for $V_2' > V_1$. We work with condition (4) with the understanding that there are parameter restrictions for which the informational gain to commitment alone is positive.

Proposition 2. *Suppose that condition (4) is satisfied. The firm always offers probation to age 1 workers in any equilibrium of this game.*

Proof: Suppose, by contradiction, that there is an equilibrium σ in which there is $\widehat{h} \in \mathcal{H}_f$ after which the firm offers $(\underline{w}, 0)$ to the available age 1 worker. Then $V(\widehat{h}|\sigma) = V_1$, and this is the equilibrium payoff to the firm. The firm, however, can always offer $(\underline{w}, 1)$ to an age 1 worker. Therefore, $V(\widehat{h}|\sigma) \geq V_2' + \beta_f(1-\beta_f)\left\{\phi_0\alpha[y(1,\gamma) - y(1,\alpha)] + (1-\phi_0\alpha)[y(\underline{\phi},\gamma) - y(\phi_0,\alpha)]\right\} > V_2'$, a contradiction. $\square$

Output-Contingent Contracts What happens when the firm can also offer output–contingent contracts? Consider first the case where it can only offer one–period output–contingent contracts. We claim that, in equilibrium, these contracts are never offered to an age 1 worker. This is immediate when $\eta(\underline{e}, \overline{e}) = 0$. This is also true when $\eta(\underline{e}, \overline{e})$ is positive but small. Indeed, Lemma 6 is still true: the key observation is that if the firm can use a one–period output–contingent contract to induce an incumbent of age $k \geq 2$ who has never produced high output to exert effort, then it can do the same with an age 1 worker. Hence, if the firm does not offer probation to an age 1 worker, then the only way it can induce this worker to exert effort is if the wage it pays after he produces high output is larger than the wage it pays after he produces low output. This, however, is too expensive when $\eta(\underline{e}, \overline{e})$ is small: the wage after $\overline{y}$ that is necessary to induce effort diverges to infinity as $\eta(\underline{e}, \overline{e})$ goes to zero.[19] Thus, when $\eta(\underline{e}, \overline{e})$ is small, probation is necessary for an age 1 worker to exert effort. Moreover, Assumption A2 ensures that probation is also sufficient.

Now notice that, in equilibrium, the firm never offers one–period output–contingent contracts to an incumbent it knows is of the high type when $\eta(\underline{e}, \overline{e})$ is small. Therefore, there is no difference between this case and the case where output–contingent contracts are not feasible. Consequently, Proposition 2 still holds.

We now consider the case where the firm can offer long–term output–contingent contracts. Let $Y_t = \{\underline{y}, \overline{y}\}^t$, with typical element y^t, and assume that the firm can offer output–contingent wage schedules $\{w_t\}_{t=1}^T$ to age 1 workers, where $w_t : Y_t \to \mathbb{R}_+$ is such that $w_t(y^t)$ is the wage the firm

[19] The restriction that the firm cannot pay less than $\underline{w}$ does not play a role in this argument. What is important is the assumption that v is bounded below.

pays to a worker when he is of age t if he is employed and his sequence of outputs is y^t. Once again Lemma 6 is true. The reason is the same as when one–period output–contingent contracts are feasible. Thus, inducing an age 1 worker to exert effort without offering probation is again too expensive for the firm when $\eta(\underline{e}, \overline{e})$ is small: $w_1(\overline{y}) = w_1(\underline{y})$ in equilibrium. Probation is still necessary and sufficient for an age 1 worker to exert effort when $\eta(\underline{e}, \overline{e})$ is small.

Now observe that, unlike one–period output–contingent contracts, long–term output–contingent contracts can be used to induce workers of age $k \leq T - 1$ who are known to be of the high type to exert effort even when $\eta(\underline{e}, \overline{e}) = 0$. Whether the firm wants to do so depends on whether effort is efficient or not for high type workers. The point is that this can only increase the informational gain of inducing age 1 workers to exert effort. Consequently, Proposition 2 is still true in this case.

6 Conclusion

This paper provides a rationale for the use of probationary appointments for workers of uncertain talent. If effort makes performance more informative about ability, a firm may benefit from offering probation to workers of uncertain talent if by doing so it induces them to exert effort. We show that probation is only beneficial if the effect of effort on performance is delayed. Without delay, probation weakens the incentives for effort exertion and prevents the firm from dismissing an unpromising worker. When there is delay, probation is valuable because it solves a time-consistency problem: without it, the firm cannot credibly commit to retain a worker of uncertain talent whose initial performance is poor, thus undermining the incentives of such workers to exert effort. A key aspect of our results is that they hold whether the firm can offer output–contingent contracts or not. In particular, the use of probationary appointments is necessary and sufficient when there is delay.

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7 Appendix

7.1 Omitted Proofs

Proof of Lemma 1: Consider an equilibrium and assume the firm has an incumbent that it knows is of the high type. Suppose this worker accepts an offer by the firm and the probability that he is employed in the next period is zero. Then $\widetilde{p}$ is equal to the wage offered regardless of his output, for otherwise the firm can profitably deviate by lowering $\widetilde{p}$. Hence, this worker does not exert effort, as his output does not affect his continuation payoff. Suppose now this incumbent receives an offer by the firm and the probability he is employed in the next period is zero. Then $\widetilde{\omega} \leq \overline{w}$, for he accepts any offer greater than $\overline{w}$ and does not exert effort if he accepts an offer. So, the firm can profitably deviate if $\widetilde{\omega} > \overline{w}$ with positive probability.

Suppose then, by induction, that there exists $\overline{s} \in \{0, \ldots, T-2\}$ such that: (i) if this incumbent accepts an offer by the firm and the maximum number of future periods he can be employed is $s \leq \overline{s}$, then he does not exert effort and $\widetilde{p} = w$, the wage offered; (ii) if this incumbent receives an offer by the firm and the maximum number of future periods he can be employed if he accepts the offer is $s \leq \overline{s}$, then $\widetilde{\omega} \leq \overline{w}$.

Suppose now this incumbent accepts an offer by the firm and the maximum number of future periods he can be employed is $\overline{s}+1$. We claim that $\widetilde{p} = w$, the wage offered. Suppose not. This means there is an output y after which $\widetilde{p} > w$ with positive probability. Notice that after y the probability this worker receives an offer by the firm in the next period must be positive, for otherwise the firm can profitably deviate by lowering $\widetilde{p}$ after y. Consider then the following deviation for the firm: pay w after y and behave from the next period on as if no deviation had occurred, except that it offers $\overline{w}+\epsilon$ to the worker under consideration as long as it is supposed to make an offer to him *and* he is supposed to accept, where $\epsilon > 0$. By the induction hypothesis, from the next period on this worker never exerts effort when employed and rejects any wage smaller than $\overline{\omega}$ (as he is never paid more than $\overline{w}$). Hence, this deviation is profitable if ϵ is close enough to zero, a contradiction. Consequently, it must be that $\widetilde{p} = w$. The induction hypothesis then implies that this incumbent does not exert effort, for his continuation payoff is independent of his output.

Finally, suppose the firm makes an offer to this incumbent and the maximum number of future periods he can be employed is $\overline{s}+1$. Then $\widetilde{\omega} \leq \overline{w}$. Indeed, by the previous paragraph, this worker does not exert effort if he accepts the offer. Hence, if $\widetilde{\omega} > \overline{w}$ with positive probability, then the induction hypothesis and the fact that a worker always accepts a wage greater than his outside option imply that the firm has a profitable deviation (constructed along the same lines as the deviation in the previous paragraph). This concludes the proof. $\square$

Proof of Lemma 6: Suppose, by contradiction, that there is an equilibrium σ with $\widehat{h} \in \mathcal{H}_f$ after which, even though not committed to do so, the firm offers $(\underline{w}, q)$, with $q \leq T - k$, to an age $k \geq 2$ incumbent who never produced high output. Notice that if $k = 2$, then it must be that the firm offered $(\underline{w}, 0)$ to this worker in the previous period. We can assume, without loss of generality, that this worker is not employed by the firm when he is of age $q + k + 1$ if he only produces low output. For the remainder of this proof, let $V = V(h|\sigma)$ for any $h \in \mathcal{H}_f$ after which the firm makes an offer to an age 1 worker. In particular, V is the equilibrium payoff to the firm.

Consider first the case where $k \geq 3$. We claim that the firm has a profitable deviation after $\widehat{h}$: offer $(\underline{w}, q)$ to the available age 1 worker and then behave as if no deviation has occurred. To see why, let V' be the present discounted payoff to the firm after $\widehat{h}$ if it plays according to the deviation just described and let $e \in \{\underline{e}, \overline{e}\}$ denote the choice of effort of the worker under consideration in his first period of employment. By Assumption A3 and Lemma 1, a worker of age $k \geq 2$ never exerts effort when employed. Hence, an upper bound for $V(\widehat{h}|\sigma)$ is

$$\underline{\underline{\phi}}(e) \sum_{j=0}^{q} (1-\alpha)^j \alpha \Big\{ (1-\beta_f)^j [\underline{y} - \underline{w}] + \beta_f^j (1-\beta_f)[\overline{y} - \underline{w}] + \beta_f^{j+1}(1 - \beta_f^{T-k-j})[y(1,\alpha) - \overline{w}] + \beta_f^{T-k+1} V \Big\} + \Big[\underline{\underline{\phi}}(e)(1-\alpha)^{q+1} + 1 - \underline{\underline{\phi}}(e) \Big] \Big\{ (1 - \beta_f^{q+1})[\underline{y} - \underline{w}] + \beta_f^{q+1} V \Big\},$$

while a lower bound for V' is

$$\phi_0 \sum_{j=0}^{q} (1-\alpha)^j \alpha \Big\{ (1-\beta_f)^j [\underline{y} - \underline{w}] + \beta_f^j (1-\beta_f)[\overline{y} - \underline{w}] + \beta_f^{j+1}(1 - \beta_f^{T-k-j})[y(1,\alpha) - \overline{w}] + \beta_f^{T-k+1} V \Big\} + \big[\phi_0(1-\alpha)^{q+1} + 1 - \phi_0 \big] \Big\{ (1 - \beta_f^{q+1})[\underline{y} - \underline{w}] + \beta_f^{q+1} V \Big\}.$$

Since $V < y(1,\alpha) - \overline{w}$, it follows that

$$\beta_f^j (1-\beta_f)[\overline{y} - \underline{w}] + \beta_f^{j+1}(1 - \beta_f^{T-k-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-k+1} V \geq \beta_f^j V$$

for all $j \geq 0$. Moreover, $\phi_0 \sum_{j=0}^{q} (1-\alpha)^j \alpha = \phi_0[1 - (1-\alpha)^{q+1}] = 1 - [\phi_0(1-\alpha)^{q+1} + 1 - \phi_0]$ and $V > \underline{y} - \underline{w}$. Therefore, $V' > V(\widehat{h}|\sigma)$ as claimed, a contradiction. We can then conclude that σ can be an equilibrium only if $k = 2$.

Let $k = 2$. We claim the firm has a profitable deviation after $\widehat{h}$ if $q \geq 1$: (i) offer $(\underline{w}, 0)$ to the incumbent; (ii) if he produces high output, behave as if no deviation has occurred; (iii) if he produces low output, pay $\underline{w}$, offer $(\underline{w}, q-1)$ to the available age 1 worker in the next period, and then behave as if no deviation has occurred. For this, let V'' be the present discounted payoff to the firm after $\widehat{h}$ if it plays according to the deviation just described and define $V(\phi)$ to be such that

$$\begin{aligned} V(\phi) \;=\; & \phi \sum_{j=0}^{q-1} (1-\alpha)^j \alpha \left\{ (1-\beta_f)^j [\underline{y} - \underline{w}] + \beta_f^j (1-\beta_f)[\overline{y} - \underline{w}] + \beta_f^{j+1}(1-\beta_f^{T-j-2})[y(1,\alpha) - \overline{w}] + \right. \\ & \left. \beta_f^{T-1} V \right\} + [\phi(1-\alpha)^q + (1-\phi)] \left\{ (1-\beta_f^q)[\underline{y} - \underline{w}] + \beta_f^q V \right\}, \end{aligned}$$

Since a worker of age $k \geq 2$ never exerts effort when employed, $V(\widehat{h}|\sigma)$ is given by

$$(1-\beta_f)[y(\underline{\phi}, \xi(e)) - \underline{w}] + \underline{\phi}\xi(e)\beta_f \left\{ (1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-2} V \right\} + (1 - \underline{\phi}\xi(e))\beta_f V(\underline{\phi}),$$

while a lower bound for V'' is

$$(1-\beta_f)[y(\underline{\phi}, \xi(e)) - \underline{w}] + \underline{\phi}\xi(e)\beta_f \left\{ (1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-2} V \right\} + (1 - \underline{\phi}\xi(e))\beta_f V(\phi_0),$$

where we recall that $\xi(\underline{e}) = \alpha$ and $\xi(\overline{e}) = \gamma$. The reasoning of the previous paragraph shows that $V(\phi_0) > V(\underline{\phi})$. Hence, $V'' > V(\widehat{h}|\sigma)$ as claimed, a contradiction.

Thus, it must be that $q = 0$ if σ is to be an equilibrium. By Assumption A2, it is then the case that the worker under consideration exerts effort in his first period of employment. Consequently, V satisfies the following recursion:

$$\begin{aligned} V \;=\; & (1-\beta_f)[y(\phi_0, \alpha) - \underline{w}] + \phi_0\alpha\{\beta_f(1-\beta_f)[y(1,\gamma) - \overline{w}] + \beta_f^2(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^T V\} + \\ & (1-\phi_0\alpha)\{\beta_f(1-\beta_f)[y(\underline{\phi}, \gamma) - \underline{w}] + \underline{\phi}\gamma\{\beta_f^2(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^T V\} + (1-\underline{\phi}\gamma)\beta_f^2 V\}. \end{aligned}$$

Rearranging terms in the above equation we obtain that $V = (1-\beta_f)[y(\phi_0, \alpha) - \underline{w}] + \beta_f \widehat{V}$, where

$$\begin{aligned} \widehat{V} = & (1-\beta_f)\left\{ \phi_0\alpha[y(1,\gamma) - \overline{y}] + (1-\phi_0\alpha)[y(\underline{\phi}, \gamma) - \underline{w}] \right\} + \\ & \phi_0(\alpha + \gamma(1-\alpha)) \left\{ \beta_f(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-1} V \right\} + [1 - \phi_0(\alpha + \gamma(1-\alpha))]\beta_f V. \end{aligned}$$

Now observe that $\phi_0(\alpha + \gamma(1-\alpha)) > \underline{\phi}\gamma$. Therefore,

$$\widehat{V} > (1-\beta_f)[y(\underline{\phi}, \gamma) - \underline{w}] + \underline{\phi}\gamma \left\{ \beta_f(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-1} V \right\} + (1-\underline{\phi}\gamma)\beta_f V,$$

and so there exists $\underline{\beta}_f$ such that if $\beta_f \geq \underline{\beta}_f$, then

$$V > (1-\beta_f)[y(\underline{\phi}, \gamma) - \underline{w}] + \underline{\phi}\gamma \left\{ \beta_f(1-\beta_f^{T-2})[y(1,\alpha) - \overline{w}] + \beta_f^{T-1} V \right\} + (1-\underline{\phi}\gamma)\beta_f V = V(\widehat{h}|\sigma)$$

This, however, implies that the firm has a profitable deviation after $\widehat{h}$ when $\beta_f \geq \underline{\beta}_f$. $\square$

7.2 Robustness

Here we discuss the restrictions that the firm cannot offer less than $\underline{w}$ to workers and that a worker who produces high output reveals himself to be of the high type. We start with the lower bound on wages.

7.2.1 Lower Bound on Wages

Suppose there is no lower bound on wage offers (which still need to be positive). We know that any worker of age $T-1$ or less who has never produced high output is willing to accept a wage that is smaller than $\underline{w}$, for by working at the firm he has the chance of revealing himself to be of the high type. Among these workers, the one who is willing to accept the smallest wage is the age 1 worker, for he has the longest lifetime and is the most optimistic about his type. Thus, an age 1 worker is still more desirable to the firm than an incumbent of age $k \geq 2$ who has never produced high output. However, Assumption A1 is no longer the relevant assumption, as now the firm can hire an age 1 worker for less than $\underline{w}$. A sufficient assumption is

(A1') $\alpha\overline{y} + (1-\alpha)\underline{y} - \overline{w} > \phi_0\overline{y} + (1-\phi_0)\underline{y}$.

Observe that without the lower bound $\underline{w}$ on wage offers, the firm can, in principle, induce a worker to which it is committed to make an offer to take his outside option. Hence, we need to increase the scope of commitment by allowing the firm to promise that it will make future offers that are not smaller than some pre–specified wage. In other words, commitment is now of the form (q, w'), where w' is the smallest one–period wage the firm can offer in the next q periods. In this way, we guarantee that the commitment to make future wage offers translates into commitment to employment.

IID Case The only difference is that now $R(y, e)$ in equation (1) in the proof of Proposition 1 depends on the worker's reputation (besides his age and his private belief). This does not matter, though, for a worker's reputation is only affected by his output. Thus, if a worker produces low output, the effect of this on his reputation is the same whether he exerted effort or not.

Non-IID Case Now, in equilibrium, the wage the firm pays an age 2 worker who produces low output in his first period of employment is such that it makes an age 1 worker indifferent between exerting effort or not.

7.2.2 High Output Reveals High Type

We now argue that our results are still true when a low type worker can also produce high output as long as the probability that this happens is small enough. We can assume, without any loss, that this probability is the same whether the worker exerts effort or not. Denote it by α_L. For simplicity, we assume that the lower bound on wage offers is in place.

The first observation that we make is that there is $\phi^* \in (0,1)$ such that Lemma 1 holds for any incumbent with reputation in the interval $[\phi^*, 1]$.[20] Now observe, by A1, that there is $\phi^{**} \in (0,1)$ such that if $\phi \geq \phi^{**}$, then $\phi^{**}\alpha\overline{y} + (1-\phi^{**}\alpha)\underline{y} - \overline{w} > \phi_0\overline{y} + (1-\phi_0)\underline{y} - \underline{w}$. Hence, since Lemma 2 still holds, it is strictly optimal for the firm to hire an incumbent with reputation in the interval $[\phi^{**}, 1]$. The proof of this fact follows the line of argument used in the proof of Lemma 3.

Let $\widehat{\phi} = \max\{\phi^*, \phi^{**}\}$. If α_L is low enough, then producing high output is a very strong signal that a worker is of the high type. Indeed, there is $\alpha_L' \in (0,1)$ such that if $\alpha_L \leq \alpha_L'$, then, once a worker produces high output, his reputation stays in the interval $[\widehat{\phi}, 1]$ until he dies.[21] This means that as long as $\alpha_L \in (0, \alpha_L')$, producing high output plays the same role as revealing oneself to be of the high type. We can then proceed in exactly the same way as we did in the main text to obtain Propositions 1 and 2.

[20] Notice that ϕ^* depends on T, the lifetime of workers.

[21] Notice that α_L' also depends on the lifetime of workers.